Mesoscopic physics society
2019 Summer school

Slides courtesy : 정제성, 김기호

Quantum control, decoherence, and quantum error correction

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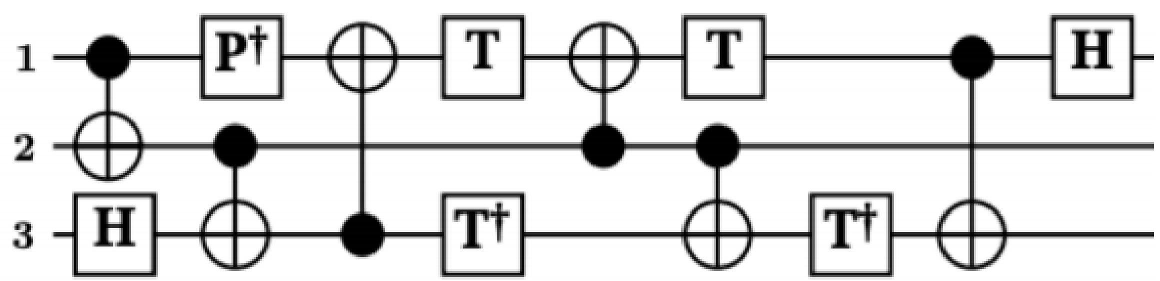
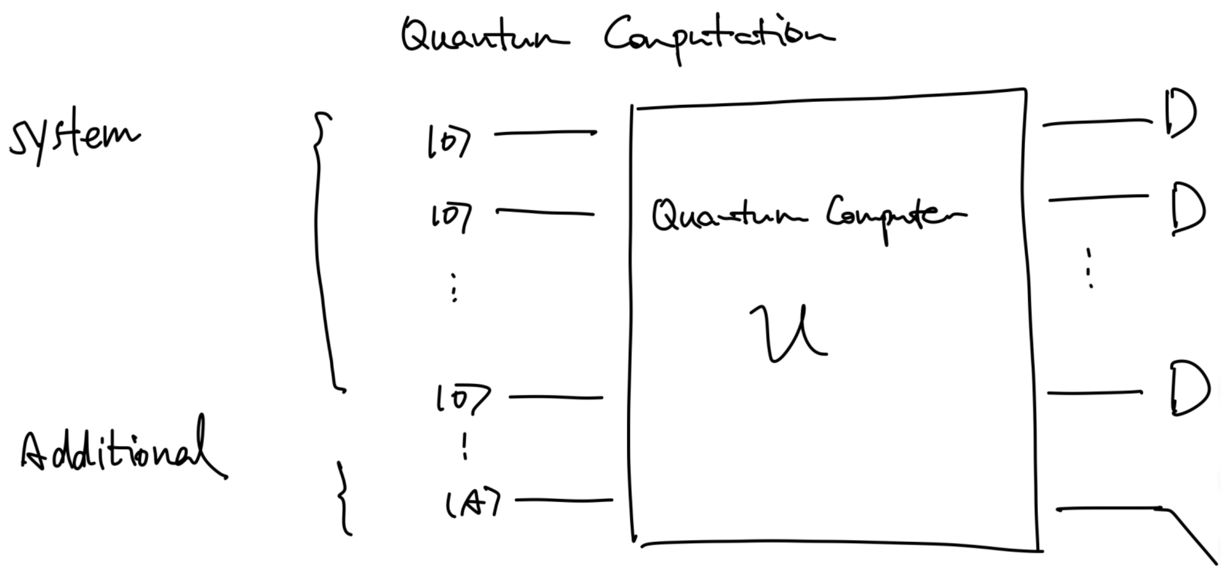
Aim of this lecture

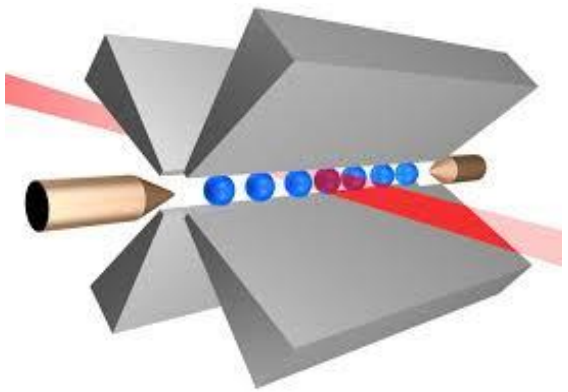
전반부

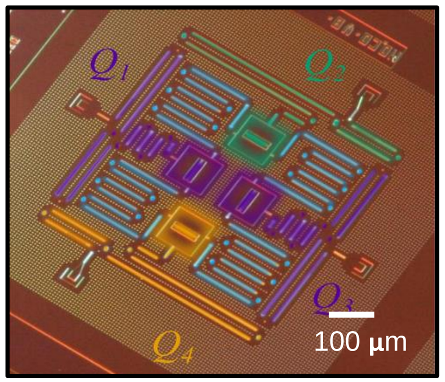


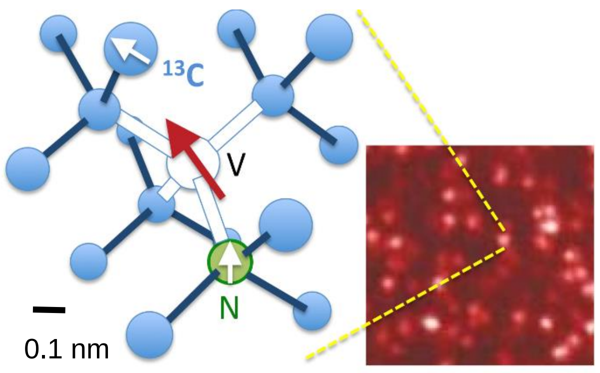


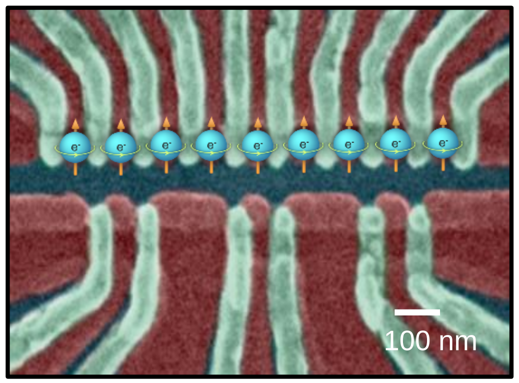
본 강의는, bridging the two part

후반부









Outline

Introduction

Quantum control and decoherence

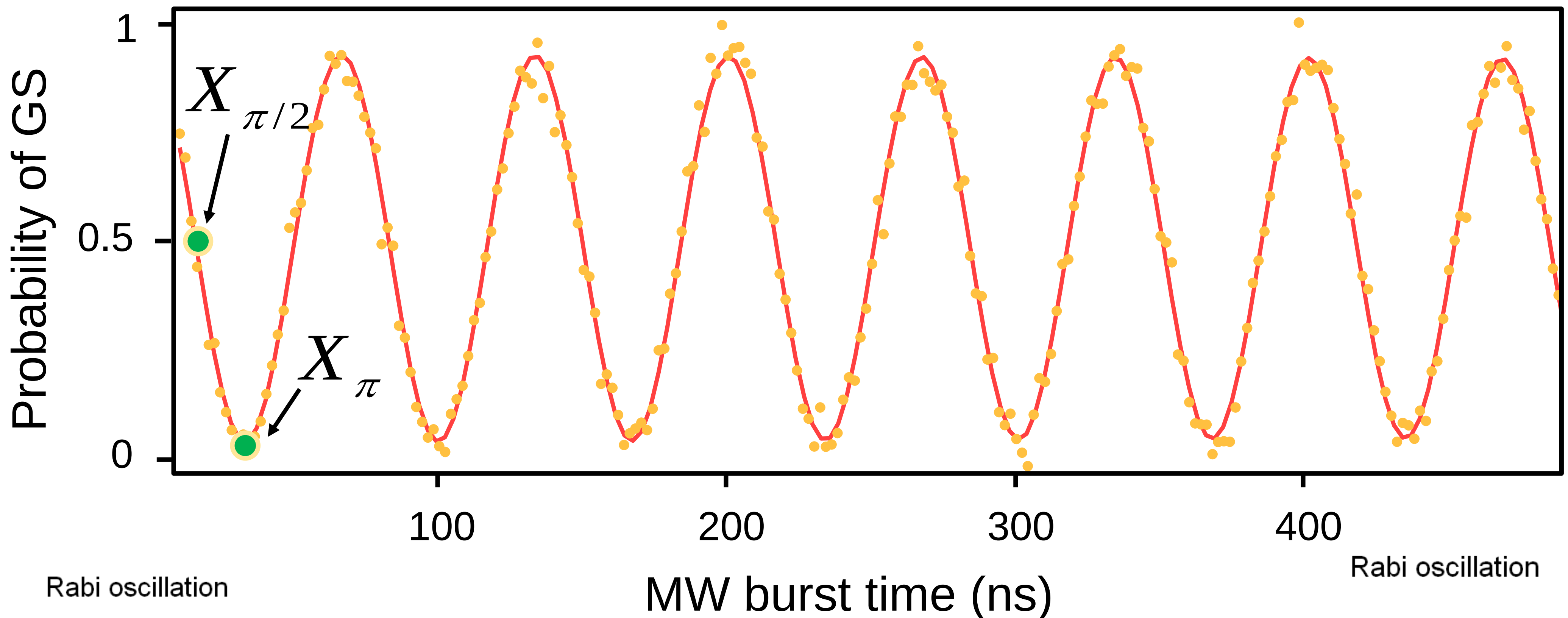
- 1Q, 2Q gate
- Master equations in the Markov approximation
- Quantum noise channels

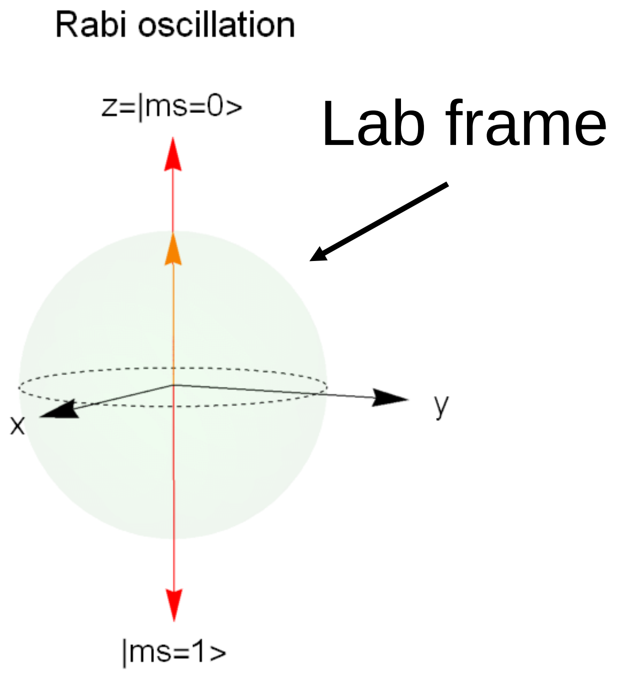
Quantum error correction

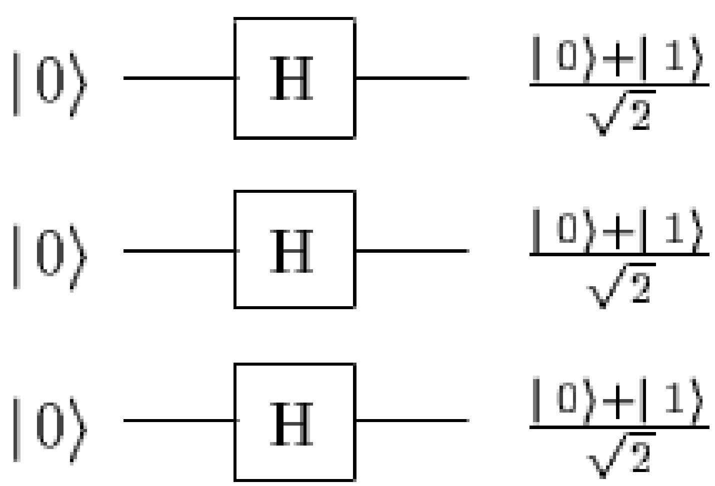
- Basic concepts : Bit-flip and Phase-flip error correction
- Quick summary of general quantum error correction
- Experimental examples

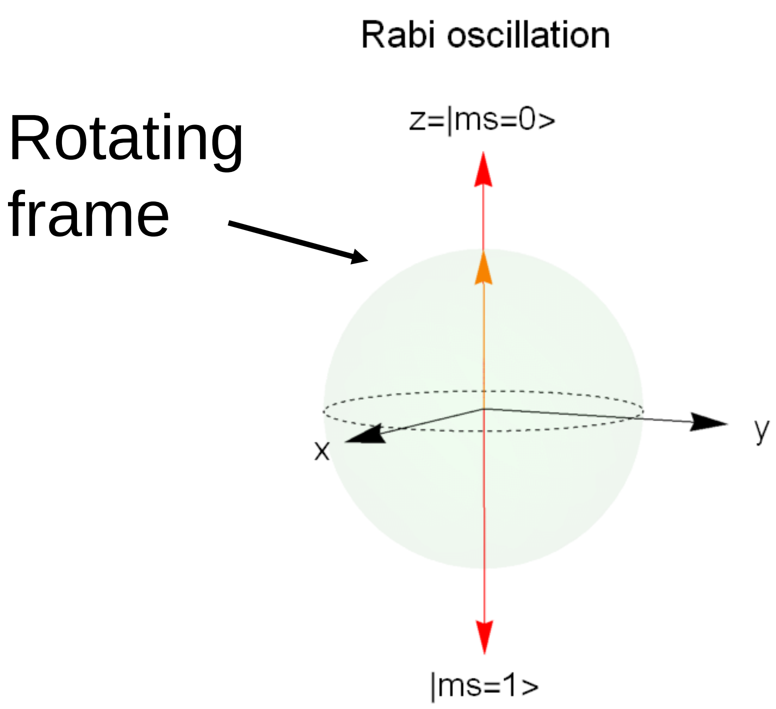
Single qubit gate : coherent rotation

Coherent Rabi pulse + Phase control = Single qubit rotation gates









Control of quantum two level system

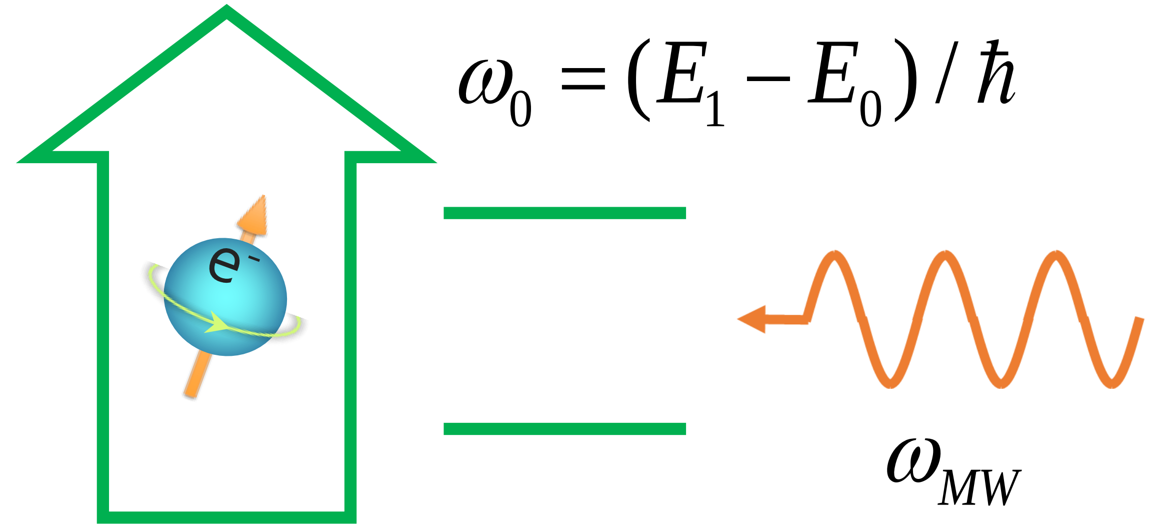
Rabi oscillation

Two level system, with

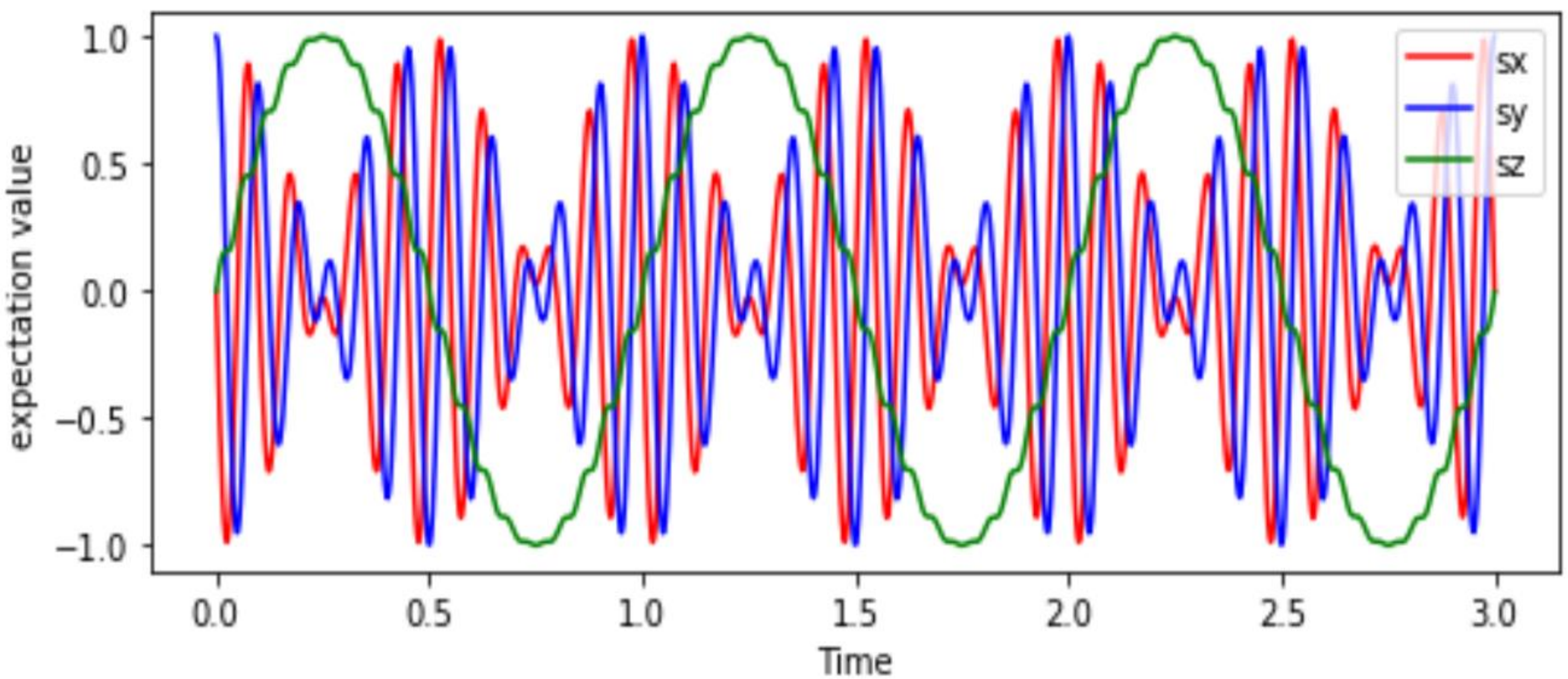
$$\hat{H} = \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \hbar\eta(\hat{\sigma}_x \cos\omega_{MW} t)$$

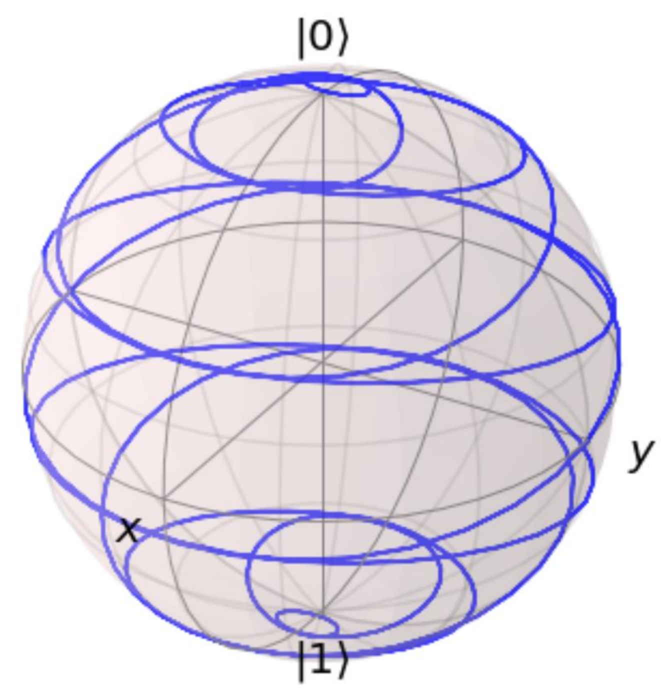
Apply harmonic radiation

$$\omega_0 = (E_1 - E_0)/\hbar$$



On resonance, $\omega_0 = \omega_{MW}$





Control of quantum two level system

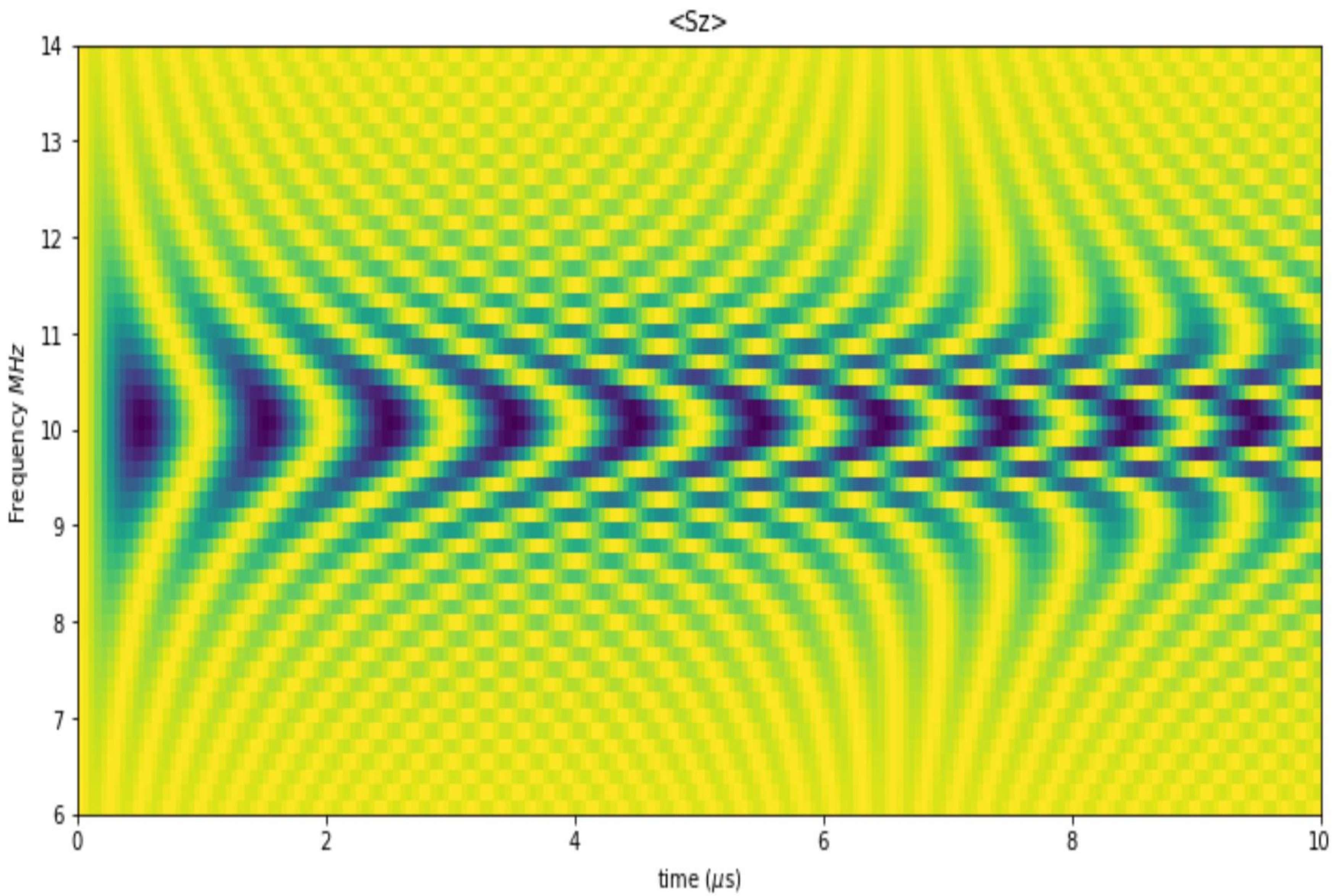
Rotating frame: RWA approximation

$$\hat{H}_{rot} = \frac{\hbar}{2}(\omega_0 - \omega_{MW})\hat{\sigma}_z + \frac{\hbar\eta}{2}\hat{\sigma}_x$$

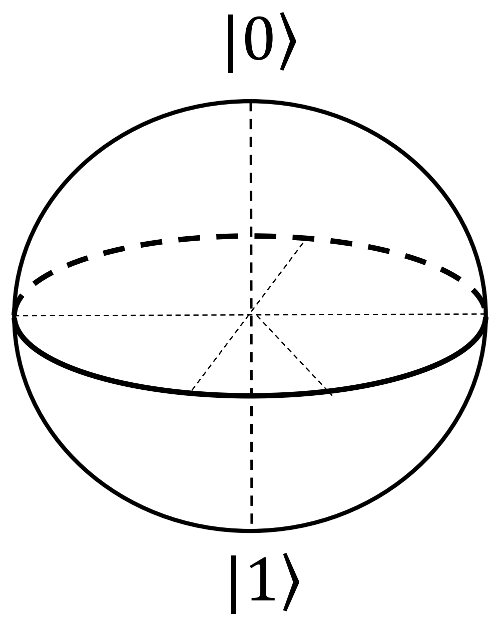
$$\delta = \omega_0 - \omega_{MW}$$

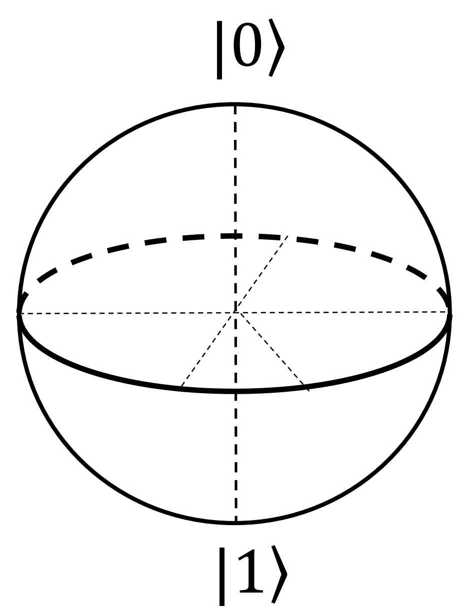
ω_0 의 intrinsic rot. 사라짐
$\hat{\sigma}_z, \hat{\sigma}_x$ 성분의 벡터합이 도는 축을 결정

Q : Hadamard Gate ?



낙서



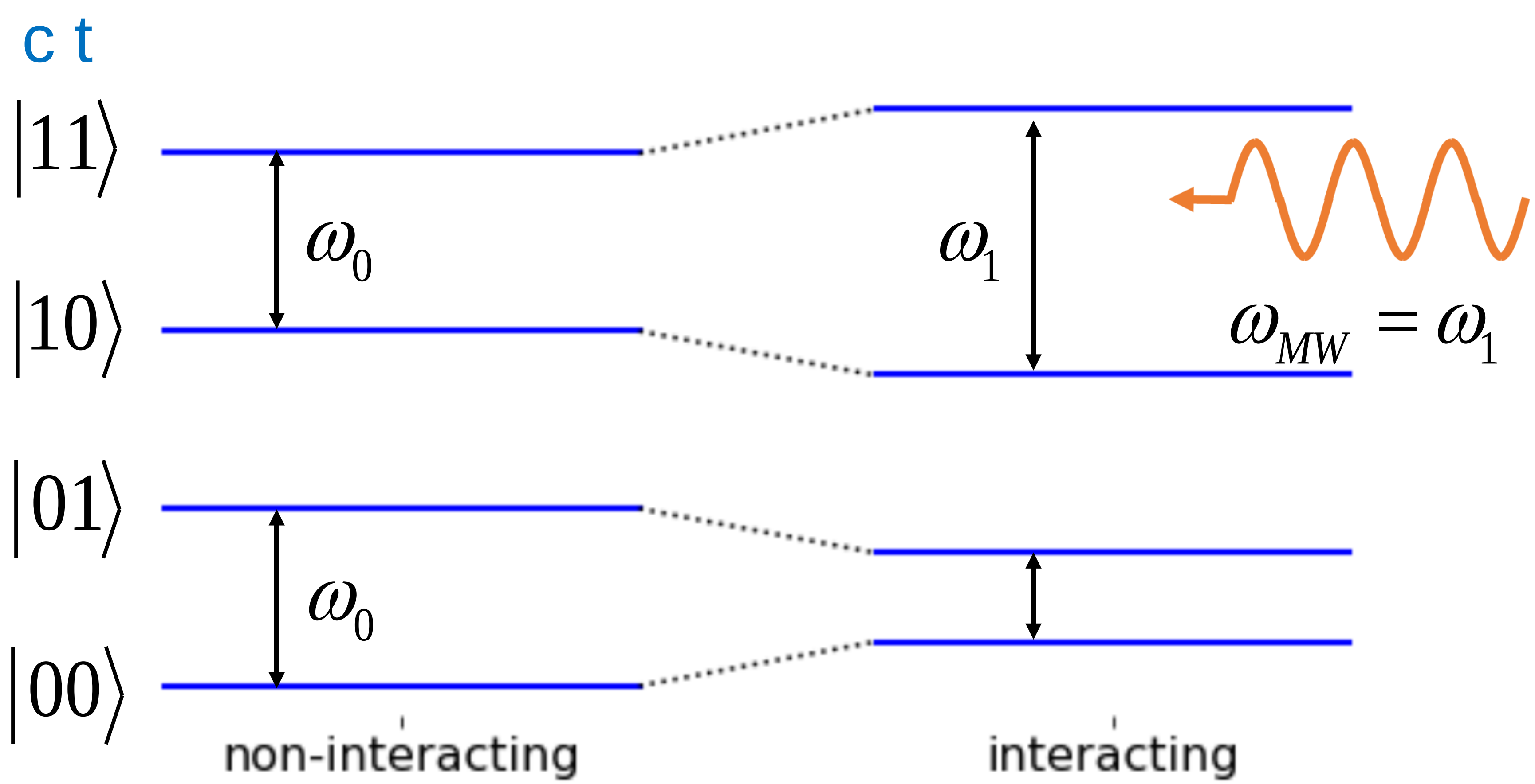


Control of quantum two level system

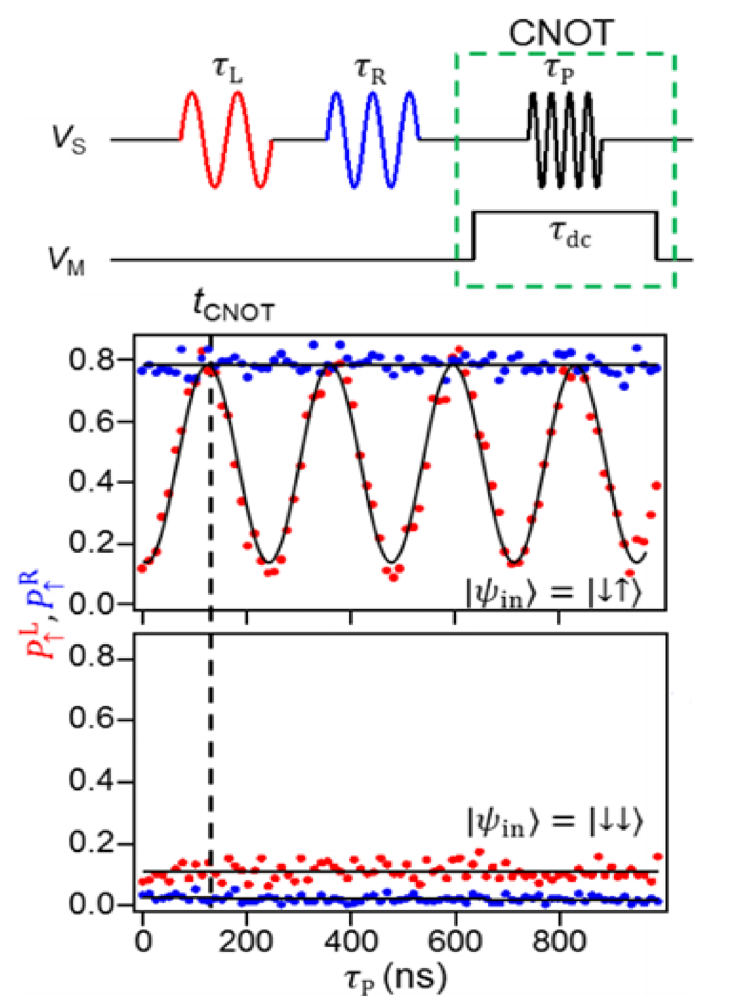
Two qubit gate

Ex. Calibrated Rabi π pulse under two body interaction = CNOT

$$\hat{H} = \frac{\hbar\omega_0}{2}(2\hat{\sigma}_{z1} \otimes I + I \otimes \hat{\sigma}_{z2}) + \hbar g(\hat{\sigma}_{z1} \otimes \hat{\sigma}_{z2})$$

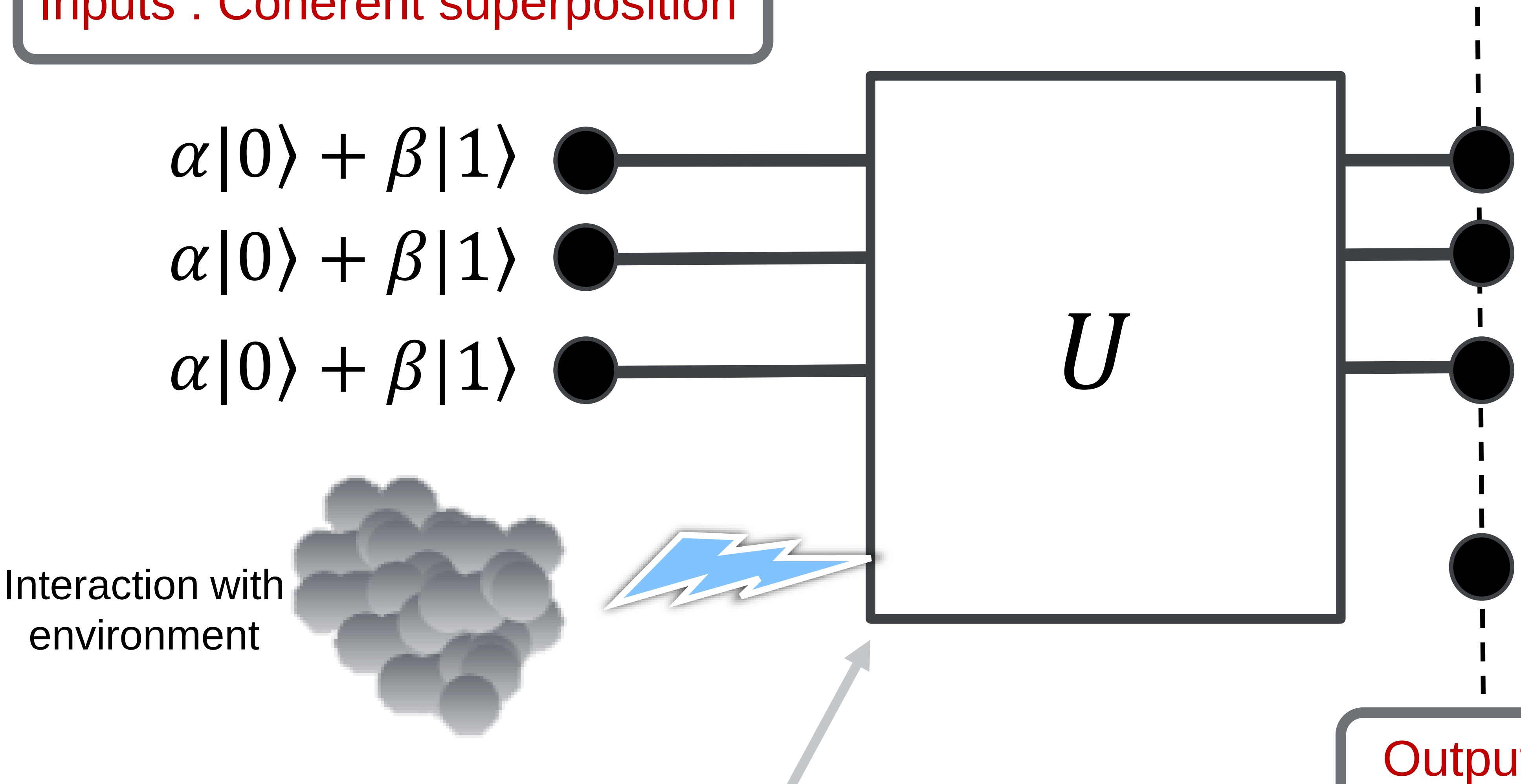


반도체 스핀 큐빗의 예



Coherent time evolution : computation

Inputs : Coherent superposition

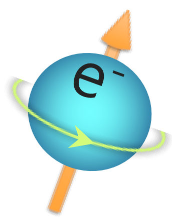


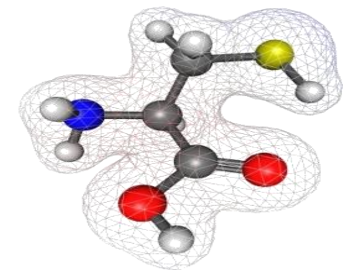
Computation = Quantum mechanical time evolution

Output : Final projective measurement

Understanding / controlling system – environment interaction is crucial

Quantum to classical transition



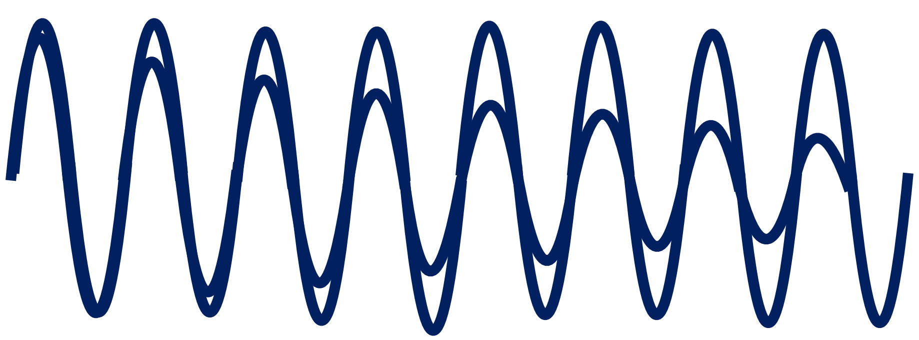


Decoherence

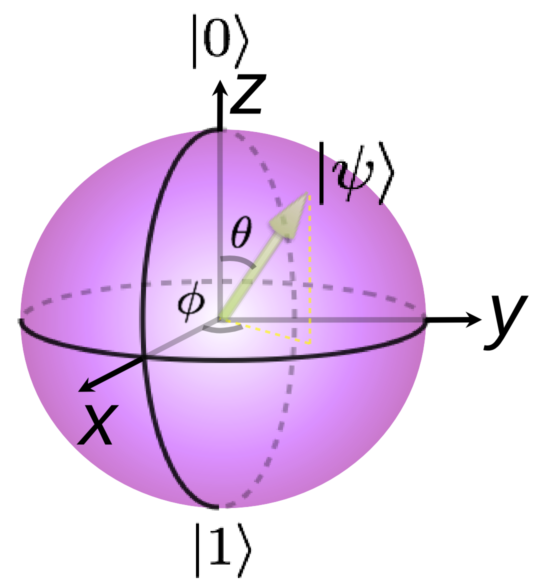
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\Downarrow$$

$$\rho = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$$



Quantum noise = decoherence, control error, etc.

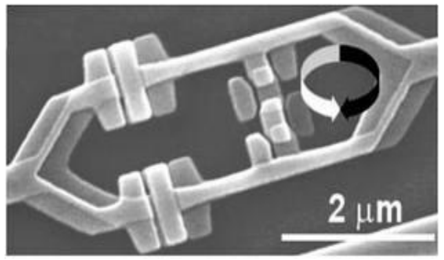


$$\alpha|0\rangle + \beta|1\rangle$$

$$\alpha = \cos(\frac{\theta}{2})$$

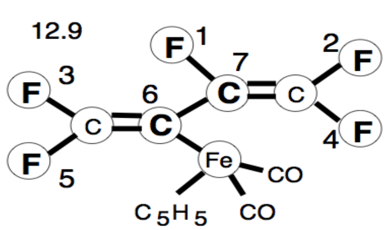
$$\beta = e^{i\phi}\sin(\frac{\theta}{2})$$

Superconductor



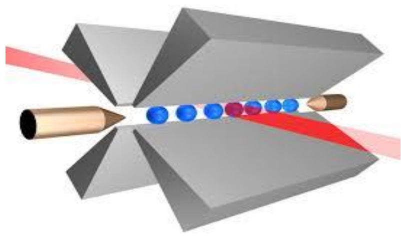
Y. Nakamura, et al., *Nature* **398**, 786 (1999)

NMR



L.M.K. Vandersypen, et al., *Nature* 414, 883 (2001)

Trapped Ions/Atoms



C. Monroe, et al., *Nature* **417**, 709 (2002)

Optics

E. Knill, et al., *Nature* **409**, 46 (2001)

Open quantum system

From closed to open quantum system

	Closed	Open
1. State	Ket vector $\left\|\Psi\right\rangle$	Density Matrix $\hat{\rho}$
2. Dynamics	Schrodinger	Master eq.
3. Measurement	Projective	Generalized (weak) Measurement

Density matrix

$$\hat{\rho} = \sum_i p_i \left|\Psi_i\right\rangle\left\langle\Psi_i\right|$$

p_i : probability to be in *i* th quantum state.

Properties

$$\hat{\rho}^\dagger = \hat{\rho}$$

$Tr(\rho^2)$ is called *purity*

$$Tr(\rho) = 1$$

$\rho^2 = \rho$ iff pure.

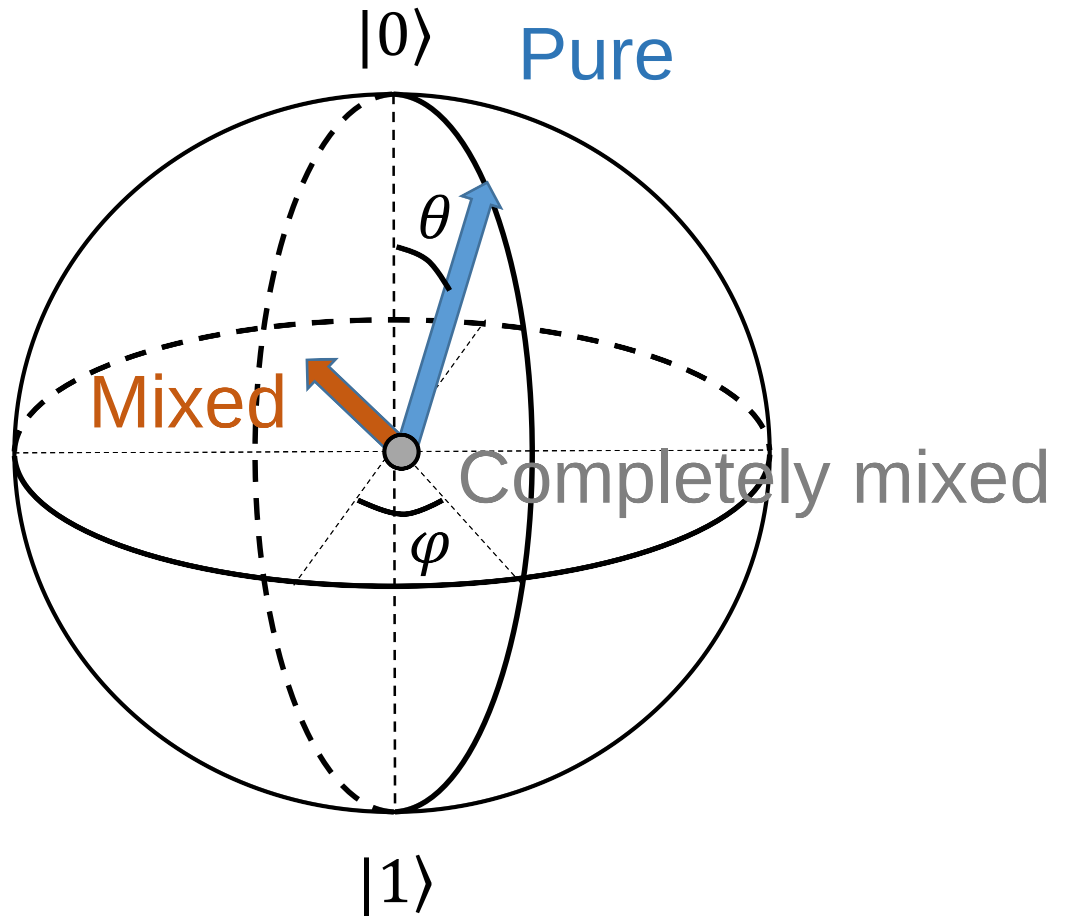
$Tr(\rho^2) = 1$ iff pure.

Open quantum system: Two level system

Pauli representation

$$\hat{\rho} = \frac{1}{2}\left(I + \sum_i m_i \sigma_i \right)$$

σ_i : Pauli matrix spans space of 2x2 matrices



Purity in Pauli rep.

$$Tr(\rho^2) = \frac{1}{4} Tr(I + 2\sum m_i \sigma_i + \sum m_i m_j \sigma_i \sigma_j)$$

$$= \frac{1}{2}(1 + \sum m_i^2)$$: By orthogonality

$$\frac{1}{2} \leq purity \leq 1 \Rightarrow |\vec{m}| \leq 1$$ Bloch Sphere rep.

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Closed : Unitary evolution

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{\mathrm{H}}, \hat{\rho}]$$

: Liouville von-Neumann equation

Open: System + Environment 의 전체 time evolution 중 system의 상태만 보면 어떻게 변화하는가?

$$\frac{d\hat{\rho}_S}{dt} = -\frac{i}{\hbar}[\hat{\mathrm{H}}, \hat{\rho}_S(t)] + \sum_{\mu}\left(-\frac{1}{2}L_{\mu}L_{\mu}^{\dagger}\hat{\rho}_S - \frac{1}{2}\hat{\rho}_S L_{\mu}L_{\mu}^{\dagger} + L_{\mu}\hat{\rho}_S L_{\mu}^{\dagger}\right)$$

: The master equation in the Lindblad form

Examples of Quantum channel

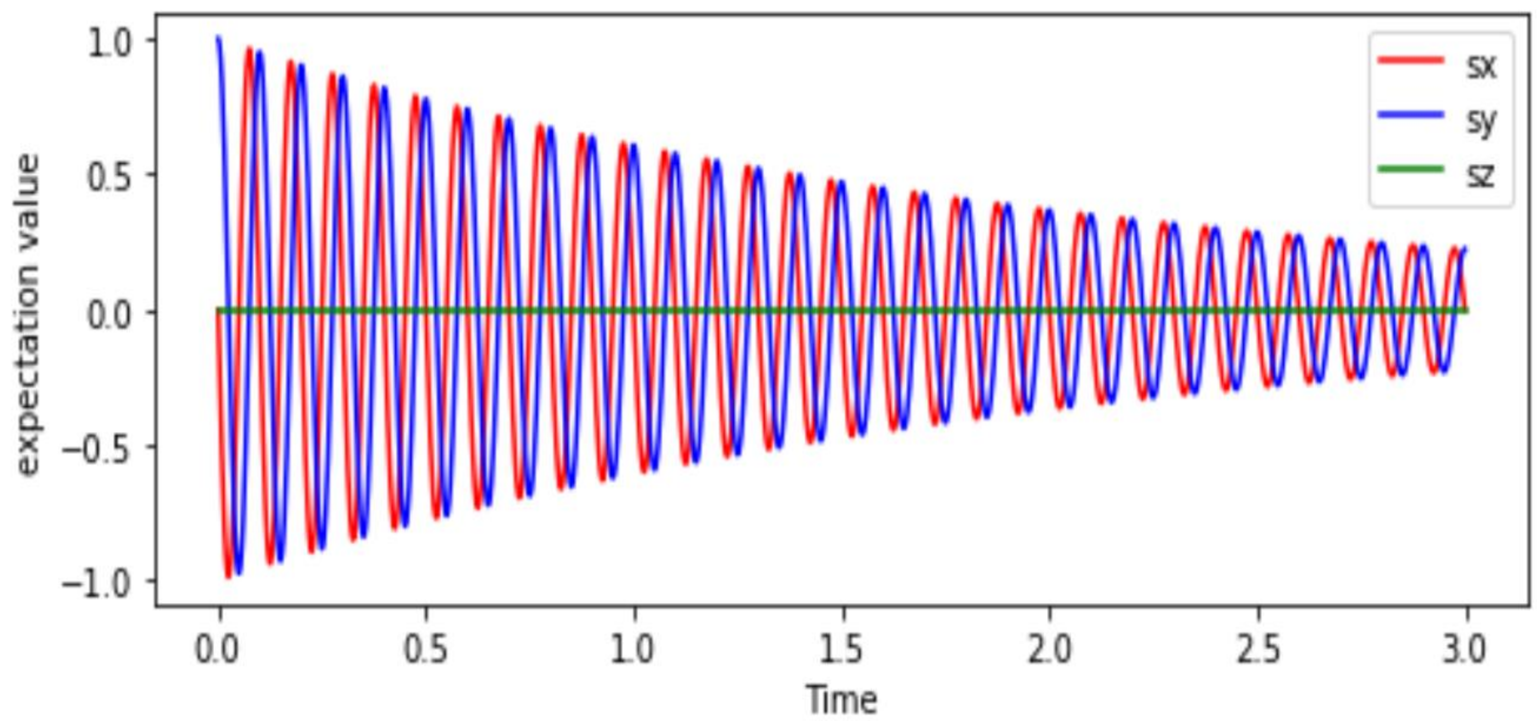
Application of L form to two-level system

Pure dephasing channel

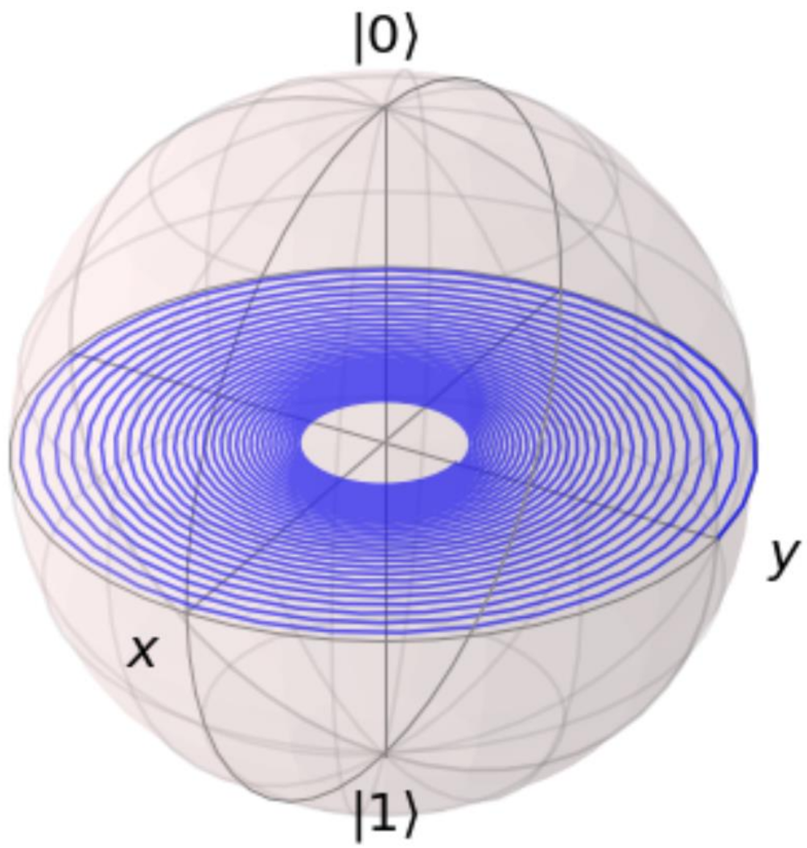
$$H_S = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix} \qquad L_1 = L_1^{\dagger} = \sqrt{\gamma}\sigma_+\sigma_- = \sqrt{\gamma}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Fluctuating energy levels

This time scale is called, T_ϕ pure dephasing time



In the Lab frame,



Examples of Quantum channel

Application of L form to two-level system

Amplitude damping (relaxation) channel

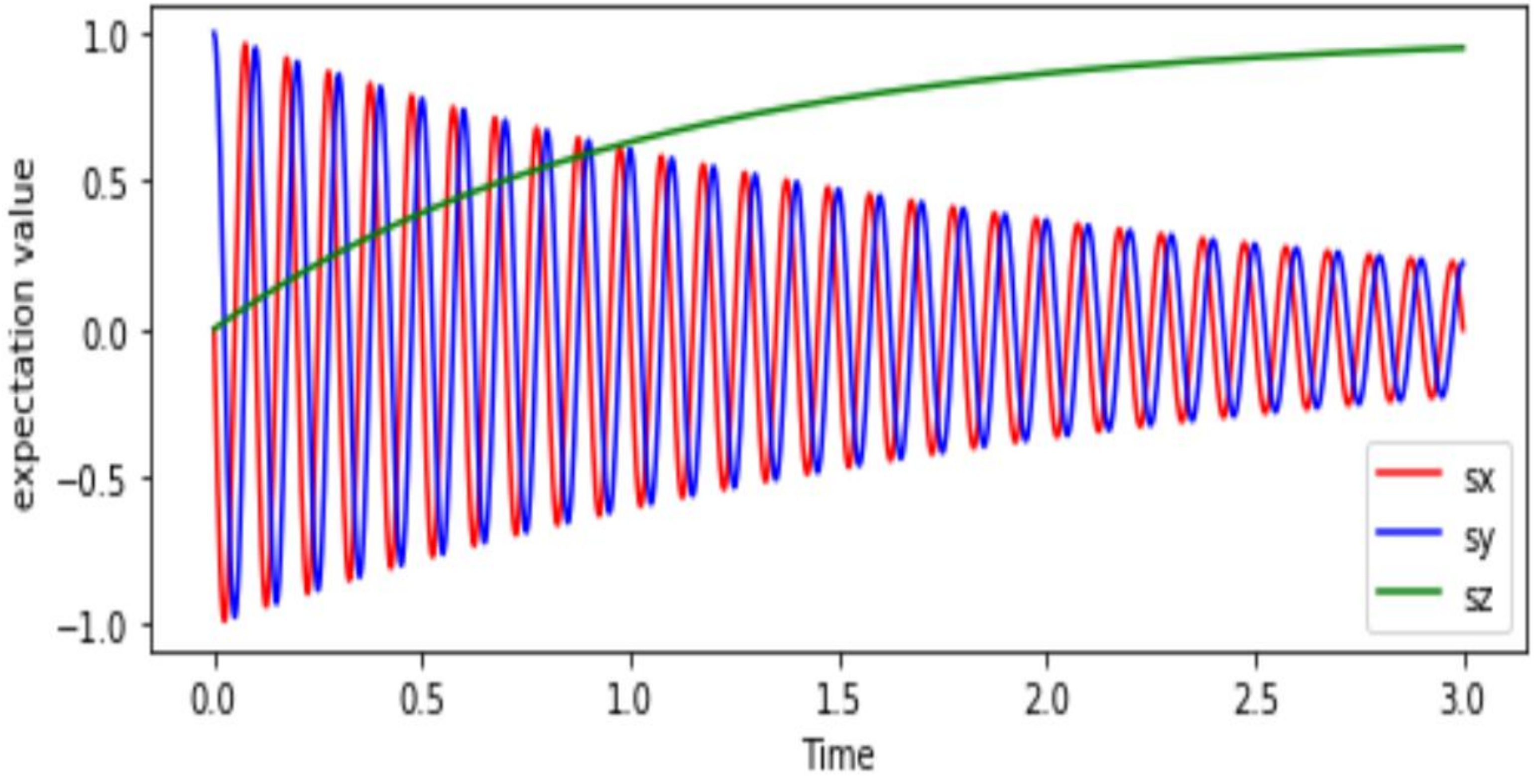
$$L_2 = \sqrt{\gamma}\sigma_- = \sqrt{\gamma}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

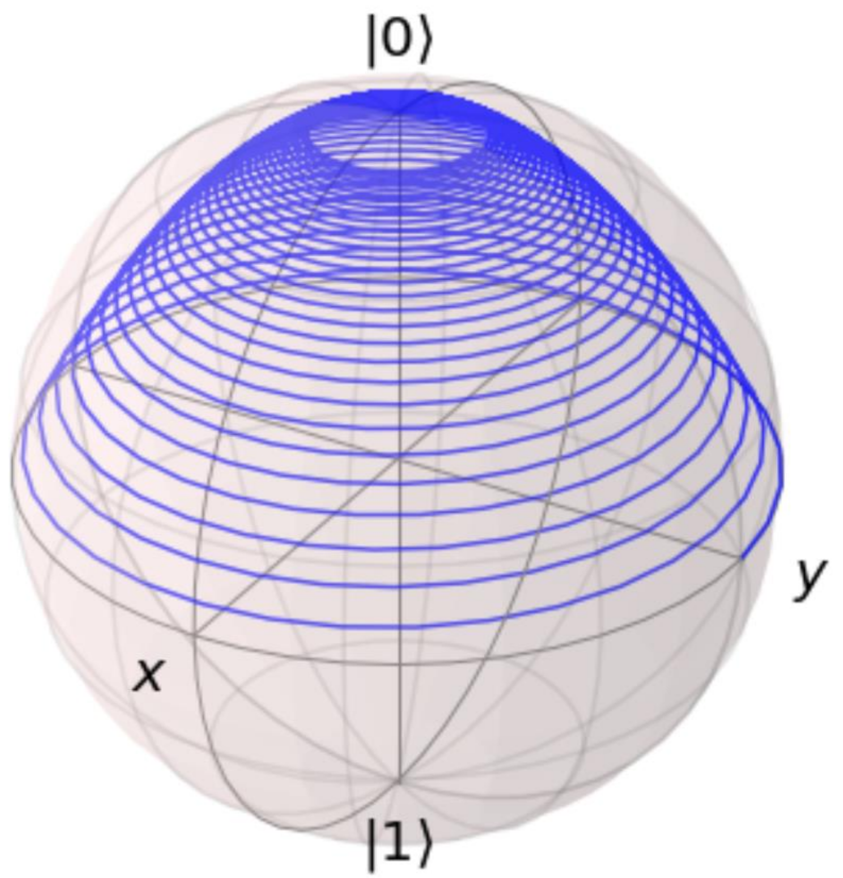
Energy relaxation

Total decoherence rate set by,

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$

This time scale is called, T_1 relaxation time





Also, depolarizing channel.. Etc.

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: The master equation in the Lindblad form

Dynamics of open quantum system

Time evolution: Kraus operators

$$H_{SE} = H_S \otimes H_E$$

Unitary evolution: U_{SE}

Environment orthonormal basis: $\left\{ \left| \mu \right\rangle_E \right\}$

Special proposition: $\hat{\rho}_{SE}(t) = \hat{\rho}_S(0) \otimes \hat{\rho}_E(0) = \hat{\rho}_S(0) \otimes \left| 0 \right\rangle_E \left\langle 0 \right|_E$

$$\hat{\rho}_{SE}(t) = U_{SE}(t)\hat{\rho}_{SE}(0)U^{\dagger}{}_{SE}(t)$$

$$\hat{\rho}_S(t) = Tr_E(U_{SE}(t)\hat{\rho}_{SE}(0)U^{\dagger}{}_{SE}(t))$$

$$= \sum_{\mu} \left\langle \mu \right|_E U_{SE}(t) \left| 0 \right\rangle_E \hat{\rho}_S(0) \left\langle 0 \right|_E U^{\dagger}{}_{SE}(t) \left| \mu \right\rangle_E$$

$$= \sum_{\mu} \hat{M}_{\mu}(t)\hat{\rho}_S(0)\hat{M}^{\dagger}_{\mu}(t)$$

$\hat{M}_{\mu}(t)$: Kraus operators

Dynamics of open quantum system

Operator-sum representation 낙서

$$\rho_s(t) = \sum_\mu M_\mu(t)\hat{\rho}_S(0)M^\dagger_\mu(t)$$
$$= a[\hat{\rho}_S(0)]$$

Unitary evolution?
in general no.

Special case : pure state

$$\rho_s(t) = \sum_\mu M_\mu(t)\left|\psi_s(0)\right\rangle\left\langle\psi_s(0)\right|M^\dagger_\mu(t)$$
$$= \sum_\mu \left|\psi_s(t)\right\rangle\left\langle\psi_s(t)\right|$$
$$= \sum_\mu p_\mu(t)\left|\psi_s(t)\right\rangle\left\langle\psi_s(t)\right|$$

$$\left|\psi_s(t)\right\rangle \equiv M_\mu(t)\left|\psi_s(0)\right\rangle$$

$$\left|\psi_s(t)\right\rangle \equiv \frac{\left|\psi_s(t)\right\rangle}{\left\|\left|\psi_s(t)\right\rangle\right\|}$$

Not unitary operator
Not diagonal rep.
Generally mixed state

action of unitary operator on quantum system in general create entanglement

General properties of quantum map

1. Linearity

$$a[\lambda\hat{\rho}_1 + (1-\lambda)\hat{\rho}_2] = \lambda a[\hat{\rho}_1] + (1-\lambda)a[\hat{\rho}_2]$$

2. Completely Positive (CP condition)

$\hat{\rho}_{out} = a[\hat{\rho}_{in}]$: Physical state $\hat{\rho}_{out} = \hat{\rho}^{\dagger}_{out}$

$\Rightarrow \hat{\rho}_{out} \geq 0$ ($\hat{\rho}^{\dagger}_{out}\hat{\rho}_{out}$ non-negative eigenvalue)

3. Trace preserving (TP condition)

$$Tr_S[a[p_S]] = Tr_S(\sum_{\mu} M_{\mu}(t)\hat{\rho}_S(0)M^{\dagger}_{\mu}(t))$$

$$= Tr_S(\sum_{\mu}(M_{\mu}(t)M^{\dagger}_{\mu}(t))\hat{\rho}_S(0))$$

$$\sum_{\mu} M_{\mu}(t)M^{\dagger}_{\mu}(t) = \sum_{\mu} \langle 0|_E U^{\dagger}_{SE}(t)(|\mu\rangle_E \langle\mu|_E)U_{SE}(t)|0\rangle_E$$

$$= \langle 0|_E U^{\dagger}_{SE}(t)U_{SE}(t)|0\rangle_E = \hat{I}_S$$

Dynamics of open quantum system

Closed quantum system

$$\rho(t+dt) = U(t+dt,t)\rho(t)U^{\dagger}(t+dt,t)$$: generator of time-translation

$$U(t+dt,t) = I - \frac{i}{\hbar}H(t)dt$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho]$$: Liouville von-Neumann equation

Open quantum system

what is generator of time-translation?

$$\hat{\rho}_S(t_2) = a[\hat{\rho}_S(t_1)]$$

$$\hat{\rho}_S(t+dt) = a(t+dt,t)[\hat{\rho}_S(t)]$$: Markov approx.

$$= \sum_{\mu} M_{\mu}(t+dt,t)\hat{\rho}_S(t)M^{\dagger}_{\mu}(t+dt,t)$$

Dynamics of open quantum system

Lindblad operator

$$M_\mu(t+dt,t) \equiv L_\mu(t)\sqrt{dt}$$

$$M_0(t+dt,t) \equiv \hat{I} + G(t)dt$$

CPTP condition

$$\sum_\mu M_\mu M^\dagger_\mu = \hat{I}$$

$$\Rightarrow M_0 M^\dagger_0 + \sum_{\mu\neq 0} M_\mu M^\dagger_\mu = \hat{I} + (G+G^\dagger)dt + \sum_\mu L_\mu L_\mu^\dagger dt = \hat{I}$$

$$\Rightarrow G + G^\dagger = -\sum_\mu L_\mu L_\mu^\dagger$$

$$\Rightarrow G \equiv K - \frac{i}{\hbar}H = -\frac{i}{\hbar}H - \frac{1}{2}\sum_\mu L_\mu L_\mu^\dagger \equiv -\frac{i}{\hbar}H_{eff}$$

Dynamics of open quantum system(Master eqn.)

$$\rho_S(t+dt) = M_0 \hat{\rho}_S(t) M^{\dagger}_0 + \sum_{\mu\neq 0} M_\mu \hat{\rho}_S(t) M^{\dagger}_\mu$$

$$= \left(\hat{I} - \frac{i}{\hbar} H_{eff} dt \right) \hat{\rho}_S(t) \left(\hat{I} - \frac{i}{\hbar} H_{eff} dt \right) + \sum_\mu L_\mu \hat{\rho}_S(t) L^{\dagger}_\mu dt$$

$$\Rightarrow \frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H_{eff}, \hat{\rho}_S(t)] + \sum_\mu L_\mu \hat{\rho}_S(t) L^{\dagger}_\mu$$

expanding effective Hamiltonian

$$\frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H, \hat{\rho}_S(t)] + \sum_\mu \left(-\frac{1}{2} L_\mu L_\mu^{\dagger} \hat{\rho}_S - \frac{1}{2} \hat{\rho}_S L_\mu L_\mu^{\dagger} + L_\mu \hat{\rho}_S L_\mu^{\dagger} \right)$$

: The master equation in the Lindblad form

The differential eqn. whose integral is CPTP map has to be this 'Lindblad form'

Composite systems

Hilbert Space of composite system

$$H = H_A \otimes H_B$$

Two observable algebra A, B

Product State of composite systems

$$|\phi_A\rangle \otimes |\phi_B\rangle,$$

$$where, |\phi_A\rangle \in H_A, |\phi_B\rangle \in H_B$$

Pure product states have density operators

$$\hat{\rho} = |\phi_A\rangle\langle\phi_A| \otimes |\phi_B\rangle\langle\phi_B| = \hat{\rho}_A \otimes \hat{\rho}_B$$

Q. All states are product state?

States which aren't products are correlated

$$\hat{\rho} = p|00\rangle\langle 00| + (1-p)|11\rangle\langle 11|$$

Q. How to check if the state is product state or not?

Schmidt decomposition for pure state

$$|\psi\rangle = \sum_{n=1}^{d} \sqrt{p_i}\,|e_i\rangle|f_j\rangle$$

$$then, \rho_A = \sum_{n=1}^{d} p_j |e_j\rangle\langle e_i|$$

Composite systems: Entanglement

Partial Trace

$$\rho_{AB} = \sum_i p_i \left|\Psi_i\right\rangle_{AB} \left\langle\Psi_i\right|_{AB}$$

'Trace out' environment -> information loss?

$$\rho_A = Tr_B \rho_{AB} = \sum_j \left\langle\psi_i\right|_B p_i \left|\Psi_i\right\rangle_{AB} \left\langle\Psi_i\right|_{AB} \left|\psi_j\right\rangle_B$$

Ex) For singlet state

$$\hat{\rho} = \frac{1}{2}(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)$$

$$\hat{\rho}_A = \frac{1}{2}\left\langle\uparrow\right|_B (\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)\left|\uparrow\right\rangle_B$$

$$+\frac{1}{2}\left\langle\downarrow\right|_B (\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)\left|\downarrow\right\rangle_B$$

$$\Rightarrow \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

Completely Mixed state

Entropy: How much entangled?

$$S = -\sum_\mu p_\mu \log p_\mu$$

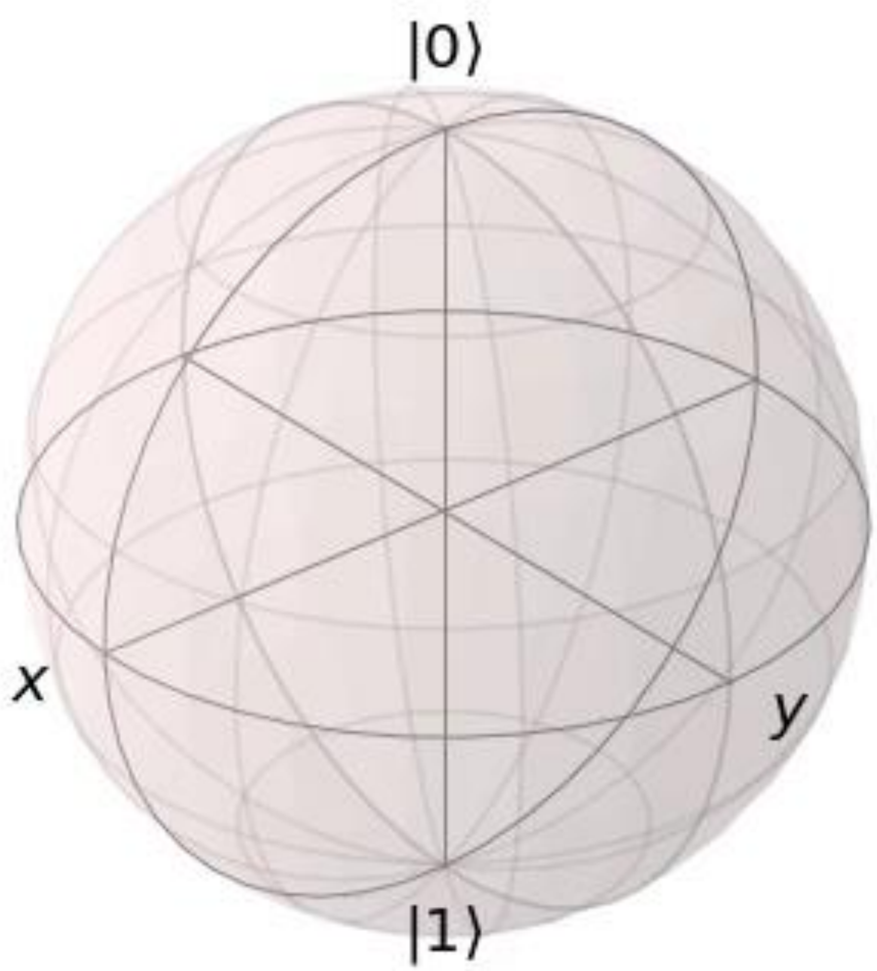
p_μ : Schmidt coefficient

More on entanglement

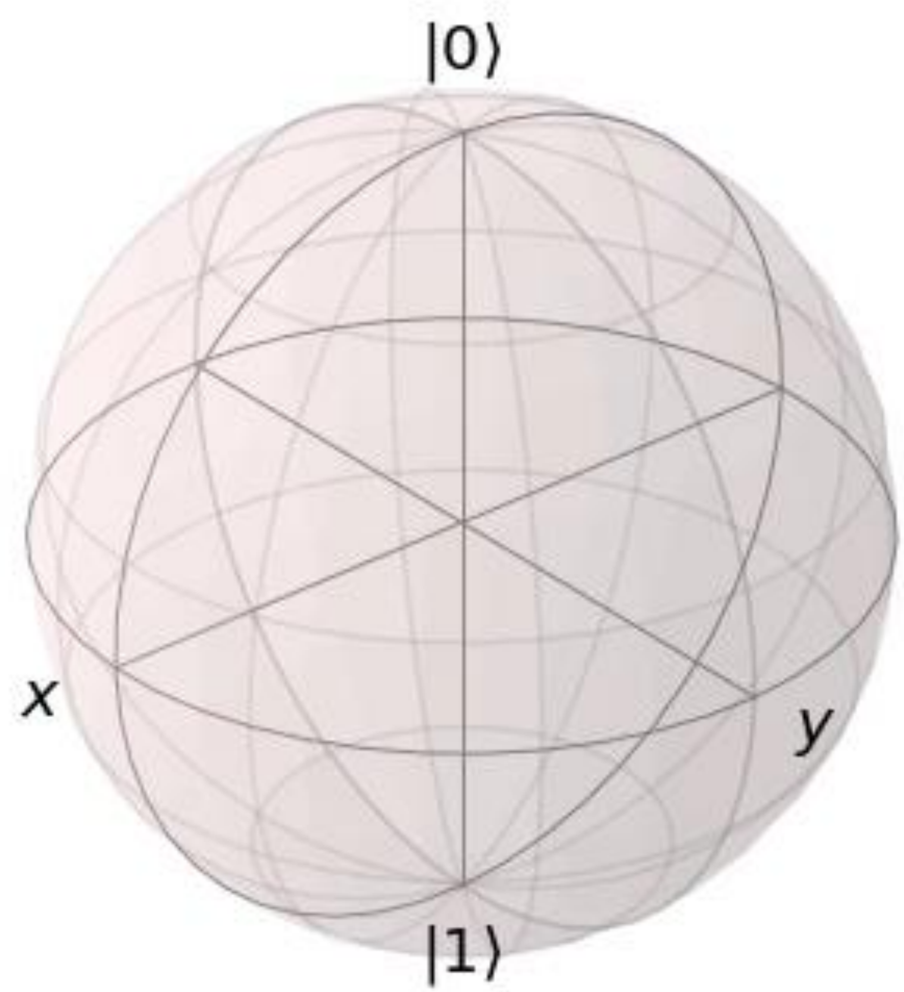
$$\hat{H} = \frac{J_1}{2}(\hat{\sigma}_{z1} \otimes I) + \frac{J_2}{2}(I \otimes \hat{\sigma}_{z2}) + \frac{J_{12}}{2}((\hat{\sigma}_{z1} - I) \otimes (\hat{\sigma}_{z2} - I))$$

Initial state : $|++\rangle$ Note : no decoherence as a whole (no L)

Qubit 1 (system)



Qubit 2 (Env.)



Lab frame

Outline

Introduction

Quantum control and decoherence

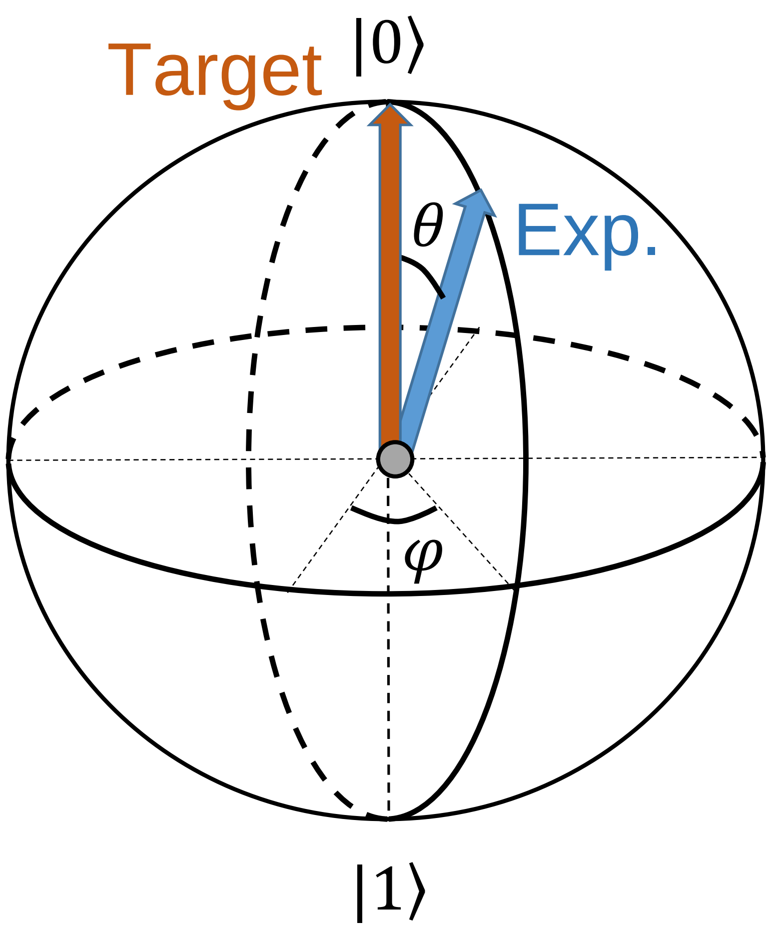
- 1Q, 2Q gate : Pulsed, Calibrated perturbation
- Master equations in the Markov approximation : Lindbald form
- Quantum noise channels : T_1 , T_2 time, Entanglement, meaning of decoherence

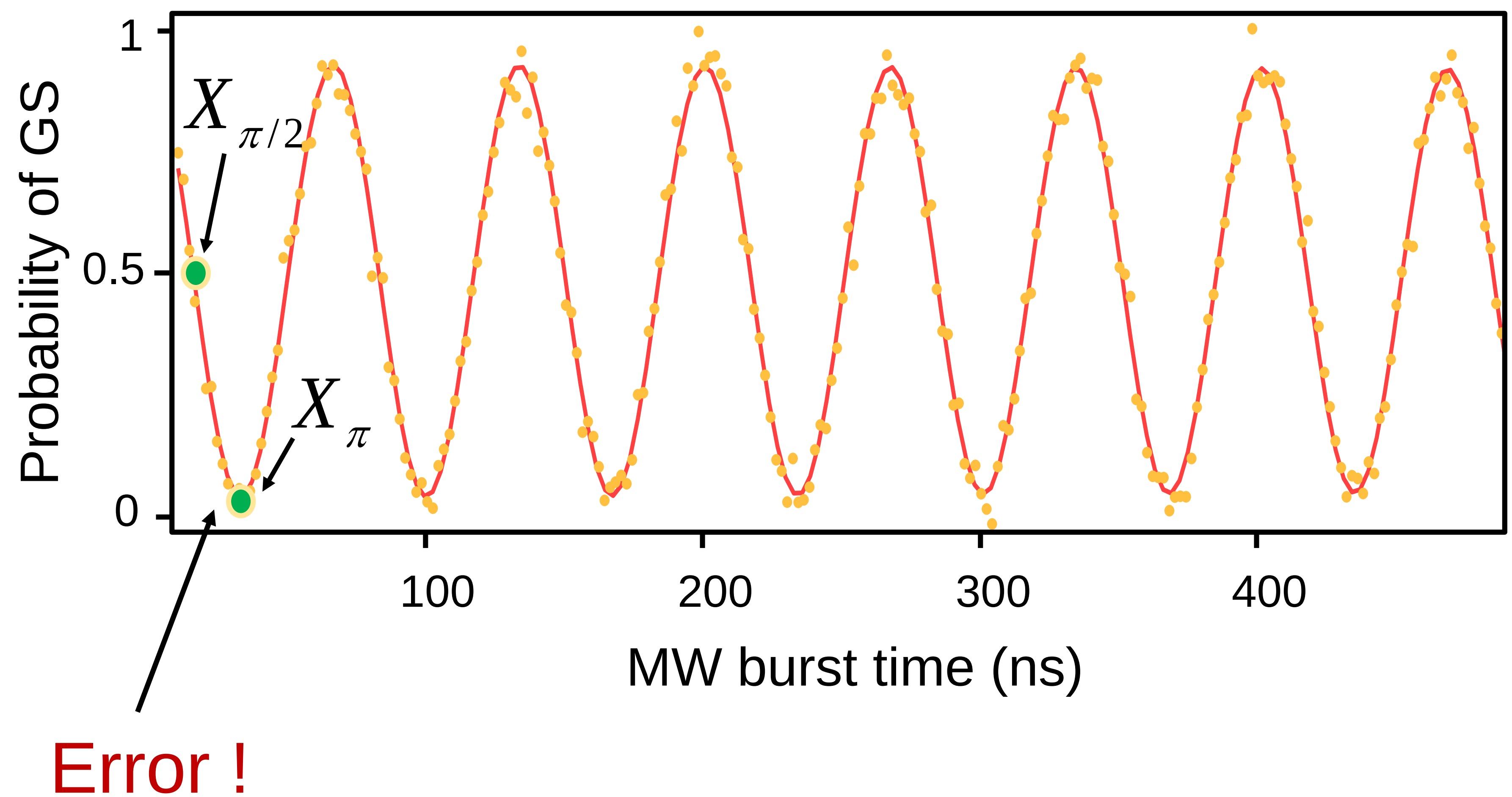
Quantum error correction

- Basic concepts
- Bit-flip and Phase-flip error correction
- Experimental examples

Quantum Errors

Quantum gate set is discrete, but quantum control generating a quantum gate is inherently analog.





Quantum Errors

A general quantum error is a superoperator (Kraus operator):

$$\rho \to \sum A_k \rho A_k^{\dagger}$$

Examples of single-qubit errors:

Bit Flip X: $X|0\rangle = |1\rangle,\ X|1\rangle = |0\rangle$

Phase Flip Z: $Z|0\rangle = |0\rangle,\ Z|1\rangle = -|1\rangle$

Complete dephasing: $\rho \to 1/2(\rho + Z\rho Z^{\dagger})$
(decoherence)

Rotation: $R_0|0\rangle = |0\rangle,\ R_0|1\rangle = e^{i\theta}|1\rangle$

Classical Repetition Code

To correct a single bit-flip error for classical data,

we can use the repetition code:

$$0 \to 000$$

$$1 \to 111$$

If there is a single bit flip error, we can correct the state

by choosing the majority of the three bits, e.g. $010 \to 0$.

When errors are rare, one error is more likely than two.

Barriers to Quantum Error Correction

1. Measurement of error destroys superpositions.

2. No-cloning theorem prevents repetition.

3. Must correct multiple types of errors (e.g., bit flip and phase errors).

4. How can we correct continuous errors and decoherence?

Measurement Destroys Superpositions?

Let us apply the classical repetition code to a quantum state to try to correct a bit flip error:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$

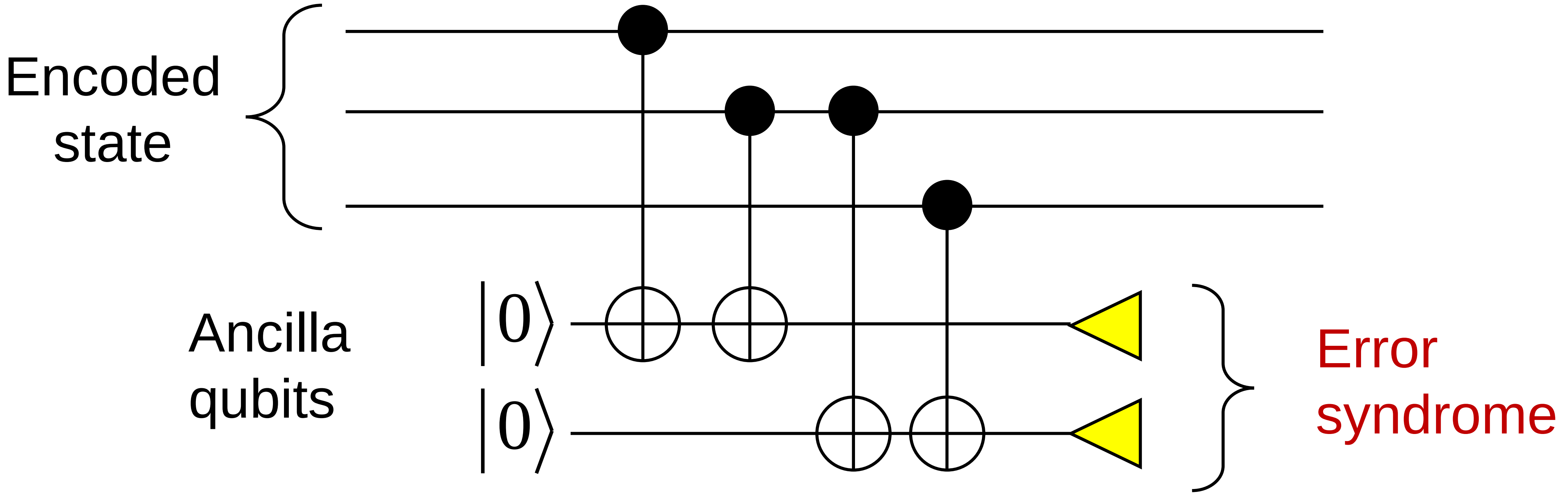
Bit flip error (X) on 2nd qubit:

$$\alpha|010\rangle + \beta|101\rangle$$

2nd qubit is now different from 1st and 3rd. We wish to measure that it is different without finding its actual value.

Measure the Error, Not the Data

Use this circuit:



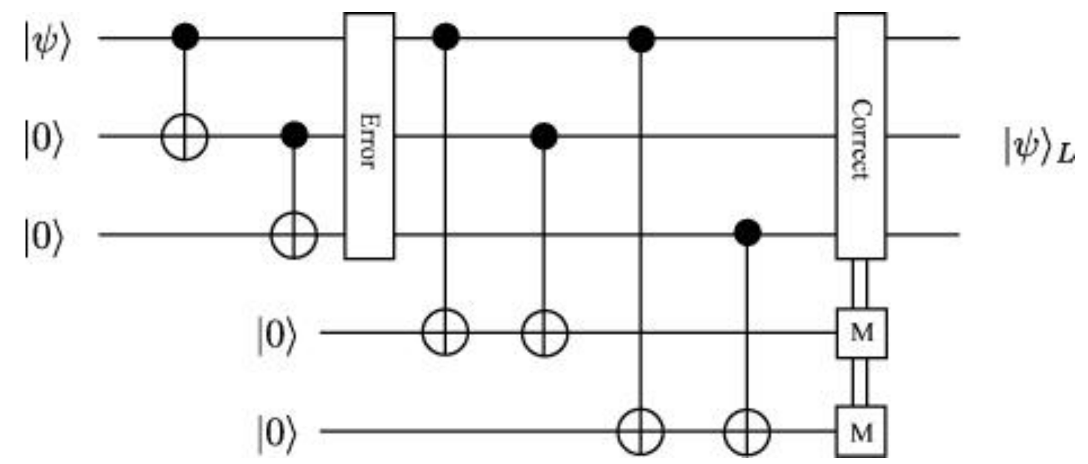
1st bit of error syndrome says whether the first two bits of the state are the same or different.

2nd bit of error syndrome says whether the second two bits of the state are the same or different.

Redundancy, Not Repetition

This encoding does not violate the no-cloning theorem:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle$$
$$\neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$$



We have repeated the state only in the computational basis; superposition states are spread out (redundant encoding), but not repeated (which would violate no-cloning).

Update on the Problems

✓ 1. Measurement of error destroys superpositions.

✓ 2. No-cloning theorem prevents repetition.

3. Must correct multiple types of errors (e.g., bit flip and phase errors).

4. How can we correct continuous errors and decoherence?

Correcting Just Phase Errors

Hadamard transform H exchanges bit flip and phase errors:

$$H(\alpha|0\rangle + \beta|1\rangle) = \alpha|+\rangle + \beta|-\rangle$$

$$X|+\rangle = |+\rangle, X|-\rangle = -|-\rangle$$ (acts like phase flip)

$$Z|+\rangle = |-\rangle, Z|-\rangle = |+\rangle$$ (acts like bit flip)

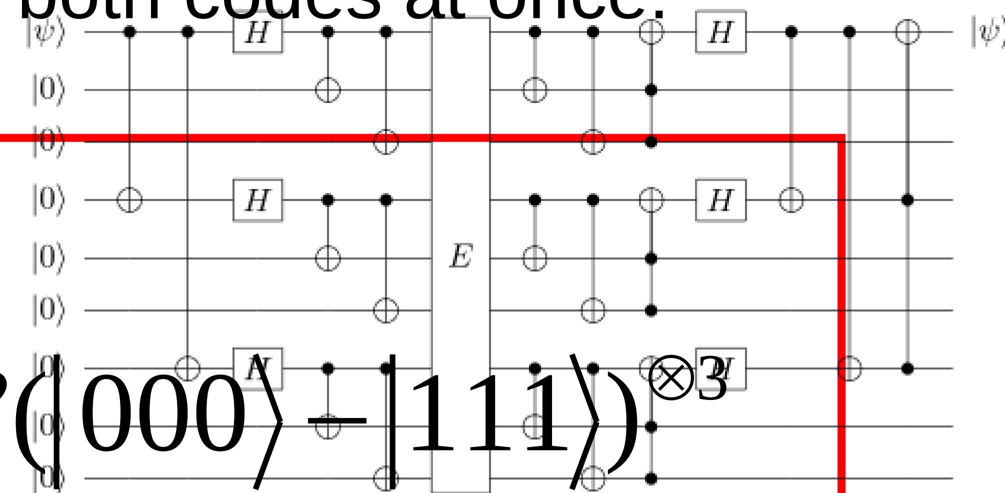
Repetition code corrects a bit flip error

⇨ Repetition code in Hadamard basis corrects a phase error.

$$\alpha|+\rangle + \beta|-\rangle \rightarrow \alpha|+++\rangle + \beta|---\rangle$$

Nine-Qubit Code

To correct both bit flips and phase flips, use both codes at once:



$$\alpha|0\rangle + \beta|1\rangle \to$$

$$\alpha(|000\rangle + |111\rangle)^{\otimes 3} + \beta(|000\rangle - |111\rangle)^{\otimes 3}$$

Repetition 000, 111 corrects a bit flip error,

repetition of phase +++, --- corrects a phase error

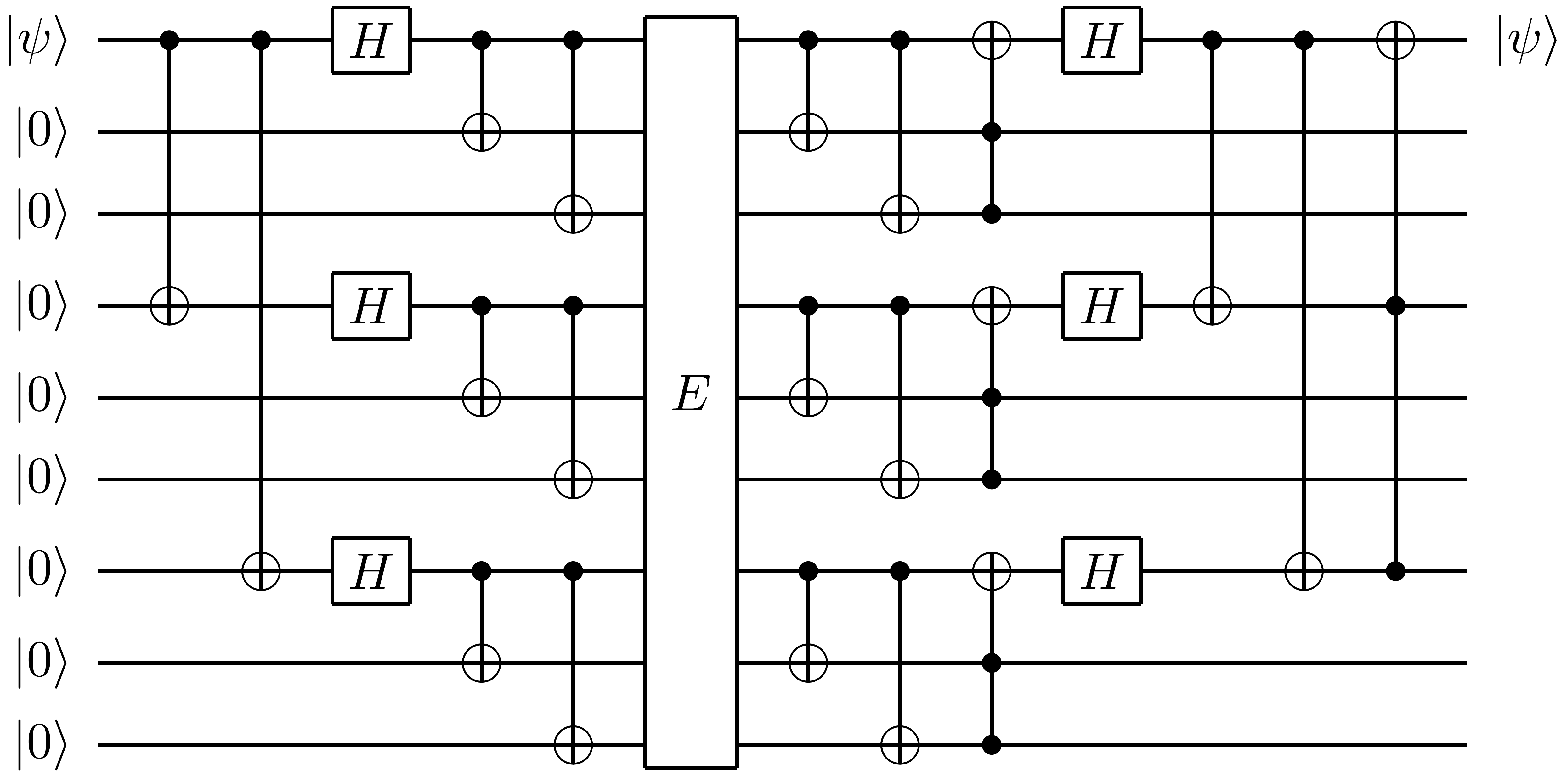
Actually, this code corrects a bit flip and a phase, so it also corrects a Y error:

$$Y = iXZ;\ Y|0\rangle = i|1\rangle,\ Y|1\rangle = -i|0\rangle$$

(global phase irrelevant)

Nine-Qubit Code

Circuit for the nine-qubit code



Update on the Problems

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Correcting Continuous Rotation

Let us rewrite continuous rotation

$$R_\theta |0\rangle = |0\rangle, R_\theta |1\rangle = e^{i\theta} |1\rangle$$

$$R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta/2} \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \cos(\theta/2)\, I - i \sin(\theta/2)\, Z$$

$$R_\theta^{(k)} |\psi\rangle = \cos(\theta/2)|\psi\rangle - i\sin(\theta/2) Z^{(k)} |\psi\rangle$$

($R_\theta^{(k)}$ is R_θ acting on the kth qubit.)

Correcting Continuous Rotations

How does error correction affect a state with a continuous rotation on it?

$$R_\theta^{(k)}|\psi\rangle = \cos(\theta/2)|\psi\rangle - i\sin(\theta/2)Z^{(k)}|\psi\rangle$$

$$\longrightarrow \quad \cos(\theta/2)|\psi\rangle|I\rangle - i\sin(\theta/2)Z^{(k)}|\psi\rangle|Z^{(k)}\rangle$$

Error syndrome

Measuring the error syndrome collapses the state:

$$\text{Prob. } \cos^2(\theta/2)\text{: } |\psi\rangle \text{ (no correction needed)}$$

$$\text{Prob. } \sin^2(\theta/2)\text{: } Z^{(k)}|\psi\rangle \text{ (corrected with } Z^{(k)}\text{)}$$

Correcting All Single-Qubit Errors

Theorem: If a quantum error-correcting code (QECC) corrects errors A and B, it also corrects $\alpha A + \beta B$.

Any 2x2 matrix can be written as $\alpha I + \beta X + \gamma Y + \delta Z$.

A general single-qubit error $\rho \to \sum A_k \rho A_k^\dagger$ acts like a mixture of $|\psi\rangle \to A_k |\psi\rangle$, and A_k is a 2x2 matrix.

Any QECC that corrects the single-qubit errors X, Y, and Z (plus I) corrects every single-qubit error.

Correcting all t-qubit X, Y, Z on t qubits (plus I) corrects all t-qubit errors.

Small Error on Every Qubit

Suppose we have a small error U_ε on every qubit in the QECC, where $U_\varepsilon = I + \varepsilon E.$

Then

$$U_\varepsilon^{\otimes n} |\psi\rangle = |\psi\rangle + \varepsilon(E^{(1)} + \ldots + E^{(n)})|\psi\rangle + O(\varepsilon^2).$$

If the code corrects one-qubit errors, it corrects the sum of the $E^{(i)}$s. Therefore it corrects the $O(\varepsilon)$ term, and the state remains correct to order ε^2.

A code correcting t errors keeps the state correct to order ε^{t+1}.

QECC is Possible

✓ 1. Measurement of error destroys superpositions.

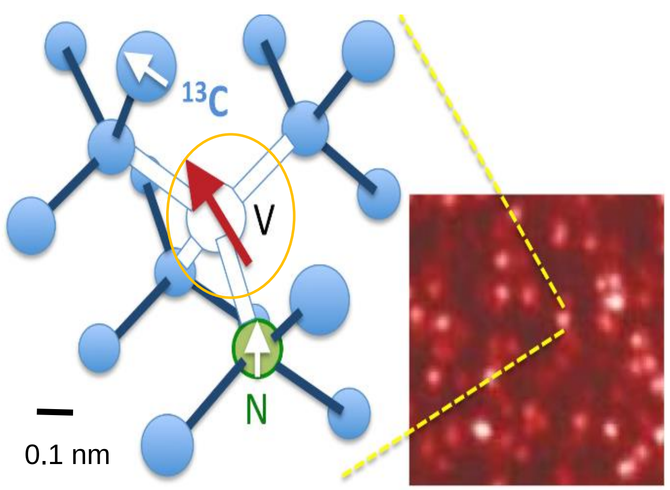
✓ 2. No-cloning theorem prevents repetition.

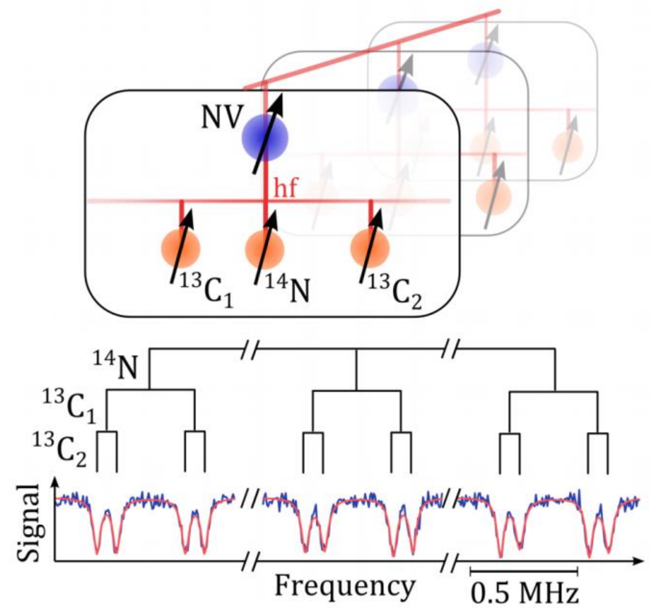
✓ 3. Must correct multiple types of errors (e.g., bit flip and phase errors).

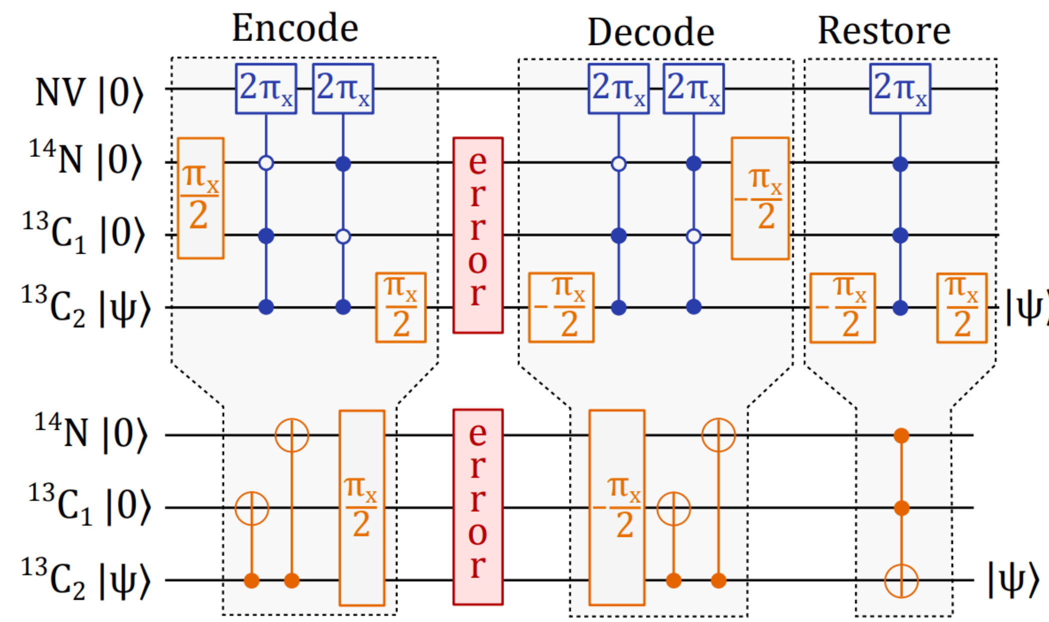
✓ 4. How can we correct continuous errors and decoherence?

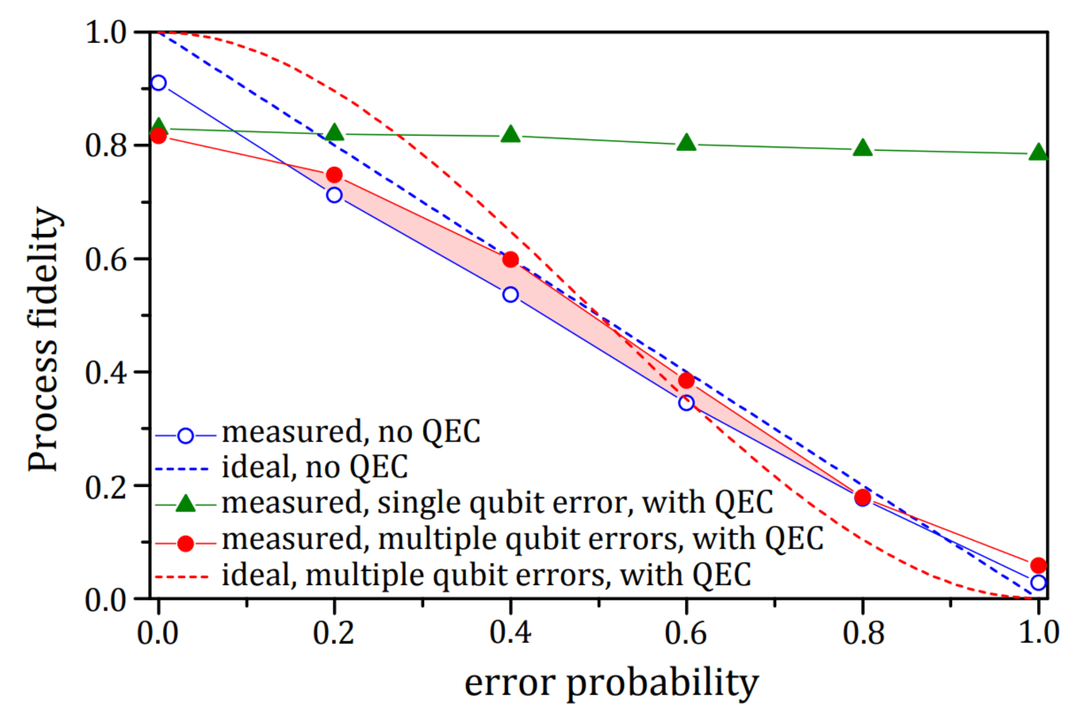
Formally, stabilizer formalism for constructing & designing error correction codes

Error correction example : Diamond NV center

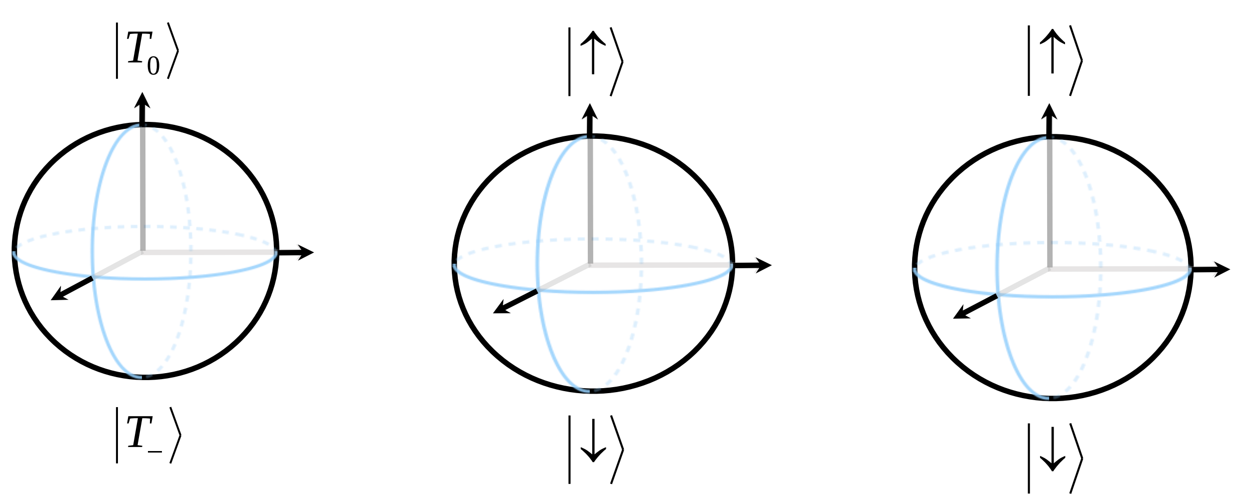






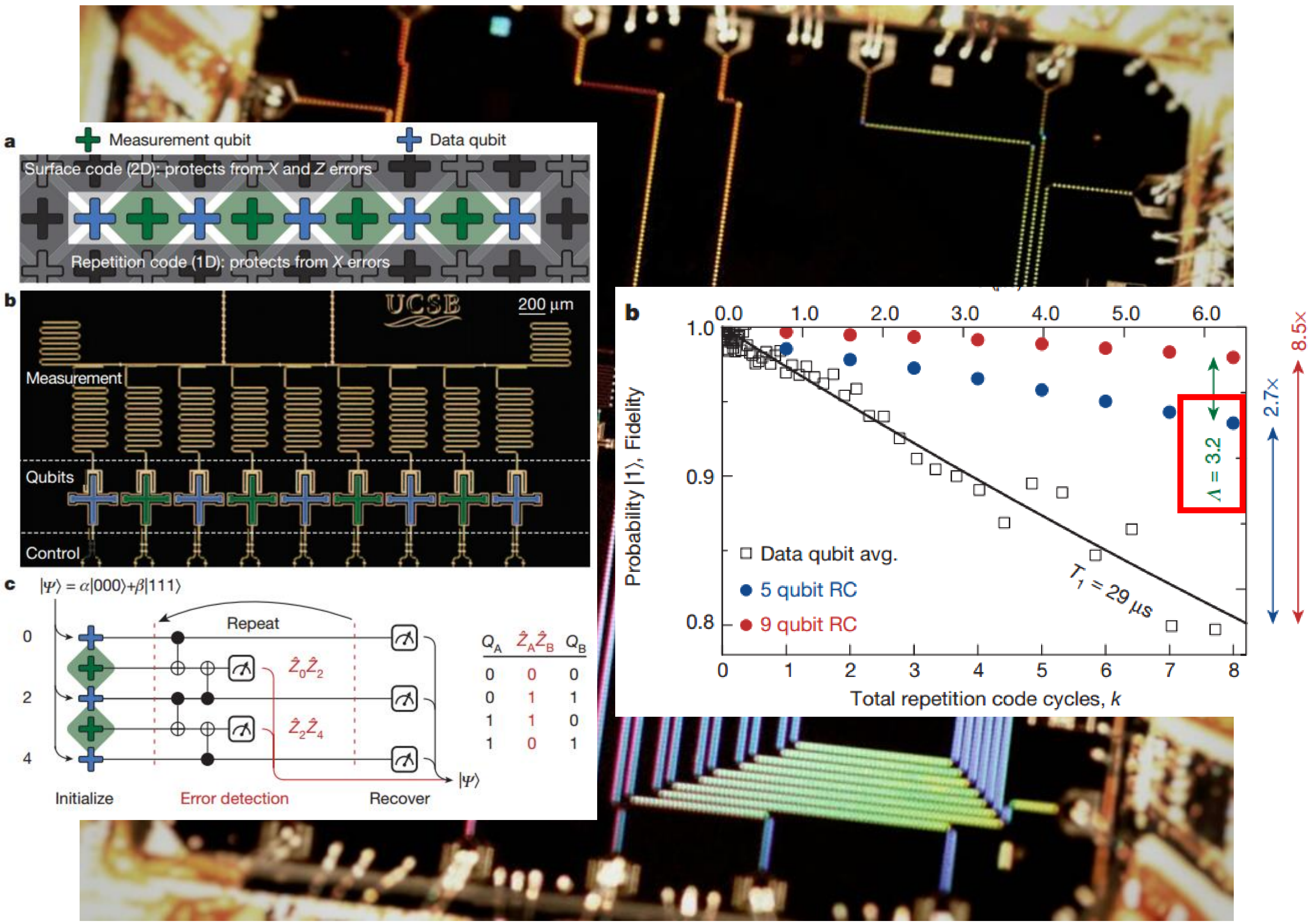


전자 스핀큐빗 질소 핵스핀큐빗 ^{13}C 핵스핀큐빗



No syndrome measurement, but proof-of-principle

Error correction example : Transmon



State preservation by repetitive measurements

- John Martinis – UCSB, Google
- 9 coupled **linear** transmon array
- Surface code error correction demonstrated (**X flip** error)
- X,Z gate time ~ 10ns
- CPHASE gate time ~ 100ns
- T_2~ 10μs
- 1Q gate error < 0.2%
- 2Q gate error < 1%
- 2Q gate enabled by direct capacitive coupling
- Frequency multiplexed single-shot dispersive readout

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- Experimental examples Quantum Error Correction is possible

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- Qubit dynamics calculation tutorial : QuTip
- Let’s use IBM machine online !