Tidal deformability for neutron and hyperon stars

Bharat Kumar

Center for Computational Sciences, University of Tsukuba, Japan

QCS2019(September 26-29, 2019)

Introduction

- Understanding the nature of dense neutron-rich matter is a major thrust of current research in both nuclear physics and astrophysics.
- Two neutron stars (NS) rotate about a common centre of mass. While rotating, they emit gravitational waves. In this process, the orbits lose energy and get close and closer, which is called inspiralling.
- When they approach, they come under the influence of each other and get distorted. The after effect is that tides are raised exactly the same way as tides are created on Earth due to Moon.
- The newly formed tides pick the energy out of the orbit resulting in the speedy motion of the inspiral. This can be detected and measured in the form of gravitational waves.
- Larger are the size of the neutron stars, bigger are the tides formed.
- From the equation of state (EoS) we can determine the size of NS alongwith its tidal deformation.

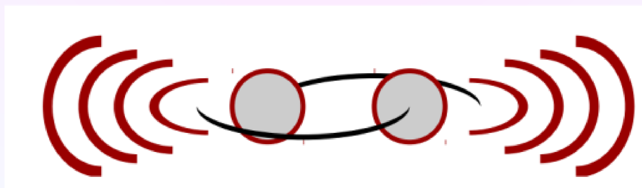


Figure: Compact stars; Credit: P. Landry

Tidal Love numbers

- In 1911, the mathematician A. E. H. Love introduced the dimensionless parameter in Newtonian theory. It is related to the tidal deformation of the Earth which is because of the gravitational attraction between the Moon and the Sun.
- In Newtonian gravity, the tidal Love number is a constant of proportionality between the tidal field applied to the body and the resulting multipole moment of its mass distribution.
- In case of quadrupole, the tidal field is characterized by tidal moment

$$\mathscr{E}_{ij} = -\partial u_{ext} = -\partial_{ij}\left(\frac{M_B}{r_{AB}}\right)$$

in which the external potential is generated by the rest of the universe.

- In the presence of a tidal field, the quadrupole moment is proportional to the tidal field

$$Q_{ij} = -\frac{2}{3}k_2 R^5 \mathscr{E}_{ij} = -\lambda \mathscr{E}_{ij}$$

where, k_2 and λ are the dimensionless tidal Love number, and the tidal deformability of the star which depend on the EoS. R is the radius of the star.

- In the absence of a tidal field the body would be spherical, and its quadrupole moment would vanish.
- In general relativity, two types of Love numbers: an electric type of Love number k_{el} that has direct analogy with the Newtonian Love number (the gravitational fields generated by masses), and a magnetic-type Love number k_{mag} (gravitational field generated by motion of masses) that has no analogue in Newtonian gravity [1].

[1] T. Binnington and E. Poisson, Phys. Rev. D **80**, 084018 (2009).

Equation of states

Lagrangian density of the baryon-meson interaction

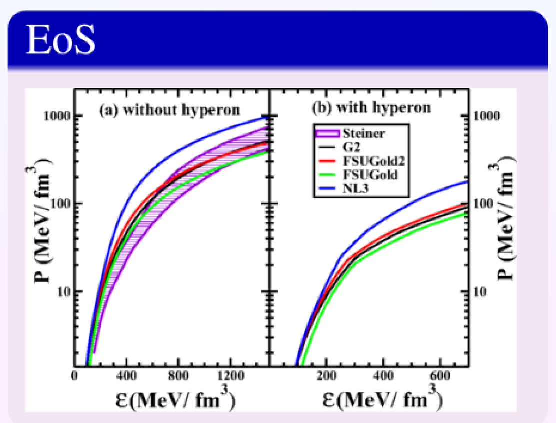
$$\mathscr{L} = \sum_B \overline{\psi}_B\left(i\gamma^\mu D_\mu - m_B + g_{\sigma B}\sigma\right)\psi_B + \frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2\left(\frac{1}{2} + \frac{g_3}{3!}\frac{g_\sigma\sigma}{m_B} + \frac{g_4}{4!}\frac{g_\sigma^2\sigma^2}{m_B^2}\right) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu\left(1+\eta_1\frac{g_\sigma\sigma}{m_B} + \frac{\eta_2}{2}\frac{g_\sigma^2\sigma^2}{m_B^2}\right) - \frac{1}{4}R_{\mu\nu}^a R^{\mu\nu a} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu}\left(1+\eta_\rho\frac{g_\sigma\sigma}{m_B}\right)$$
$$+\frac{1}{4!}\zeta_0\left(g_\omega\omega_\mu\omega^\mu\right)^2 + \Lambda_v(g_\rho^2\rho_\mu^a\rho^{\mu a})(g_\omega^2\omega_\mu\omega^\mu) + \sum_l \overline{\psi}_l\left(i\gamma^\mu\partial_\mu - m_l\right)\psi_l.$$

Energy and pressure density

$$\mathscr{E} = \sum_B \frac{2}{(2\pi)^3}\int_0^{k_B} d^3k E_B^*(k) + m_\sigma^2\sigma_0^2\left(\frac{1}{2} + \frac{g_3}{3!}\frac{g_\sigma\sigma_0}{m_B} + \frac{g_4}{4!}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right) + \frac{1}{2}m_\omega^2\omega_0^2\left(1+\eta_1\frac{g_\sigma\sigma_0}{m_B} + \frac{\eta_2}{2}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right)$$
$$+\frac{1}{4!}\zeta_0 g_\omega^2\omega_0^4 + \frac{1}{2}m_\rho^2\rho_{03}^2\left(1+\eta_\rho + \frac{g_\sigma\sigma_0}{m_B}\right) + 3\Lambda_v(g_\rho\rho_{03})^2(g_\omega\omega_0)^2 + \sum_l \mathscr{E}_l,$$

$$P = \sum_B \frac{2}{3(2\pi)^3}\int_0^{k_B} d^3k\frac{k^2}{E_B^*(k)} - m_\sigma^2\sigma_0^2\left(\frac{1}{2} + \frac{g_3}{3!}\frac{g_\sigma\sigma_0}{m_B} + \frac{g_4}{4!}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right) + \frac{1}{2}m_\omega^2\omega_0^2\left(1+\eta_1\frac{g_\sigma\sigma_0}{m_B} + \frac{\eta_2}{2}\frac{g_\sigma^2\sigma_0^2}{m_B^2}\right)$$
$$+\frac{1}{4!}\zeta_0 g_\omega^2\omega_0^4 + \frac{1}{2}m_\rho^2\rho_{03}^2\left(1+\eta_\rho\frac{g_\sigma\sigma_0}{m_B}\right) + \Lambda_v(g_\rho\rho_{03})^2(g_\omega\omega_0)^2 + \sum_l P_l.$$

EoS

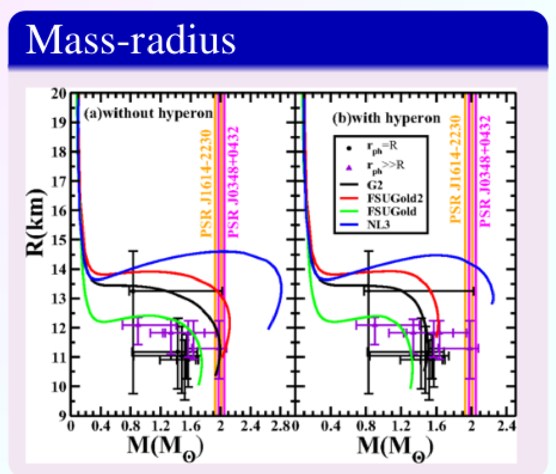


TOV equation

$$\frac{dP(r)}{dr} = -\frac{[\mathscr{E}(r)+P(r)][M(r)+4\pi r^3 P(r)]}{r^2(1-\frac{2M(r)}{r})},$$
$$\frac{dM(r)}{dr} = 4\pi r^2\mathscr{E}(r).$$

- For a given EoS, the TOV equation must be integrated from the boundary conditions $P(0) = P_c$ and $M(0) = 0$, $P(R) = 0$ and $M(R) = M$.

Mass-radius



Various tidal Love numbers

- To estimate the Love numbers k_l ($l = 2,3,4$), alongwith the evolution of TOV equation, we have to compute $y = y_l(R)$ with initial boundary condition $y(0) = l$ from the following differential equation iteratively:

$$r\frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2 Q(r) = 0,$$

$$F(r) = \frac{r - 4\pi r^3[\mathscr{E}(r) - P(r)]}{r - 2M(r)}, \quad Q(r) = \frac{4\pi r(5\mathscr{E}(r) + 9P(r) + \frac{\mathscr{E}(r)+P(r)}{\partial P(r)/\partial\mathscr{E}(r)} - \frac{l(l+1)}{4\pi r^2})}{r - 2M(r)} - 4\left[\frac{M(r)+4\pi r^3 P(r)}{r^2(1-2M(r)/r)}\right]^2.$$

- The electric tidal Love number k_l :

Quadrupole Love number

$$k_2 = \frac{8}{5}(1-2C)^2C^5[2C(y_2-1)-y_2+2]\Big\{2C(4(y_2+1)C^4 + (6y_2-4)C^3 +$$
$$(26-22y_2)C^2 + 3(5y_2-8)C - 3y_2 + 6) - 3(1-2C)^2(2C(y_2-1)-y_2+2)log\left(\frac{1}{1-2C}\right)\Big\}^{-1},$$

Octupole Love number

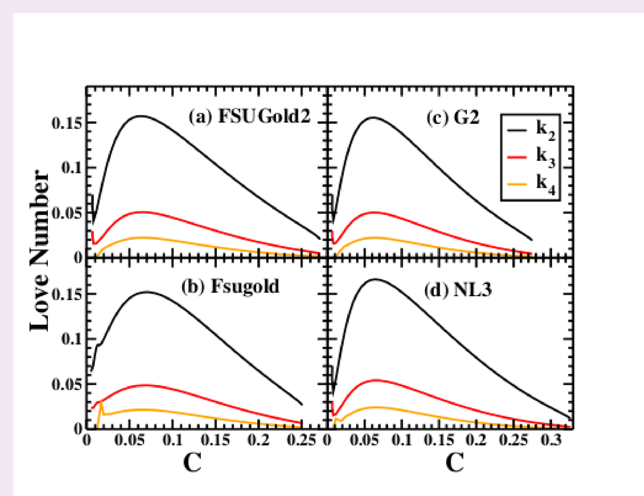
$$k_3 = \frac{8}{7}(1-2C)^2C^7[2(y_3-1)C^2 - 3(y_3-2)C + y_3 - 3]\Big\{2C[4(y_3+1)C^5 + 2(9y_3-2)C^4 - 20(7y_3-9)C^3$$
$$+5(37y_3-72)C^2 - 45(2y_3-5)C + 15(y_3-3)] - 15(1-2C)^2(2(y_3-1)C^2 - 3(y_3-2)C + y_3 - 3)log\left(\frac{1}{1-2C}\right)\Big\}^{-1},$$

[2] Bharat Kumar, S. K. Biswal, and S. K. Patra, Phys. Rev. C **95**, 015801 (2017)

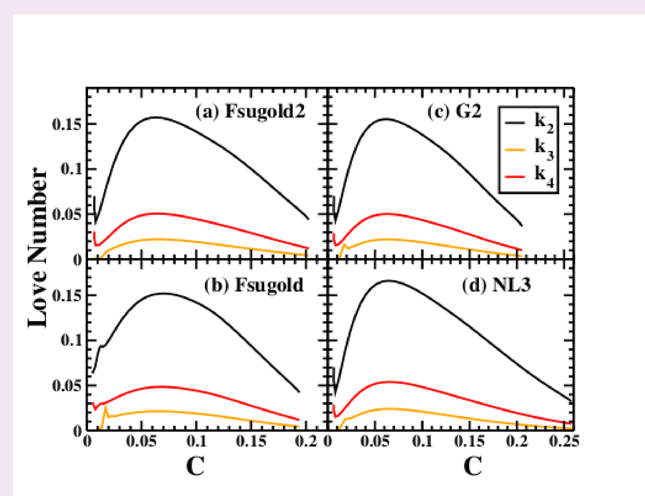
Hexadecapole Love number

$$k_4 = \frac{32}{147}(1-2C)^2C^9[12(y_4-1)C^3 - 34(y_4-2)C^2 + 28(y_4-3)C - 7(y_4-4)]\Big\{2C[8(y_4+1)C^6$$
$$+(68y_4-8)C^5 + (1284-996y_4)C^4 + 40(55y_4-116)C^3 + (5360-1910y_4)C^2 + 105(7y_4-24)C - 105(y_4-4)]$$
$$-15(1-2C)^2[12(y_4-1)C^3 - 34(y_4-2)C^2 + 28(y_4-3)C - 7(y_4-4)]log\left(\frac{1}{1-2C}\right)\Big\}^{-1},$$

Neutron stars



Hyperon stars



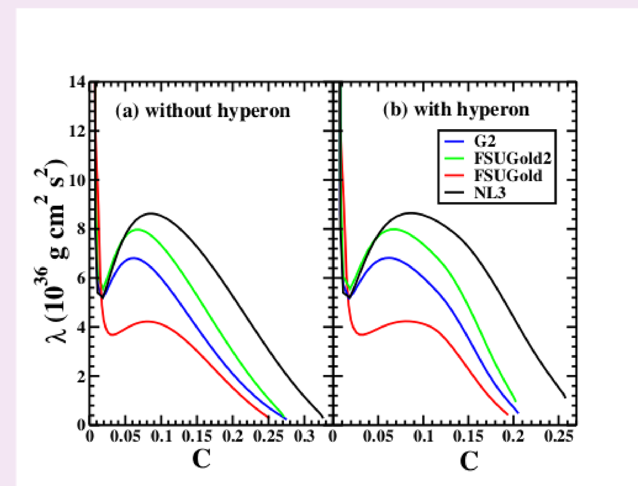
Tidal deformability

- Tidal deformability $\lambda = \frac{2}{3}k_2R^5$.
- Dimensionless tidal deformability is defined as $\Lambda = \frac{2k_2}{3C^5}$

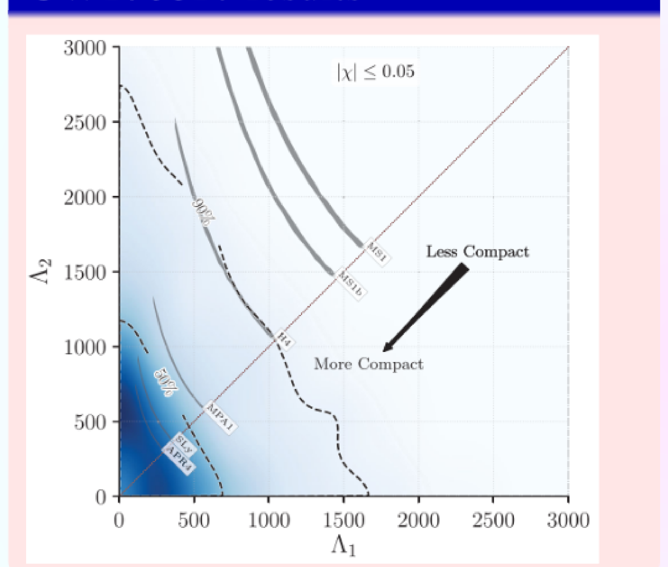
Properties of a $1.4M_\odot$ star

EOS	$R(km)$	C	k_2	k_3	k_4	λ	Λ
Neutron Star							
NL3	14.422	0.144	0.1197	0.0353	0.0142	7.466	1288.81
G2	13.148	0.157	0.0934	0.0265	0.0103	3.668	652.76
FSUGold2	13.850	0.149	0.1040	0.0301	0.0119	5.299	944.08
FSUGold	12.236	0.170	0.0882	0.0244	0.0071	2.418	414.13
Hyperon Star							
NL3	14.430	0.143	0.1203	0.0355	0.0143	7.527	1341.20
G2	12.686	0.163	0.0804	0.0229	0.0088	2.641	465.83
FSUGold2	13.690	0.151	0.0988	0.0287	0.0113	4.750	839.04
$(FSUGold)_{1.3M_\odot}$	9.922	0.194	0.0421	0.0116	0.0042	0.4048	102.14

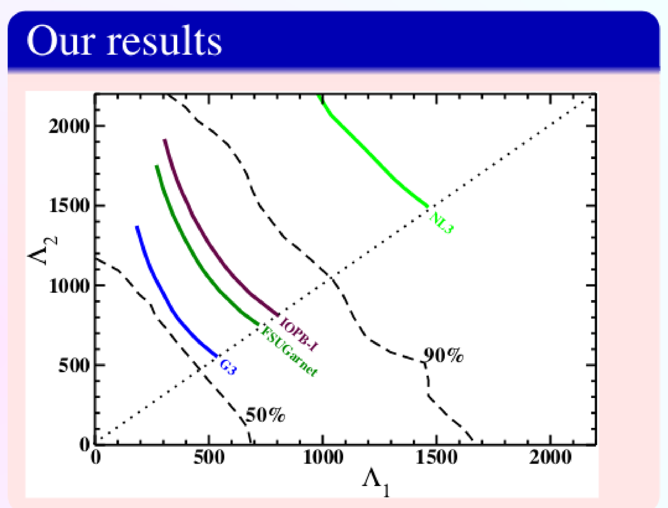
- Recently, aLIGO and VIRGO detectors measured a value of $\Lambda \leq 800$ in the low-spin case.



GW170817 results



Our results



- The LIGO/VIRGO limit on the tidal deformabilities favour soft EoSs, the $2M_\odot$ constraints requires the EoS to be stiff, thus setting a very restrictive bound for the quantity.
- The gravitational waveform depends on the weighted tidal deformability:

$$\tilde{\Lambda} = \frac{16}{13}\left(\frac{(M_1+12M_2)M_1^4}{(M_1+M_2)^5}\Lambda_1 + \frac{(M_2+12M_1)M_2^4}{(M_1+M_2)^5}\Lambda_2\right)$$

[3] Bharat Kumar, S. K. Patra, and B. K. Agrawal, Phys. Rev. C **97**, 045806 (2018)

Thank You