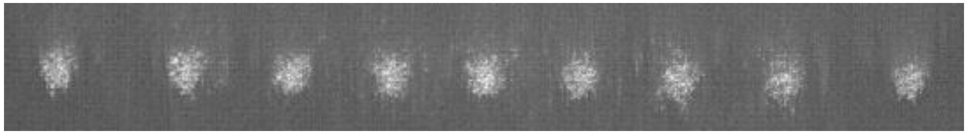
Quantum Computing

– Basic concepts and ion-trap experiment



The 8th School of Mesoscopic Physics

Introduction to Quantum Computing

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2019/05/24

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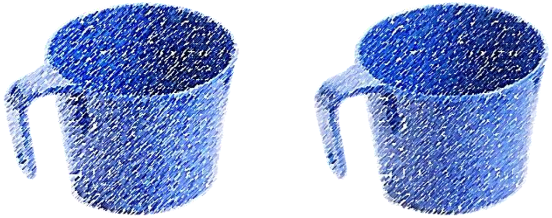
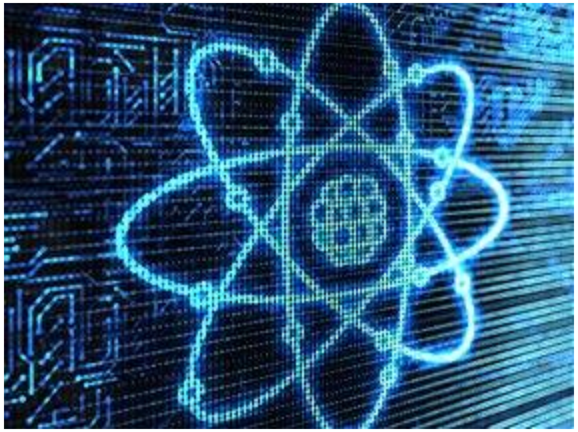
Outline

1. Two rules of Quantum mechanics



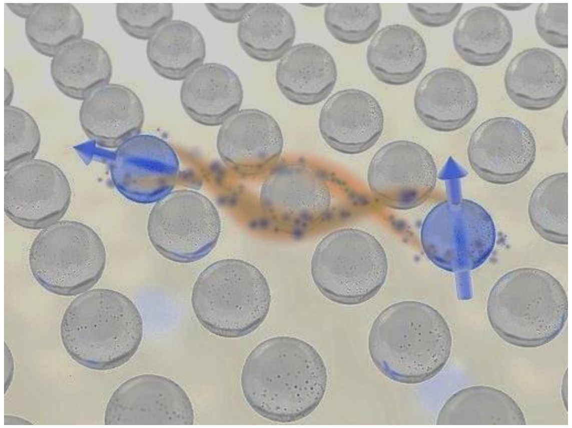
2. What is Quantum computing ???

- Quantum superposition: parallel computation
- Quantum computer:
  Quantum superposition + entanglement
- Quantum entanglement: universal logic gate



3. Basic physics for Quantum computing

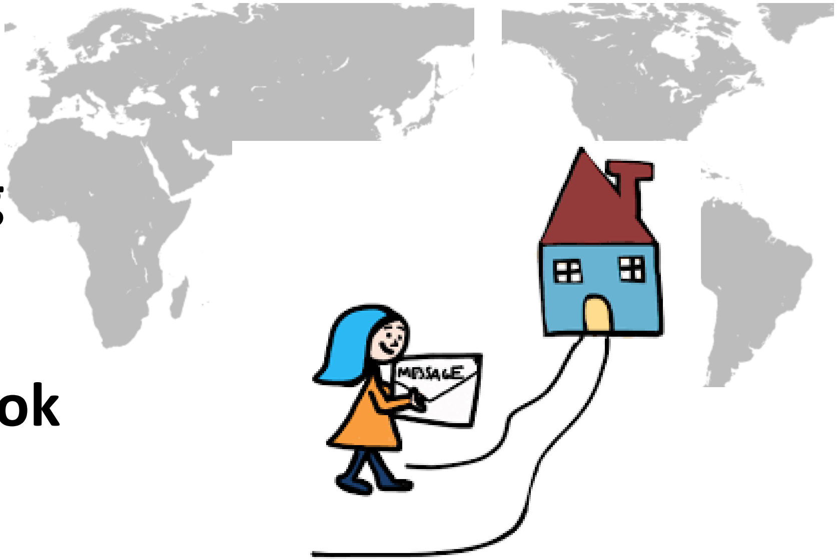
- Light-matter interaction
- Bloch sphere picture
- Completeness of single qubit rotation
- Rabi and Ramsey oscillation and T_1, T_2
- Two qubit gate and Quantum algorithm



Outline

4. Ion trap Quantum computing

- Why ion trap quantum computing
- Ion trapping (Doppler cooling)
- 171Yb+ qubit, preparation and detection
- Ion trap experimental setup
- Qubit manipulation (single qubit + two qubit gate)
- State of art experiments



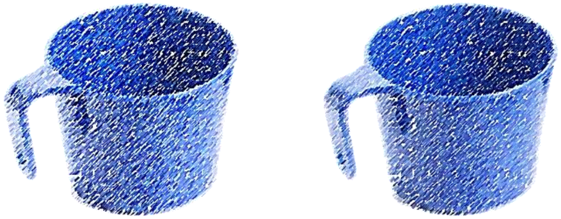
5. World trend of Quantum computing

6. Take home message and outlook

1. Quantum mechanics

= Wave mechanics + statistics + measurement action

Rule #1



[0]   &   [1]

- Quantum objects are waves and can be in states of superposition "Qubit": [0] & [1]

ex) 50%[0] + 50%[1]

Rule #2

[0]   or   [1]

- Rule #1 holds as long as you don't look !!!

with probability !!

ex) 50% for being [0]
50% for being [1]

2. What is Quantum computing ???

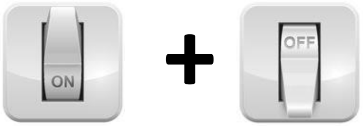
- Quantum superposition: parallel computation

Superposition

- What if we store information in quantum systems?

- classical bit: 0 or 1

- quantum bit: $a[0] + b[1]$



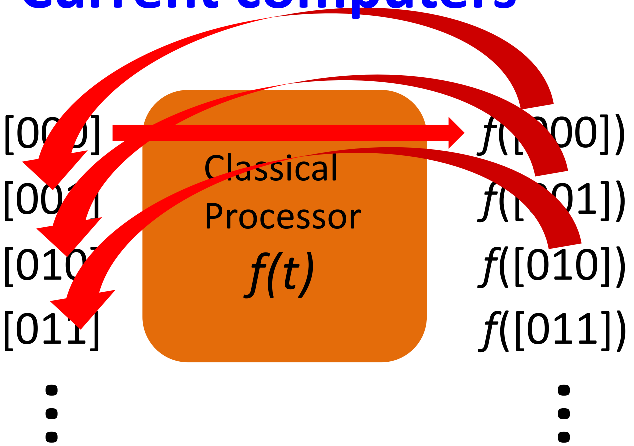
- More than one qubit ?

- classical bit: 00 or 01 or 10 or 11

- quantum bit: [00] + [01] + [10] + [11]

- Superposition: parallel computation

Current computers



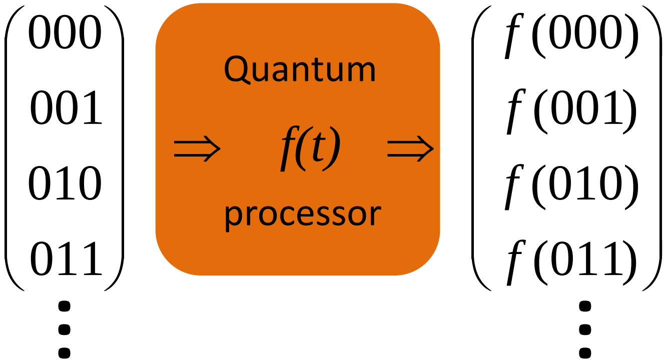
[000]
[001]
[010]
[011]
⋮

Classical
Processor
f(t)

f([000])
f([001])
f([010])
f([011])
⋮

CPU clock speed (1GHz)

Quantum computers



$$\begin{pmatrix} 000 \\ 001 \\ 010 \\ 011 \end{pmatrix} \quad \Rightarrow \quad f(t) \quad \Rightarrow \quad \begin{pmatrix} f(000) \\ f(001) \\ f(010) \\ f(011) \end{pmatrix}$$

⋮ ⋮

Quantum
f(t)
processor

Massive quantum parallel computation

Example: N=3 qubits

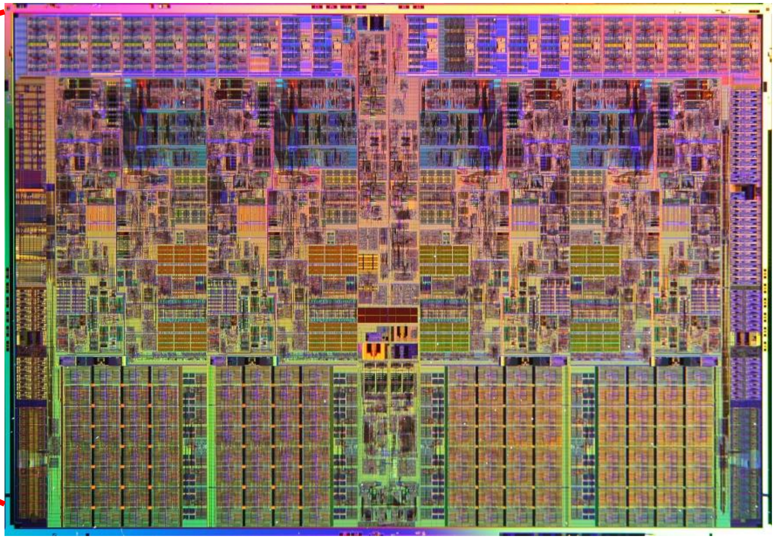
$$\begin{aligned} \Psi = \; & a_0 [000] + a_1 [001] + a_2 [010] + a_3 [011] \\ & a_4 [100] + a_5 [101] + a_6 [110] + a_7 [111] \end{aligned}$$

GOOD NEWS…

quantum parallel processing on 2^N inputs

- Conventional computer: sequential computation

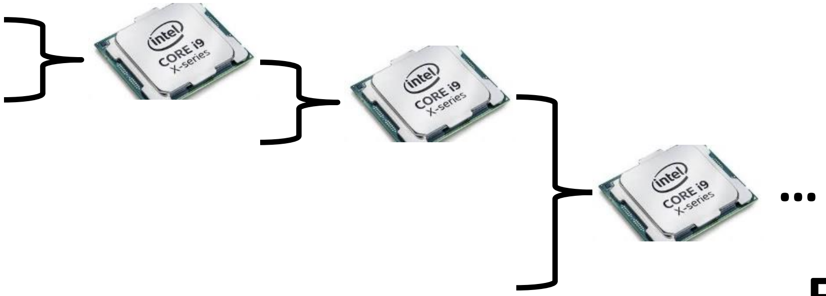
Conventional computer



Computing: Logic operation

- Billions of input and computation

0
0
1
1
0
…

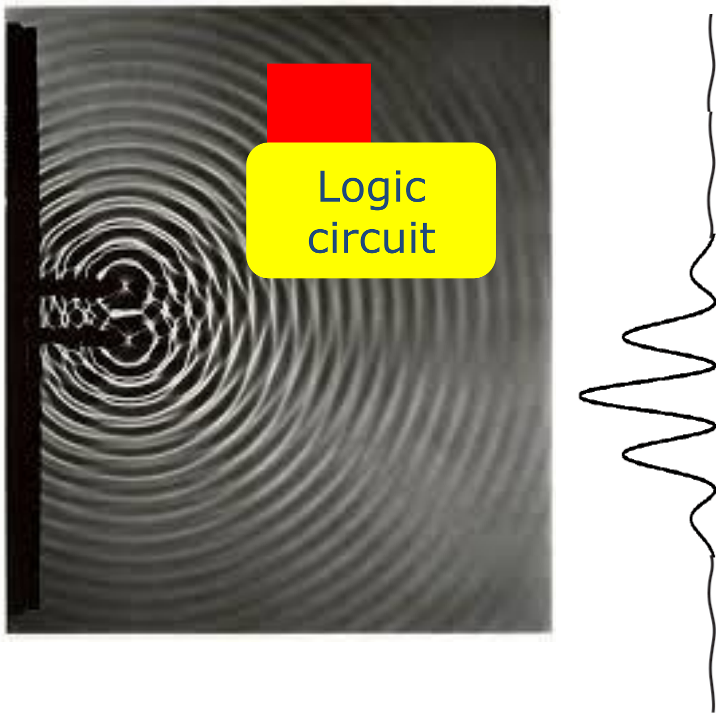


Billions of transistors

CPU clock speed: 1GHz = 1 ns
Billions of computation = 1 sec

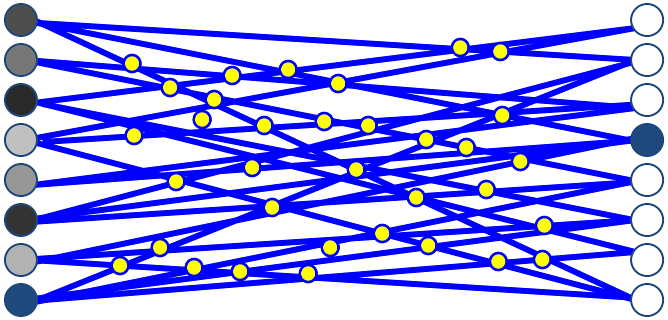
Factoring number: takes too long

- Quantum computer: parallel computation



...GOOD NEWS!

quantum interference
(Wave mechanics)



Quantum logic gate

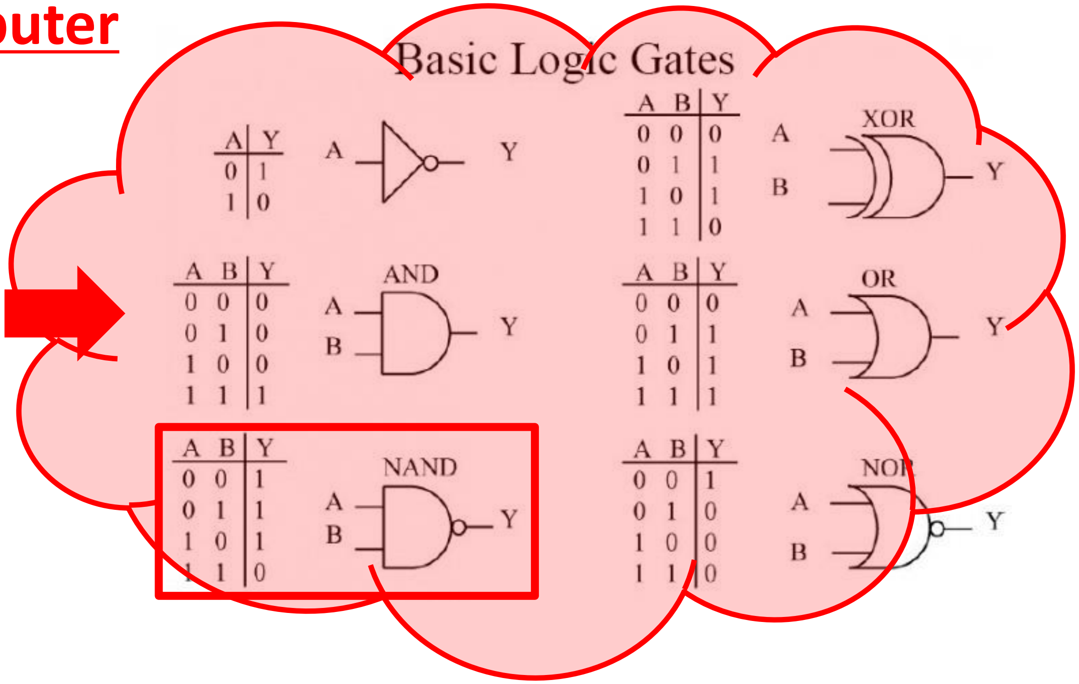
Massive parallel computation

Quantum circuits

Deutsch (1985)
Shor (1994) fast number factoring $N = p \times q$
Grover (1996) fast database search

- What computation means ?

Universal logic gate for computer



A	Y
0	1
1	0

XOR

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

AND

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR

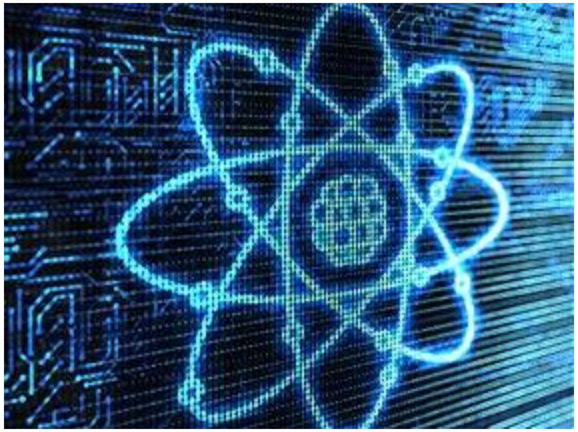
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NAND

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0



1) $[0] \rightarrow a[0] + b[1]$

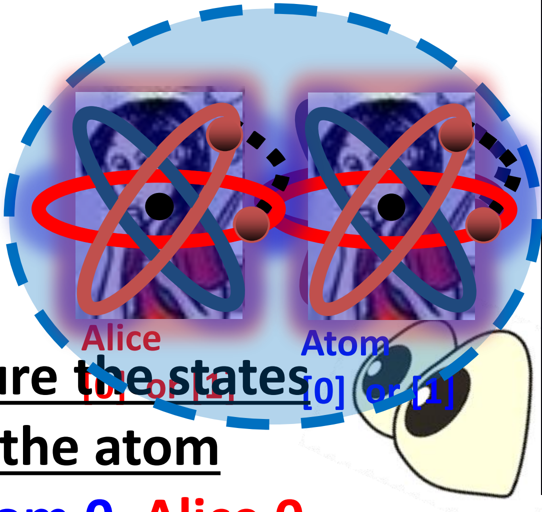
Quantum superposition
(Single qubit gate)

2) $[00] \rightarrow a[00] + b[11]$

Quantum entanglement
(Two qubit gate)

- Quantum entanglement: universal logic gate

2) [00] → [00] + [11]



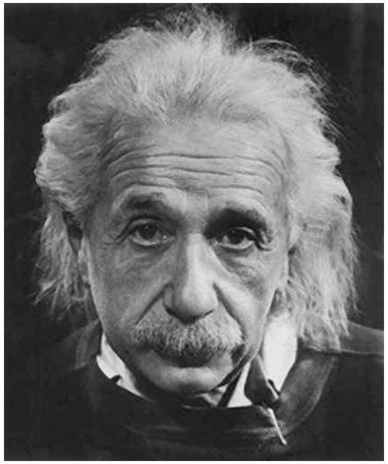
Measure the states
of the atom

Atom 0, Alice 0
Atom 1, Alice 1

→ Information of atom is the same as the one of Alice

→ Entangled information is not limited by the speed of light
→ Quantum teleportation

Quantum Entanglement

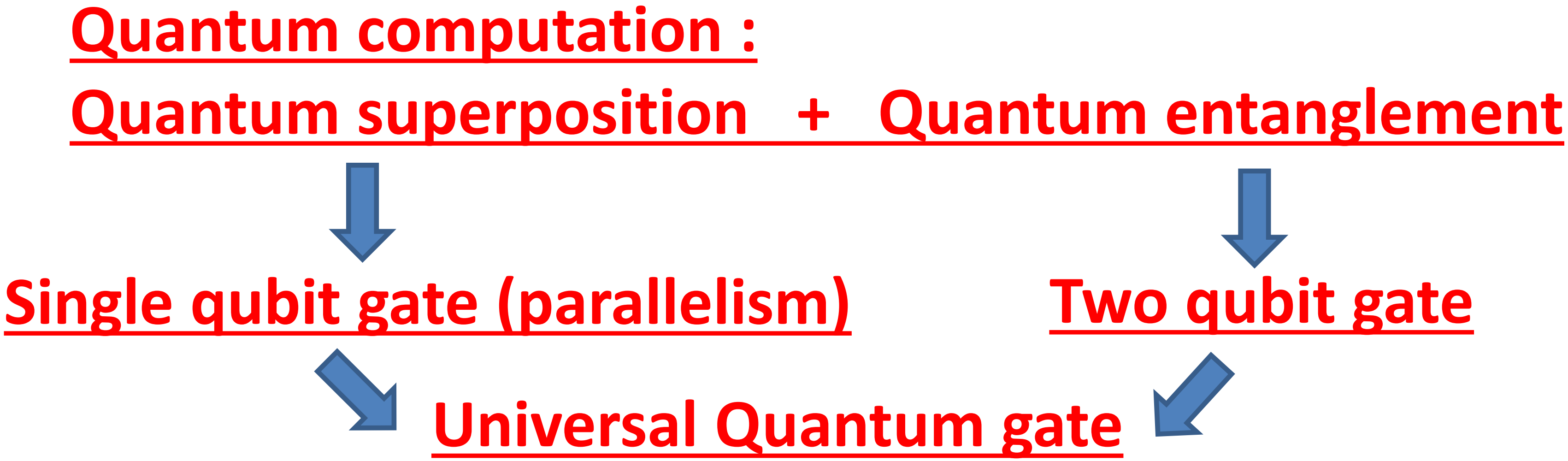


“Spooky action-at-a-distance”

(A. Einstein)

“God does not play dice”

(A. Einstein)



Quantum computation :

Quantum superposition + Quantum entanglement

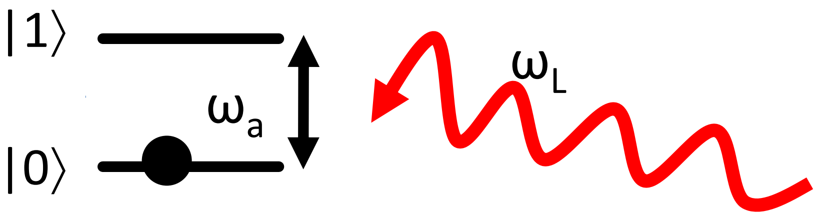
Single qubit gate (parallelism)

Two qubit gate

Universal Quantum gate

3. Basic physics for Quantum computing

- Light-matter interaction



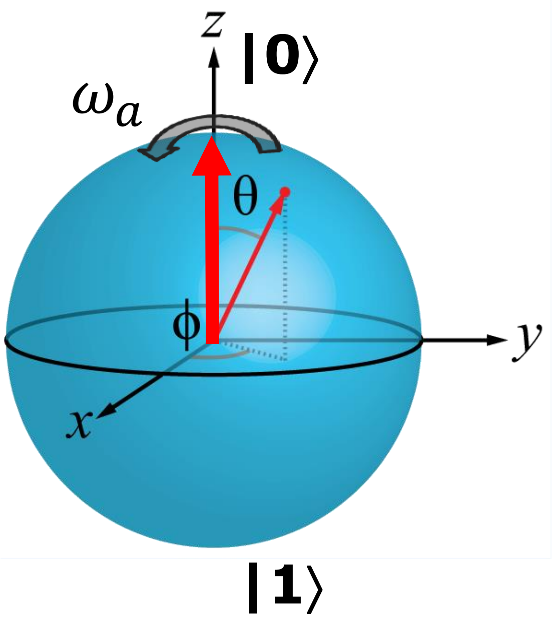
$$H_{Int}(t) = -\vec{\mu}\cdot\vec{B}(t)$$

$$H_{int}(t) = -\vec{p}\cdot\vec{E}(t)$$

- 1st order interaction with environment
: ex) electric and magnetic dipole interaction

- Two lowest ground states $|0\rangle$ and $|1\rangle$ compared to k_BT
: ex) Zeeman, atomic orbitals, superconducting RLC circuit

➔ **We need good two level Quantum systems**



[Lab frame]

Bloch sphere

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$)$

(t)

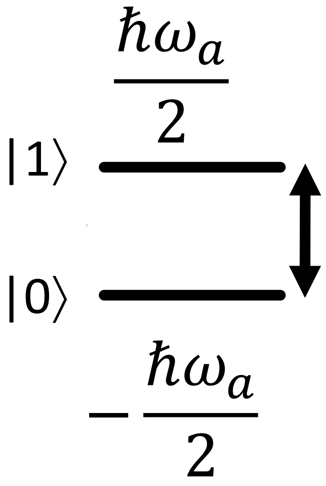
$B(t)$

$$\vec{\mu} = \frac{g_e e}{2m_e}\vec{S} = \gamma\vec{S}$$

$indep\ B\ field$

$$H_{int}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\gamma\frac{\hbar}{2}B_z\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\gamma\frac{\hbar}{2}B_z\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

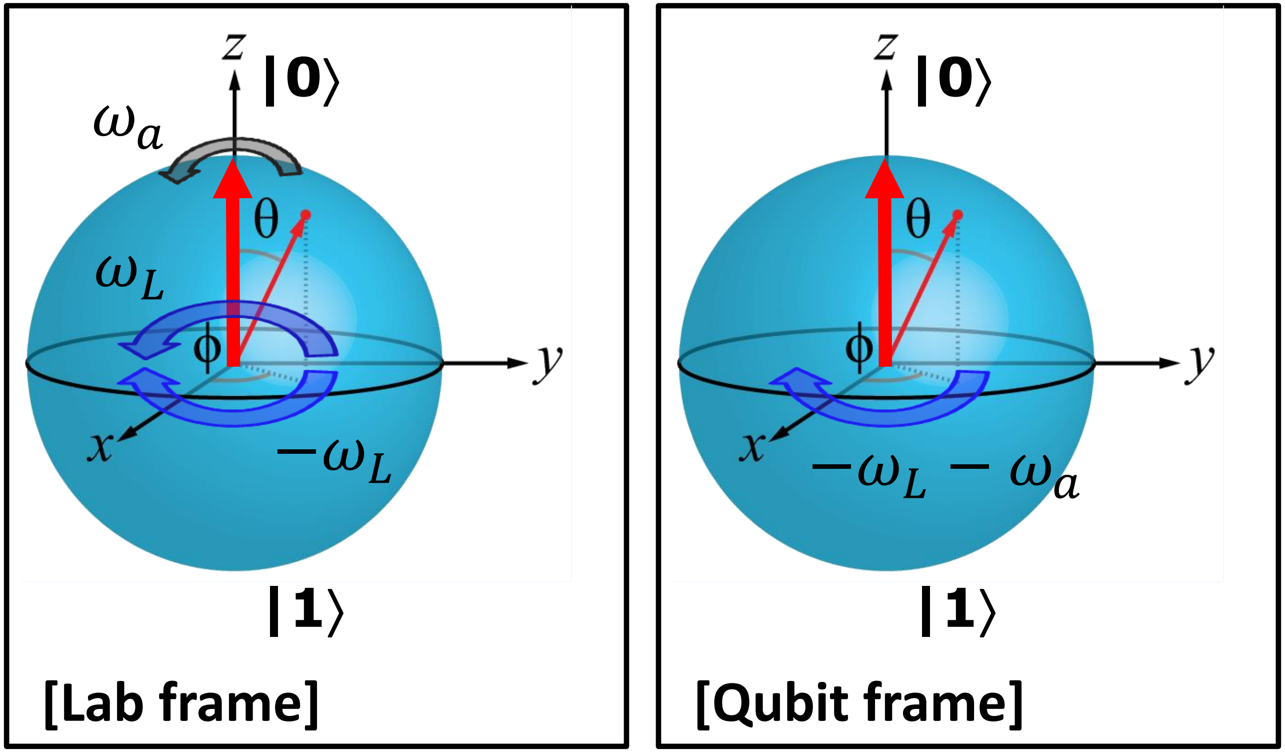
$$H_{int}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\gamma\frac{\hbar}{2}B_z\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = +\gamma\frac{\hbar}{2}B_z\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



$$\omega_a = \gamma B_z$$

2) $\vec{B} = (B_1 \cos(\omega_L t), 0, 0)$: *time dep & in − plane B_1 field*

$$= (B_1 \cos(\omega_L t), B_1 \sin(\omega_L t), 0) + (B_1 \cos(\omega_L t), -B_1 \sin(\omega_L t), 0)$$



➔ Rotating Wave Approximation: fast oscillating term averages out

➔ $B_1\,cos(\omega_L t)$ term becomes time-independent

$$
\begin{aligned}
H_{int}(t) &= -\vec{\mu}\cdot\vec{B}(t) \\
&= -\gamma\vec{S}\cdot\vec{B}(t) \\
&= -\gamma\frac{\hbar}{2}\vec{\sigma}\cdot\vec{B}(t)
\end{aligned}
$$

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

2) $\vec{B} = (B_1, 0, 0): time\ indep\ \&\ in - plane\ B_1\ field\ @\ qubit\ frame$

$$H_{int}\begin{pmatrix}1\\0\end{pmatrix} = -\gamma\frac{\hbar}{2}B_1\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = -\gamma\frac{\hbar}{2}B_1\begin{pmatrix}0\\1\end{pmatrix}$$

$$H_{int}\begin{pmatrix}0\\1\end{pmatrix} = -\gamma\frac{\hbar}{2}B_1\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = -\gamma\frac{\hbar}{2}B_1\begin{pmatrix}1\\0\end{pmatrix}$$

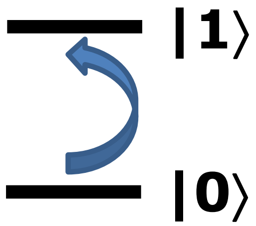
$$\hbar\Omega = \hbar\gamma B_1/2$$

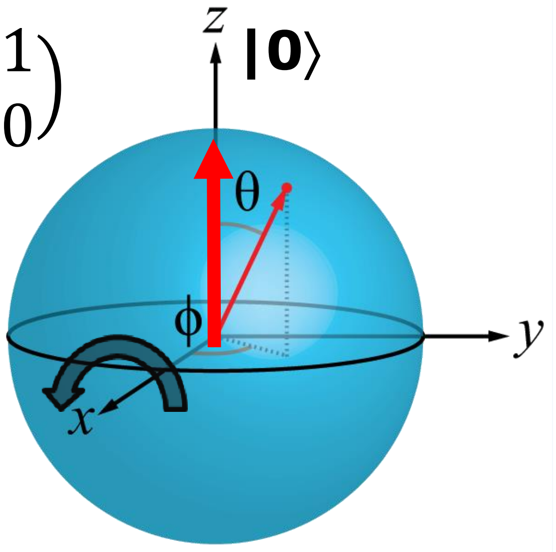
$$\Omega = Rabi\ frequency$$

How fast we rotate qubit

What if off resonance?

$$\omega_L - \omega_a = \delta(detuning)$$





[Qubit frame]

- Single qubit rotation

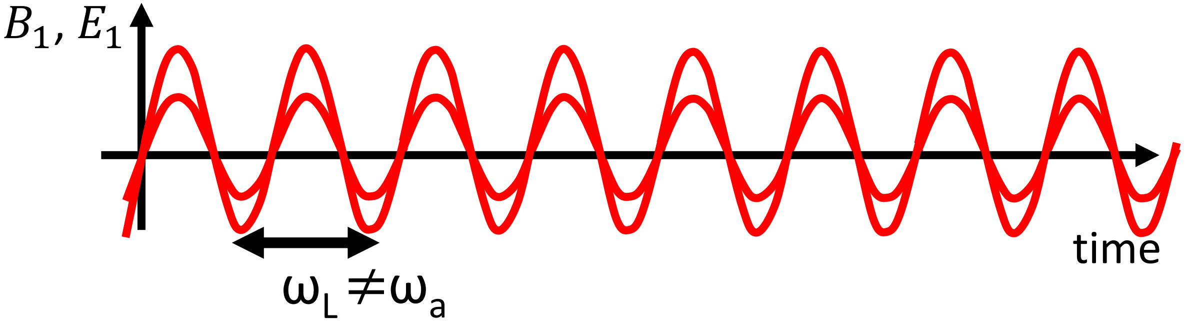
- Coherent driving source
: single freq sine wave
Ex) μ-wave: 0.3 to 300GHz
400 nm laser: 750 THz
1000 nm laser: 300 THz

Quiz: What is measured Quantum state when we apply π/2 pulse ?

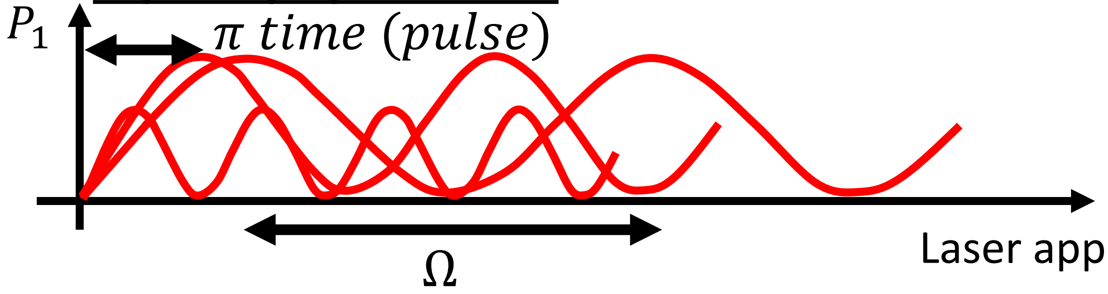
$$\hbar\Omega = \hbar\gamma B_1\langle 1|\mu|0\rangle$$

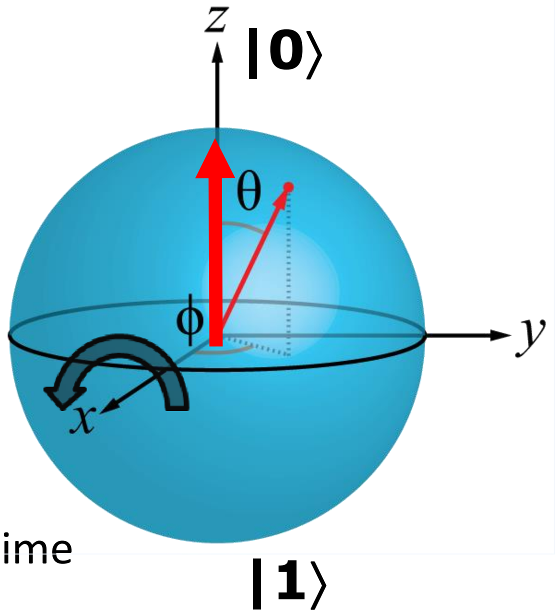
$$\hbar\Omega = \hbar\gamma E_1\langle 1|p|0\rangle$$

<Laser>



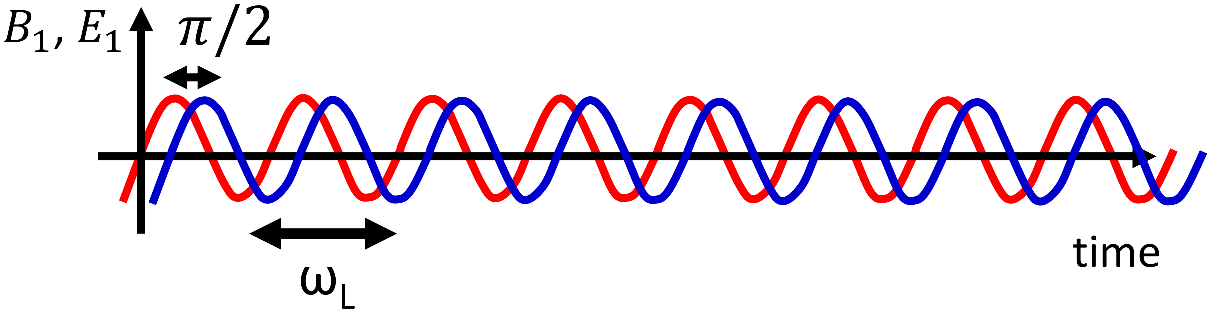
<qubit population>

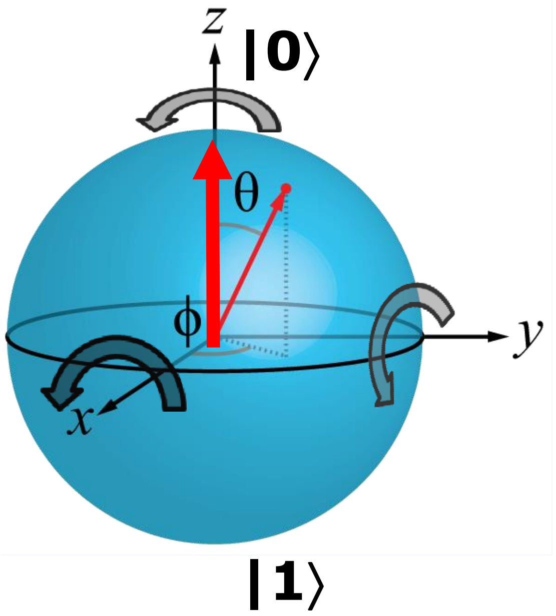




- X, Y, Z rotation

<Laser> **X rotation** **Y rotation** **Z rotation**

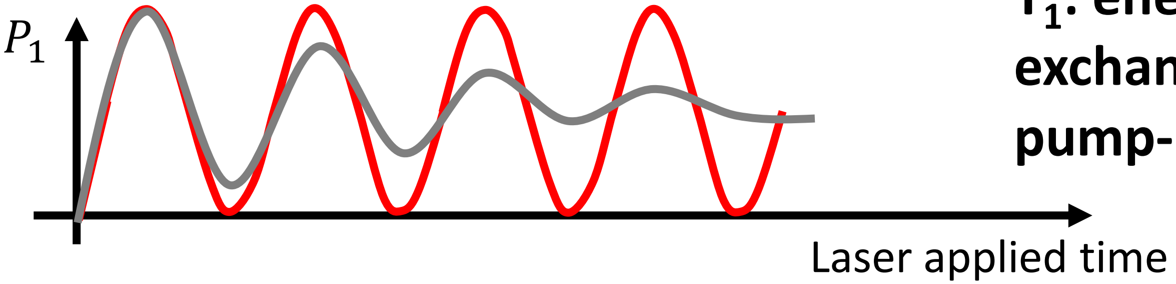




$$\phi\ in\ Z = \frac{\pi}{2}\ in\ X + \phi\ in\ Y + -\frac{\pi}{2}\ in\ X$$

- Rabi oscillation: coherent single qubit control

<Qubit>

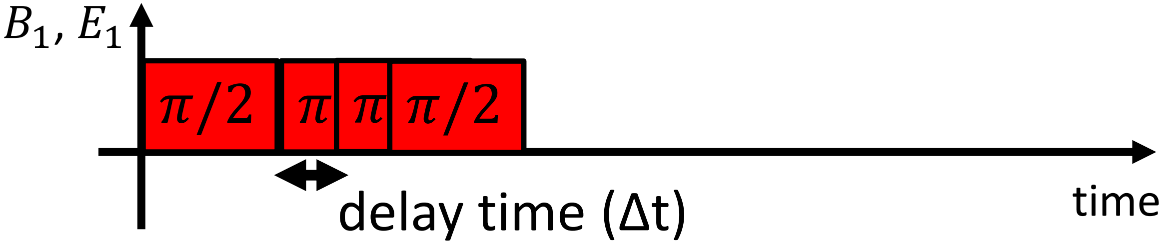


T_1: energy relaxation in z (energy exchange with environment): pump-probe experiment

T_2: phase coherence of xy-plane (how long rotation axis is good) : Ramsey experiment

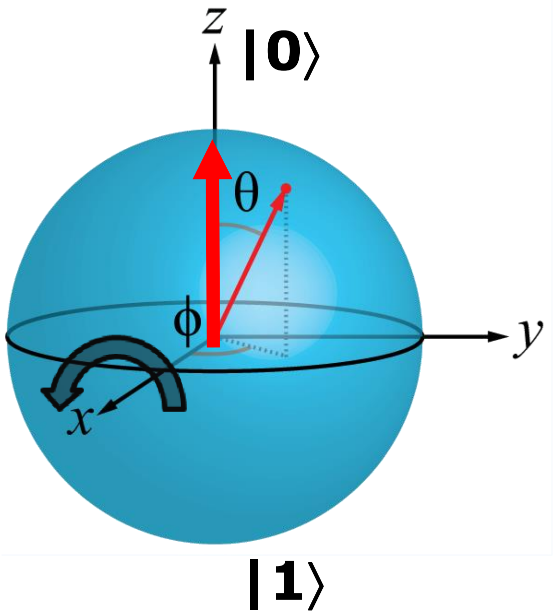
- Ramsey experiment

<Laser>

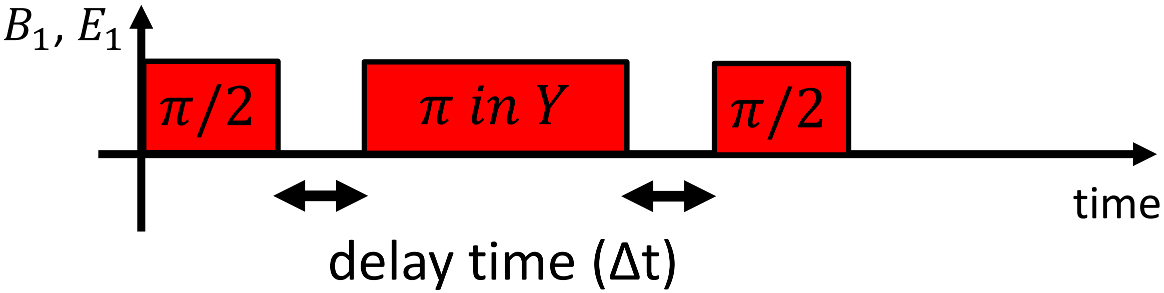


<Qubit>

Exponential decay with T_2*



- Hahn echo (low pass filter of phase noise)



Focusing qubit signal: T_2

- Two qubit gate and Quantum algorithm

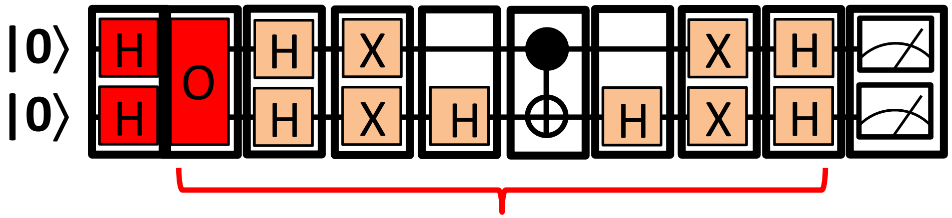
$$|00\rangle \xrightarrow{\pi/2 \text{ on } 1^{st}} (|0\rangle+|1\rangle)|0\rangle \xrightarrow{CNOT} |00\rangle+|11\rangle$$

Entanglement = CNOT

CNOT gate
$\|00\rangle \rightarrow \|00\rangle$
$\|01\rangle \rightarrow \|01\rangle$
$\|10\rangle \rightarrow \|11\rangle$
$\|11\rangle \rightarrow \|10\rangle$

- Grover search algorithm

$\sqrt{N}$ factor faster



How fast search database

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$\boxed{H} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

0 → 0+1

1 → 0-1

$$\boxed{X} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

00 → (0+1)(0+1)=00+01+10+11

→ 00+01+10-11=0(0+1)+1(0-1)

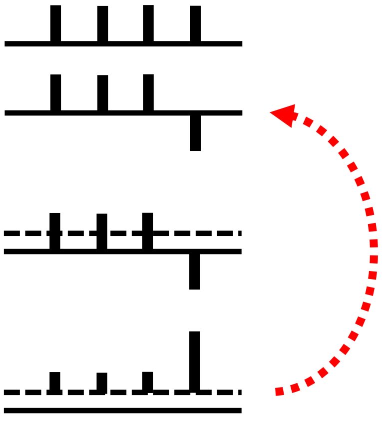
→ (0+1)0+(0-1)1

→ (1+0)1+(1-0)0=1(1+0)+0(1-0)

→ 10-01 → 11-01=(1-0)1

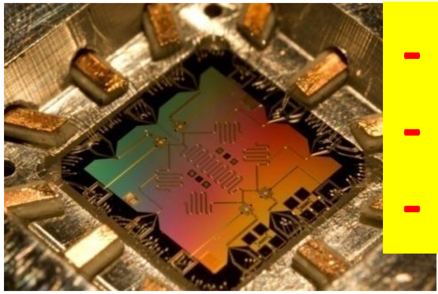
→ (1-0)(0-1) → (0-1)(1-0)

→ -11



4. Ion trap Quantum computing

Condensed matter



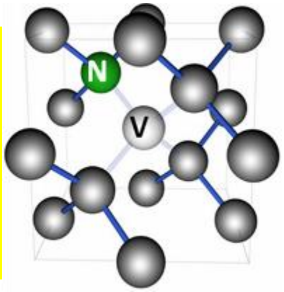
- Short coherence time (<0.1 ms)
- Difficult to control interaction
- Better scalability

Superconducting qubit

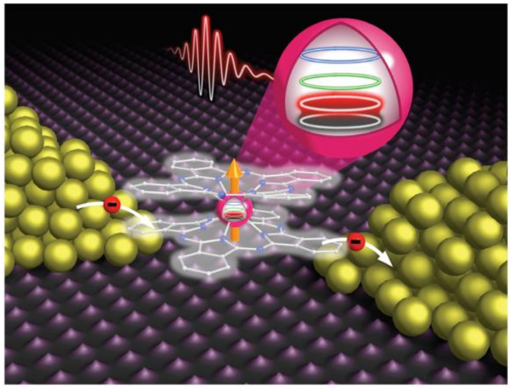
John Martinis group at UCSB

Spin/charge qubit in Q.D.

Yacoby group at Harvard

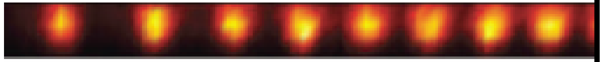


NV center in diamond

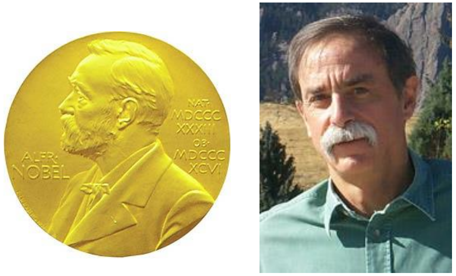


SMM in break junction

Atomic Molecular opt



Ultracold trapped ions

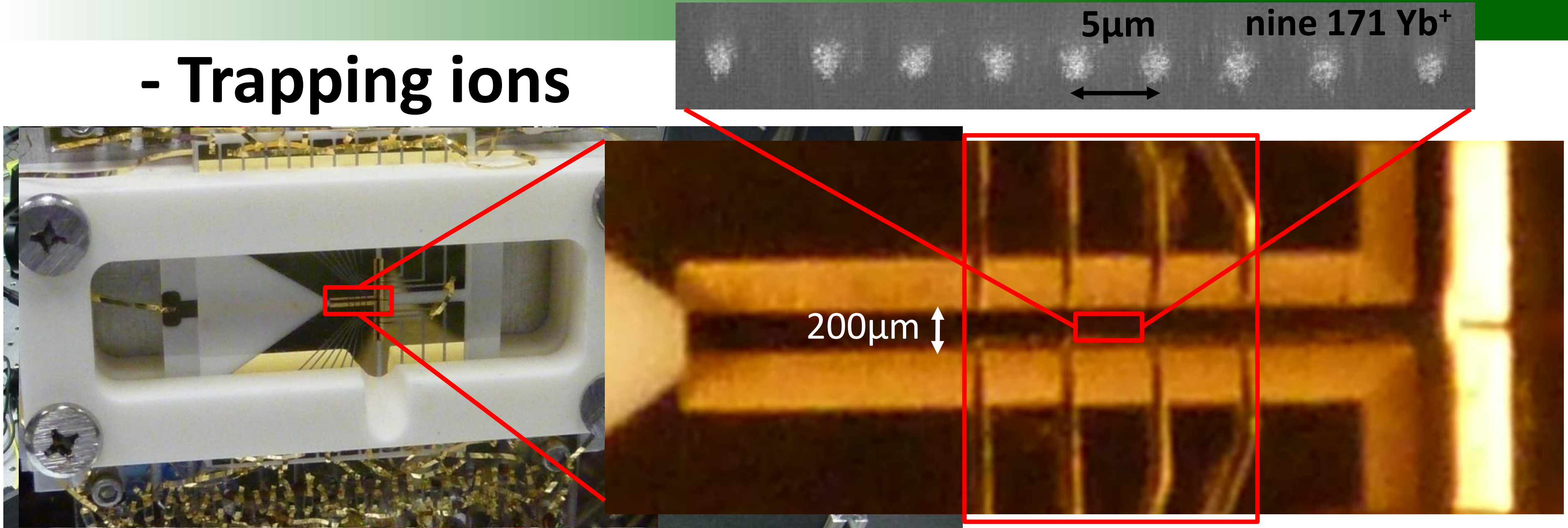


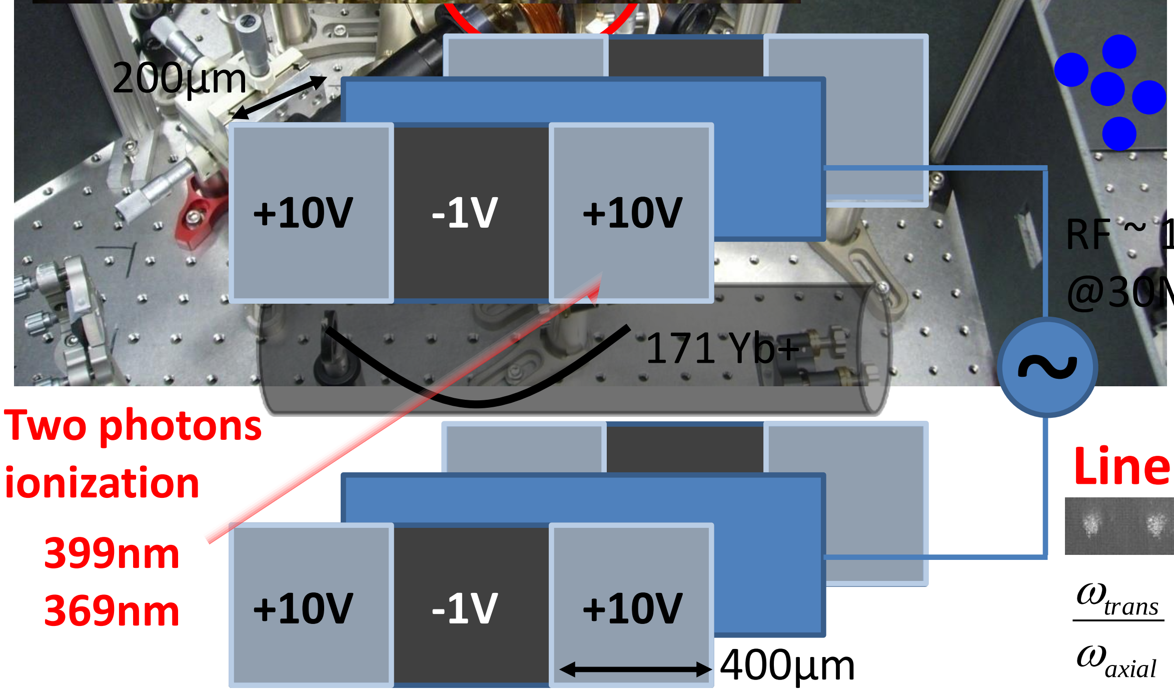
Dr. David Wineland

- Long coherence time (>1 sec)
- Easier to control interaction
- Difficult to scale (may improved)

Neutral atom qubits

- Trapping ions



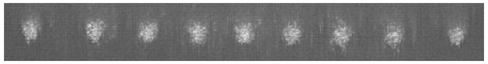


Two photons ionization

399nm

369nm

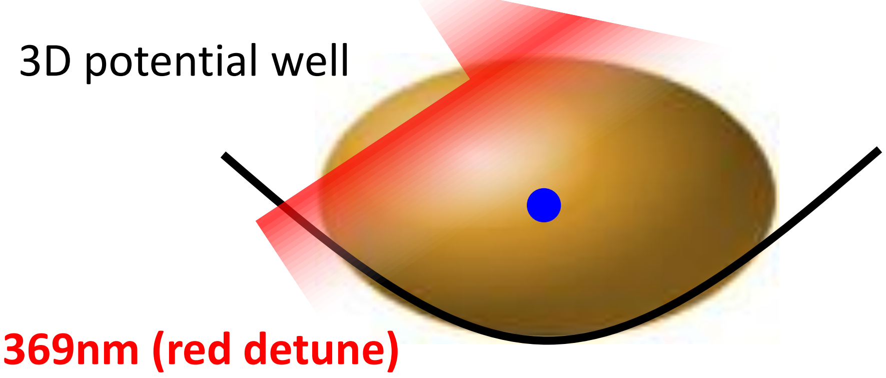
Linear chain ??



$$\frac{\omega_{trans}}{\omega_{axial}} > 0.73N^{0.86} \quad (N:\#\text{ions})$$

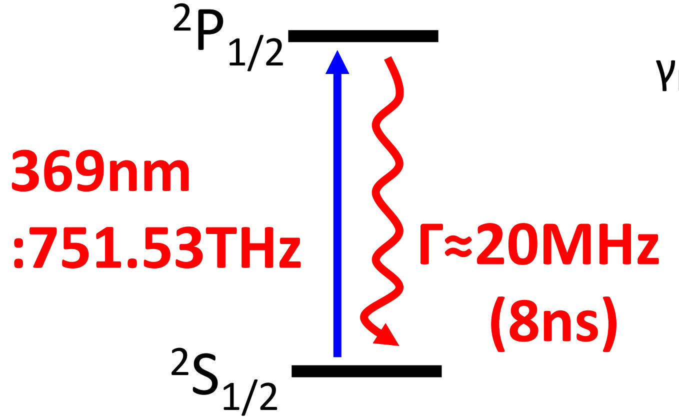
$\omega_{axial} \approx 1\text{MHz}$, $\omega_{trans} \approx 5\text{MHz}$

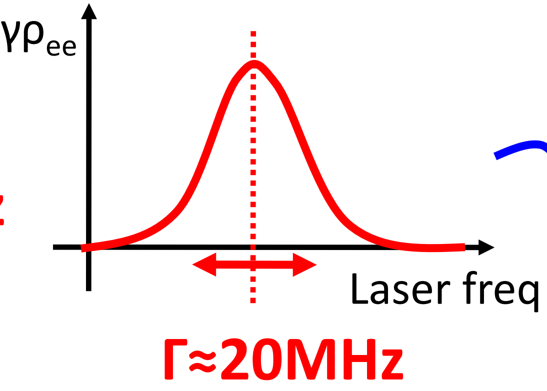
- Doppler Laser cooling

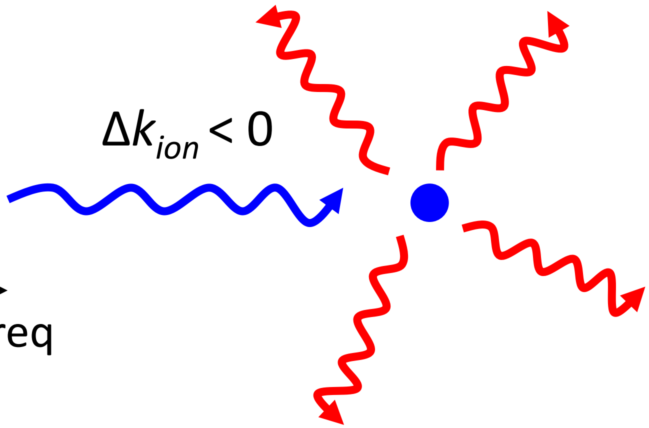


- **Energy scale**
  - Thermally evaporated (1000K ≈ 0.1eV ≈ 300 m/s)
- **Ion must be static in space**
  - Use laser

171 Yb$^+$:[Xe]4f^{14}6s^1



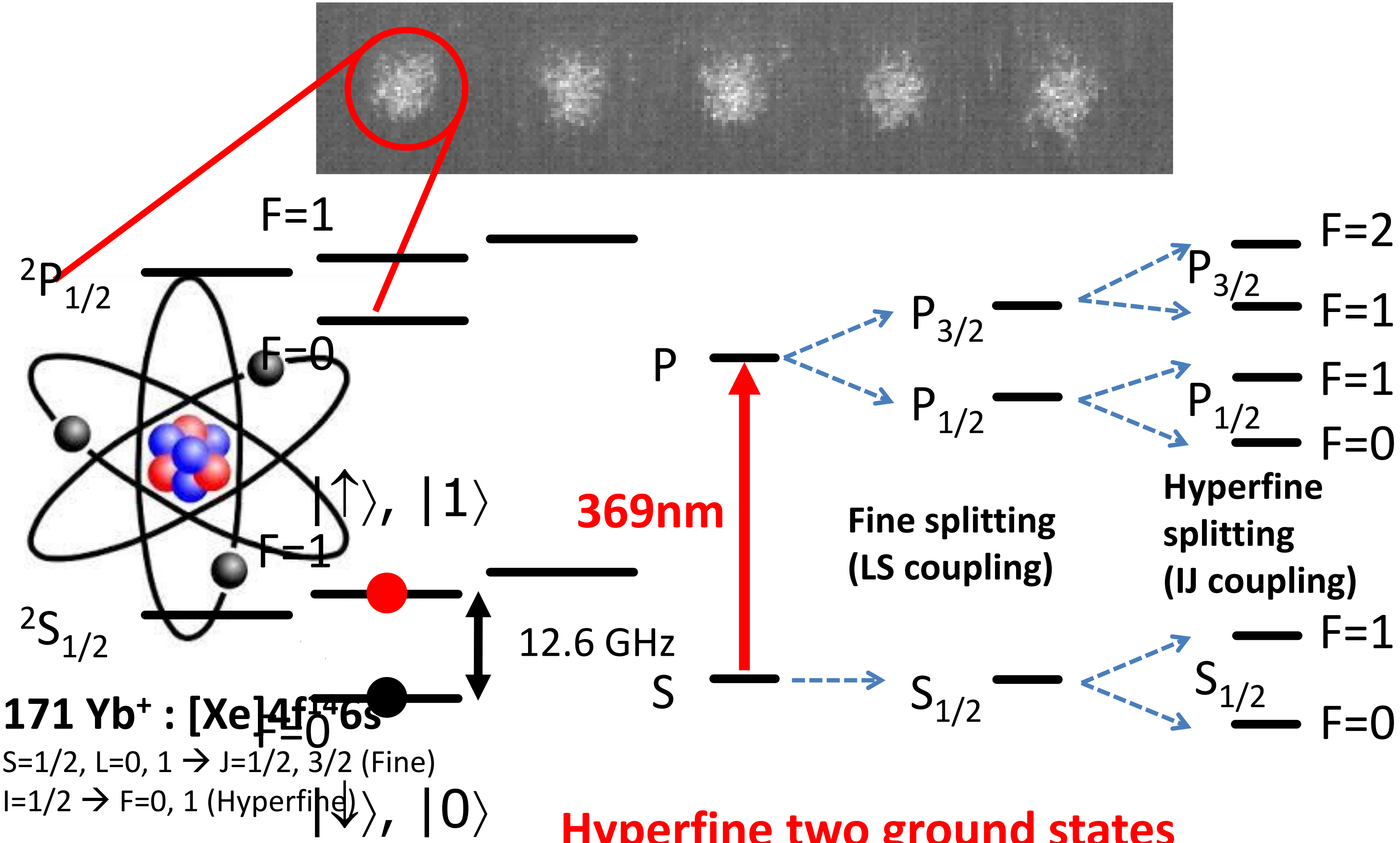




- **Doppler effect**
  - Red-detuned light freq
  - Photon scattering profile
  - Beam angled w.r.t 3-axis

- Cooling limit : 10MHz ≈ 100μK ≈ motional quanta (n≈3): Γ = nħω

- Qubit 171 Yb^+



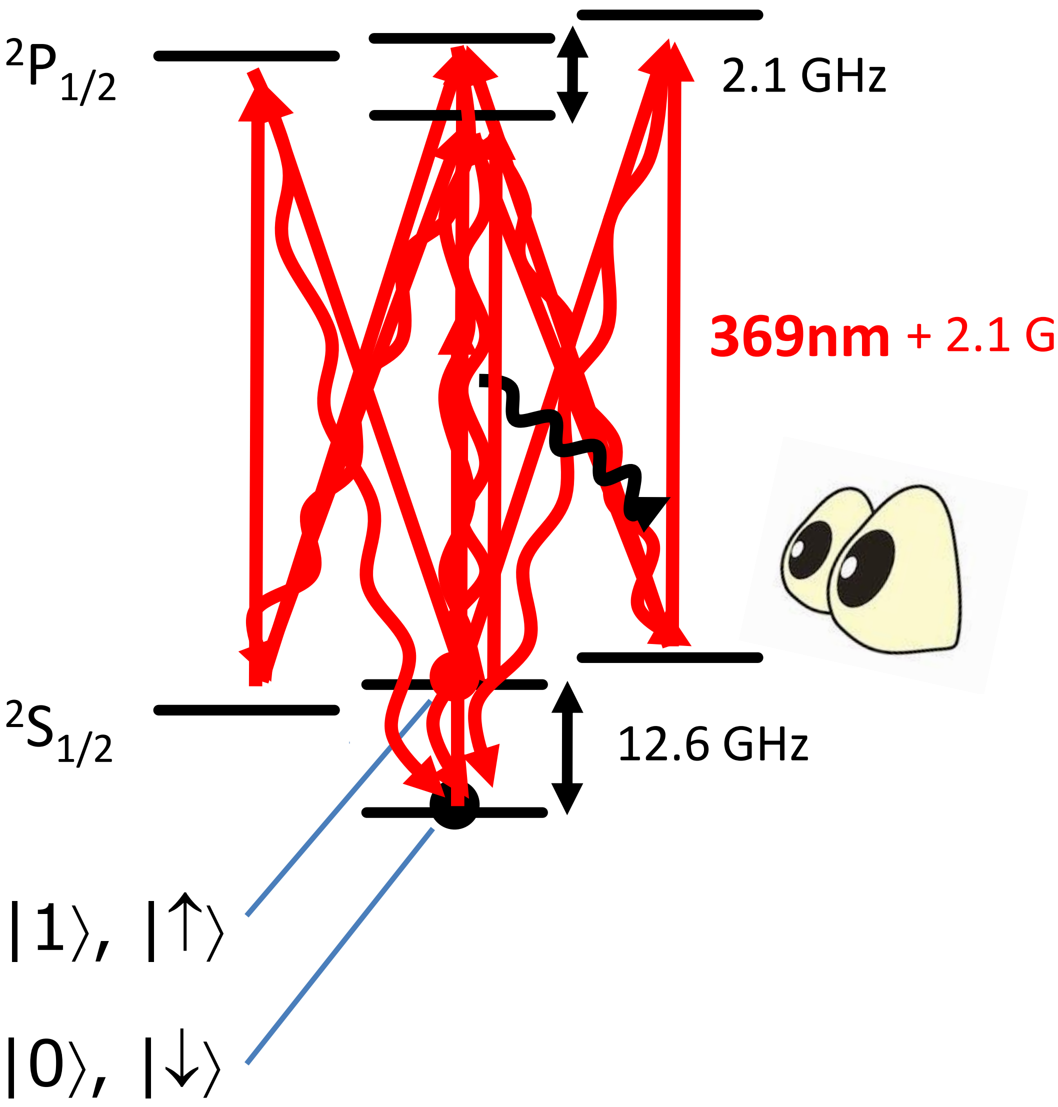
171 Yb^+ : $[Xe]4f^{14}6s$

S=1/2, L=0, 1 → J=1/2, 3/2 (Fine)

I=1/2 → F=0, 1 (Hyperfine)

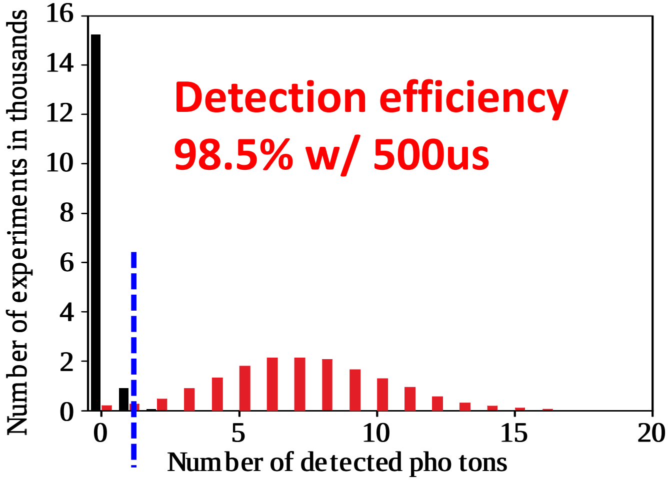
Hyperfine two ground states

- Prepare and Detect the qubit (171Yb$^+$)



- **171 Yb$^+$: [Xe]4f^{14}6s^1**
  - Hydrogen-like
- **Preparation of |↓⟩**
  - Optical pumping

Quiz: What is detection histogram looks when we apply π/2 pulse ?



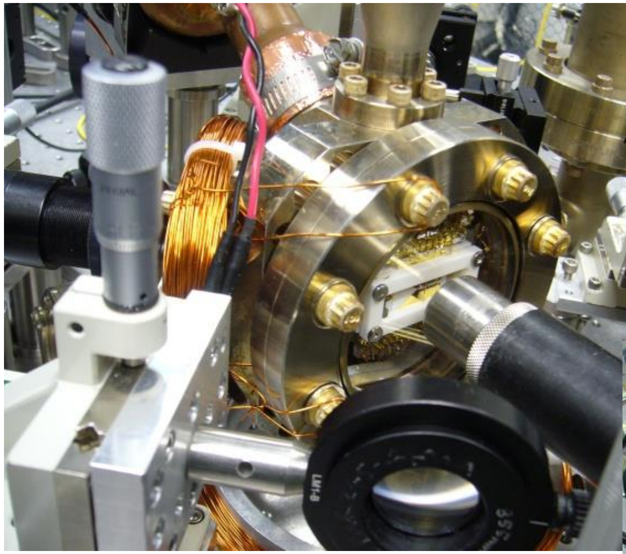
Detection efficiency 98.5% w/ 500us

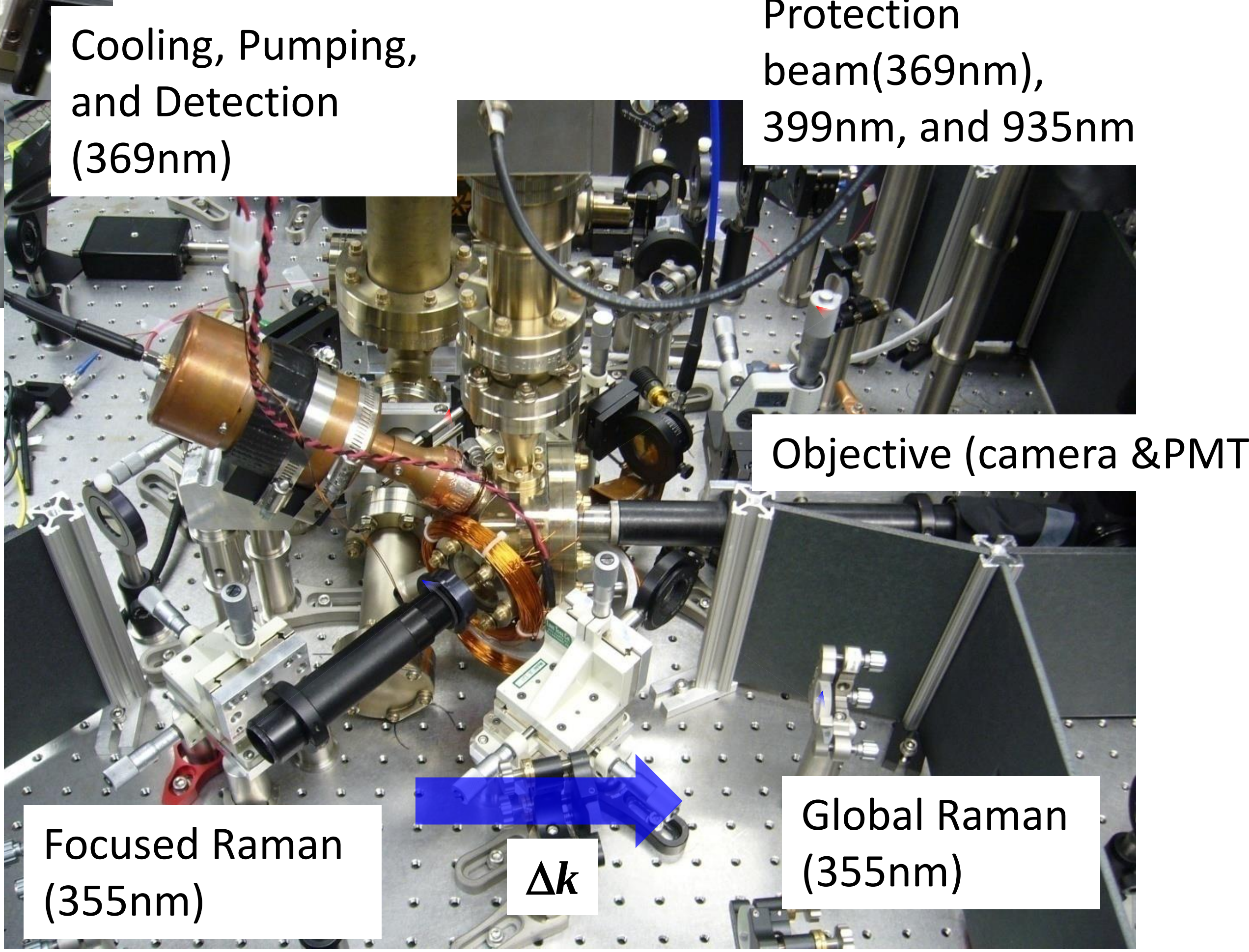
S. Olmschenk et al., PRA **76**, 052314 (2007)

- Experimental setup

- **Quantum gate by 355nm laser**

  - Raman transition





Cooling, Pumping, and Detection (369nm)

Protection beam(369nm), 399nm, and 935nm

Objective (camera &PMT

Focused Raman (355nm)

Δk

Global Raman (355nm)

Lasers :

369nm (Cooling, Pumping, Detection)

399nm (ionization)

935nm (re-pumping)

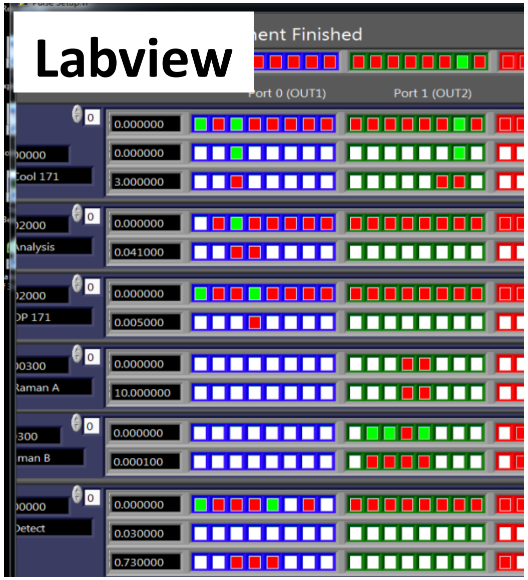
355nm (Qubit operation)

: Two beams (beatnote)

Room Temperature & Light

- Experimental Chapter

- Cooling ions (ms)
- Pump ions (μs) (prepare states)
- Quantum operation (100μs) (microwave/Raman pulsed laser)
- Detect qubits (500μs)

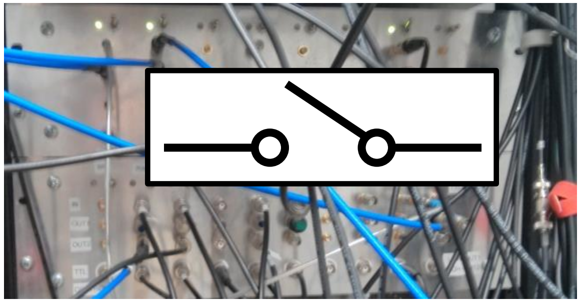

Labview

0.000000
0.000000
3.000000
0.000000
0.041000
0.000000
0.005000
0.000000
10.000000
0.000000
0.000100
0.000000
0.030000
0.730000

FPGA Digital TTL (1GHz clock)

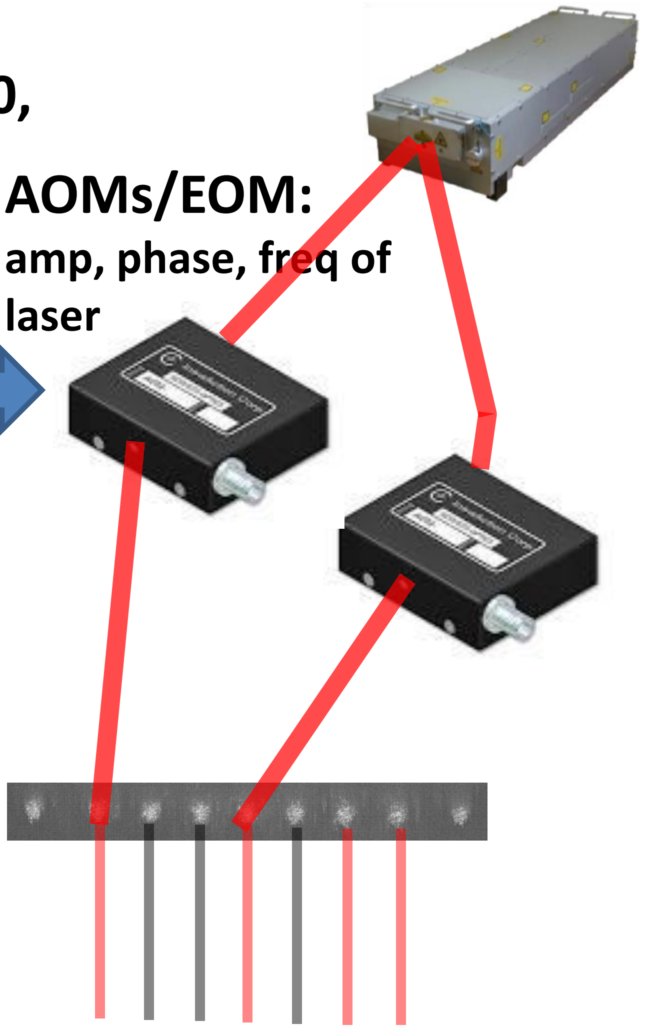
TTL signal

RF switch box



RF sources: PTS, DDS, HP8640, HP8672A

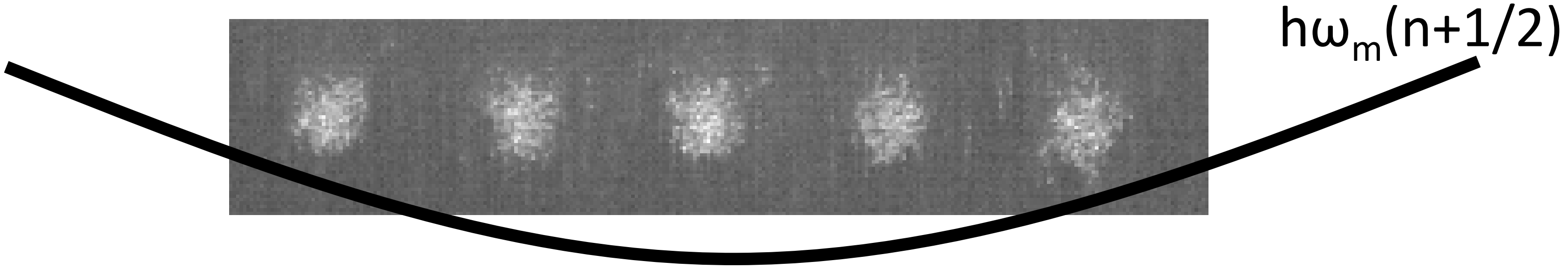
AOMs/EOM: amp, phase, freq of laser

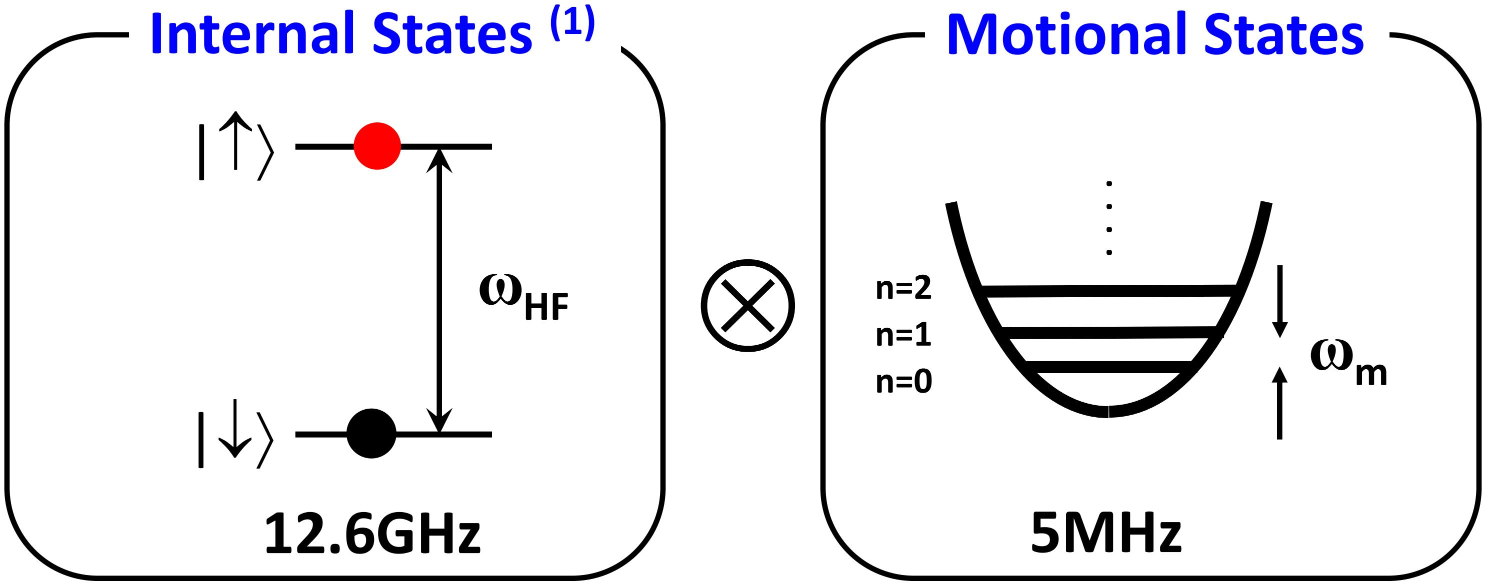


Camera/PMT

Repeat experiment

- Single qubit & two qubit gates



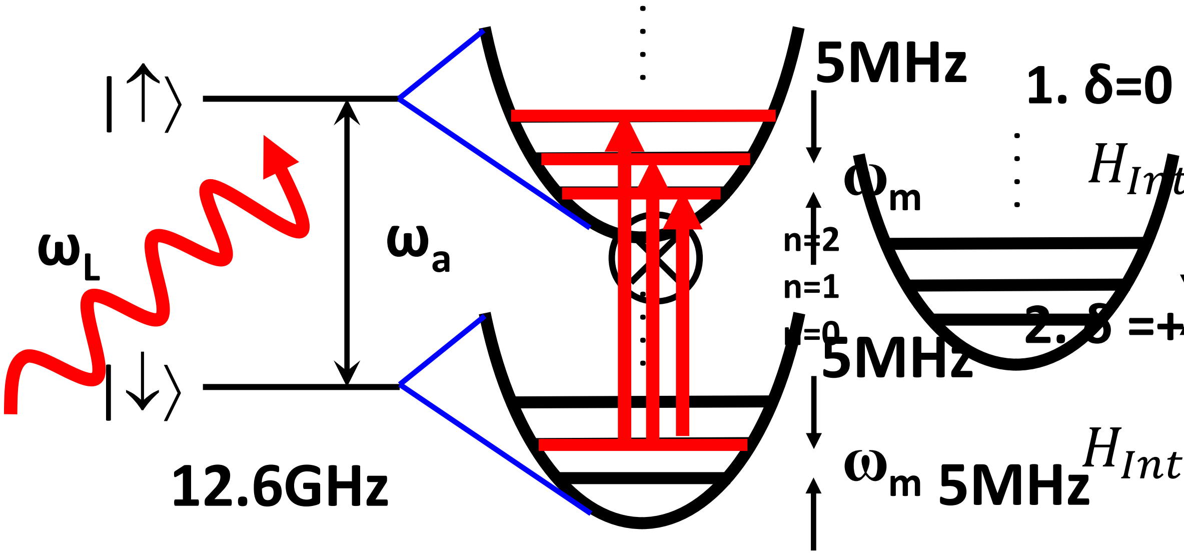


Internal States (2)

•••

Qubits share the motion
Entanglement use this !!
(Motion = Quantum bus)

$$H_{Int}(t) = -\vec{\mu}\cdot\vec{E}(t) \qquad\qquad \omega_L - \omega_a = \delta(detuning)$$
$$= -\hbar\Omega\left(\sigma_+ e^{-i\delta t} + \sigma_- e^{+i\delta t}\right) +$$
$$-i\eta\hbar\Omega\left(\sigma_+ e^{-i\delta t}\cdot\left(a^+ e^{i\omega_m t} + a e^{-i\omega_m t}\right) + \sigma_- e^{i\delta t}\cdot\left(a^+ e^{i\omega_m t} + a e^{-i\omega_m t}\right)\right)$$



1. δ=0 (Resonance)

$$H_{Int}(t) = -\hbar\Omega(\sigma_+ + \sigma_-)$$
$$= -\hbar\Omega\sigma_x \cdots Carrier$$

2. δ =+ω$_m$ (add motional quanta)

$$H_{Int}(t) = -i\eta\hbar\Omega(\sigma_+ a^+ + \sigma_- a)$$
$$\cdots Blue\ side\ band$$

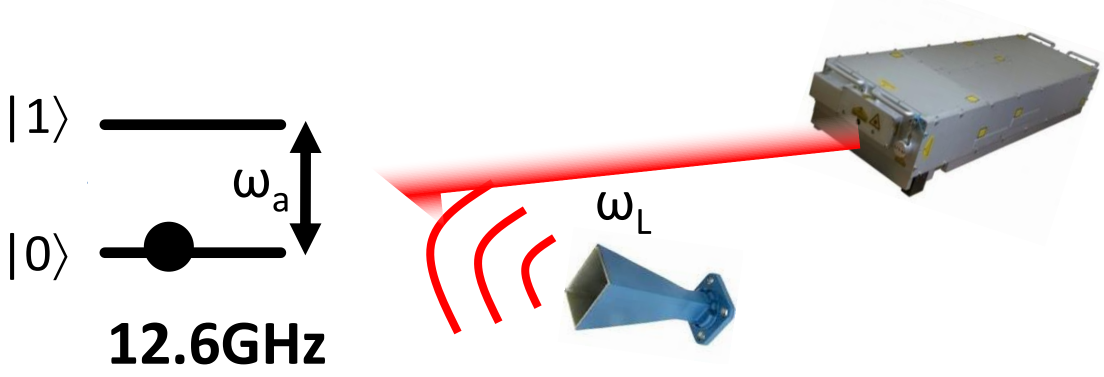
3. δ =-ω$_m$ (remove motional quanta)

$$H_{Int}(t) = -i\eta\hbar\Omega(\sigma_+ a + \sigma_- a^+)$$
$$\cdots Red\ side\ band$$

- Full control of qubit and motional states

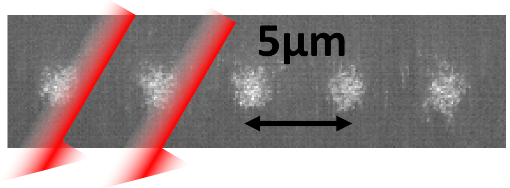
- η: Lamb-Dicke parameter (coupling of qubit and motion)

- Why laser for qubit control ??



- Coherent driving source
: single freq sine wave
Ex) μ-wave: 0.3 to 300GHz
400 nm laser: 750 THz
1000 nm laser: 300 THz

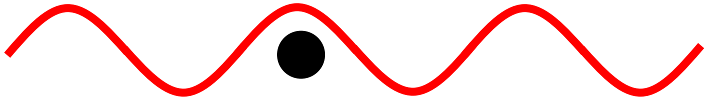
- Wavelength of 12 GHz vs 750 THz
: 2.5 cm vs 0.4 μm



- Focus to individual qubits

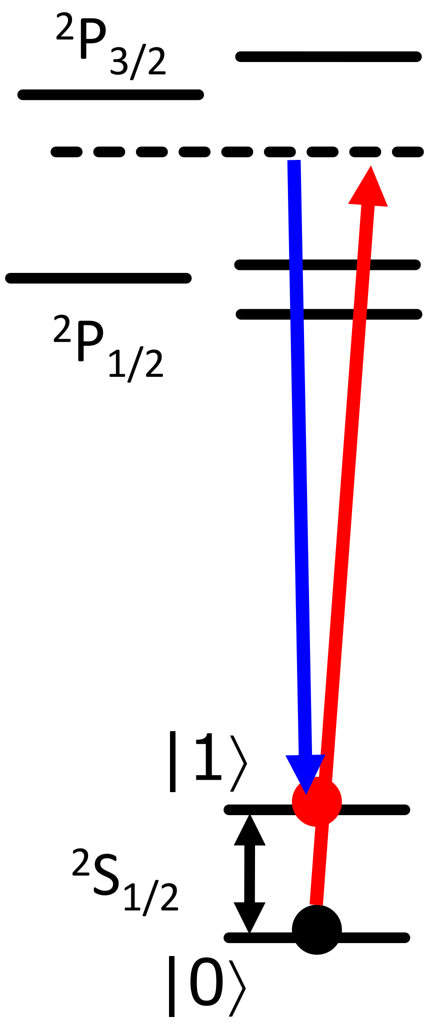
- Coupling strength of qubit and motion
η (Lamb-Dicke) of 12 GHz vs 750 THz
: 10^{-6} vs 10^{-2}

$$\eta = kx_0 = \frac{2\pi x_0}{\lambda} << 1$$



- Small η is good, too small is bad

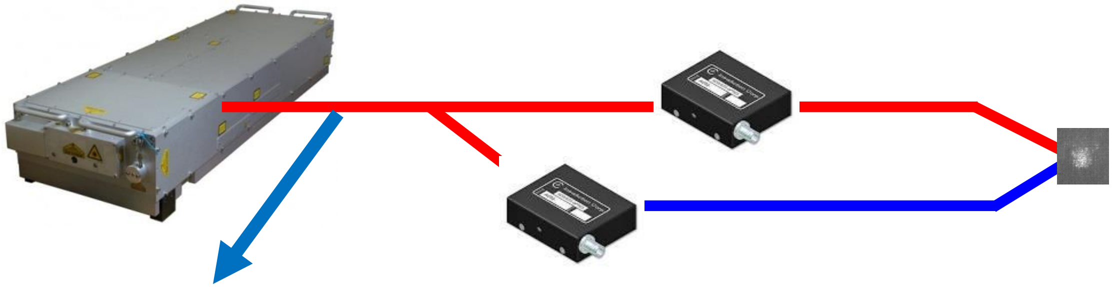
- Two photon stimulus Raman transition

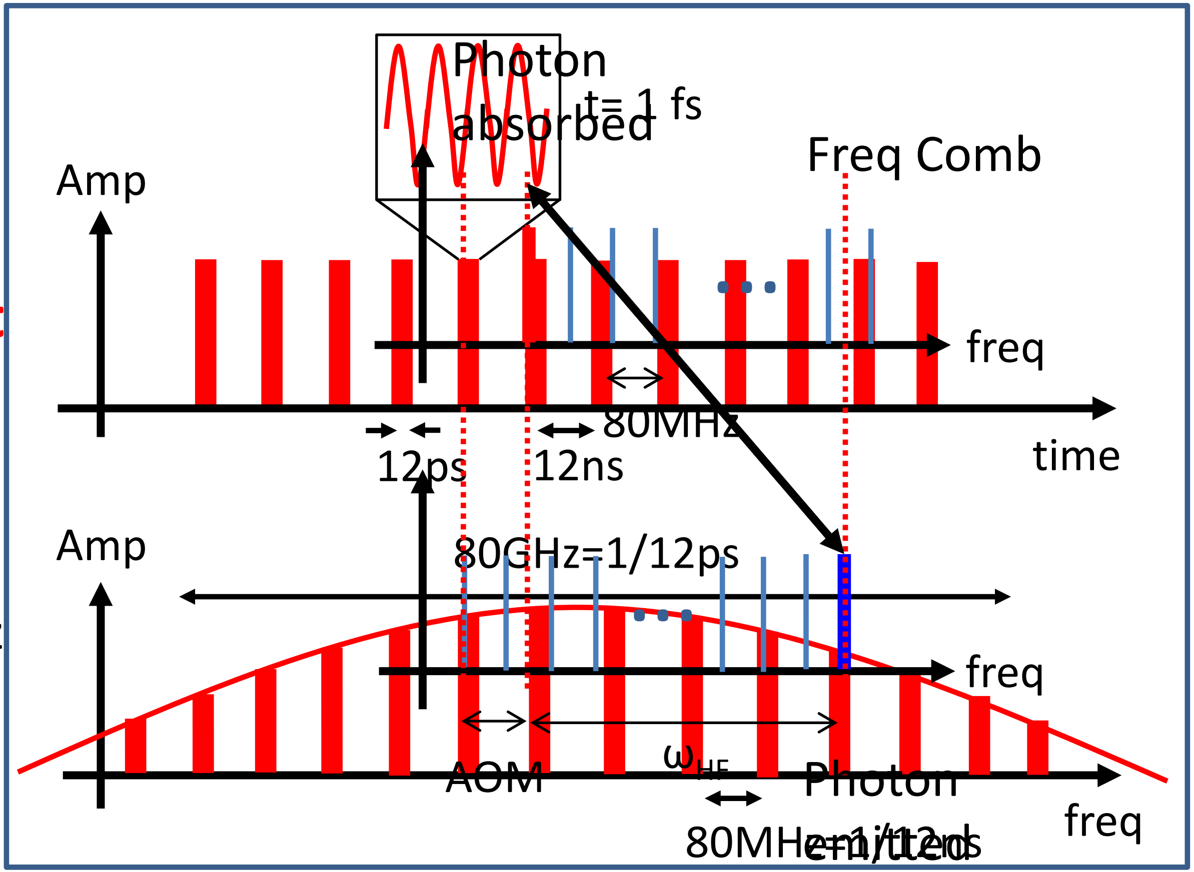


355nm (set

80MHz rep

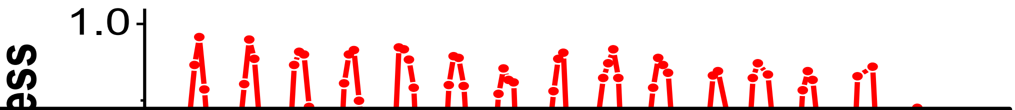
ω_{HF}=12.6 GHz

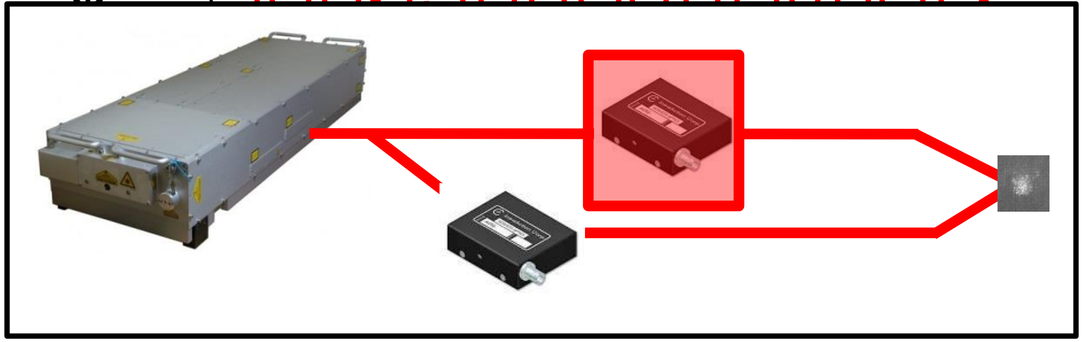




- Single qubit rotation

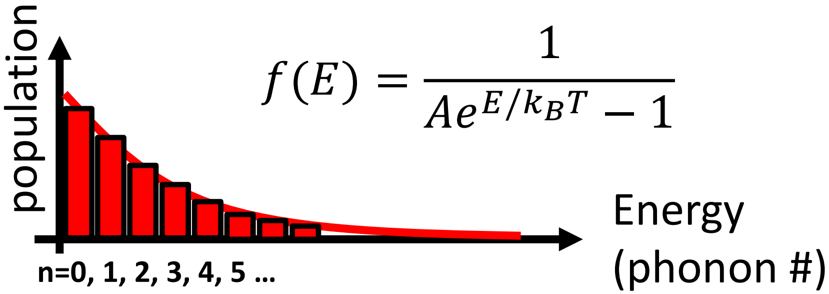
- Rabi oscillation of carrier





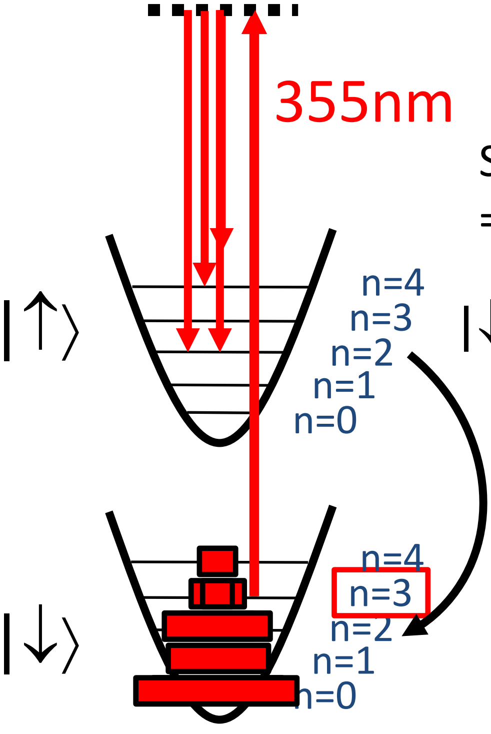
- Control of motions

<BE statistics>



$$f(E) = \frac{1}{Ae^{E/k_BT} - 1}$$

- Cooling limit : 10MHz ≈ 100μK
≈ motional quanta (n≈3): Γ = nħω

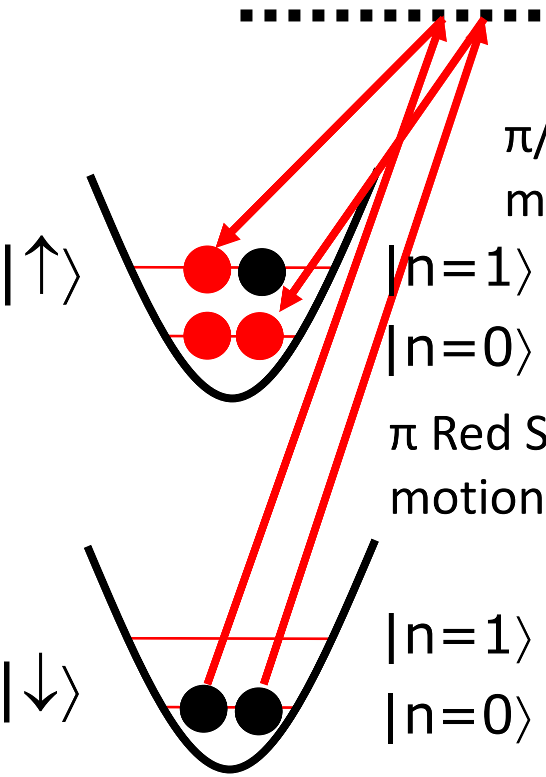


Sideband cooling
= RSB + π pulse

|↓⟩|n⟩ → |↓⟩|n-1⟩
⋮
→ |↓⟩|0⟩

- Quantum entanglement using Motional modes

- Use motional modes as Entangling “bus”



$|\uparrow\rangle$

$|n=1\rangle$

$|n=0\rangle$

π/2 Blue Side (add 1/2 motional quanta)

π Red Side (reduce 1 motional quanta)

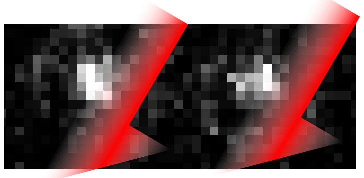
$|\downarrow\rangle$

$|n=1\rangle$

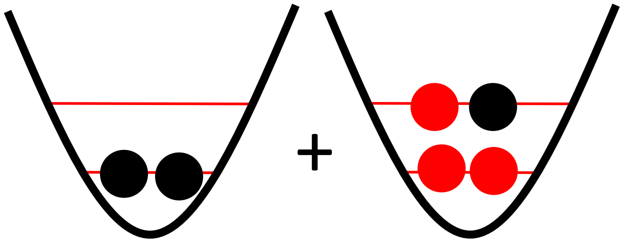
$|n=0\rangle$

$|\downarrow\downarrow\rangle|n=0\rangle$

- Two photon process
(Stimulus Raman transition)



$|\text{com}\rangle$ ● ● $|n=0,\ 1,\ \ldots\rangle$



$|\downarrow\downarrow\rangle|n=0\rangle+|\uparrow\downarrow\rangle|n=1\rangle$

$|\downarrow\downarrow\rangle|n=0\rangle+|\uparrow\uparrow\rangle|n=0\rangle$

- Entanglement !!

J.I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995)

- Current status of ion trap Quantum computing

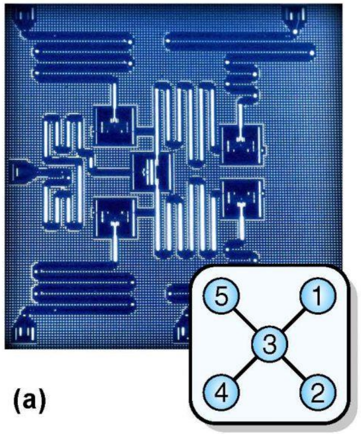
Chris Monroe's group @ UMD (2016)

Full Quantum computing with 5 qubits

- Deutsch-Jozsa algorithm
- Bernstein-Vazirani algorithm
- Quantum Fourier transform

} Fidelity >90%

VS



IBM 5 qubit (Superconducting circuit) Quantum Experience (2016)

Weak points of Ion trap Q.C.

- Many unstable factors (laser, man-power,…)
- Difficult to scale up (background gas collision)
  (Hard to trap more than 20 ions)

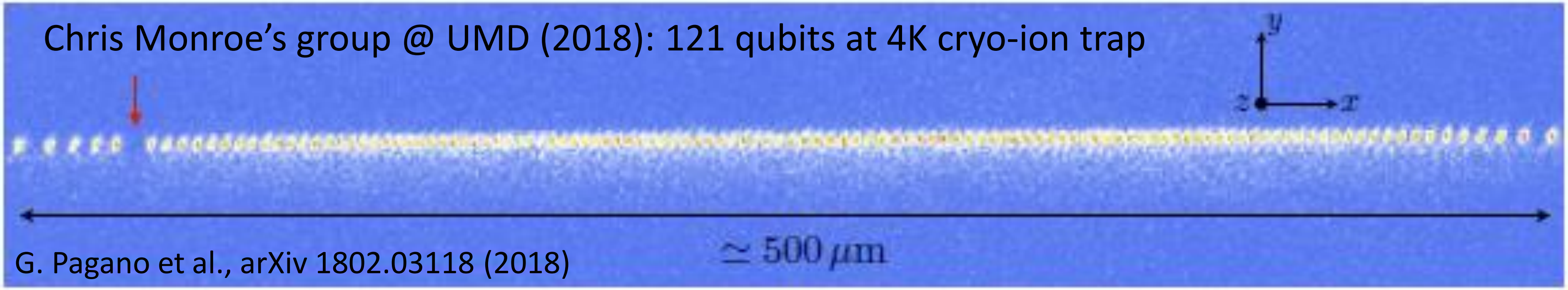
S. Debnath et al., Nature **536**, 63 (2016)

- Current status of ion trap Quantum computing

- Superconducting circuit: we can print them out (>70 qubits)

Weak points of S.C Q.C.

- Connectivity
- Fidelity (coherence: 100μs)
- Qubit freqs are all different



Chris Monroe's group @ UMD (2018): 121 qubits at 4K cryo-ion trap

G. Pagano et al., arXiv 1802.03118 (2018)

Ion trap system is still one of the most promising platforms for Quantum computing

- Connectivity
- Scalable ?
- High fidelity (coherence: >1s)
- Ion-photon network
- Qubit freqs are all the same
- Quantum simulation

5. World trend of Quantum computing

Europe

- Trapped ion (optics)
- Superconducting circuit
- Quantum dot
- NV center

Asia

- Quantum dot

Canada

US

- Trapped ion (optics)
- Superconducting circuit
- Quantum dot
- NV center

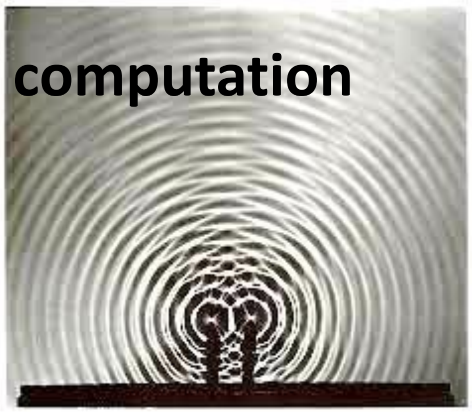
6. Take home message and Outlook

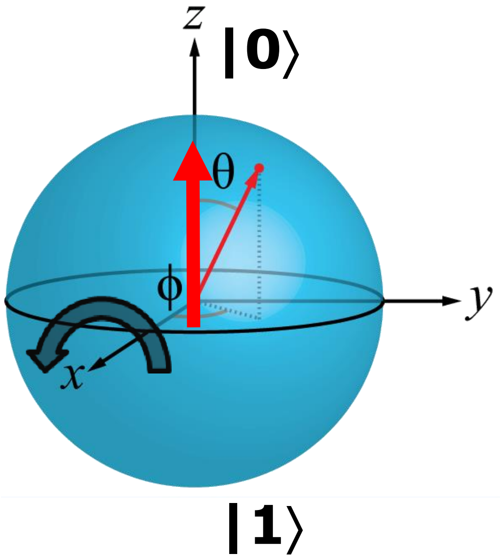
6. Take home message and Outlook

1. Quantum mechanics: Wave + measurement + probability

2. Quantum computing: multiple input and parallel computation via Quantum interference

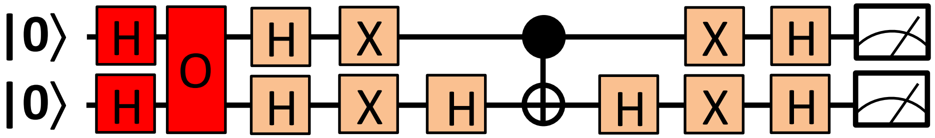
- Single qubit rotation + Quantum entanglement are what we all need for complete computing



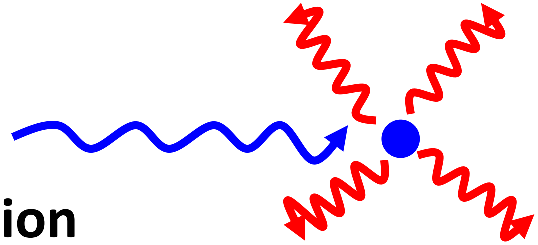


3. Basic physics for Quantum computing

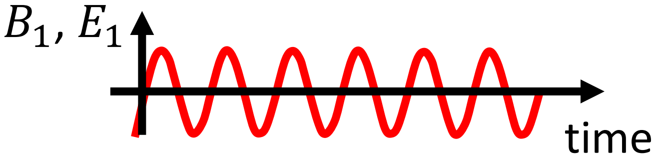
- Light-matter dipole interaction is the key
- Rabi (coherent qubit rotation) and Ramsey (T_2)
- Two qubit gate and Grover search algorithm

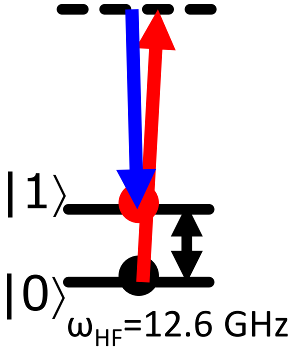


4. Ion trap Quantum computing

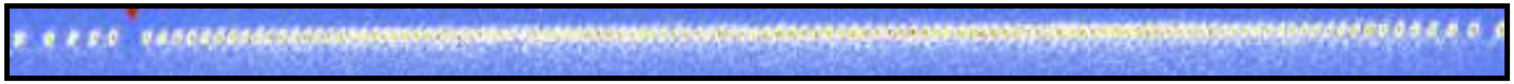


- Ion trapped by Doppler and DC+RF
- 171Yb+ qubit, optical preparation and detection
- Ion trap experimental setup
  (RF + TTL controller, lasers: phase is the key)





- Two photon stimulus Raman transition
  for single qubit + two qubit gate
- More than 100 qubits and full control of 5 qubits



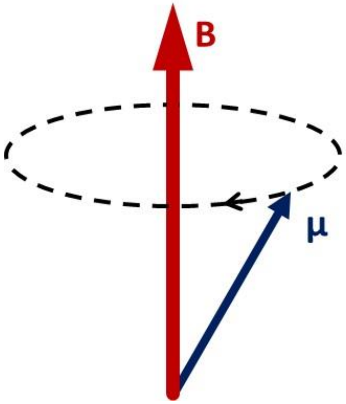
5. World is working pretty hard on Quantum computing

Korea has started this research in Univs and Govern institutes. A very promising research field needs your challenge and may promise a new opportunity !!!

Supplementary Materials

- Bloch sphere (classical pic vs quantum pic)
- Requirement and importance of Quantum computer
- Detail of two photon Raman transition

- Classical picture vs Quantum picture

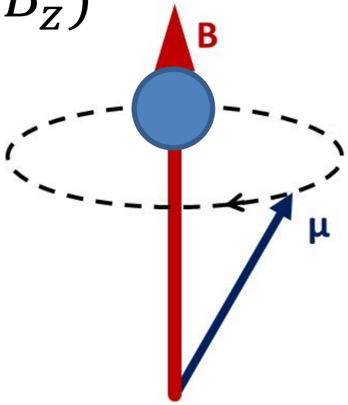


$$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B} \quad \Rightarrow \quad \frac{d\vec{\mu}}{dt} = \gamma_e \vec{\mu} \times \vec{B}$$

Electron orbits (angular momentum) creates magnetic moment

$$\dot{\mu}_x = \gamma_e \mu_y B_z - \gamma_e \mu_z B_y$$
$$\dot{\mu}_y = -\gamma_e \mu_x B_z + \gamma_e \mu_z B_x$$
$$\dot{\mu}_z = \gamma_e \mu_x B_y - \gamma_e \mu_y B_x$$

1) $\vec{B} = (0,0,B_z)$



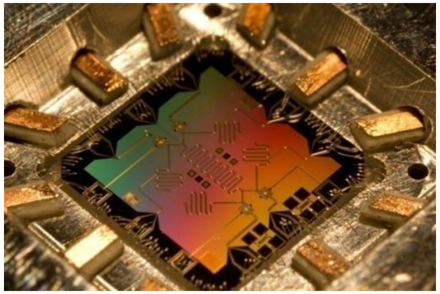
2) With relaxation (τ_2 $in\ x, y\ plane, \tau_1\ in\ z$)

$$\dot{\mu}_x = \gamma_e \mu_y B_z - \mu_x/\tau_2$$
$$\dot{\mu}_y = -\gamma_e \mu_x B_z - \mu_y/\tau_2$$
$$\dot{\mu}_z = (\mu_{z0} - \mu_z)/\tau_1$$

Requirement and Good Quantum systems ??

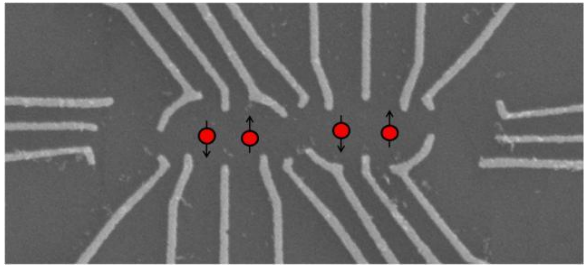
- **Requirements**
  - Quantum states
  - Prepare/Detect
  - Universal Quantum gate
  - Gate time << Coherence time
  - Scalable (# qubit ↑)

Condensed matter physics



Superconducting qubit

John Martinis group at UCSB

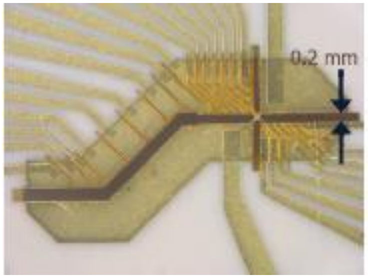


Spin qubit in Quantum dot

Yacoby group at Harvard

- **Physical system**
  - Atoms (isolated atoms, defects in semiconductors, NV centers, Single Molecular Magnets)
  - Superconducting current, phase, magnetic field
  - Electron spins/charges in Quantum Dots
  - Nuclear spins in NMR
  - Photon polarization

Atomic molecular optics

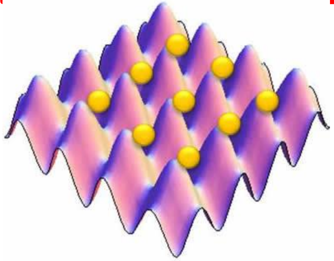


Trapped ion qubit

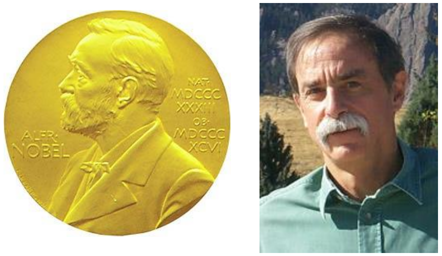
David Wineland group at NIST

Chris Monroe group at UMD : >16 qubits

- Laser cooling
- Each atom is 1 qubit
- Coherence time ~ secs



Neutral atom qubits



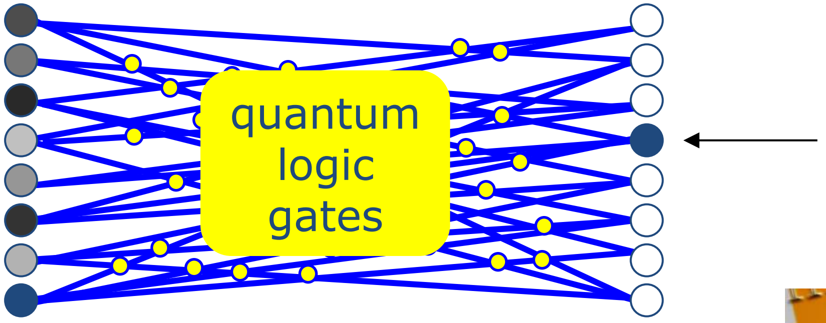
Dr. David Wineland

Qauntum computer

...GOOD NEWS!

quantum interference

(Wave mechanics)

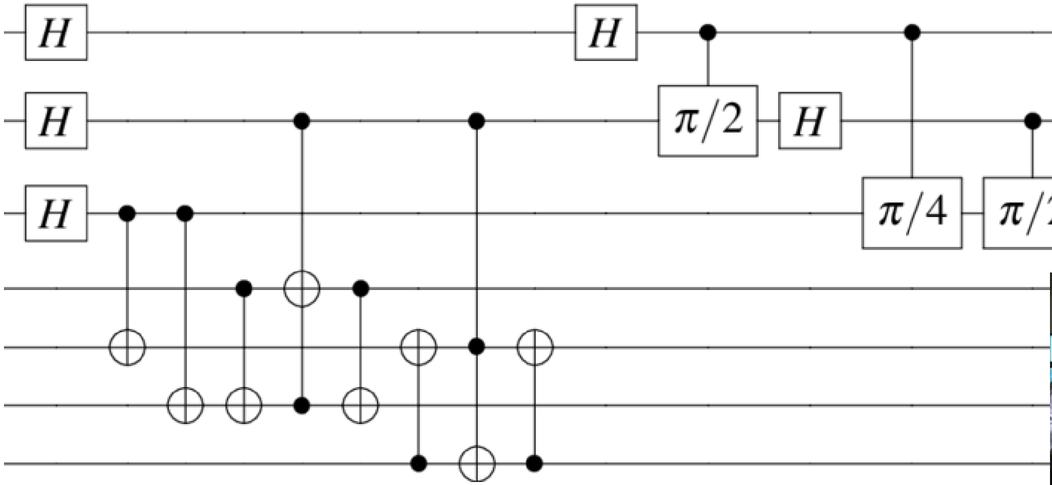


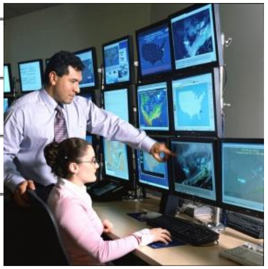
depends
on *all*
inputs

Deutsch (1985)

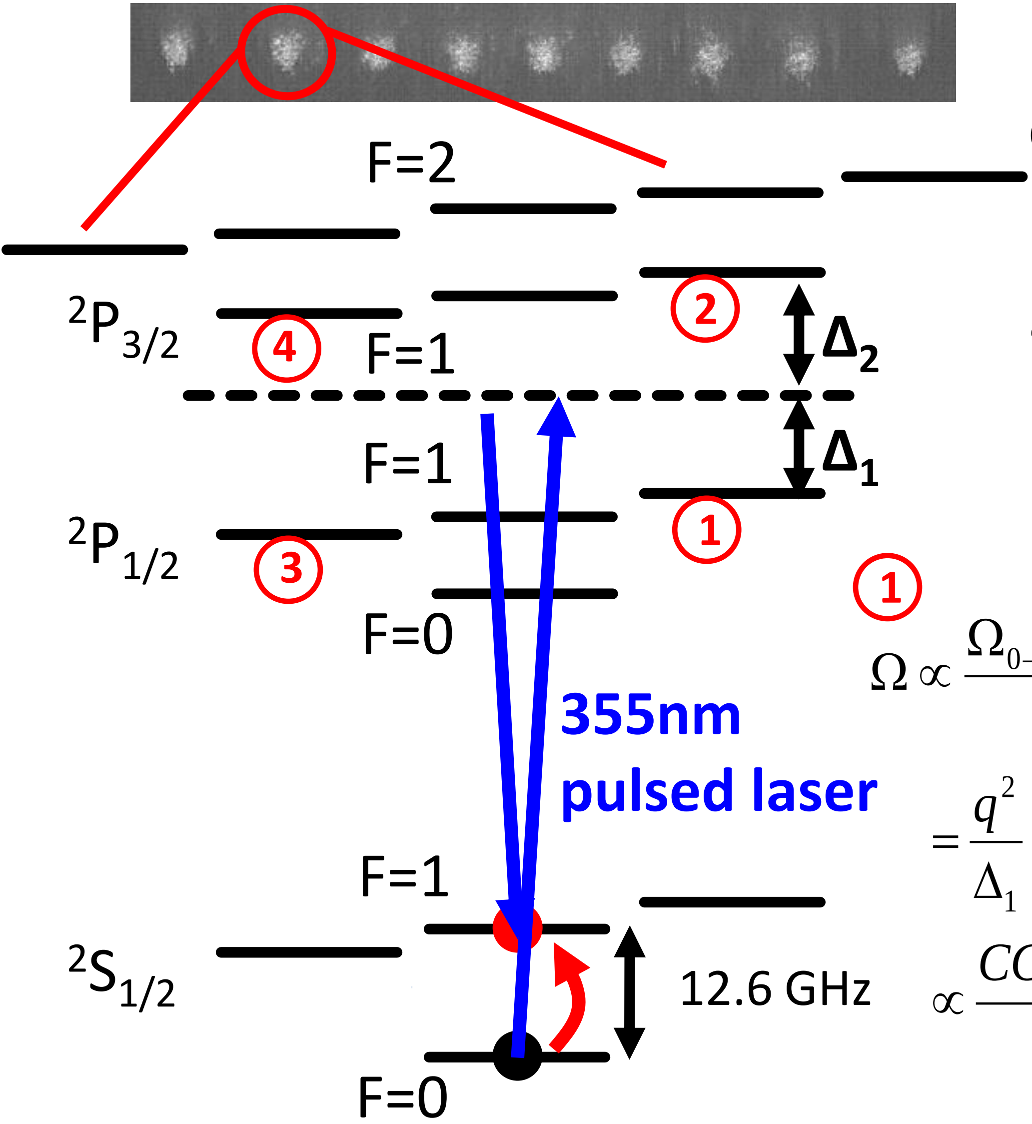
Shor (1994) fast number factoring $N = p \times q$

Grover (1996) fast database search





How to manipulate qubit ?



1. μ-wave (wavelength too long, coupling to motion weak)

$$\Omega \equiv \mu_d \cdot E_0 / \hbar \qquad \mu_d \equiv q\langle 1 | \hat{r} | 0 \rangle$$

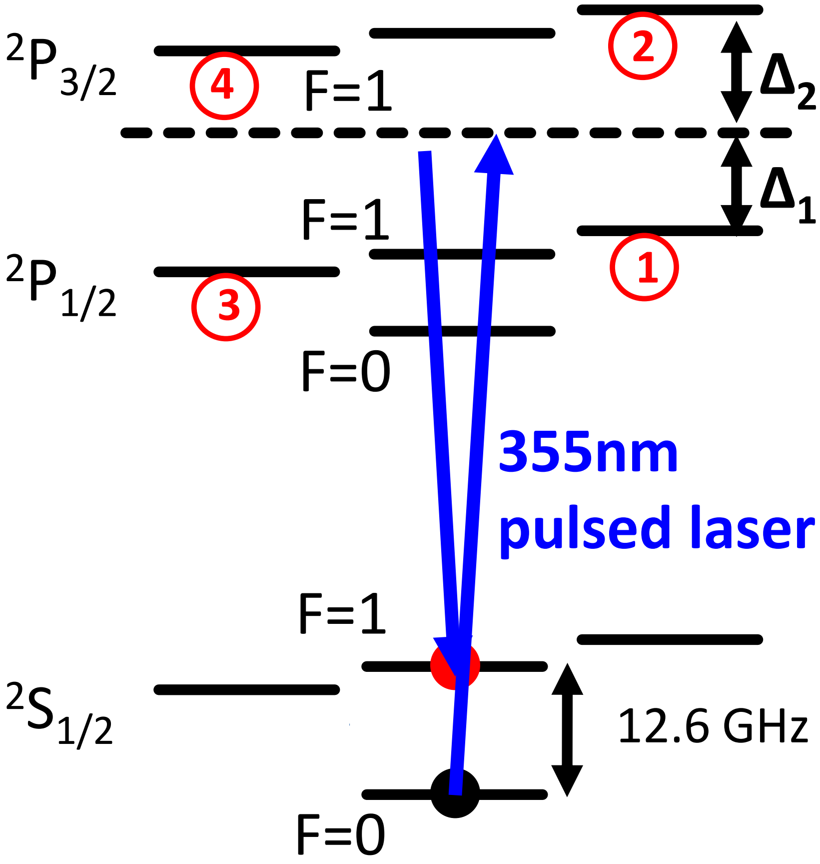
2. Raman process with pulsed laser

①

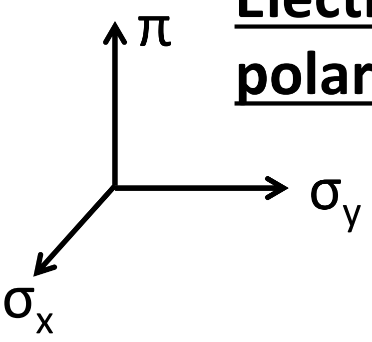
$$\Omega \propto \frac{\Omega_{0\to aux}\Omega_{aux\to 1}}{\Delta_1} = \frac{1}{\Delta_1}\mu_{d\,0\to aux}\cdot\mu_{daux\to 1}\cdot\frac{E_{0\to aux}E_{aux\to 1}}{\hbar^2}$$

$$= \frac{q^2}{\Delta_1}\cdot\langle aux | \hat{r} | 0\rangle\cdot\langle 1 | \hat{r} | aux\rangle\cdot\frac{E_{0\to aux}E_{aux\to 1}}{\hbar^2}$$

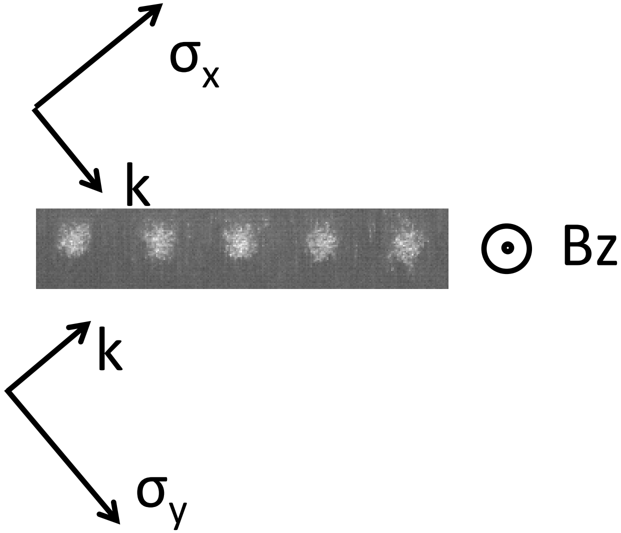
$$\propto \frac{CG_{0\to aux}CG_{aux\to 1}\sigma_+\sigma_+}{\Delta_1}$$



Electric field polarization



π||B



$$\Omega \propto \frac{CG_{(1)}\sigma_+\sigma_+ + CG_{(2)}\sigma_+\sigma_+ + CG_{(3)}\sigma_-\sigma_- + CG_{(4)}\sigma_-\sigma_-}{\Delta}$$

$$\propto \frac{\sigma_+\sigma_+ - \sigma_-\sigma_-}{\Delta} = \frac{\sigma_x\sigma_y}{\Delta}$$

Raman process = two photon process on multi- level system !!