Nuclear many-body theory from microscopic chiral 2N and 3N forces

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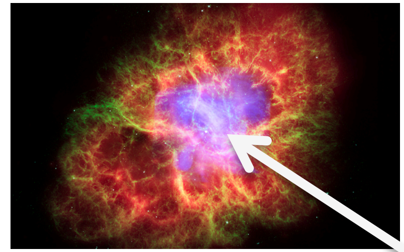
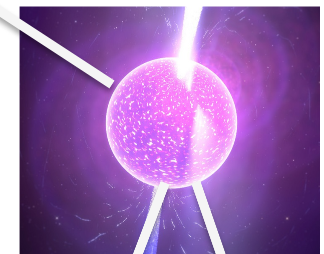
APCTP Focus Program in Nuclear Physics 2019
Nuclear Many-Body Theories: Beyond the Mean Field Approaches (7/1/2019)

Matter under extreme conditions

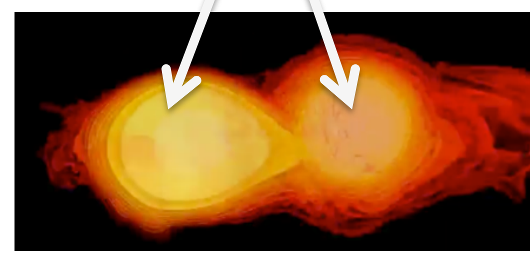
How do stars explode?
(Birthplace of neutron stars)



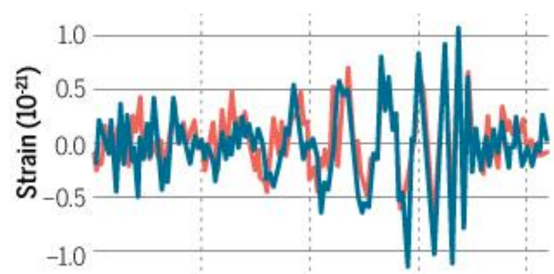


What is the nature of the densest observable matter in the universe?

Where are the heavy elements synthesized?



What new insights can gravitational wave astronomy reveal?

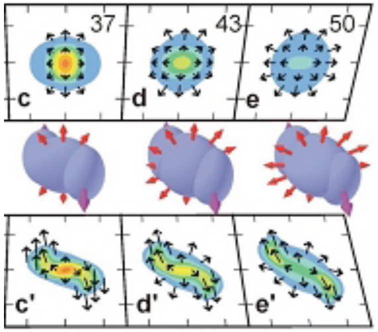


Progress on many fronts

Astronomical observations

Radio, optical,
X-ray, gamma ray,
gravitational wave,
neutrino astronomy

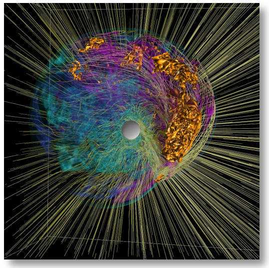
Nuclear experiments



Heavy ion collisions,
exotic isotope masses,
neutron skin thickness,
nuclear polarizabilities,

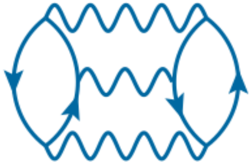
Astrophysical simulations

3-dimensional,
full general relativity,
neutrino transport,
magnetohydrodynamics

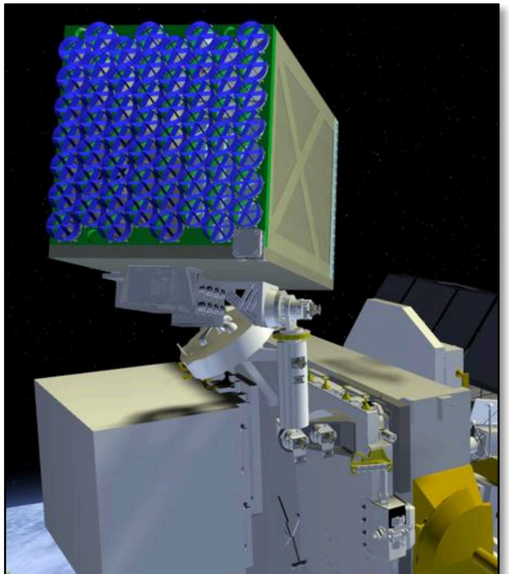


Nuclear Theory

Improved mean field
phenomenology &
chiral EFT: Equation of
state, neutrino response,
single-particle potentials



Observational campaigns of neutron stars

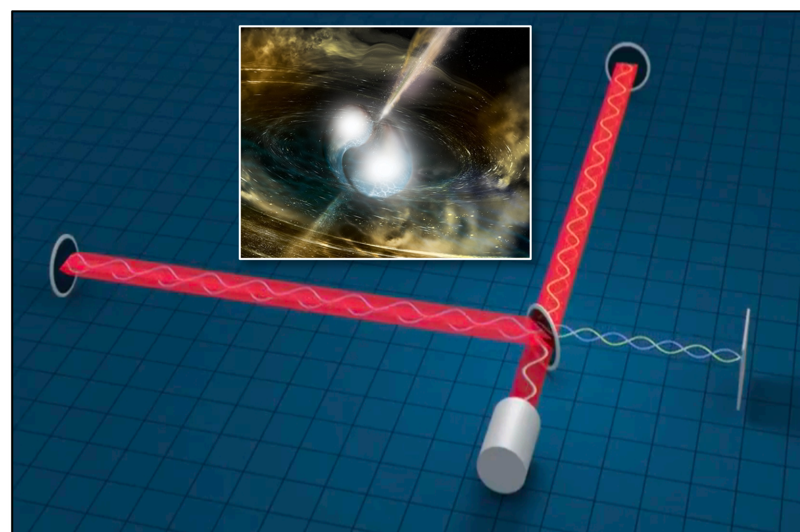


Neutron Star Interior Composition Explorer (NICER)

- Neutron star radii: $\pm 5\%$
- Neutron star masses: $\pm 10\%$
- Combined timing and spectral resolution in the soft X-ray band
- First dedicated targets: $\begin{cases} \text{PSR\_J0437-4715} \\ \text{PSR\_J0030+0451} \end{cases}$

LIGO/VIRGO

- Late-inspiral gravitational waveform related to neutron star tidal deformability
- Poster-merger peak frequency sensitive to neutron star radius



Multidimensional numerical simulations

- Parameter studies too expensive: ***incentive for improved microphysics modeling***

- **Nuclear equation of state:** $F = F(\rho, T, Y_p)$
(supernova shock wave energy, neutron star mass-radius relation, tidal deformability and gravitational waveforms,...)

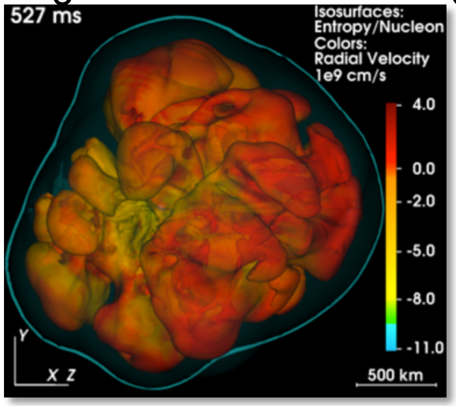
- **Single-particle properties and response functions:**

$$\bar{\nu}_e + n \longrightarrow \bar{\nu}_e + n$$

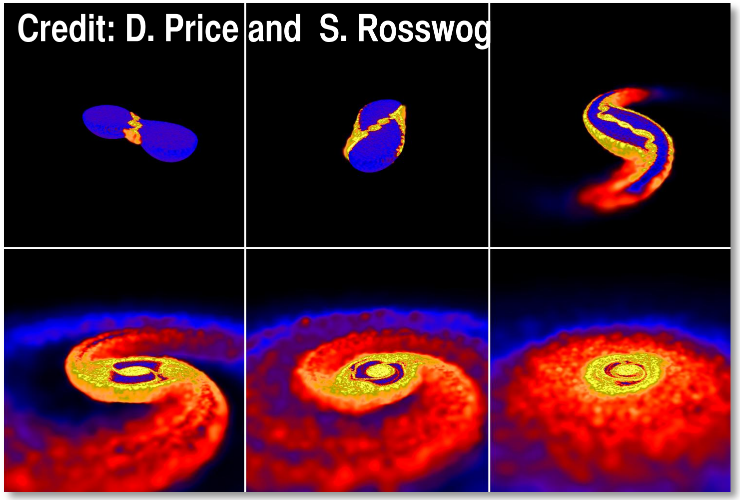
$$\nu_e + n \longleftrightarrow e^- + p$$

(supernova stalled shock wave revival, nucleosynthesis outcome, late-time supernova neutrino signal,…)

Progenitor mass = 20 $M_\odot$

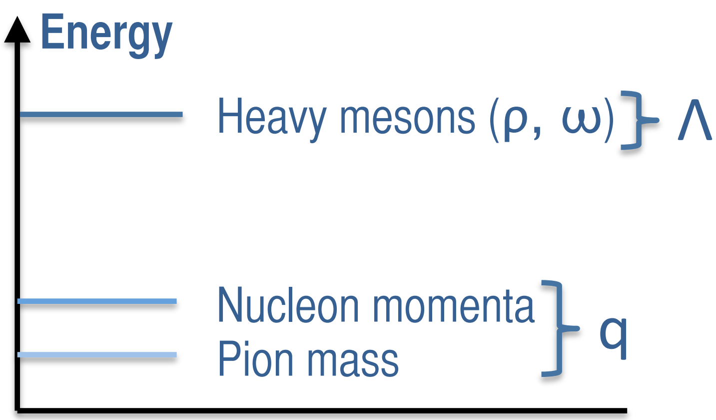


Melson et al., ApJ (2015)



Nuclear forces from chiral effective field theory

NATURAL SEPARATION OF SCALES



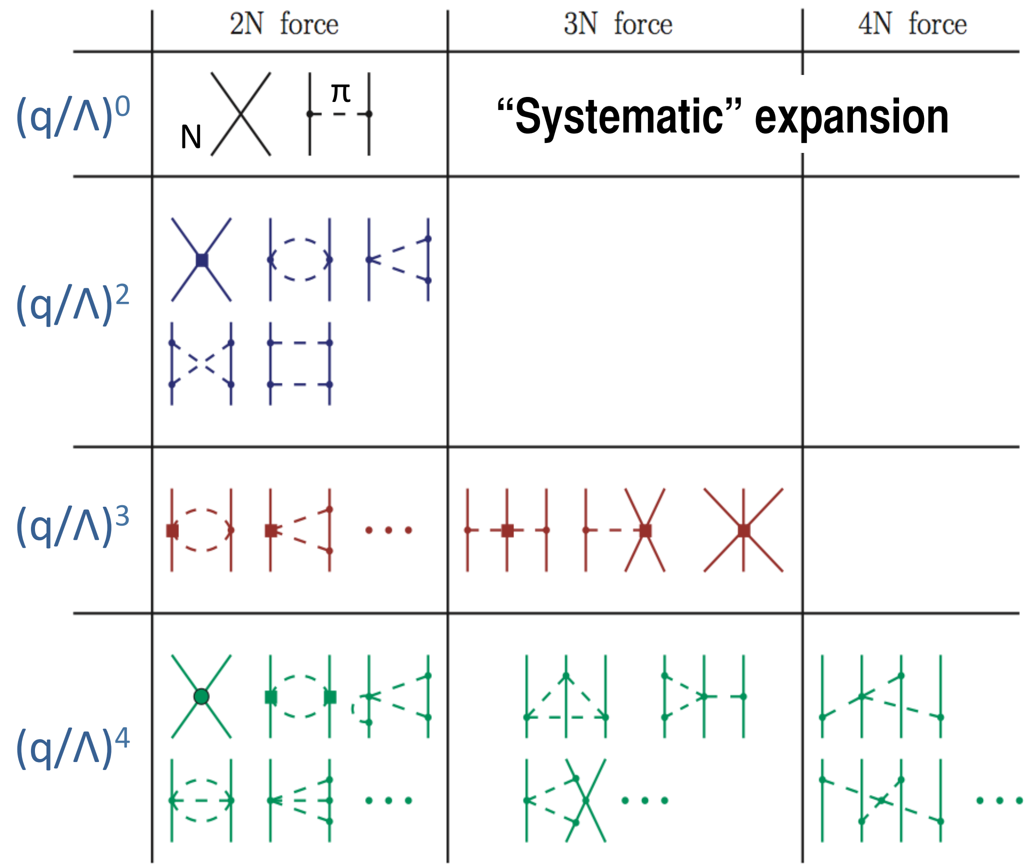
Pions weakly-coupled at low momenta

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{1}{2}\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{1}{2f_\pi^2}(\partial_\mu \vec{\pi} \cdot \vec{\pi})^2$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(i\gamma^\mu D_\mu - m - \frac{g_A}{2f_\pi}\gamma^\mu\gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\pi}\right) N$$

CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions



Progress, but still open questions: RG invariance, power counting, explicit $\Delta' s \ldots$

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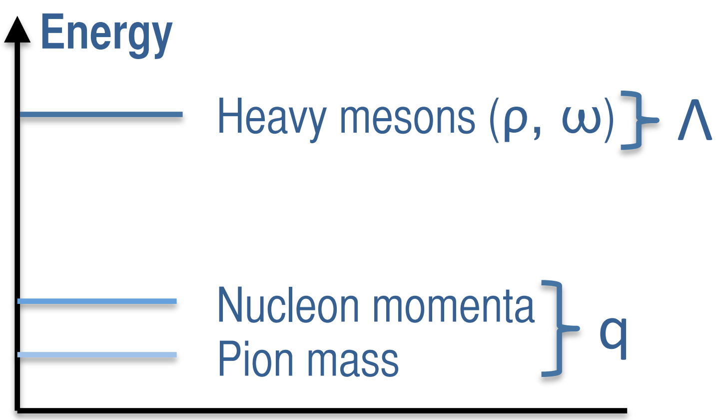

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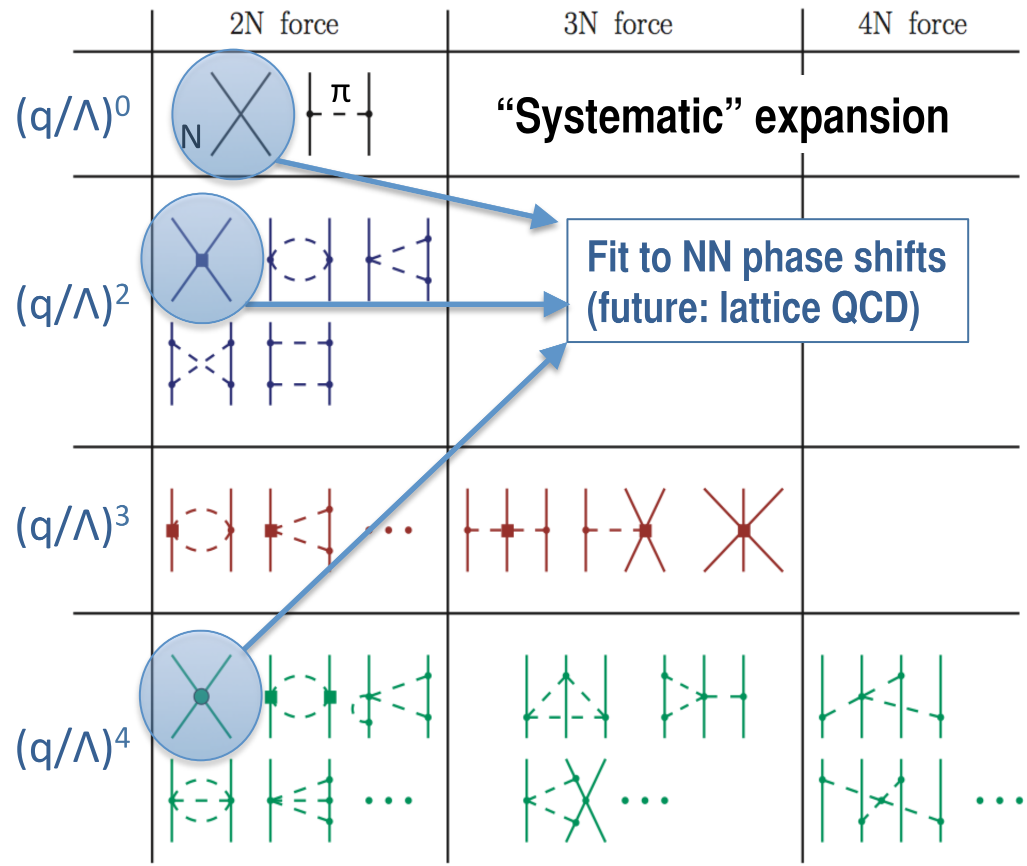

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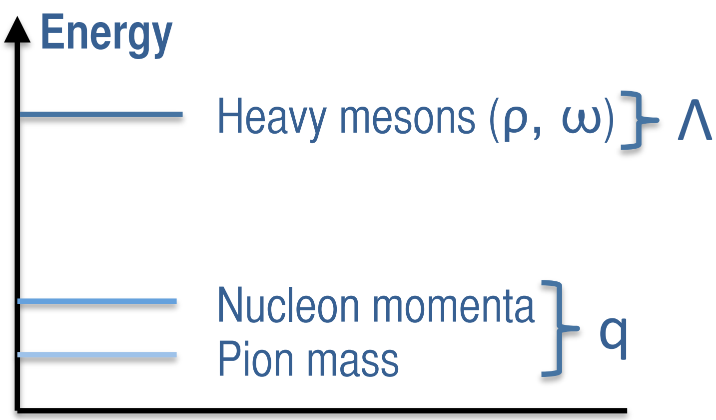

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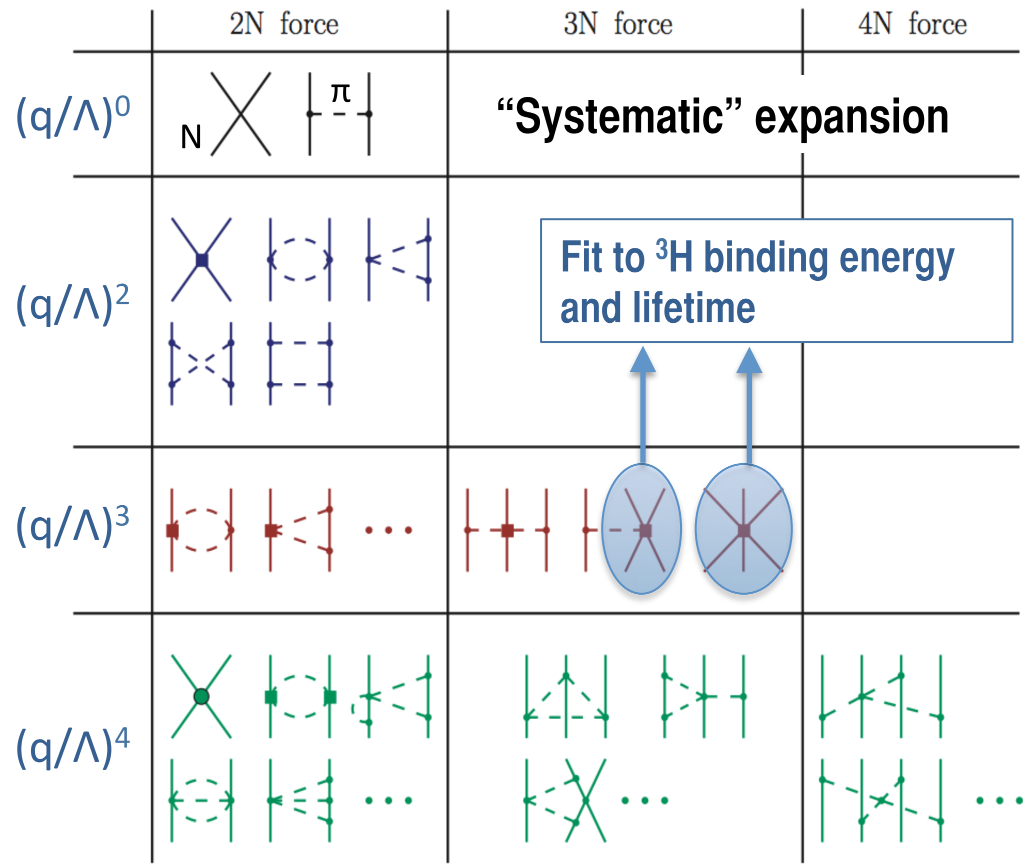

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Progress, but still open questions: RG invariance, power counting, explicit $\Delta's$ …

Choice of resolution scale

Variations in regulator

- Estimate of theoretical uncertainty

(red solid line) $\Lambda = 410\,\mathrm{MeV}\,(\Delta x \sim 1.50\,\mathrm{fm})$

(blue dashed line) $\Lambda = 450\,\mathrm{MeV}\,(\Delta x \sim 1.38\,\mathrm{fm})$

(black dotted line) $\Lambda = 500\,\mathrm{MeV}\,(\Delta x \sim 1.25\,\mathrm{fm})$

$$\langle \vec{p}\,'|V|\vec{p}\rangle \; \exp[-(p/\Lambda)^{2n} - (p'/\Lambda)^{2n}]$$

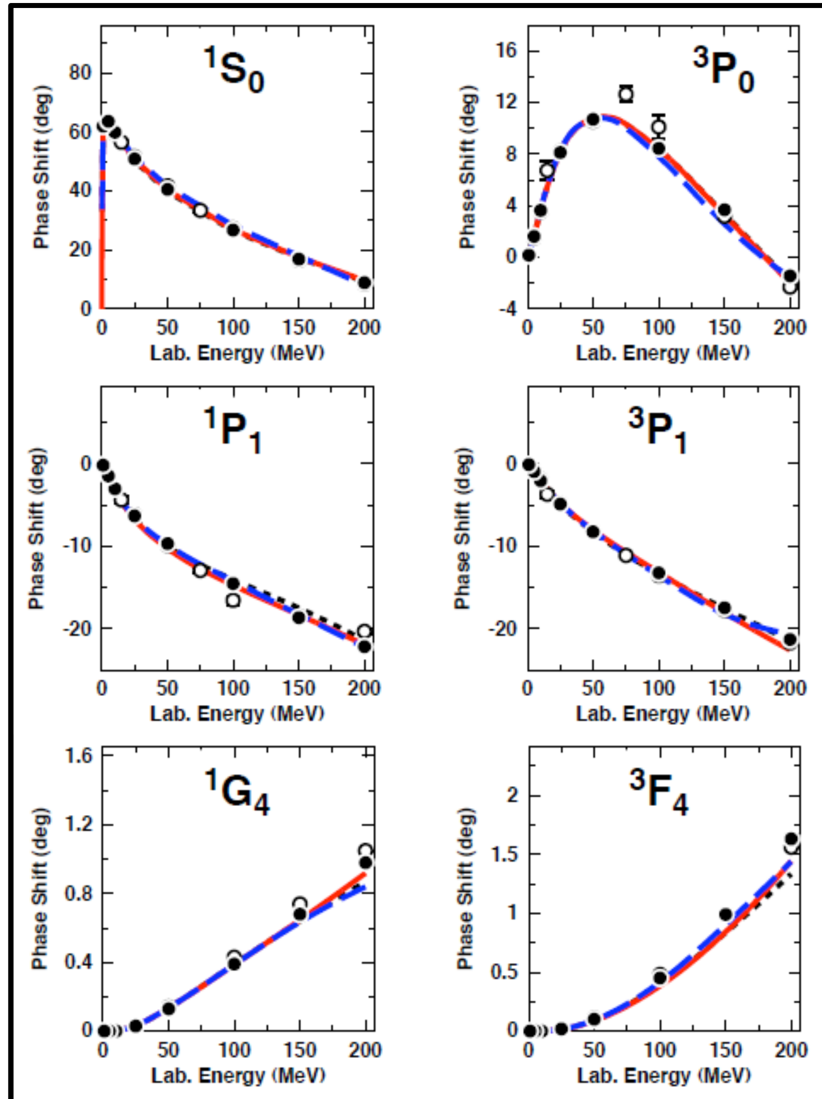
sets resolution scale

Symmetric nuclear matter:

$$k_f(2\rho_0) = 330\,\mathrm{MeV}$$

Pure neutron matter:

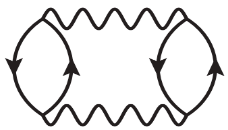
$$k_f(2\rho_0) = 420\,\mathrm{MeV}$$



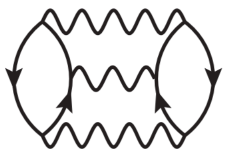
Coraggio, Holt, Itaco, Machleidt and Sammarruca, PRC (2013)

Many-body perturbation theory

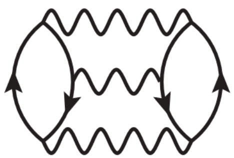
$$\rho E^{(1)} = \frac{1}{2} \sum_{12} n_1 n_2 \langle 12 | \left(\overline{V}_{NN} + \overline{V}_{NN}^{\rm med}/3 \right) | 12 \rangle,$$



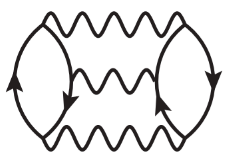
$$\rho E^{(2)} = -\frac{1}{4} \sum_{1234} |\langle 12 | \overline{V}_{\rm eff} | 34 \rangle|^2 \frac{n_1 n_2 \bar{n}_3 \bar{n}_4}{e_3 + e_4 - e_1 - e_2},$$



$$\rho E^{(3)}_{\rm pp} = \frac{1}{8} \sum_{123456} \langle 12 | \overline{V}_{\rm eff} | 34 \rangle \langle 34 | \overline{V}_{\rm eff} | 56 \rangle \langle 56 | \overline{V}_{\rm eff} | 12 \rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 \bar{n}_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_5 + e_6 - e_1 - e_2)},$$



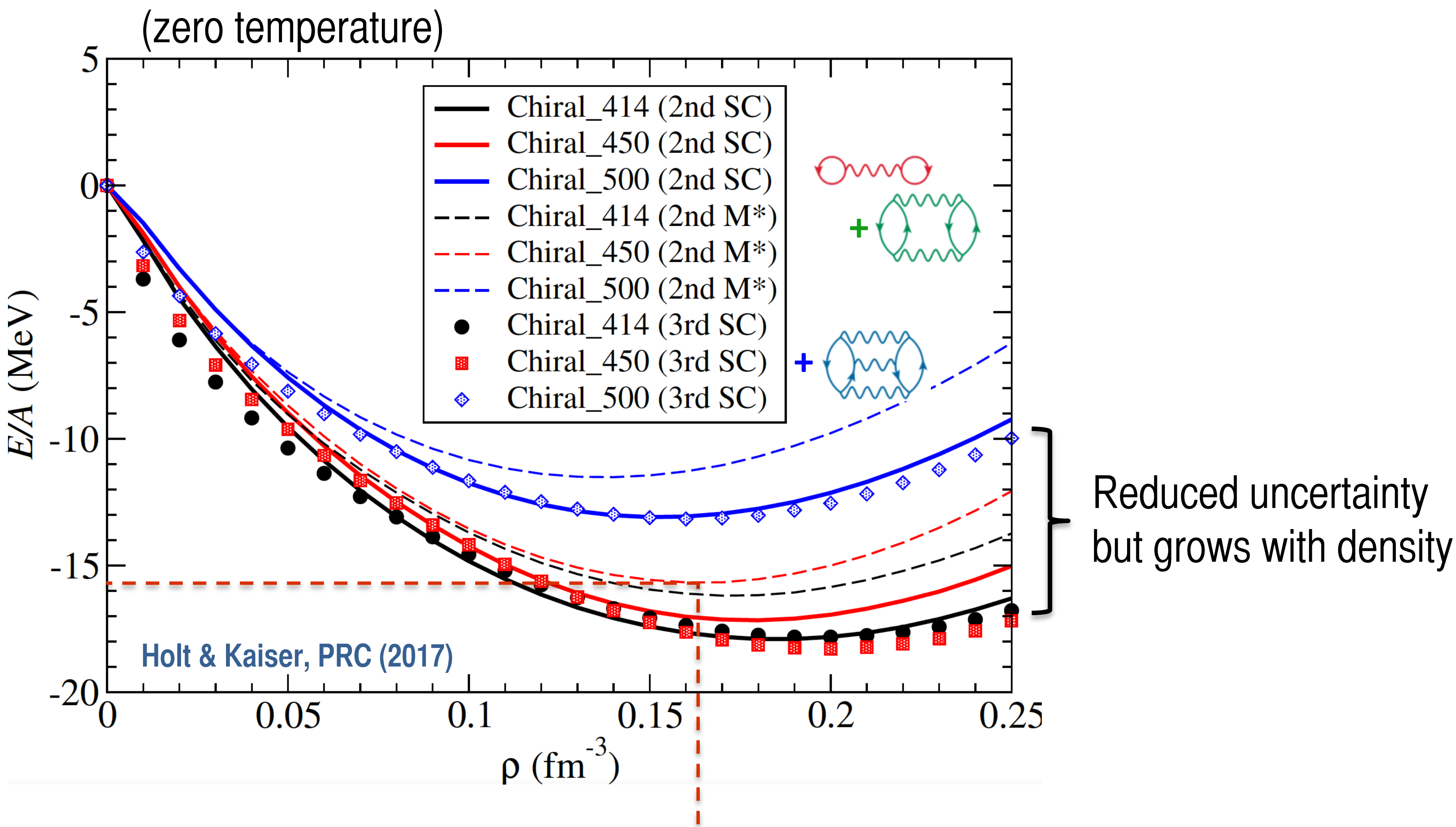
$$\rho E^{(3)}_{\rm hh} = \frac{1}{8} \sum_{123456} \langle 12 | \overline{V}_{\rm eff} | 34 \rangle \langle 34 | \overline{V}_{\rm eff} | 56 \rangle \langle 56 | \overline{V}_{\rm eff} | 12 \rangle \times \frac{\bar{n}_1 \bar{n}_2 n_3 n_4 n_5 n_6}{(e_1 + e_2 - e_3 - e_4)(e_1 + e_2 - e_5 - e_6)},$$



$$\rho E^{(3)}_{\rm ph} = -\sum_{123456} \langle 12 | \overline{V}_{\rm eff} | 34 \rangle \langle 54 | \overline{V}_{\rm eff} | 16 \rangle \langle 36 | \overline{V}_{\rm eff} | 52 \rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 n_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_3 + e_6 - e_2 - e_5)},$$

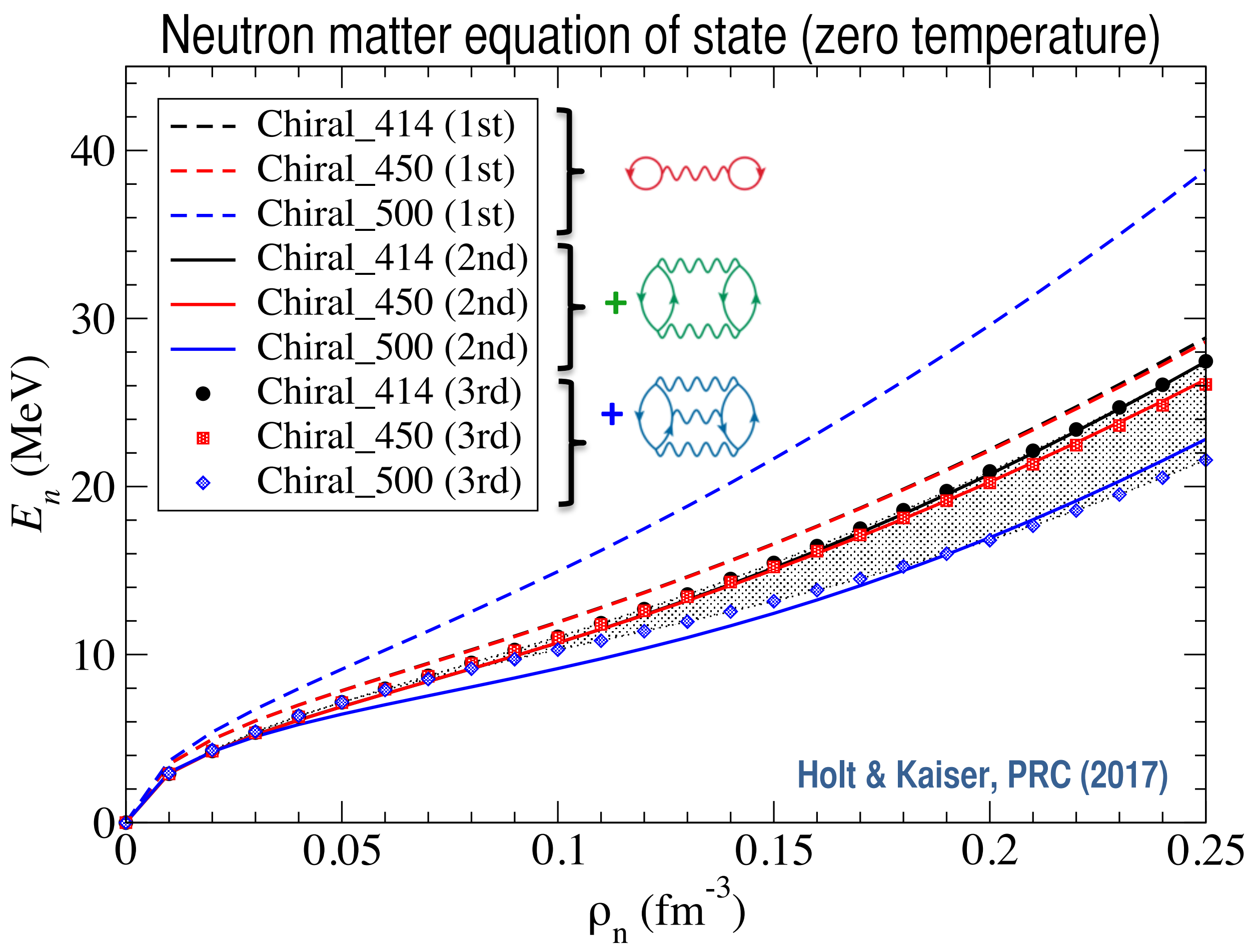
Plus: consistent response functions, single-particle potentials, …

Symmetric nuclear matter equation of state



Several approximations give good saturation properties

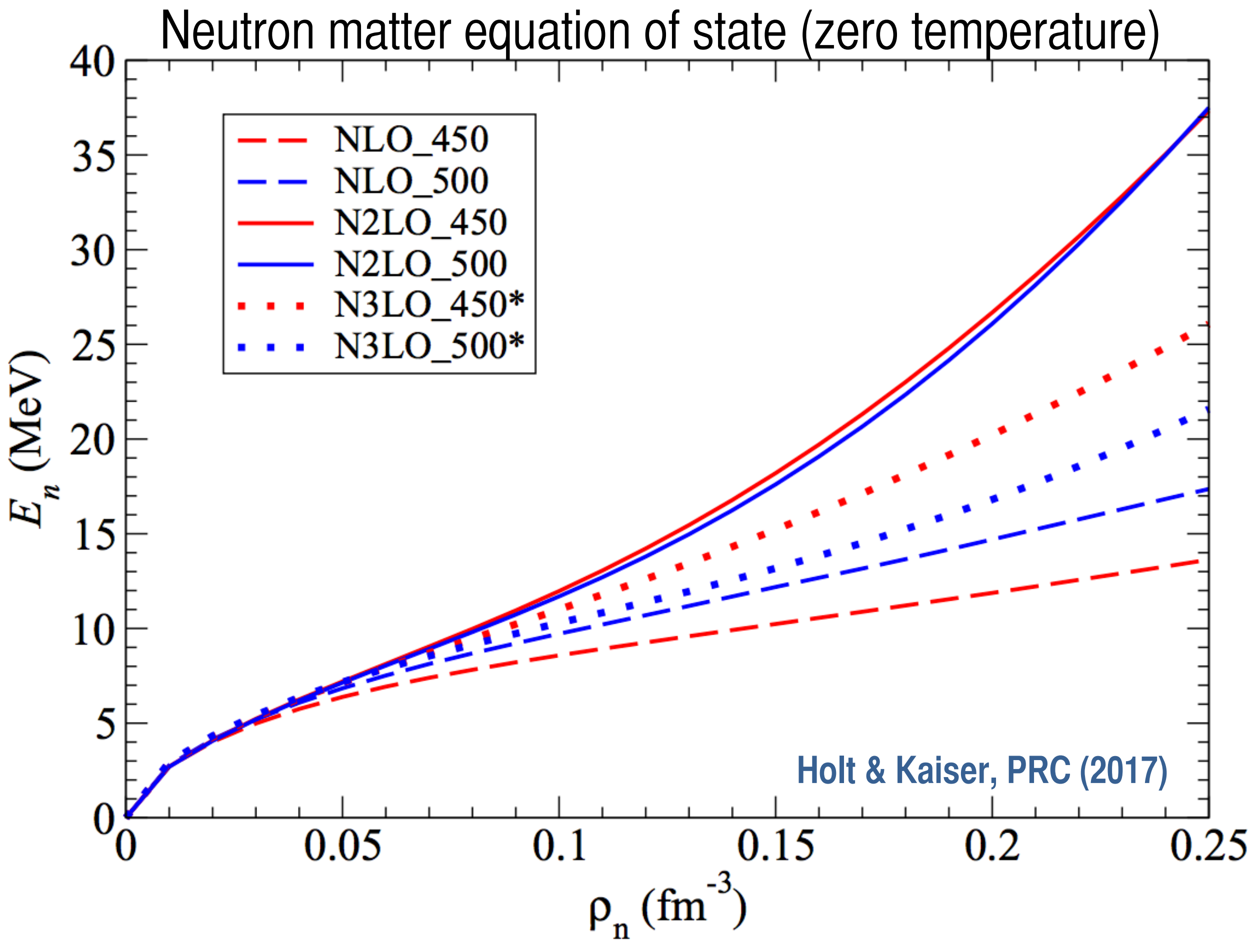
Pure neutron matter equation of state



Sources of uncertainty

- Scale dependence
- Convergence in many-body perturbation theory

Convergence in the chiral expansion



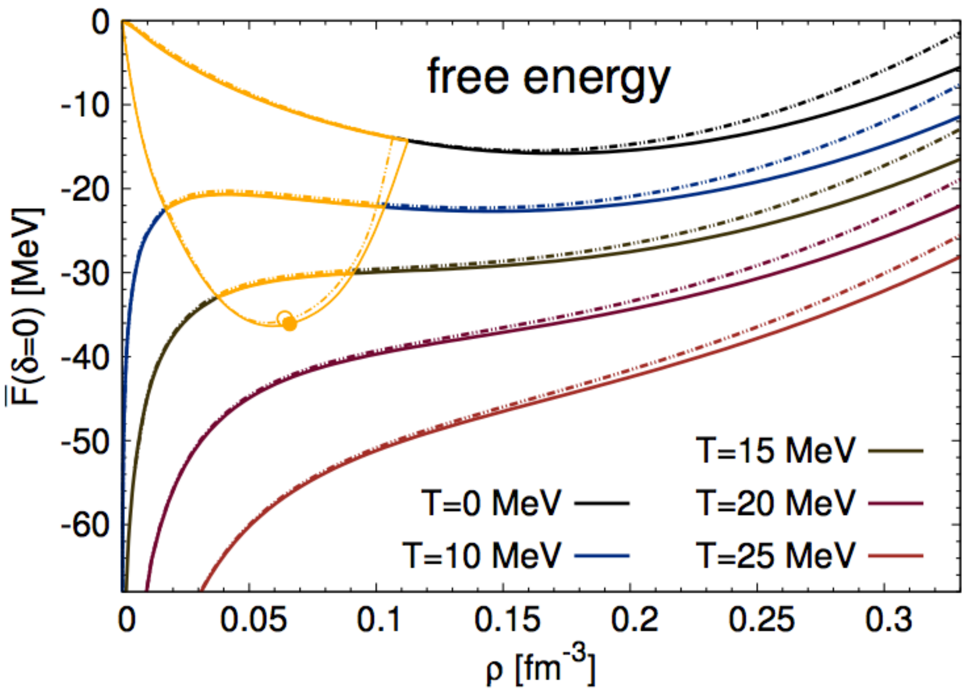
Sources of uncertainty

- Scale dependence
- Convergence in many-body perturbation theory
- Convergence in chiral expansion

Extension to finite temperature

- Perturbation series of free-energy density in terms of grand canonical potential Ω

$$F(\mu_0,T) = F_0(\mu_0,T) + \lambda\Omega_1(\mu_0,T) + \lambda^2\left(\Omega_2(\mu_0,T) - \frac{1}{2}\frac{(\partial\Omega_1/\partial\mu_0)^2}{\partial^2\Omega_0/\partial\mu_0^2}\right) + \mathcal{O}(\lambda^3)$$



Wellenhofer, Holt, Kaiser & Weise, PRC (2014)

- All thermodynamic quantities derived from free energy, e.g., $P(\rho,T) = \rho^2\frac{\partial\bar{F}(\rho,T)}{\partial\rho}$

Benchmark: nuclear liquid-gas phase transition

Predicted critical endpoint

- Critical temperature:

$$T_c = 17.2 - 19.1\,\mathrm{MeV}$$

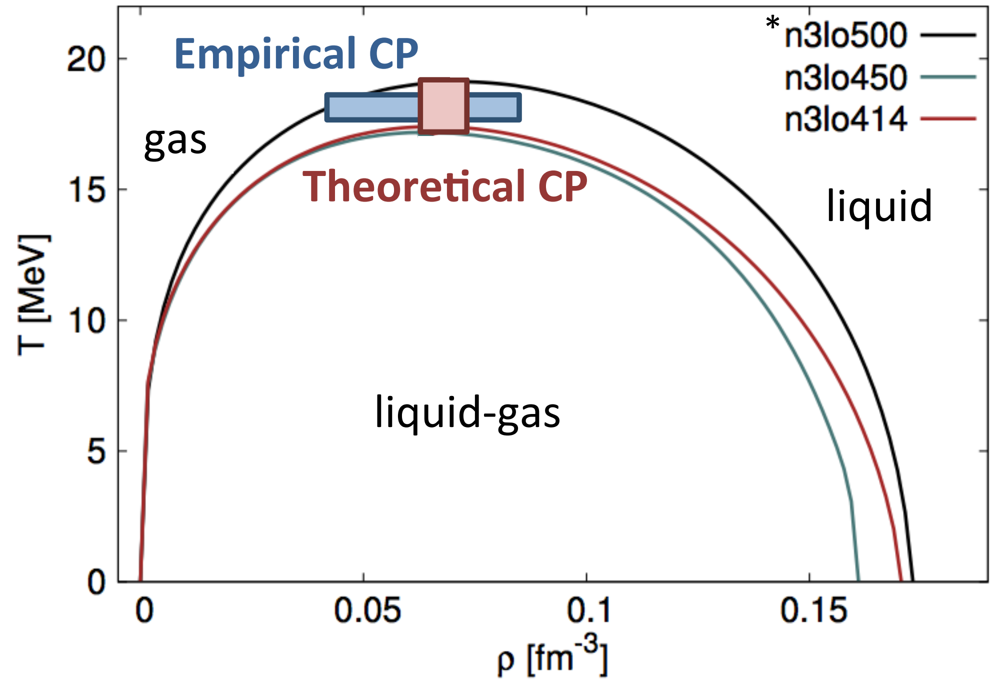
- Critical density:

$$\rho_c = 0.064 - 0.072\,\mathrm{fm}^{-3}$$

- Critical pressure:

$$P_c = 0.3 - 0.4\ \mathrm{MeV\ fm}^{-3}$$

Wellenhofer, Holt, Kaiser & Weise, PRC (2014)

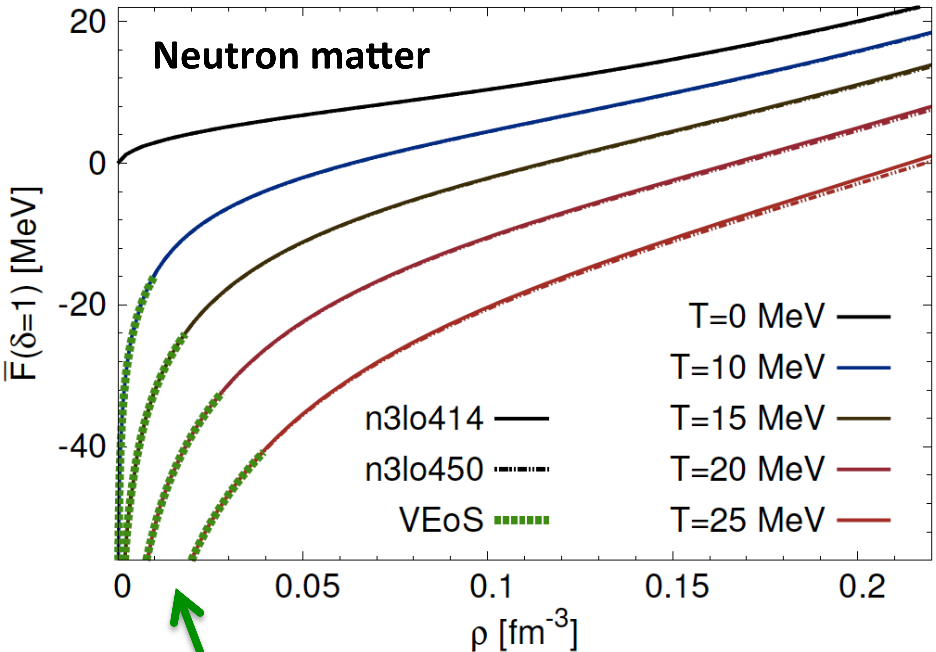


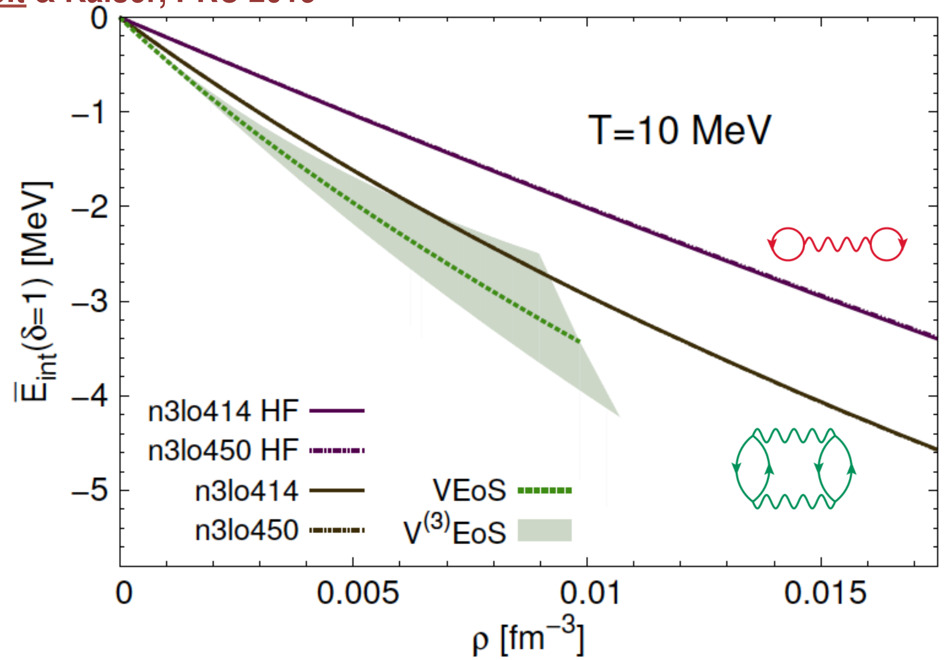
- Experiment (compound nucleus & multifragmentation) [J. B. Elliott et al., PRC (2013)]

$$T_c = 17.9 \pm 0.4\,\mathrm{MeV} \quad \rho_c = 0.06 \pm 0.02\ \mathrm{fm}^{-3} \quad P_c = 0.31 \pm 0.07\ \mathrm{MeV\ fm}^{-3}$$

Benchmark: virial EOS for hot and dilute neutron matter

Wellenhofer, Holt & Kaiser, PRC 2015





Matches model-independent virial EoS
at low densities and high temperatures

Asymmetric nuclear matter

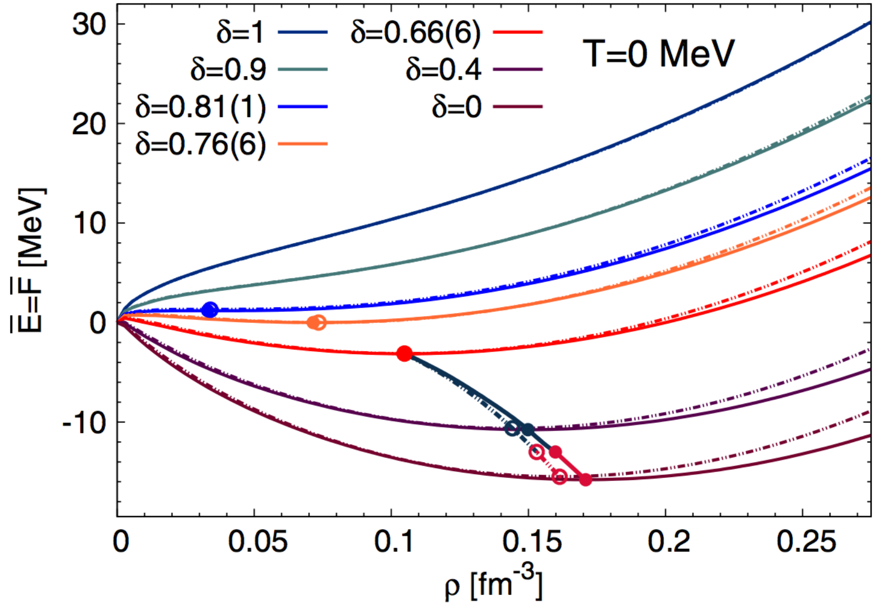
$$E(\rho,\delta) = A_0(\rho) + A_2(\rho)\delta^2 + \mathcal{O}(\delta^4)$$

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$A_0(\rho) = E_0 + \frac{1}{6}K\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots$$

$$A_2(\rho) = J + \frac{1}{3}L\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{1}{6}K_{sym}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots$$

Wellenhofer, Holt and Kaiser, PRC (2015)



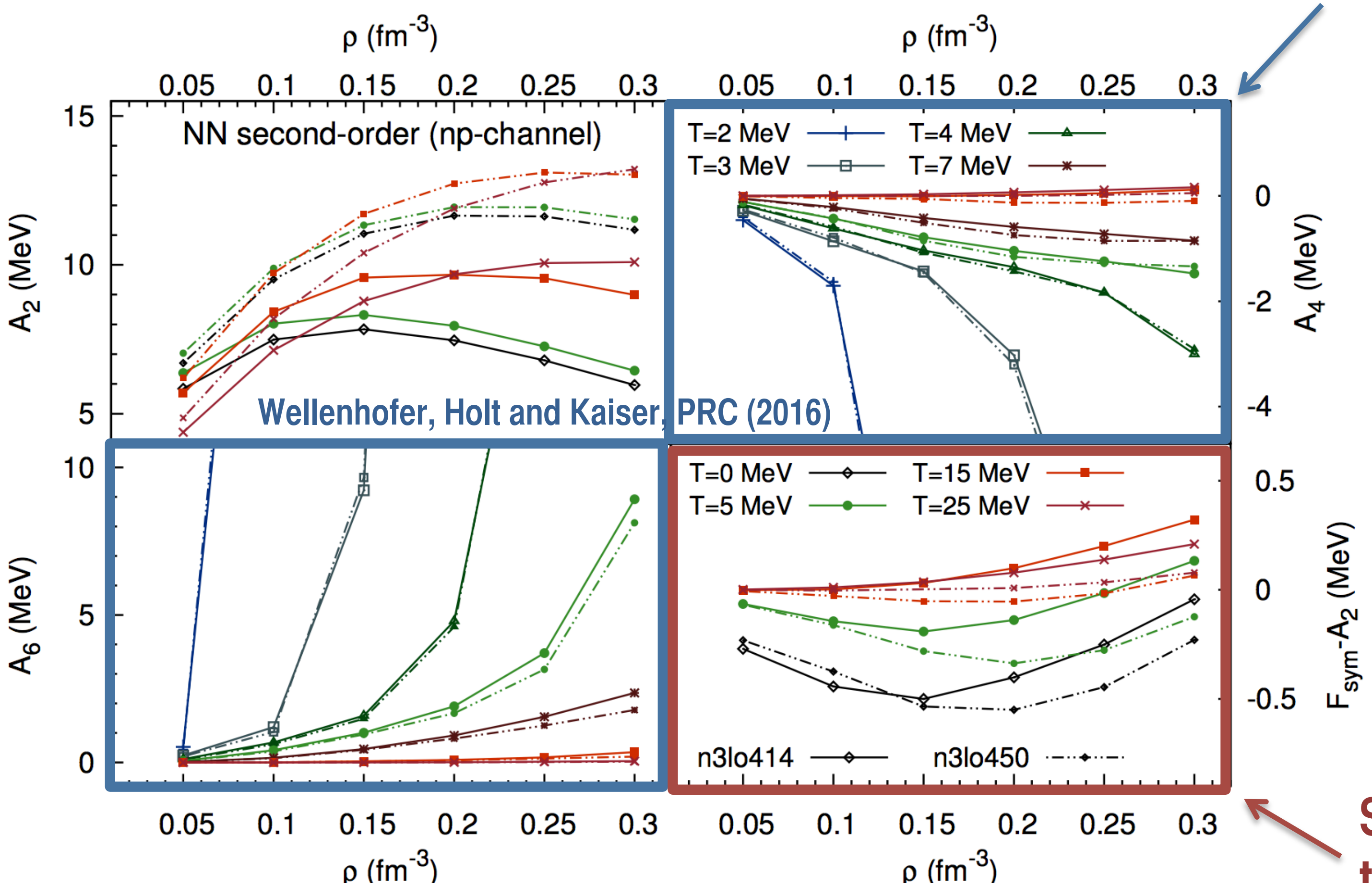
Role of higher-order δ^4 terms?

Crust-core transition density,...

Maclaurin series expansion in the isospin asymmetry

$$F(T,\rho,\delta) \simeq \sum_{n=0}^{N} A_{2n}(T,\rho)\,\delta^{2n}$$

Divergent expansion at low temperature



Sum of higher-order terms is finite

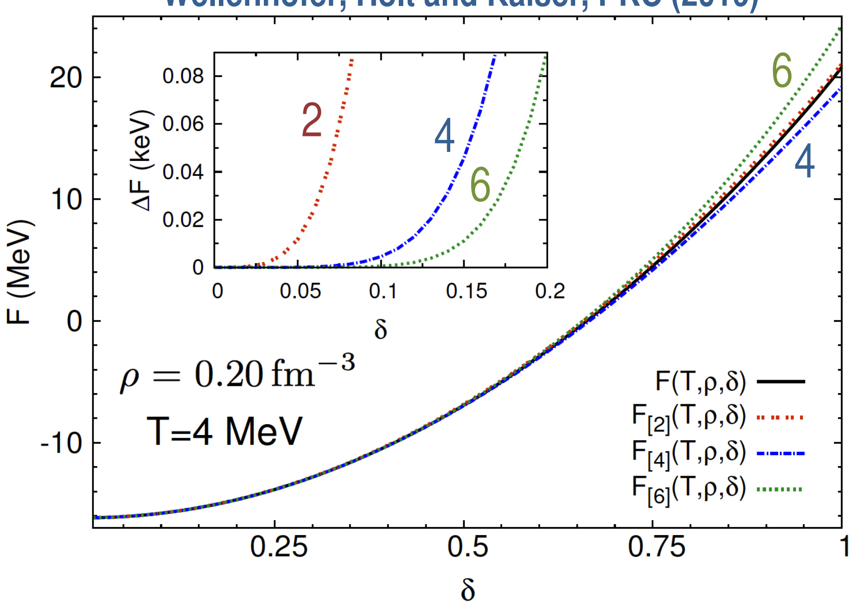
Nonanalytic terms arise beyond the mean field level

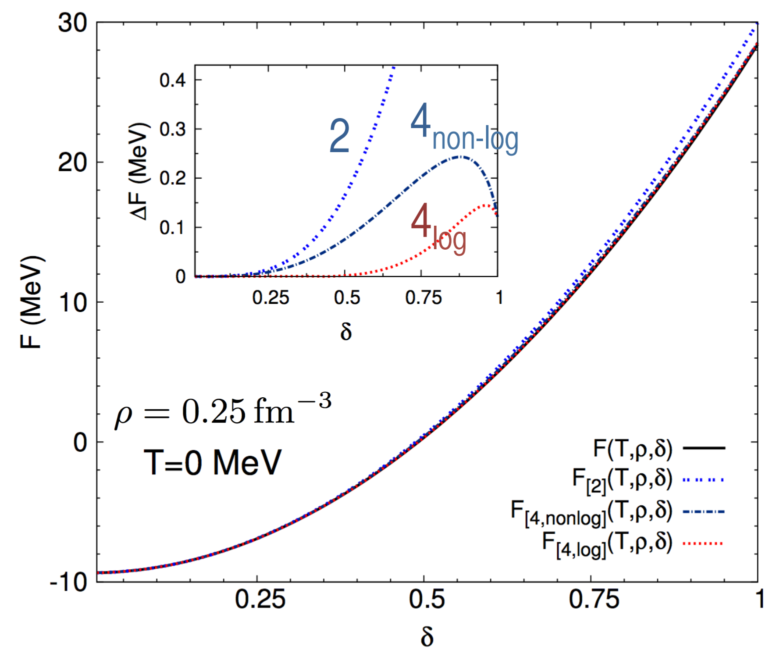
$$F(T=0,\rho,\delta) = A_0(T=0,\rho) + A_2(T=0,\rho)\,\delta^2 + \sum_{n=2}^{\infty} A_{2n,\mathrm{reg}}(\rho)\,\delta^{2n} + \sum_{n=2}^{\infty} A_{2n,\log}(\rho)\,\delta^{2n}\ln|\delta|$$

Kaiser, PRC 2015

Logarithmic but finite

Wellenhofer, Holt and Kaiser, PRC (2016)





Importance of nucleon single-particle potentials

- **R-process nucleosynthesis**
  - Neutron-capture rates in cold r-process environments
  - *Global optical potentials* from infinite matter calculations (update JLM)
  - Charged-current reactions in the supernova neutrinosphere

- **Transport model simulations of heavy-ion collisions**
  - Needed to extract equation of state at high density
  - FRIB experimental program

Global optical potentials

$$\mathcal{U}(r,E) = -\mathcal{V}_V(r,E) - i\mathcal{W}_V(r,E) - i\mathcal{W}_D(r,E) + \mathcal{V}_{SO}(r,E).\mathbf{l}.\sigma + i\mathcal{W}_{SO}(r,E).\mathbf{l}.\sigma + \mathcal{V}_C(r).$$

$$\mathcal{V}_V(r,E) = V_V(E) f(r, R_V, a_V),$$

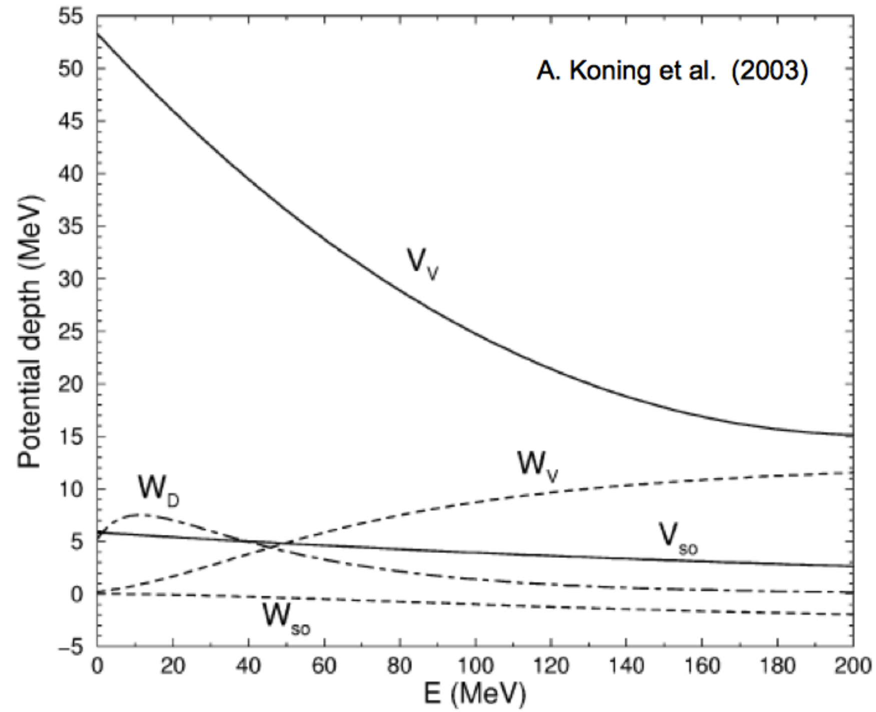
$$\mathcal{W}_V(r,E) = W_V(E) f(r, R_V, a_V),$$

$$\mathcal{W}_D(r,E) = -4a_D W_D(E) \frac{d}{dr} f(r, R_D, a_D),$$

$$\mathcal{V}_{SO}(r,E) = V_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r, R_{SO}, a_{SO}),$$

$$\mathcal{W}_{SO}(r,E) = W_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r, R_{SO}, a_{SO}).$$

$$f(r, R_i, a_i) = \left(1 + \exp[(r - R_i)/a_i]\right)^{-1}$$



$$V_V(E) = v_1\left[1 - v_2(E - E_f) + v_3(E - E_f)^2 - v_4(E - E_f)^3\right]$$

$$W_V(E) = w_1 \frac{(E - E_f)^2}{(E - E_f)^2 + (w_2)^2},$$

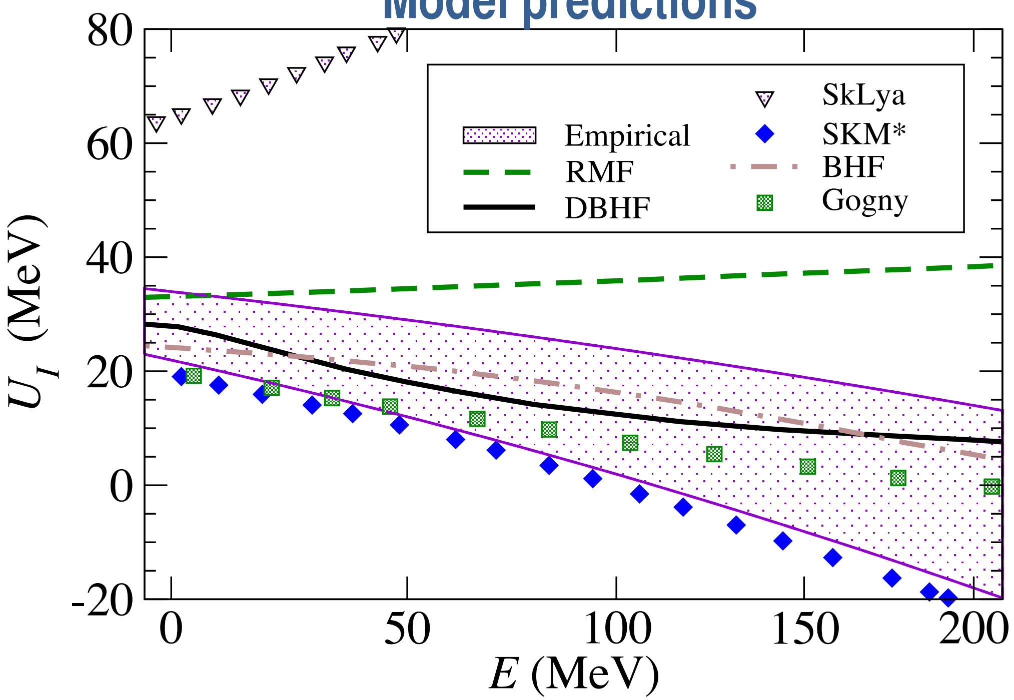
Energy dependence

Isospin asymmetry dependence

- Isovector part of optical potential linear in the isospin asymmetry

$$U = U_0 - U_I \delta_{np} \tau_3 \qquad \delta_{np} = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Model predictions

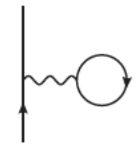


- Very little is known/predicted about **isovector imaginary part**

Optical potential in homogeneous matter

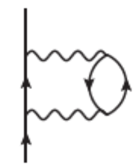
- Identified with the on-shell nucleon self-energy $\Sigma(\vec{r}_1, \vec{r}_2, \omega)$

- Hartree-Fock contribution (real, energy-independent):



$$\Sigma^{(1)}_{2N}(q; k_f) = \sum_1 \langle \vec{q}\vec{h}_1 s s_1 t t_1 | \bar{V}_{2N} | \vec{q}\vec{h}_1 s s_1 t t_1 \rangle n_1$$

- Second-order perturbative contibutions (complex, energy-dependent):

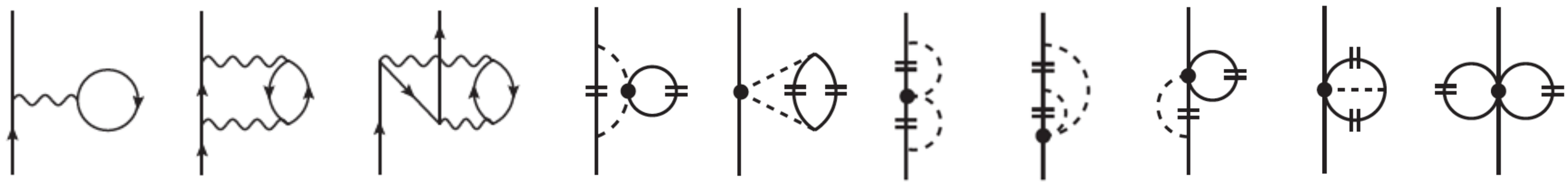


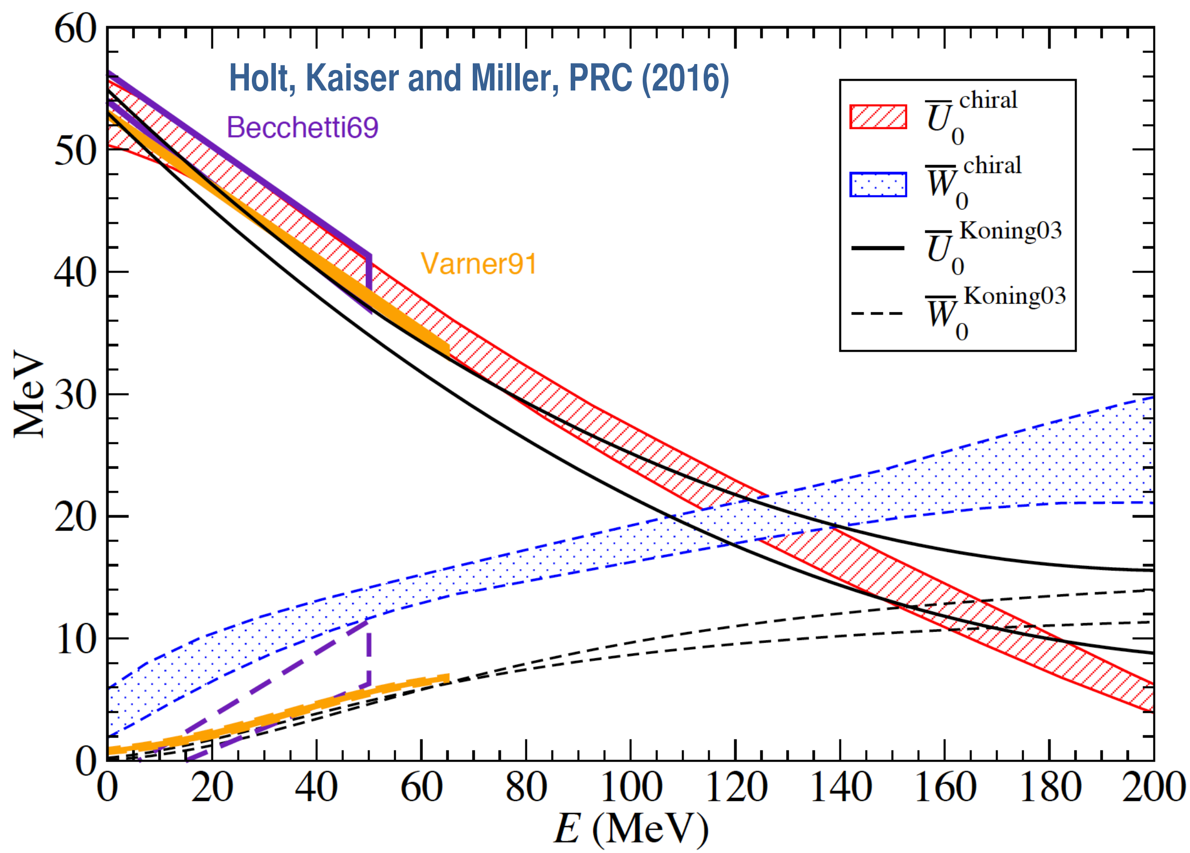
$$\Sigma^{(2a)}_{2N}(q, \omega; k_f) = \frac{1}{2} \sum_{123} \frac{|\langle \vec{p}_1 \vec{p}_3 s_1 s_3 t_1 t_3 | \bar{V} | \vec{q}\vec{h}_2 s s_2 t t_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 + i\eta} \bar{n}_1 n_2 \bar{n}_3 (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_3 - \vec{q} - \vec{h}_2)$$

Benchmarks:

- Depth and energy dependence of phenomenological volume parts (including isospin dependence)

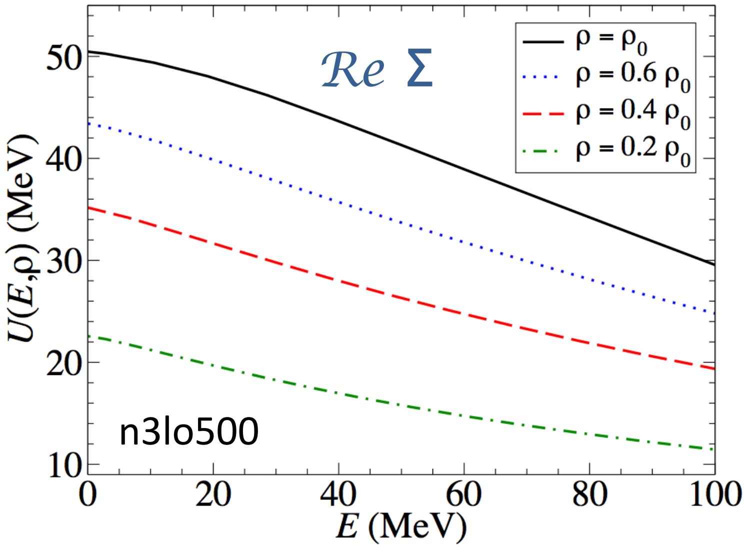
Optical potential in symmetric nuclear matter

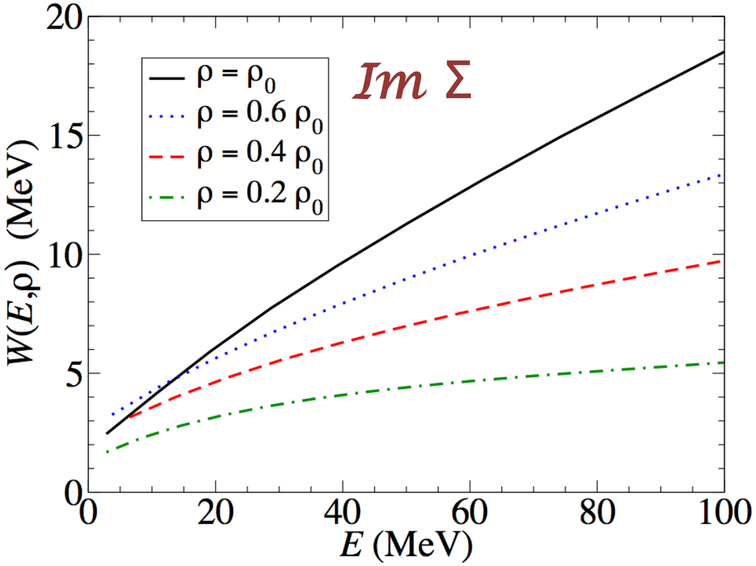




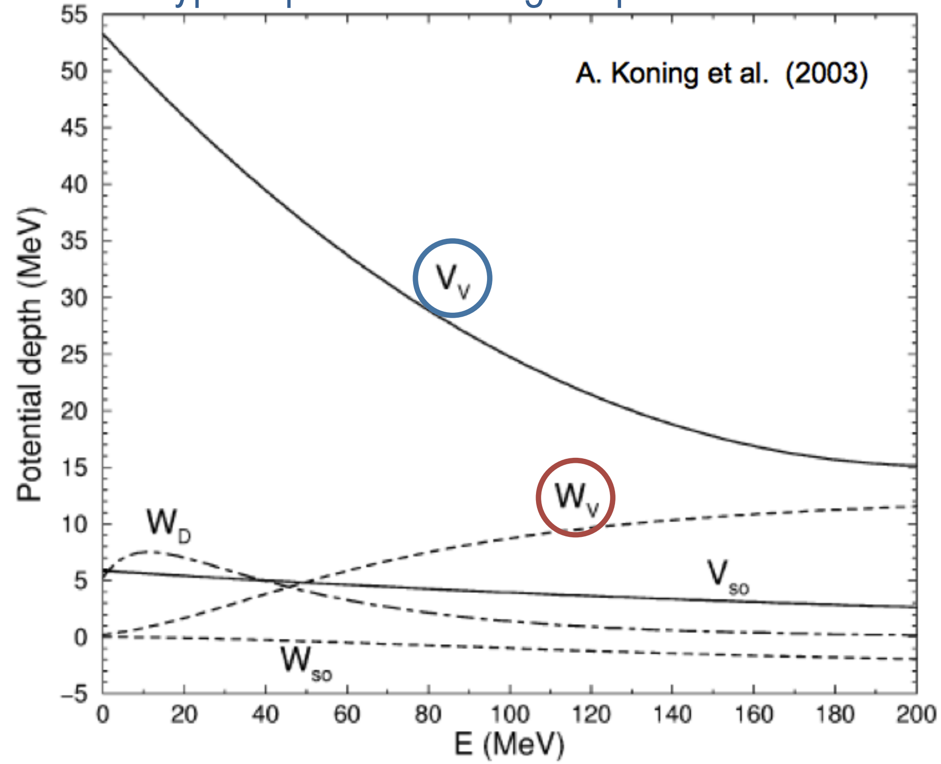
Density dependence of real and imaginary optical potentials

Holt, Kaiser, Miller & Weise, PRC (2013)

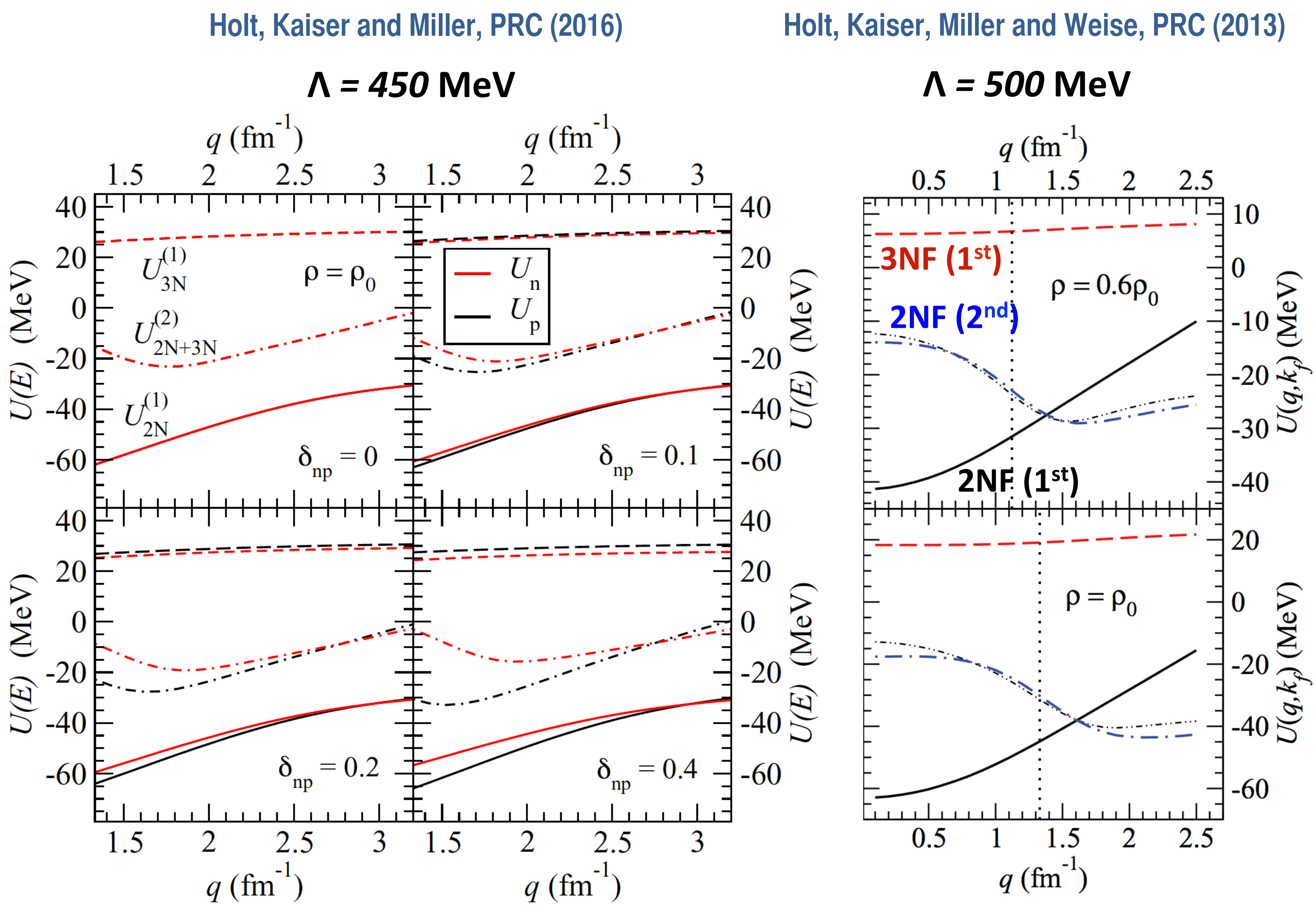




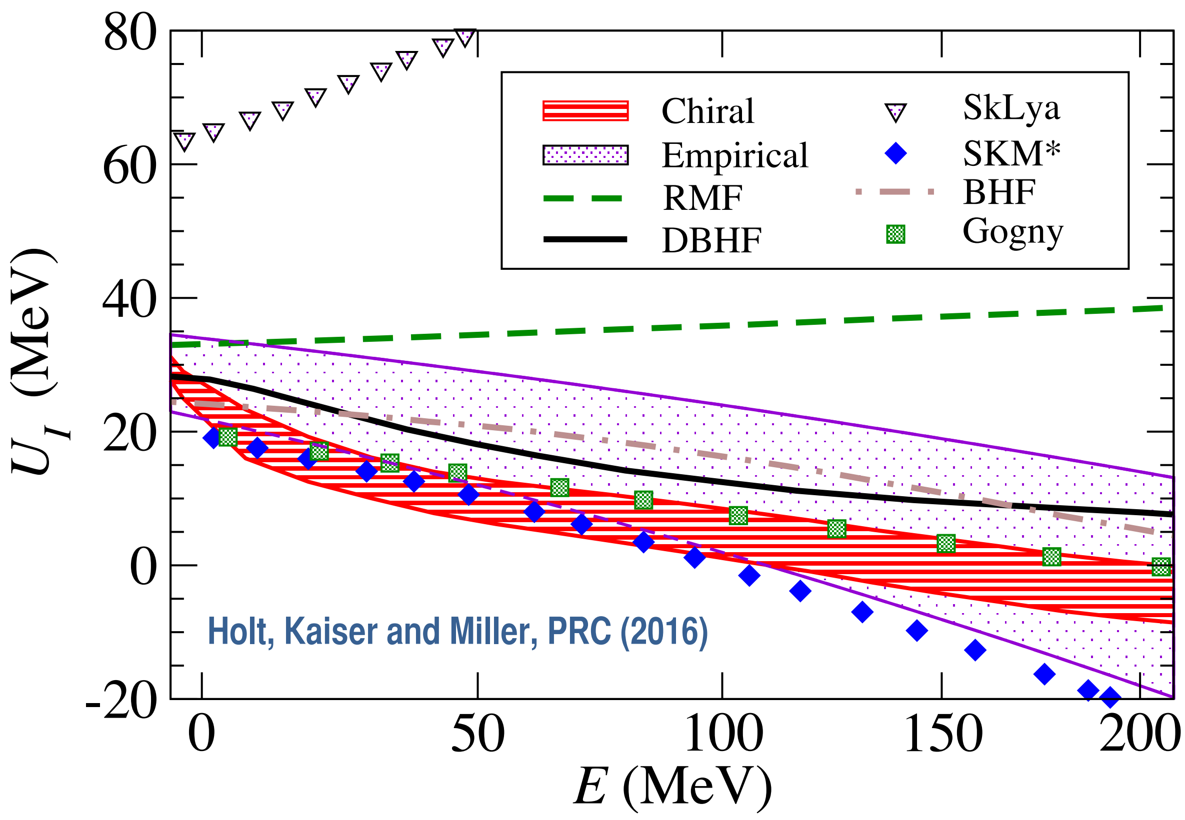
Typical phenomenological potential: ^{56}Fe



Convergence in perturbation theory



Isovector real optical potential

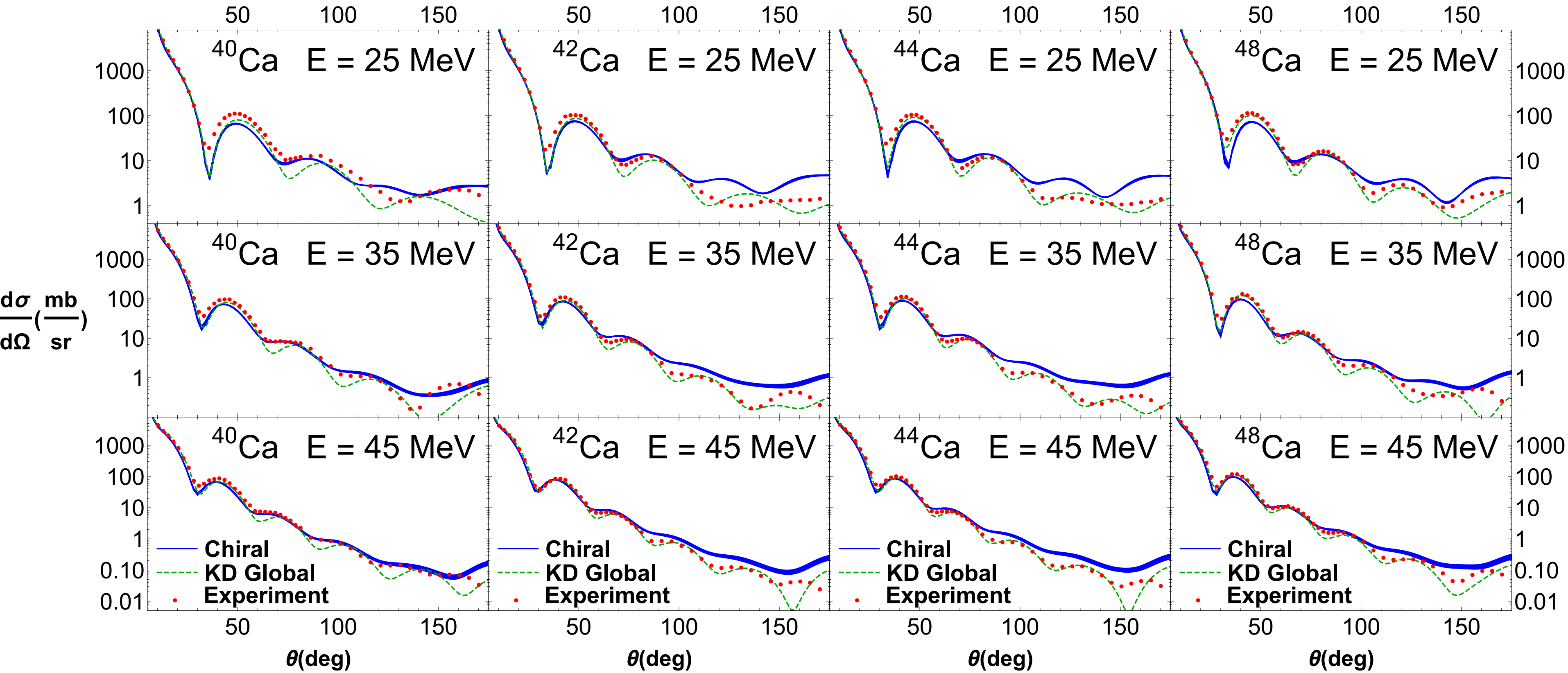


- Chiral EFT prediction consistent with broad empirical constraints

Calculated differential elastic scattering cross sections

$$p + A \rightarrow p + A$$

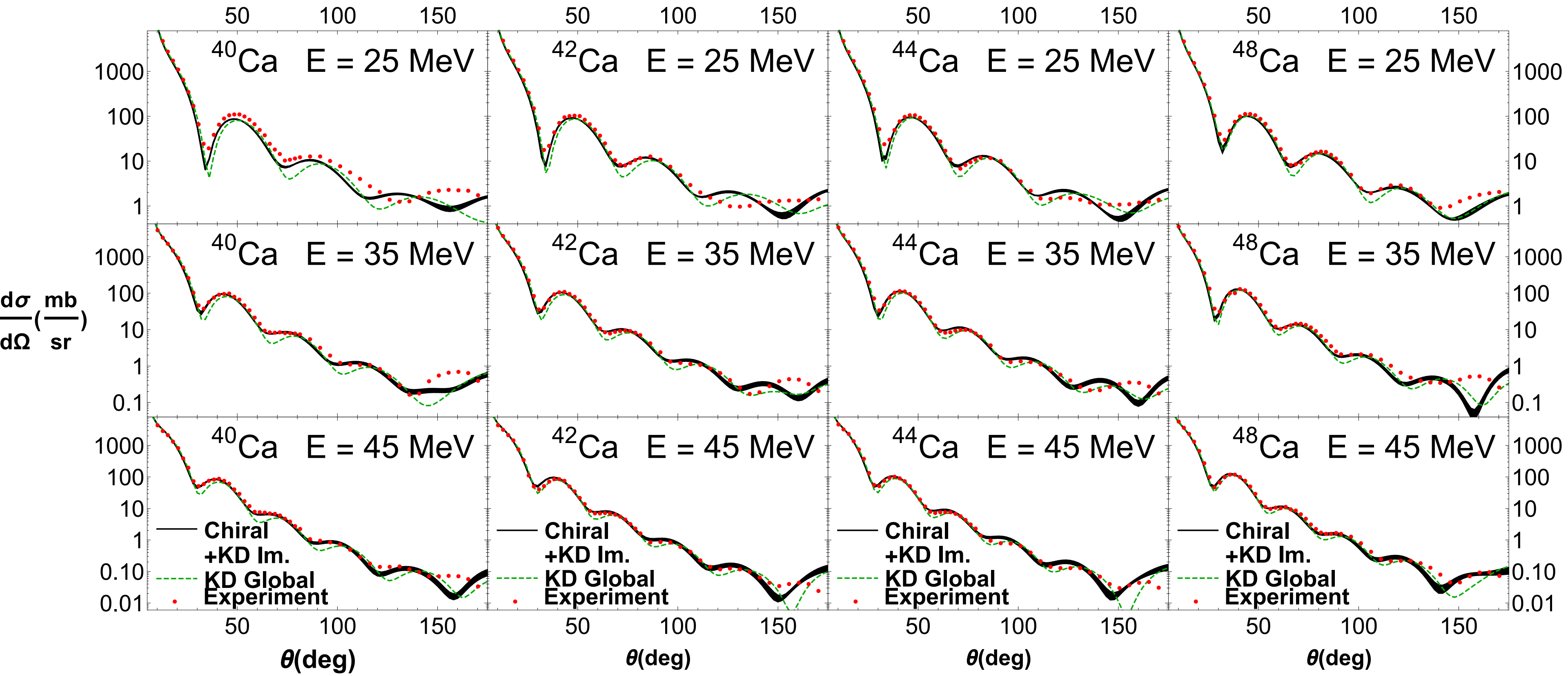
Whitehead, Lim and Holt, PRC (2019)



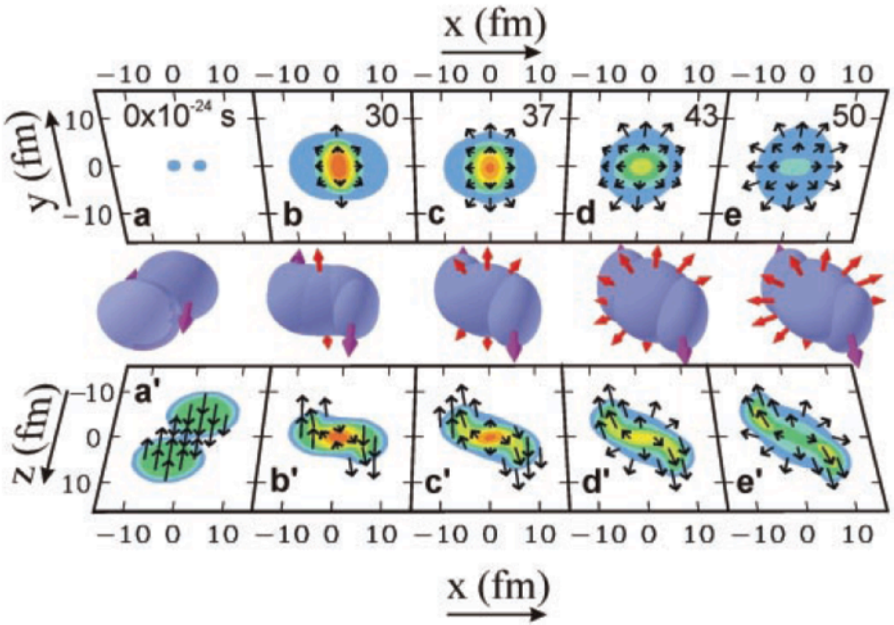
Too strong microscopic absorptive imaginary part

$$p + A \rightarrow p + A$$

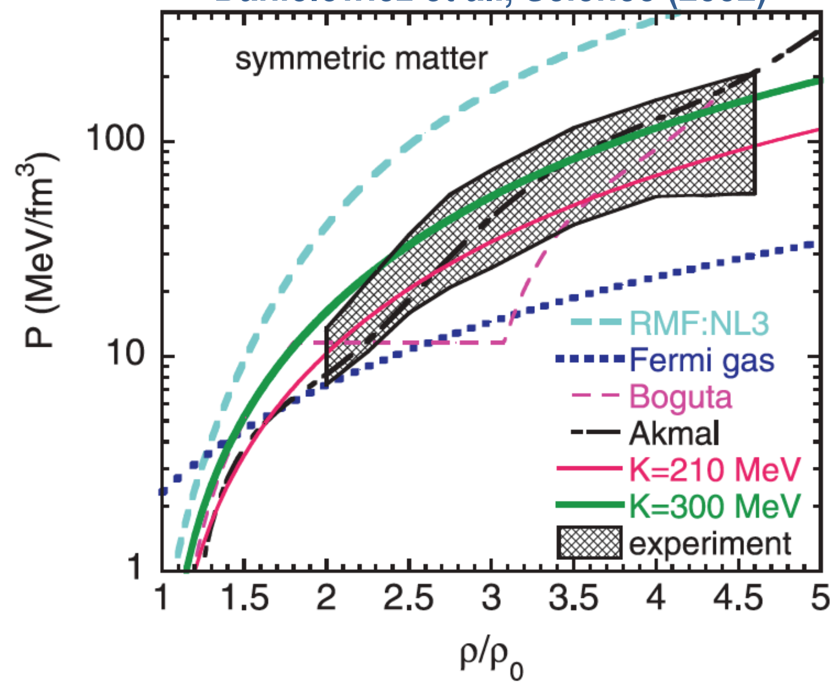
Whitehead, Lim and Holt, PRC (2019)



Probing the nuclear equation of state in the lab



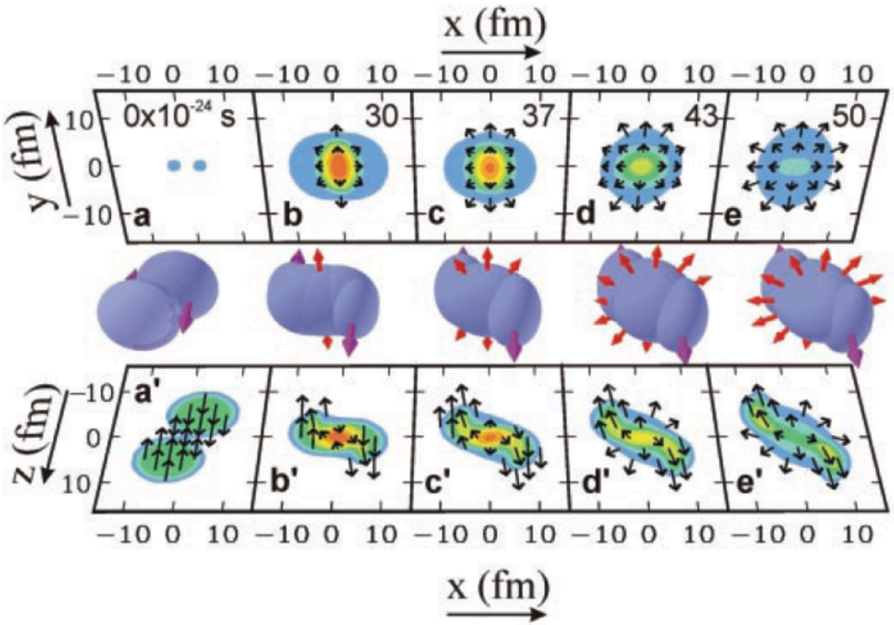
Danielewicz et al., Science (2002)



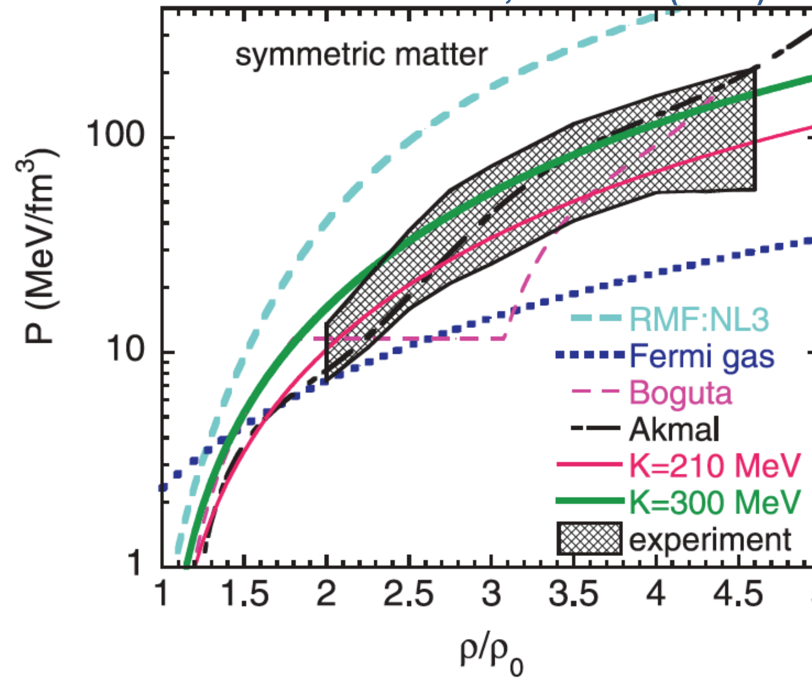
- Observables: elliptic flow, transverse flow, fragment yields
- Analyze with Boltzmann-like transport equation:

$$\frac{\partial f}{\partial t} + \nabla_p \varepsilon \cdot \nabla_r f - \nabla_r \varepsilon \cdot \nabla_p f = I$$

Probing the nuclear equation of state in the lab



Danielewicz et al., Science (2002)



- Observables: elliptic flow, transverse flow, fragment yields
- Analyze with Boltzmann-like transport equation:

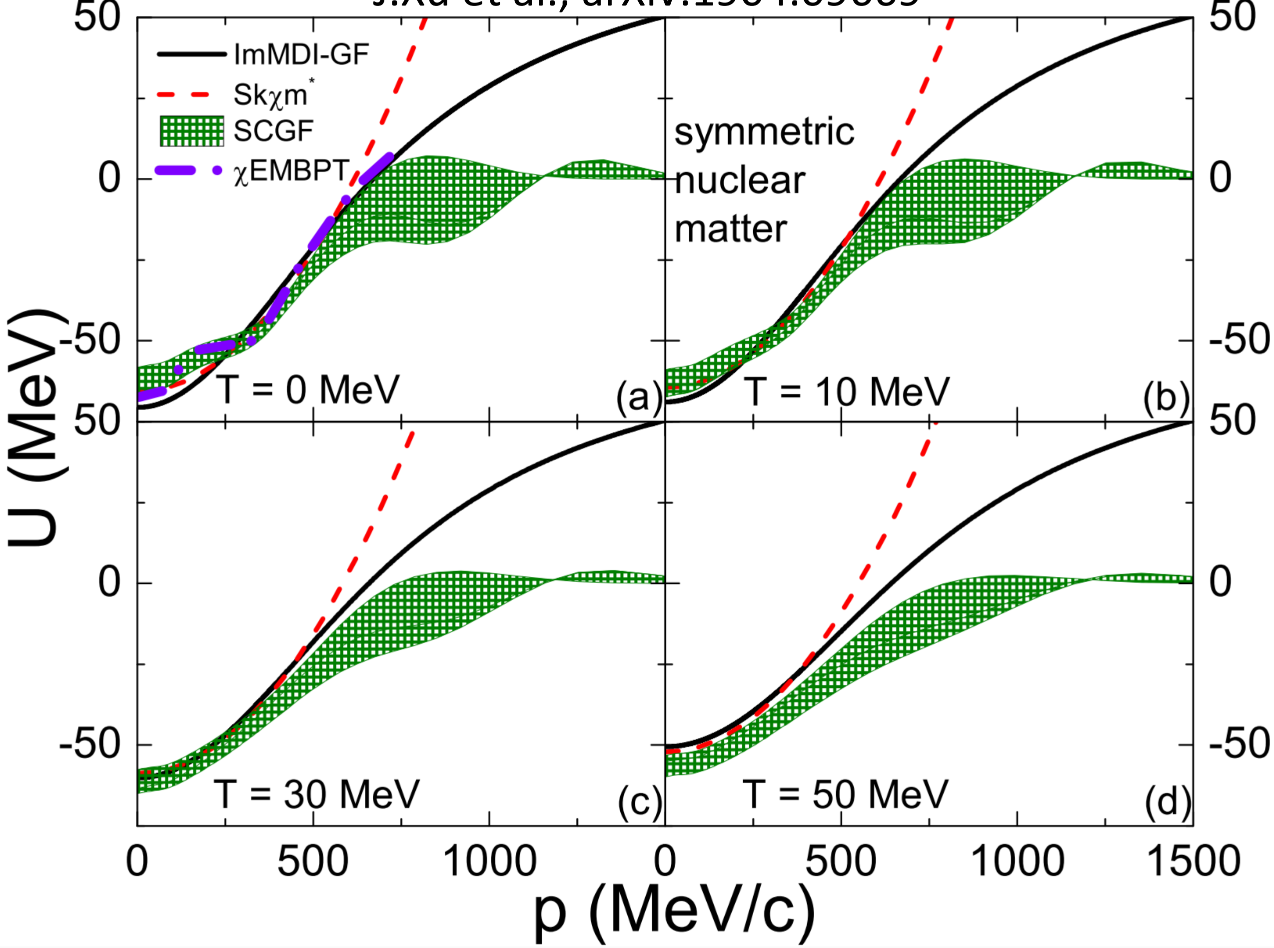
$$\varepsilon = p^2/2M + U(r,p,t)$$

$$\frac{\partial f}{\partial t} + \nabla_p \varepsilon \cdot \nabla_r f - \nabla_r \varepsilon \cdot \nabla_p f = I$$

Probing the nuclear equation of state in the lab

- Quadratic momentum dependence of nucleon single-particle potential

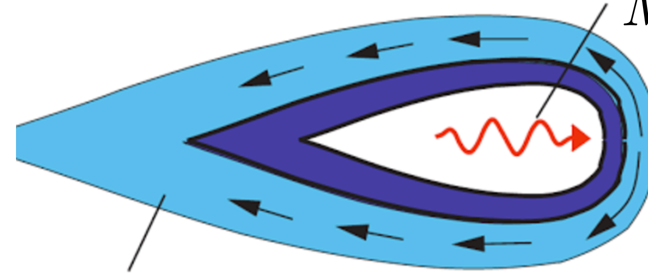
J.Xu et al., arXiv:1904.09669



Effective mass in medium

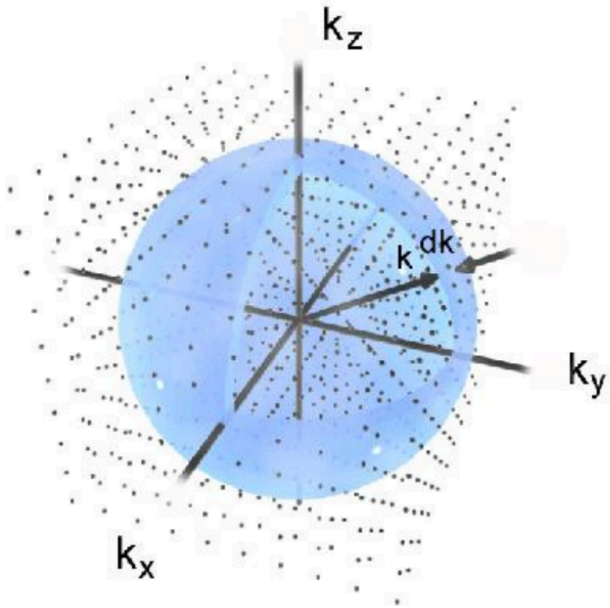
$$v = \frac{d}{dk}E(k) = \frac{k}{M^*}$$

$$= \frac{k}{M} - \underbrace{\frac{k}{M}\left(\frac{F_1}{3+F_1}\right)}_{\text{Interaction \& medium effects}}$$



$\frac{M^*}{M} > 1$	**Medium "drag"**
$0 < \frac{M^*}{M} < 1$	**Medium "backflow"**
$\frac{M^*}{M} < 0$	**Total momentum opposite to velocity**

Astrophysical applications: supernovae



$$g(k)dk = 2(4\pi k^2 dk)\left(\frac{\Omega}{8\pi^3}\right)$$

$$\frac{d}{dk}E(k) = \frac{k}{M^*}$$

$$\boxed{N_0 = \frac{kM^*}{\pi^2}}$$

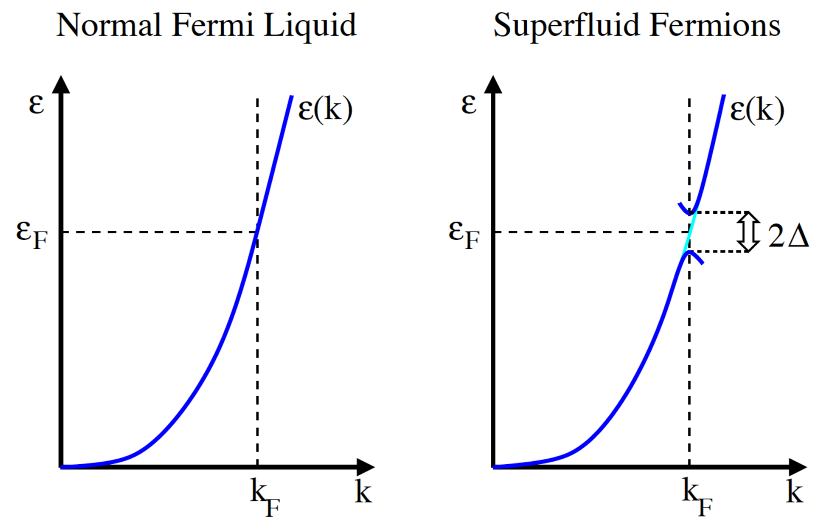
Density of states

- Thermal properties and entropy generation in core-collapse supernovae
- Supernova composition and its evolution (electron capture processes)
- R-process nucleosynthesis and neutron-capture reactions

Astrophysical applications: neutron stars

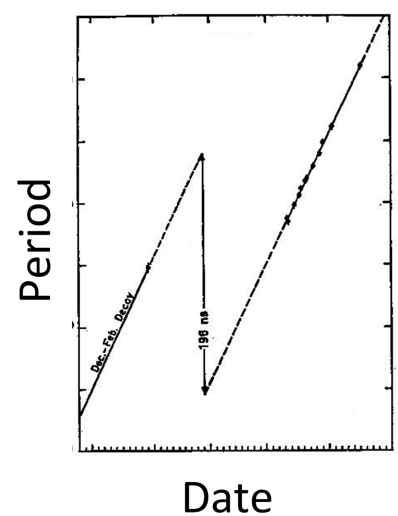
- **Neutron superfluidity** and **proton superconductivity** in neutron stars

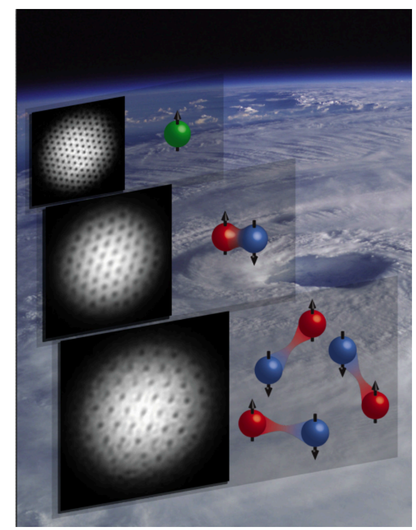
$$\Delta_k = -\frac{1}{2}\sum_{k'} \frac{V_{\text{eff}}(k,k')\Delta_{k'}}{\left(\frac{k^2}{2M^*} - \frac{k_F^2}{2M^*}\right)^2 + \Delta_{k'}^2}$$



- **Superfluid entrainment:** sensitive to "isovector effective mass"

- ***Enough angular momentum in the superfluid vortices to produce glitches??***



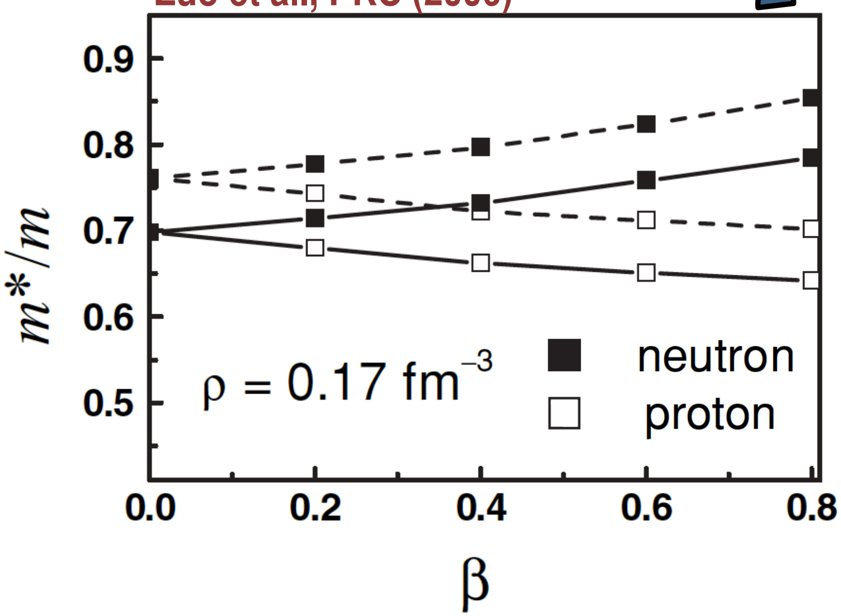


Isospin-asymmetric systems

- **Isoscalar effective mass:** $$\frac{M}{M_s^*} = \frac{1}{2}\left(\frac{M}{M_p^*} + \frac{M}{M_n^*}\right)$$
- **Isovector effective mass:** $$\frac{M}{M_v^*} = \frac{\frac{M}{M_p^*}\rho_n - \frac{M}{M_n^*}\rho_p}{\rho_n - \rho_p}$$

$$\beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

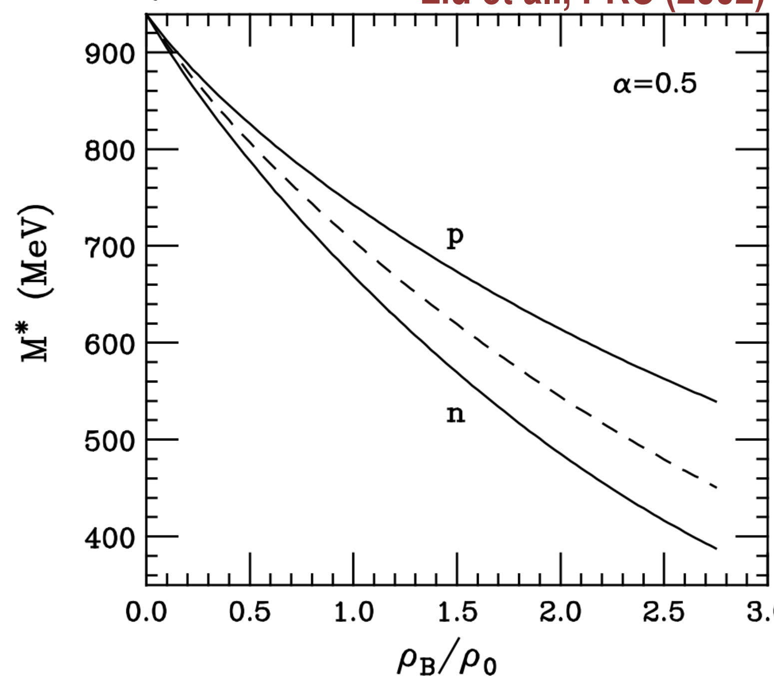
Zuo et al., PRC (2006)



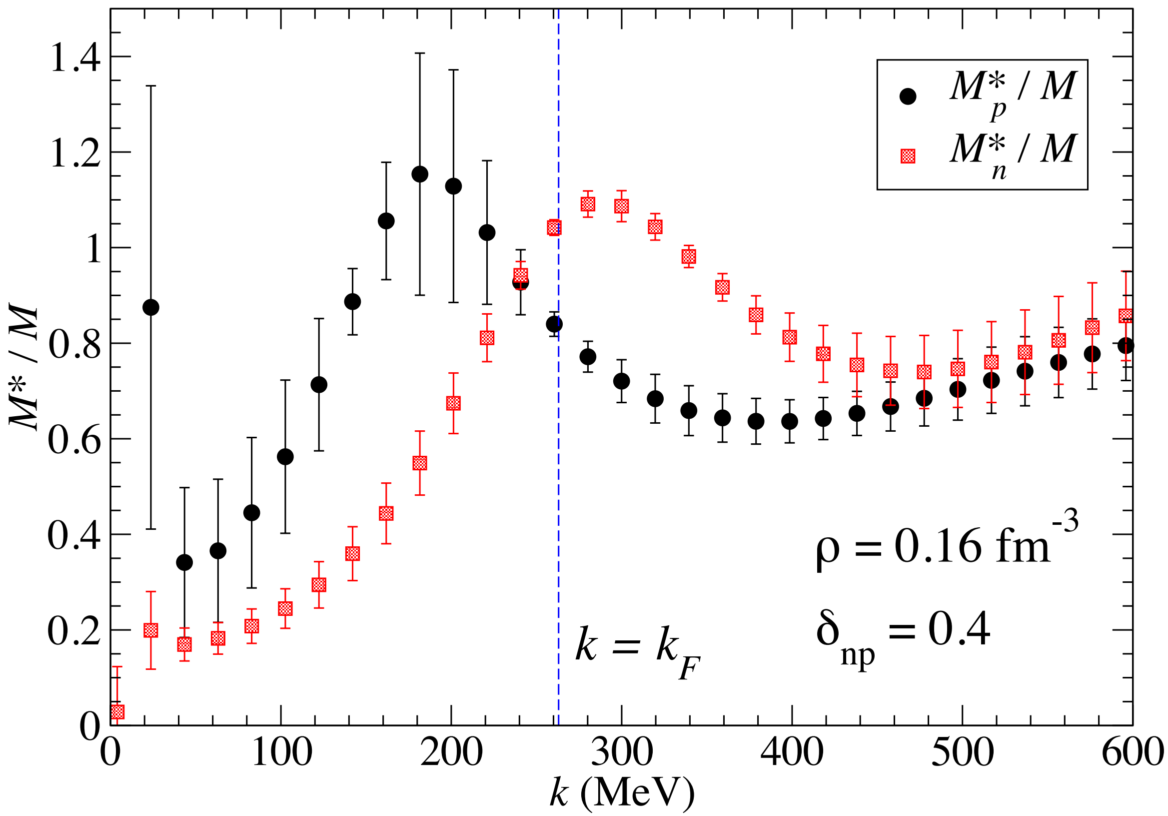
Disagreement

$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Liu et al., PRC (2002)



Momentum-dependent proton and neutron effective masses



- Uncertainty estimates from equal weighting of results from {n2lo450, n2lo500, n3lo410, n3lo450, n3lo500} chiral potentials

Beyond mean field contributions to response functions

- Cross sections for neutrino scattering, production, and absorption needed in supernova simulation codes
- Neutron star cooling, spectral & temporal features of observable neutrino signal
- Need for improved estimation of strong-interaction physics uncertainties from realistic two-body and three-body forces

- Neutrino-nucleon scattering and absorption in the neutrinosphere
- Updates to Fermi liquid theory from chiral nuclear potentials
- Mean field corrections to charged-current reactions
- Vertex corrections to density and spin response functions

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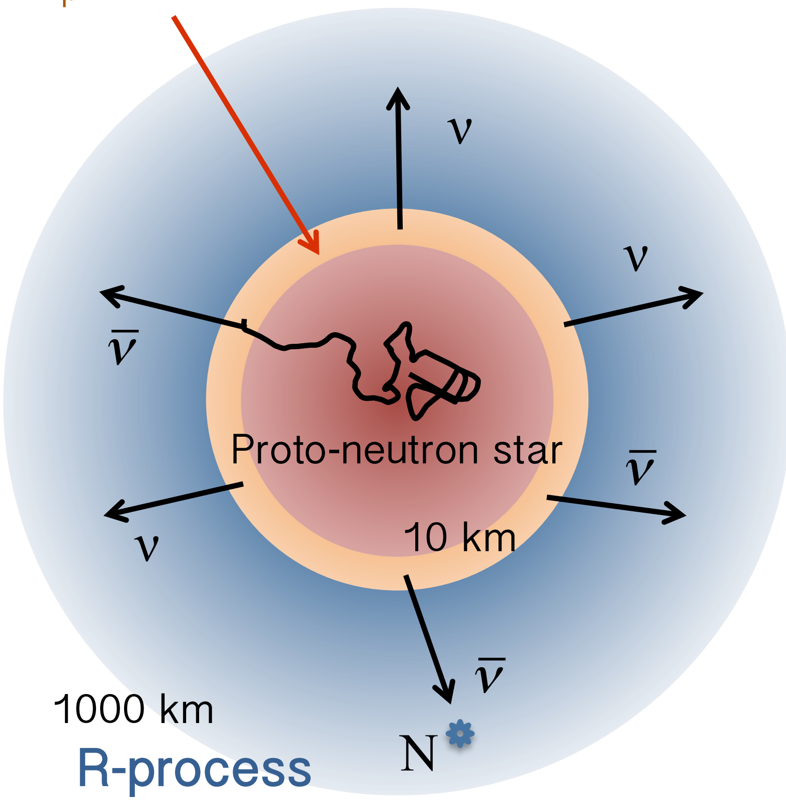
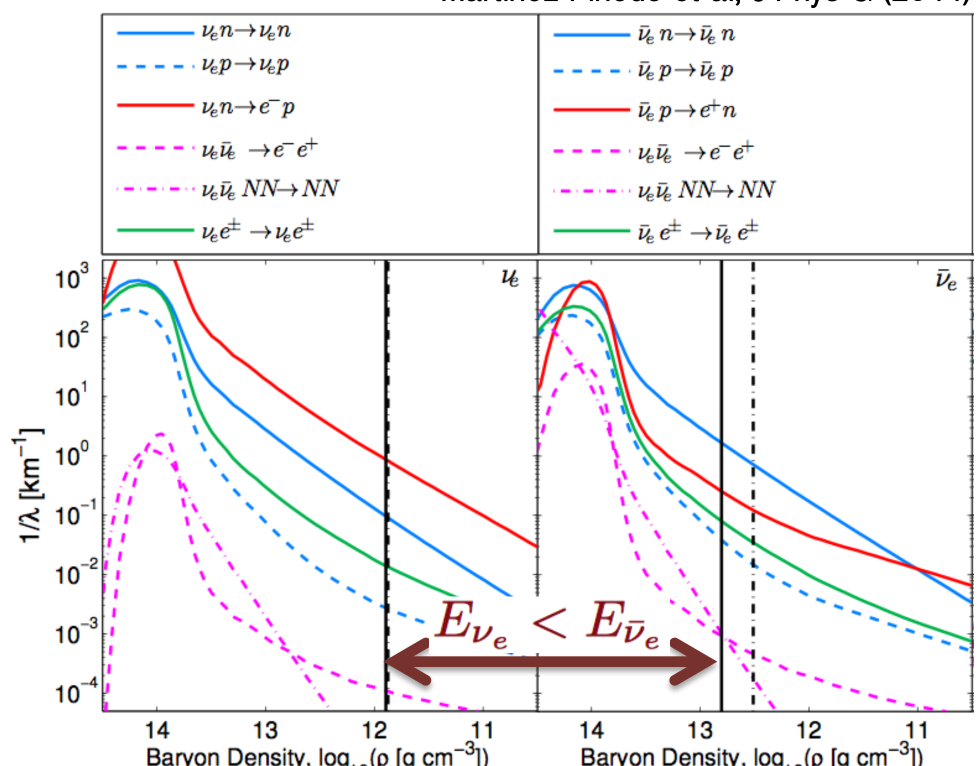
Late-time supernova neutrinos

Neutrinosphere
$\rho = 10^{11} - 10^{13}$ g/cm^3,
T = 4 – 8 MeV,
$Y_p \sim 0.05 - 0.10$



Neutrino opacity	Anti-neutrino opacity
$\nu_e + n \longleftrightarrow e^- + p$	$\bar{\nu}_e + n \longrightarrow \bar{\nu}_e + n$

Martinez-Pinedo et al, J Phys G (2014)



Governs energy distribution of free-streaming neutrinos

Differential scattering cross sections

- Neutrino-nucleon scattering (weak neutral-current reaction)

$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta\, d\omega} = \frac{G_F^2}{4\pi^2}E_3^2\left[c_V^2(1+\cos\theta)S_V(\omega,q) + c_A^2(3-\cos\theta)S_A(\omega,q)\right]$$

- Neutrino absorption (weak charged-current reaction)

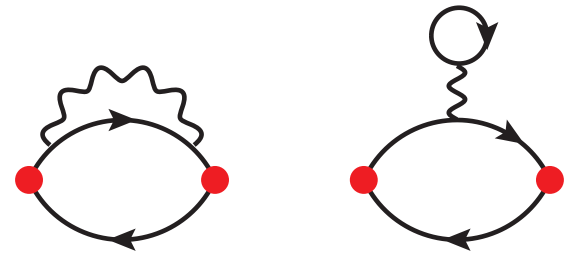
$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta\, d\omega} = \frac{G_W^2}{4\pi^2}p_3E_3\left(1-f_e(E_3)\right)\left[g_V^2(1+\cos\theta)S_V(\omega,q) + g_A^2(3-\cos\theta)S_A(\omega,q)\right]$$

- Relation to response function χ (fluctuation-dissipation theorem):

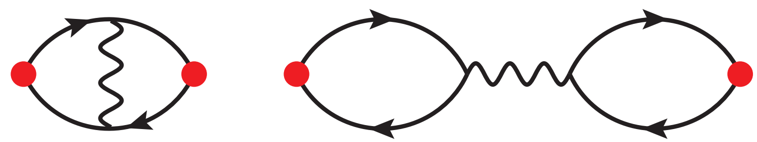
$$S(\omega,q) = \frac{-2}{n}\frac{1}{1-e^{-\beta\omega}}\mathrm{Im}\,\chi(q,\omega)$$

Spin and density response functions

Propagator (mean field) corrections:



Vertex corrections:



<u>Challenges</u>: consistency with EOS and consistency across different density/temperature domains

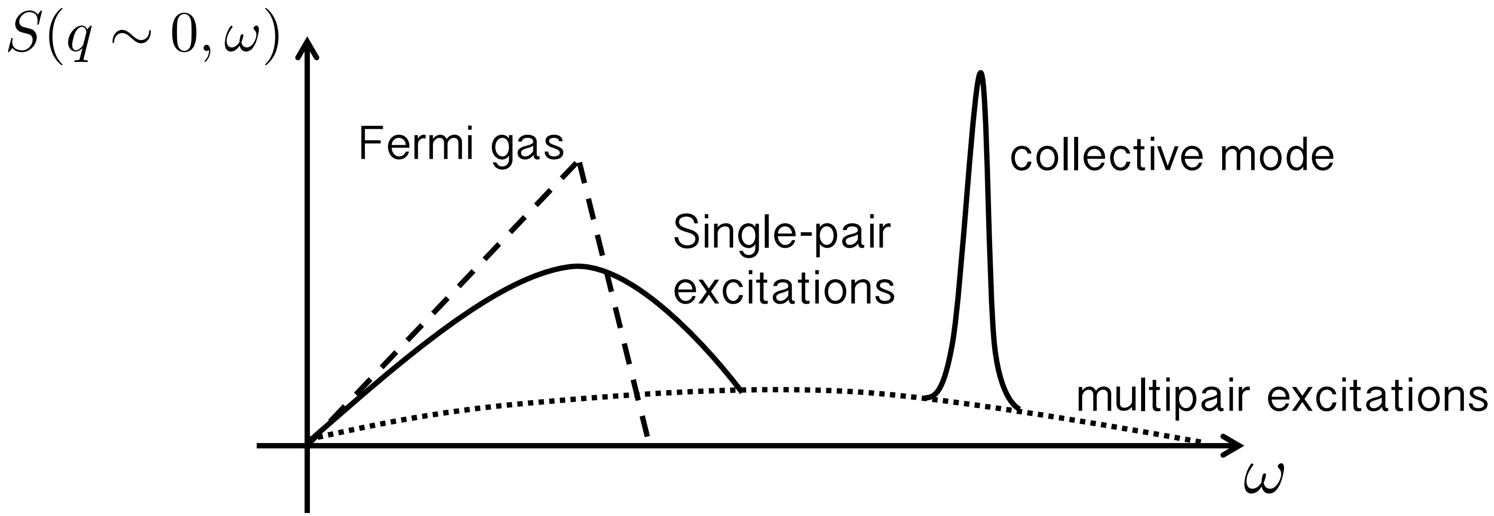
Different theoretical approaches:

(1) Fermi liquid theory with chiral nuclear forces

(2) Order-by-order perturbation theory, RPA,... with chiral nuclear forces

(3) Mean field theory with pseudopotentials

Structure of spectrum



$$\chi(q,\omega) = \underbrace{\sum_{n\neq 0}{}' \left|\langle \Psi_n | \rho_q^\dagger | \Psi_0 \rangle\right|^2 \frac{2(E_n - E_0)}{(\omega + i\eta)^2 - (E_n - E_0)^2}} + \chi_{\mathrm{multipair}}$$

Landau Fermi liquid theory

(applicable in high-density regime: $q \sim 0.15\,\mathrm{fm}^{-1} << k_F^n$)

Quasiparticle interaction (effective particle-hole interaction)

- Extracted from functional derivatives of the ground state energy density

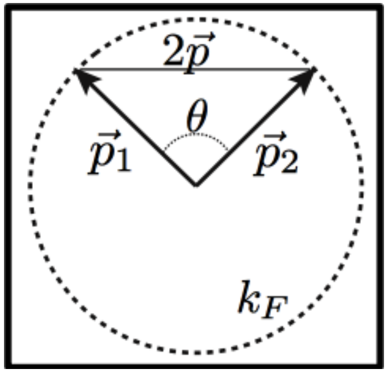
$$\delta\mathcal{E} = \sum_{\vec{p}_1} \epsilon^{(0)}_{\vec{p}_1} \delta n(\vec{p}_1) + \frac{1}{2} \sum_{\vec{p}_1,\vec{p}_2} \mathcal{F}(\vec{p}_1,\vec{p}_2)\delta n(\vec{p}_1)\delta n(\vec{p}_2) + \cdots$$

- Quasiparticle energies $\epsilon_{\vec{p}} = \epsilon^{(0)}_{\vec{p}} + \sum_{\vec{p}'} \mathcal{F}(\vec{p},\vec{p}')\delta n(\vec{p}') + \cdots$

- Legendre polynomial decomposition

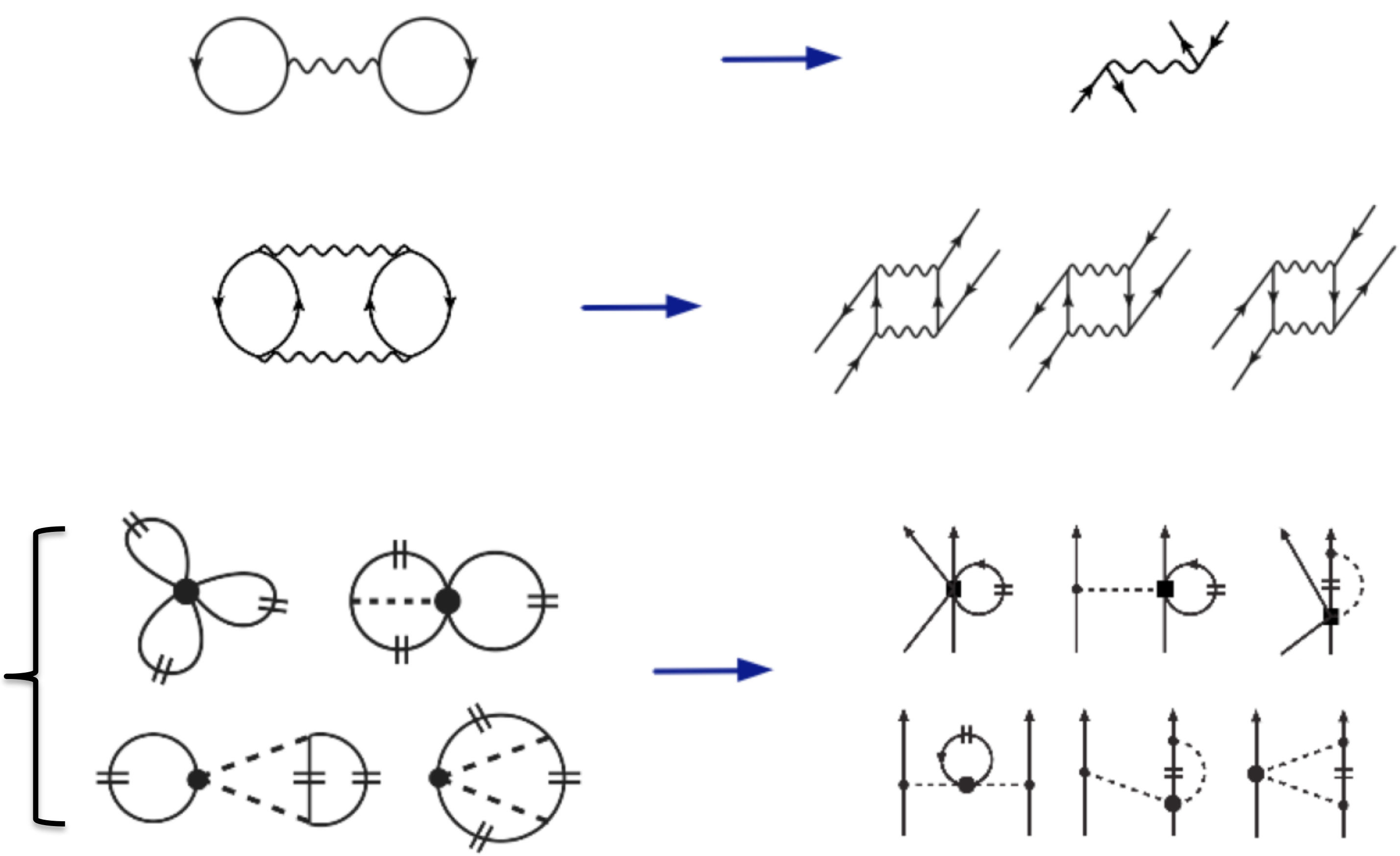
$$\mathcal{F}(\vec{p}_1,\vec{p}_2) = f(\vec{p}_1,\vec{p}_2) + f'(\vec{p}_1,\vec{p}_2)\vec{\tau}_1\cdot\vec{\tau}_2 + [g(\vec{p}_1,\vec{p}_2) + g'(\vec{p}_1,\vec{p}_2)\vec{\tau}_1\cdot\vec{\tau}_2]\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$
$$+ \text{ noncentral components}$$

$$f(\vec{p}_1,\vec{p}_2) = \sum_L f_L\, P_L(\cos\theta)$$
$$g(\vec{p}_1,\vec{p}_2) = \sum_L g_L\, P_L(\cos\theta)$$



Perturbation theory description

Nuclear equation of state | **Quasiparticle interaction**



3-body forces

Fermi liquid parameters in symmetric matter (benchmarks)

$\frac{M^*}{M_N} = 1 + \frac{F_1}{3}$	$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*}(1+F_0)$	$\beta = \frac{\hbar^2 k_F^2}{6M^*}(1+F_0')$	$\delta g_l = \frac{F_1' - F_1}{3(1+F_1/3)}$
Effective mass	**Compression modulus**	**Symmetry energy**	**Anomalous orbital g-factor**
$[0.7 - 1.0]$	$[220 - 260\ \text{MeV}]$	$[29 - 34\ \text{MeV}]$	$[0.20 - 0.26]$

$V_{\text{N3LO}}^{(1+2)} \quad (k_F = 1.33\ \text{fm}^{-1})$

l	F_l	G_l	F_l'	G_l'
0	-1.64	0.35	1.39	1.59
1	-0.13	0.50	0.58	0.47

$V_{\text{N3LO}}^{(1+2)} + V_{3N}^{(1)} \quad (k_F = 1.33\ \text{fm}^{-1})$

l	F_l	G_l	F_l'	G_l'
0	-0.15	0.35	1.36	1.20
1	-0.22	0.21	0.28	0.24

[Holt, Kaiser & Weise (2012)]

$$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} = 0.93$$

$$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*}(1+F_0) = 200\ \text{MeV}$$

$$\beta = \frac{\hbar^2 k_F^2}{6M^*}(1+F_0') = 31\ \text{MeV}$$

$$\delta g_l = \frac{F_1' - F_1}{3(1+F_1/3)} = 0.09$$

Non-central QP interaction in neutron matter

- Exchange tensor interaction: $S_{12}(\hat{q}) = 3\vec{\sigma}_1 \cdot \hat{q}\vec{\sigma}_2 \cdot \hat{q} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \qquad \vec{q} = \vec{p}_1 - \vec{p}_2$
  [Haensel & Dabrowski (1975)]

- Presence of Fermi sea allows additional contributions to the effective interaction that are absent in free space **[Friman & Schwenk (2004)]**

  Center of mass tensor: $S_{12}(\hat{P}) = 3\vec{\sigma}_1 \cdot \hat{P}\vec{\sigma}_2 \cdot \hat{P} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \qquad \vec{P} = \vec{p}_1 + \vec{p}_2$

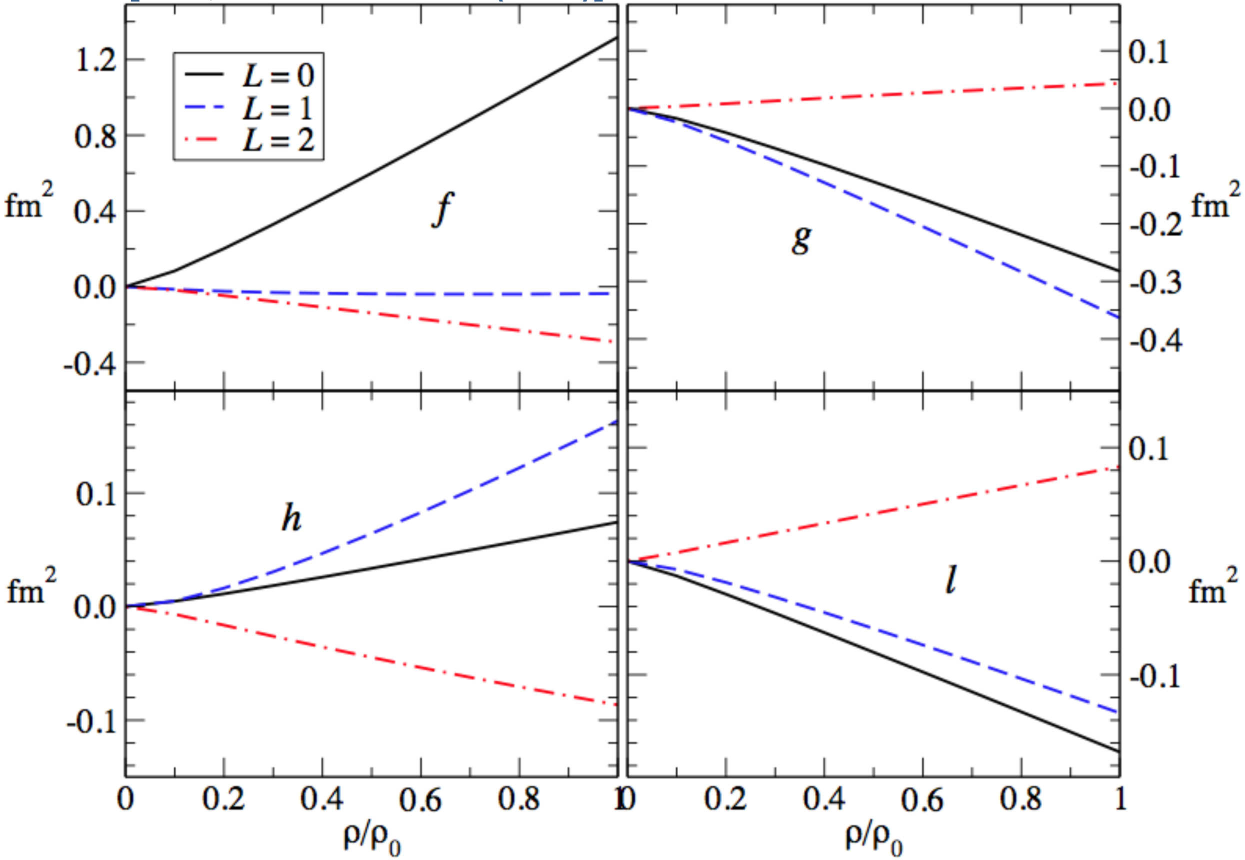
  Cross-vector: $A_{12}(\vec{q}, \vec{P}) = (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot (\vec{q} \times \vec{P})$

- Pure neutron matter particle-hole interaction:

$$\mathcal{F}(\vec{p}_1, \vec{p}_2) = f(\vec{p}_1, \vec{p}_2) + g(\vec{p}_1, \vec{p}_2)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + h(\vec{p}_1, \vec{p}_2)S_{12}(\hat{q})$$
$$+ k(\vec{p}_1, \vec{p}_2)S_{12}(\hat{P}) + l(\vec{p}_1, \vec{p}_2)(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot (\vec{q} \times \vec{P})$$

Density dependence of three-nucleon force contributions

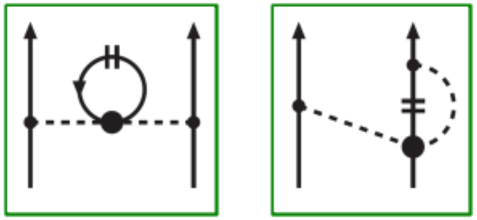
[Holt, Kaiser & Weise (2013)]



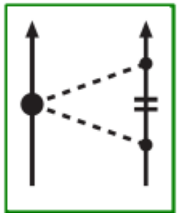
- Low-energy constants

$$c_1 = -0.81\,\mathrm{GeV}^{-1}$$
$$c_3 = -3.2\,\mathrm{GeV}^{-1}$$

- Contributions to exchange-tensor

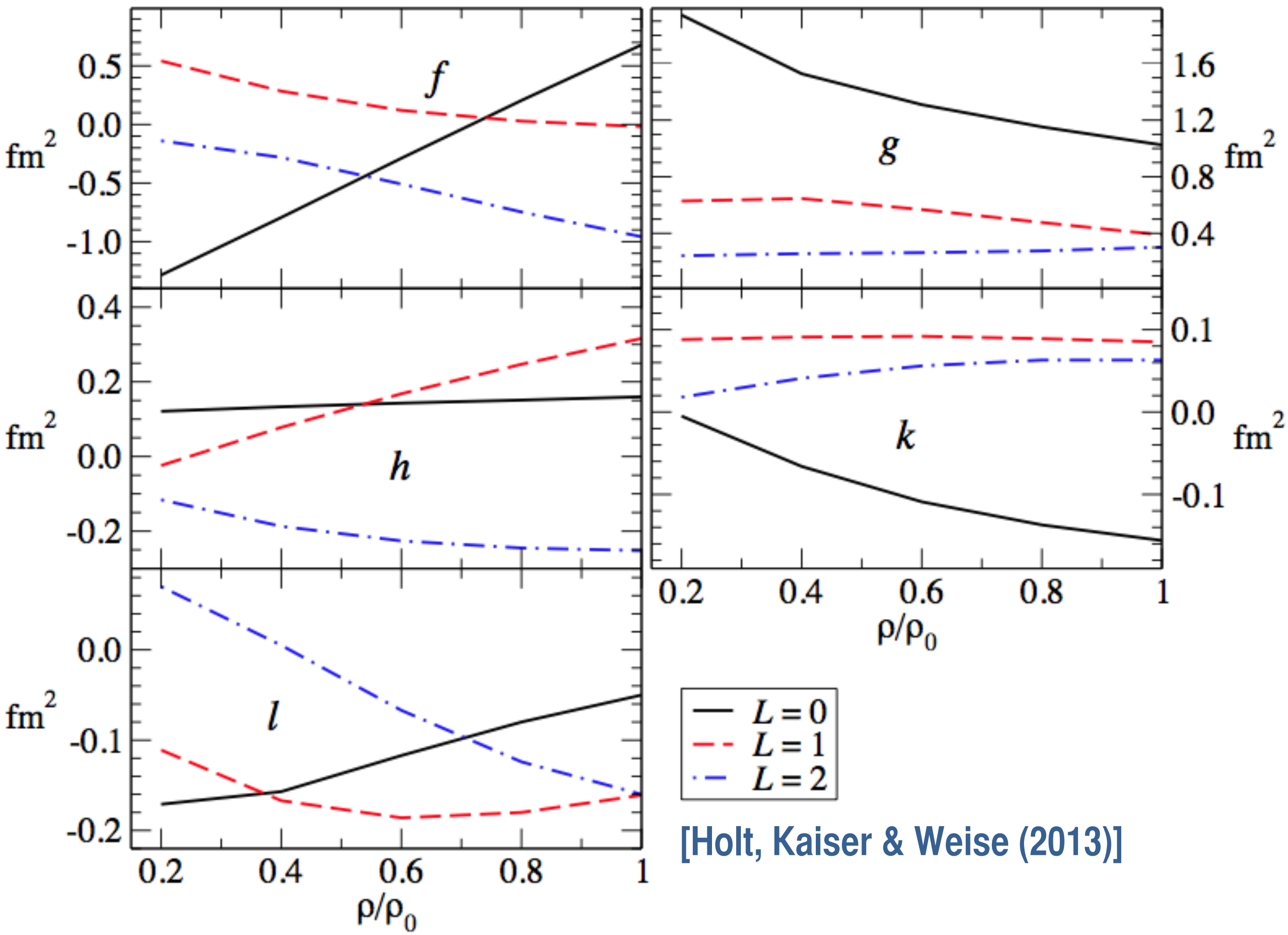


- Contribution to cross vector



Density dependence of full Landau parameters

$$\mathcal{F}(\vec{p}_1,\vec{p}_2) = f(\vec{p}_1,\vec{p}_2) + g(\vec{p}_1,\vec{p}_2)\vec{\sigma}_1\cdot\vec{\sigma}_2 + h(\vec{p}_1,\vec{p}_2)S_{12}(\hat{q})$$
$$+k(\vec{p}_1,\vec{p}_2)S_{12}(\hat{P}) + l(\vec{p}_1,\vec{p}_2)(\vec{\sigma}_1\times\vec{\sigma}_2)\cdot(\vec{q}\times\vec{P})$$



[Holt, Kaiser & Weise (2013)]

Relation to response functions

- Response functions in the Landau limit $(q \to 0, \;\; \omega/q \to \text{fixed})$

$$\chi_\sigma(\vec{q},\omega) = \frac{N_0}{V}\frac{g(\lambda)}{1+[G_0+\lambda^2 G_1/(1+G_1/3)]g(\lambda)}$$

$$g(\lambda) = 1 - \frac{\lambda}{2}\ln\left|\frac{\lambda+1}{\lambda-1}\right| + \frac{i\pi}{2}\lambda\theta(1-|\lambda|) \qquad \lambda = \frac{\omega}{q v_F}$$

- Including noncentral interactions in RPA (static limit):

$$\frac{\chi_{HF}(0)}{\chi_{RPA}^{(S=0)}(0)} = 1 + F_0 \qquad \frac{\chi_{HF}(0)}{\chi_{RPA}^{(S=1)}(0)} = 1 + G_0 + \frac{T_1}{T_2}$$

$$T_1 = -\frac{1}{8}\left(H_0 - H_1 + K_0 + K_1\right)^2$$

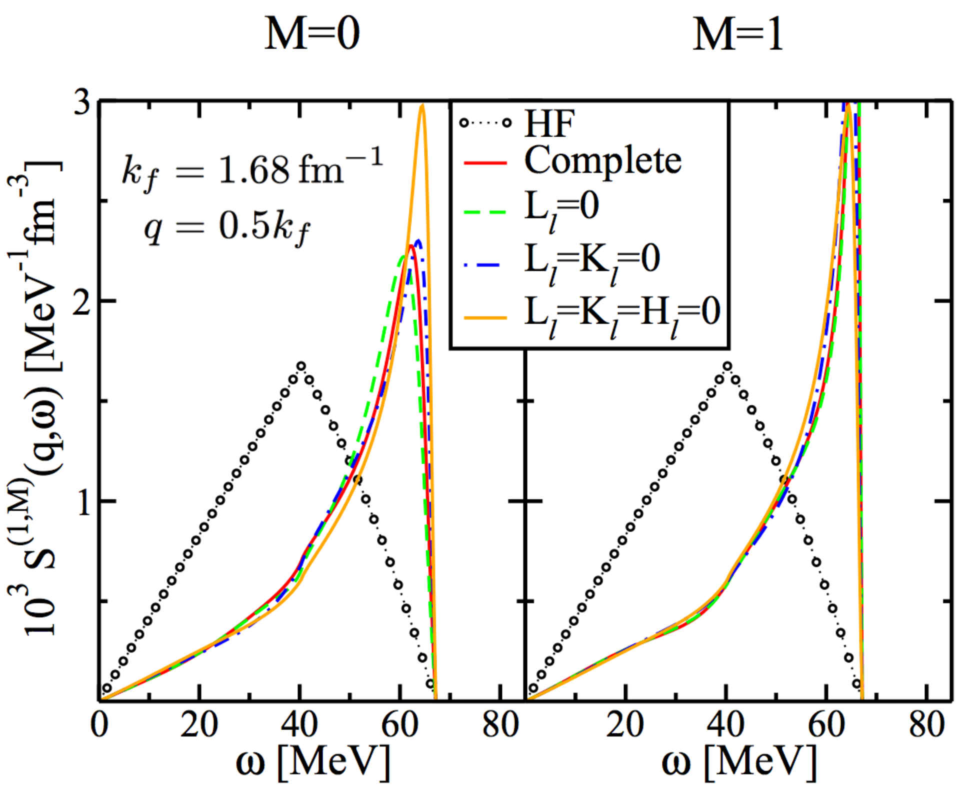
$$T_2 = 1 + \frac{1}{5}G_2 - \frac{1}{4}H_0 - \frac{1}{4}H_1 + \frac{1}{10}H_2 - \frac{1}{4}K_0 + \frac{1}{4}K_1 + \frac{1}{10}K_2 + \frac{2}{5}\tilde{L}_1 - \frac{6}{35}\tilde{L}_3$$

Results from chiral two- and three-body interactions

- Terms up to $l_{max} = 3$ included

- Only the exchange tensor contribution is relevant

- Exchange tensor contributions smaller than in mean field models

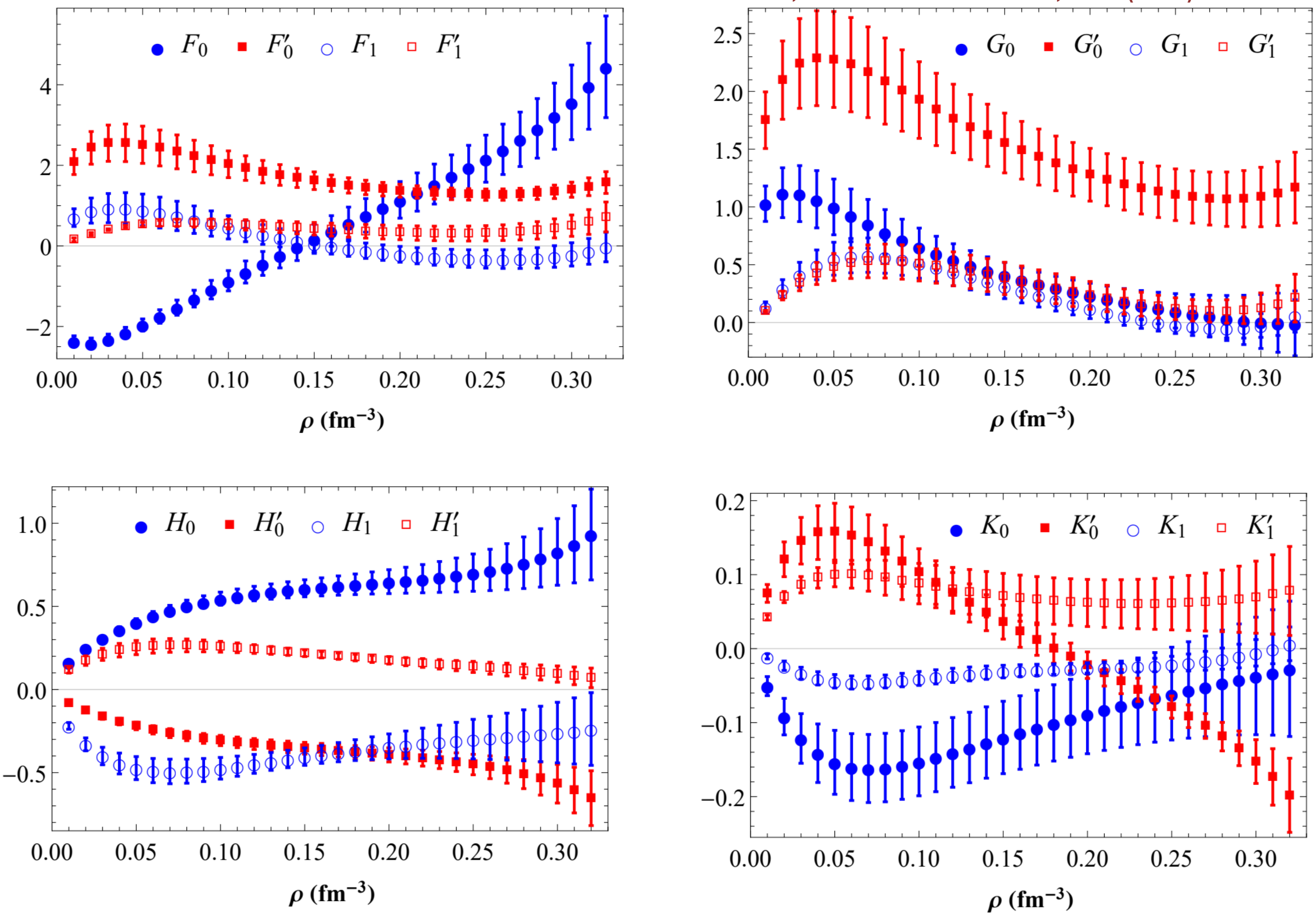


Davesne, Holt, Pastore & Navarro, PRC (2015)

- Future efforts: extend to finite temperature and arbitrary isospin asymmetry

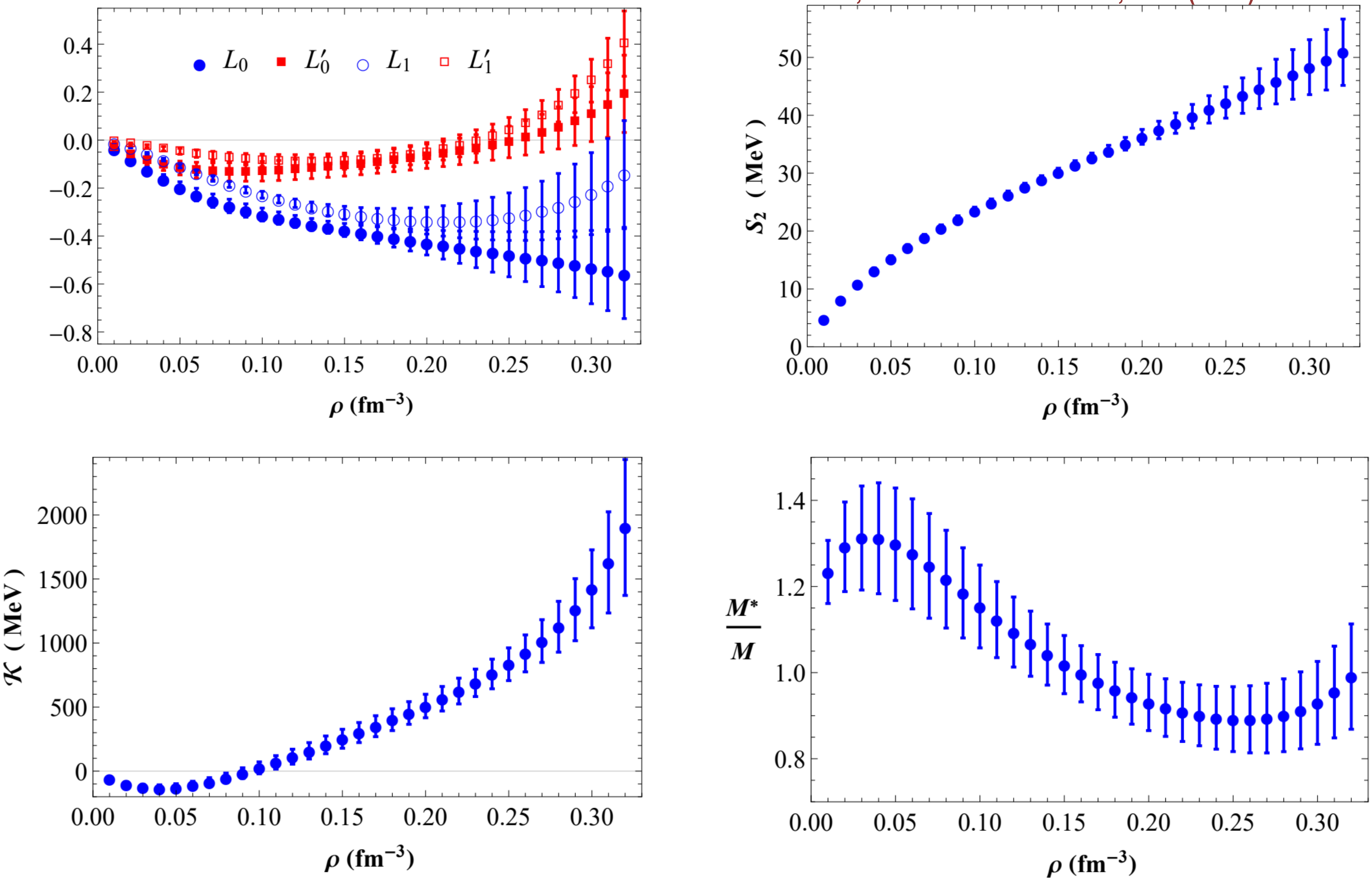
Fermi liquid parameters in symmetric nuclear matter

Holt, Kaiser and Whitehead, PRC (2018)



Fermi liquid parameters and derived empirical quantities

Holt, Kaiser and Whitehead, PRC (2018)



Charged-current reactions in neutrinosphere

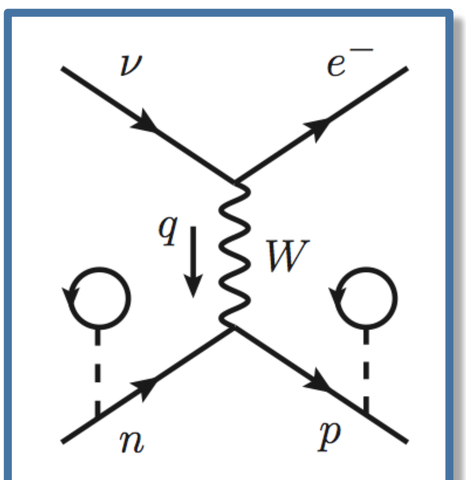
Neutrino-antineutrino spectral difference crucial for nucleosynthesis

$$\nu_e + n \longleftrightarrow e^- + p$$
$$\bar{\nu}_e + p \longleftrightarrow e^+ + n$$

Set proton fraction in region of r-process

$$N_p \lesssim 0.4$$
$$\langle E_{\bar{\nu}_e} \rangle - \langle E_{\nu_e} \rangle > 4(m_n - m_p)$$

Determines nucleosynthesis outcome



Nuclear mean fields enhance neutrino absorption

Skyrme & RMF calculations: Martinez-Pinedo et al, PRL (2012); Roberts et al, PRC (2012)

Resonant nucleon-nucleon interactions may enhance effect ($a_{nn} = -18\,\mathrm{fm}$)

Neutrino absorption cross section

$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta\, dE_e} = \frac{G_F^2\cos^2\theta_C}{4\pi^2}|\vec{p}_e|\,E_e\,(1-f_e(\xi_e))$$

Electron phase space

$$\times\left[(1+\cos\theta)S_\tau(q_0,q) + g_A^2(3-\cos\theta)S_{\sigma\tau}(q_0,q)\right]$$

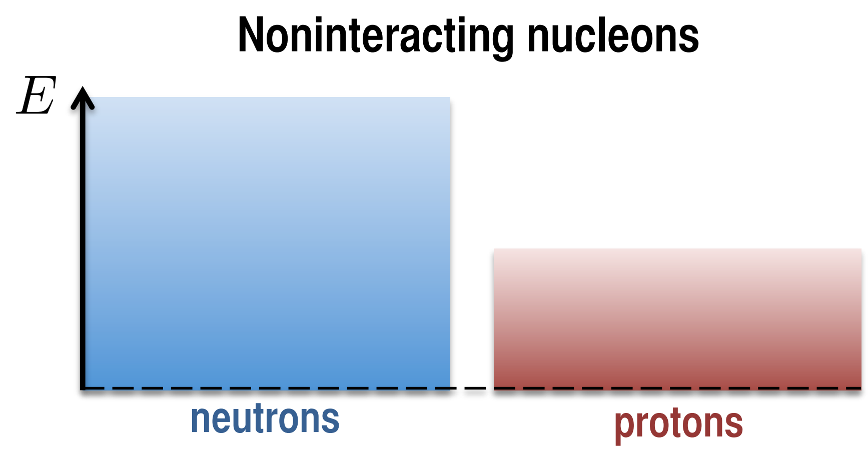
Nucleon response

- Nuclear interactions attractive at low momenta and

  $$|\langle np|V_{NN}|np\rangle| > |\langle nn|V_{NN}|nn\rangle|$$

- Mean field effects widen the energy gap between protons and neutrons

- **Q-value** for neutrino absorption changes significantly



Neutrino absorption cross section

$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta\, dE_e} = \frac{G_F^2\cos^2\theta_C}{4\pi^2}\, |\vec{p}_e|\, E_e\,(1 - f_e(\xi_e))$$

Electron phase space

$$\times\left[(1+\cos\theta)S_\tau(q_0,q) + g_A^2(3-\cos\theta)S_{\sigma\tau}(q_0,q)\right]$$

Nucleon response

- Nuclear interactions attractive at low momenta and

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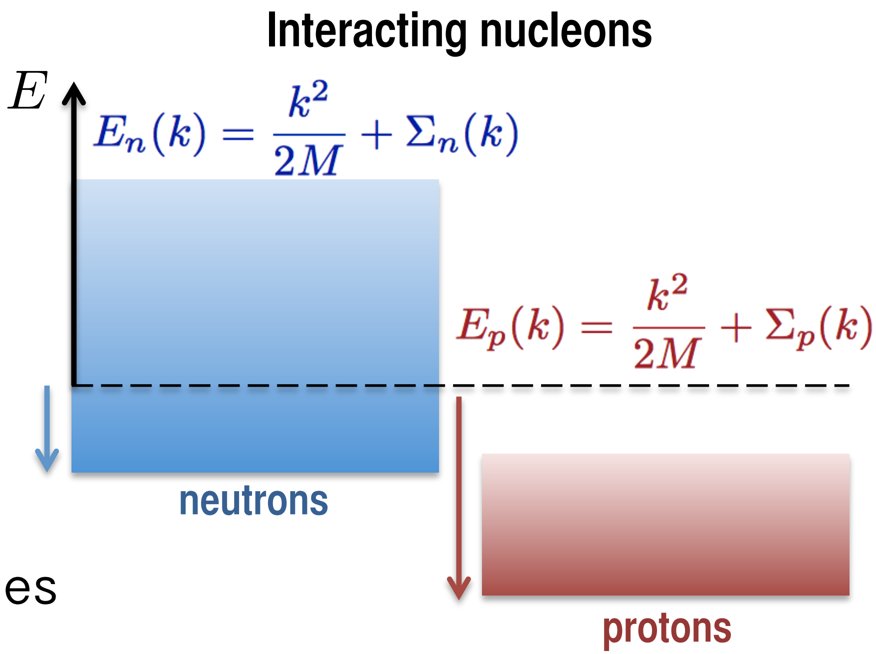

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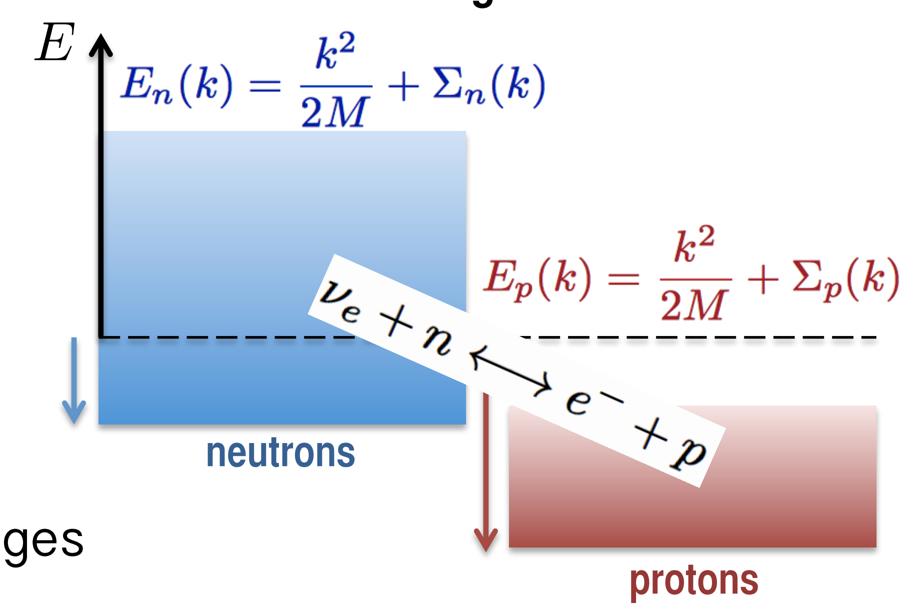
Interacting nucleons



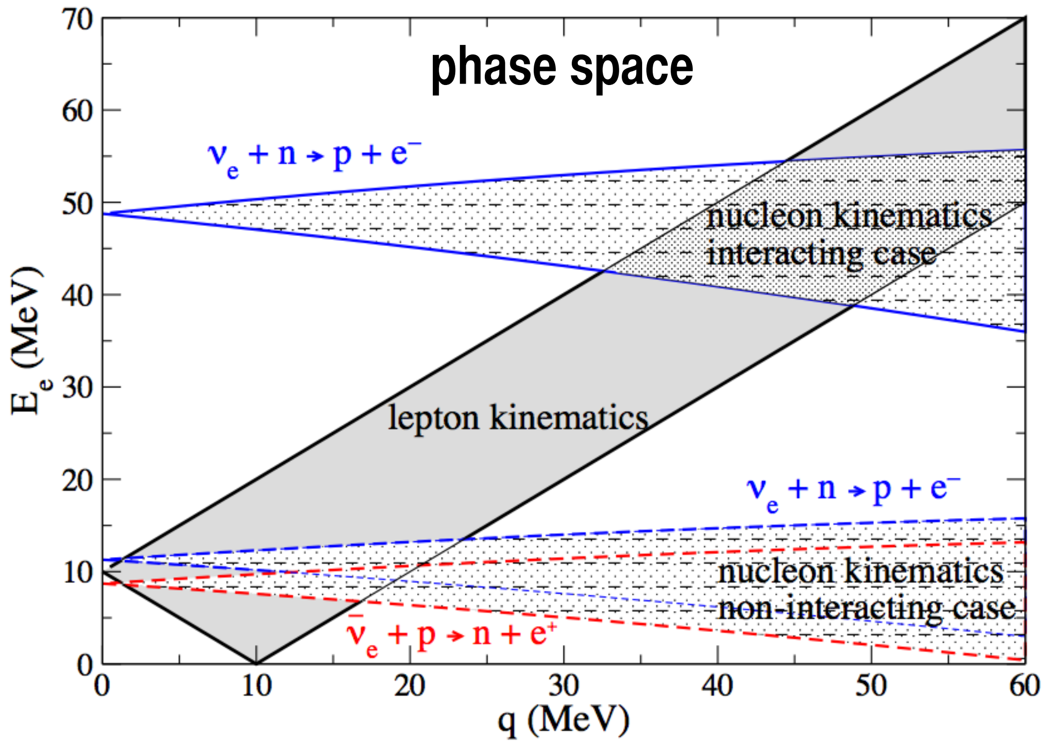
Phase space considerations

Charged-current reactions ($\nu_e n \to e^- p$) with $E_\nu = 10\ \mathrm{MeV},\ p_n = 100\ \mathrm{MeV}$

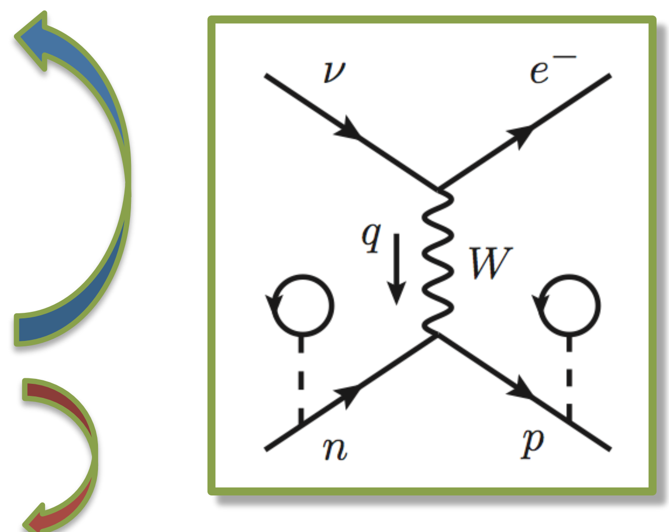
$$E_e = \sqrt{E_\nu^2 - 2E_\nu q \cos\theta + q^2 + m_e^2}$$ **lepton**

$$E_e = E_\nu + (E_n - E_p) = E_\nu - \frac{1}{2\bar{M}}(q^2 + 2p_N q \cos\theta) + (M_n - M_p)$$ **nucleon**

kinematic regions



Mean-field effects



Neutrino mean free path

(1) Pseudo-potential (reproduces **exact energy shift** when used at the mean field level)

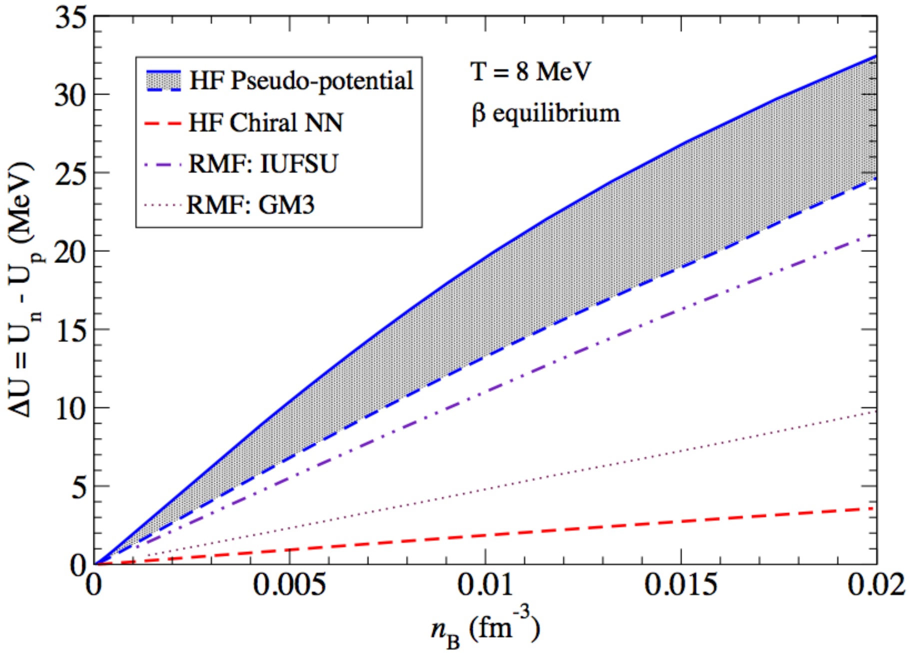
$$\langle p | V_{llSJ}^{pseudo} | p \rangle = -\frac{\delta_{lSJ}(p)}{p M_N}$$

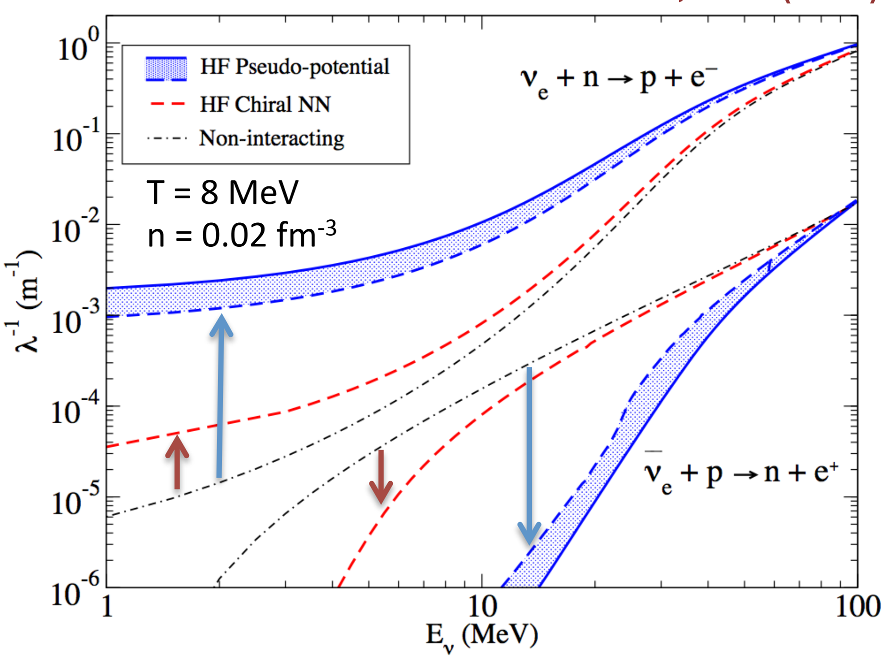
Fumi (1955),
Fukuda & Newton (1956)

(2) Chiral NN potential at mean-field level

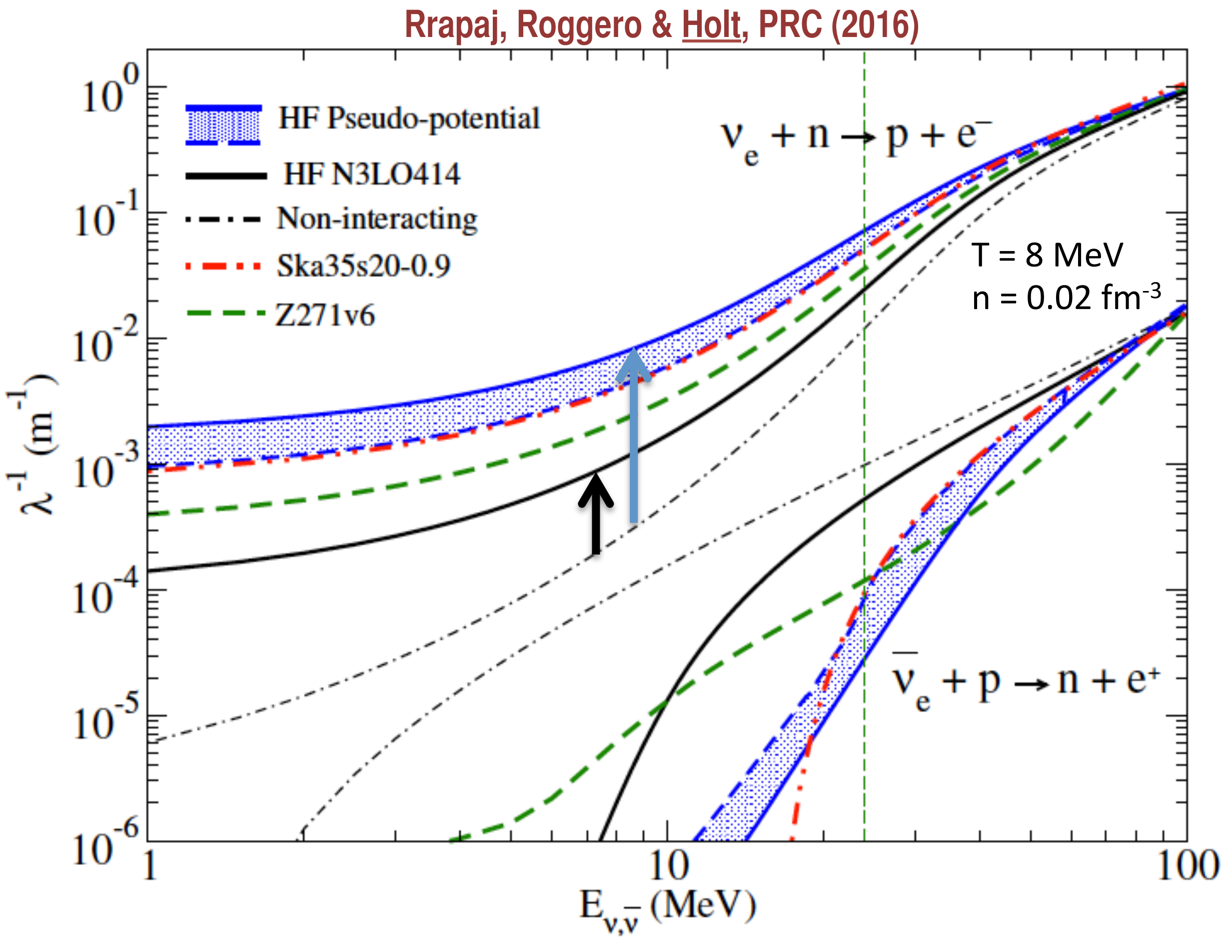
- Nucleon energies: $E_N(k) = \frac{k^2}{2M} + \Sigma_N(k) \simeq \frac{k^2}{2M^*} - U_N$

Rrapaj, Holt, Bartl, Reddy & Schwenk, PRC (2015)



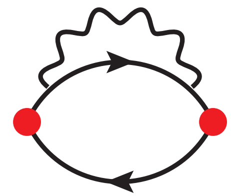
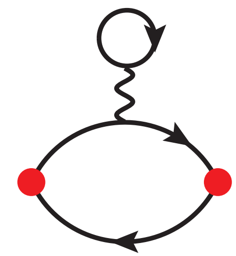


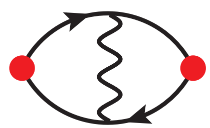
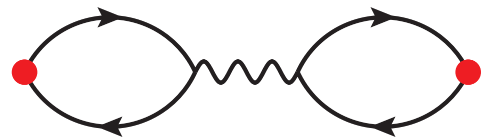
Comparison to common mean field models



- Use of pseudopotential generally leads to enhancement of neutrino absorption

Extend to include vertex corrections to response function

Propagator (mean field) corrections:  

Vertex corrections:  

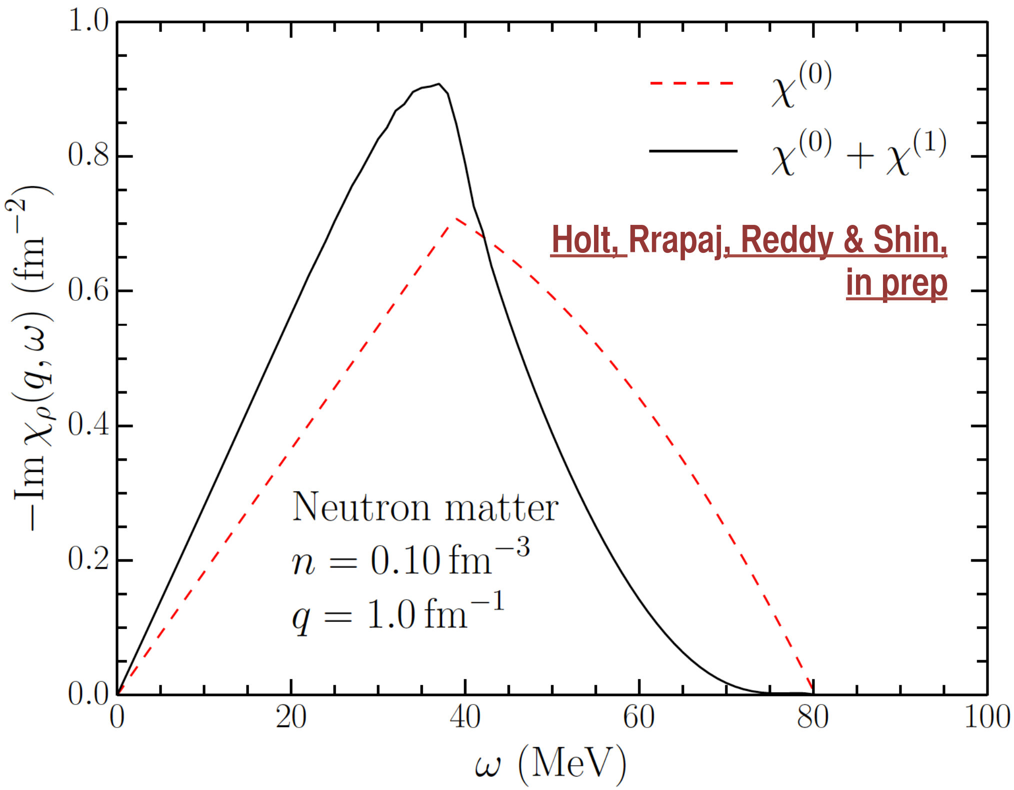
Pseudopotential only defined for on-shell matrix elements:

$$\langle p | V_{llSJ}^{pseudo} | p \rangle = -\frac{\delta_{lSJ}(p)}{p M_N}$$

Vertex corrections require off-shell matrix elements:

$$\chi^{(1)}_{\rho\rho}(\omega, \vec{q}) = \frac{M^2}{8\pi^4 q^2} \int dk_1 k_1 \int d\cos\theta_1 \left[\frac{n_{k_1} - n_{k_1+q}}{\cos\theta_1 - \frac{M\omega}{k_1 q} + \frac{q}{2k_1} - i\eta} \right] \int dk_2 k_2 \int d\cos\theta_2 \left[\frac{n_{k_2} - n_{k_2+q}}{\cos\theta_2 - \frac{M\omega}{k_2 q} + \frac{q}{2k_2} - i\eta} \right]$$
$$\int d\phi_2 \sum_{LSJ} (2J+1) P_L(\hat{q}_1 \cdot \hat{q}_2)(1 - (-1)^{L+S+1}) \underline{\langle q_1 LSJ | V | q_2 LSJ \rangle}.$$

First-order vertex correction to density response



N3LO_500 chiral nucleon-nucleon potential

Effects from 3NF currently neglected

- Strong enhancement of low-frequency density response
- Suppression of high-frequency density response

SUMMARY

- Need for theoretical frameworks capable of computing consistent equations of state and response functions in hot/dense matter for astrophysical simulations

- Consistent single-particle potentials needed for transport simulations of medium-energy heavy-ion collisions for extracting equation of state

- Nucleon mean fields from resonant NN interactions in the neutrinosphere can strongly enhance electron neutrino absorption

- Inclusion of vertex corrections to density and spin response functions in progress