Topological Quantum Computation: Past and Present

Isaac Kim(PsiQuantum)

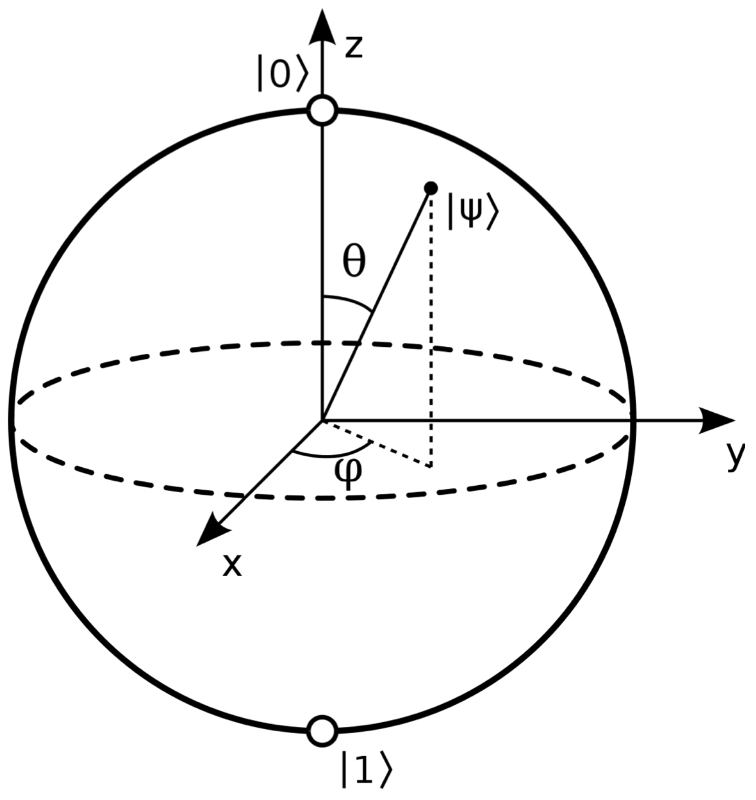
Disclaimer: None of this work is related to what I do in PsiQuantum.

Breakdown

15 min	Challenges of quantum computing
15 min	Fault-tolerance
15 min	Topological quantum computation
15 min	Examples
30 min	Modern view

Classical vs. Quantum computing

	Classical	Quantum
Elementary unit	bit	qubit
Computation	discrete	continuous
Gate set	discrete	discrete



Discrete gate set is necessary

- Useful quantum computation often requires 100,000,000 gates or more. In order for the computation to be reliable, the precision per operation must be much smaller than 1/100,000,000.
- With the fault-tolerant quantum computation technique, one can boost the precision from ~1/100 to an arbitrarily small number.
- However, the **fault-tolerant gate set is discrete**.

Discrete gate set is sufficient

- Popular fault-tolerant gate set: $T, H, CNOT$.
  - Any single qubit gate can be approximated with error ϵ by using at most $3\log\frac{1}{\epsilon} + O(\log\log\frac{1}{\epsilon})$ T-gates. (Same scaling for H-gate.) [Ross and Selinger (2014)]

ε	T-count	T-bound	Actual error	Runtime	Candidates	Time/Candidate
10^{-10}	102	$\geqslant 102$	$0.91180 \cdot 10^{-10}$	$0.0190s$	3.0	$0.0064s$
10^{-20}	200	$\geqslant 198$	$0.87670 \cdot 10^{-20}$	$0.0433s$	7.0	$0.0061s$
10^{-30}	298	$\geqslant 298$	$0.99836 \cdot 10^{-30}$	$0.0600s$	7.0	$0.0085s$
10^{-40}	402	$\geqslant 400$	$0.77378 \cdot 10^{-40}$	$0.0976s$	11.7	$0.0084s$
10^{-50}	500	$\geqslant 500$	$0.82008 \cdot 10^{-50}$	$0.1353s$	20.3	$0.0067s$
10^{-60}	602	$\geqslant 596$	$0.61151 \cdot 10^{-60}$	$0.1548s$	16.0	$0.0097s$
10^{-70}	702	$\geqslant 698$	$0.40936 \cdot 10^{-70}$	$0.1931s$	20.9	$0.0093s$
10^{-80}	804	$\geqslant 794$	$0.92372 \cdot 10^{-80}$	$0.2402s$	27.2	$0.0088s$
10^{-90}	898	$\geqslant 898$	$0.96607 \cdot 10^{-90}$	$0.2696s$	22.2	$0.0121s$
10^{-100}	1000	$\geqslant 998$	$0.78879 \cdot 10^{-100}$	$0.3443s$	31.2	$0.0110s$
10^{-200}	1998	$\geqslant 1994$	$0.73266 \cdot 10^{-200}$	$1.1423s$	62.3	$0.0183s$
10^{-500}	4990	$\geqslant 4986$	$0.67156 \cdot 10^{-500}$	$8.6509s$	170.4	$0.0508s$
10^{-1000}	9974	$\geqslant 9966$	$0.80457 \cdot 10^{-1000}$	$47.9300s$	270.4	$0.1773s$
10^{-2000}	19942	$\geqslant 19934$	$0.88272 \cdot 10^{-2000}$	$383.1024s$	556.7	$0.6881s$

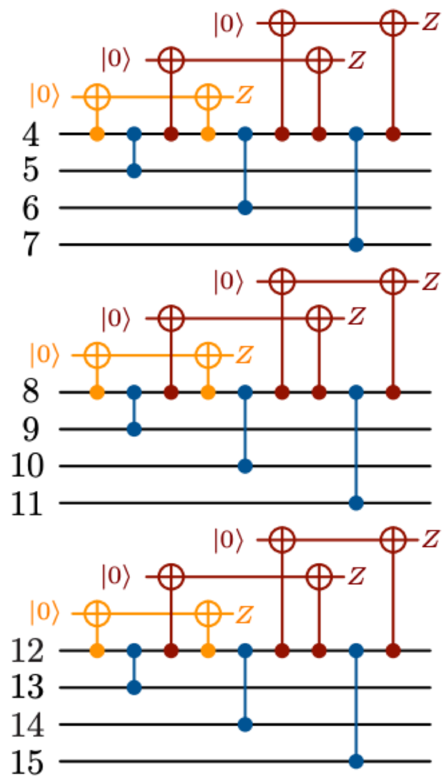
From Ross and Selinger(2014)

Fault-tolerant quantum computation

- Because a discrete gate set can approximate any continuous gate set arbitrarily well with small($O(\log\frac{1}{\epsilon})$) overhead, it suffices to make those gates really, really good.
- The theory of fault-tolerant quantum computation attempts to achieve this goal.

Fault-tolerance: Active

- Approach: Check if an error occurs in the middle of the computation. If an error is detected, identify the most likely scenario and correct.
- Tricky part
    - Theory: The check may also suffer an error. So we need to verify the checks as well.
    - Experiment: For n-qubit system, one needs to calibrate $O(n)$ frequencies, pulses, etc. Because a useful quantum computation requires $\sim 10^6$ qubits, this poses a significant engineering challenge.



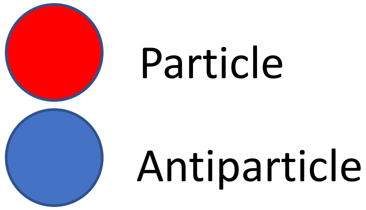
From Reichardt and Cao(2017)

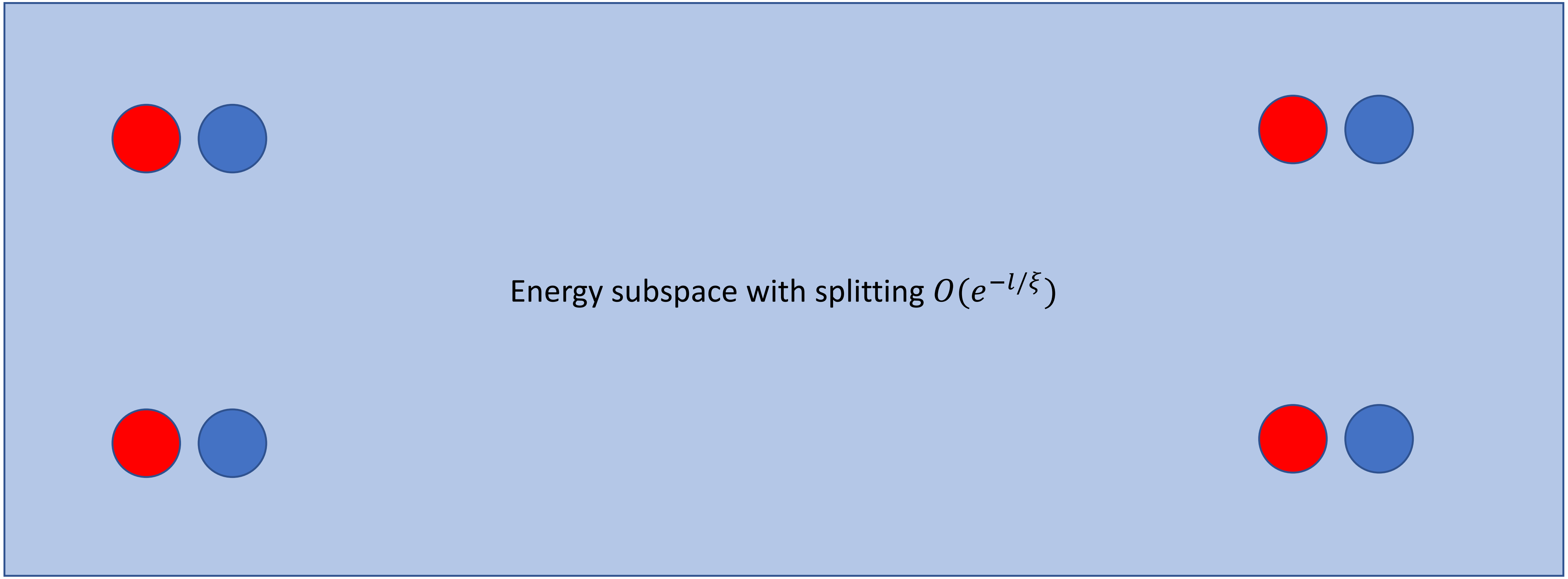
Fault-tolerance: Passive

- Approach: Build a physical device that is fault-tolerant by its nature.
- Most of these approaches has been pioneered by Kitaev.
    - Anyons(1996)
    - Majorana wire(2000)
    - Superconducting current mirror(2006)
- Idea: Once you are in some quantum phase, then all of your gates are protected.

Topological quantum computation

- Zero temperature
- Adiabatic transport of quasiparticles





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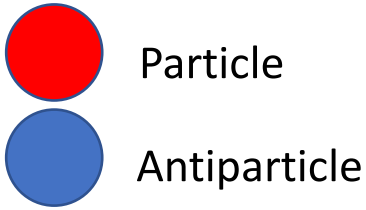

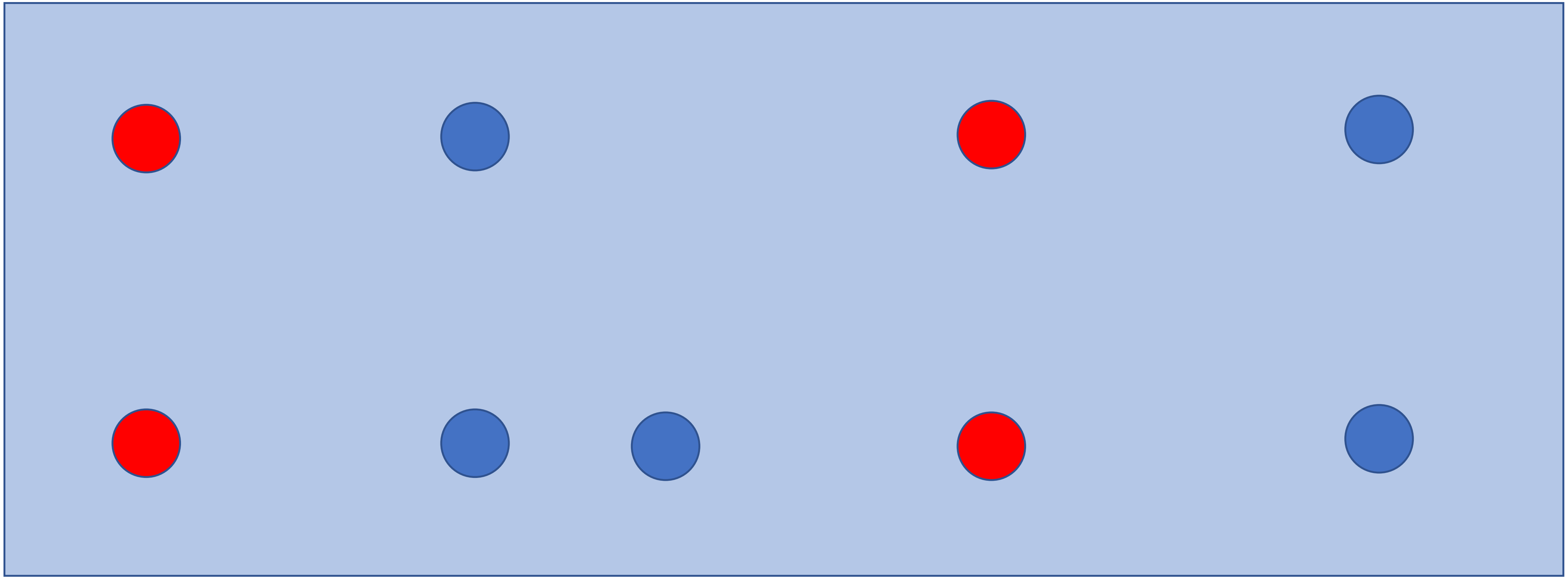

Topological quantum computation: Error source

- Zero error in the zero temperature limit and infinite separation between quasiparticles.
- Finite temperature effect: $O(e^{-\beta\Delta})$, where β inverse temperature and Δ quasiparticle gap.
- Finite separation effect: $O(e^{-l/\xi})$. Typically $\xi \sim 1/\Delta$.

Topologically-Protected Qubits from a Possible Non-Abelian Fractional Quantum Hall State

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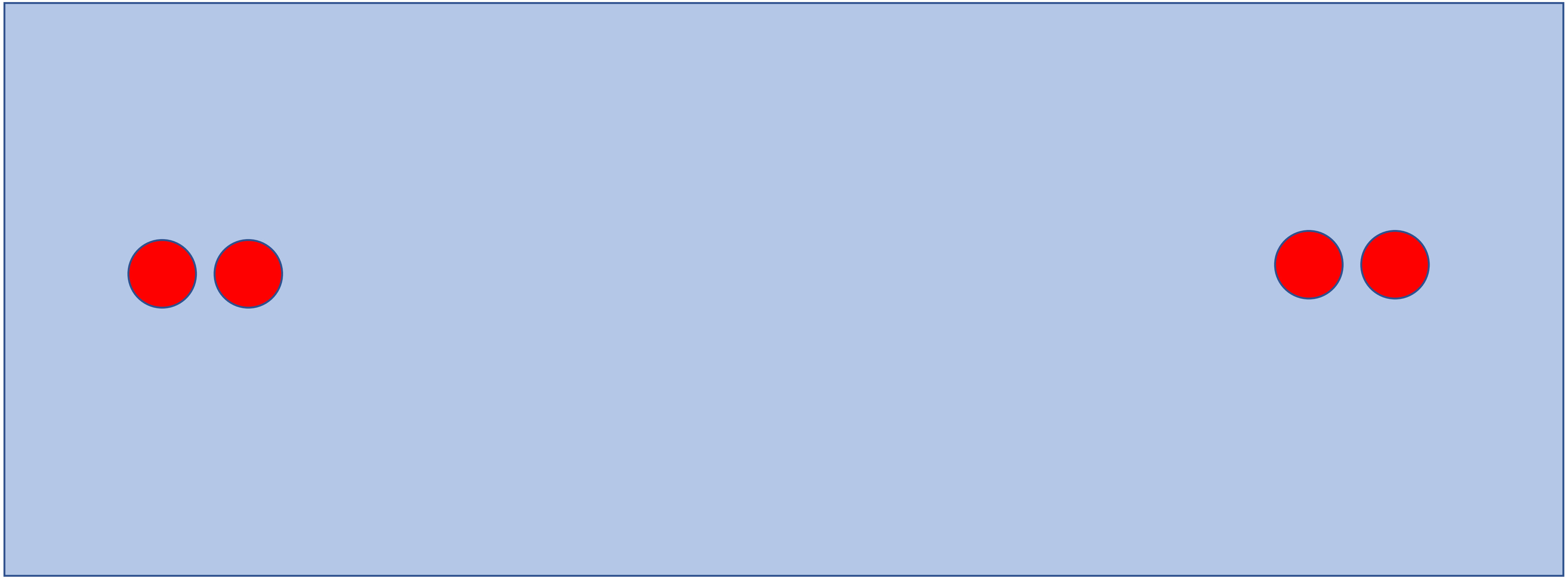
(Dated: September 10, 2018)

The Pfaffian state is an attractive candidate for the observed quantized Hall plateau at Landau level filling fraction $\nu = 5/2$. This is particularly intriguing because this state has unusual topological properties, including quasiparticle excitations with non-Abelian braiding statistics. In order to determine the nature of the $\nu = 5/2$ state, one must measure the quasiparticle braiding statistics. Here, we propose an experiment which can simultaneously determine the braiding statistics of quasiparticle excitations and, if they prove to be non-Abelian, produce a topologically-protected qubit on which a logical NOT operation is performed by quasiparticle braiding. Using the measured excitation gap at $\nu = 5/2$, we estimate the error rate to be 10^{-30} or lower.

Example 1: Majorana fermion

- A fermion that is its own antiparticle.

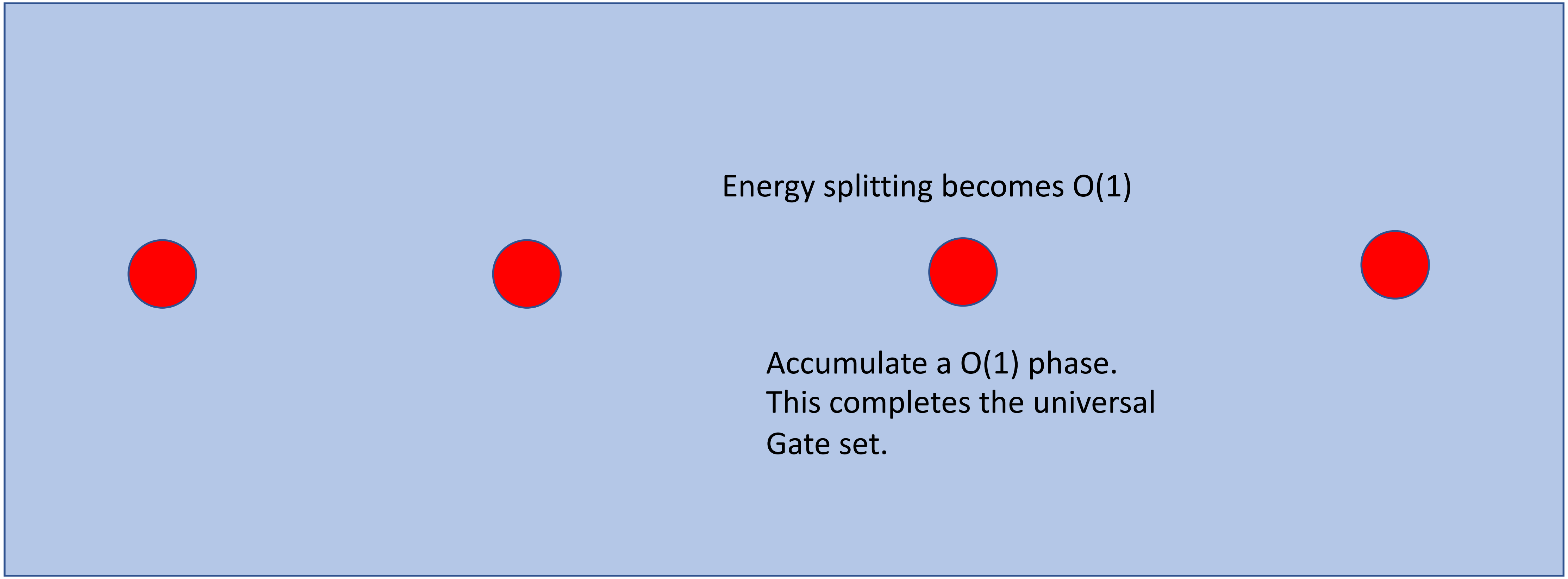


Gate set

- Operators: $c_m, m = 1,2,3,4, \ldots$
  - $\{c_n, c_m\} = \delta_{nm}$.
- Braid Gates: $\exp(\frac{\pi}{4} i\, c_n c_m)$.
  - This gate exchanges $c_n \to c_m, c_m \to -c_n$.
  - This gate set generates a finite group.
  - Therefore, this gate set is not universal.
- Measurement: Parity of n, m.
  - Observable: $-i c_n c_m$.

Universal gate set

- In order to complete a universal gate set, a non-topological gate must be included.
- Difference
  - Topological gates(Braid gates and parity measurement): Universal property of the phase. Assume to be nearly perfect.
  - Non-topological gates: Non-universal, depends on the details of the experiment.

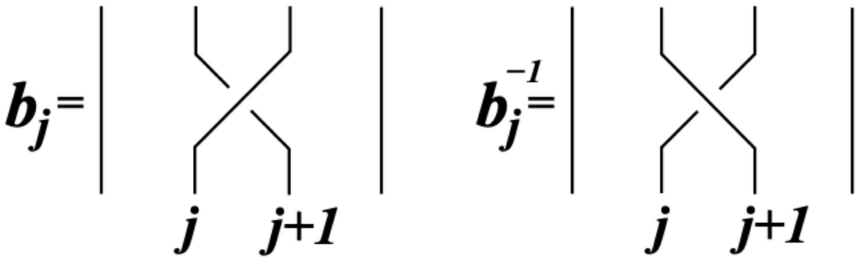


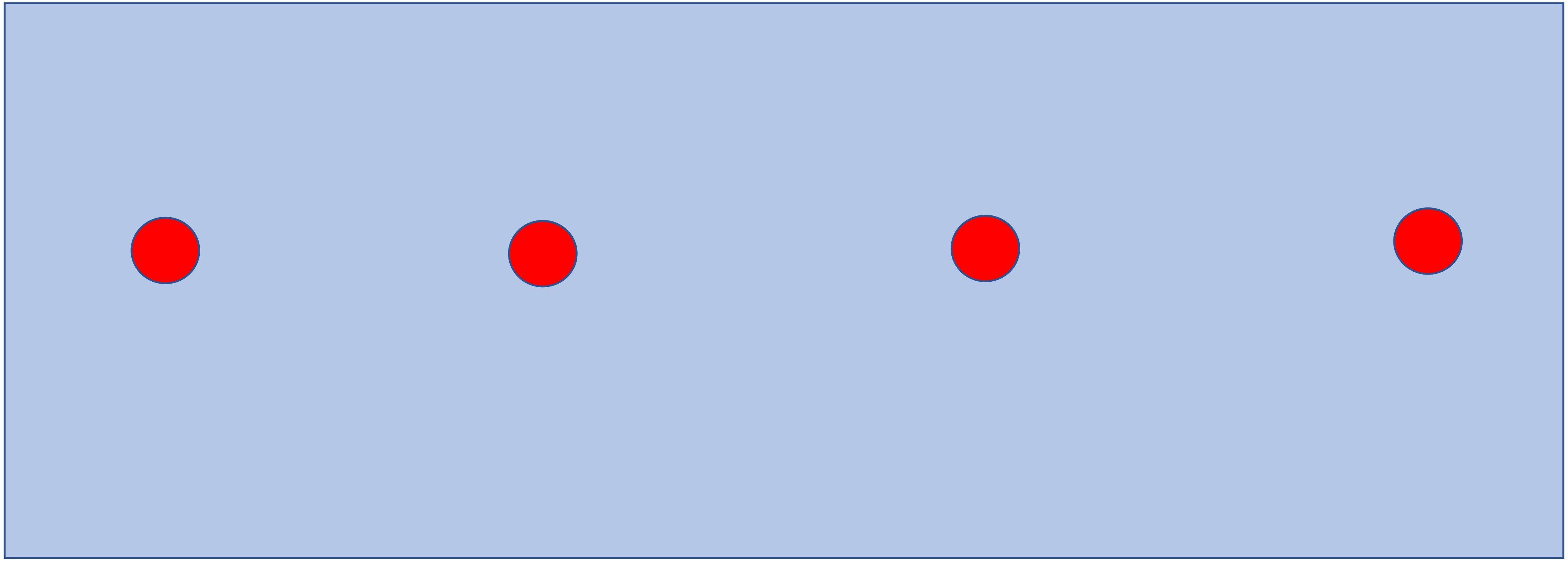
Universal gate set: Why is this reasonable?

- Non-topological gates will be generally much worse than topological gates, because it requires fine-tuning.
- However, there is a "software-approach" to mitigate the fine tuning.
  - If topological gates are perfect, the error rate for non-topological gates only need to be below 14%. [Bravyi (2005)]
  - Note: If every gate suffers the same error, the highest error rate that we can tolerate is ~3%. [Knill (2005)]. But Knill's approach is a bit unrealistic. For realistic schemes, the best one out there can tolerate ~0.7%[Raussendorf, Harrington, Goyal (2007)].

Generality: Braid group

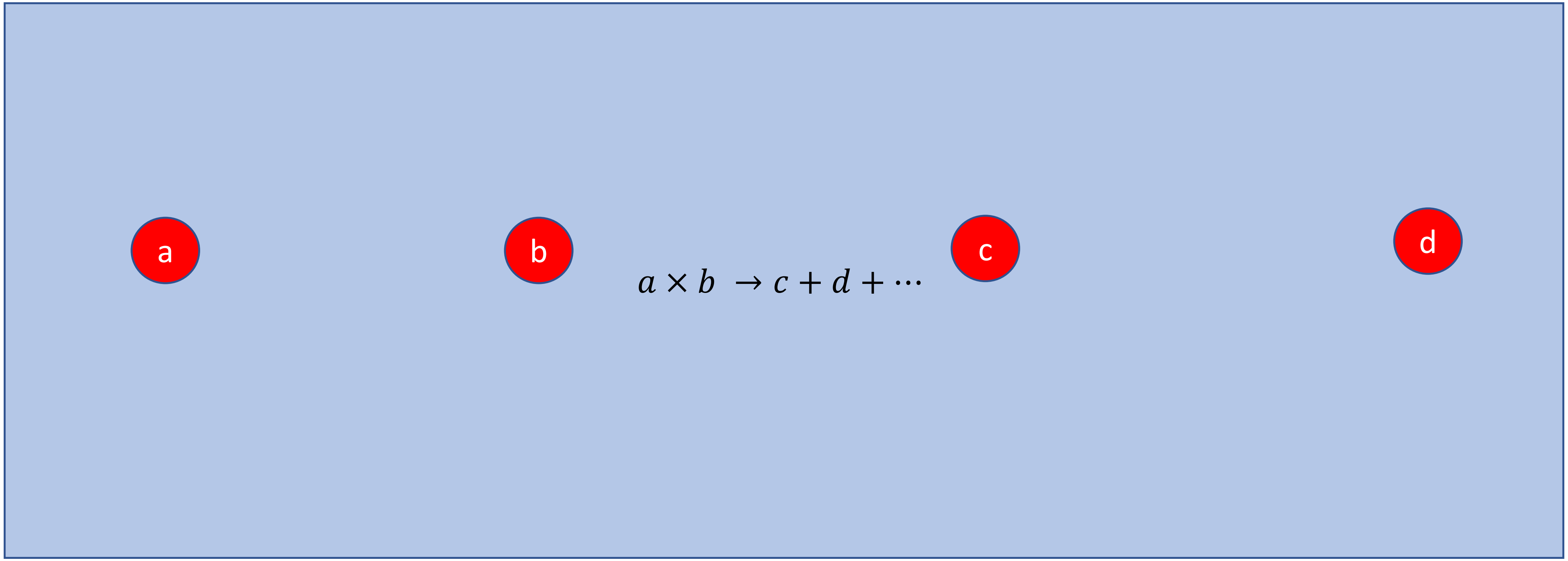
Braid group is infinite!





Generality: Particle type

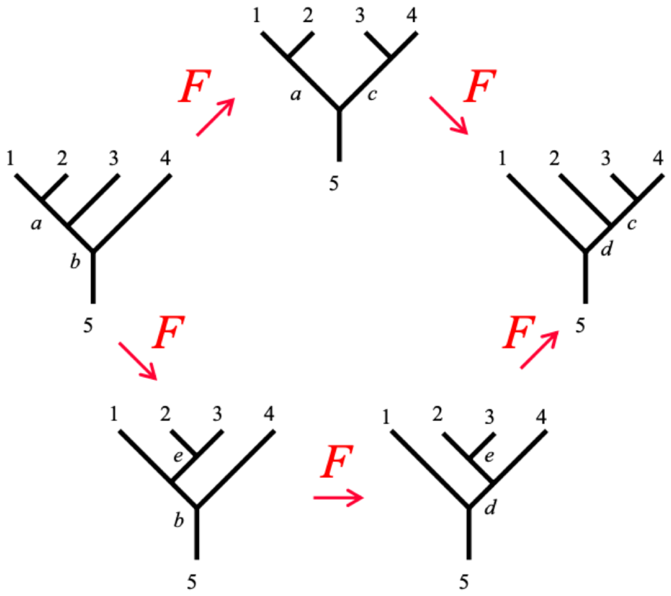
There are finite numbers of particle types, and they can fuse and become another particle.



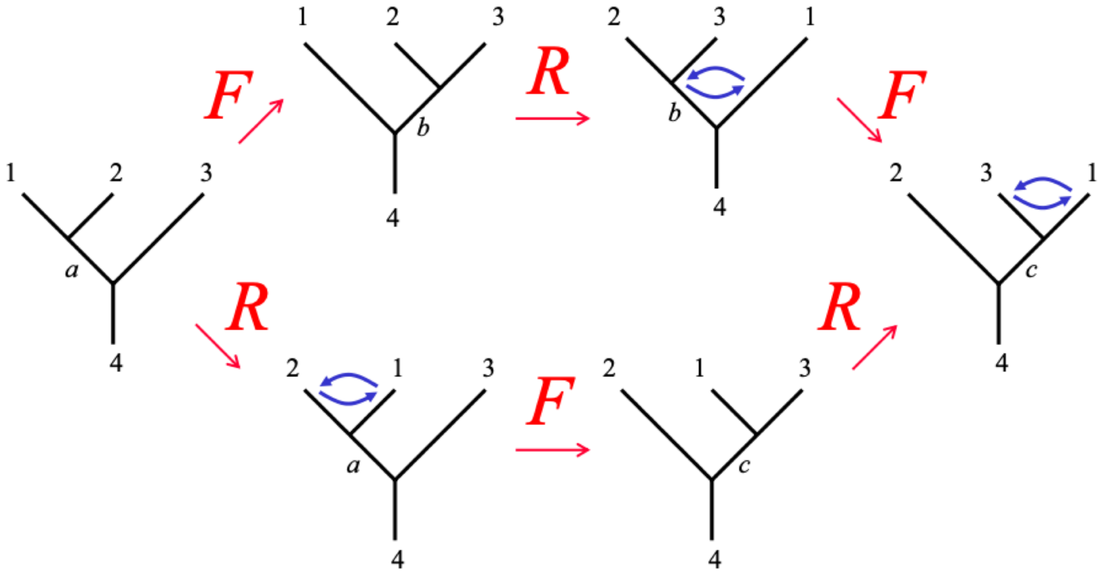
$$a \times b \rightarrow c + d + \cdots$$

Generality: How to determine the gates?

1. Based on what particle type fuses into what particle type, we can constrain F.
2. Solve the consistency equations.
3. If solution exists, our gates are $R, FRF^{-1}, \ldots$



$$(F^5_{12c})^d_a \, (F^5_{a34})^c_b = \sum_e \left(F^d_{234}\right)^c_e \, (F^5_{1e4})^d_b \left(F^b_{123}\right)^e_a$$



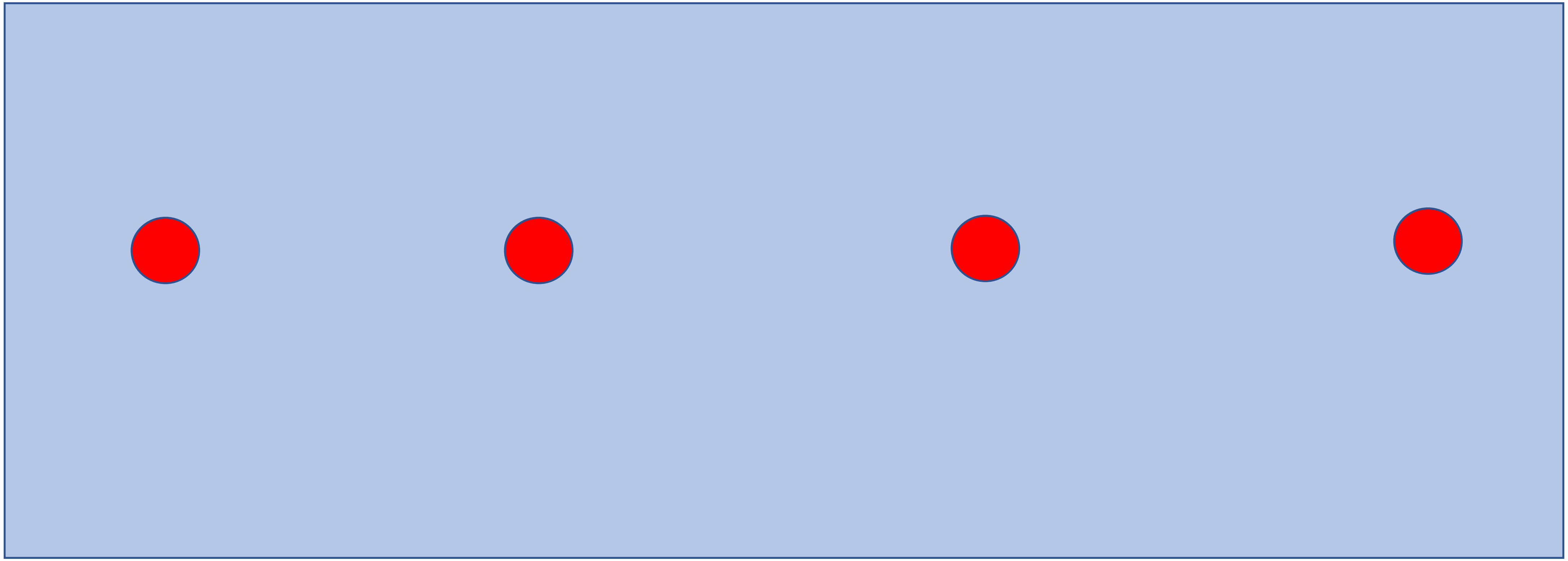
$$R^c_{13} \, (F^4_{213})^c_a \, R^a_{12} = \sum_b (F^4_{231})^c_b \, R^4_{1b} \, (F^4_{123})^b_a$$

These equations are not easy to solve in general…

Example 2: Fibonacci anyon

- 4 anyons encode a qubit.
- Braiding is universal.
- Potentially available for $\nu = 12/5$ FQHE state.
- Much less experimental effort.

$$R = \begin{pmatrix} e^{4\pi i/5} & 0 \\ 0 & -e^{2\pi i/5} \end{pmatrix} , \quad F = \begin{pmatrix} \tau & \sqrt{\tau} \\ \sqrt{\tau} & -\tau \end{pmatrix}$$



Original proposal

- Das Sarma, Freedman, Nayak(2005): Use $\nu = \frac{5}{2}$ fractional quantum Hall state.
  - Experimentally, excitation gap of 100mK observed.
  - The ground state may host an Ising anyon,[Read and Moore (1991)] whose computational power is exactly equal to the Majorana fermion.

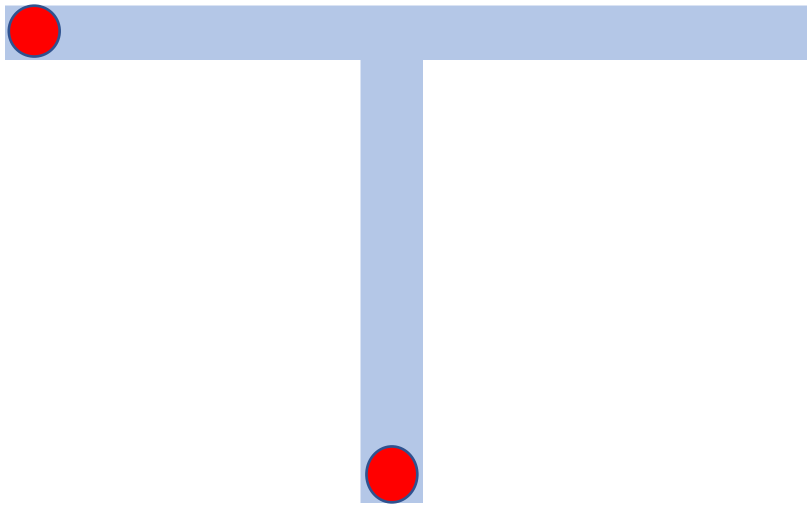
- Drawbacks
  - Both quasiparticle transport and charge measurement difficult in practice.
  - The existence of anyon is not confirmed yet.

Leading modern proposal

- Majorana nanonwire [Alicea et al. (2012)]

Leading modern proposal

- Majorana nanonwire [Alicea et al. (2012)]



Majorana nanowire

- Zero error in the zero temperature limit and infinite separation between quasiparticles.
- Finite temperature effect: $O(e^{-\beta\Delta})$, where β inverse temperature and Δ quasiparticle gap.
- Finite separation effect: $O(e^{-l/\xi})$. Typically $\xi \sim 1/\Delta$.

- Manufacturing yield is not very good yet.
- Quasi-particle transport is challenging in practice.

Important lessons

- Non-topological gates will be generally much worse than topological gates, because it requires fine-tuning.
- However, there is a "software-approach" to mitigate the fine tuning.
  - If topological gates are perfect, the error rate for non-topological gates only need to be below 14%. [Bravyi (2005)]
  - If every gate suffers the same error, the highest error rate that we can tolerate is ~3%. [Knill (2005)]. But Knill's approach is a bit unrealistic. For realistic schemes, the best one out there can tolerate ~0.7%.
  - If every single-qubit gate is perfect, the highest noise rate one can tolerate is not too different from 0.7% ~3% range.
- **Lesson 1: If we have very good two-qubit gates, fault-tolerant quantum computation becomes much easier.**
- **Lesson 2: Topological quantum computation is attractive because we can have very good two-qubit gates.**

Summary

- In principle, topological quantum computation can yield very low error rate.
  - Finite temperature effect: $O(e^{-\beta\Delta})$, where β inverse temperature and Δ quasiparticle gap.
  - Finite separation effect: $O(e^{-l/\xi})$. Typically $\xi \sim 1/\Delta$.
- The existing manufacturing process is not mature enough.
- However, once the process becomes scalable, we can expect to have very good qubits.

Topological quantum error correction

- Topological quantum computation: Cool to zero temperature by dissipating heat → Small error
- Topological quantum error correction: Cool to zero temperature by classical feedback → Vanishing error

Anyon model
$H = \Sigma_i h_i$

Cool to zero temperature.

Measure h_i. Lower energy based on the measurement outcome.

Topological quantum computation

Error=$\sim e^{-\beta\Delta}$

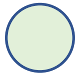
No classical computation necessary

Topological quantum error correction

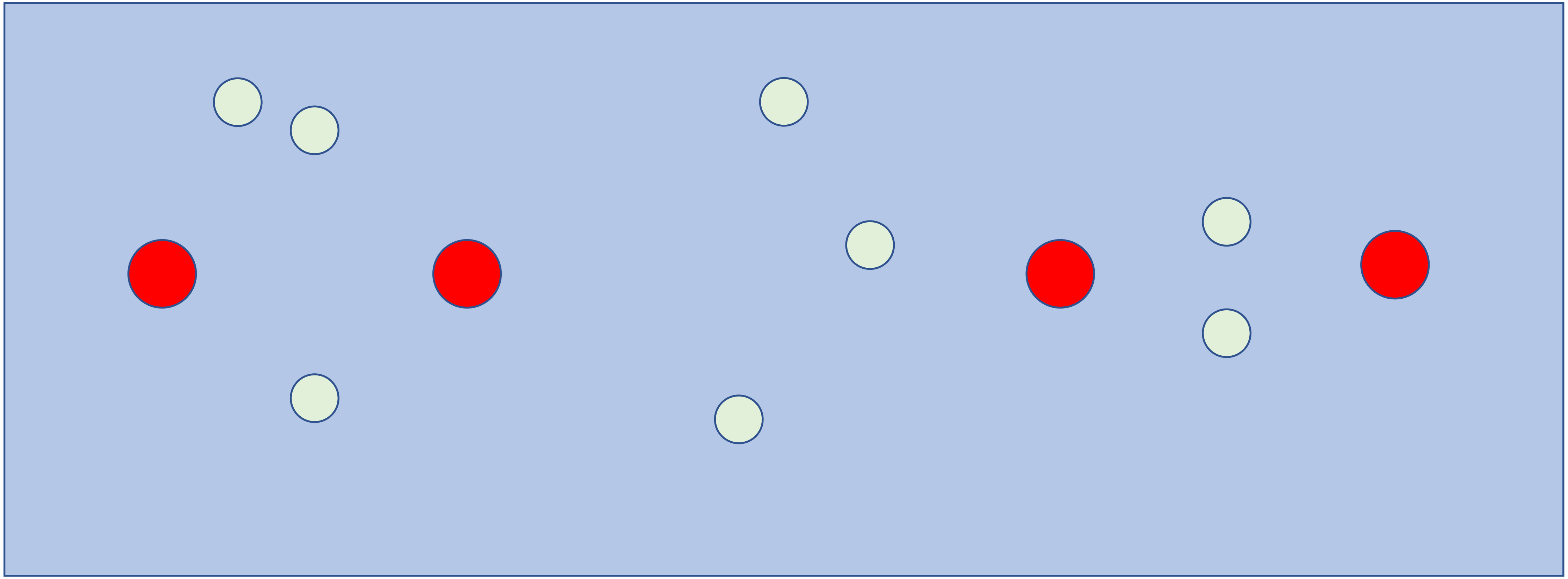
Error=$\sim e^{-cL}$ for LxL system.

Classical computation necessary.

Topological quantum error correction

 Thermal excitations

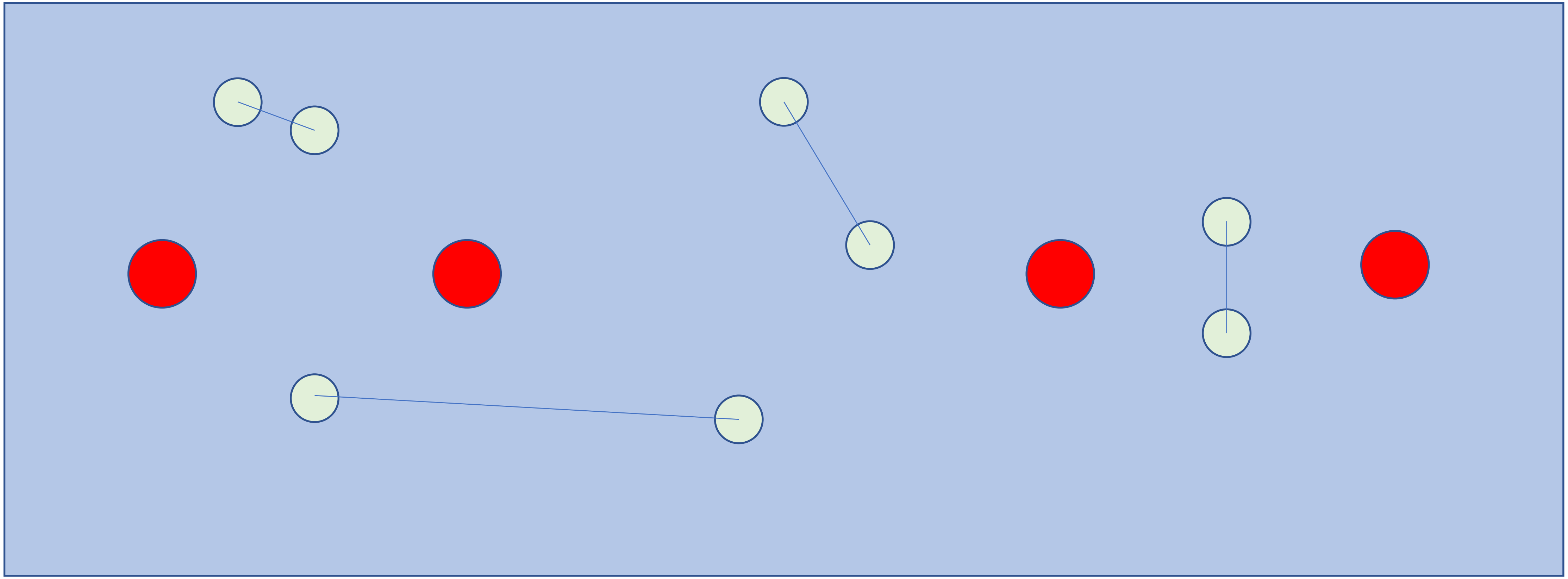
In a fixed time, thermal excitations are created and travel.



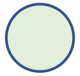
Topological quantum error correction

Thermal excitations

One can reverse this effect by annihilating the thermal excitations.

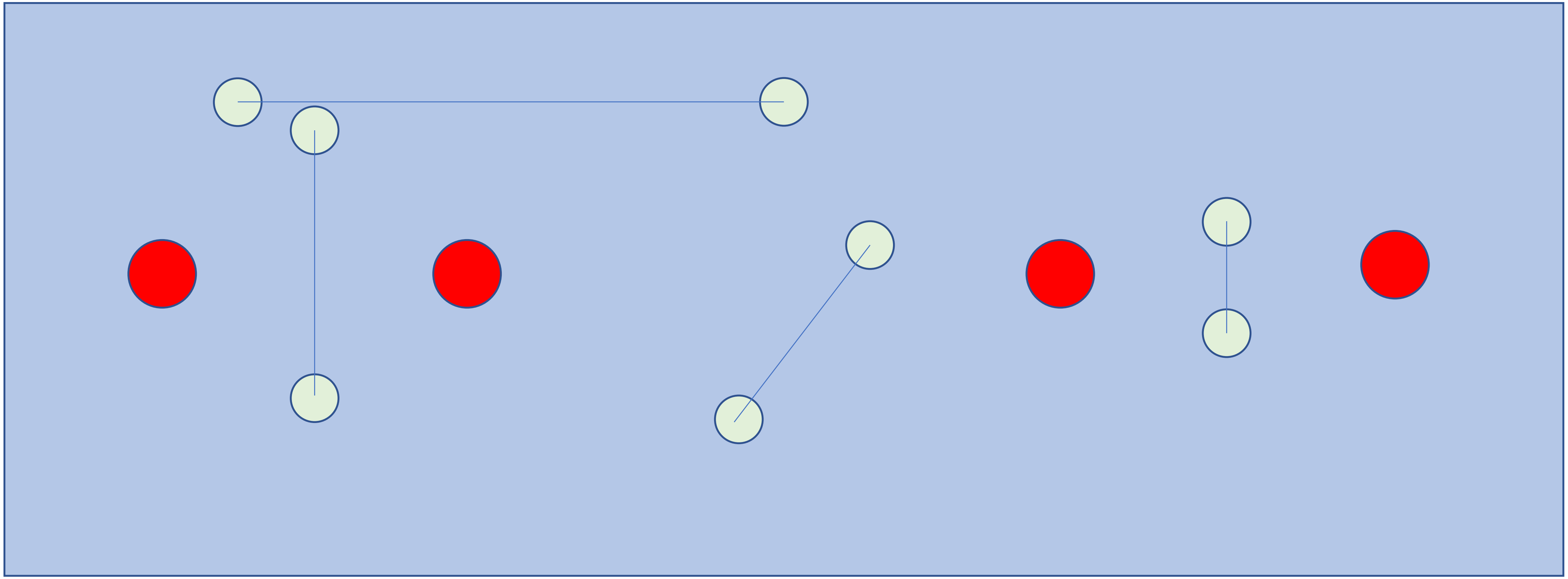


Topological quantum error correction

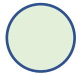


Thermal excitations

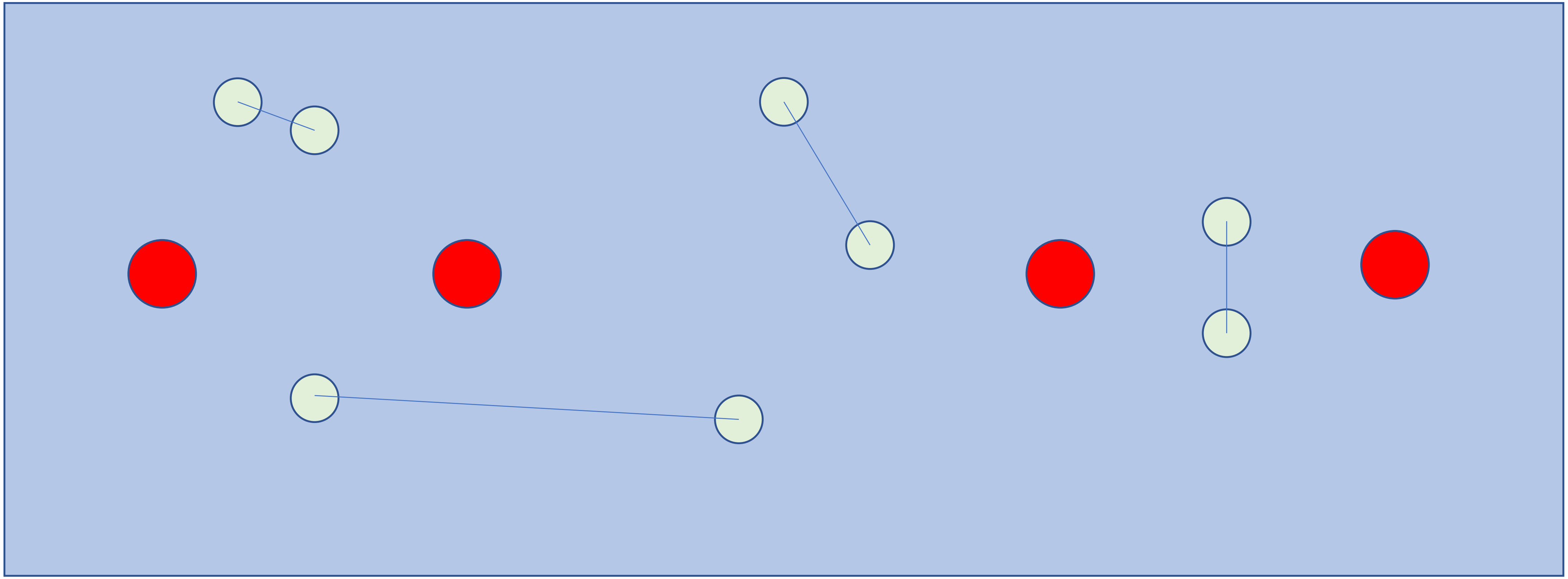
But there is an ambiguity…



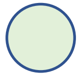
Topological quantum error correction



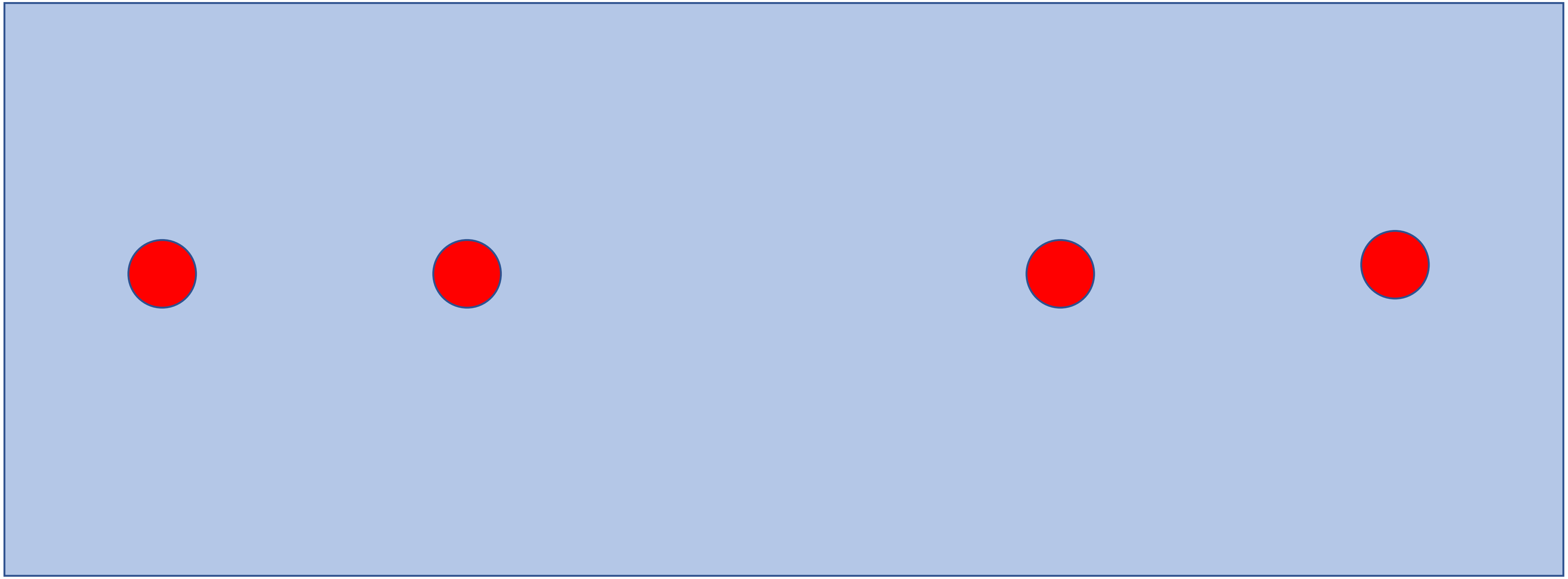
The most likely scenario: A path with the shortest travel distance.



Topological quantum error correction



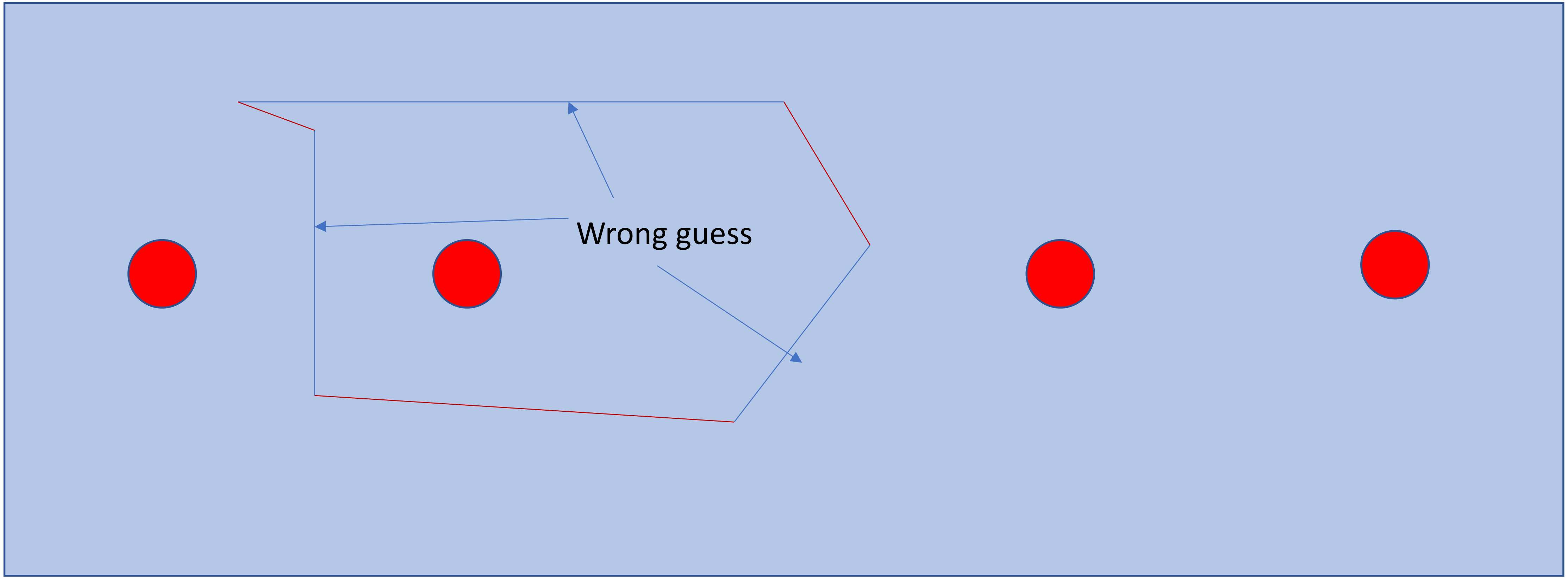
Most of the time, this will work.



Topological quantum error correction

Thermal excitations

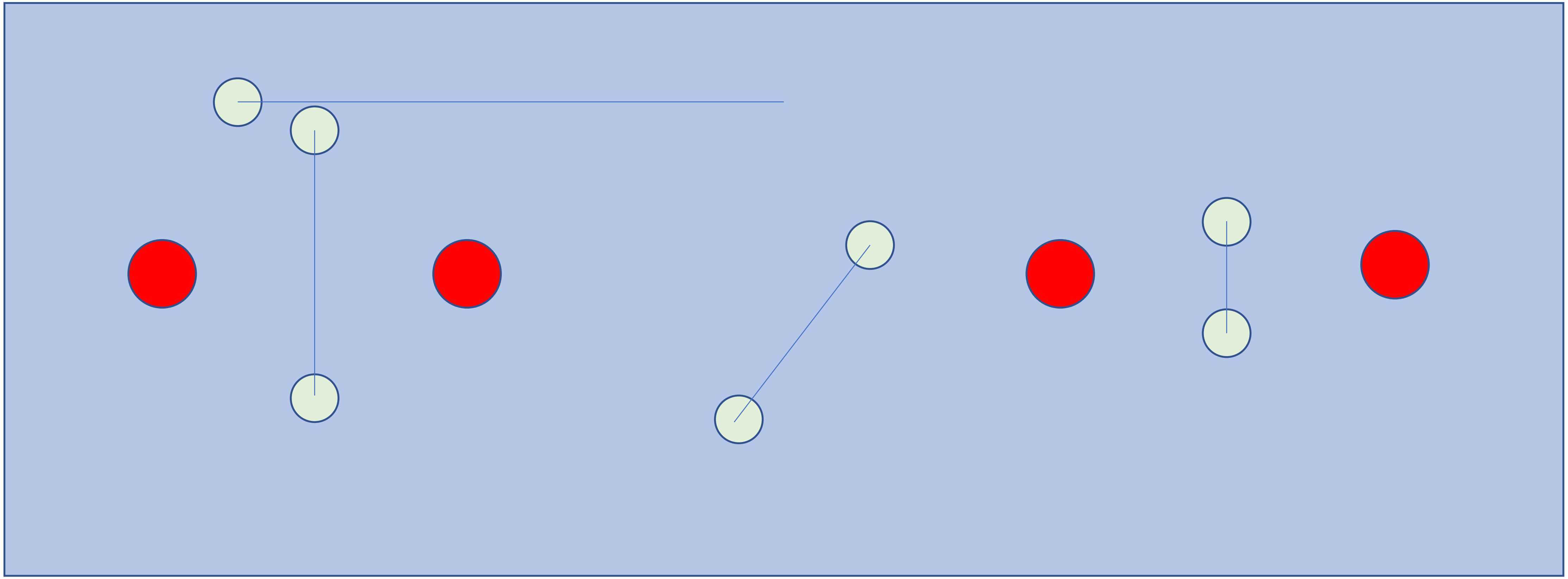
But sometimes we will get it wrong.



Topological quantum error correction: Measurement error

Thermal excitations

Sometimes, you might fail to detect the excitation.



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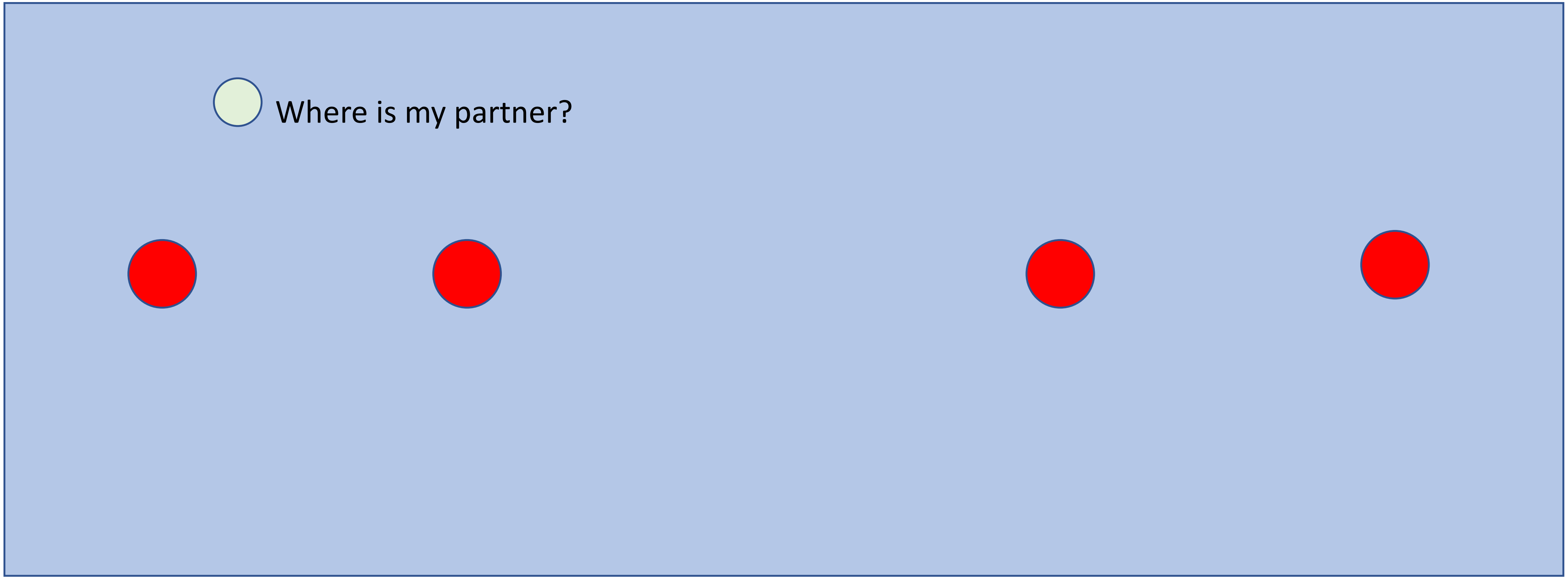
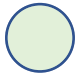
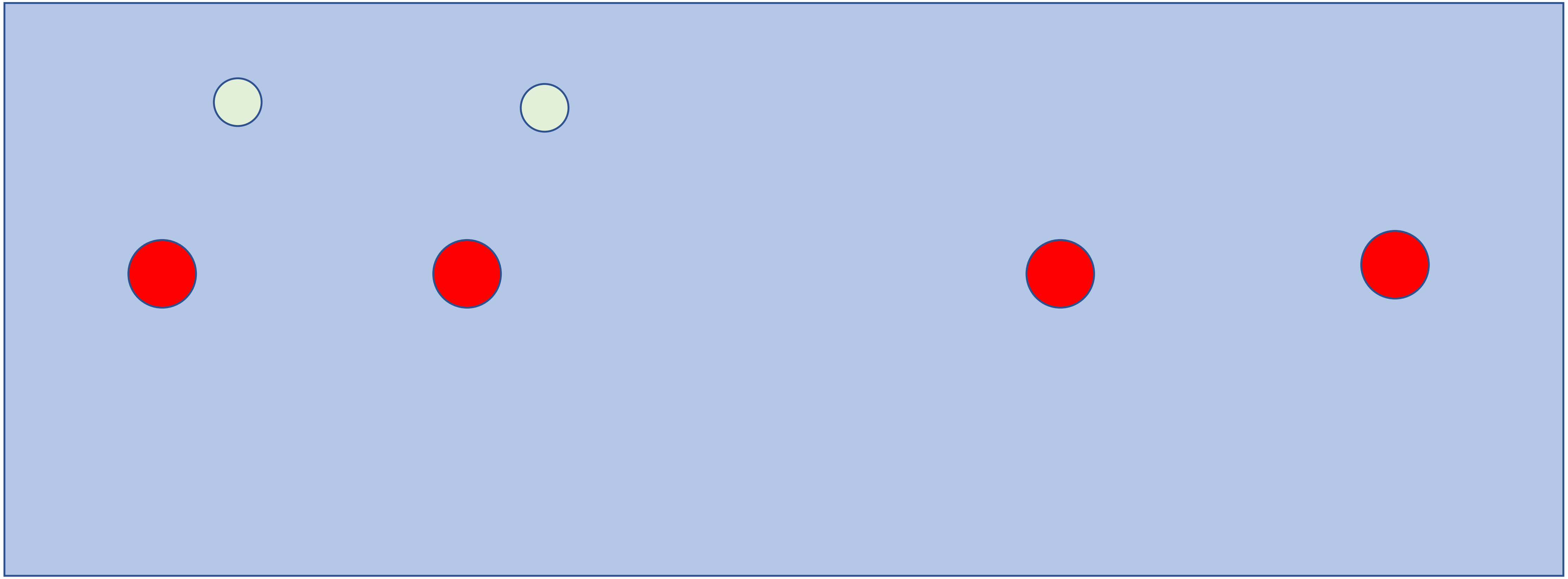


Topological quantum error correction: Measurement error

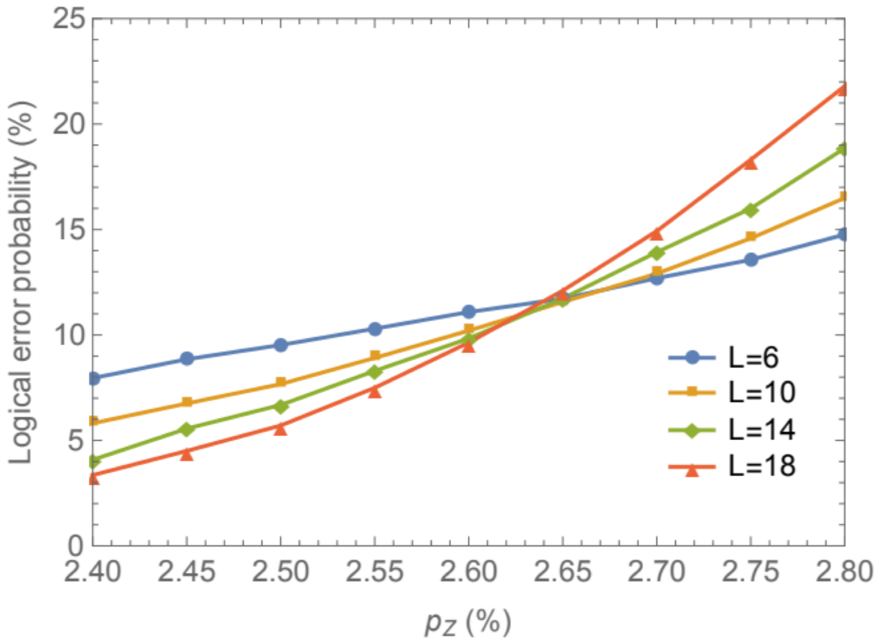
 Thermal excitations

Solution: Repeat the measurement many times



Topological quantum error correction: Complete picture

- For a $l \times l$ system, measure and locate the excitations for $O(l)$ time.
- Guess the most likely path of the excitations.
- Fuse excitations with their partners.



Below p~2.7%, error decays exponentially with l.

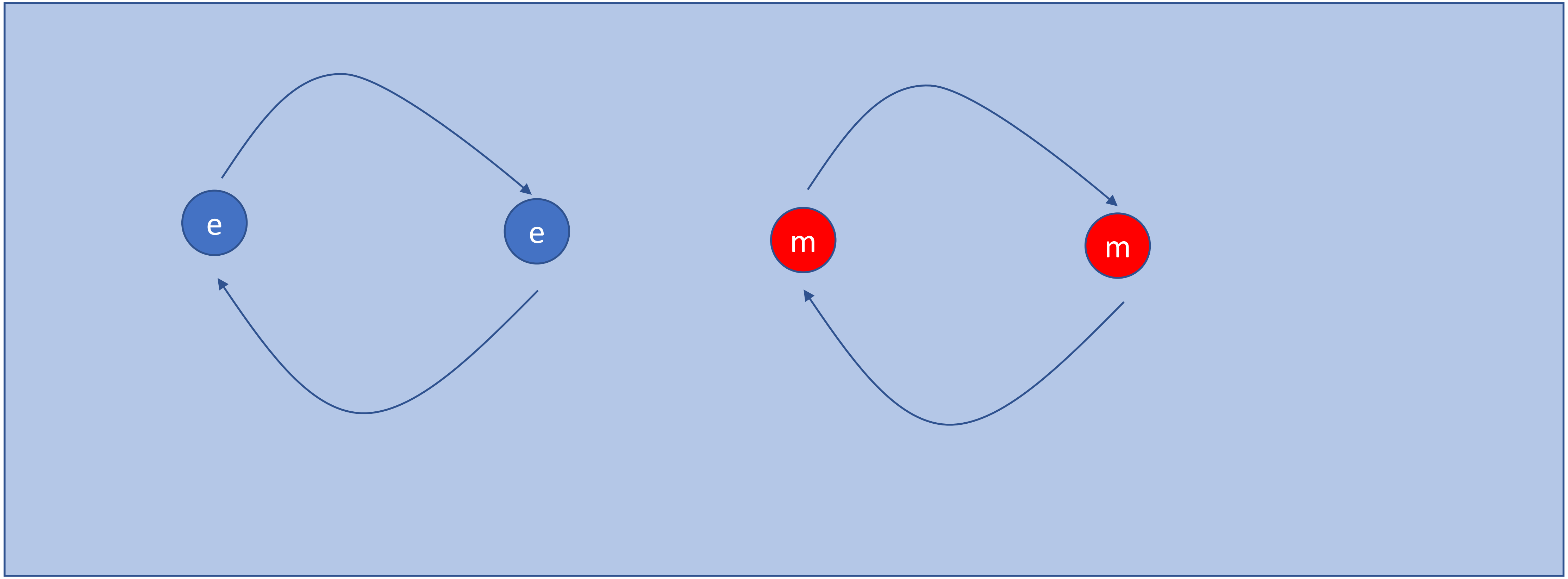
$O(l^3)$-time algorithm exists. [Nickerson and Delfosse(2018)]

From [Nickerson and Delfosse(2018)]

Topological quantum error correction: Fault-tolerant gate

- Problem: Underlying anyon model is Abelian.

Nothing happens…



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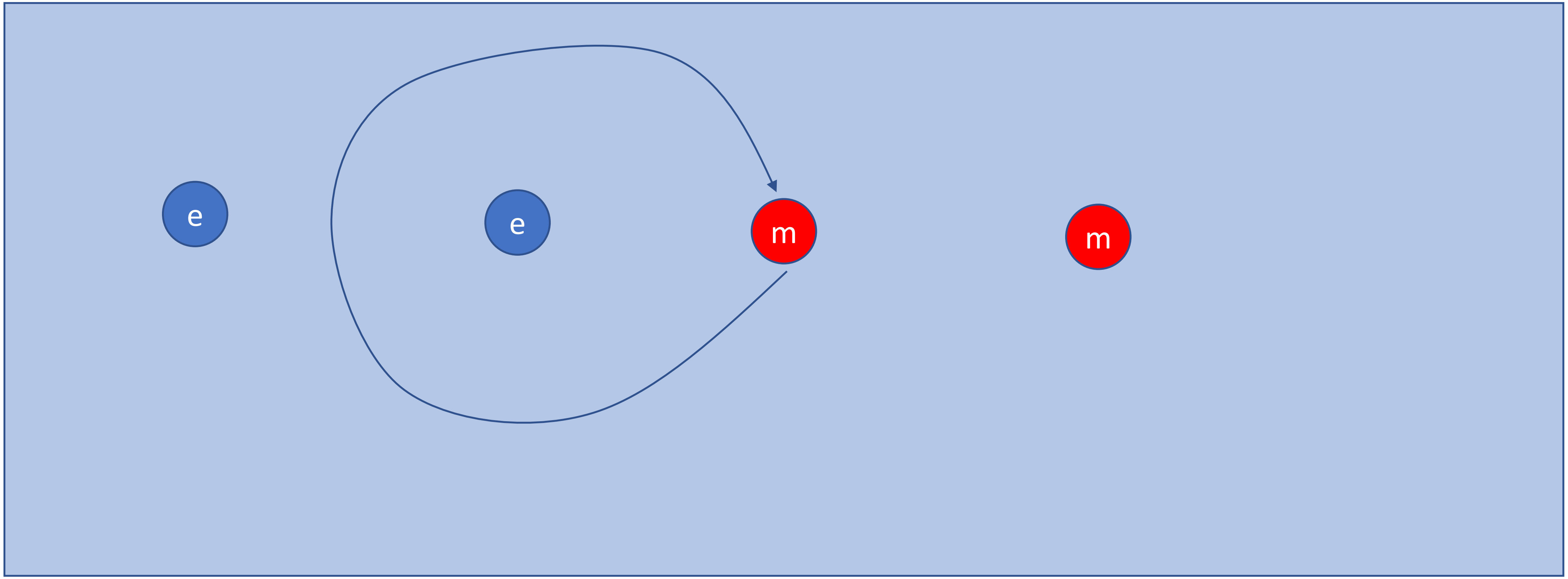
Phase of -1 is accumulated.



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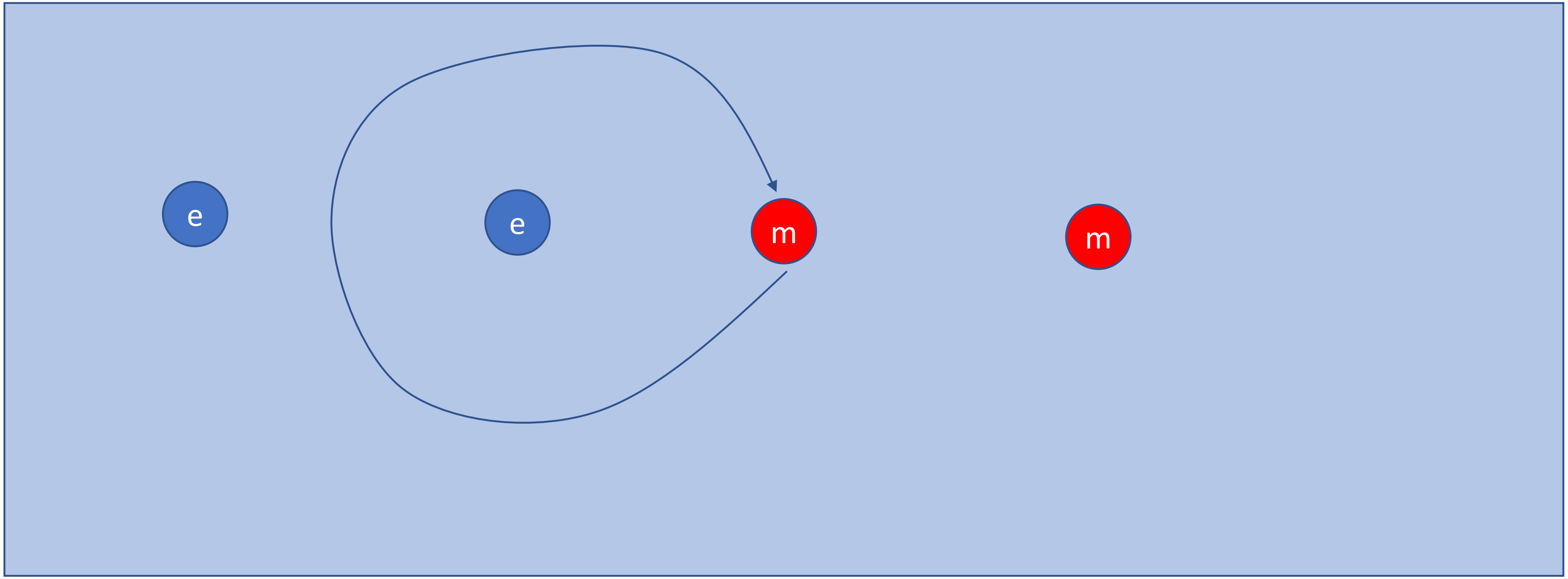
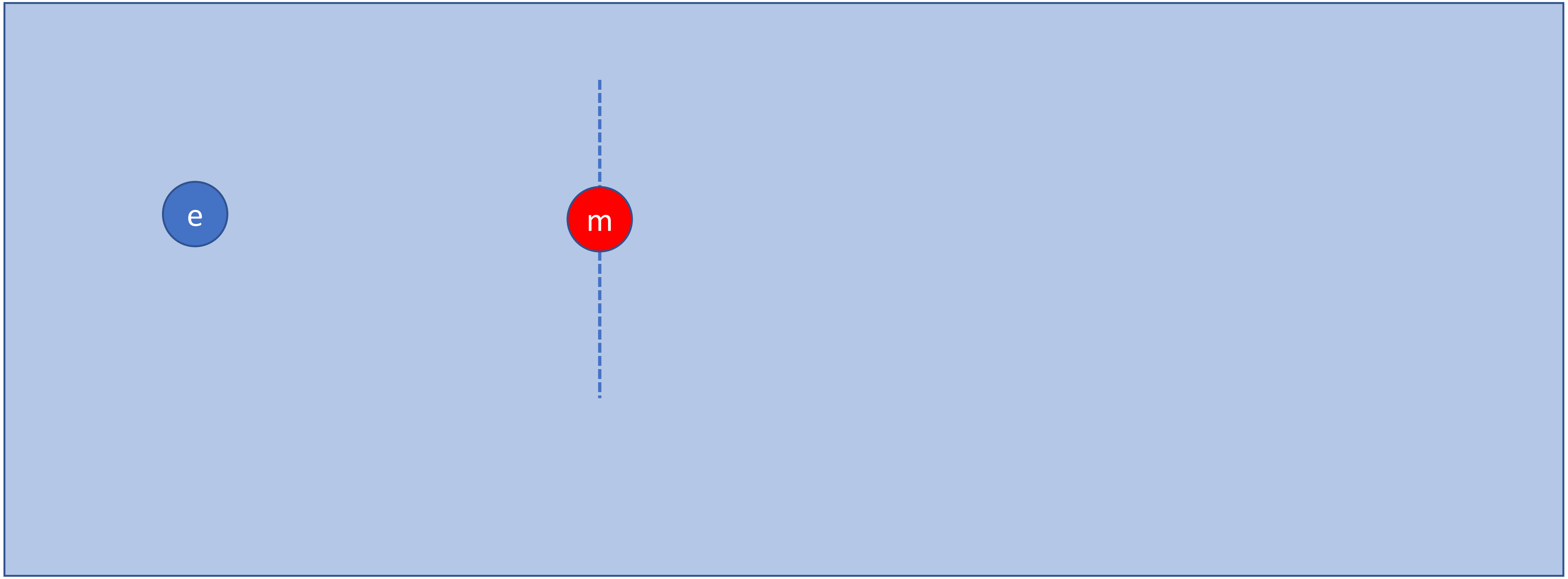
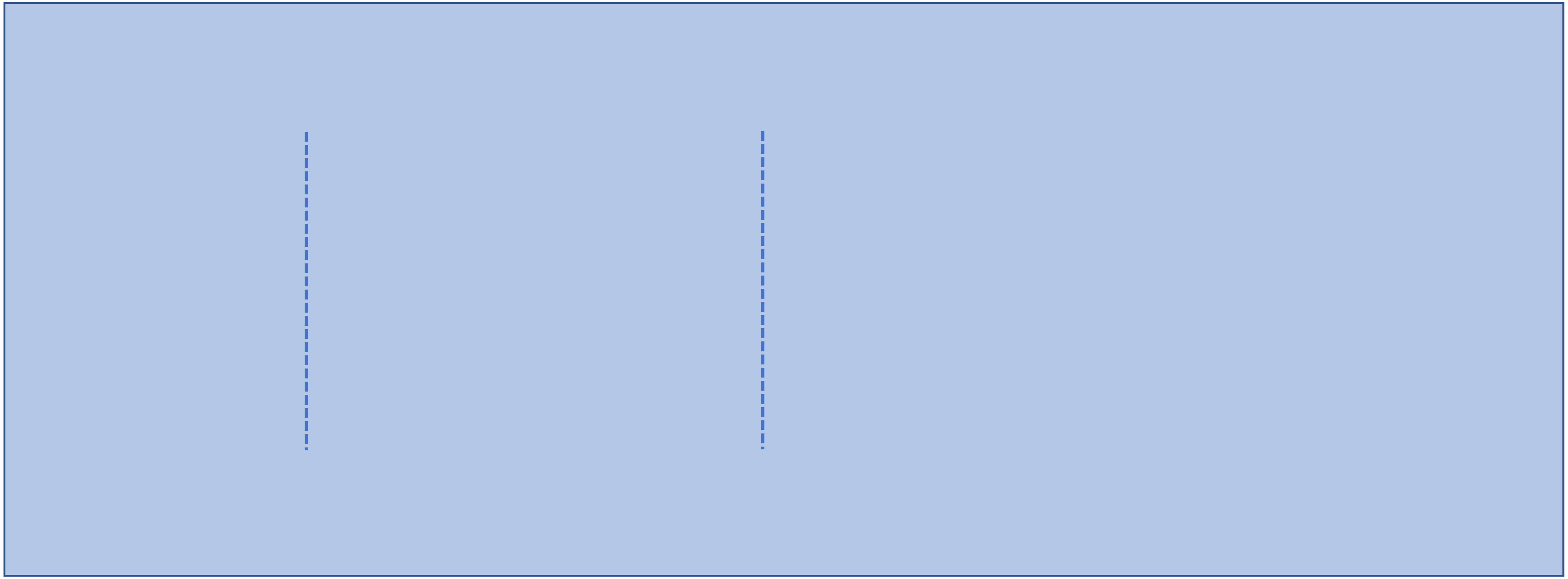


Topological quantum error correction: Defect



Topological quantum error correction: Defect

A pair of defect line encodes a qubit.
Braiding the end of defect line realizes the entire Clifford group.
[Bombin (2006)]



Summary

1. Topological quantum computation makes quantum gates robust by their physical nature.
2. While the gate sets are often not universal, there are schemes to complete the universal gate set.
3. Topological quantum computing approaches are often classified into two groups
    1. Original approach: Cool down the system in braid anyons.
    2. Active approach: Actively measure excitations and fuse them back.