Quantum Computing Basic 2

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Breakdown

15 min	Classical vs. Quantum computing
15 min	Pauli group
15 min	Clifford group
15 min	Non-Clifford gate
15 min	Single-qubit gates
30 min	Universal gate set
15 min	Quantum circuits

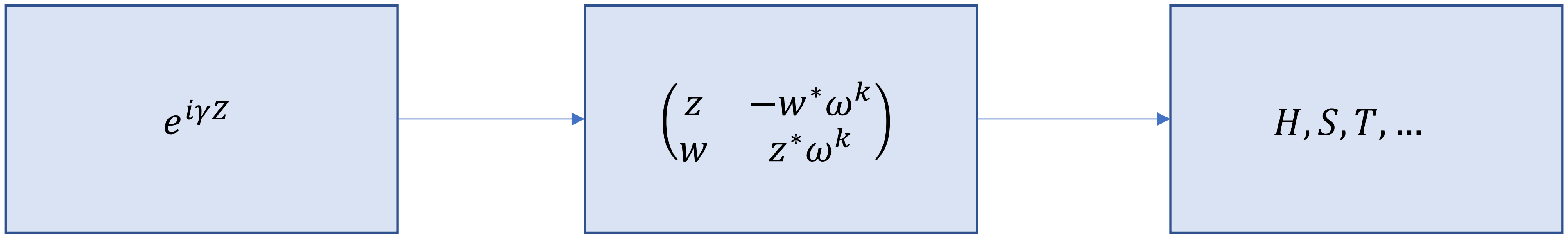
Single-qubit gates

- Pauli: X, Y, Z
- Clifford: H, S
- Non-Clifford: T
- Any single-qubit unitary gate can be approximated by a sequence of these gates.

$$U = e^{i\alpha X} e^{i\beta Y} e^{i\gamma Z} \rightarrow e^{i\gamma Z} \rightarrow H, S, T, \ldots$$

Single-qubit gates

- Fact: Any gate composed of Clifford+T has the following form[Kliuchnikov, Maslov, Mosca (2013)]:
  - $U = \begin{pmatrix} z & -w^*\omega^k \\ w & z^*\omega^k \end{pmatrix}$, where ω is the 8th root of unity.



Approximate Synthesis:
ε- approximation

Exact Synthesis

Single-qubit gates

- Fact: Any gate composed of Clifford+T has the following form[Kliuchnikov, Maslov, Mosca (2013)]:
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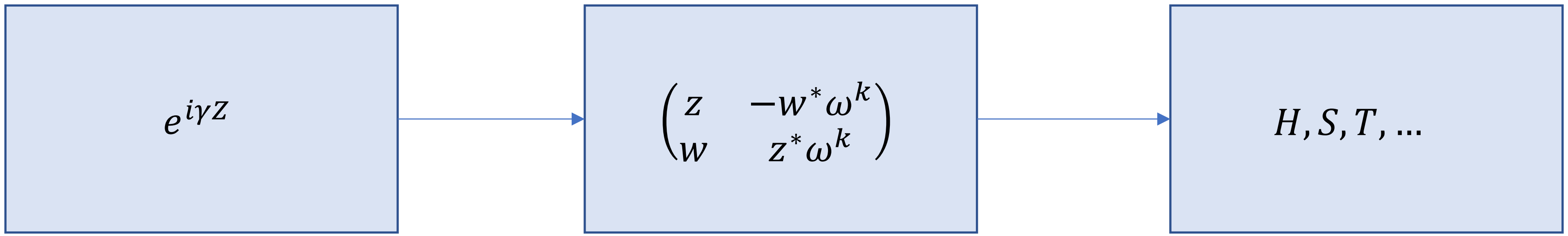


Approximate Synthesis:
ε- approximation

Exact Synthesis

- Ross and Selinger: ε-approximation possible with ~$3 \log \frac{1}{\epsilon}$ gates. (Both H and T.)
- This is nearly optimal.

Single-qubit gates

- n-bit precision single-unitary can be done with O(n) elementary gates.

ε	T-count	T-bound	Actual error	Runtime	Candidates	Time/Candidate
10^{-10}	102	$\geqslant 102$	$0.91180 \cdot 10^{-10}$	$0.0190s$	3.0	$0.0064s$
10^{-20}	200	$\geqslant 198$	$0.87670 \cdot 10^{-20}$	$0.0433s$	7.0	$0.0061s$
10^{-30}	298	$\geqslant 298$	$0.99836 \cdot 10^{-30}$	$0.0600s$	7.0	$0.0085s$
10^{-40}	402	$\geqslant 400$	$0.77378 \cdot 10^{-40}$	$0.0976s$	11.7	$0.0084s$
10^{-50}	500	$\geqslant 500$	$0.82008 \cdot 10^{-50}$	$0.1353s$	20.3	$0.0067s$
10^{-60}	602	$\geqslant 596$	$0.61151 \cdot 10^{-60}$	$0.1548s$	16.0	$0.0097s$
10^{-70}	702	$\geqslant 698$	$0.40936 \cdot 10^{-70}$	$0.1931s$	20.9	$0.0093s$
10^{-80}	804	$\geqslant 794$	$0.92372 \cdot 10^{-80}$	$0.2402s$	27.2	$0.0088s$
10^{-90}	898	$\geqslant 898$	$0.96607 \cdot 10^{-90}$	$0.2696s$	22.2	$0.0121s$
10^{-100}	1000	$\geqslant 998$	$0.78879 \cdot 10^{-100}$	$0.3443s$	31.2	$0.0110s$
10^{-200}	1998	$\geqslant 1994$	$0.73266 \cdot 10^{-200}$	$1.1423s$	62.3	$0.0183s$
10^{-500}	4990	$\geqslant 4986$	$0.67156 \cdot 10^{-500}$	$8.6509s$	170.4	$0.0508s$
10^{-1000}	9974	$\geqslant 9966$	$0.80457 \cdot 10^{-1000}$	$47.9300s$	270.4	$0.1773s$
10^{-2000}	19942	$\geqslant 19934$	$0.88272 \cdot 10^{-2000}$	$383.1024s$	556.7	$0.6881s$

From [Ross, Selinger (2014)]

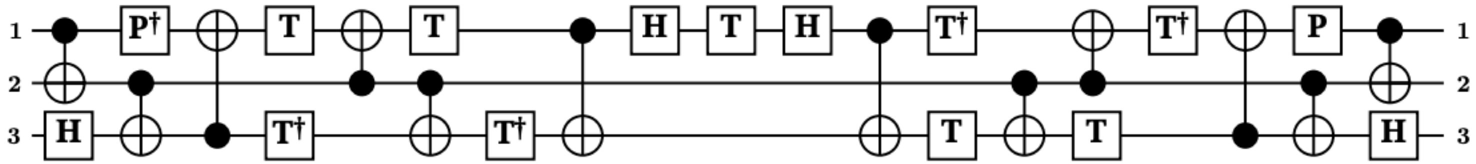
T-gate is the dominant cost of fault-tolerant quantum computation.

Universal gate set

- Generally, arbitrary single-qubit unitary + a fixed two-qubit unitary is universal.
- In particular, {H, T, CNOT} is a universal gate set.
- While this is not the only choice, this is one of the most popular choices.

Quantum circuits

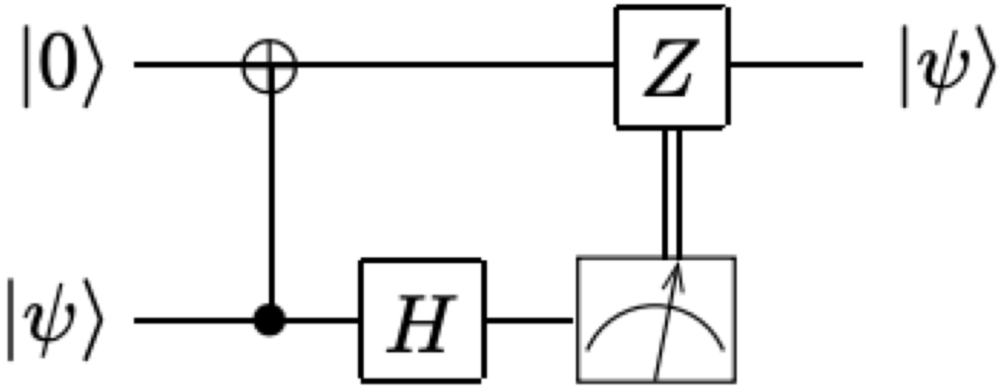
How do we actually apply these gates in practice?



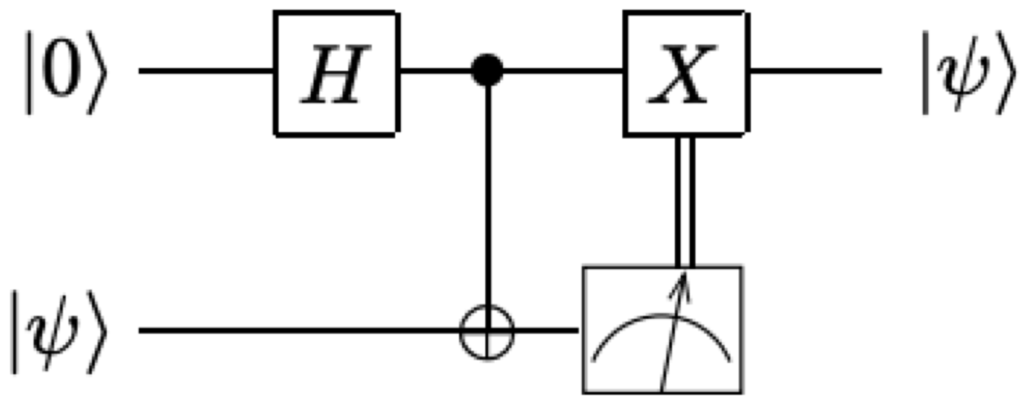
Gate teleportation

- Ordinary gates: Unitary operation
- Teleportation: Measurement + unitary
    - End result: Identity operation
- Gate teleportation: Measurement + unitary
    - End result: nontrivial unitary operation

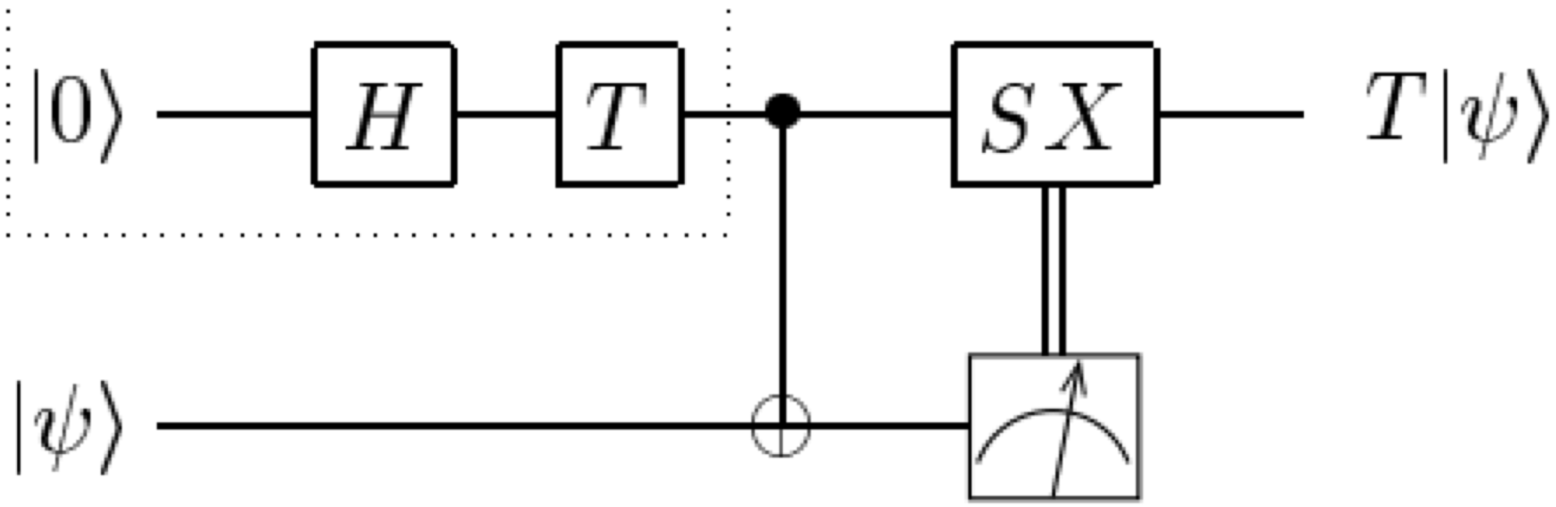
One-bit teleportation



One-bit teleportation



Phase/T-gate teleportation



Lessons

- T-gate can be applied by gate teleportation.
- The ”fix” operation is the phase gate, which is Clifford.
- Generally speaking, $\begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{\pi}{2^n}i} \end{pmatrix}$ gate can be applied by gate teleportation followed by a fix operation $\begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{\pi}{2^{n-1}}i} \end{pmatrix}$.
- $\begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{\pi}{2^n}i} \end{pmatrix}$ gate belongs to the (n+1)th level of Clifford hiercharchy.