Hydrostatic equilibria of rotating stars in Lagrangian coordinate

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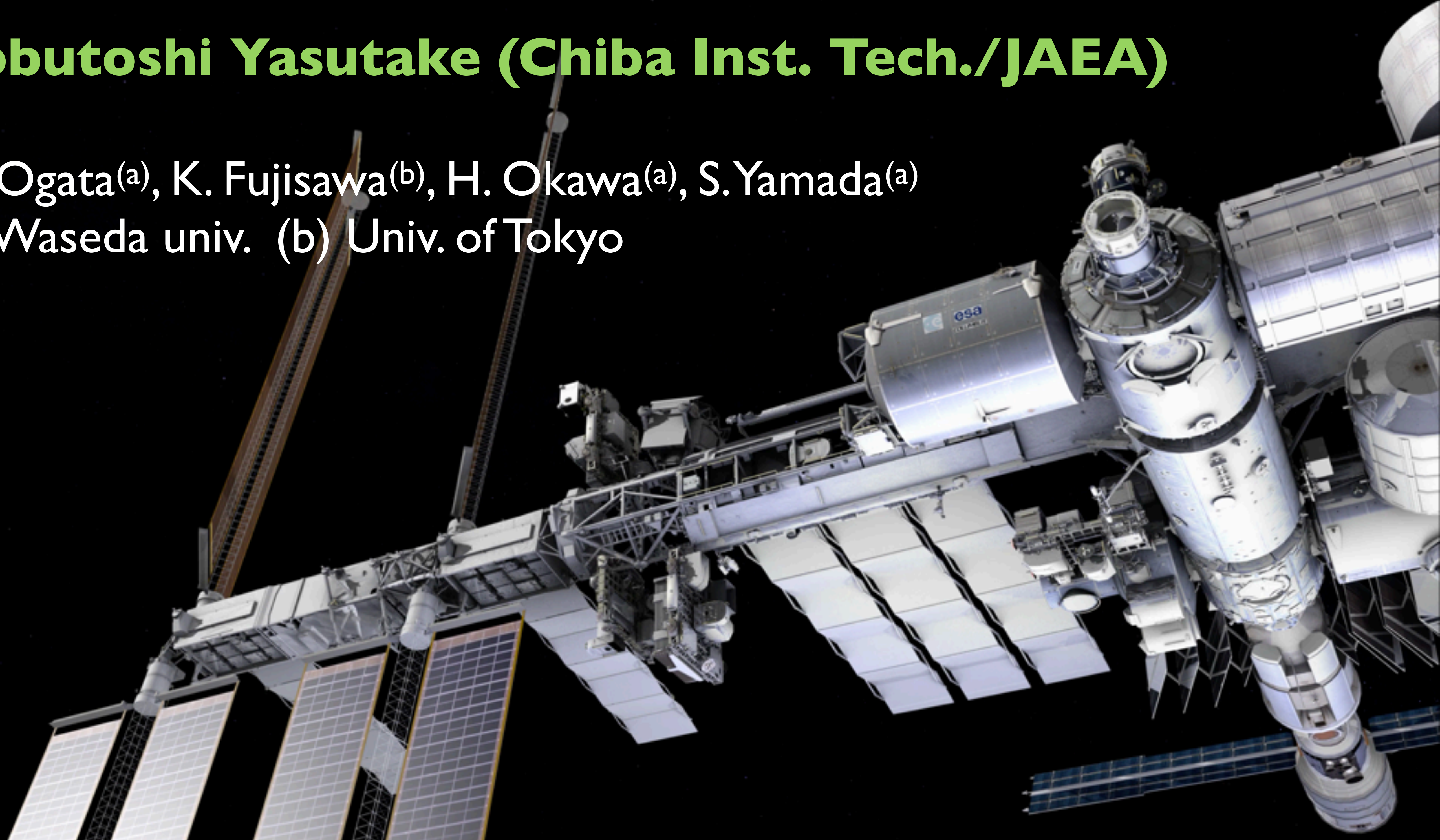
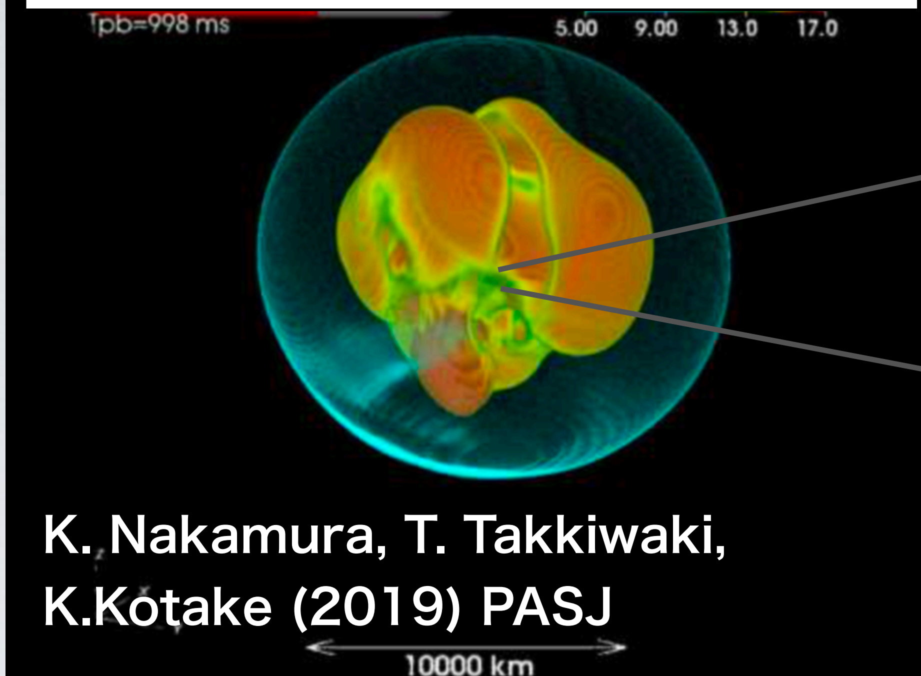


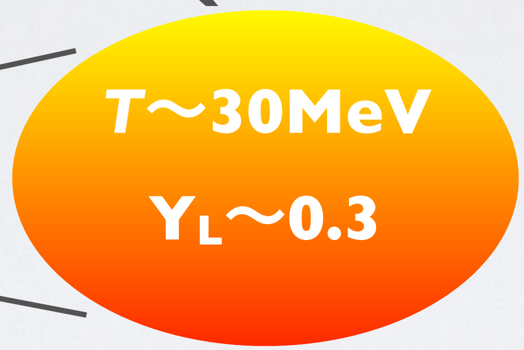
A big problem in astrophysics
There is no scheme of time evolution for *``deformed stars”* in quasi-equilibrium !!

Supernovae
dynamical simulation(3D)



K. Nakamura, T. Takkiwaki, K.Kotake (2019) PASJ

PNSs(R〜50km)



?

evolution (1D)

Heyney method (1964) in GR

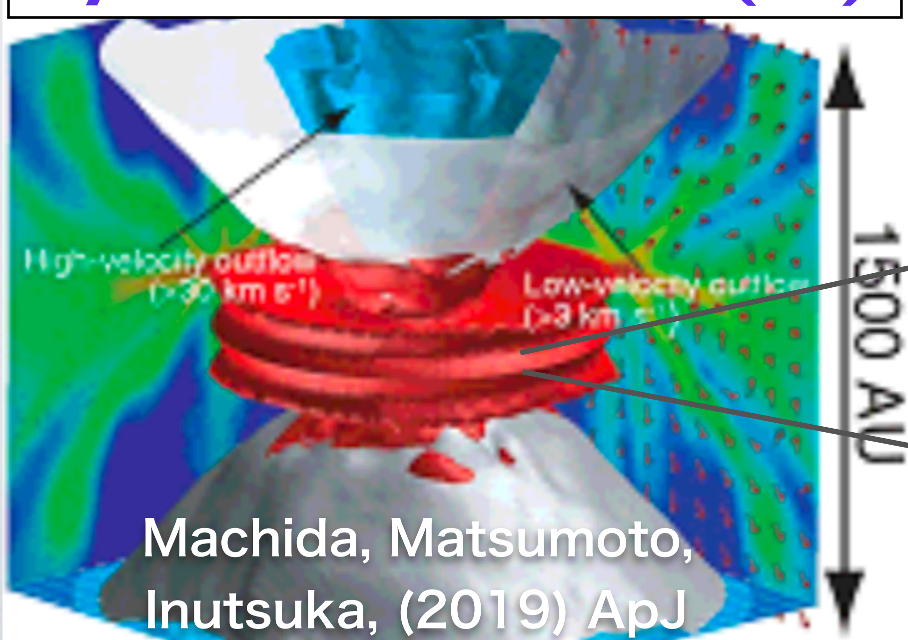
NSs(R〜10km)

T〜0 MeV

Yν〜0

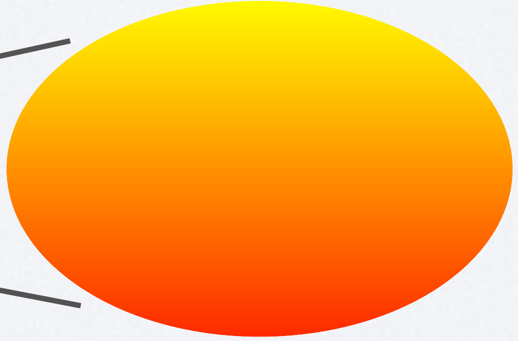
C. Jacobi, R. Dedekind, P.L. Dirichlet, B. Riemann, Chandrasekhar…

Star formations
dynamical simulation(3D)



Machida, Matsumoto, Inutsuka, (2019) ApJ

Proto-stars

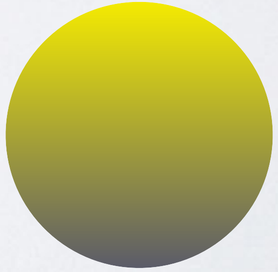


?

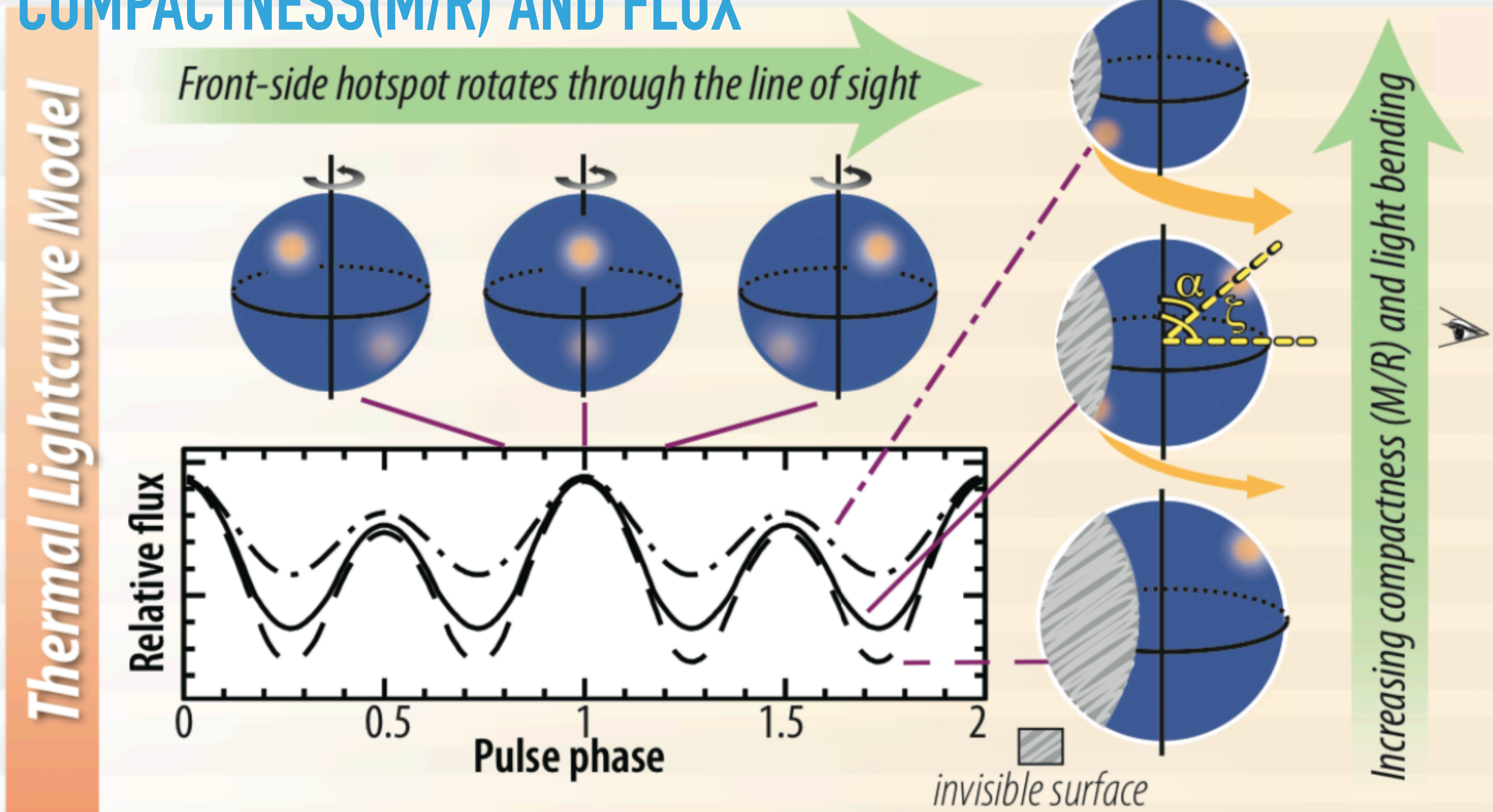
evolution (1D)

Heyney method (1964)

Massive Stars, Planets, ... etc

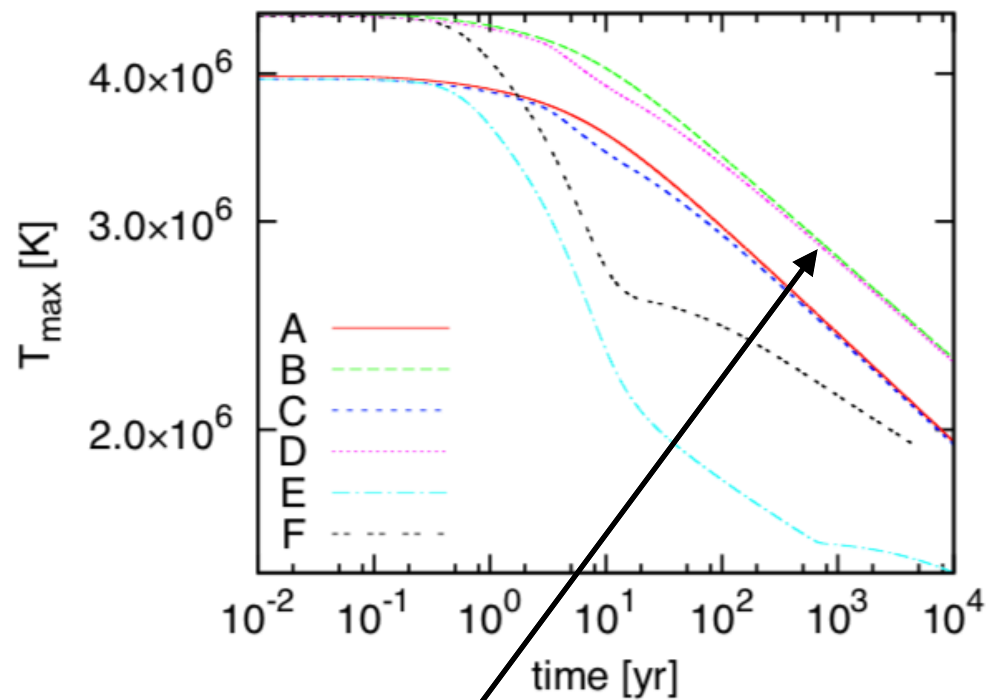


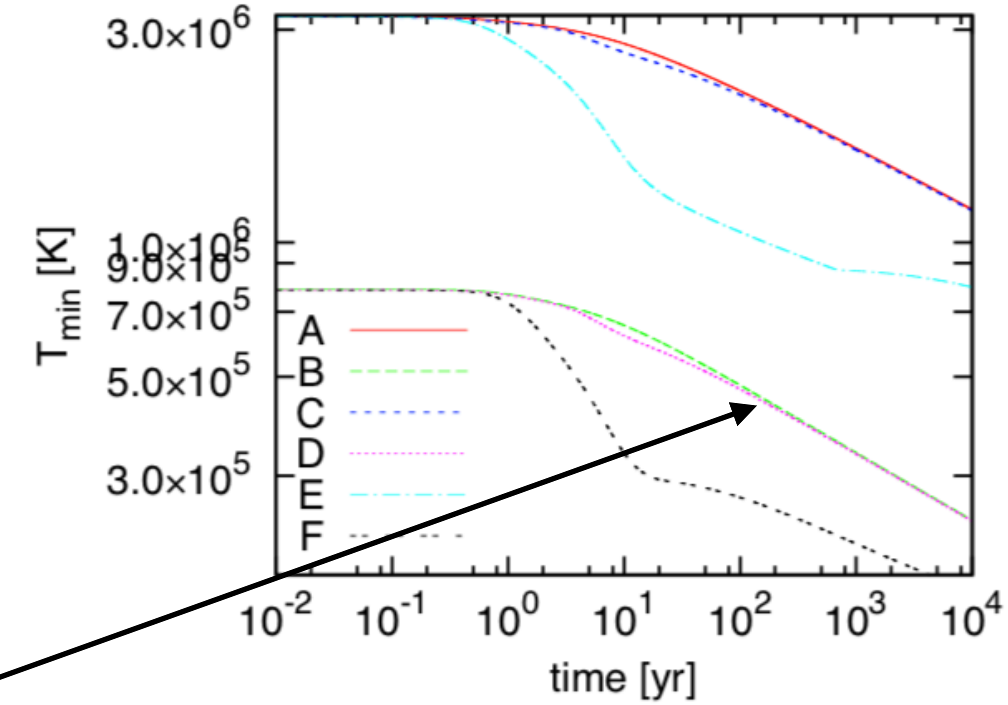
COMPACTNESS(M/R) AND FLUX



(C) KEITH GENDREAW@NASA

Thermal evolutions with magnetic field in 2,3D





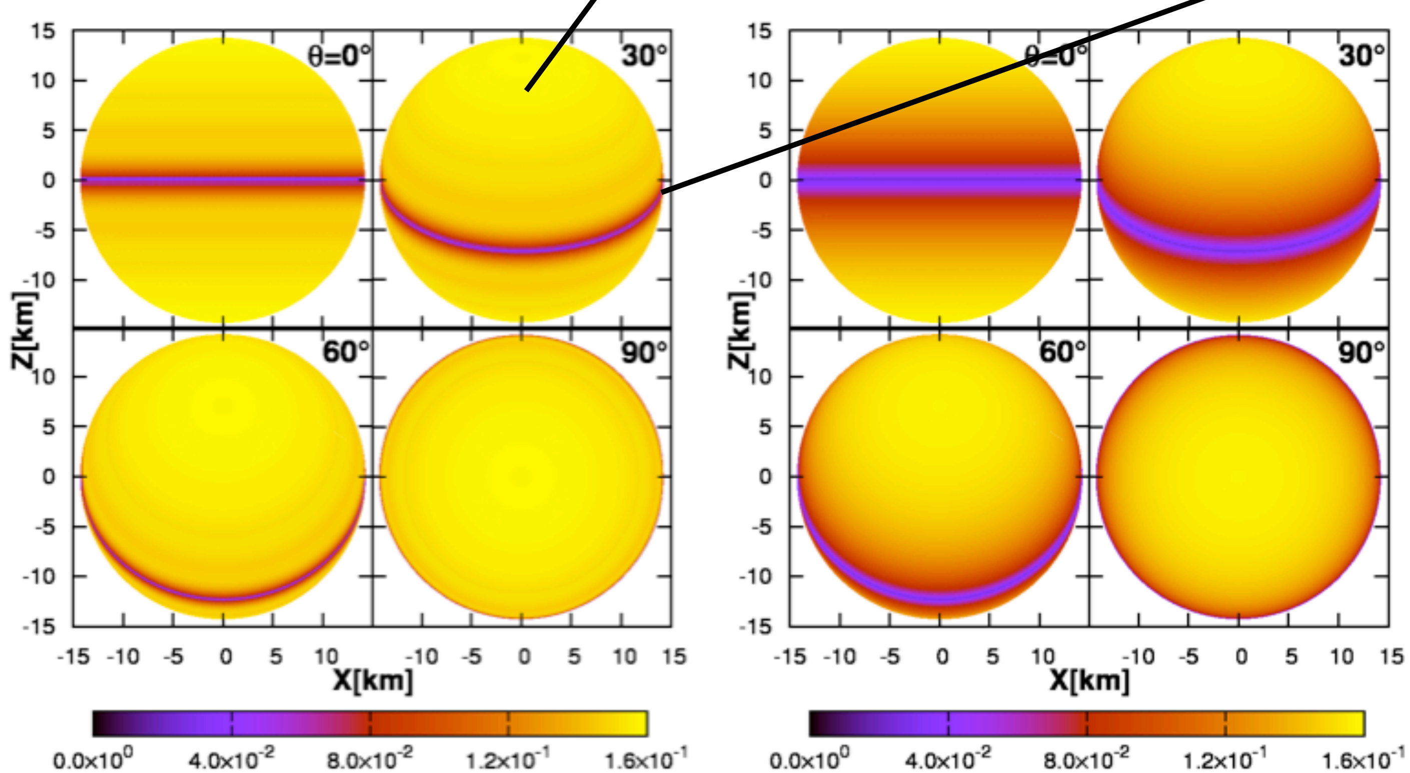
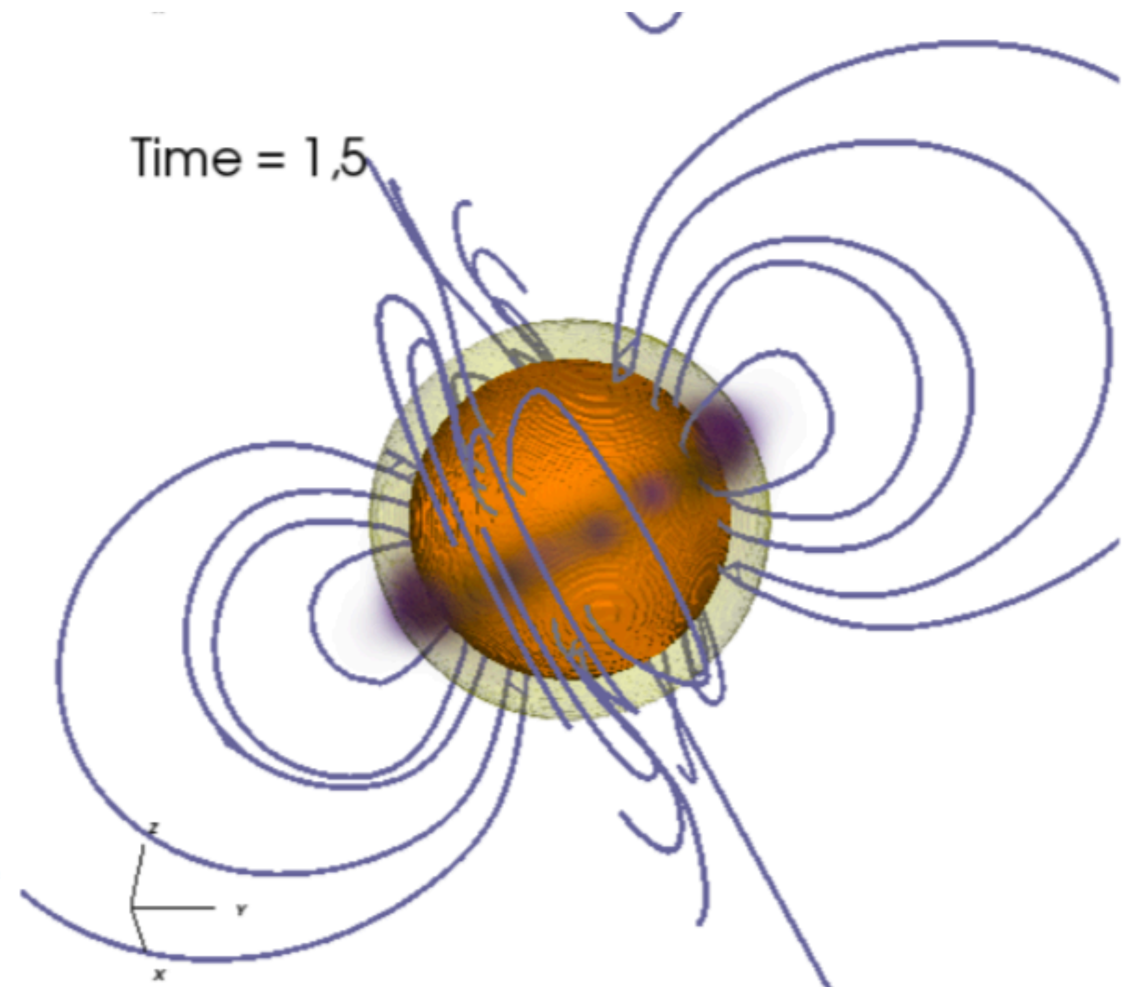


FIG. 7: (Color online) Temperature distribution for model "mSUK" after 10^4 years depended on the inclination angle θ . The unit of color contour is [keV].



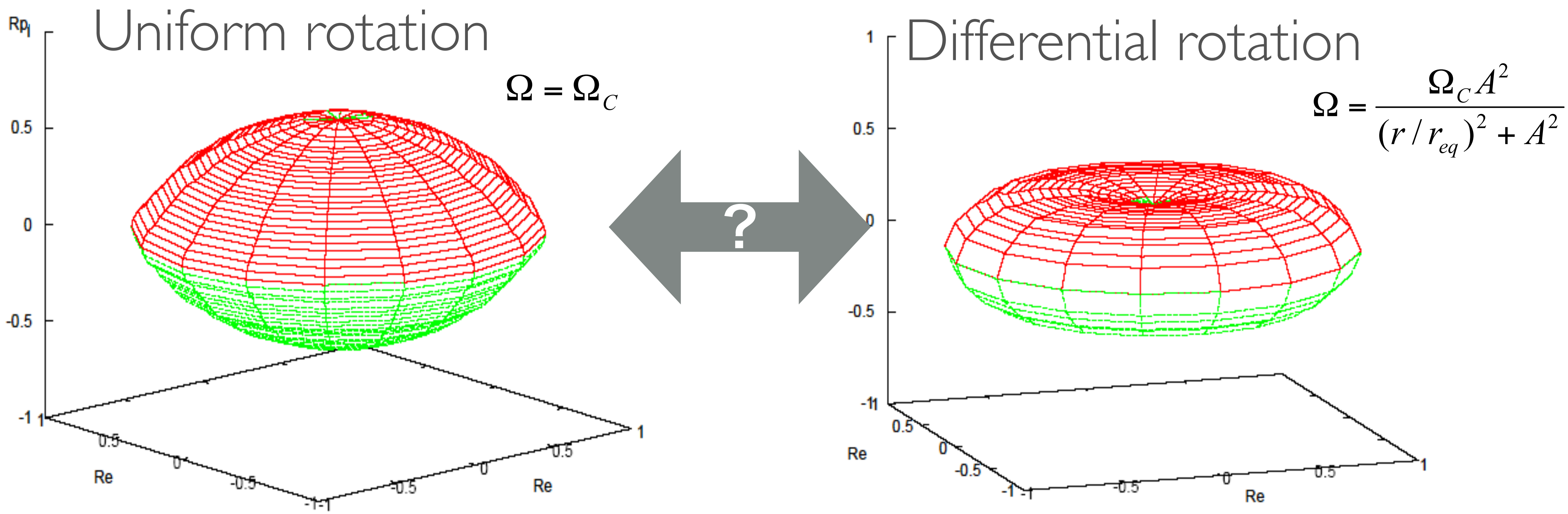
NY, Kotake, Kutsuna, Shigeyama (2014) PASJ

Vigano et al. (2018)

Without rotational law, no one can get rotating structures.

Rotating structures of neutron stars in fully GR formulation

NY, Hashimoto, Eriguchi, PTP(2005)



Komatsu, Eriguchi, Hachisu(1989), Cook, Shapiro, Teukolsky(1992)

Shaprio, Teukolsky, Nakamura(1994), Gourgoulhon et al.(1999)

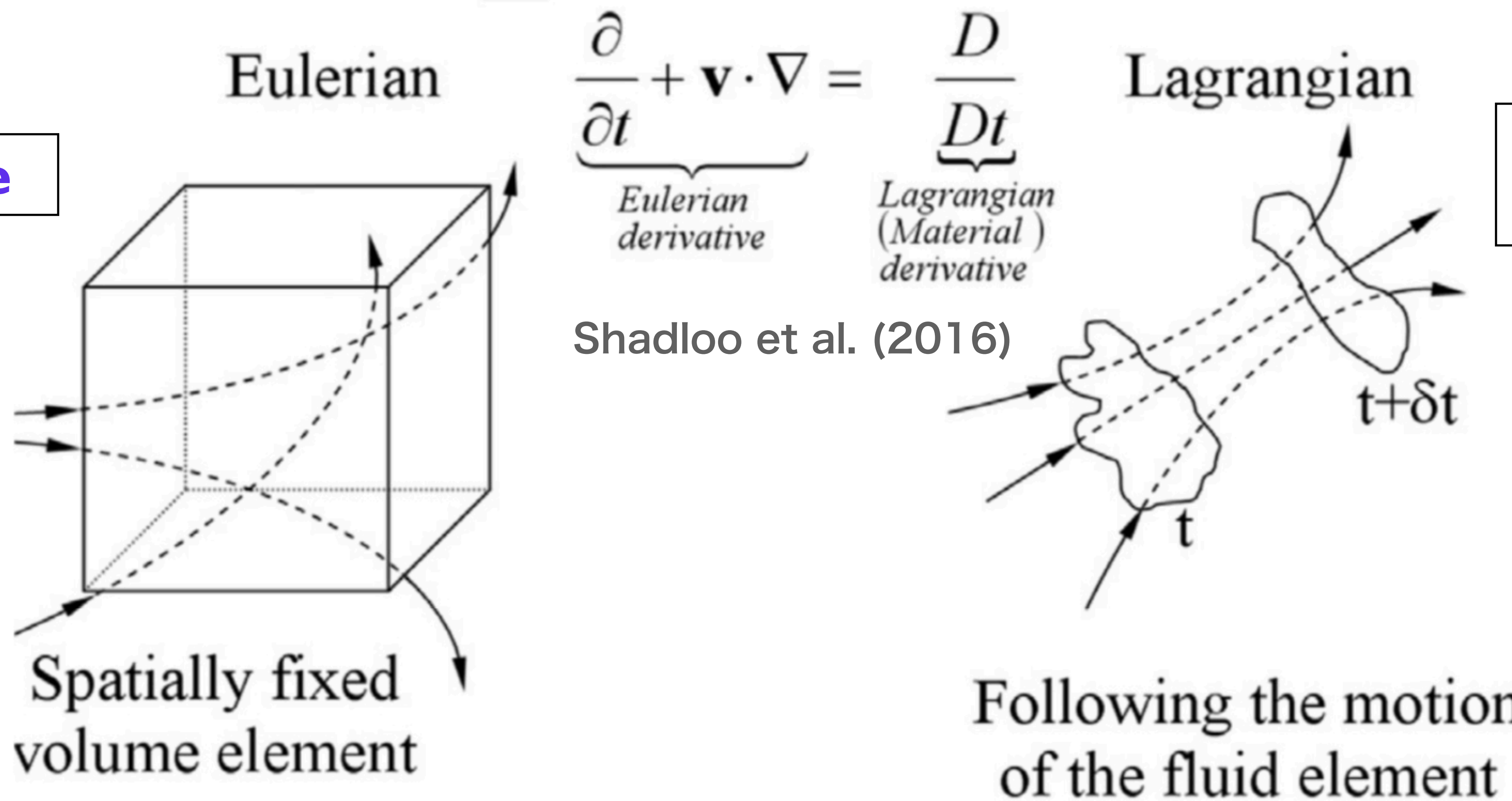
Usui, Uryu, Eriguchi(1999)….Uryu et al.(2017)

also in E. Zhou's talk

SPACE(+TIME) OR PARTICLES

supernovae

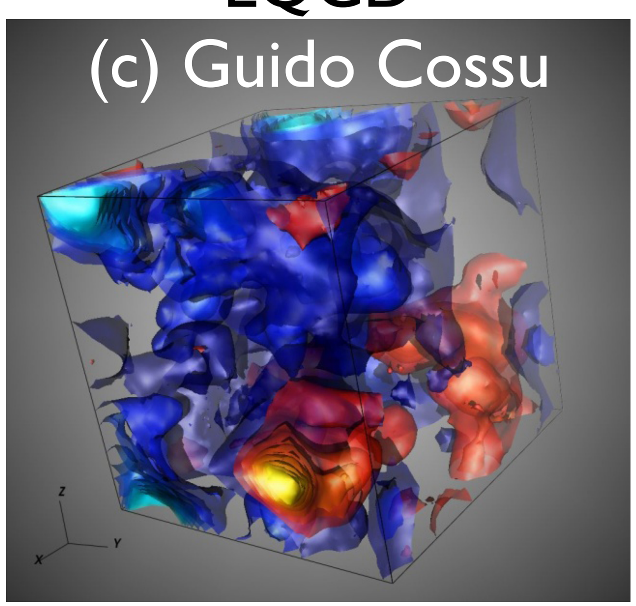
stellar evolutions



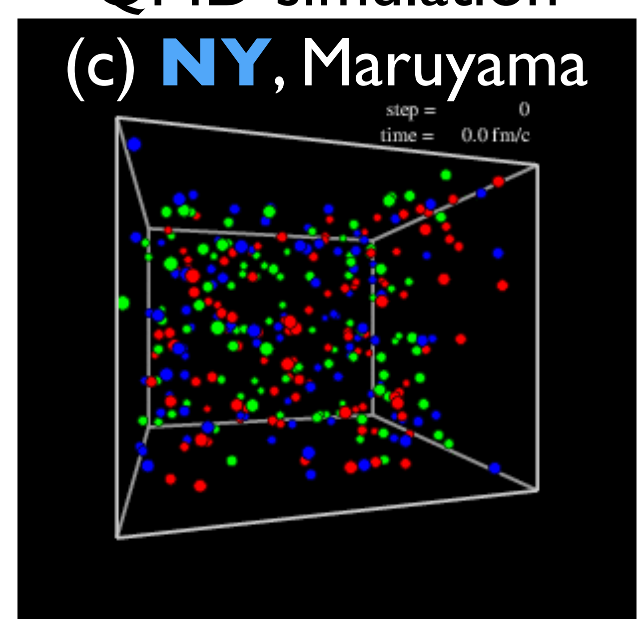
Shadloo et al. (2016)

c.f.)

LQCD



QMD simulation



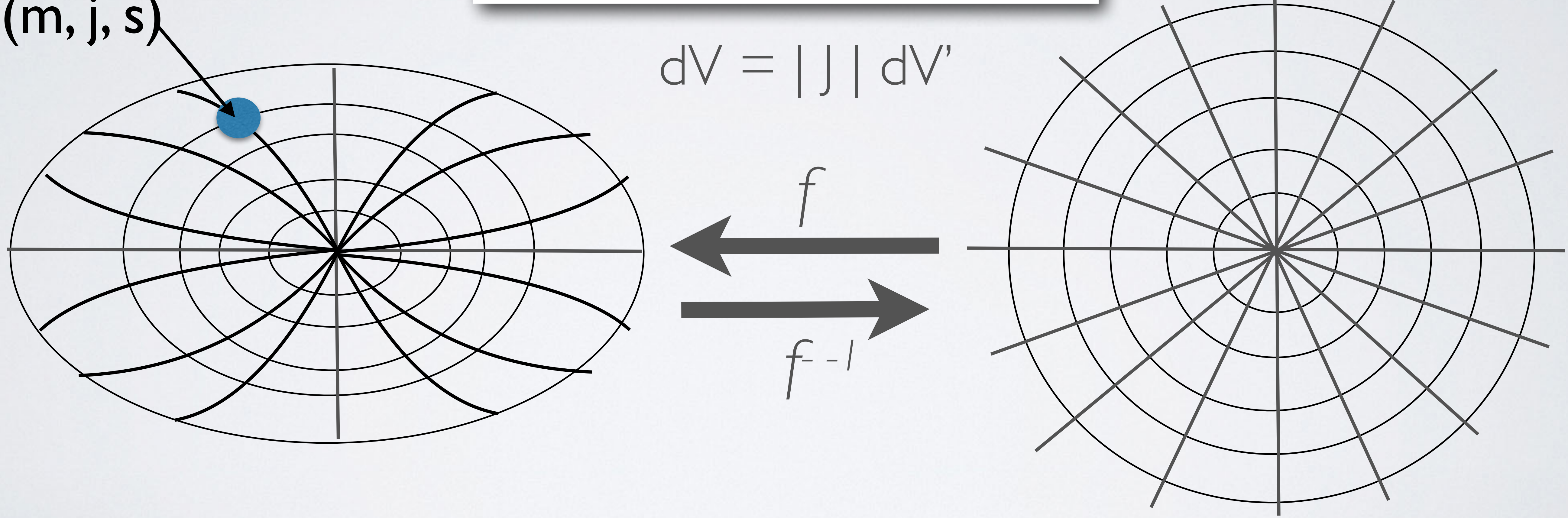
HOW TO SOLVE ?

IN LAGLANGIAN COORDINATE

We have to solve this eq. in **mass-coordinate** for **baroclinic cases with arbitrary angular momentum profiles**.

$$\nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3} \mathbf{e}_\varpi = 0,$$

(Newtonian case)



Problems

①**Uncertainty because of the gauge freedom. (Freedman&Shutz 1978)**

②**Numerical problems cf. Hour-glass problem**

SOME APPROACHES HYDROSTATIC EQUILIBRIA IN LAGRANGIAN COORDINATE

① **Simple discretization**
Ogata, Fujisawa, Okawa, **NY**, Yamada

$$\frac{\partial P}{\partial r} \to \frac{P_{i+1} - P_i}{r_{i+1} - r_i}$$

② **Spectral method**
Fujisawa, Ogata, Okawa, **NY**, Yamada

$$P = P(r,\theta) = \sum_i \alpha_{ij}\sigma_i(r)\sigma_j(\theta)$$

③ **GR case**
Okawa, Ogata, Fujisawa, **NY**, Yamada

$$\phi(r,\theta) \to f_0(r,\theta), f_1(r,\theta), f_2(r,\theta), \omega(r,\theta)$$

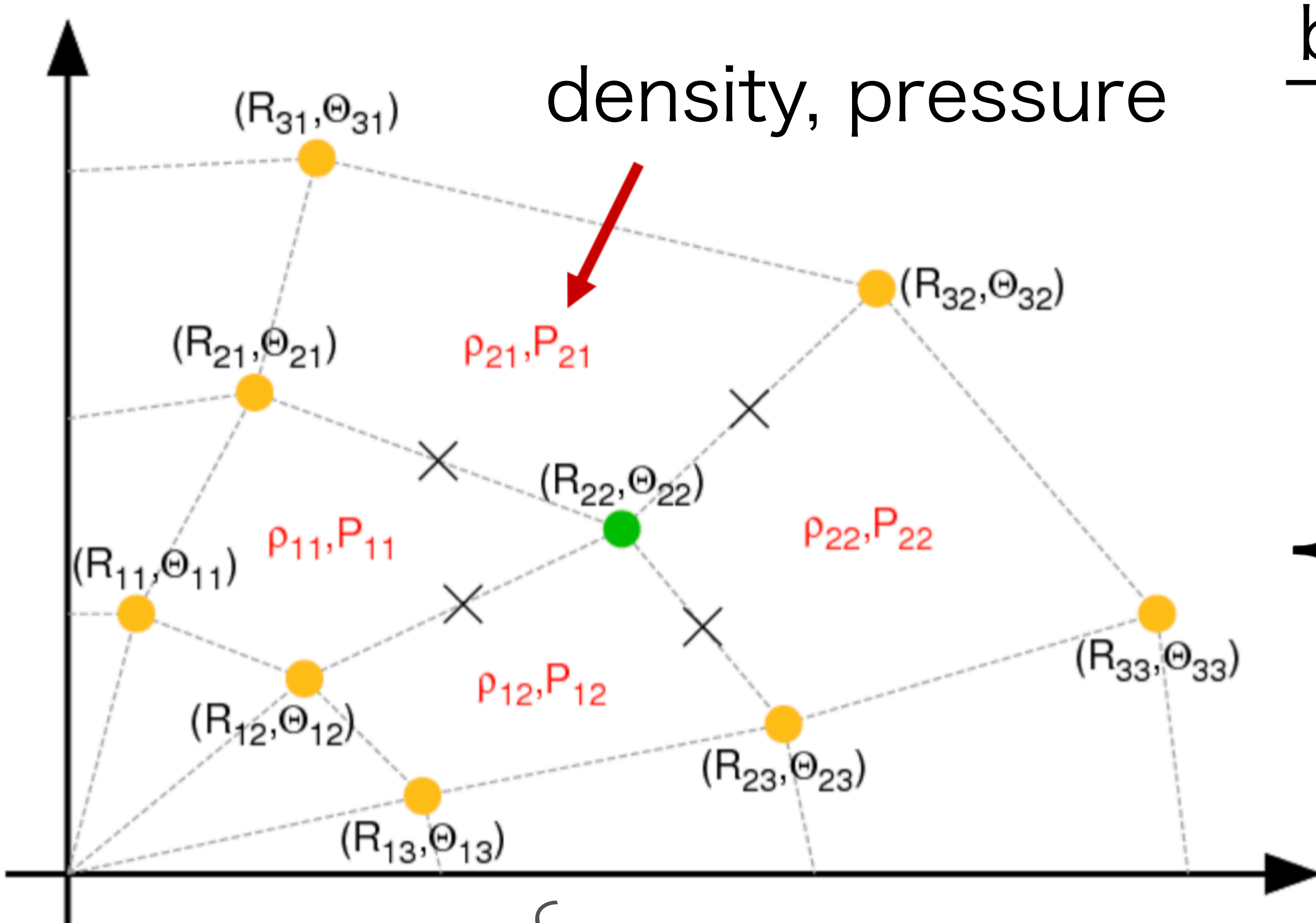
gravitational potential — Metric

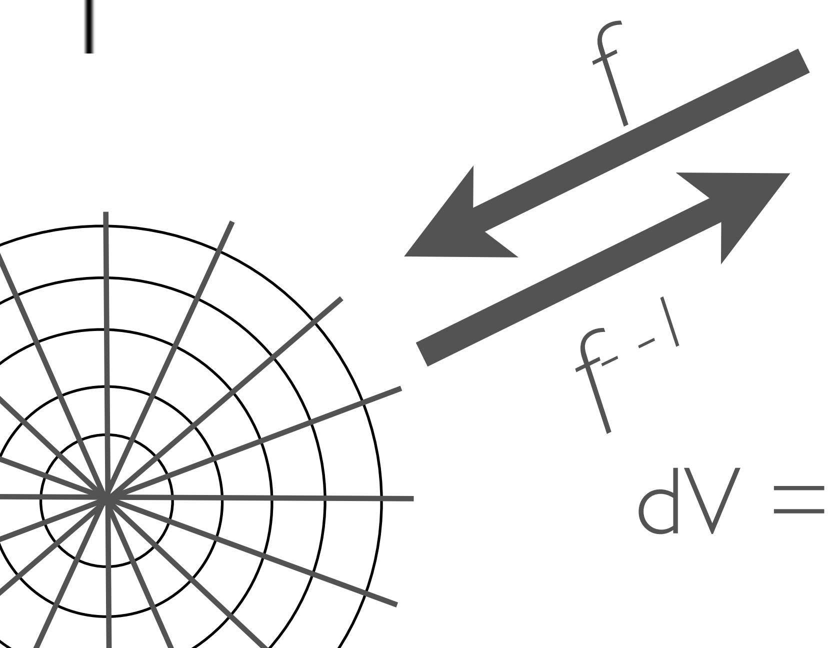
④ **Neural Network**
NY, Ogata, Fujisawa, Okawa, Yamada

① Simple discretization

Ogata, Fujisawa, Okawa, **NY**, Yamada

$$\frac{\partial P}{\partial r} \rightarrow \frac{P_{i+1} - P_i}{r_{i+1} - r_i}$$



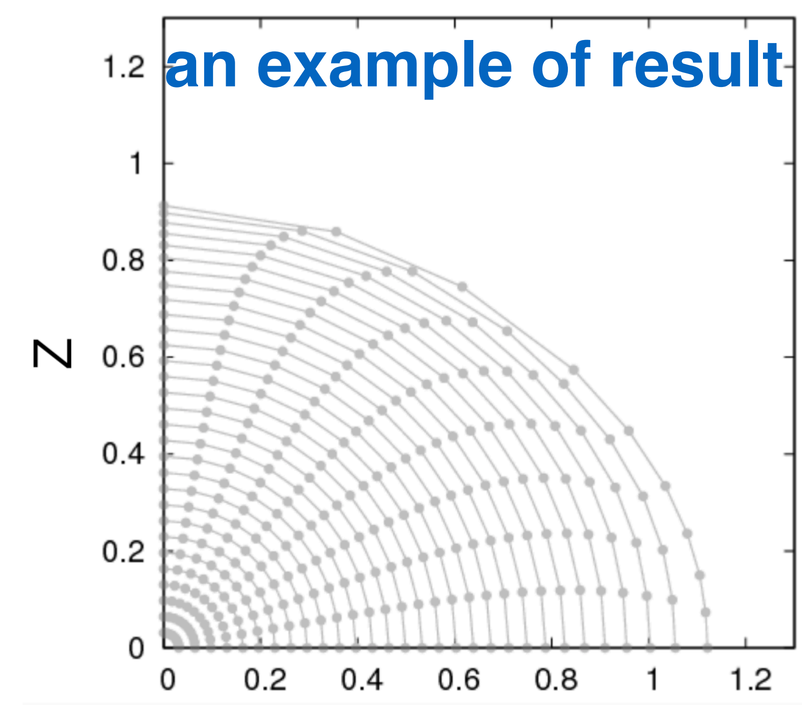


dV = | J | dV'

balance equation

$$\frac{1}{\rho}\boldsymbol{\nabla}P - \boldsymbol{\nabla}\phi + \omega^2 (R\sin\theta)\boldsymbol{e_{\varpi}} = \boldsymbol{0}$$

$$\begin{cases} F_{\mathrm{R}} = \dfrac{1}{\rho}\dfrac{\partial \mathrm{P}}{\partial R} - \dfrac{\partial \phi}{\partial R} + \omega^2 (R\sin\Theta)\sin\Theta = 0 \\ F_{\Theta} = \dfrac{1}{\rho}\dfrac{\partial \mathrm{P}}{\mathrm{R}\partial\Theta} - \dfrac{\partial \phi}{\mathrm{R}\partial\Theta} - \omega^2 (R\sin\Theta)\cos\Theta = 0 \end{cases}$$



② Spectral method

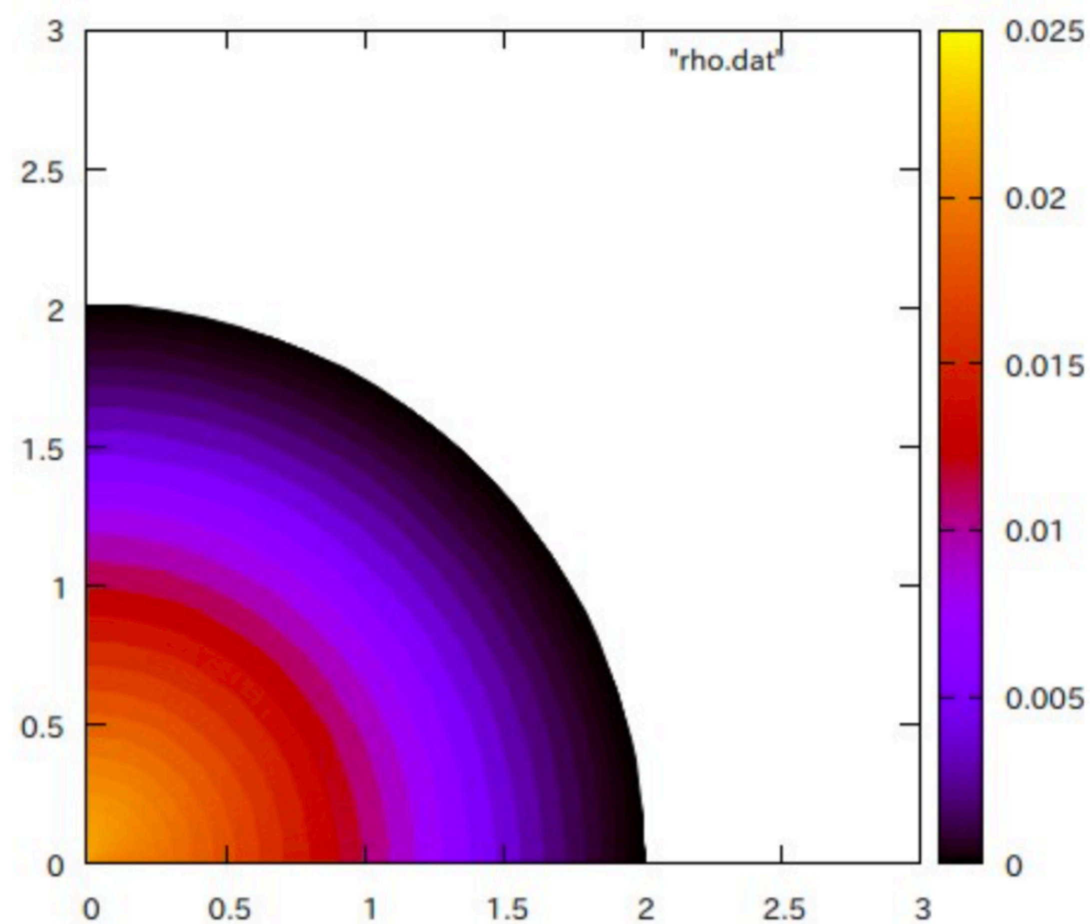
Fujisawa, Ogata, Okawa, **NY**, Yamada

Chebyshev's expansions

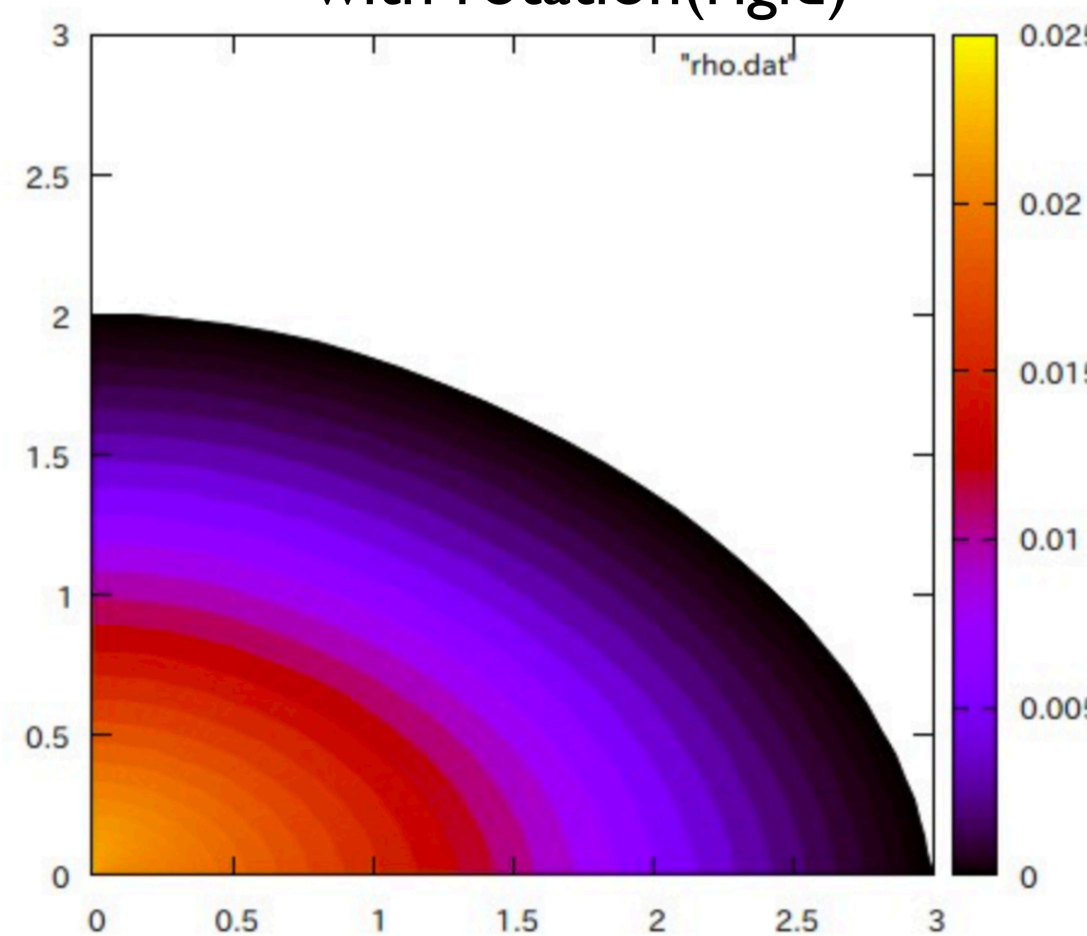
$$X(r,\nu) = \sum_{n=0}^{N_{\max}} \sum_{\ell=0}^{L_{\max}} a_{n\ell} T_n(r) T_\ell(\nu) \qquad X = r, \cos\theta, P, \phi \dots$$

$$\int\int T_n(r) T_\ell(\nu) \left\{ \nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3} \mathbf{e}_\varpi \right\} dr\, d\nu = 0$$

without rotation



with rotation(rigid)



We are ready to conduct evolution calculations !

③ GR case

Okawa, Ogata, Fujisawa, **NY**, Yamada

$\phi(r,\theta) \rightarrow f_0(r,\theta), f_1(r,\theta), f_2(r,\theta), \omega(r,\theta)$

gravitational potential Metric

Under the axial symmetric metric

$$ds^2 = -f_0^2(r,\theta)dt^2 + f_1^{-2}(r,\theta)\left(dr^2 + r^2 d\theta^2\right) + f_2^{-2}(r,\theta) r^2 \sin^2\theta \left(d\varphi - \omega(r,\theta)dt\right)^2$$

Highly non-linear equations (Not integral form)

$E_{\mu\nu}(\equiv G_{\mu\nu} - \kappa_G T_{\mu\nu})$ Einstein eqs.$(E_{tt}, E_{\theta\theta}, E_{\varphi\varphi}, E_{t\varphi})$,

Conservation $(\nabla_\mu T^\mu_r, \nabla_\mu T^\mu_\theta)$ and EOS.

Example

$$\begin{aligned}
\mathbf{G_{tt}} &= a_1^{(tt)}\left(\frac{\partial^2 f_0}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 f_0}{\partial \theta^2}\right) + a_2^{(tt)}\left(\frac{\partial^2 f_1}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 f_1}{\partial \theta^2}\right) + a_3^{(tt)}\left(\frac{\partial^2 f_2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 f_2}{\partial \theta^2}\right) \\
&+ a_4^{(tt)}\left(\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \theta^2}\right) + a_5^{(tt)}\left(\frac{\partial f_0}{\partial r}\frac{\partial \omega}{\partial r} + \frac{1}{r^2}\frac{\partial f_0}{\partial \theta}\frac{\partial \omega}{\partial \theta}\right) \\
&+ a_6^{(tt)}\left(\frac{\partial f_2}{\partial r}\frac{\partial \omega}{\partial r} + + \frac{1}{r^2}\frac{\partial f_2}{\partial \theta}\frac{\partial \omega}{\partial \theta}\right) + a_7^{(tt)}\frac{\partial f_0}{\partial r} + a_8^{(tt)}\frac{\partial f_1}{\partial r} + a_9^{(tt)}\frac{\partial f_2}{\partial r} + a_{10}^{(tt)}\frac{\partial \omega}{\partial r} \\
&+ a_{11}^{(tt)}\frac{\partial f_1}{\partial \theta} + a_{12}^{(tt)}\frac{\partial f_2}{\partial \theta} + a_{13}^{(tt)}\frac{\partial \omega}{\partial \theta}
\end{aligned}$$

$$\mathbf{T_{tt}} = \frac{\rho f_0^4 f_2^4 + \rho\omega^2 r^4 \sin^4\theta\,(\Omega-\omega)^2 + f_0^2 f_2^2 r^2 \sin^2\theta\left(\Omega^2 P + 2\Omega\omega\rho - 2\omega^2\rho\right)}{f_0^2 f_2^4 + f_2^2 r^2 \sin^2\theta\,(\Omega-\omega)^2}$$

W4 methods

A method to solve non-linear equations (arXiv1809.04495)

H.Okawa, K.Fujisawa, R.Hirai, Y.Yamamoto, NY, H.Nagakura, Yamada

Naive Newton-Raphon methods does not work for multi-dimensional hydrostatic equilibria in Lagrange coordinate [Ref. Friedman & Schutz 1978]. We, then, introduce a new method, named as ``W4 method'', which is based on the fixed point theorem.

$$\mathbf{F} = \nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3}\mathbf{e}_\varpi$$

$$\ddot{x}_1 + A_1 \dot{x}_1 + B_1 F = 0$$

$$\ddot{x}_2 + A_2 \dot{x}_2 + B_2 F = 0$$

$$\vdots$$

Here, $F=F(x_1,x_2,\ldots y_1,y_2,\ldots)$.

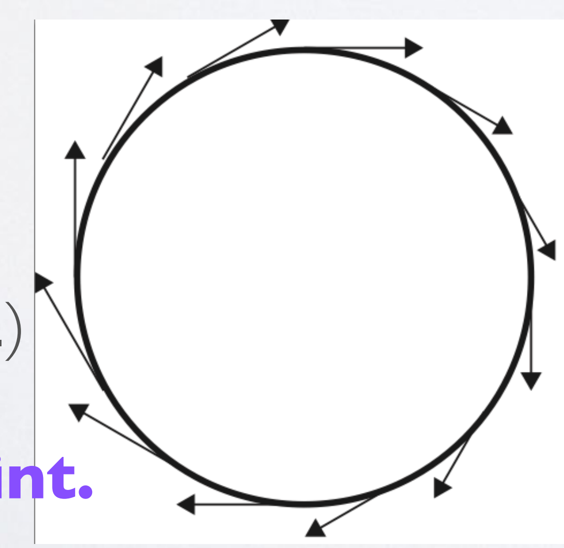
In Lagrange scheme, the variables are ``coordinates'' of nodes, which have mass, entropy, and angular momenta.

【tangent vector space on S^1】

Find a fixed point⇔

Find a solution for
1st order differential eq.
(SOR, Newton method etc.)

→**Sometime, there is no fixed point.**

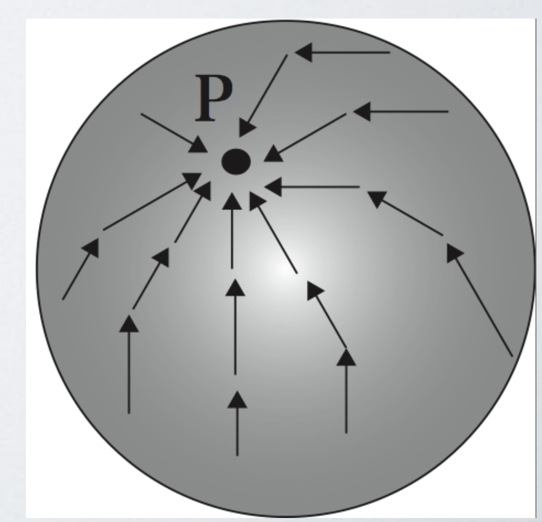


【 tangent vector space on S^2】

Find a fixed point⇔

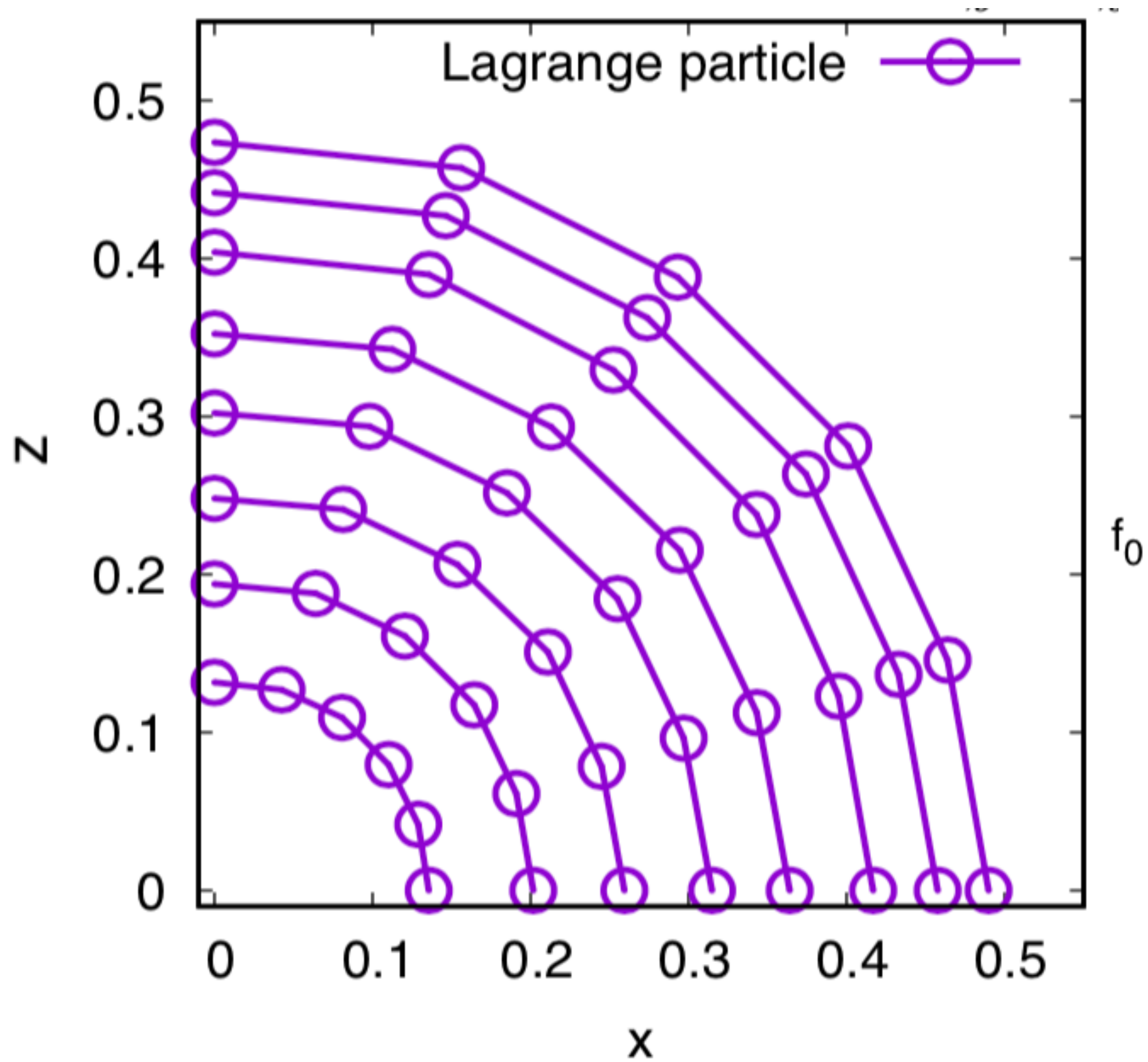
Find a solution for
2nd order differential eq.
(W4, MD calculation etc.)

→ **There must be fixed points.**

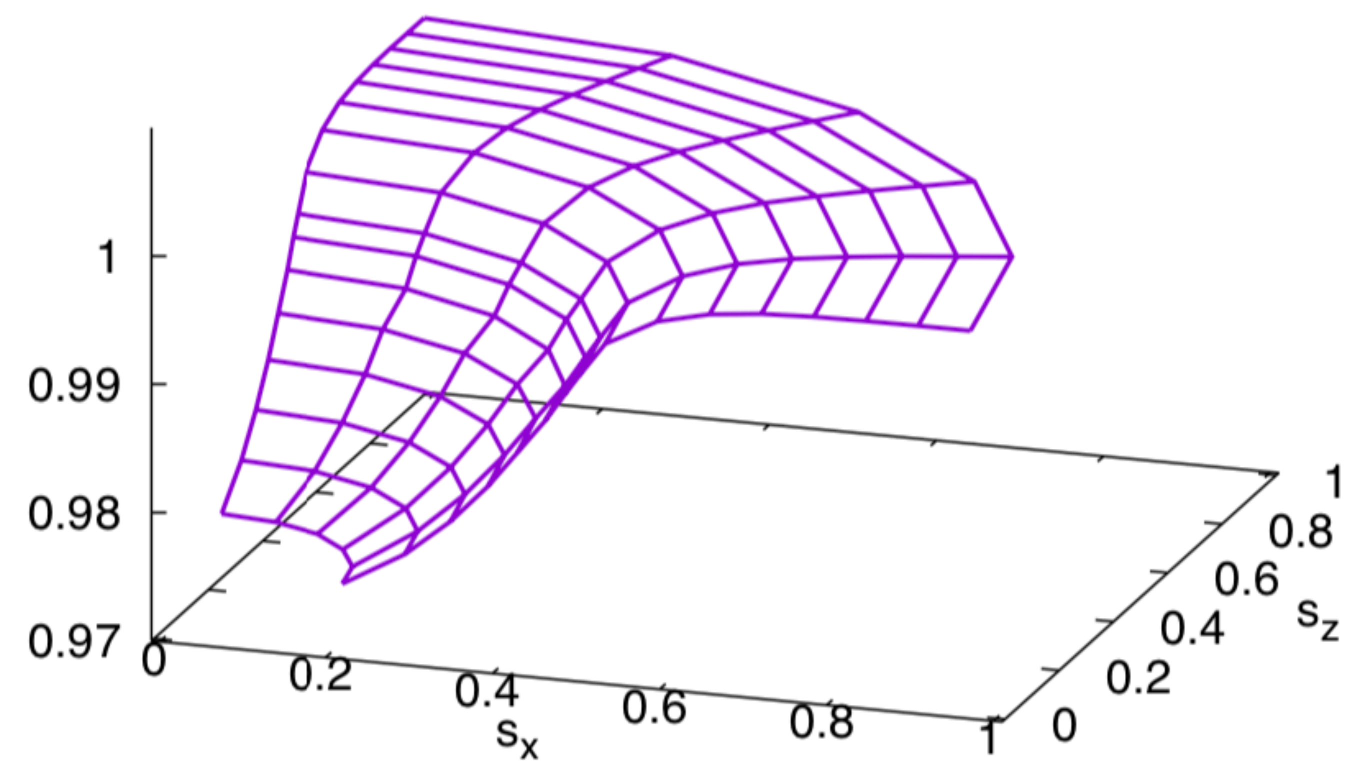


Current status of GR case

with slow rotation(rigid)



metric



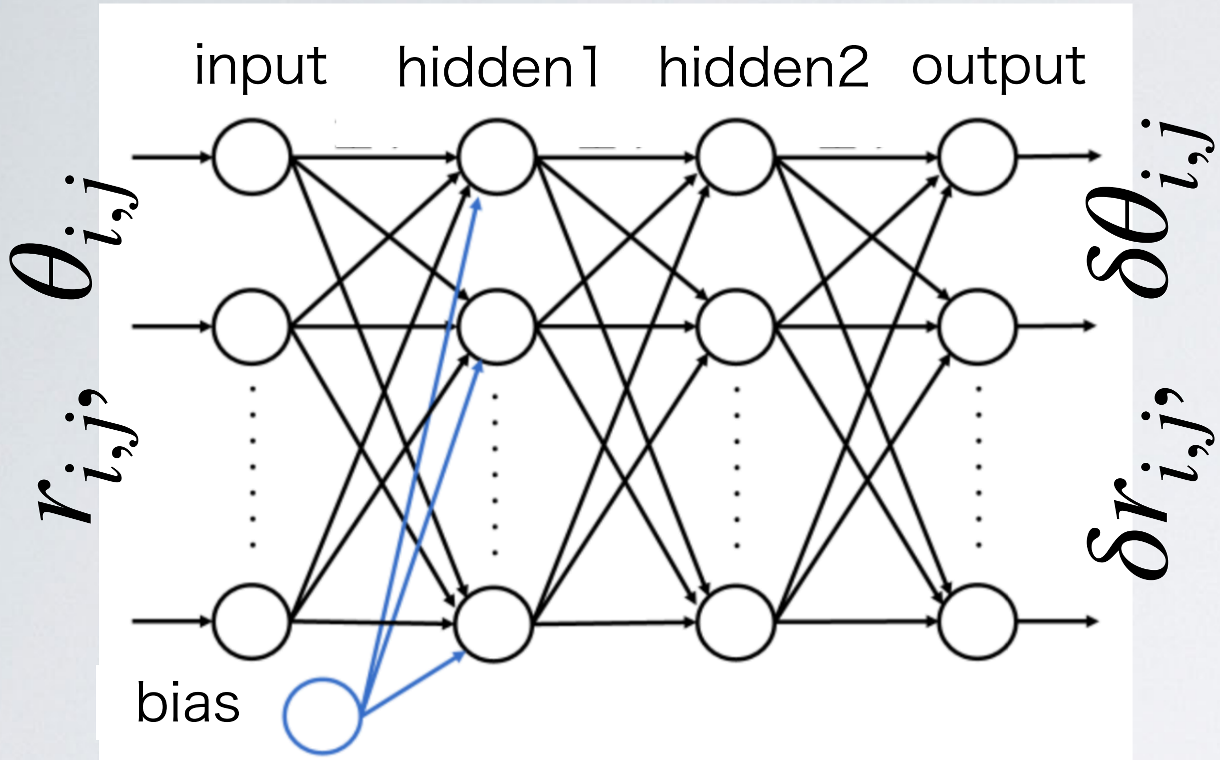
We already have some solutions with rotation.

→ Adaptable to proto-neutron stars: **τ〜20s**.

x10²

cf.) dynamical simulations for supernovae with super computers: **τ〜0.2s**.

④ NEURAL NETWORK



Equilibrium eq.

↓

Optimization problem

$$E = \sum_{i,j} \left(\nabla P + \rho \nabla \phi - \frac{\rho j^2}{\varpi^3} \mathbf{e}_\varpi \right)^2$$

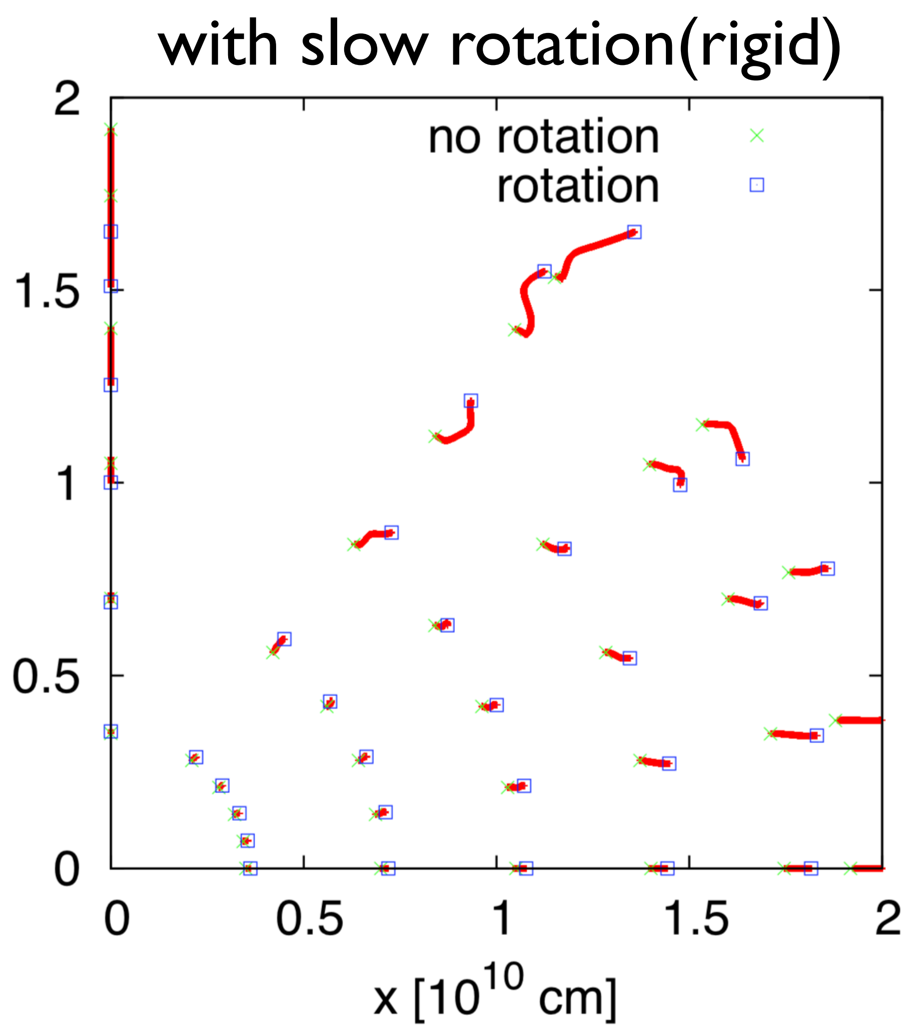
Why Neural network?

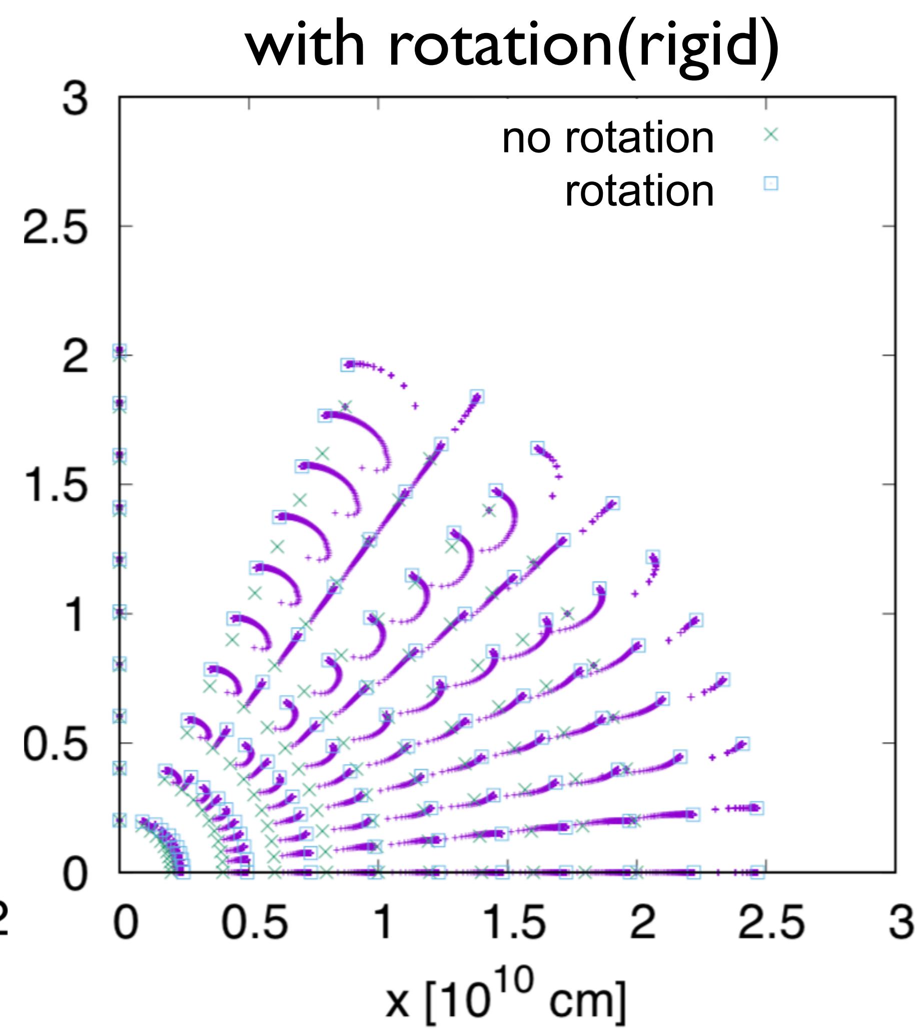
① **Universal approximation theorem**
→ **Useful for "non-linear equations".** **cf. Sign problem (K. Kashiwa's talk)**

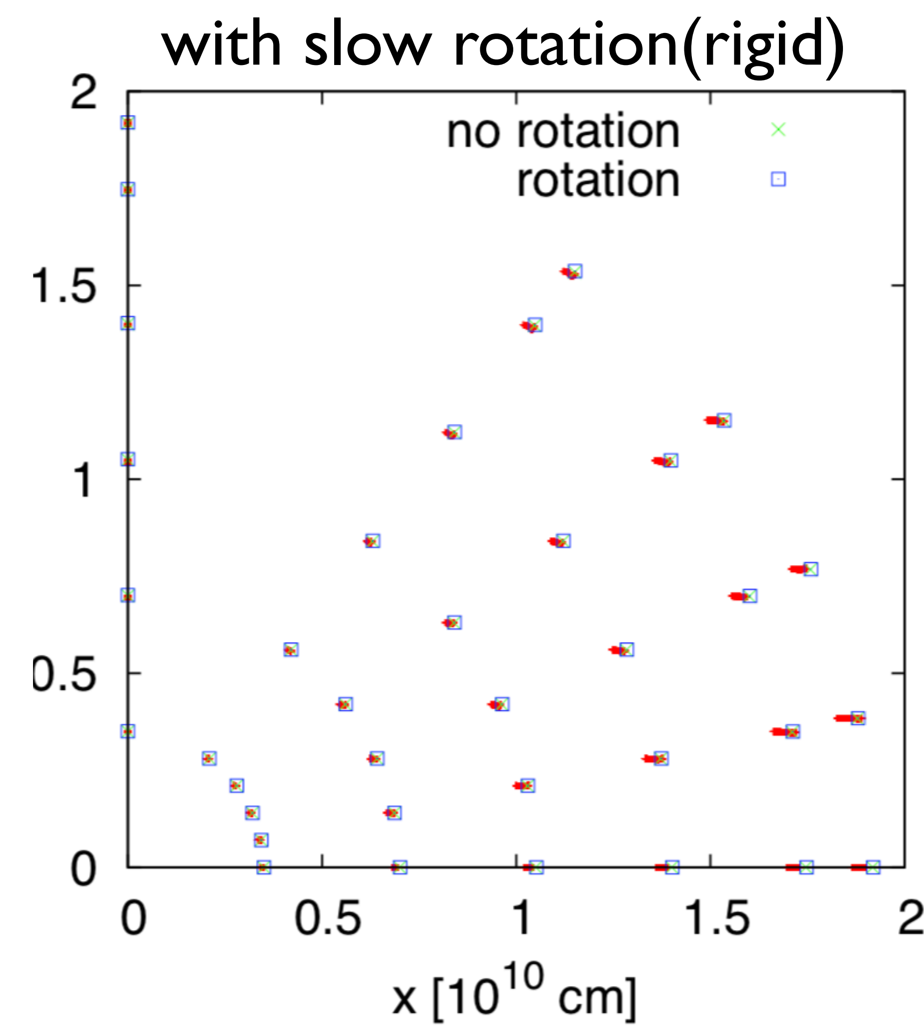
② **Development of Hardware and Software** **cf. cuDNN(Numerical Libraries) tensorflow, Keras, cafe… etc.**

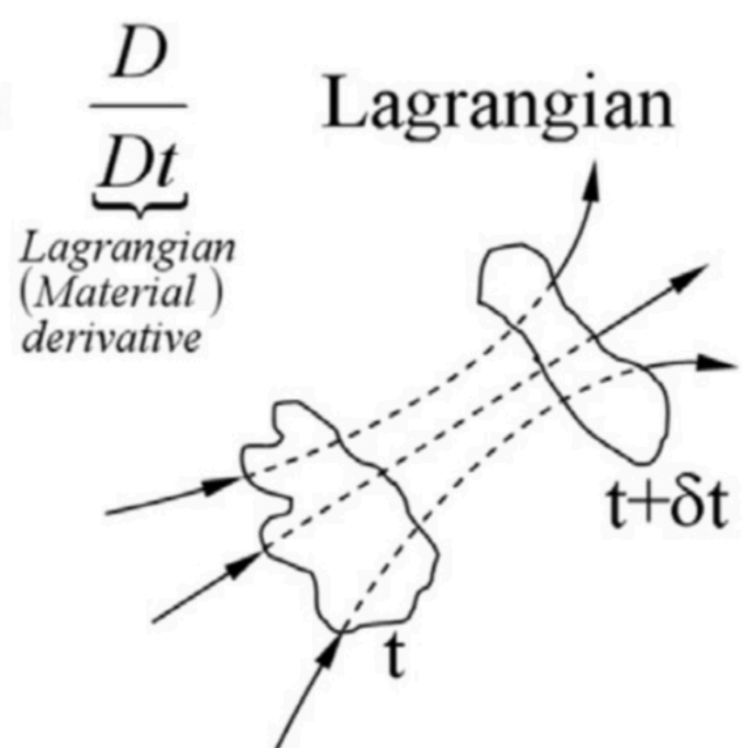
③ **Future high computings method such as quantum computers.** **cf. D-wave,..etc.**

EXAMPLES OF ROTATING STARS









Blue or **red** paths show trajectories (**vectors in left figure**) of Lagrangian mass elements, which are searched by **NN**.

↓

These trajectories strongly depends on **NN**, and initial guess.

Accuracy is not so good. Optimization doesn't equal to solve equations. High numerical costs are also problems.

SUMMARY&DISCUSSION

It is the most important for stellar evolutions to obtain (baroclinic) hydrostatic equilibria with arbitral angular-momentum profiles.
But they are highly non-linear balance equation.

We have introduced 4 approaches in this talk.

1. **Simple discretization**
2. **Spectral method**
3. **GR case**
4. **Neural Network**

Optimization ≠ Solving non-linear equations

NN W4 method (arXiv: 1809.04495)

Now, our topic moves to **“stellar evolutions in 2D,3D”**.