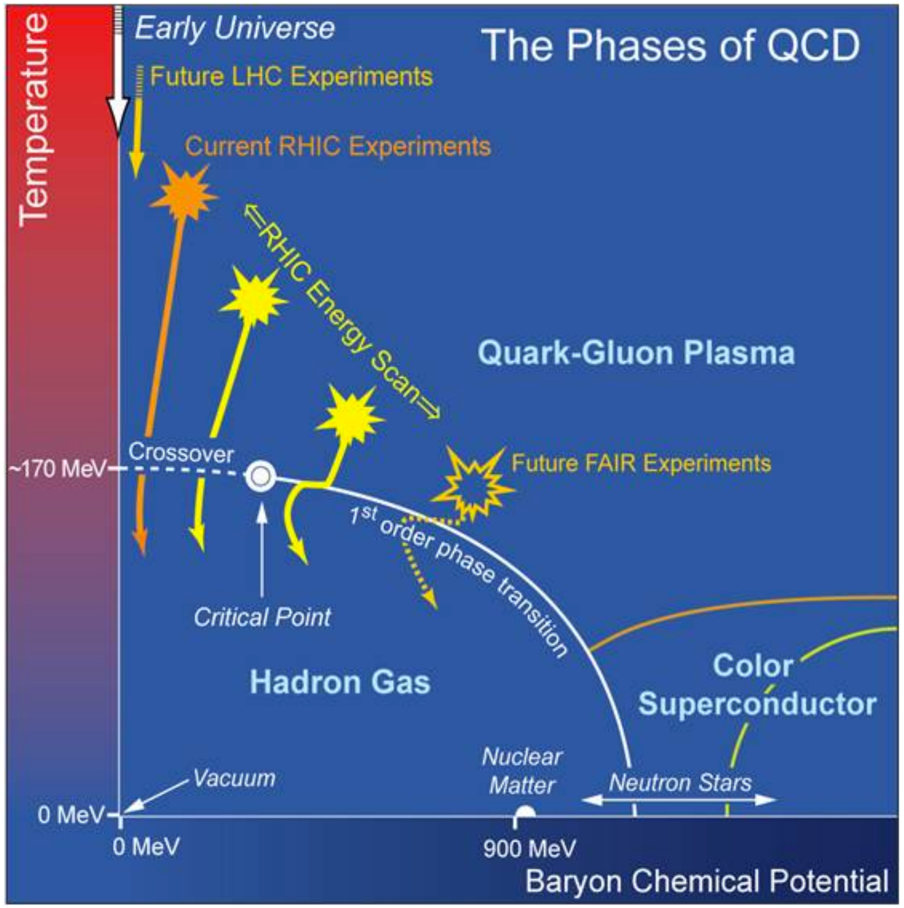
EoS (and some more) for quark matter from instanton QCD vacuum

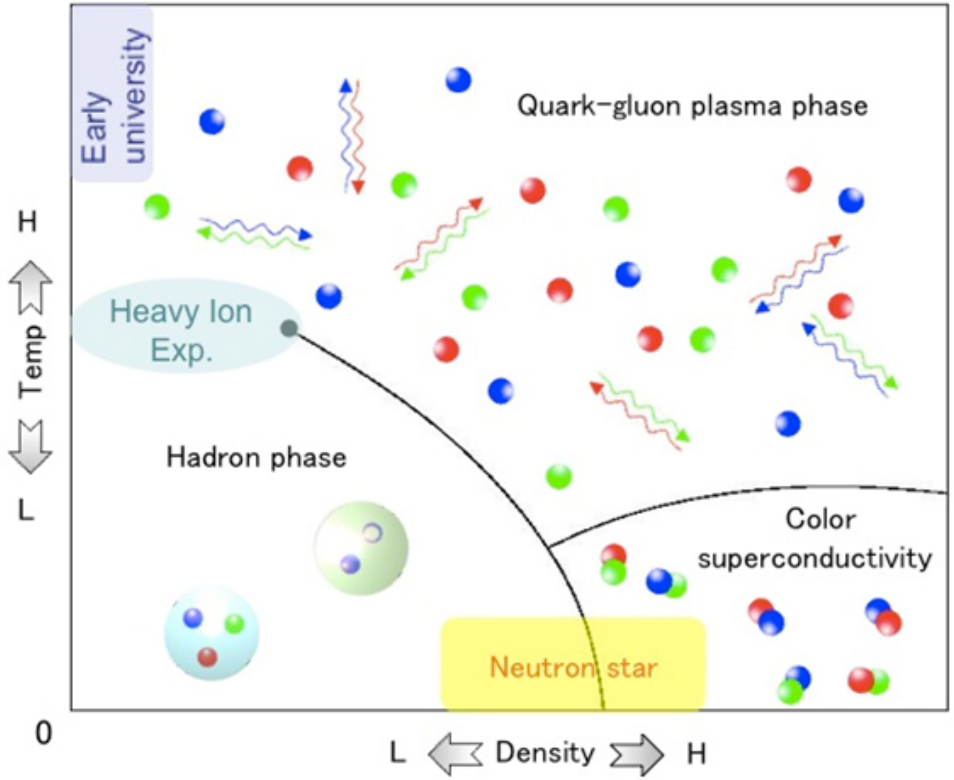
Seung-il Nam

Department of Physics, Pukyong National University (PKNU),
Center for Extreme Nuclear Matters (CENuM), Korea University,
Asia Pacific Center for Theoretical Center Physics (APCTP),
Republic of Korea

QCD at extreme conditions

QCD has complicated phase structure as a function of temperature and density





QCD at extreme conditions

I. Each QCD phases defined by its own order parameters

II. Behavior of order parameters governed by dynamics of symmetry

III. Symmetry and its breakdown governed by vacuum structure

Chiral symmetry ↔ Quark (chiral) condensate: Hadron or not?

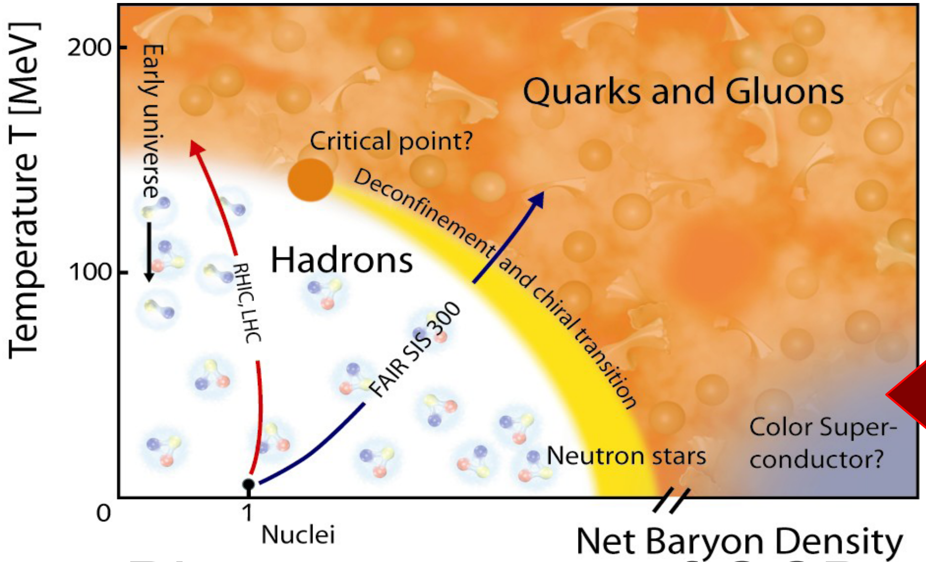
Center symmetry ↔ VEV of Polyakov loop: Confined or not?

Color symmetry ↔ Diquark condensate: Superconducting or not?

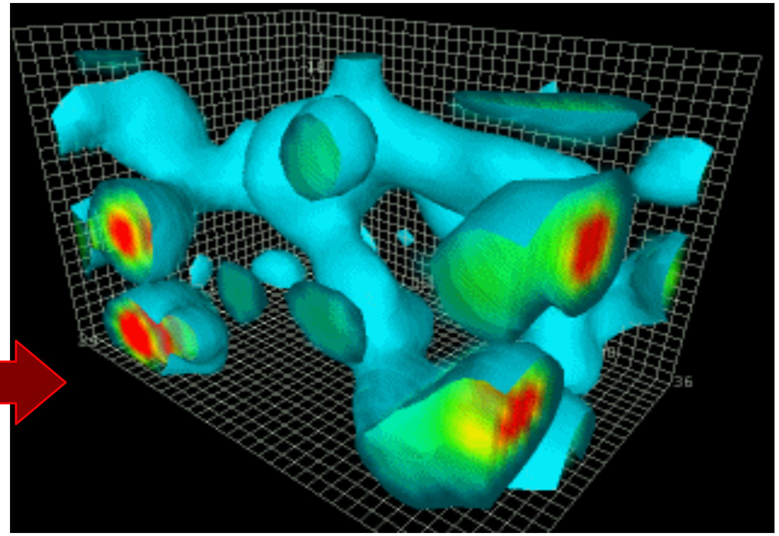
Color-flavor symmetry (locking) ↔ Diquark condensate at high density

QCD phase ↔ Symmetries of QCD ↔ QCD vacuum

Why are heavy-ion collision experiments special for QCD?



Phase structure of QCD



Nontrivial QCD vacuum

Color symmetry

C/P/T-invariance

Z(N) center symmetry

Chiral symmetry

Flavor symmetry

Symmetries and their breakdown

QCD at extreme conditions

SCSB results in nonzero chiral (quark) condensate due to nonzero effective quark mass even in the chiral limit, i.e. m=0

$$-\langle\bar{\psi}\psi\rangle_{\text{Mink}} = i\langle\psi^{\dagger}\psi\rangle_{\text{Eucl}} = 4N_c \int \frac{d^4p}{(2\pi)^4} \frac{M(p)}{p^2 + M^2(p)}$$

Nonzero <qq> indicates hadron (Nambu-Goldstone) phase, whereas zero <qq> does non-hadronic phase, not meaning deconfinement

Thus, <qq> is an order parameter for chiral symmetry

In the real world with nonzero quark current mass ~ 5 MeV, at low density, there appears crossover near T ~ 0, and it becomes 1st-order phase transition as density increases

In the vicinity of critical density, there are various and complicated phases, such as color-superconducting, quarkyonic phase, etc.

QCD and effective models

Instanton interprets well the spontaneous chiral symmetry breaking (SCSB) and U(1) axial anomaly (Witten-Veneziano theorem), etc.

Technically, it has only two model parameters for light-flavor sector in the large Nc limit: Average instant on size & inter-instanton distance

Unfortunately, there is NO confinement!!!

Some suggestions for the confinement with instanton physics: Dyon, nontrivial-holonomy caloron, etc.

It has been believed that confinement is not so relevant in ground-state hadron spectra, in contrast to resonances, Regge behavior, Hagedorn spectrum, etc.

Introduction: QCD and effective models

A sophiscated QCD-like model: Liquid-Instanton Model (LIM)

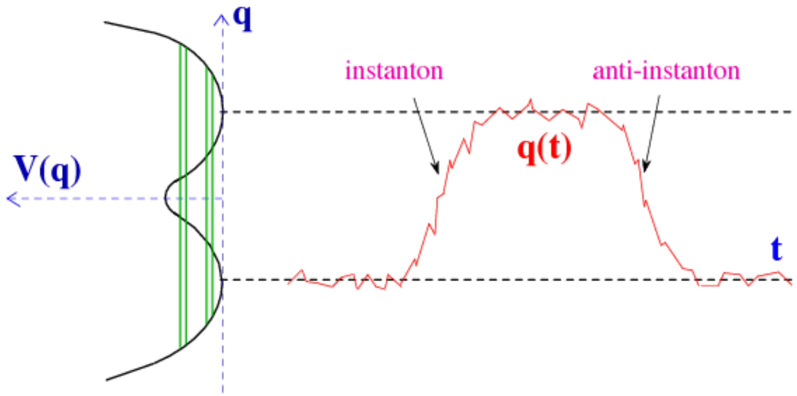
Instanton: A semi-classical solution which minimize YM action

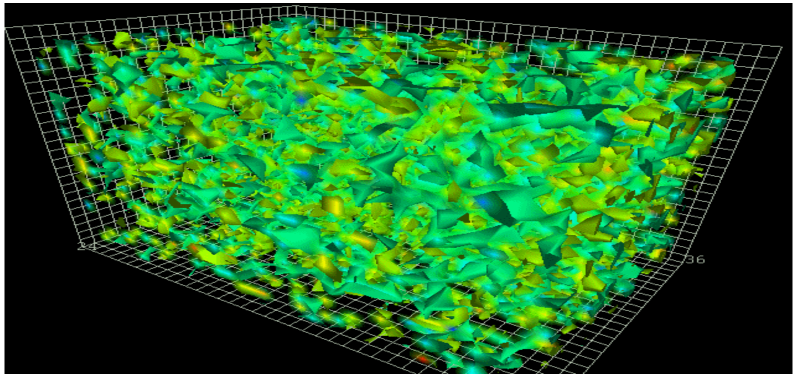
$$F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}. \quad \Rightarrow \quad A^a_\mu(x) = \frac{2\bar{\eta}^\mu_{\nu a}\rho^2}{x^2(x^2+\rho^2)}.$$

Simpler understanding of instanton: Tunneling path of vacua

Or, instanton is a low-energy effective-nonperturbative gluon

$$i\nabla\!\!\!/ = \gamma_\mu(i\partial_\mu + A^{\mathrm{I\bar{I}}}_\mu + a_\mu) \quad \Rightarrow \quad i\nabla\!\!\!/\Phi_n = \lambda_n\Phi_n,$$





Introduction: QCD and effective models

Partition function of LIM

$$\mathcal{Z}_{\rm eff}[q,q^\dagger] = \int \frac{d\lambda_\pm}{2\pi}\int DqDq^\dagger \exp\left[\int_x \sum_q q^\dagger(i\partial\!\!\!/ + m_q)q + \sum_{a=\pm}\left[\lambda_a Y^a_{N_f}(\bar\rho) + N_a\left(\ln\frac{N_a}{\lambda_a V\Lambda^{N_f}}\right)\right]\right],$$

$$Y^a_{N_f}(\bar\rho) = \frac{1}{N_c^{N_f}}\int_x \det{}_f\left[iJ^a_{qq'}(x,\bar\rho)\right] = \int_x \det{}_f\left[\frac{i}{N_c}J^a_{qq'}(x,\bar\rho)\right],$$

$$J^a_{qq'}(x,\bar\rho) = \int_k\int_p e^{i(k-p)\cdot x}F(k)F(p)\left[q^\dagger(k)\frac{1+a\gamma_5}{2}q'(p)\right]_{N_f\times N_f}.$$

$$\frac{N}{V} = 4N_c\int_k \frac{M_0^2(k)}{k^2+M_0^2(k)}, \quad M_0(k) = \sigma_0 F^2(k),$$

$$\lambda = \frac{N}{2V}\left(\frac{2\sigma_0 V N_c}{N}\right)^{N_f},$$

Example: Pion weak-decay constant

$$F^2_{q\bar q} = 4N_c\int_k \frac{M^2_{q\bar q}(k) - \frac{k}{2}M_{q\bar q}(k)\frac{\partial M_{q\bar q}(k)}{\partial k} + \frac{k^2}{4}\left[\frac{\partial M_{q\bar q}(k)}{\partial k}\right]^2}{[k^2+M^2_{q\bar q}(k)]^2},$$

$(\bar R,\bar\rho)$ [fm]	F_π [MeV]	F_K [MeV]	F_η [MeV]	$F_{\eta'}$ [MeV]
$(0.95, 0.35)$ [29]	89.07	97.87	94.65	100.23
$(0.89, 0.36)$ [30–32]	87.96	96.60	93.44	98.92
$(0.76, 0.32)$ [33]	92.67	101.96	98.56	104.47

Medium-modified Effective models

T-modified LIM:(mLIM) Instanton parameters are modified with trivial-holonomy caloron solution (Not dyon, vortex, or something)

Caloron is an instanton solution for periodic in Euclidean time, i.e temperature, but no confinement

Distribution func. via trivial-holonomy (Harrington-Shepard) caloron

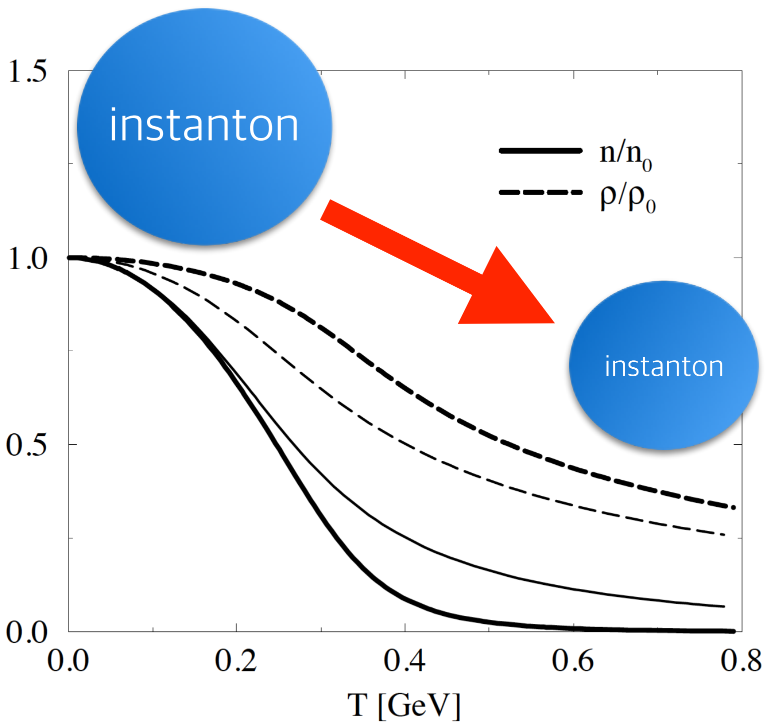
$$d(\rho, T) = \mathcal{C}\,\rho^{b-5}\exp\left[-\mathcal{F}(T)\rho^2\right], \quad \mathcal{F}(T) = \frac{1}{2}A_{N_c}T^2 + \left[\frac{1}{4}A_{N_c}^2 T^4 + \nu\bar{\beta}\gamma n\right]^{\frac{1}{2}}$$

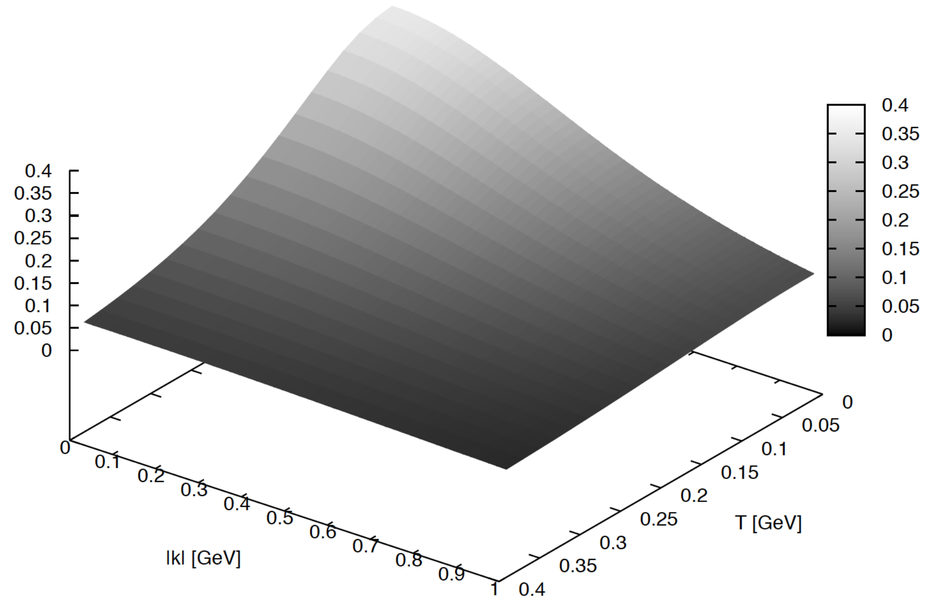
$$A_{N_c} = \frac{1}{3}\left[\frac{11}{6}N_c - 1\right]\pi^2, \quad \gamma = \frac{27}{4}\left[\frac{N_c}{N_c^2 - 1}\right]\pi^2, \quad b = \frac{11N_c - 2N_f}{3}.$$

Using this, we modify the two instanton parameters as functions of T

Medium-modified Effective models

mLIM parameters (left) and effective quark mass M (right)





Hence, effective quark mass plays the role of UV regulator

QGP and transport coefficients

- Recent heavy-ion collision experiment showed possible evidence of QGP

J. Adams et al. [STAR Collaboration], Nucl. Phys. A, 102 (2005)

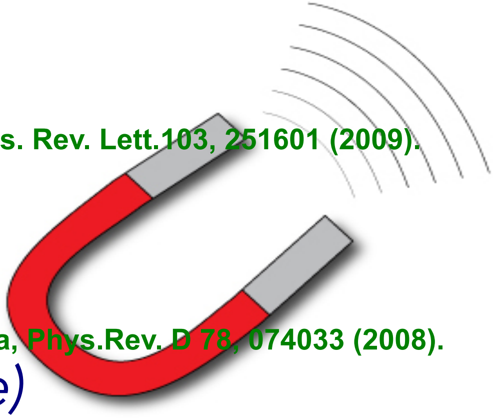
- Interpreted well by hydrodynamics with small viscosity: ~ perfect fluid
- Properties of QGP can be understood by transport coefficients:

Bulk and sheer viscosities, electrical conductivity, and so on

- They can be studied using Kubo formulae via linear response theory

F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B 663, 217 (2008).

Strong magnetic (B) field in QGP



B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett.103, 251601 (2009).

- RHIC experiments observed strong B field ~ (pion mass)^2
- Strong B field modify nontrivial QCD vacuum structure

K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys.Rev. D 78, 074033 (2008).

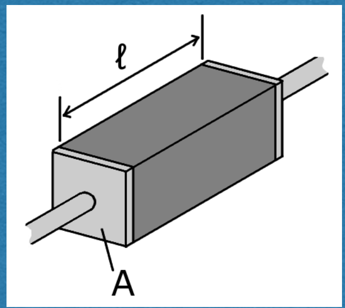
- Charged-current asymmetry: *Chiral magnetic effect (wave)*

D. P. Menezes, M. Benghi Pinto, S. S. Avancini, A. Perez Martinez, and C. Providencia, Phys. Rev. C 79, 035807 (2009).

- B field enhances SBCS: *Magnetic catalysis*

Various transport coefficients

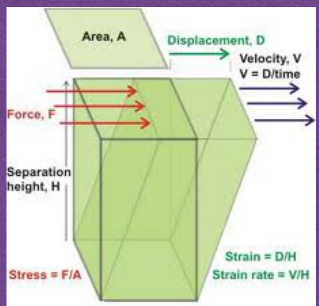
Electric conductivity



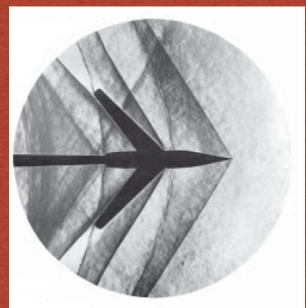
Heat conductivity

Kubo formula: Current-current correlation

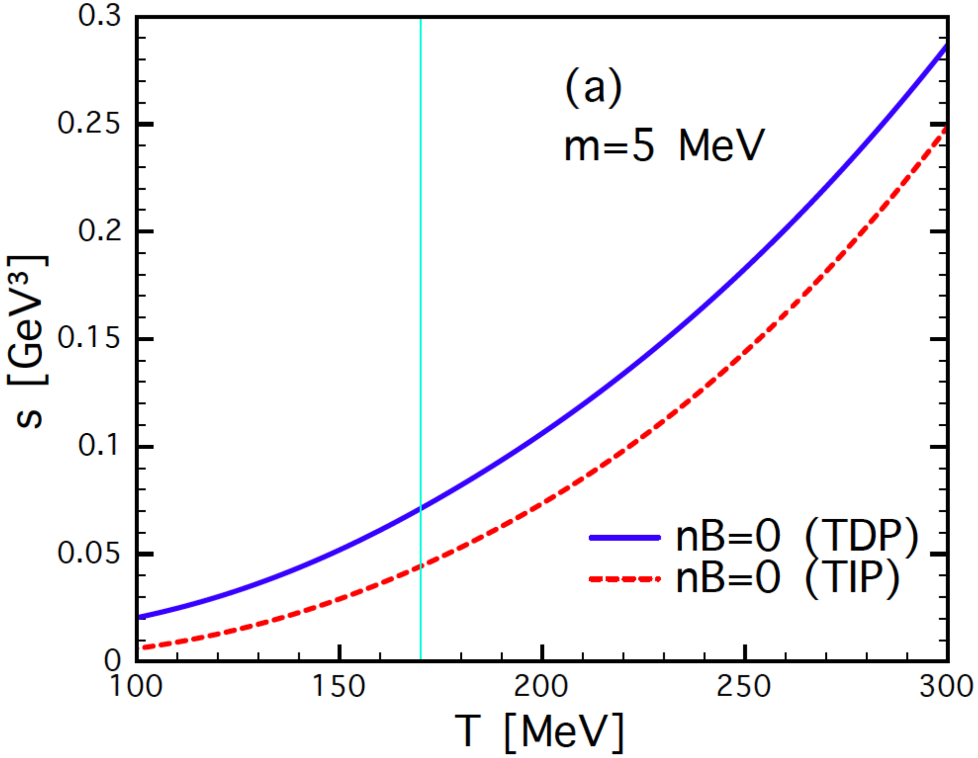
Shear viscosity

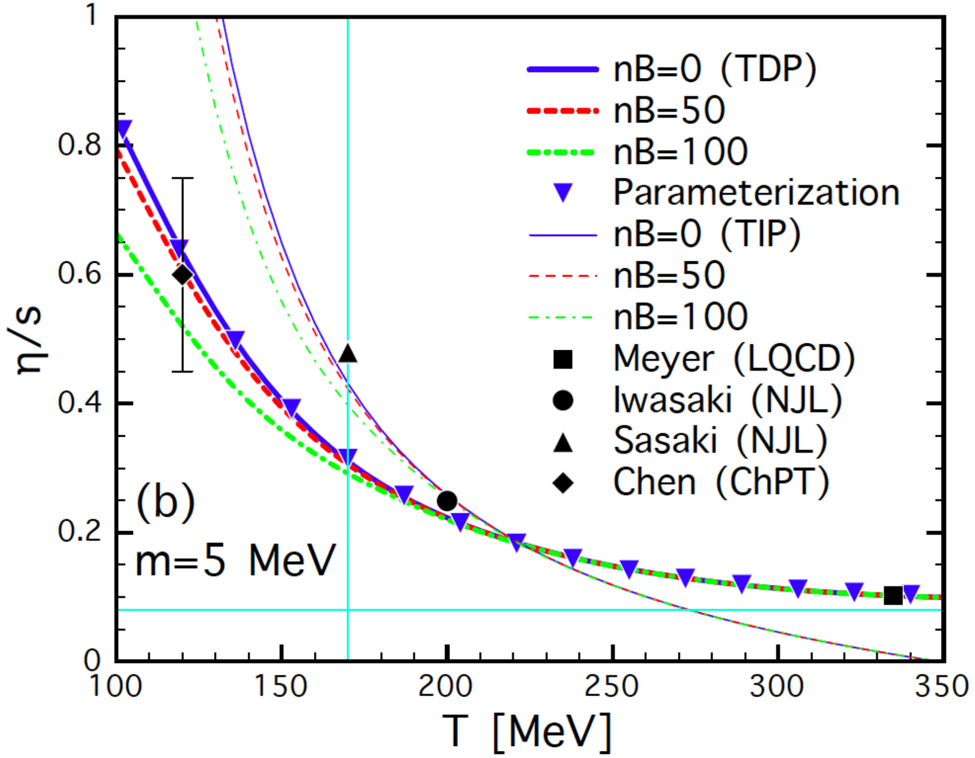


Bulk viscosity



Shear viscosity of QGP (Instanton vacuum)



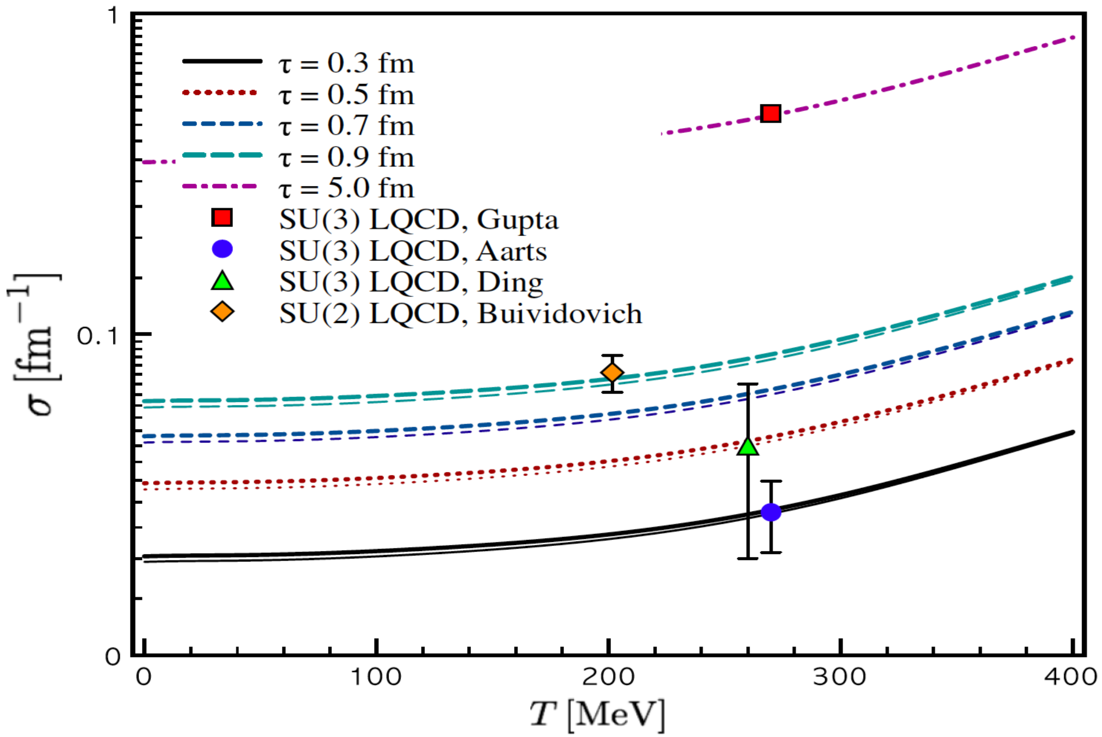


Entropy density shows increasing functions of T

Min[η/s] ~ 1/(4π):KSS bound (Kovtun, Son, and, Starinets)

LQCD, NJL, and ChPT results are compatible with ours

Electric conductivity of QGP (Instanton vacuum)



Gupta et al., PLB597 (2004)
SU(3). Unrenormalized VC

Aarts et al., PRL99 (2007)
SU(3). Unrenormalized VC

Ding et al., PRD83 (2011): SU(3)
SU(3). Unrenormalized VC

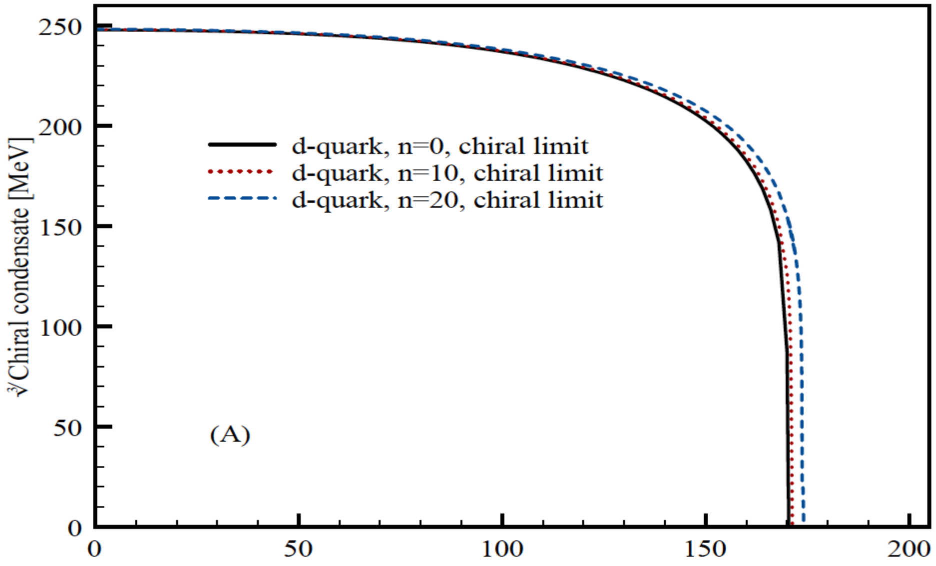
Buividovich et al., PRL105 (2010): **SU(2)**

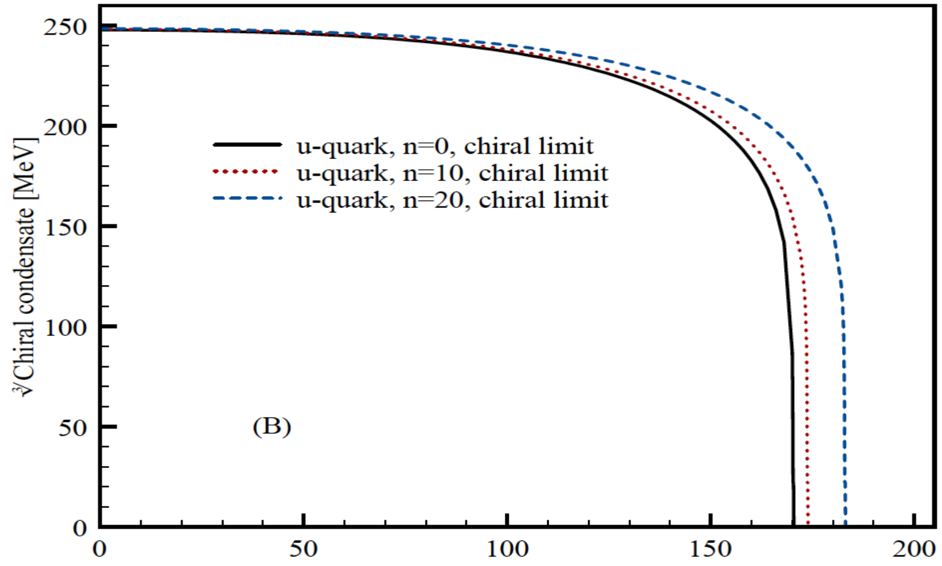
The numerical results compatible with LQCD data for various τ

Effects of B field is negligible (thick and thin lines)

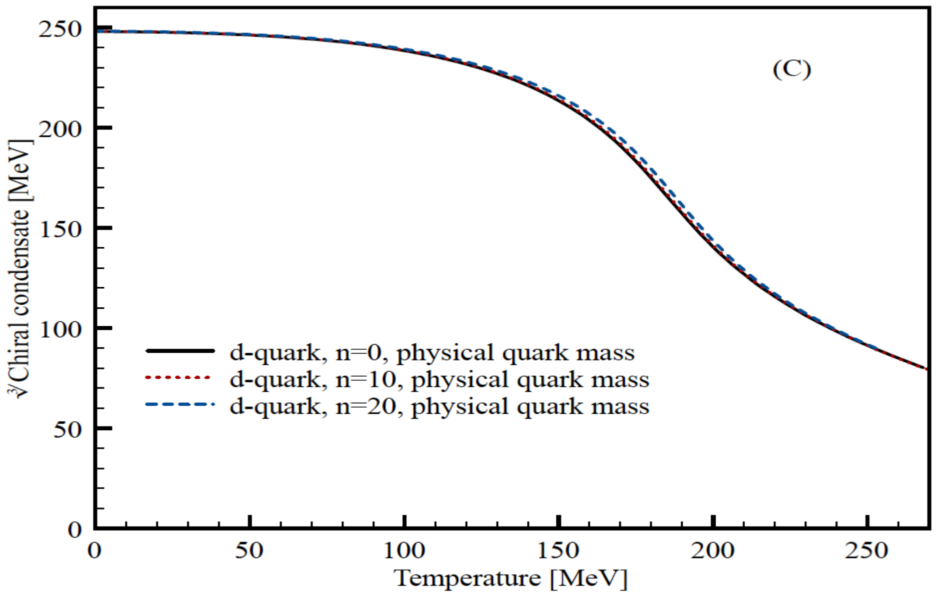
EC increases obviously beyond T ~ 200 MeV B. Kerbikov and M. Andreichikov, arXiv:1206.6044.

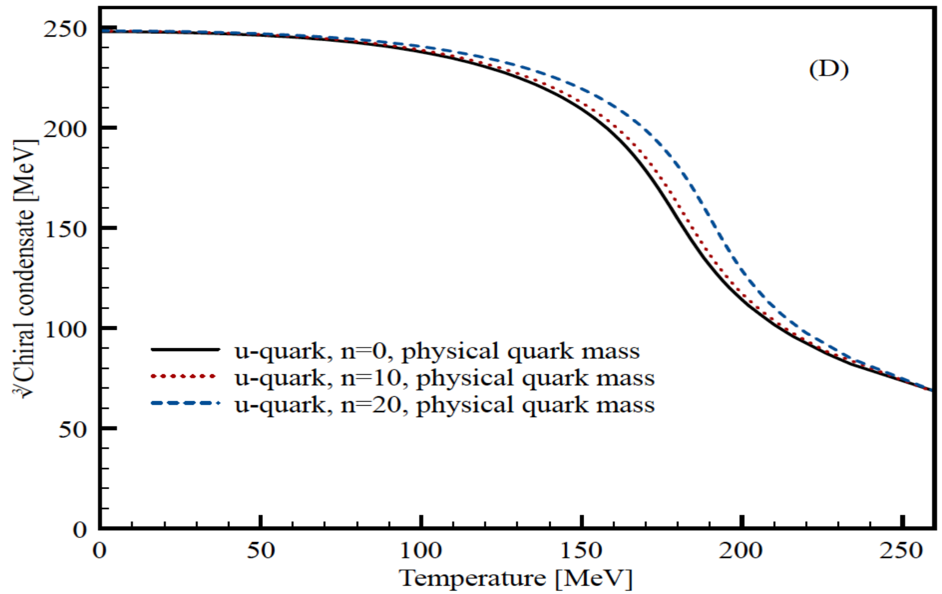
Chiral condensate under B field (Instanton vacuum)



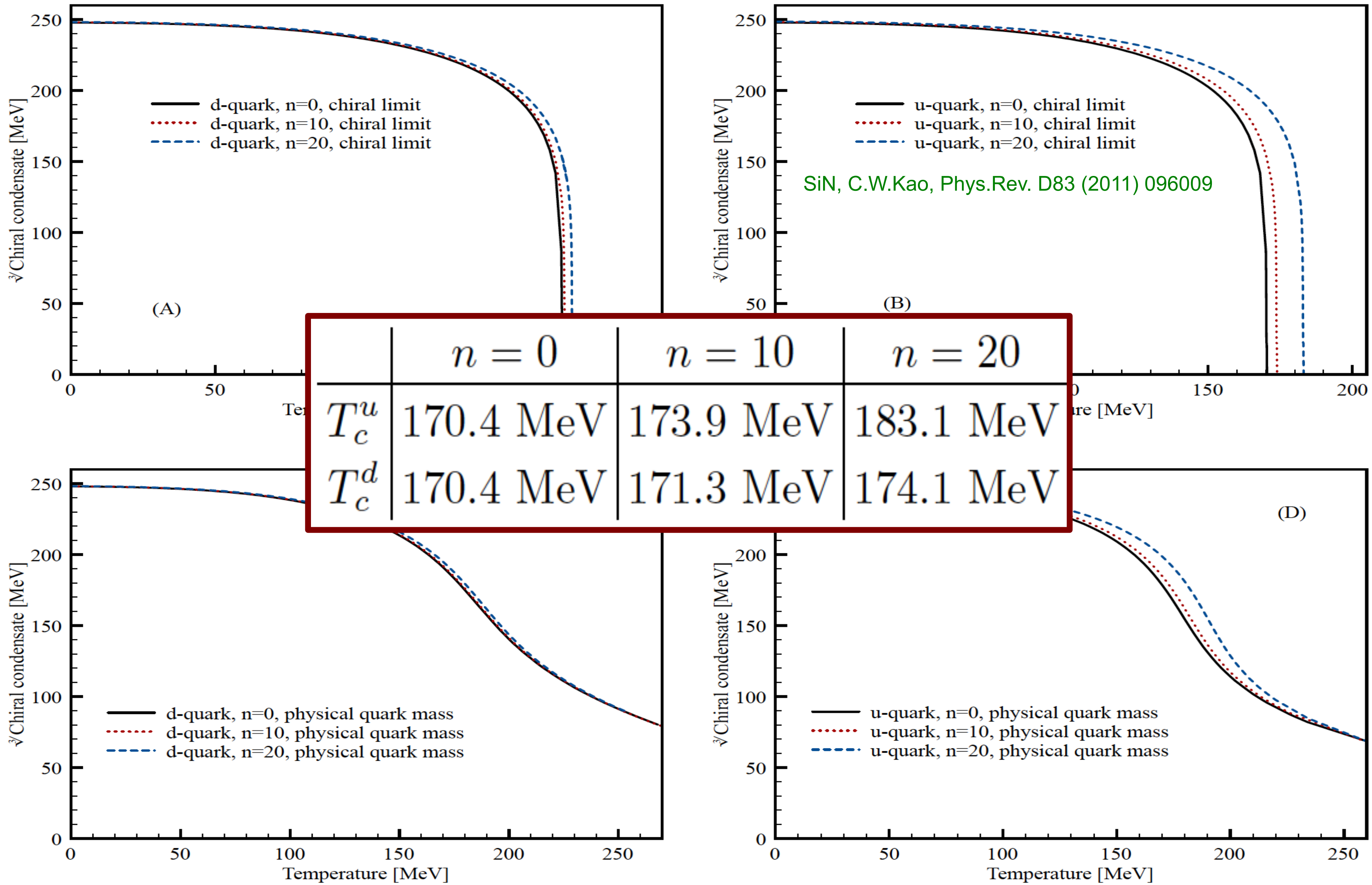


SiN, C.W.Kao, Phys.Rev. D83 (2011) 096009





Chiral condensate for u and d flavors under B field

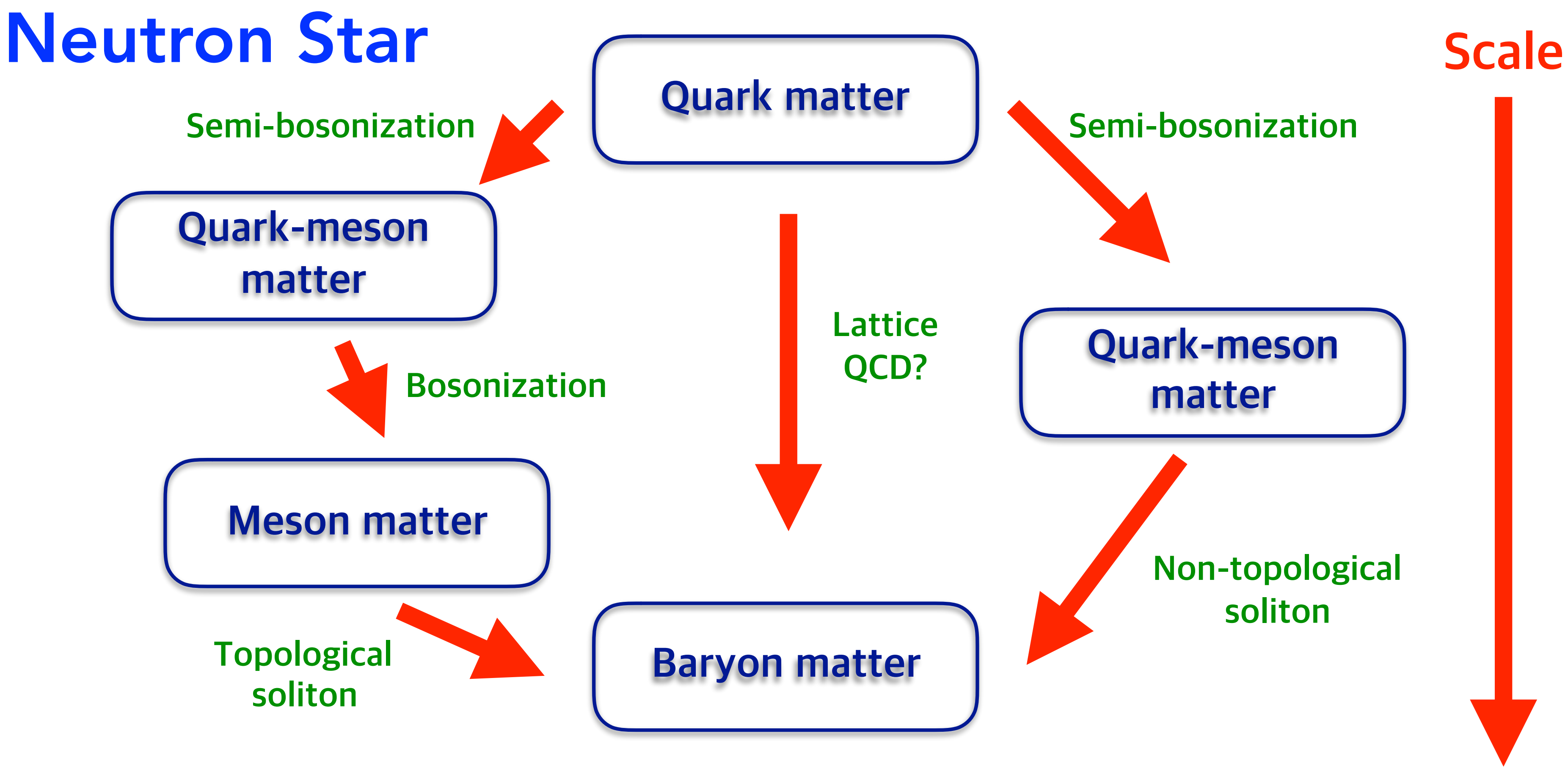


	$n=0$	$n=10$	$n=20$
T_c^u	170.4 MeV	173.9 MeV	183.1 MeV
T_c^d	170.4 MeV	171.3 MeV	174.1 MeV

Medium-modified Effective model: EoS

Thermodynamic properties of matter can be understood by EoS

Neutron star in terms of effective Dofs: Smooth transition possible?



Medium-modified Effective model in SU(2f)

Effective action from liquid-instanton vacuum (Euclidean)

$$\mathcal{S}_{\rm eff} = -\frac{N}{V}\ln\left[\frac{N}{V}\frac{2\pi^2\bar{\rho}^2}{N_c M_0 \mathrm{M}}\right] - 2N_c\int\frac{d^4k}{(2\pi)^4}\ln\left[\frac{k^2+\bar{M}_k^2}{k^2+m^2}\right]$$

Matsubara frequency for fermions

$$\int\frac{d^4k}{(2\pi)^4}f[k_4,\boldsymbol{k}] \to T\sum_{n=-\infty}^{\infty}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}f[(2n+1)\pi T,\boldsymbol{k}]$$

Thermodynamic potential from LIM and NJL

$$\Omega^{\rm LIM}_{\rm eff} = \Omega^g_{\rm eff}+\Omega^q_{\rm eff} = -\frac{N_f N}{V}\ln\left[\frac{N}{V}\frac{2\pi^2\bar{\rho}^2}{N_c M_0 \mathrm{M}}\right] - 2N_cN_f\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[E+T\ln\left[(1+Y)(1+X)\right]\right],$$

$$\Omega^{\rm NJL}_{\rm eff} = \frac{(\mathcal{M}-m)^2}{4G} - 2N_cN_f\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\mathcal{E}+T\ln\left[(1+\mathcal{Y})(1+\mathcal{X})\right]\right].$$

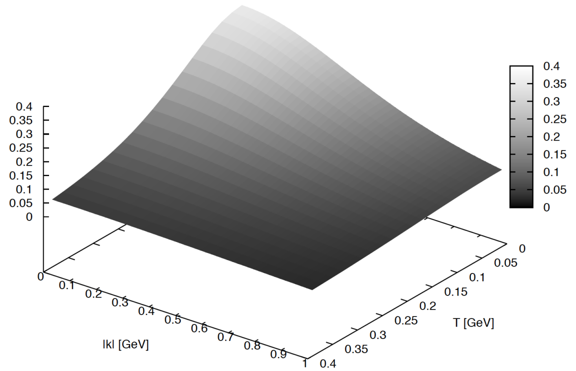
$$X = e^{-E_+/T},\quad Y = e^{-E_-/T},\quad E_\pm \equiv E\pm\mu = \sqrt{\boldsymbol{k}^2+(m+M^2)}\pm\mu,$$

$$\mathcal{X} = e^{-\mathcal{E}_+/T},\quad \mathcal{Y} = e^{-\mathcal{E}_-/T},\quad \mathcal{E}_\pm \equiv \mathcal{E}\pm\mu = \sqrt{\boldsymbol{k}^2+(m+\mathcal{M})^2}\pm\mu.$$

Medium-modified Effective model

Momentum-dependent effective quark mass

$$M = M_0(\mu, T)\left[\frac{2}{2+\bar{\rho}^2\,\boldsymbol{k}^2}\right]^{\mathcal{N}}$$



Gap (saddle-point) equations for LIM and NJL

$$\frac{NN_f}{VM_0} = 2N_cN_f\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{(m+M)F^{\mathcal{N}}}{E}\left[\frac{(1-XY)}{(1+X)(1+Y)}\right],$$

$$\frac{\mathcal{M}-m}{2G} = 2N_cN_f\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{(m+\mathcal{M})}{\mathcal{E}}\left[\frac{1-\mathcal{X}\mathcal{Y}}{(1+\mathcal{X})(1+\mathcal{Y})}\right],$$

Parameterization of instanton packing fraction in medium

$$\frac{N}{V}\to\frac{N}{V}\left[\frac{M_0}{M_{0,\mathrm{vac.}}}\right]^2$$

Medium-modified Effective model

Standard representations for thermodynamic properties of QCD matter

$$p(T,\mu) = -(\Omega - \Omega_{\mathrm{vac.}}), \quad n(T,\mu) = -\frac{\partial\Omega}{\partial\mu},$$

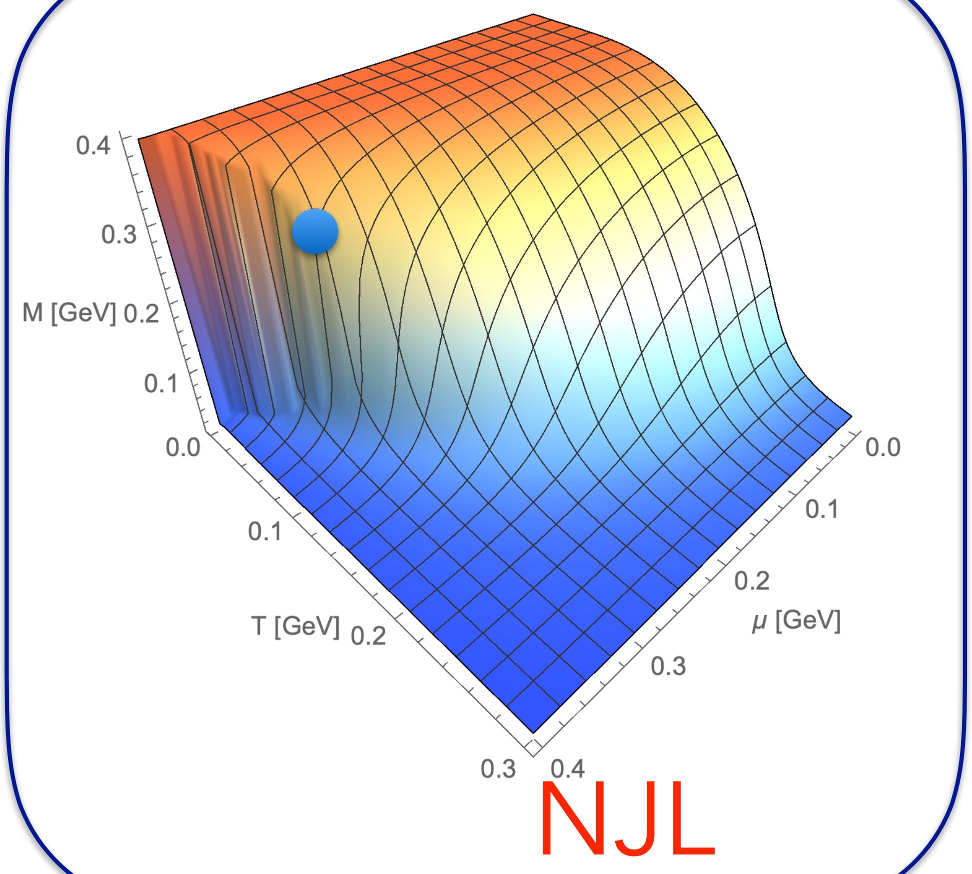
$$s(T,\mu) = -\frac{\partial\Omega}{\partial T}, \quad \epsilon(T,\mu) = T\,s(T,\mu) + \mu\,n(T,\mu) - p(T,\mu),$$

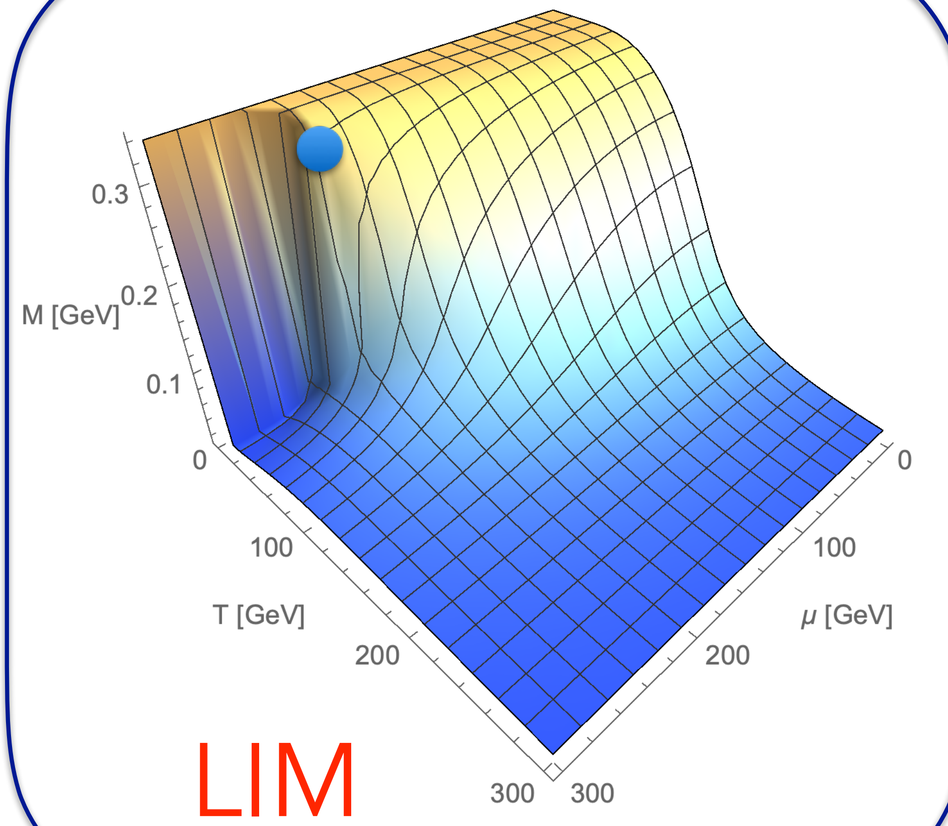
Thermodynamic properties of QCD matter for LIM and NJL

$$\begin{aligned}
p_{\mathrm{NJL}} &= -(\Omega^{\mathrm{NJL}}_{\mathrm{eff}} - \Omega^{\mathrm{NJL}}_{\mathrm{eff,vac.}}),\\
n_{\mathrm{NJL}} &= 2N_fN_c\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{\mathcal{E}(\mathcal{Y}-\mathcal{X})+(1-\mathcal{X}\mathcal{Y})\mathcal{M}\mathcal{M}^{(\mu)}}{\mathcal{E}(1+\mathcal{X})(1+\mathcal{Y})}\right] - \frac{(\mathcal{M}-m)\mathcal{M}^{(\mu)}}{2G},\\
s_{\mathrm{NJL}} &= 2N_fN_c\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\ln\left[(1+\mathcal{X})\,(1+\mathcal{Y})\right] + \frac{\mathcal{E}[\mathcal{E}_-(1+\mathcal{X})\mathcal{Y}+\mathcal{E}_+(1+\mathcal{Y})\mathcal{X}]+T(1-\mathcal{X}\mathcal{Y})\mathcal{M}\mathcal{M}^{(T)}}{\mathcal{E}T(1+\mathcal{X})(1+\mathcal{Y})}\right]\\
&\quad - \frac{(\mathcal{M}-m)\mathcal{M}^{(T)}}{2G}.
\end{aligned}$$

$$\begin{aligned}
p_{\mathrm{LIM}} &= -(\Omega^{\mathrm{LIM}}_{\mathrm{eff}} - \Omega^{\mathrm{LIM}}_{\mathrm{eff,vac.}}),\\
n_{\mathrm{LIM}} &= 2N_fN_c\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{E(Y-X)+(1-XY)MM^{\mu}}{E(1+X)(1+Y)}\right] - \frac{2M_0M_0^{\mu}}{M^2_{0,\mathrm{vac.}}}\frac{N}{V}\\
s_{\mathrm{LIM}} &= 2N_fN_c\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\ln\left[(1+X)\,(1+Y)\right] + \frac{E[E_-(1+X)Y+E_+(1+Y)X]+T(1-XY)MM^{(T)}}{ET(1+X)(1+Y)}\right] - \frac{2M_0M_0^{(T)}}{M^2_{0,\mathrm{vac.}}}\frac{N}{V}
\end{aligned}$$

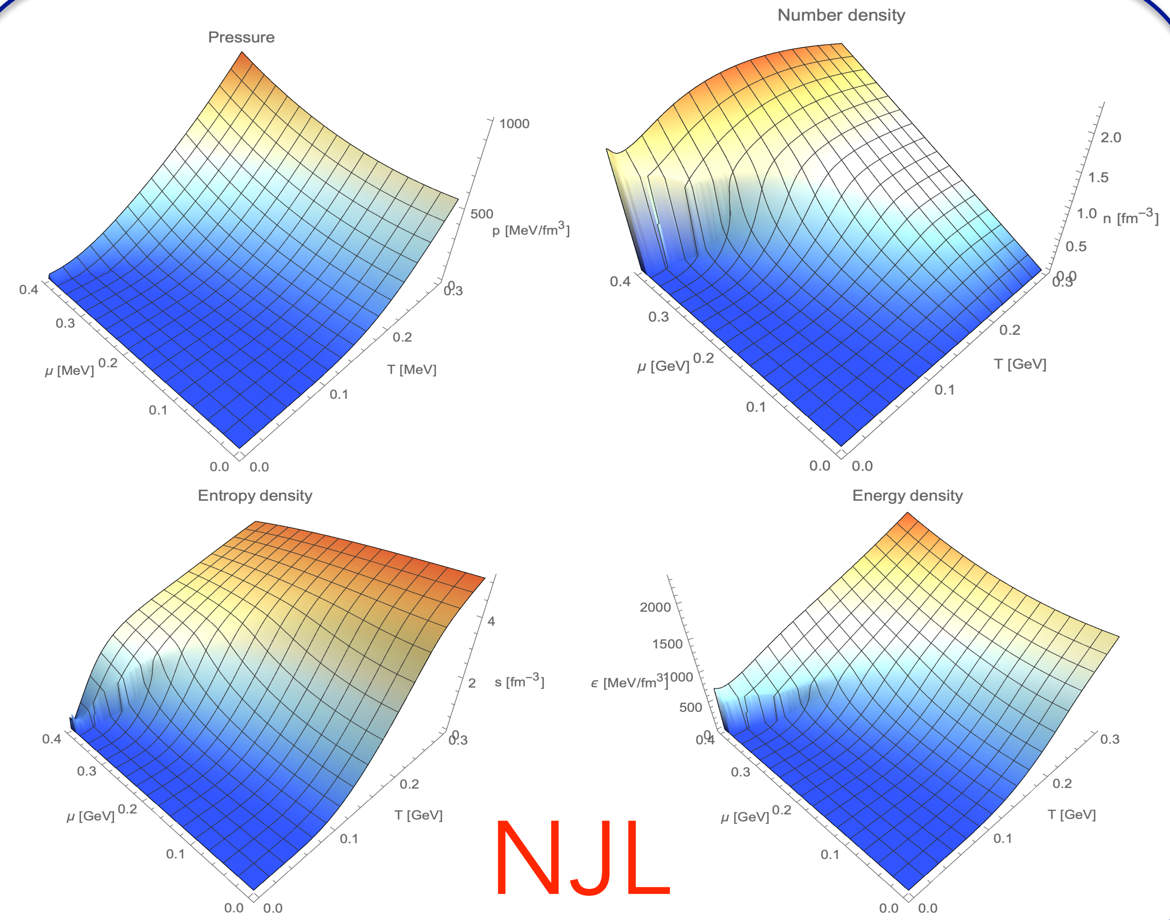
Thermodynamic properties : NJL vs. LIM

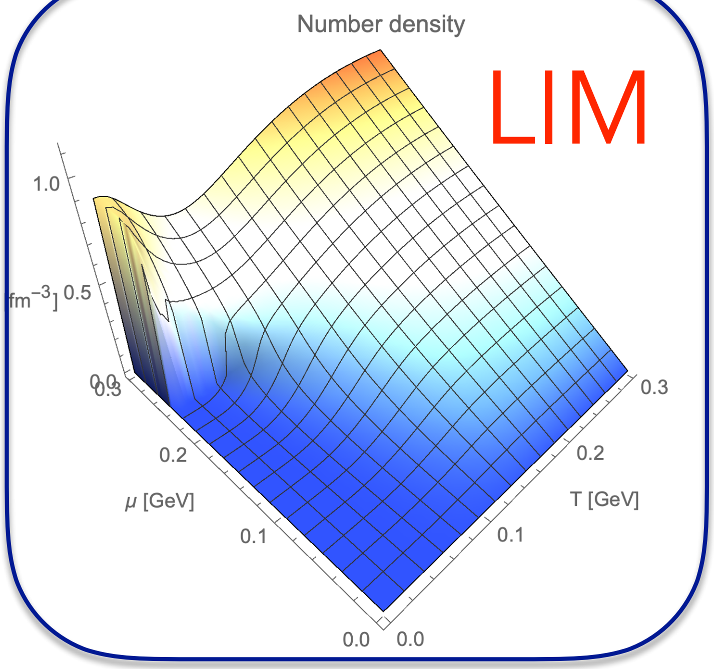




Chiral phase diagram via effective quark mass

Thermodynamic properties: NJL vs. LIM





Summary

Along with lattice QCD and theory beyond QFT, QCD-like EFT plays a important role to understand strongly-interacting systems

Strongly-interacting QGP believed to be created in HIC is a good place to test QCD in extreme conditions, i.e. hot and dense QCD matter

QCD-like EFTs are modified in medium with helps of lattice QCD, Euclidean-time formula, nonperturbative gluonic correlations, etc.

Various physical properties of QGP investigated using QCD-like EFTs, such as transport coefficients, EoS, effects of B-fields, etc: Instanton.

There are still insufficient understandings and obvious distinctions between EFTs, and they can be resolved along with lattice QCD

Thank you for your attention

This talk supported by the National Research Foundation of Korea (NRF) grants funded by the Korea government
(No.2018R1A5A1025563 and No.2019R1A2C1005697)