Quarks and Compact Stars 2019
Sep. 26-28, 2019, Hanwha Resort, Haeunde, Busan, South Korea

Time-Dependent Density Functional Theory for Superfluid Dynamics in the Neutron Star Crust

Kazuyuki Sekizawa (Niigata Univ.)

The main results were reported in: Phys. Rev. Lett. **117**, 232701 (2016)
in collaboration with

G. Wlazłowski[1,2]

P. Magierski[1,2]

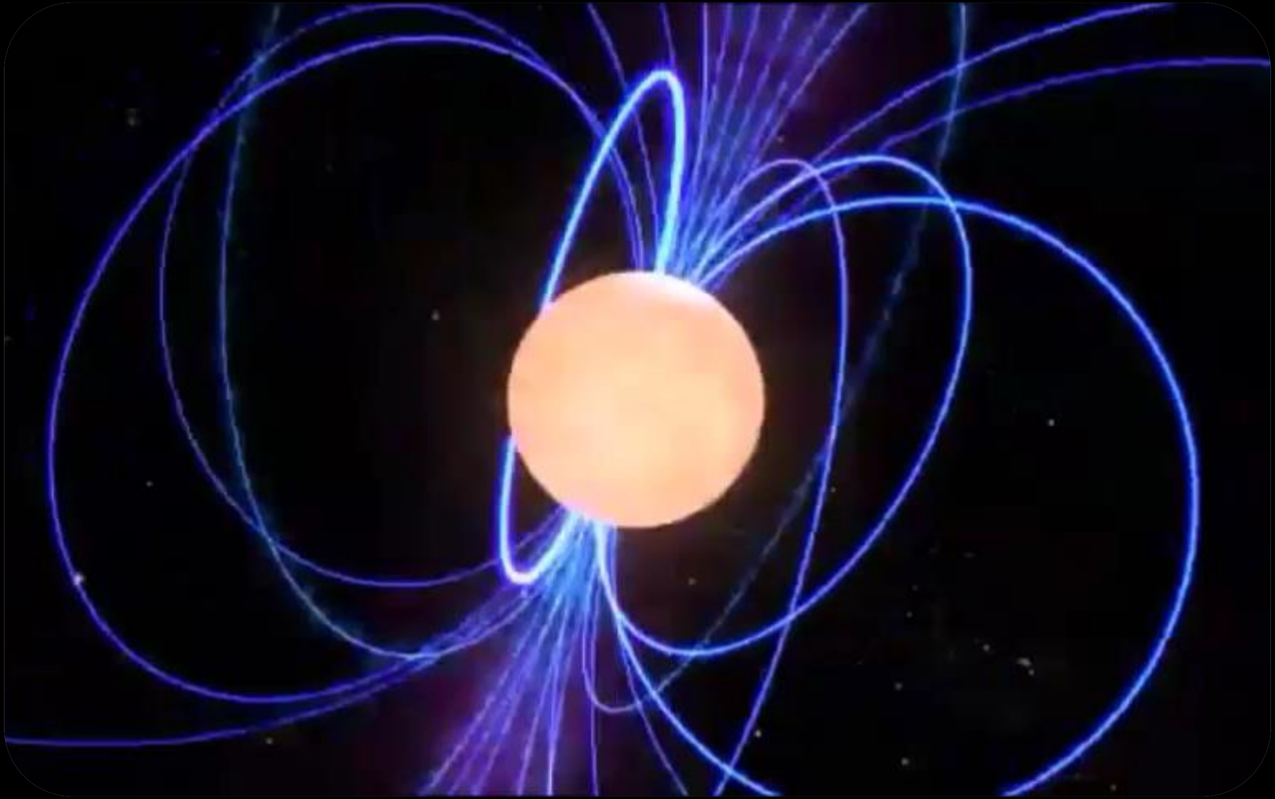
A. Bulgac[2]

M.M. Forbes[2,3]

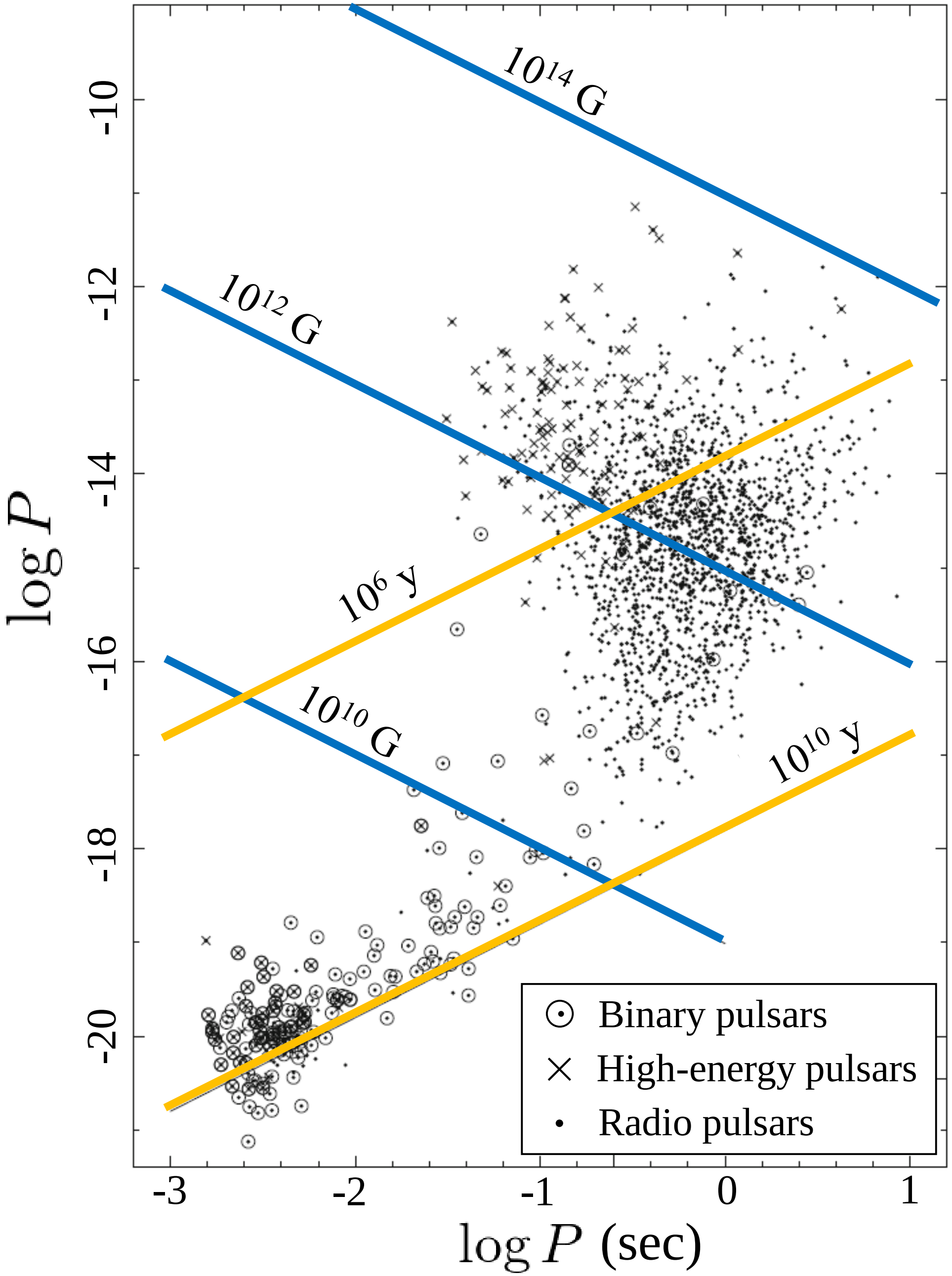
Faculty of Physics, Warsaw University of Technology[1]
Department of Physics, University of Washington[2]
Department of Physics & Astronomy, Washington State University[3]

Pulsar - a rotating neutron star

- ✓ First discovery in August 1967
- ✓ Since then, more than 2650 pulsars have been observed
- ✓ It gradually spins down due to the EM radiation



P-$\dot{P}$ diagram for pulsars (*not necessarily glitchers)



- ✓ Period (P): milliseconds to seconds
- ✓ Gradually spins down ($\dot{P} > 0$) due to the EM radiations
- ✓ Very stable "clock", especially for millisecond pulsars, i.e. $\dot{P} \sim 10^{-20}$

- ➢ Characteristic age:

$$\tau_c = P/(2\dot{P})$$

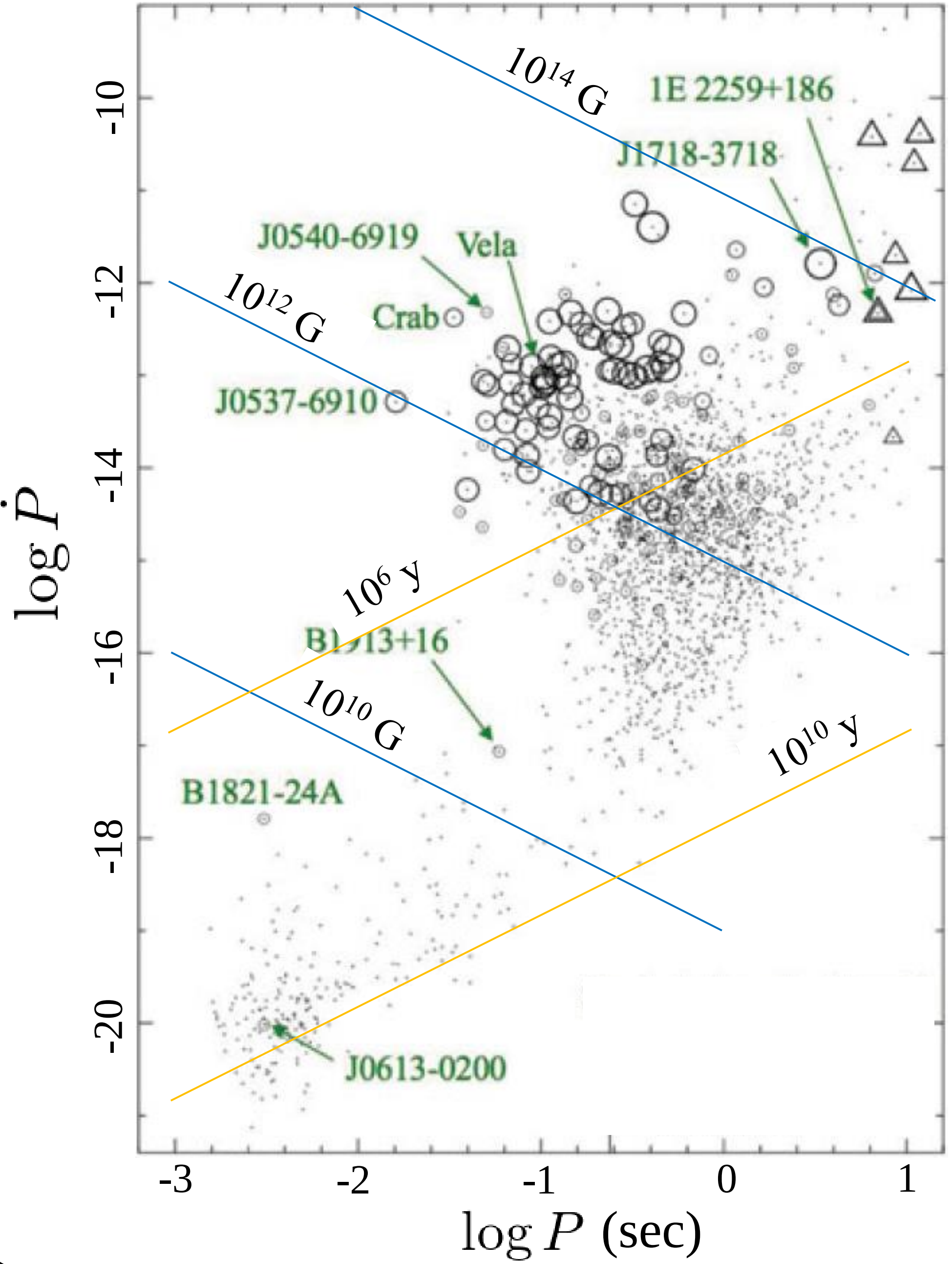
- ➢ Surface dipole magnetic field strength:

$$B = 3.2 \times 10^{19} (P\dot{P})\ \mathrm{G}$$

Figure taken from:
R.N. Manchester, J. Astrophys. Astr. **38**, 42 (2017)

Glitch Table: http://www.jb.man.ac.uk/pulsar/glitches/gTable.html
Glitch Catalogue: http://www.jb.man.ac.uk/pulsar/glitches/gTable.html

P-$\dot{P}$ diagram for pulsars (*not necessarily glitchers)



- ✓ **More than 548 glitches** have been observed **in more than 180 pulsars**
- ✓ Symbol size: glitch size (typically, log(Δν/ν) ~ 10^{-10}-10^{-5})
- ✓ Young pulsars (including magnetars) exhibit larger glitches than older ones

- ➢ Characteristic age:

$$\tau_c = P/(2\dot{P})$$

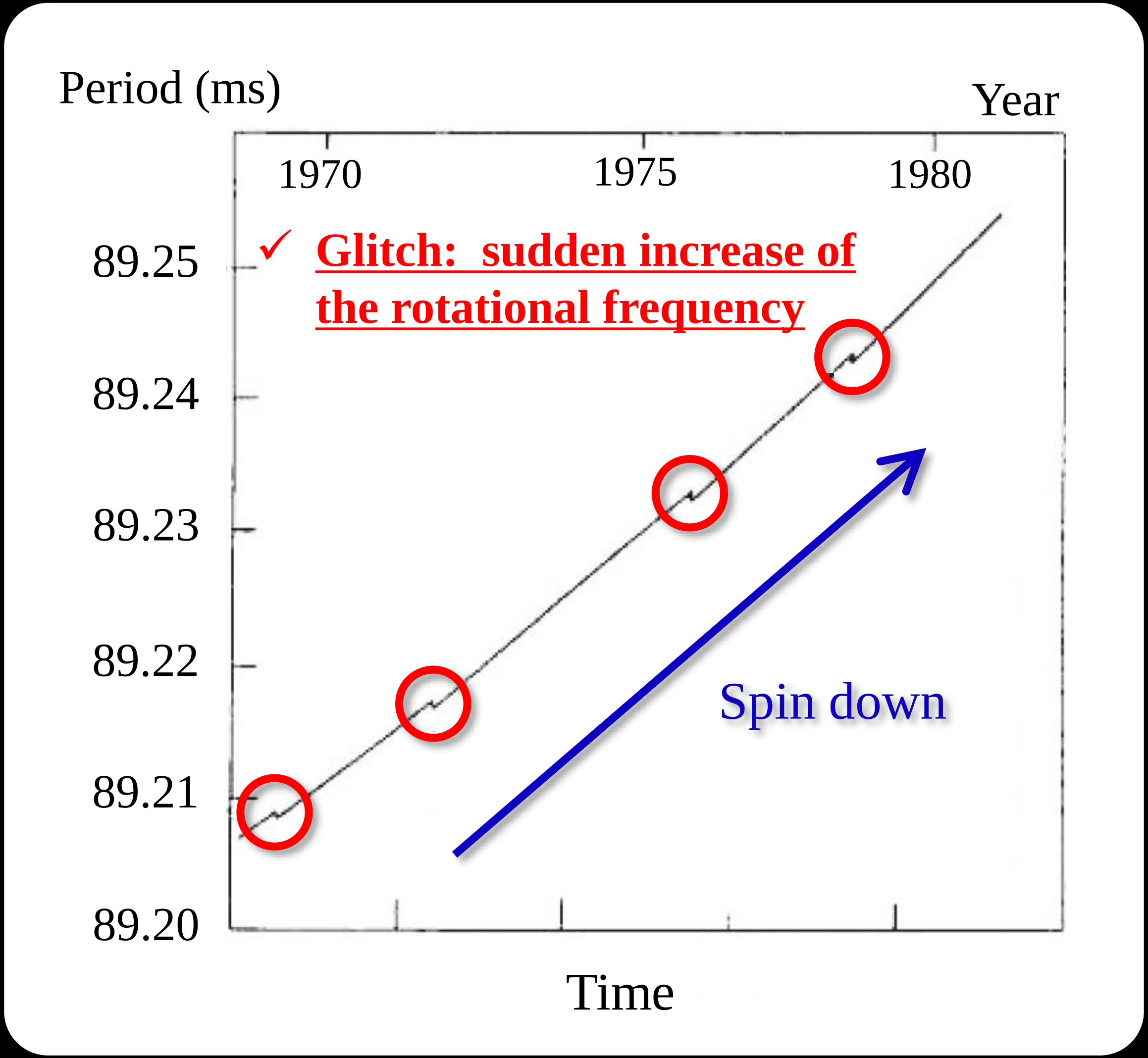
- ➢ Surface dipole magnetic field strength:

$$B = 3.2 \times 10^{19} (P\dot{P})\ \mathrm{G}$$

Figure taken from:
R.N. Manchester, Proc. IAU Symp. **337**, 197 (2017)

Typical example: glitches in the Vela pulsar

- Irregularity has been observed from continuous monitoring of the pulsation period

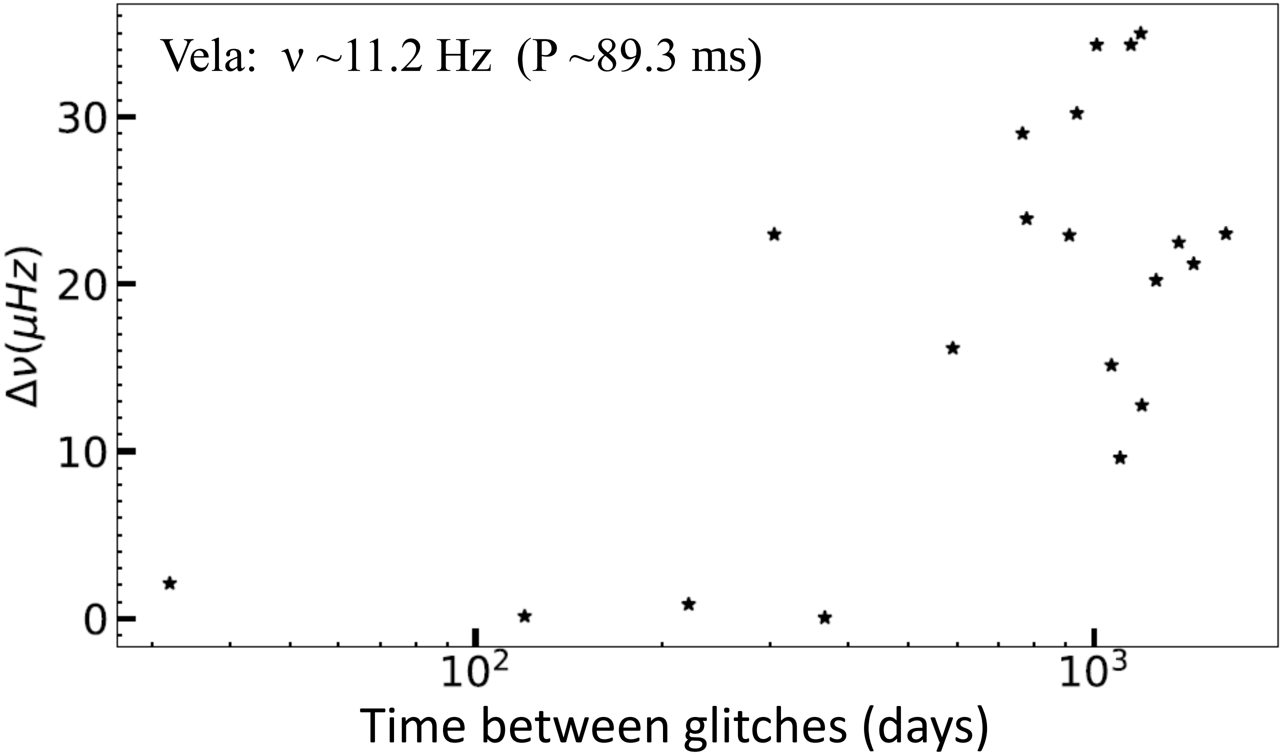


✓ **Glitch: sudden increase of the rotational frequency**

Typical example: glitches in the Vela pulsar

- ✓ Vela glitches roughly every 3 years (quasi-periodic)
- ✓ 21 glitches have been observed since its discovery in 1969
- ✓ Most of them are "large" ($\Delta\nu/\nu \sim 10^{-6}$), but there are a few small glitches
- ✓ Increase of spin down rate have also been observed: $\Delta\dot{\nu}/\dot{\nu} \sim 10^{-3}$-$10^{-1}$

*Δν: change of the rotation frequency due to a glitch

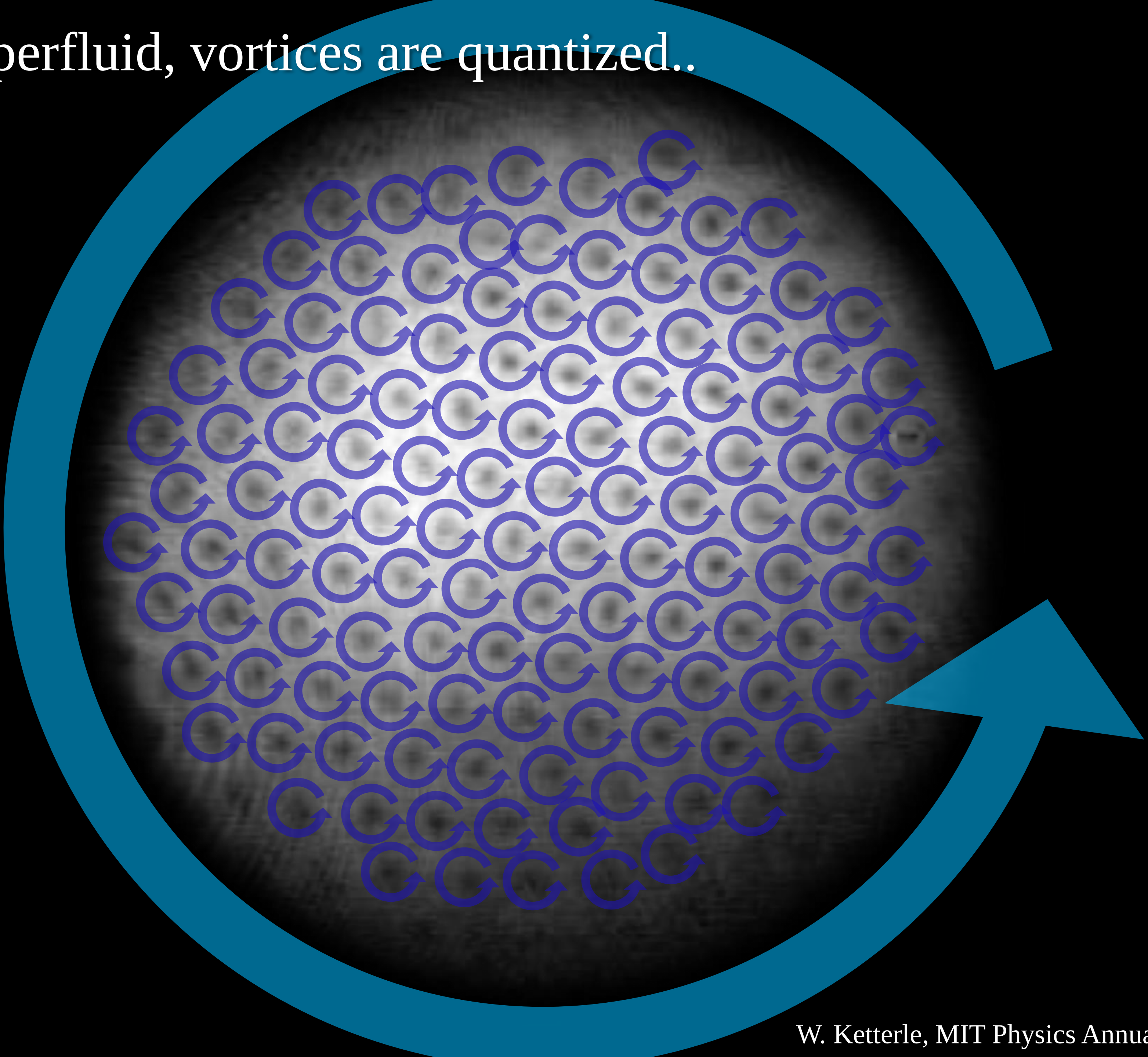


What’s the cause of the glitches?

Dynamics of quantized vortices play a key role!

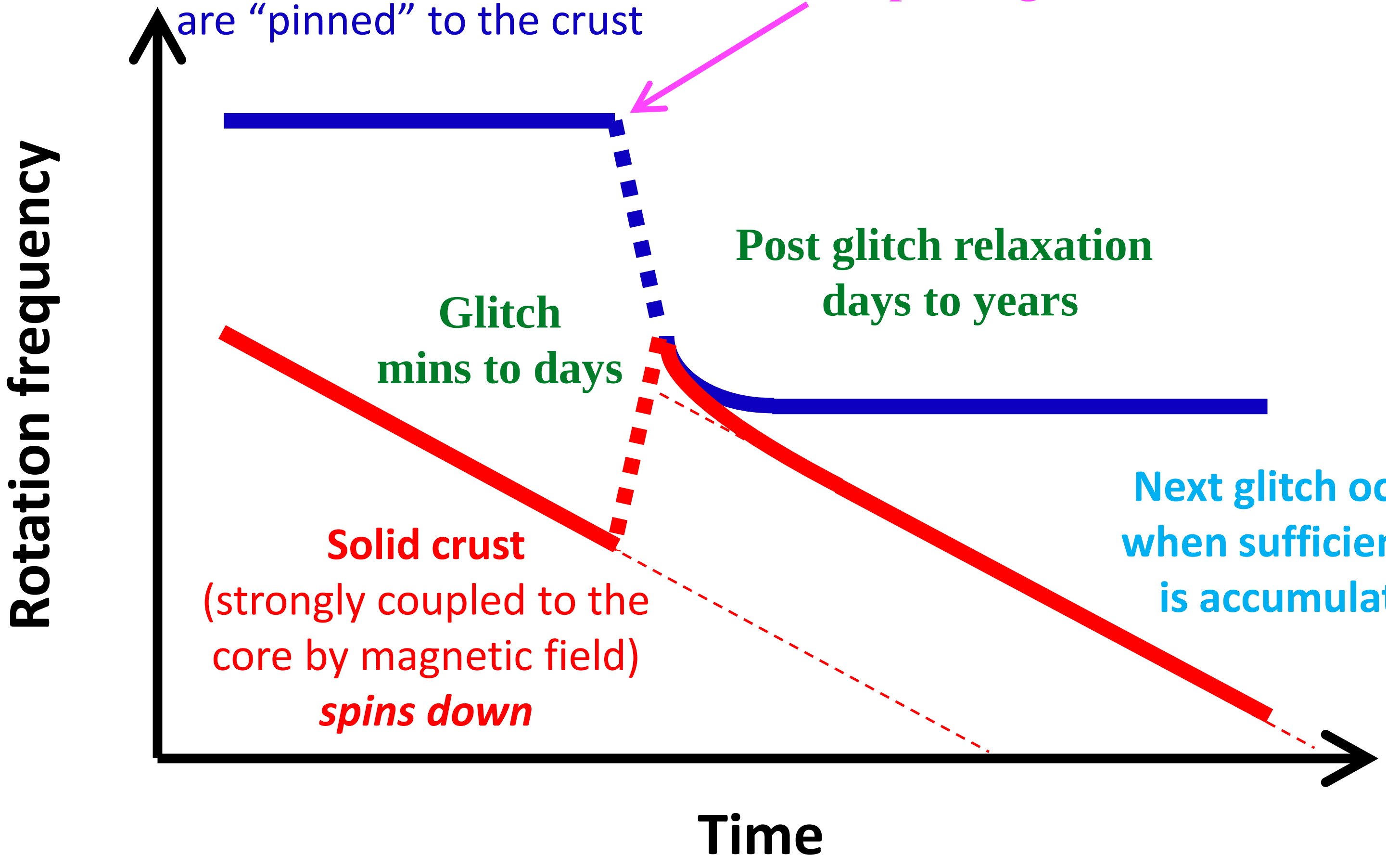
In daily life, a vortex is continuous..

In superfluid, vortices are quantized..



W. Ketterle, MIT Physics Annual. 2001

The vortex mediated glitch: Naive picture



Superfluid neutrons
(decoupled from the crust)
cannot spin down if vortices
are "pinned" to the crust

"Unpinning" occurs

Glitch
mins to days

Post glitch relaxation
days to years

Solid crust
(strongly coupled to the
core by magnetic field)
spins down

Next glitch occurs
when sufficient lag
is accumulated

To fully understand the glitches, we need to clarify:

Glitch dynamics

Spin-up followed by
post-glitch relaxation, ...

and, of course,
details of NS matter..

Pinning mechanism

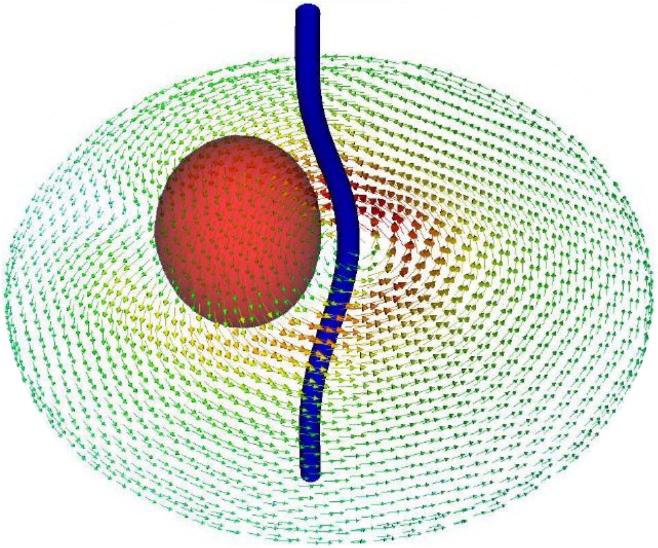
Nuclear pinning, interstitial pinning,
pinning force, ...

Trigger mechanism

Magnus force,
hydrodynamic instabilities, ...

**We attacked this problem using
the state-of-the-art microscopic nuclear theory**

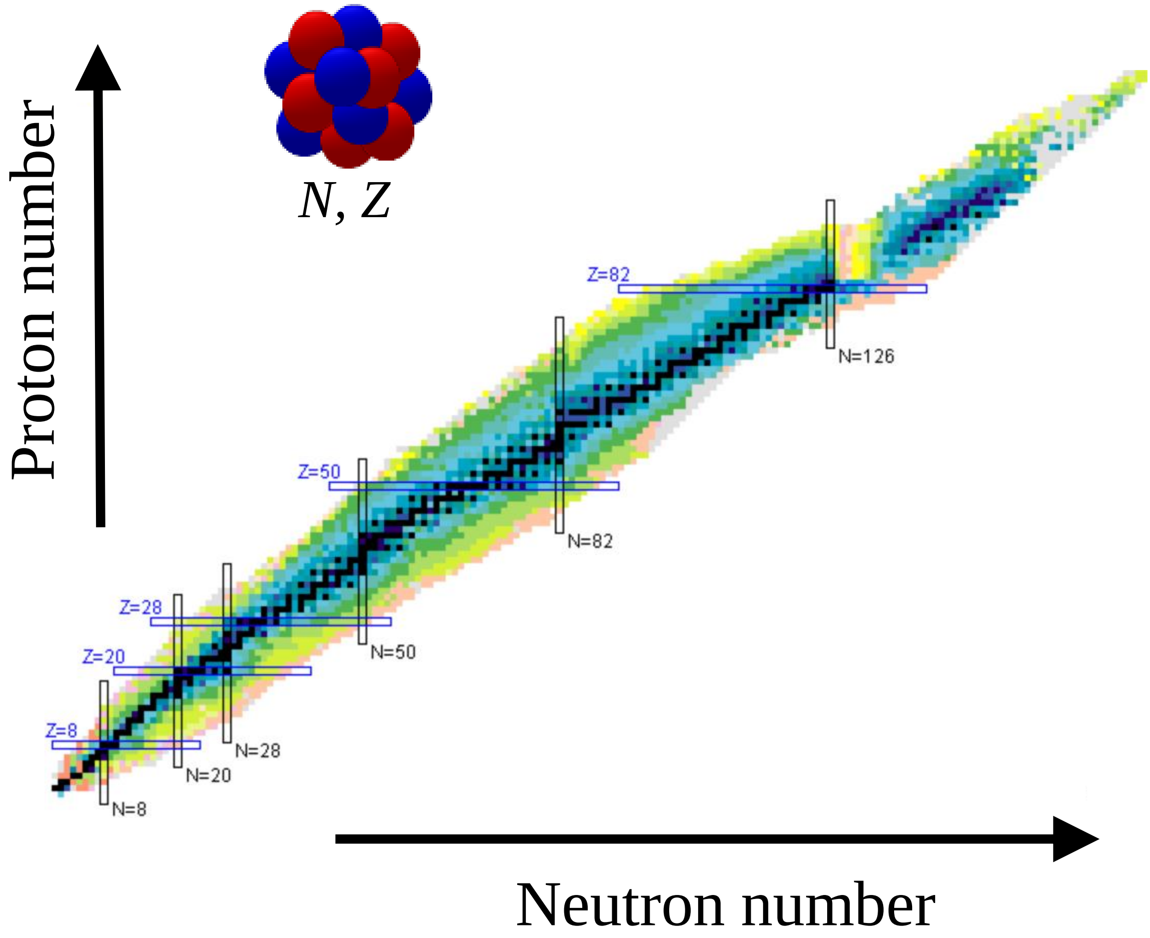
Vortex-nucleus dynamics with in TDSLDA



G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes,
Phys. Rev. Lett. **117**, 232701 (2016)

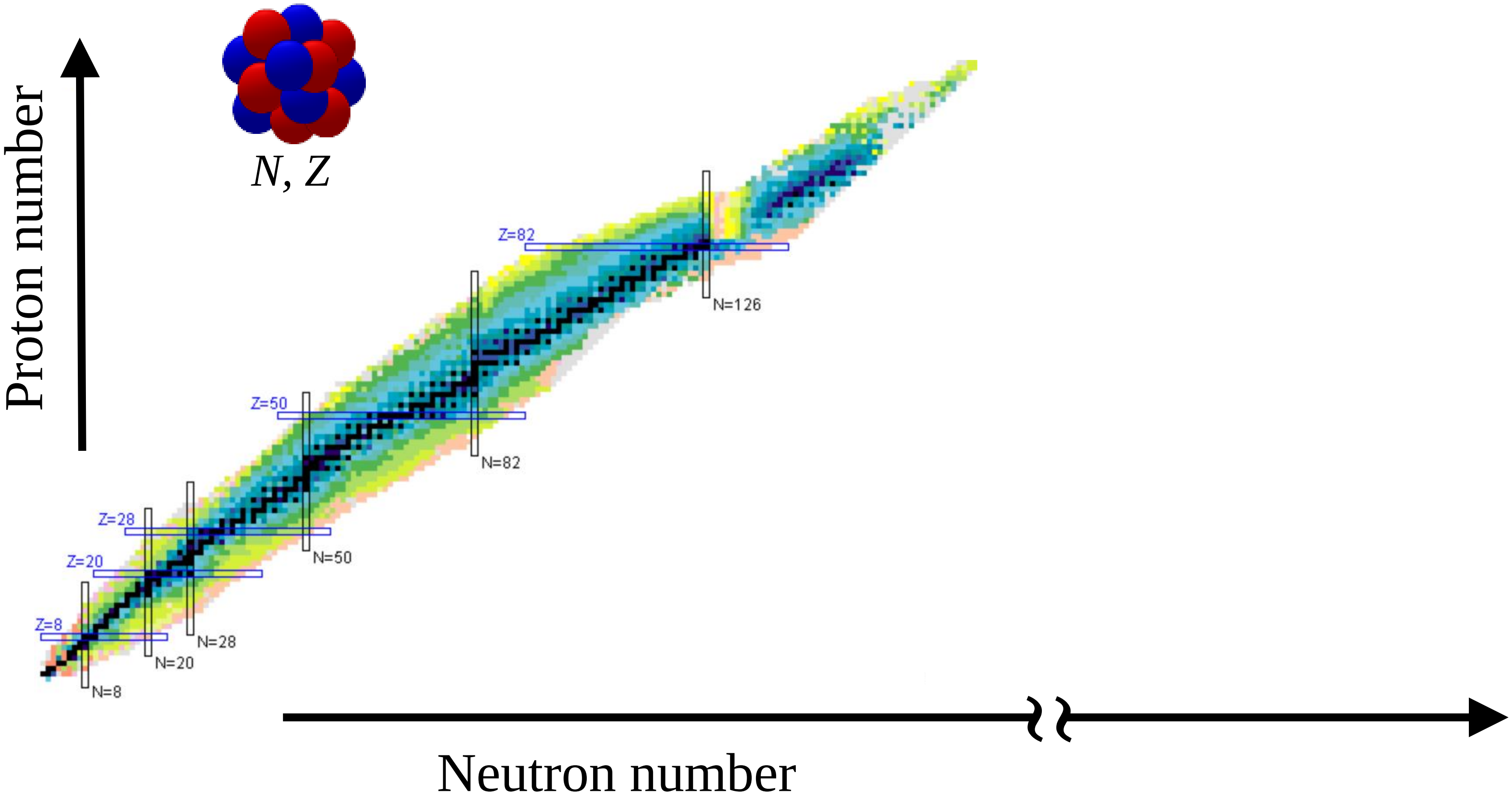
DFT in Nuclear Physics

All nuclei can be described with a single EDF



DFT in Nuclear Physics

All nuclei can be described with a single EDF

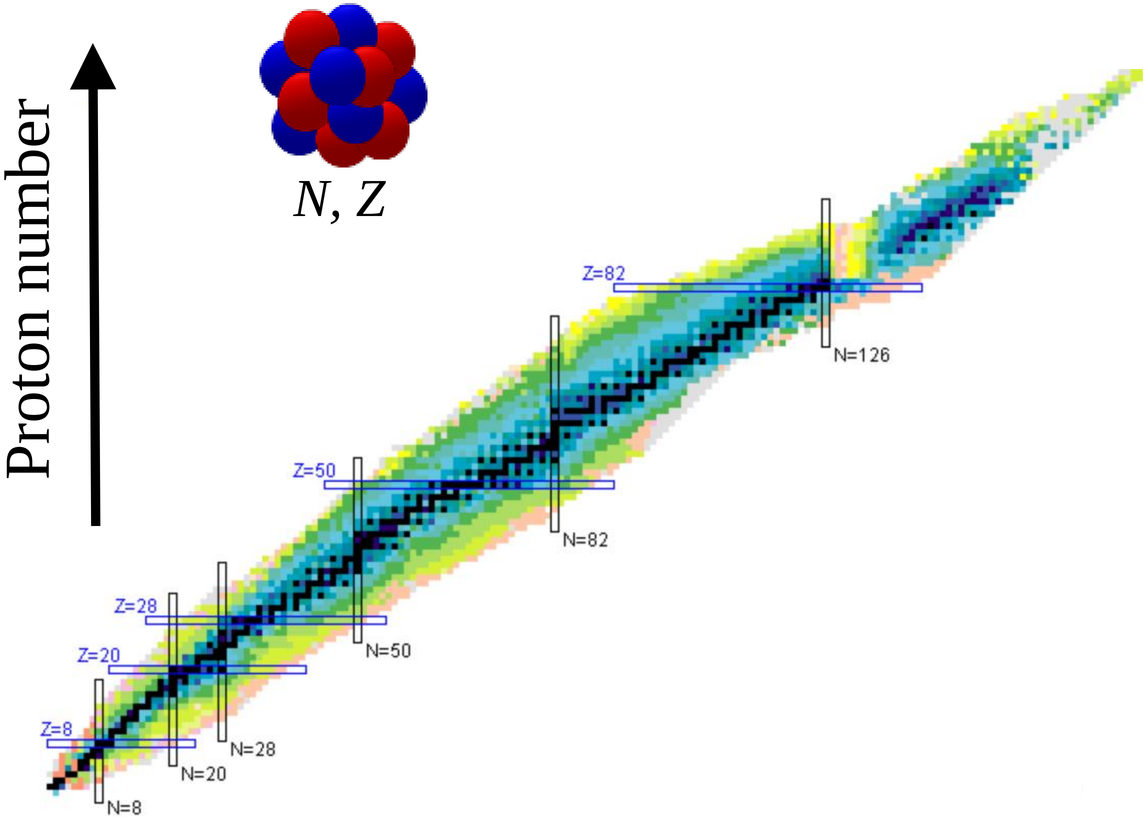


DFT in Nuclear Physics

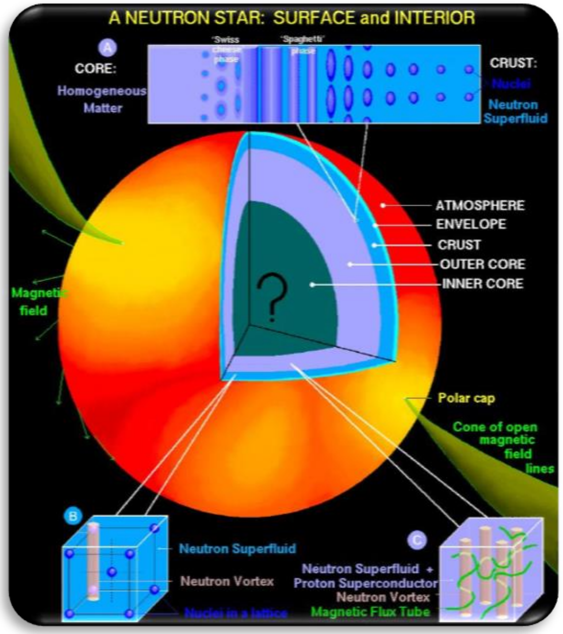
All nuclei can be described with a single EDF

N, *Z*

Proton number



Neutron star



Neutron number

TDSLDA (Time-Dependent Superfluid Local Density Approximation)

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

- TDSLDA equations (formally equivalent to TDHFB or TD-BdG equations)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix} \begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

$h_\sigma = \dfrac{\delta E}{\delta n_\sigma}$: s.p. Hamiltonian

$\Delta = -\dfrac{\delta E}{\delta \nu^*}$: pairing field

$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$: number density

$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$: anomalous density

$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar \sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla} v_{k,\sigma}(\boldsymbol{r},t)]$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

# of qp orbitals ~ # of grid points

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

- TDSLDA equations (formally equivalent to TDHFB or TD-BdG equations)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & & & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix} \begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

Supercomputing!!

$$h_\sigma = \frac{\delta E}{\delta n_\sigma}$$: s.p. Hamiltonian

$$\Delta = -\frac{\delta E}{\delta \nu^*}$$: pairing field

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$$: number density

$$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$$: anomalous density

$$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar\sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla}v_{k,\sigma}(\boldsymbol{r},t)]$$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

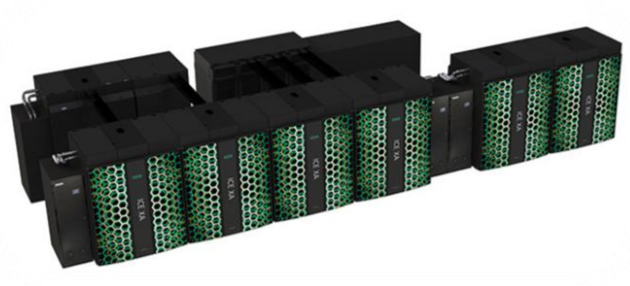
# of qp orbitals ~ # of grid points

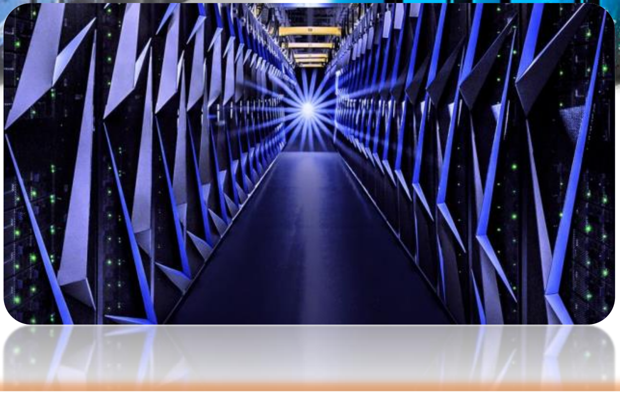
*The number indicates the rank according to the TOP500 list (June 2019)

Piz Daint, CSCS, Switzerland (No. 6)

TITAN, ORNL, USA (No. 12)

TSUBAME3.0, Japan (No. 25)

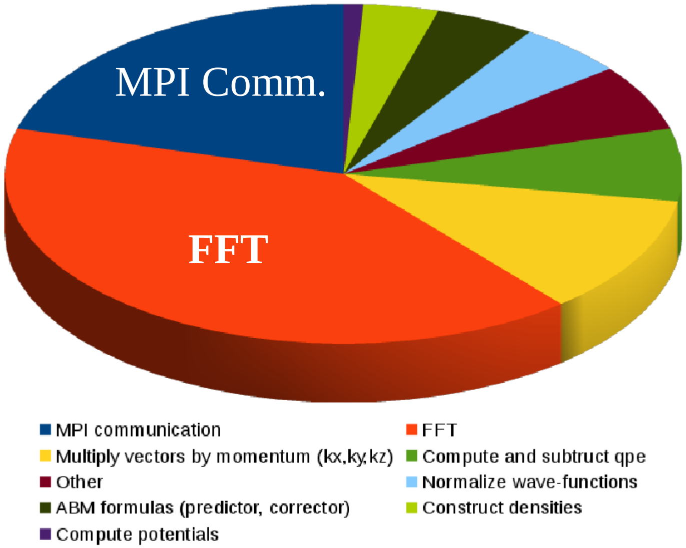




The fastest machine:
Summit, ORNL, USA
GPU, 188 PFlops/s

Present computing capabilities:

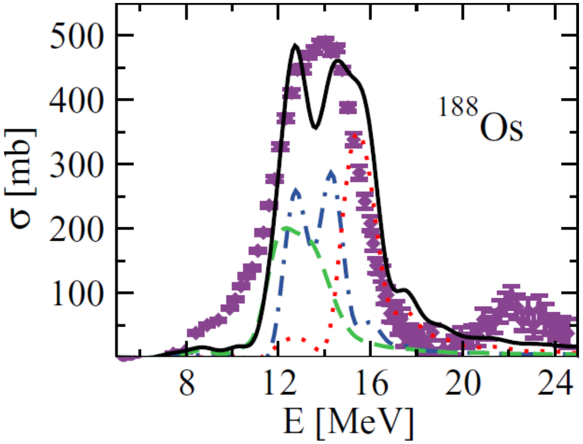
- ✓ Full 3D (w/o symmetry restrictions)
- ✓ Volume as large as 100^3 lattice points
- ✓ Evolution up to 10^6 time steps (as long as 10^{-19} sec)



Applications of TDSLDA: Nuclear systems

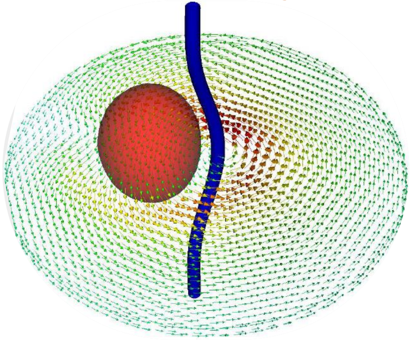
TDSLDA is a versatile tool!!

IVGDR



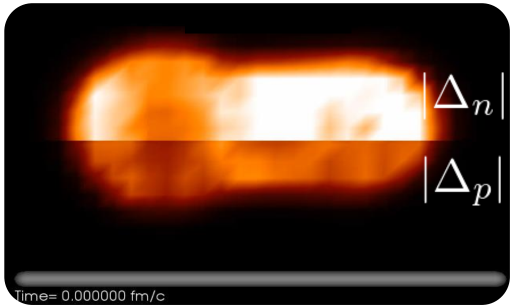
Phys. Rev. C **84**, 051309(R) (2011)
I. Stetcu, A. Bulgac, P. Magierski, and K.J. Roche

Vortex-nucleus dynamics



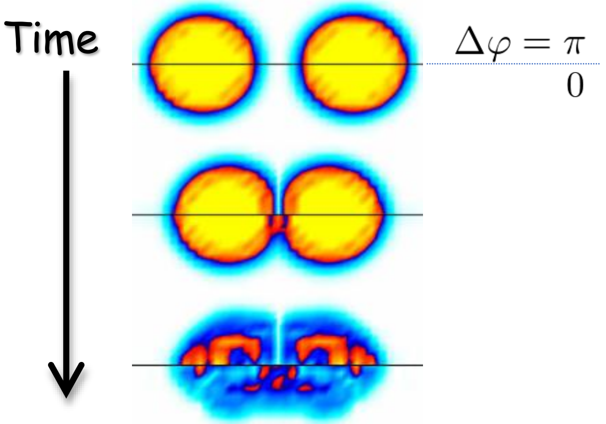
Phys. Rev. Lett. **117**, 232701 (2016)
G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes

Induced fission of ^{240}Pu



Phys. Rev. Lett. **116**, 122504 (2016)
A. Bulgac, P. Magierski, K.J. Roche, and I. Stetcu

Low-energy heavy ion reactions



Phys. Rev. Lett. **119**, 042501 (2017)
P. Magierski, K.S., and G. Wlazłowski

A key to understand the glitches is:

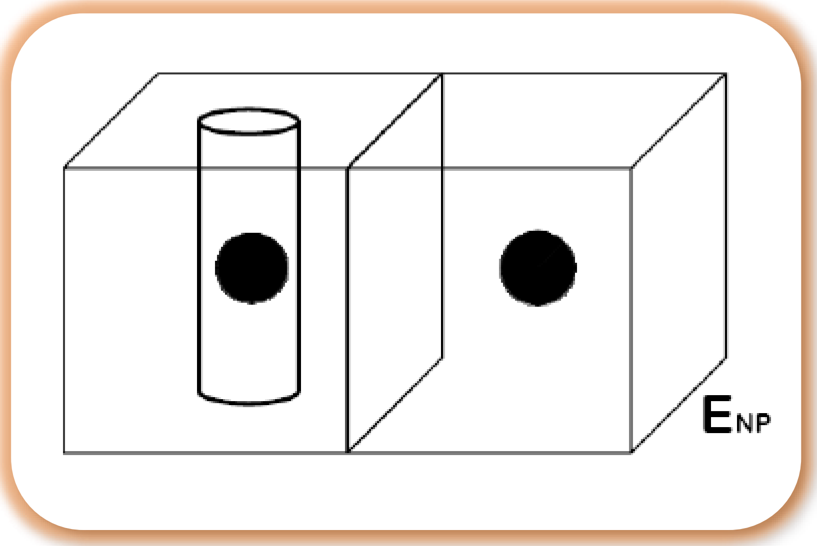
Vortex pinning mechanism in the inner crust of neutron stars

Q. Is the vortex-nucleus interaction

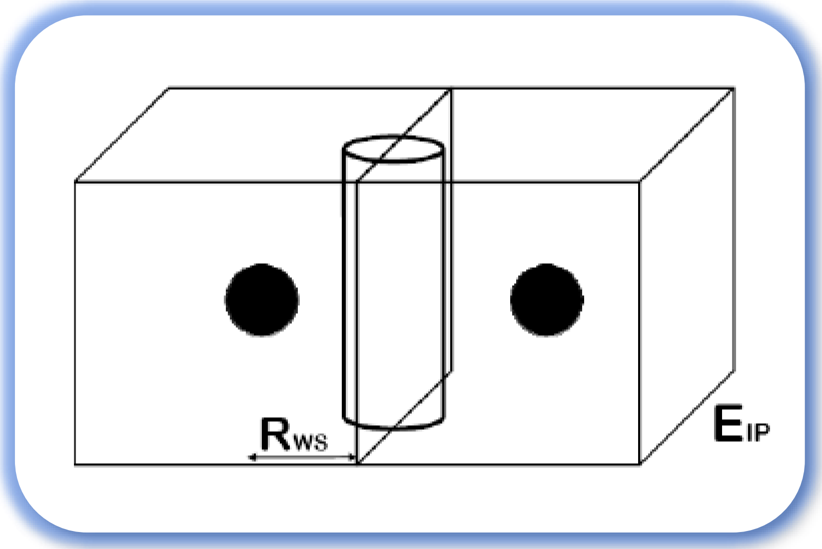
Attractive?

or

Repulsive?



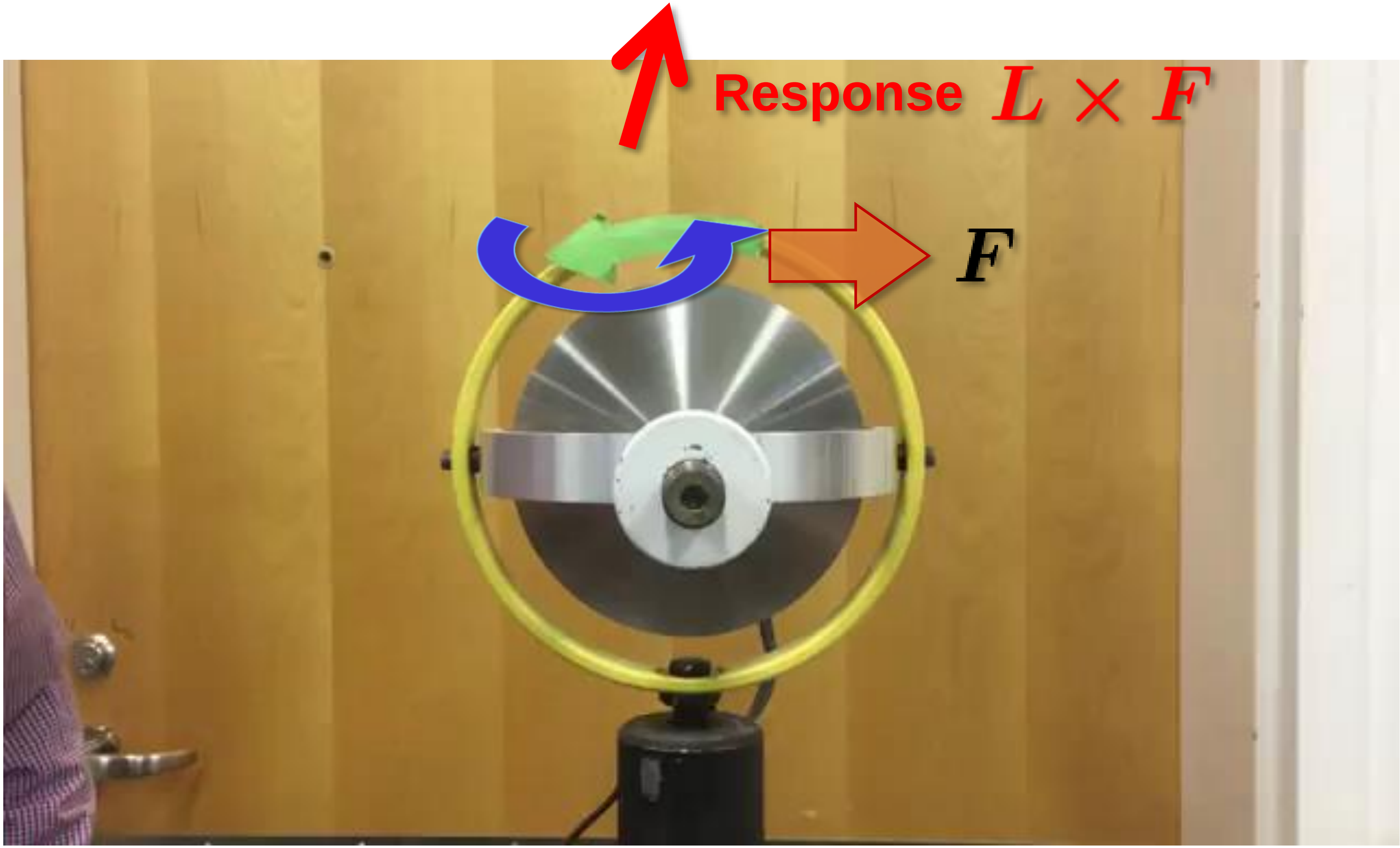
“Nuclear pinning”



“Interstitial pinning”

What we investigated - Vortex-nucleus dynamics

Response of a spinning gyroscope when pushed



Method - TDSLDA

We performed 3D, dynamical simulations by TDDFT with superfluidity

☐ TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r})\\ \Delta^*(\mathbf{r}) & -h(\mathbf{r})\end{pmatrix}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}$$

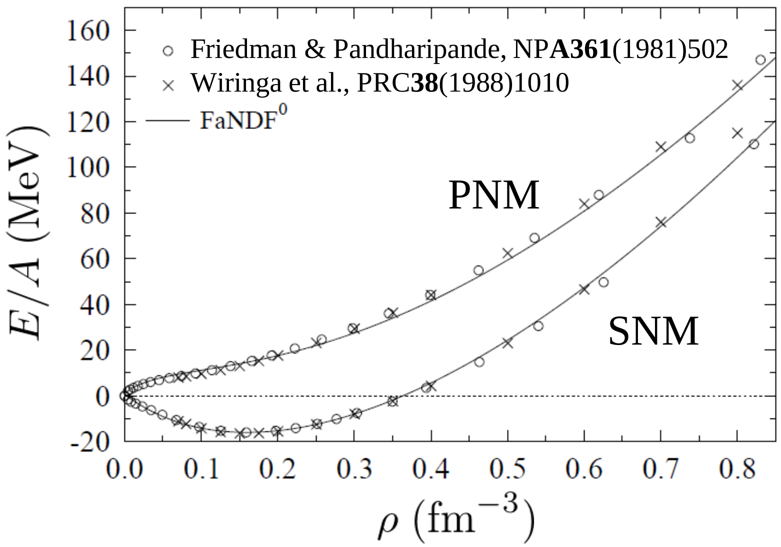
☐ Energy density functional (EDF)

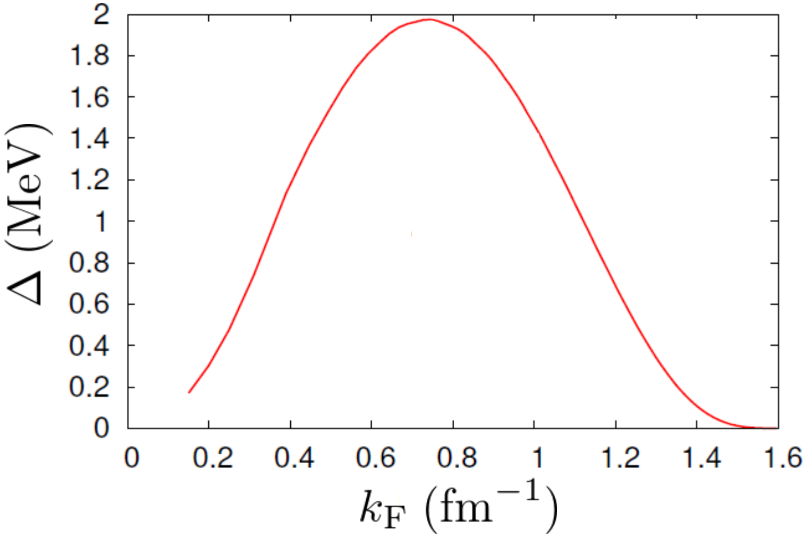
$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\text{pair}}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r}) = \sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





Method - TDSLDA

We performed 3D, dynamical simulations by TDDFT with superfluidity

☐ TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

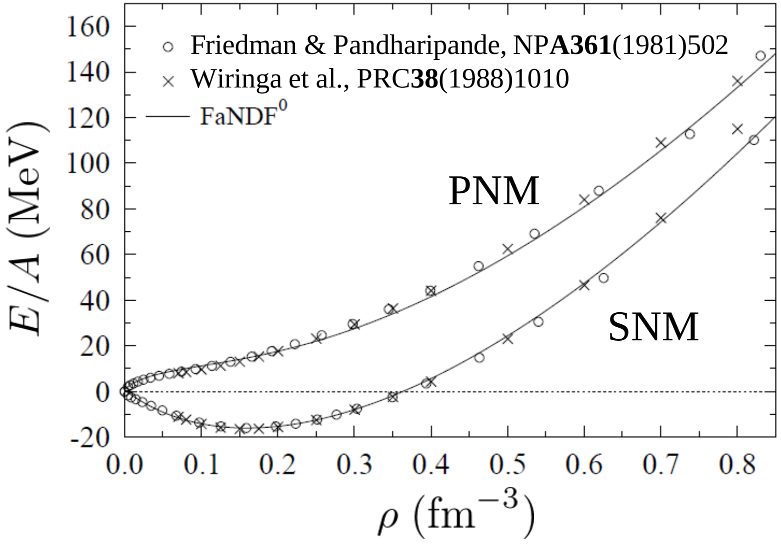
☐ Energy density functional (EDF)

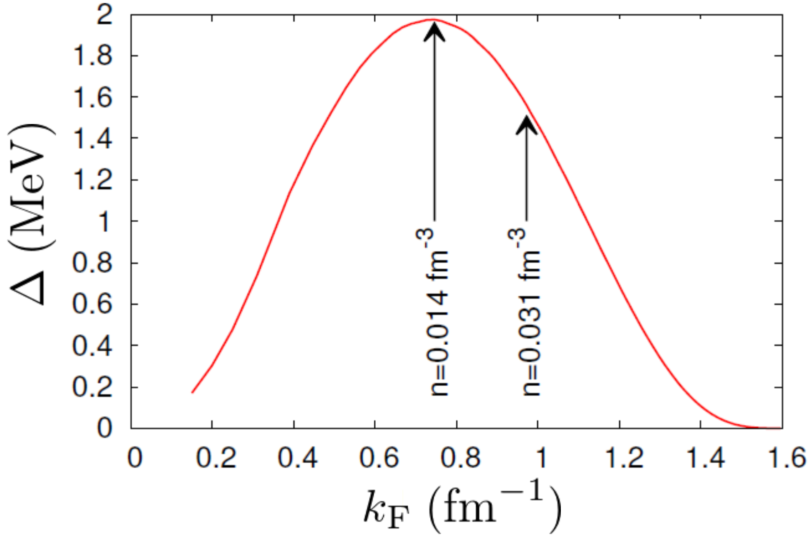
$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\text{pair}}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r}) = \sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





Method - TDSLDA

We performed 3D, dynamical simulations by TDDFT with superfluidity

☐ TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

☐ Computational details

$75\ \mathrm{fm} \times 75\ \mathrm{fm} \times 60\ \mathrm{fm}$

$(50 \times 50 \times 40,\ \Delta x = 1.5\ \mathrm{fm})$

$k_\mathrm{c} = \pi/\Delta x > k_\mathrm{F} \qquad k_\mathrm{F} = (3\pi^2\rho_n)^{1/3}$

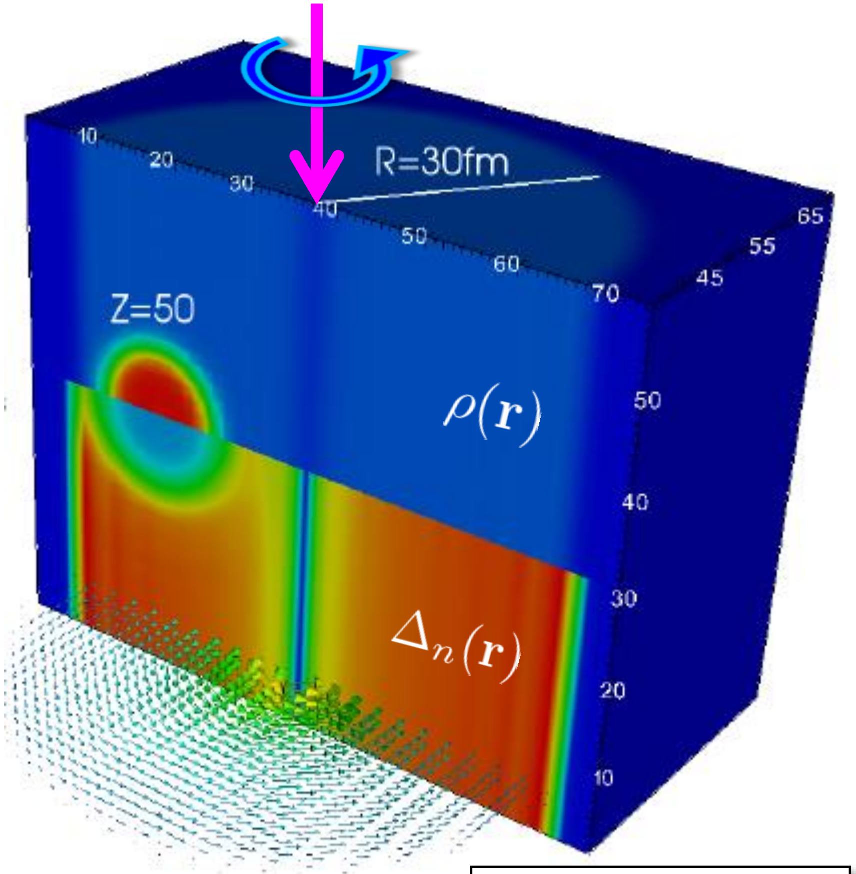
Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\ \mathrm{fm}^{-3}\ (N \simeq 2{,}530)$

$\rho_n \simeq 0.031\ \mathrm{fm}^{-3}\ (N \simeq 5{,}714)$

# of quasi-particle w.f. $\approx 100{,}000$

a vortex line exists here



$\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

Method - TDSLDA

We performed 3D, dynamical simulations by TDDFT with superfluidity

- ☐ TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

- ☐ Computational details

$75\ \text{fm} \times 75\ \text{fm} \times 60\ \text{fm}$

$(50 \times 50 \times 40,\ \Delta x = 1.5\ \text{fm})$

$k_\mathrm{c} = \pi/\Delta x > k_\mathrm{F} \qquad k_\mathrm{F} = (3\pi^2\rho_n)^{1/3}$

Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\ \text{fm}^{-3}\ (N \simeq 2{,}530)$

$\rho_n \simeq 0.031\ \text{fm}^{-3}\ (N \simeq 5{,}714)$

\# of quasi-particle w.f. $\approx 100{,}000$

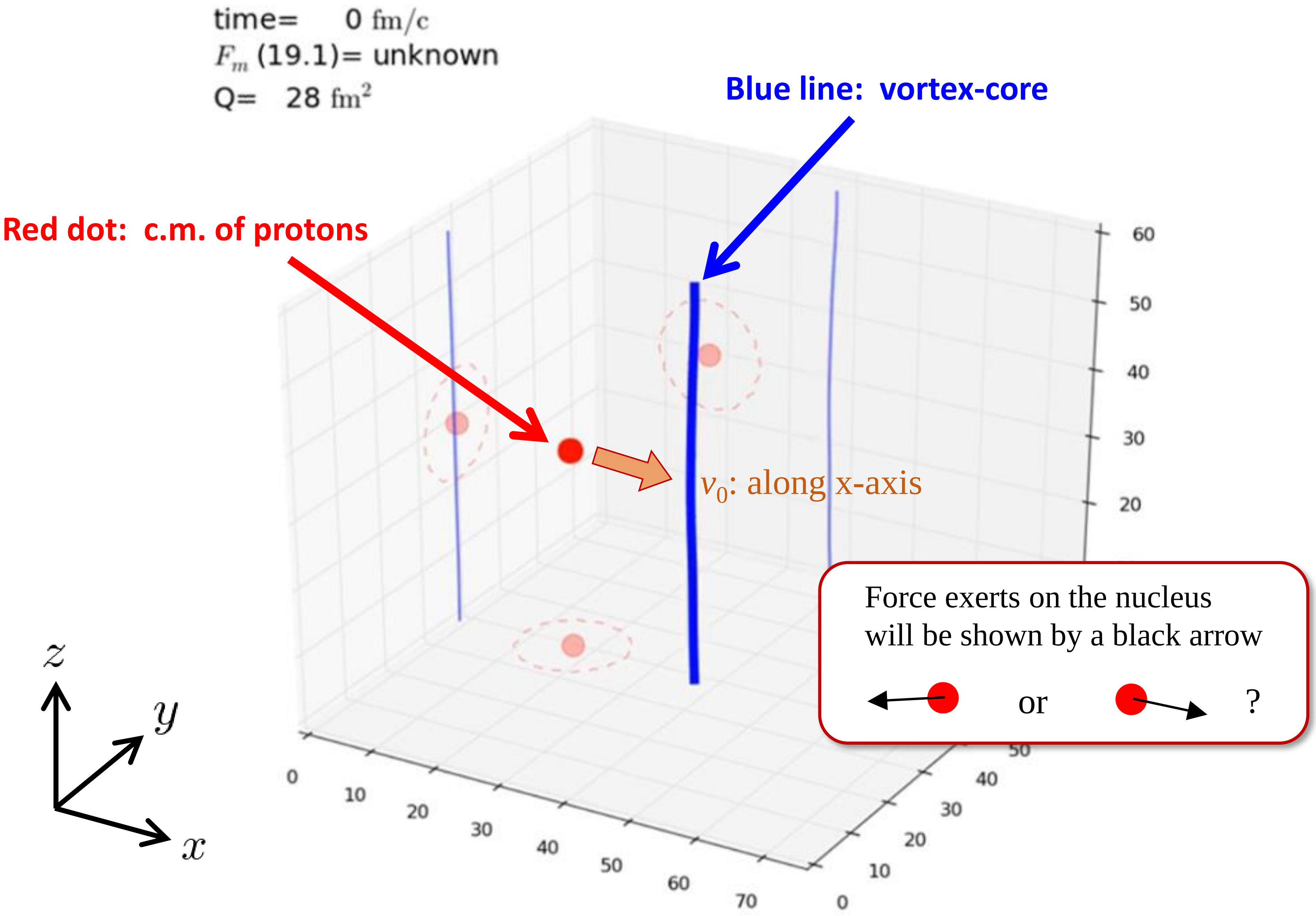
MPI+GPU
→ 48h w/ 200GPUs
for 10,000 fm/c

TITAN, Oak Ridge

NERSC Edison, Berkeley

HA-PACS, Tsukuba

Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



Red dot: c.m. of protons

Blue line: vortex-core

v_0: along x-axis

Force exerts on the nucleus will be shown by a black arrow

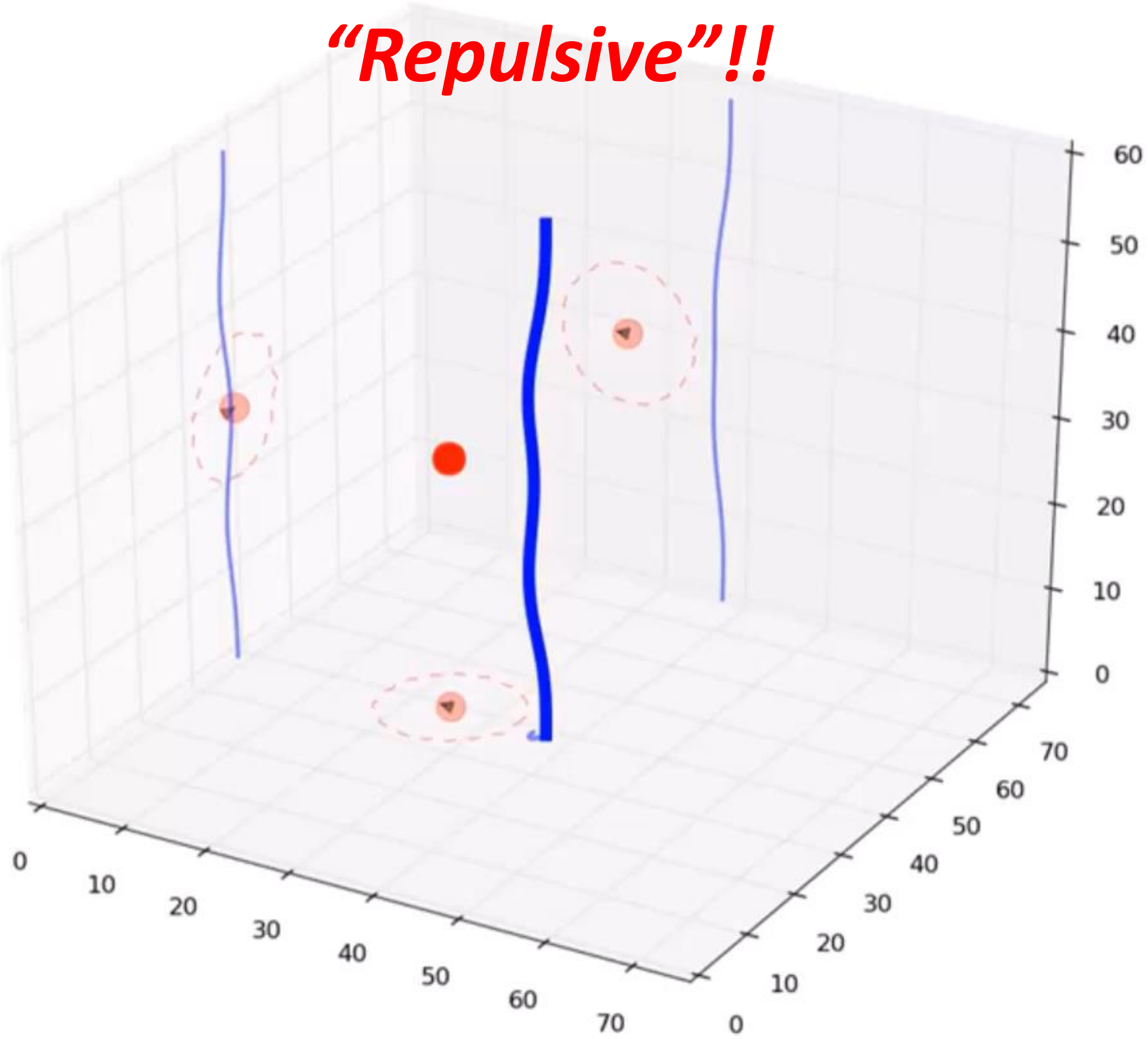
or ?

Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

time= 8032 $\mathrm{fm/c}$

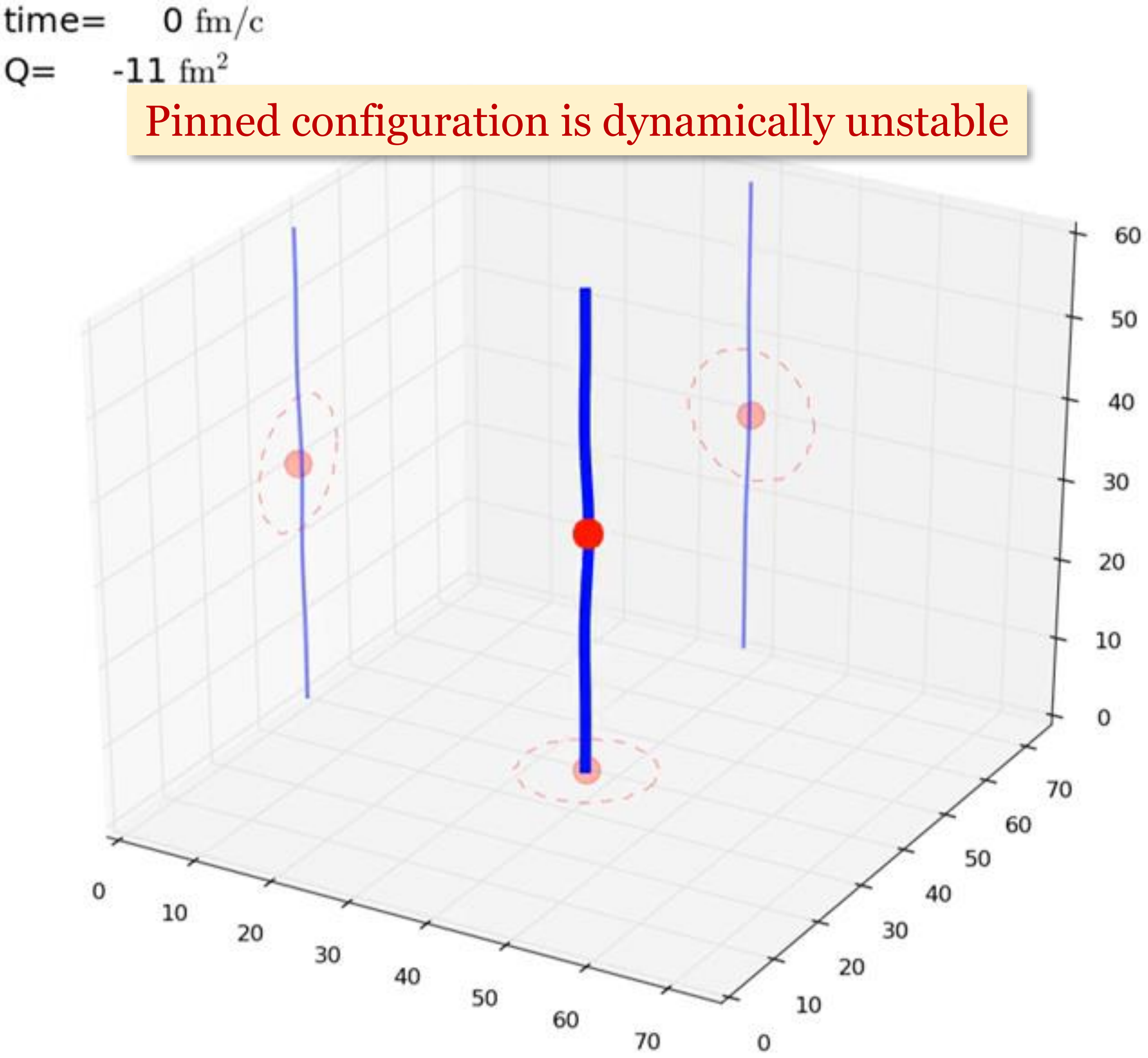
F_m (10.6)= 0.17 $\mathrm{MeV/fm}$

Q= 13 fm^2



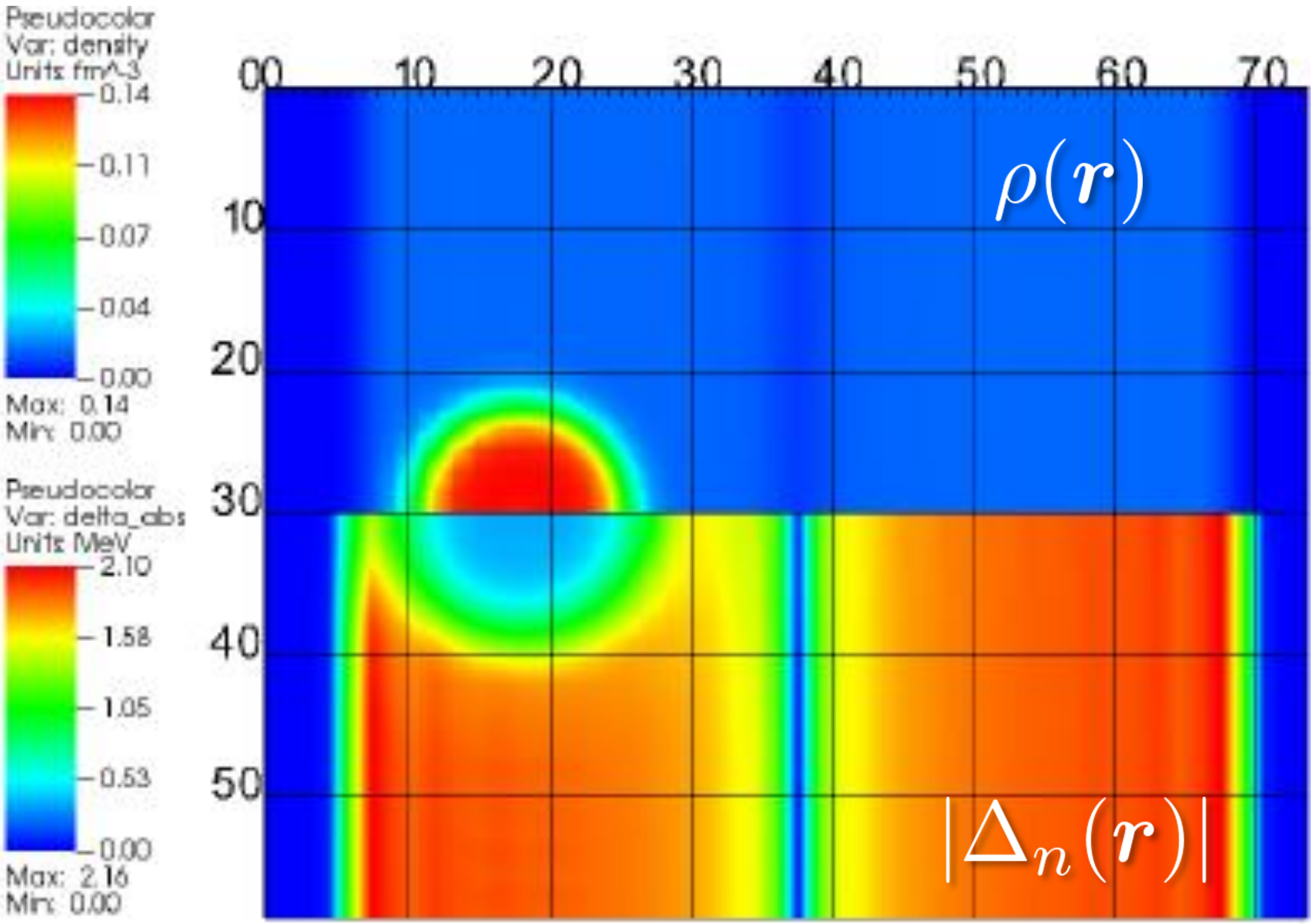
“Repulsive”!!

Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

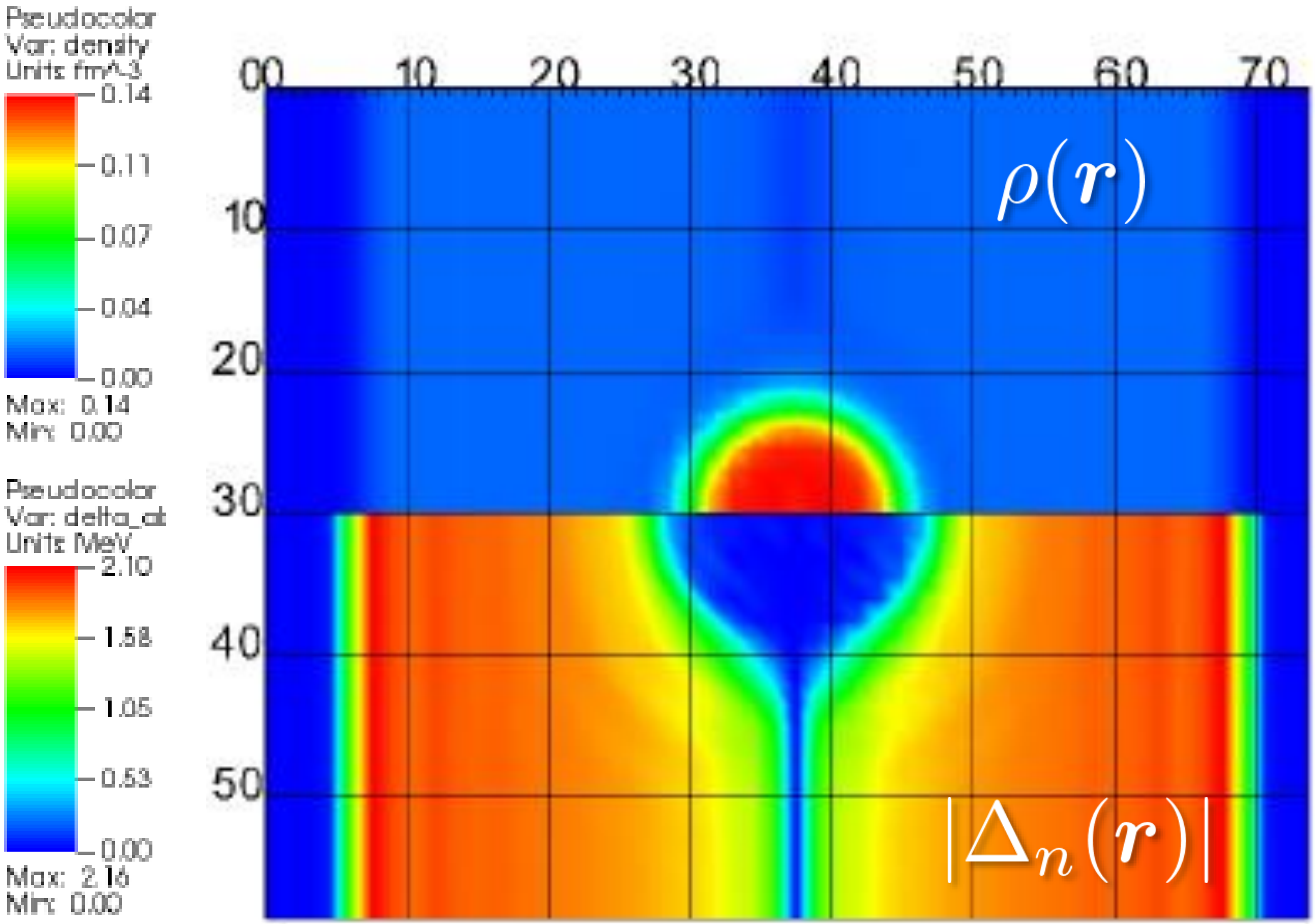


Pinned configuration is dynamically unstable

"Unpinned configuration"

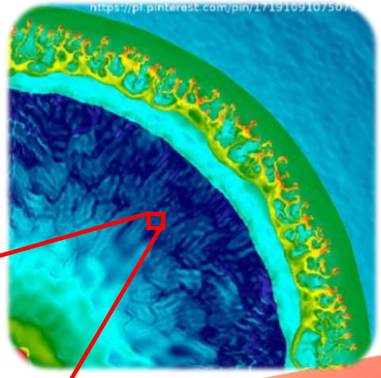


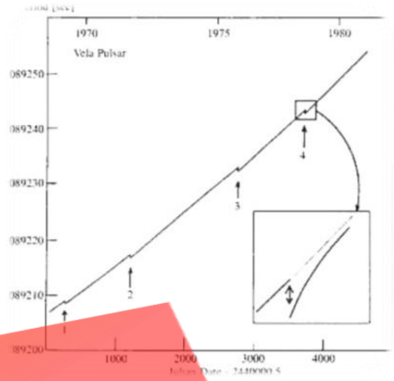
“Pinned configuration”

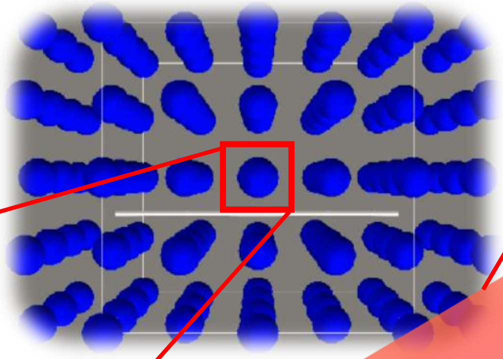


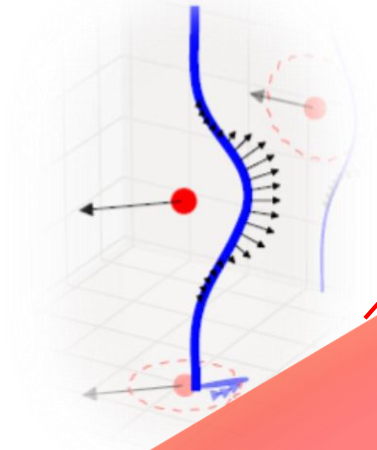
Our goal and strategy

Goal: Unveil the mechanism of glitches









10^{4}m

Macroscopic

- observations
- hydrodynamics

$\sim 10^{-10}$m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei*

(e.g. filament model)

10^{-15}-10^{-13}m

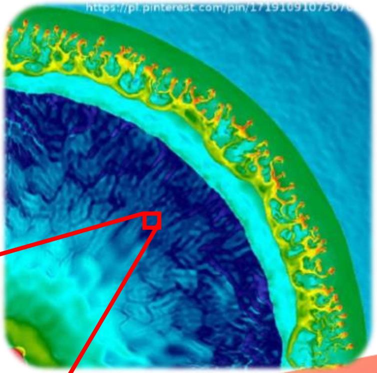
Microscopic

Nuclear Physics!!

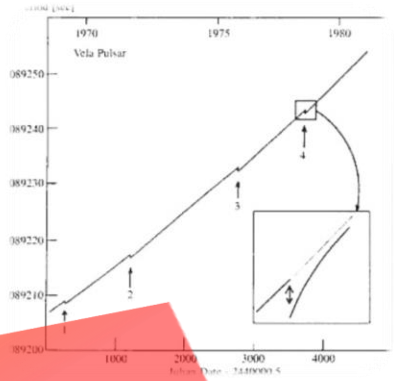
- vortex-nucleus dynamics from *neutrons and protons*

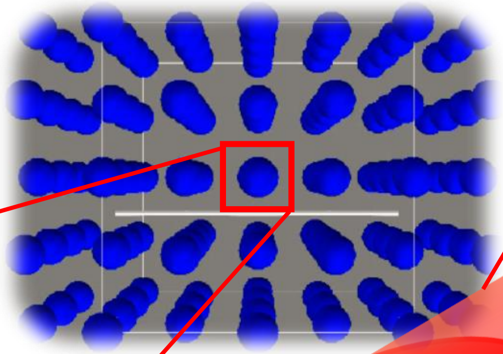
Our goal and strategy

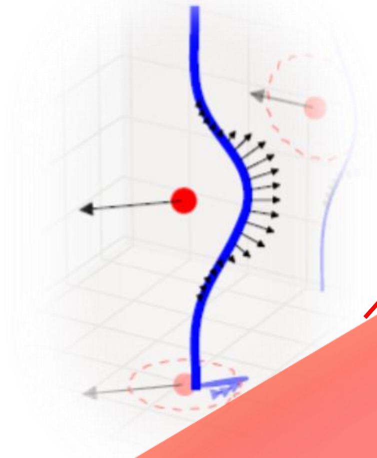
Goal: Unveil the mechanism of glitches



https://pl.pinterest.com/pin/17191091075070







10^{4}m

Macroscopic

- observations
- hydrodynamics

~10^{-10}m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

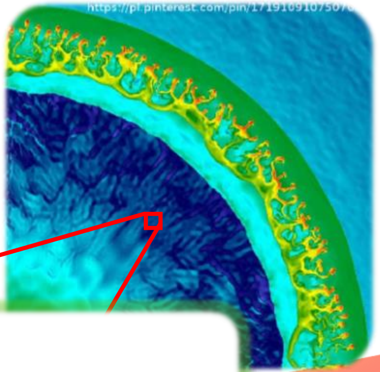
Microscopic

Nuclear Physics!!

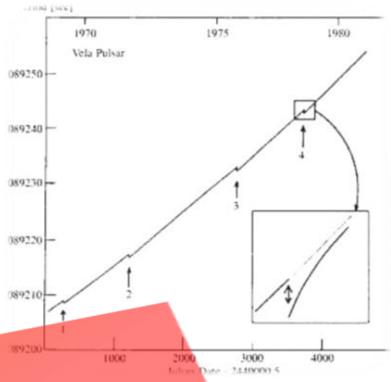
- vortex-nucleus dynamics from *neutrons and protons*

Our goal and strategy

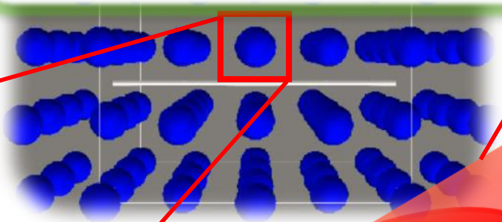
Goal: Unveil the mechanism of glitches

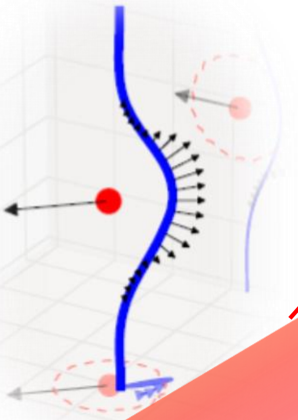


https://pl.pinterest.com/pin/1719109107507



New collaboration started:
Nicolaus Copernicus Astronomical Centre
B. Haskell et al.





10^{4}m

Macroscopic

- observations
- hydrodynamics

~10^{-10}m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

Microscopic

Nuclear Physics!!

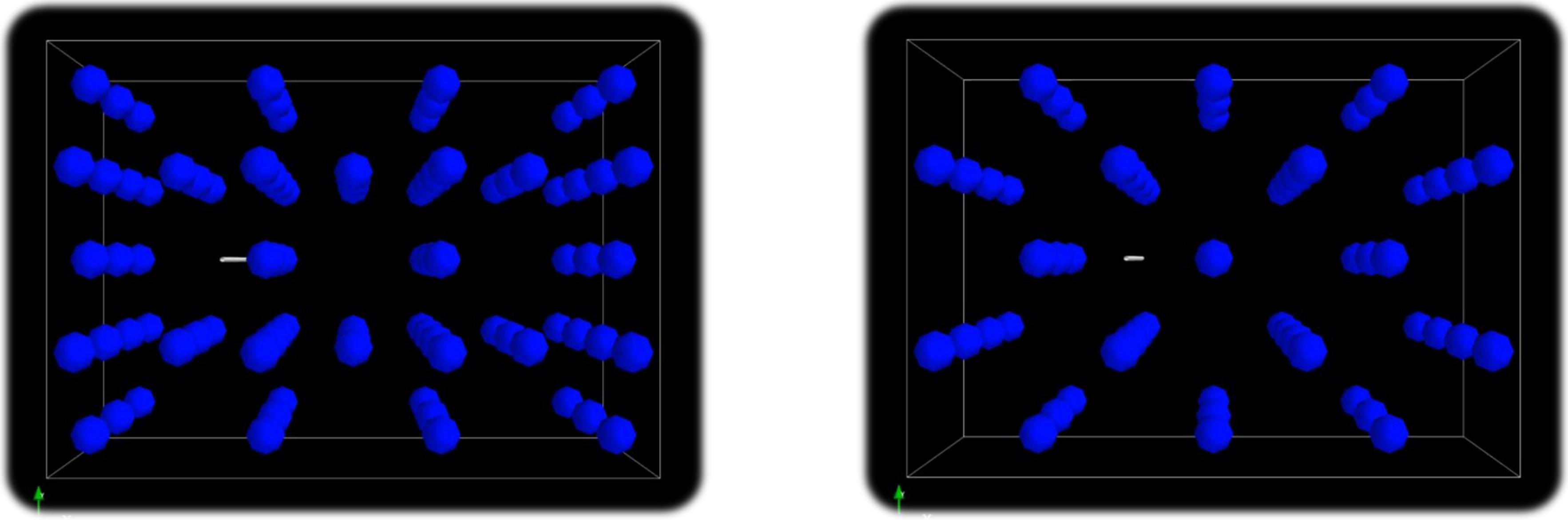
- vortex-nucleus dynamics from *neutrons and protons*

Ongoing project

Mesoscopic simulation of pinning force with the vortex filament model

Simulations by K. Kobczewski (PhD student at WUT)

$$n_s \boldsymbol{\kappa} \times \left(\boldsymbol{v}_{vor} - \boldsymbol{v}_{ext}\right) = \boldsymbol{f}_{VN} + \boldsymbol{f}_{tension} + \boldsymbol{f}_{dissipation}$$



Talk by K. Kobczewski at POLNS18, March 26-28, 2018: https://indico.camk.edu.pl/event/10/contribution/8

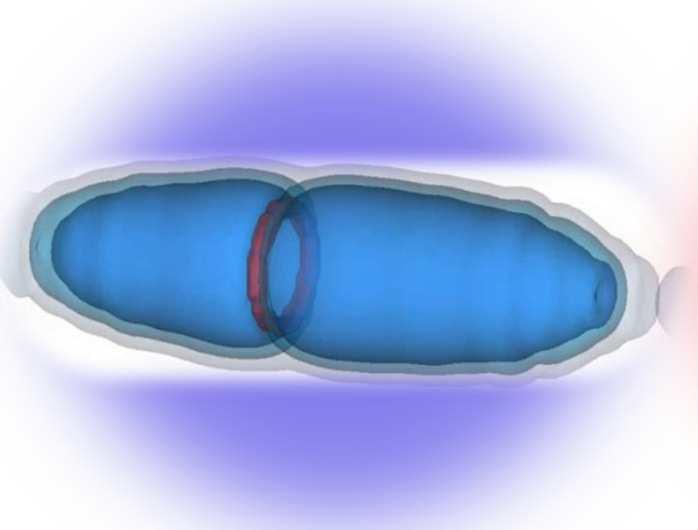
Very preliminary

Summary

Summary

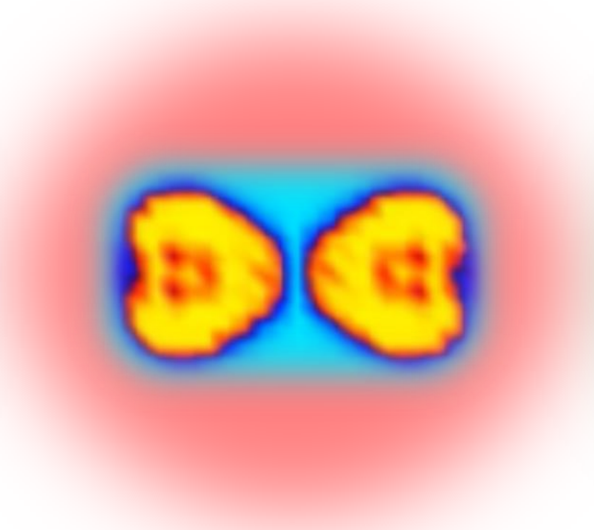
Takeaway message

✓ TDSLDA is a powerful tool to study a variety of dynamics in superfluid Fermi systems!



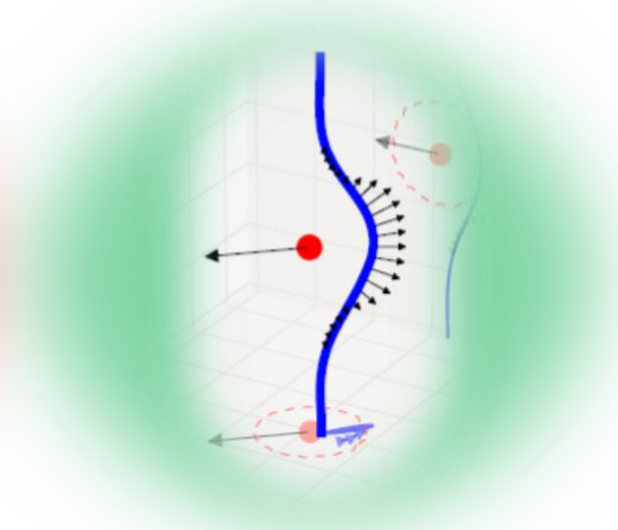
Cold atoms:
UFG

- solitonic cascades
- quantum turbulence



Atomic nuclei:
Nuclear dynamics

- solitonic excitations
- fusion hindrance



Neutron star:
Inner crust

- vortex-nucleus interaction
- tension, M^*, dissipations

Summary

Takeaway message

✓ TDSLDA is a powerful tool to study a variety of dynamics in superfluid Fermi systems!

✓ We found **Repulsive** interaction for n_n = 0.014 & 0.032 fm^{-3}

→ Needs systematic calculations: densities, EDFs

→ Develop a mesoscopic model for vortices in a lattice

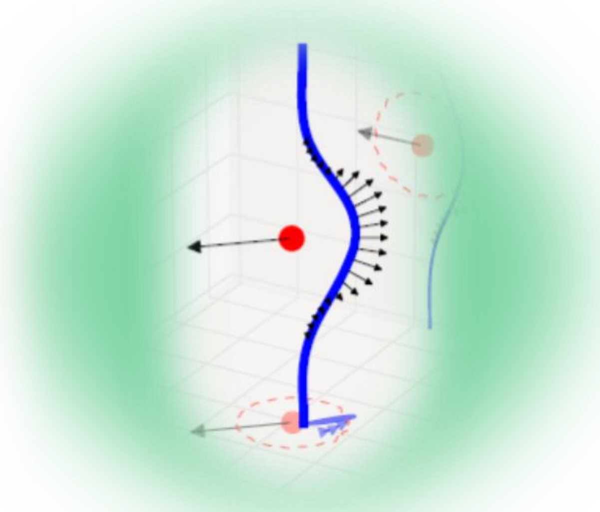
➢ Vortex dynamics in the pasta phase?

➢ Extension to core region? n-$^3P_2$, p-$^1S_0$ and proton flux tubes

➢ .. any ideas??

- quantum turbulence

- fusion hindrance



Neutron star:
Inner crust

- vortex-nucleus interaction
- tension, M^*, dissipations

Kazuyuki Sekizawa
Specially Appointed Assistant Professor
Center for Transdisciplinary Research
Institute for Research Promotion, Niigata University
8050, Ikarashi Ninoho, Nishi-ku, Niigata City, Niigata 950-2181, Japan
sekizawa @ phys.sc.niigata-u.ac.jp
http://sekizawa.fizyka.pw.edu.pl/english/