Short Range Correlations in Nuclei

Hans Feldmeier

GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt

Overview

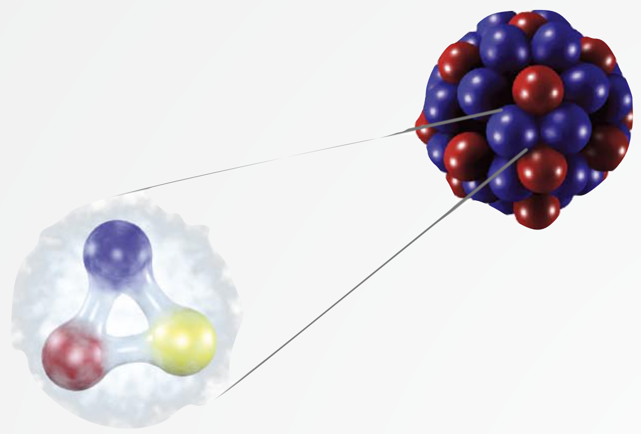
A) NN-Interactions & Short Range Correlations (SRC)

- Visualize nucleon-nucleon potential V_{NN}
- Solve many-body problem exactly for A=2,3,4
- V_{NN} leaves telling „footprints“ in densities
- Central and tensor correlations
- Universality of short range correlations
- Seeing all this, why does shell model work?

B) Similarity Transformation of Hamiltonian and Observables

- AV18/Chiral EFT $V_{NN} \rightarrow V_{\alpha}$ with SRG (Similarity Renormalization Group) transformation
- Solve many-body problem with NCSM (No Core Shell Model) for A=4,6,9,12 with soft V_{α}
- Recover short-range physics with SRG transformed observables
- Dominant role of deuteron-like S=1,T=0 pairs and tensor correlations at high relative momenta (dominance of pn over pp pairs, data)
- Many-body correlations leave traces in 2-body and 1-body densities
- Shell model works: SRC are only visible in appropriate observables

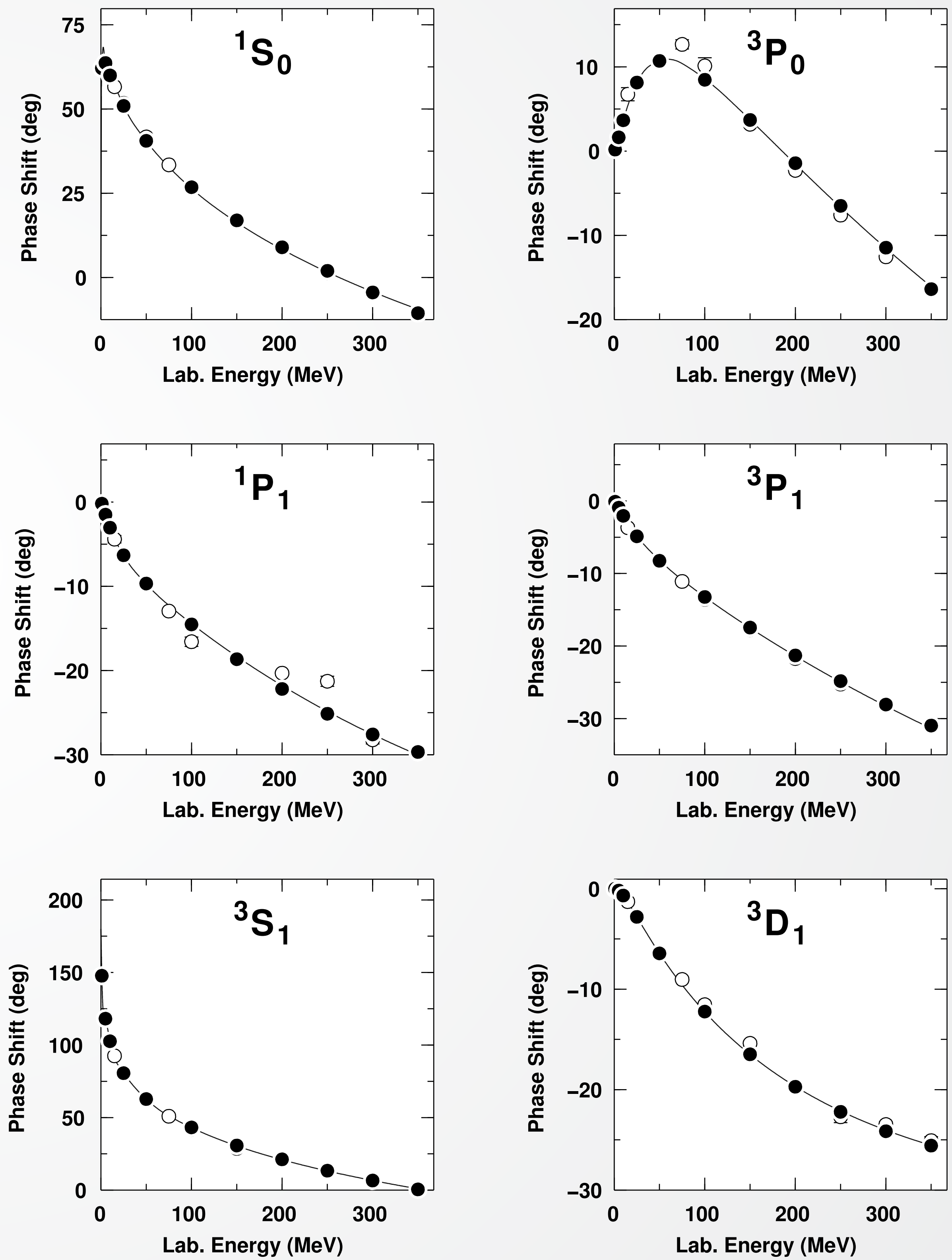
Nucleon-Nucleon Interactions



- Nucleons are not point-like, complicated quark and gluon sub-structure
- Nuclear interaction $V_{NN}+V_{NNN}$: residual interaction
- Calculation within QCD not possible yet
- Construct **realistic NN potentials** that describe two-nucleon properties (scattering, Deuteron) with high accuracy
- **high-momentum and off-shell behavior not constrained by scattering data**

↓

- e.g. Argonne V18, Chiral N3LO
- short-range repulsion, strong tensor force



Nucleon-Nucleon Interactions

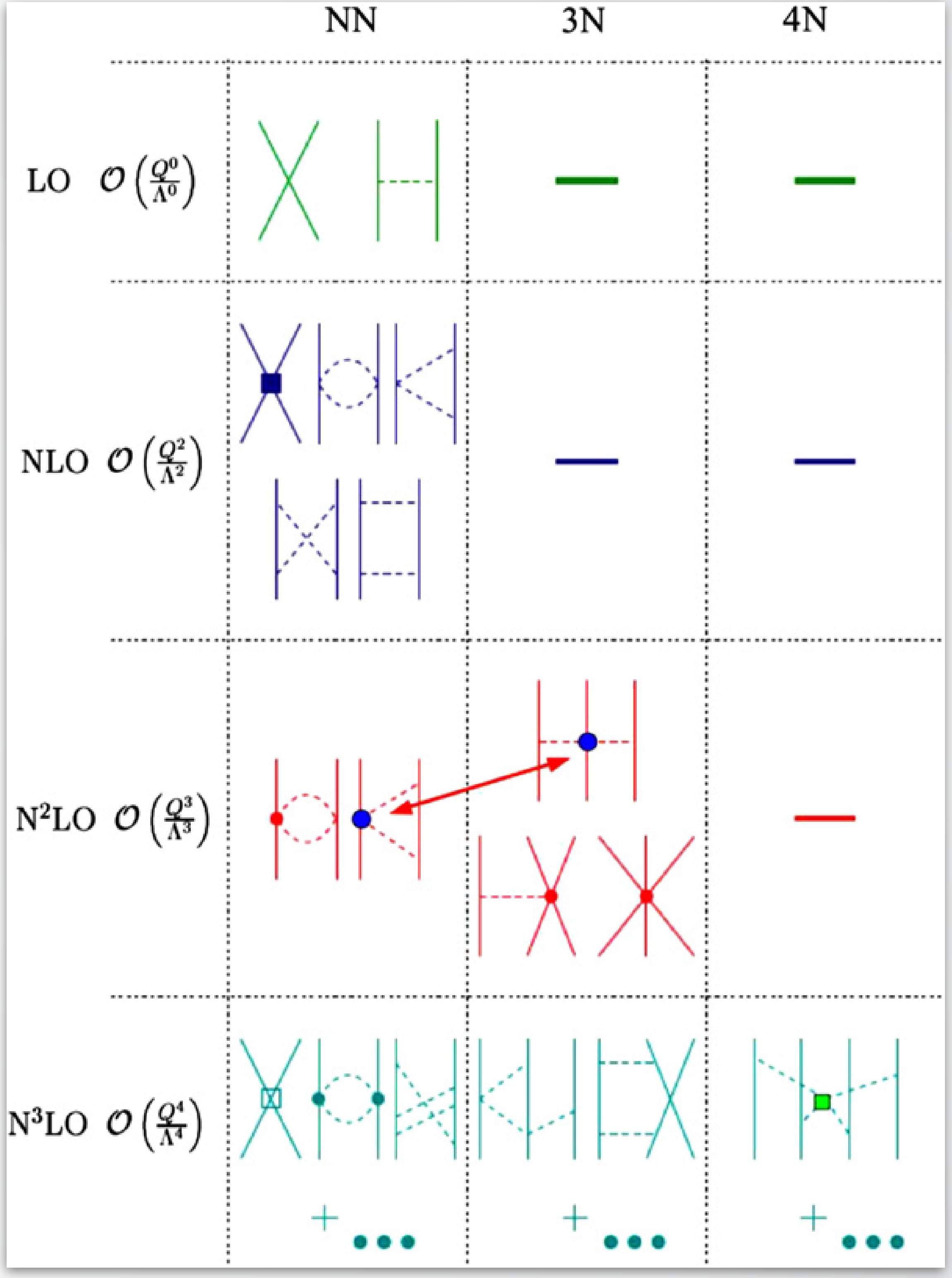
- **N³LO**
  - potential derived using chiral EFT
  - includes full π dynamics
  - power counting
  - short-range behavior given by contact-terms
  - regulated by non-local cut-off (500 MeV)

Entem, Machleidt, Phys. Rev. C **68**, 041001 (2003)

ongoing developments in chiral EFT → lecture by J. W. Holt

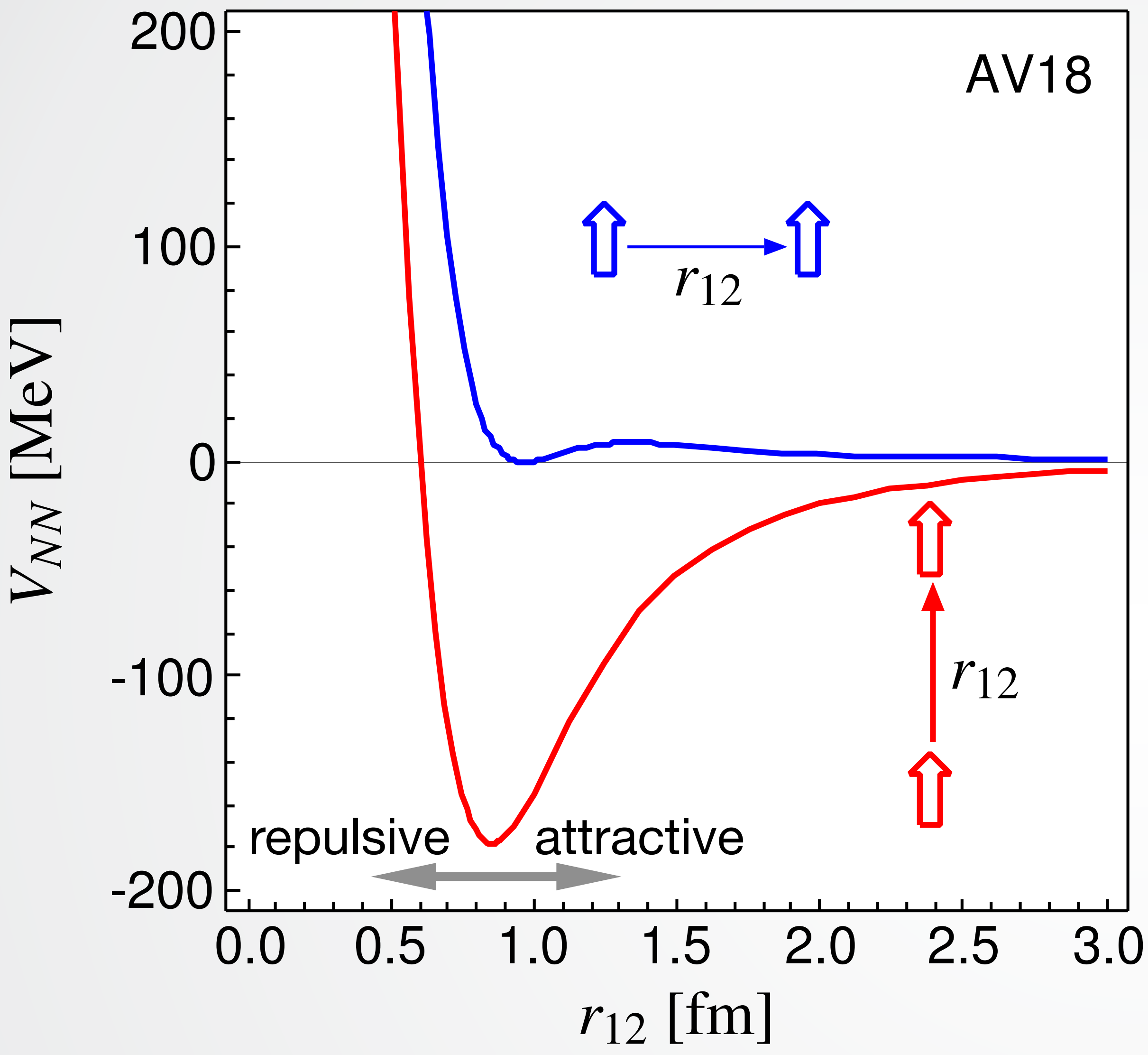
- **Argonne V18/V8’**
  - π-exchange, phenomenological short-range
  - "as local as possible”
  - fitted to phase shifts up to 350 MeV, but describes elastic phase shifts up to 1 GeV

Wiringa, Stoks, Schiavilla, Phys. Rev. C **51**, 38 (1995)



Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. **65**, 94 (2010)

Nucleon-Nucleon Interaction AV18



- **Argonne Potential AV18** (for **p**=0)
- **repulsive for r<0.9 fm:** nucleons are expelled from repulsive core
- strong dependence on the orientation of the spins due to the **tensor force** (mainly from π-exchange)
- the nuclear force will induce strong short-range correlations in the nuclear wave function

π-Exchange

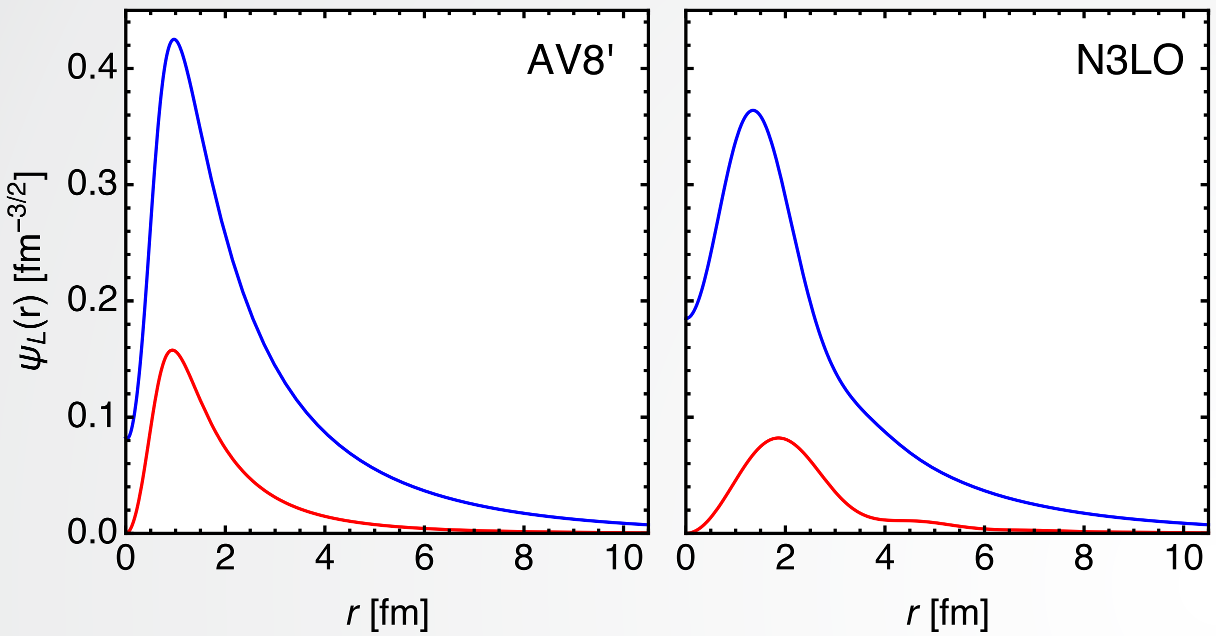
$$V_\pi(\mathbf{q}) = -\frac{f_{\pi NN}^2}{m_\pi^2}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q})(\boldsymbol{\sigma}_j \cdot \mathbf{q})}{m_\pi^2 + q^2}$$

$$V_\pi(\mathbf{r}) = \frac{1}{3} m_\pi \frac{f_{\pi NN}^2}{4\pi}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\left[(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + S_{ij}(\hat{\mathbf{r}}, \hat{\mathbf{r}})\left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right)\right]\frac{e^{-m_\pi r}}{m_\pi r}$$

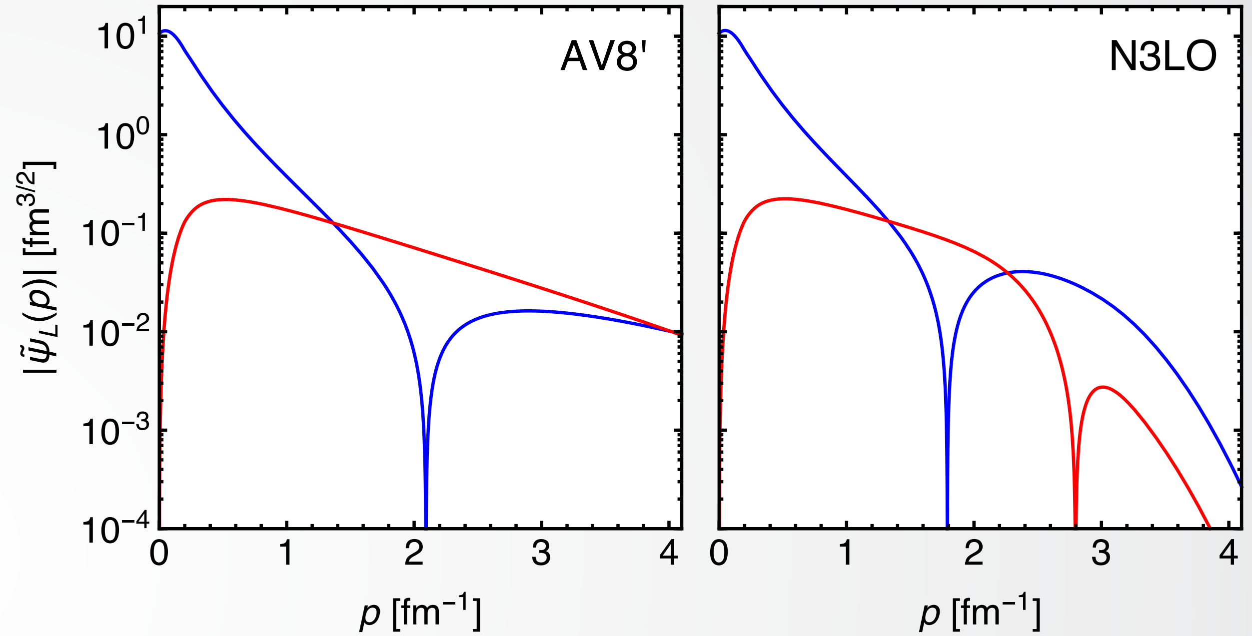
$$S_{ij}(\hat{\mathbf{r}}, \hat{\mathbf{r}}) = 3(\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}})(\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}) - (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$$

Deuteron Wave Functions

Coordinate Space



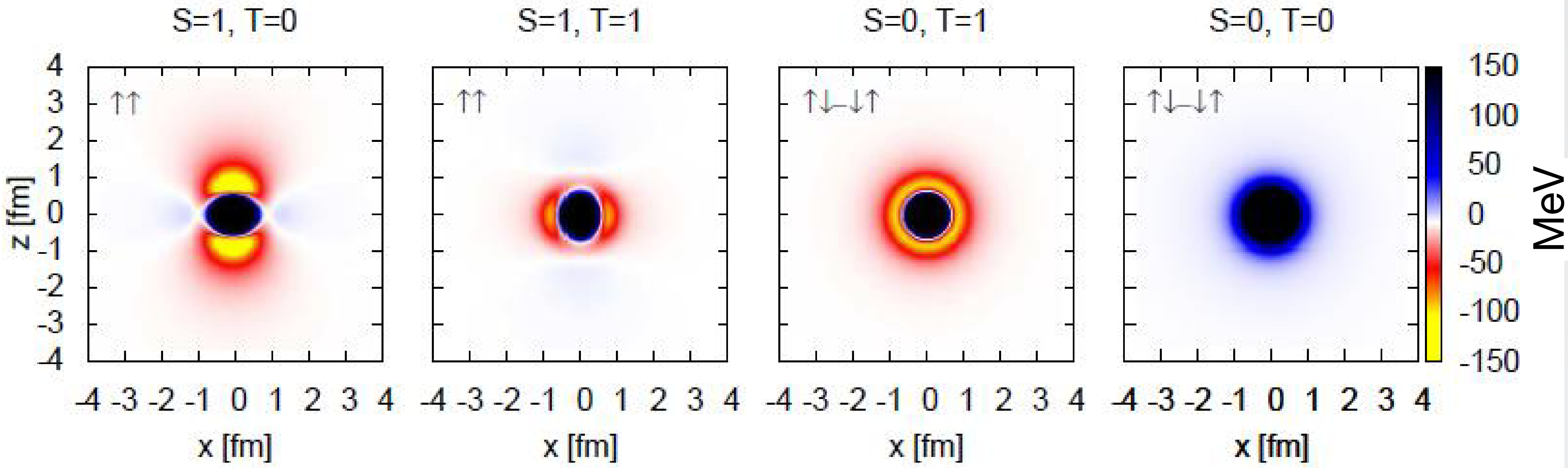
Momentum Space



- Suppression of the wave function at short distances due to repulsion
- *D*-wave admixture due to tensor force
- *D*-wave dominates high-momentum region around 2 fm^{-1}

- Short-range repulsion stronger for AV8',
  500 MeV cut-off in N3LO reflected in momentum space wave function
- N3LO wave function shows “kinks" at large distances — artifact of sudden cut-off

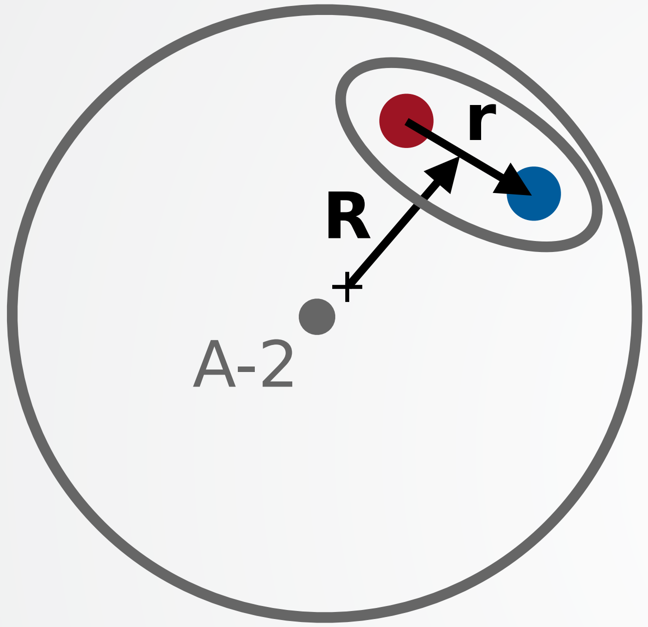
Argonne V8' Potential



- V8' in different spin-isospin channels as function of distance vector **r**=(x,y=0,z)
- In S=1 channels total spin align with z-axis

Coordinate Space Two-Body Density

- Probability to find a nucleon-pair with S and T at distance **r** inside a nucleus



$$\rho^{\text{rel}}_{SM_S,TM_T}(\mathbf{r}) = \left\langle \Psi \right| \sum_{i<j}^{A} \hat{P}^{SM_S}_{ij} \hat{P}^{TM_T}_{ij} \delta^3(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j - \mathbf{r}) \left| \Psi \right\rangle$$

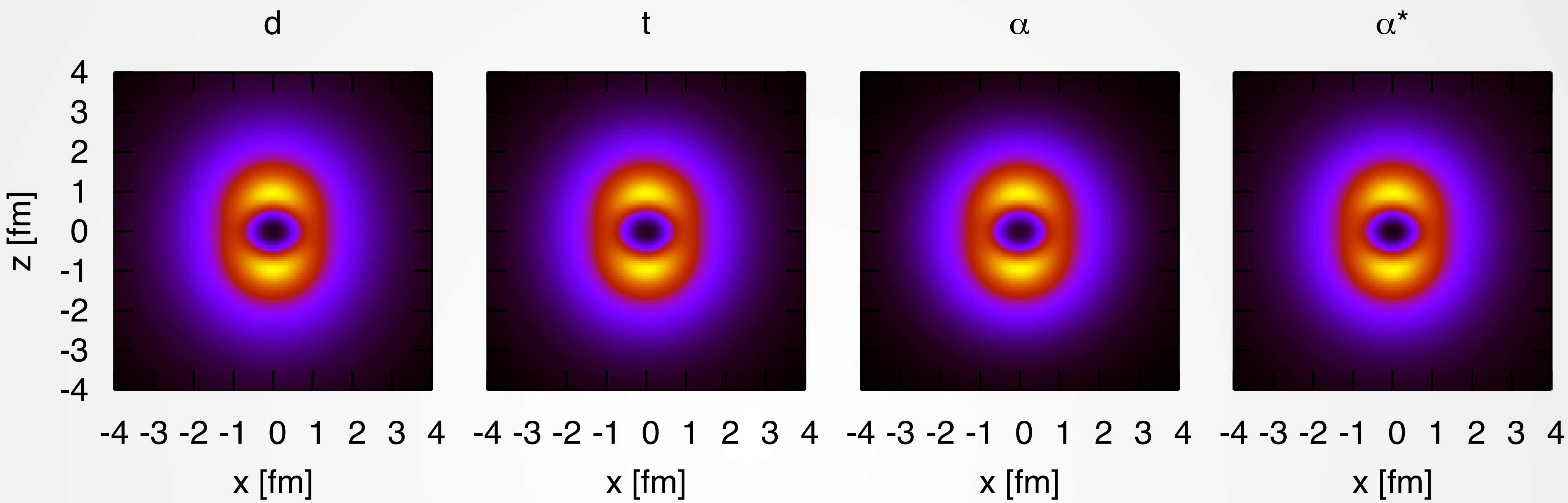
$|\Psi\rangle$ nuclear many-body state

R is measured from center-of-mass

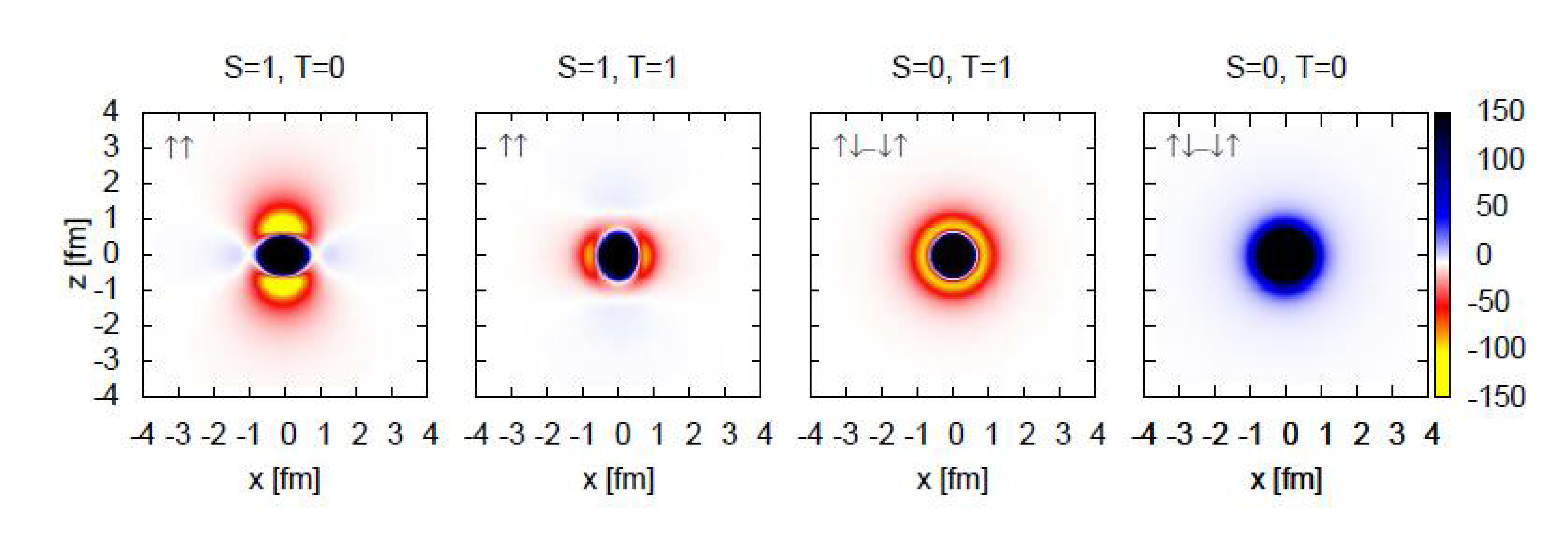
- coordinate space two-body densities will reveal correlation hole and tensor correlations

Two-body density for $S=1, M_S=1, T=0$ pairs

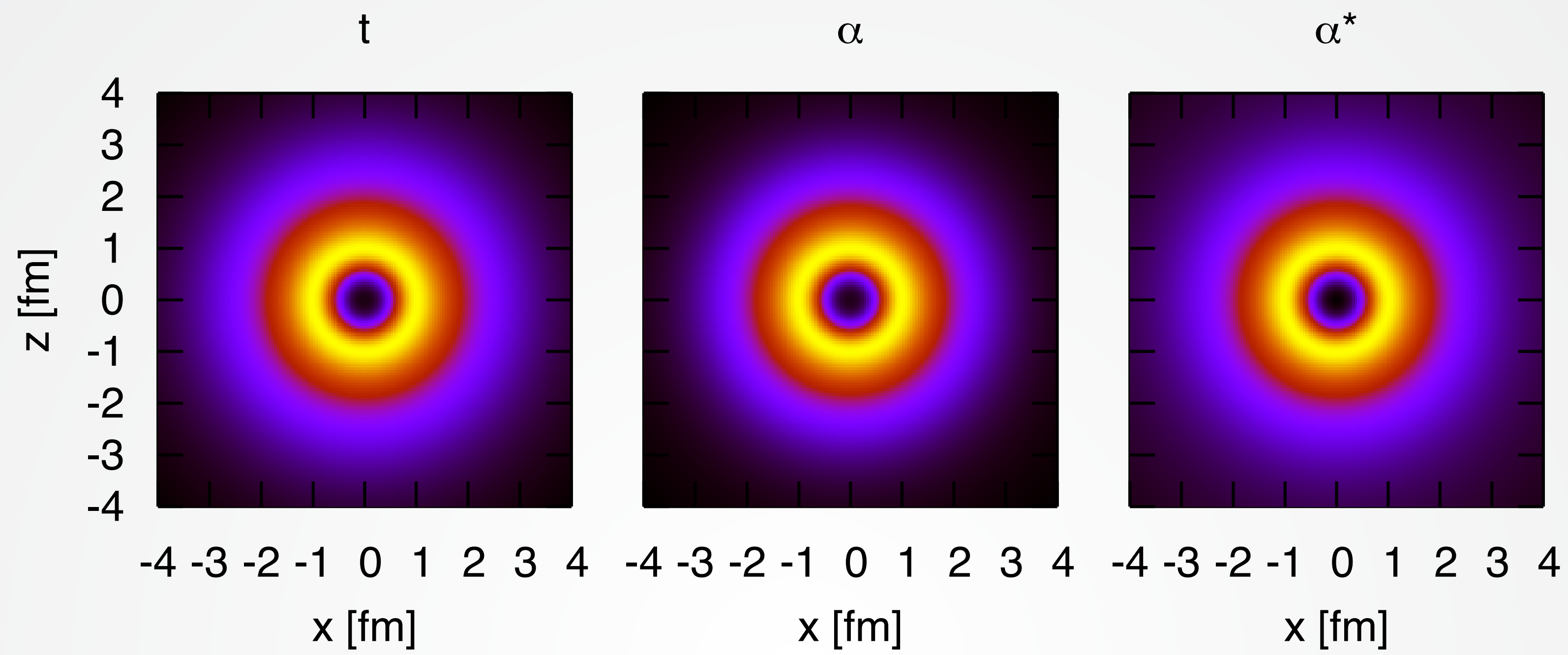
- Exact many-body calculations for d = ^{2}H, t = ^{3}H, α = ^{4}He, α^* = ^{4}He*



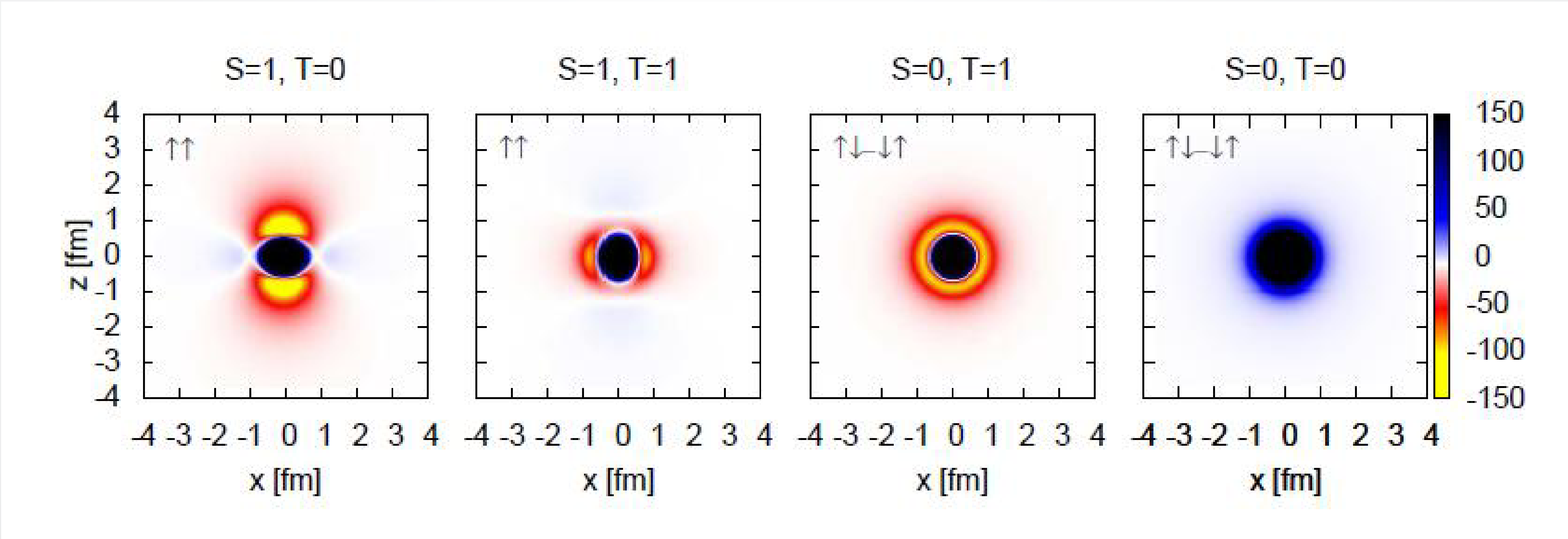
- Potential leaves one-to-one imprint on 2-body density



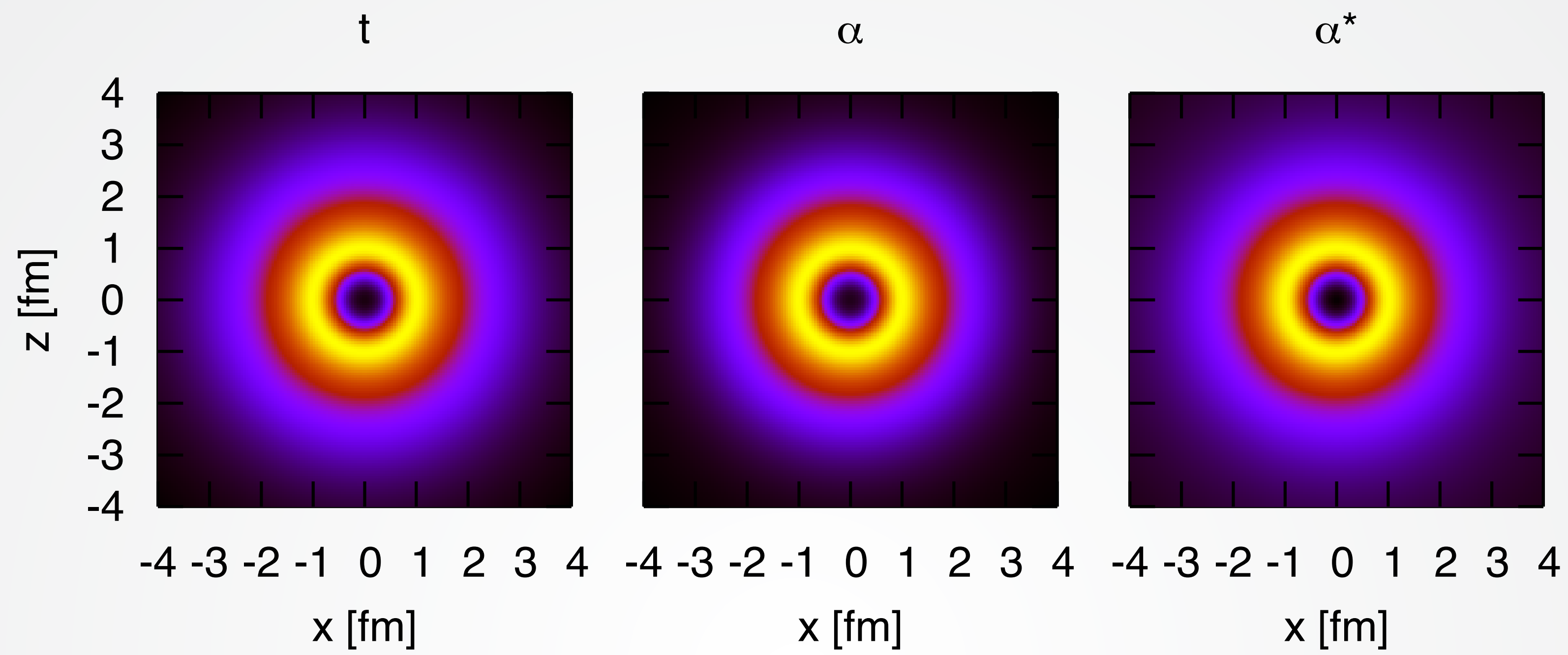
Two-body density for ?



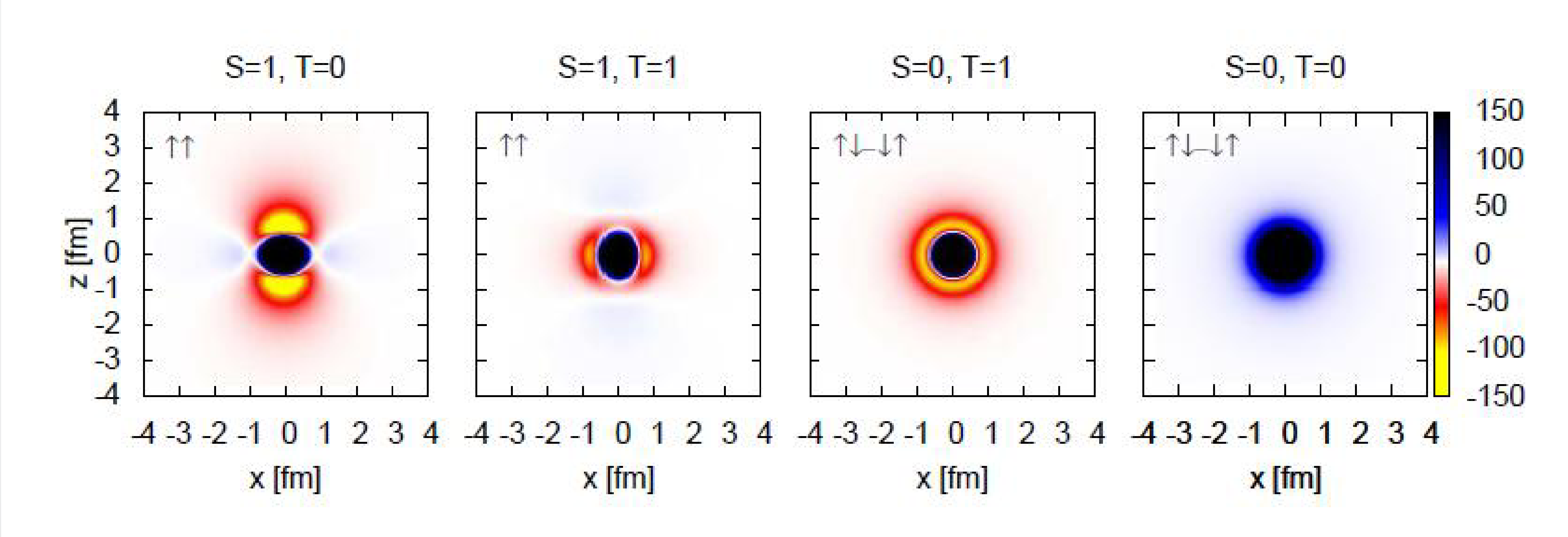
- Potential leaves one-to-one imprint on density



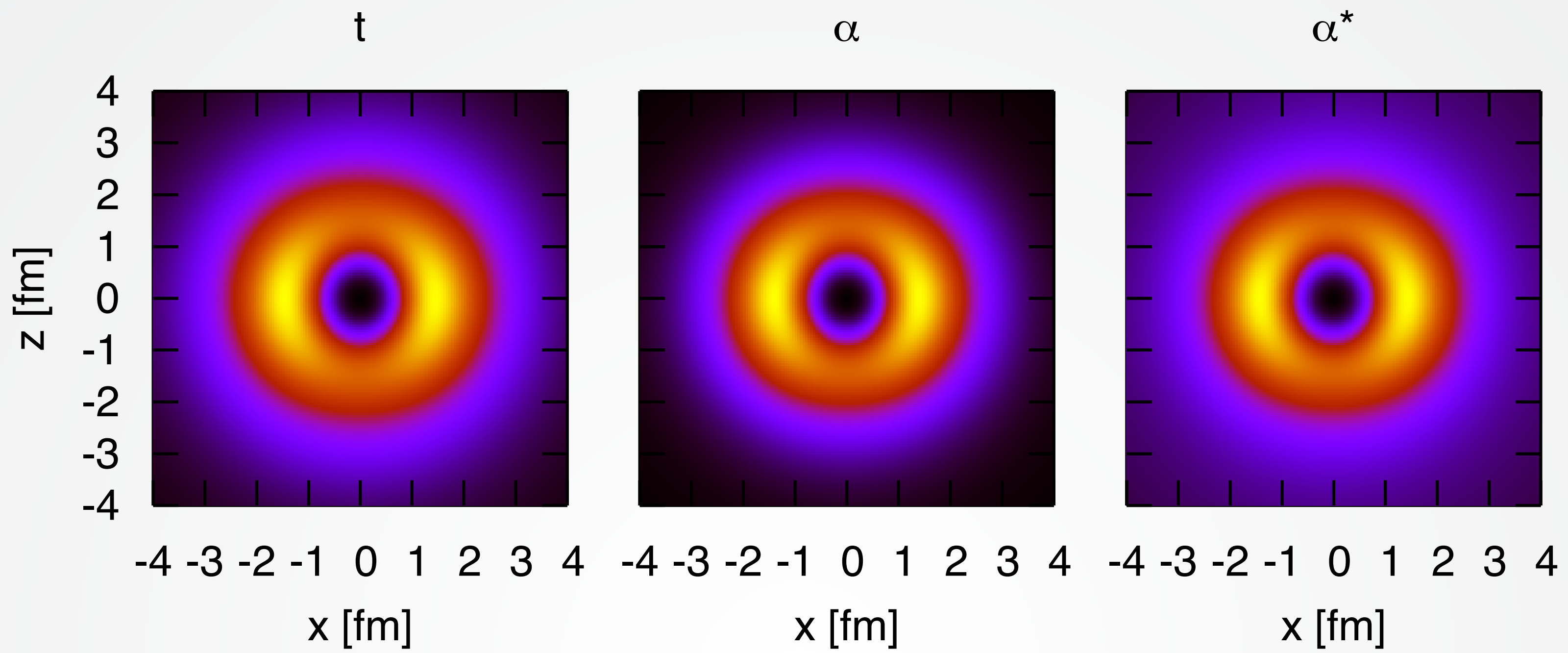
Two-body density for S=0, T=1 pairs



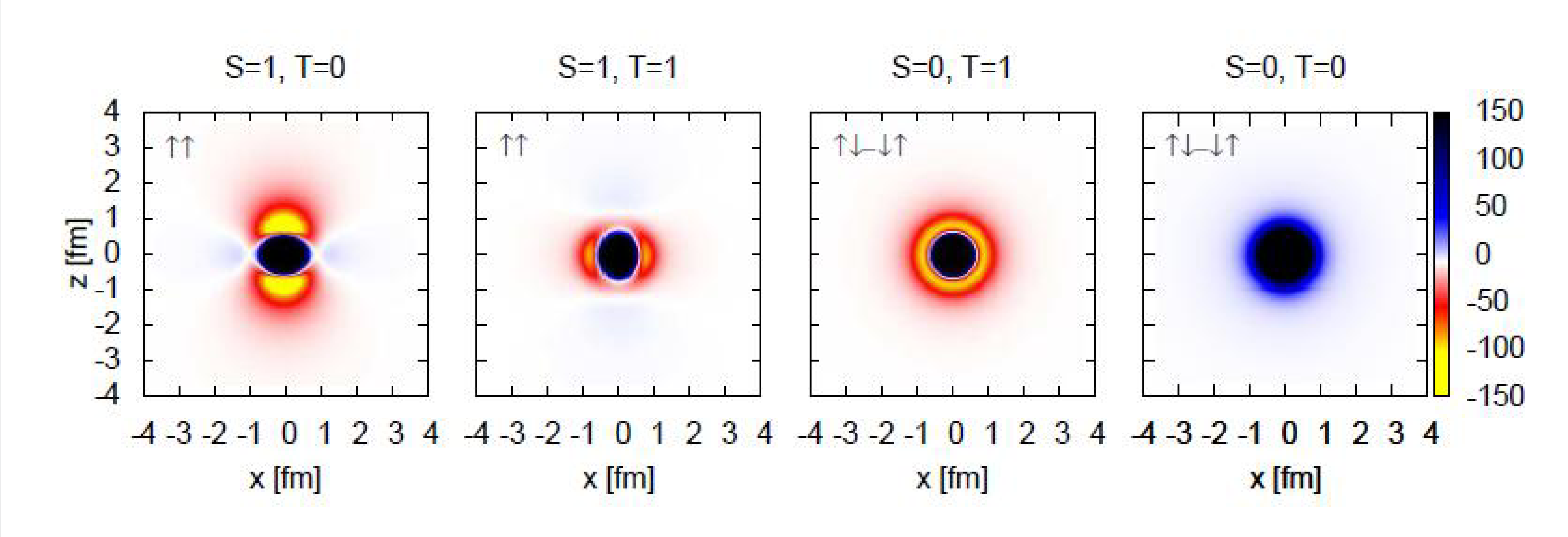
- Potential leaves one-to-one imprint on density



Two-body density for S=1, M_S=1, T=1 pairs



- Potential leaves one-to-one imprint on density

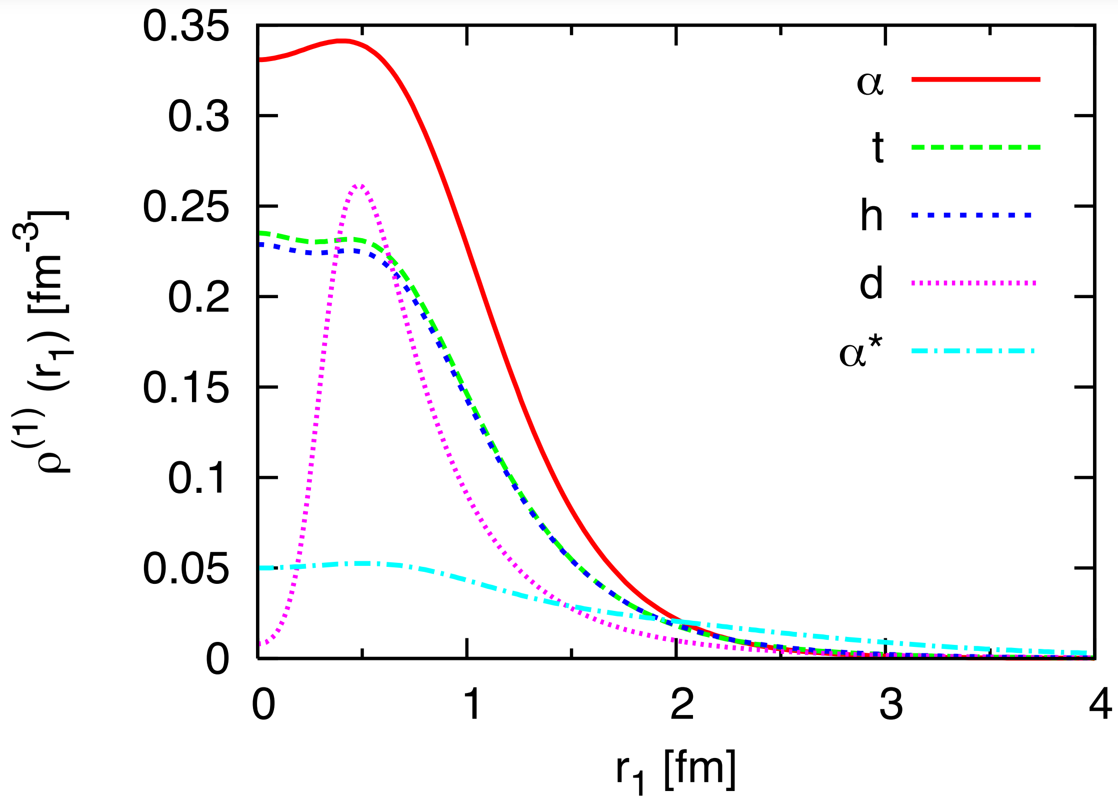


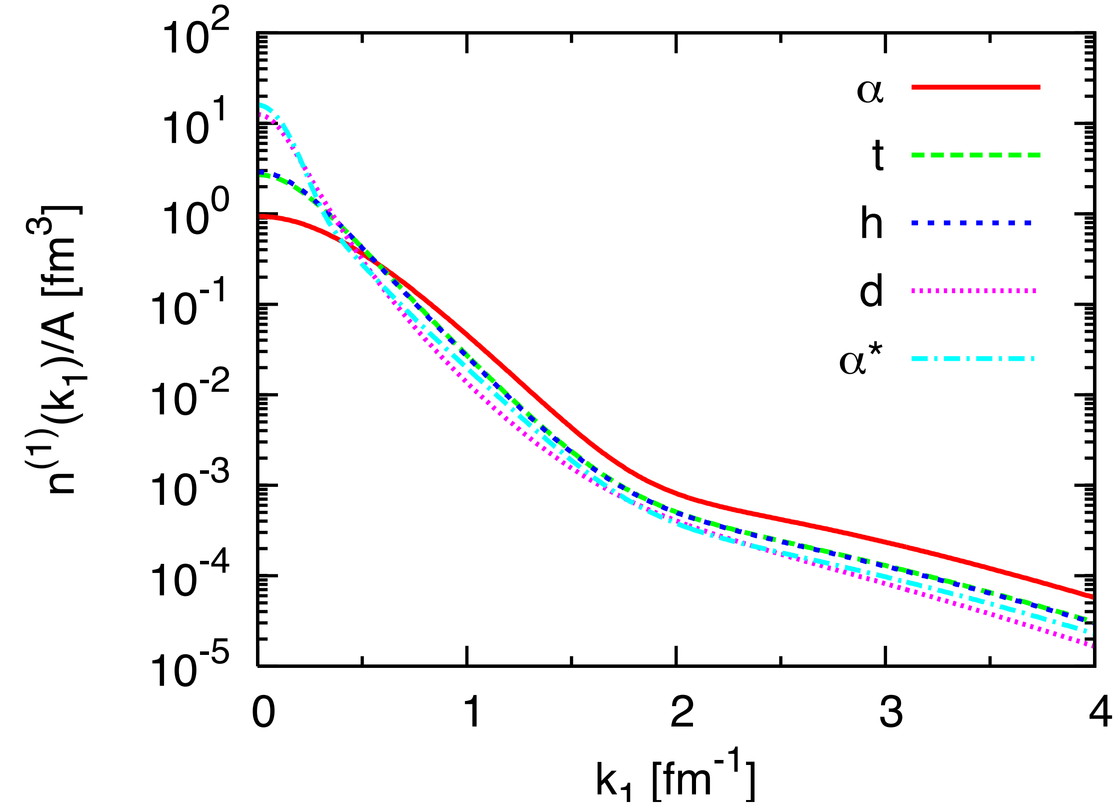
Universality of short-range correlations

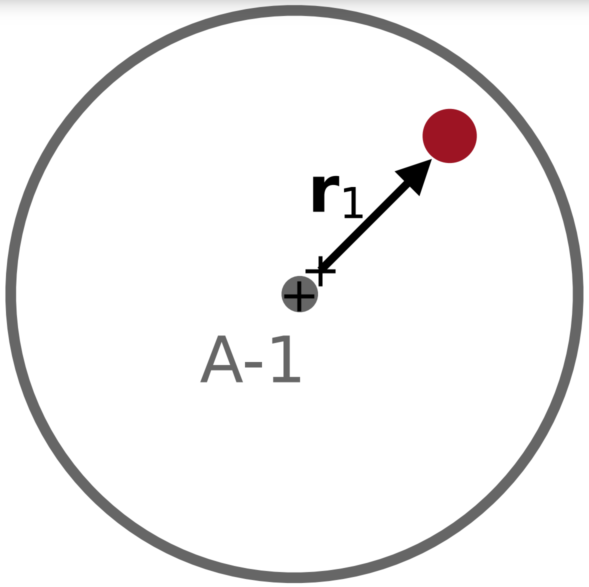
Exact solutions for A=2,3,4 nuclei with AV8’ interaction

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

One-Body Densities for *A*=2,3,4 Nuclei







$$\rho^{(1)}(\mathbf{r}_1) = \left\langle \Psi \middle| \sum_{i=1}^{A} \delta^3(\hat{\mathbf{r}}_i - \mathbf{r}_1) \middle| \Psi \right\rangle$$

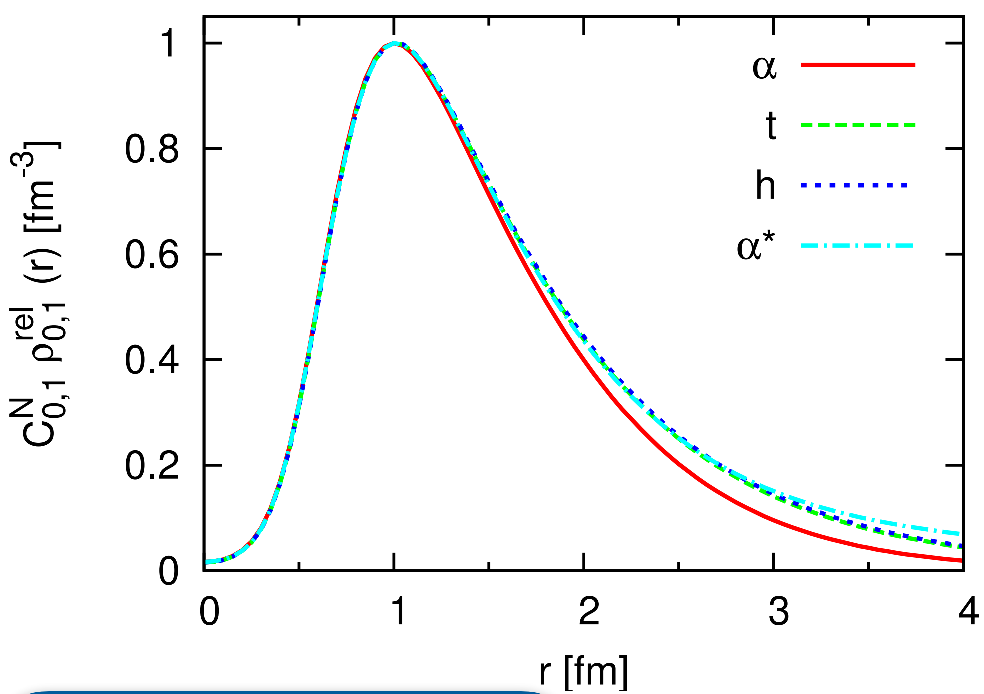
$$n^{(1)}(\mathbf{k}_1) = \left\langle \Psi \middle| \sum_{i=1}^{A} \delta^3(\hat{\mathbf{k}}_i - \mathbf{k}_1) \middle| \Psi \right\rangle$$

- One-body densities calculated from **exact wave functions** (Correlated Gaussian method) for AV8’ interaction
- coordinate space densities reflect different sizes and densities of ^{2}H, ^{3}H, ^{3}He, ^{4}He and the excited 0^+ state in ^{4}He
- similar high-momentum tails in the one-body momentum distributions

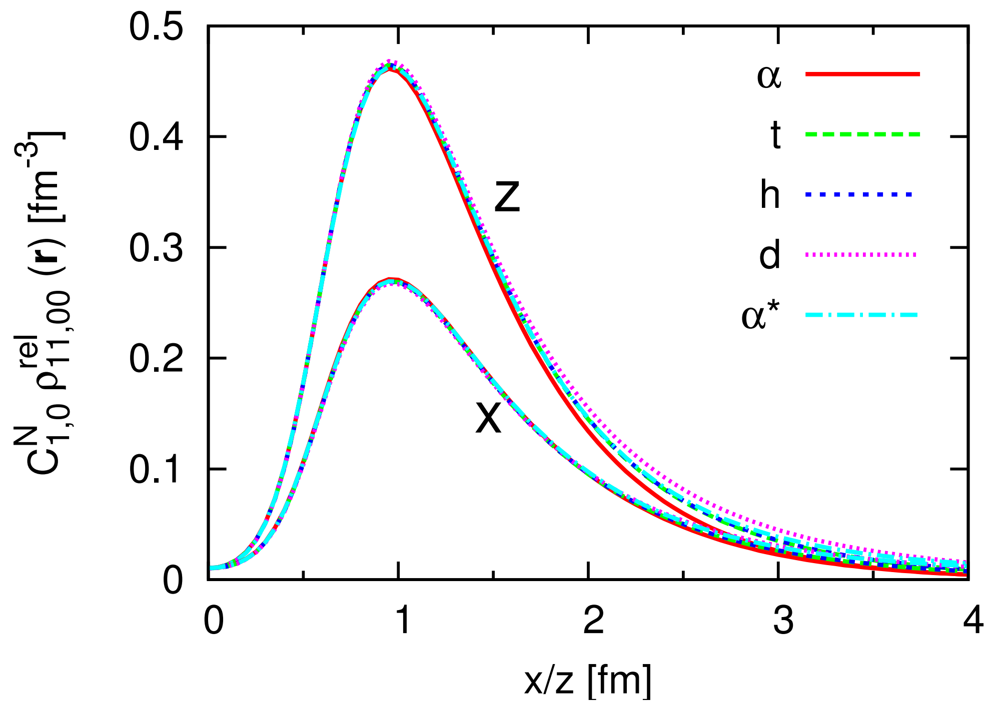
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

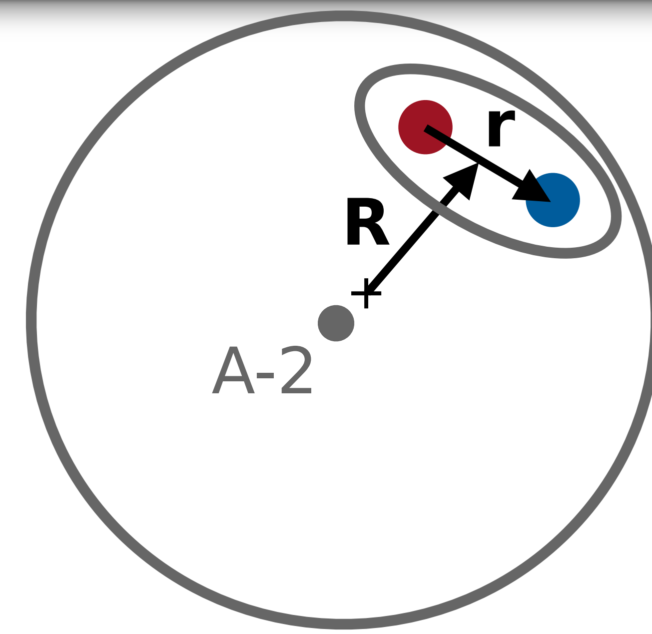
Two-Body Coordinate Space Densities

S=0, T=1



S=1, M_S=+1, T=0





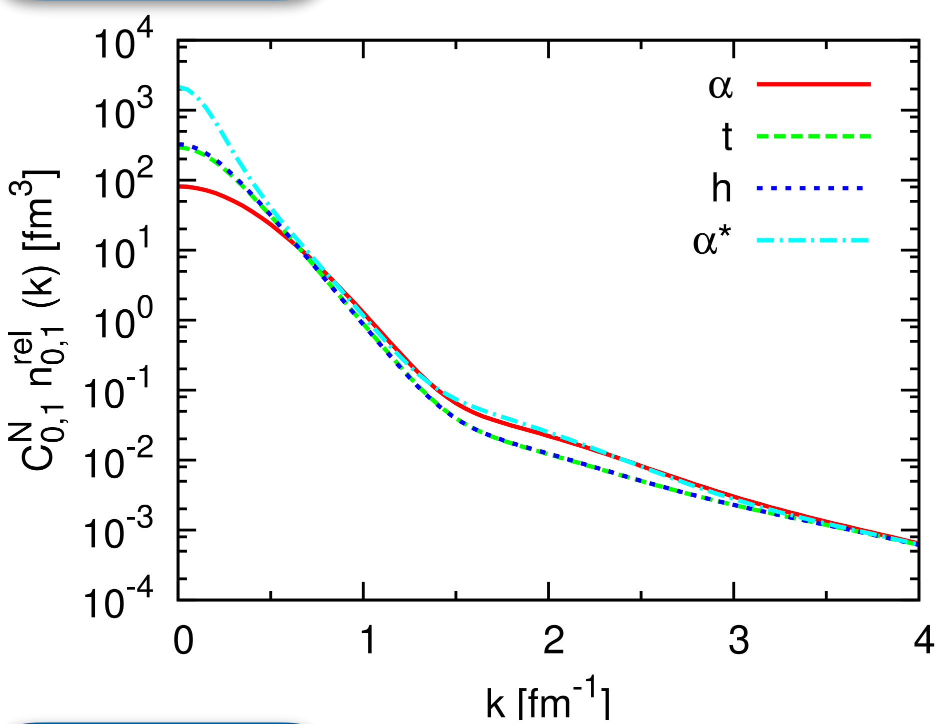
$$\rho^{rel}_{SM_S,TM_T}(\mathbf{r}) = \left\langle \Psi \right| \sum_{i<j}^{A} \hat{P}^{SM_S}_{ij} \hat{P}^{TM_T}_{ij} \delta^3(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j - \mathbf{r}) \left| \Psi \right\rangle$$

- coordinate space two-body densities show correlation hole and tensor correlations
- **normalize** two-body density in coordinate space at ***r*=1.0 fm**
- normalized two-body densities in coordinate space are **identical at short distances** for all nuclei
- also true for angular dependence in the deuteron channel

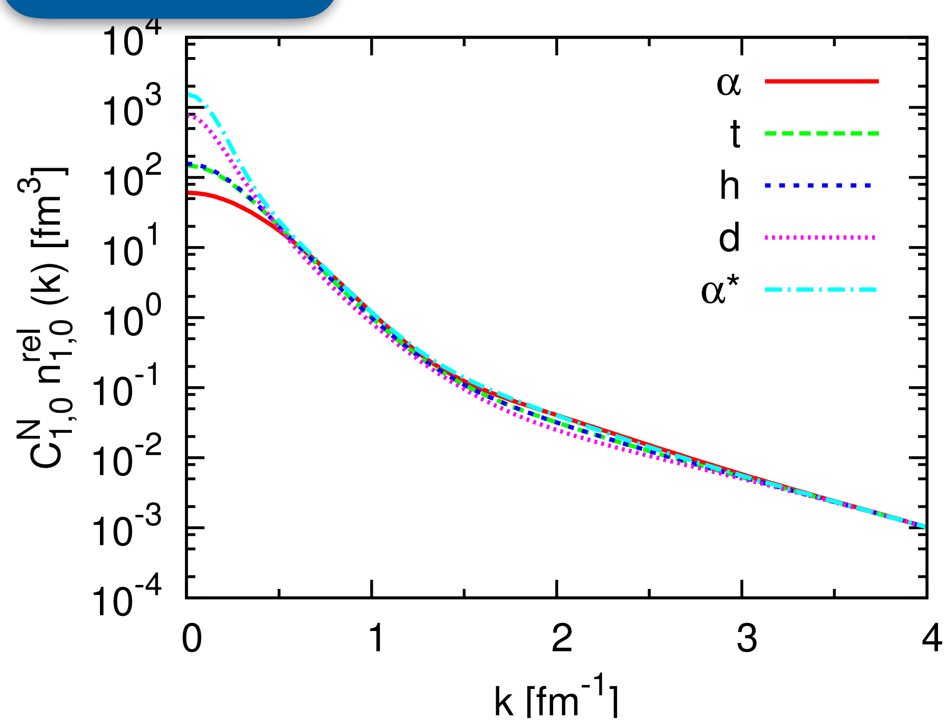
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

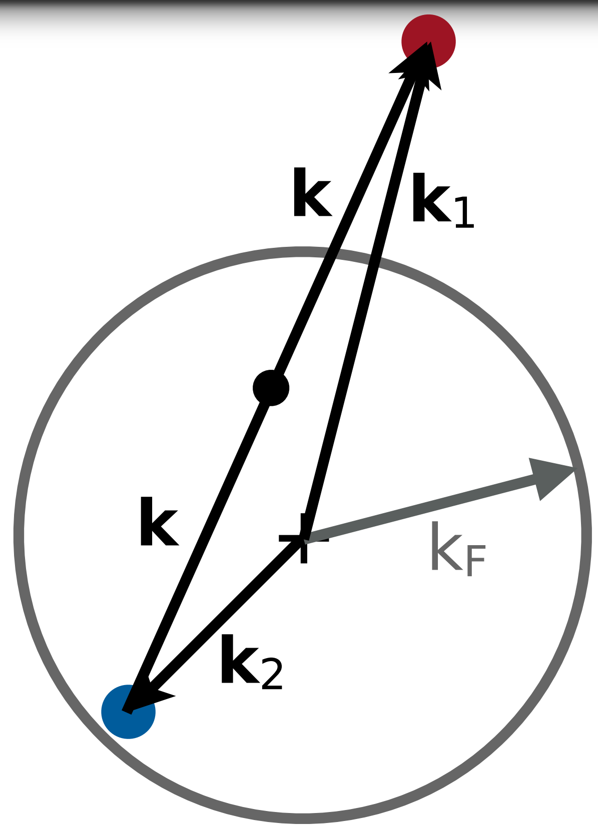
Two-Body Momentum Space Densities

$S{=}0,\ T{=}1$



$S{=}1,\ T{=}0$





$$n^{\mathrm{rel}}_{SM_S,TM_T}(\mathbf{k}) = \langle \Psi | \sum_{i<j}^{A} \hat{P}^{SM_S}_{ij} \hat{P}^{TM_T}_{ij} \delta^3\big(\tfrac{1}{2}(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_j) - \mathbf{k}\big) | \Psi \rangle$$

- use normalization factors fixed in coordinate space
- two-body densities in momentum space agree for momenta $k > 3\ \mathrm{fm}^{-1}$
- moderate nucleus dependence in momentum region $1.5\ \mathrm{fm}^{-1} < k < 3\ \mathrm{fm}^{-1}$

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Many-Body Correlations show up in 2-Body Density

number of pairs in ST channels

^{4}He

(ST)	(10)	(01)	(11)	(00)
L	even	even	odd	odd
exact AV8’	2.992	2.572	0.428	0.008
$(s_{1/2})^4$	3.000	3.000	0	0

- (ST)=(01) with L even gives away 0.428 pairs to (ST)=(01) with L odd. Why?
- odd channel is less attractive
- V_{NN} does not scatter from even to odd

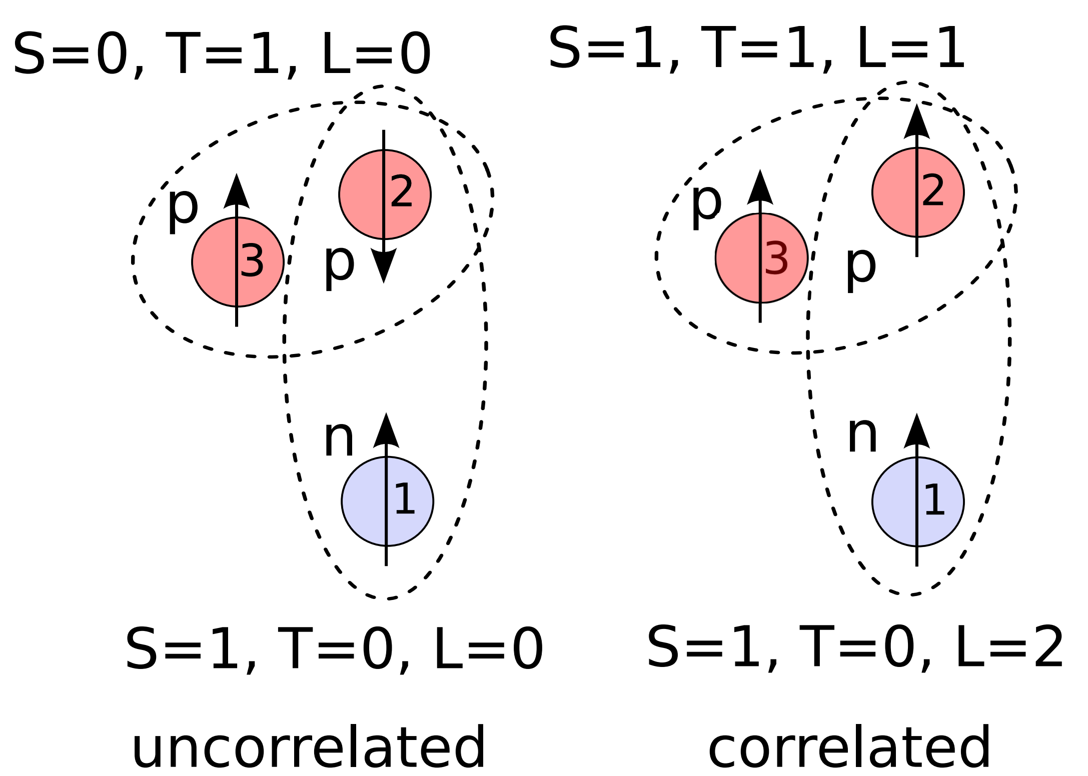
Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Many-Body Correlations show up in 2-Body Density

number of pairs in ST channels

^{4}He

(ST)	(10)	(01)	(11)	(00)
L	even	even	odd	odd
exact AV8'	2.992	2.572	0.428	0.008
$(s_{1/2})^4$	3.000	3.000	0	0



- (ST)=(01) with L even gives away 0.428 pairs to (ST)=(01) with L odd. Why?
- odd channel is less attractive
- V_{NN} does not scatter from even to odd

Answer: 3-body correlations

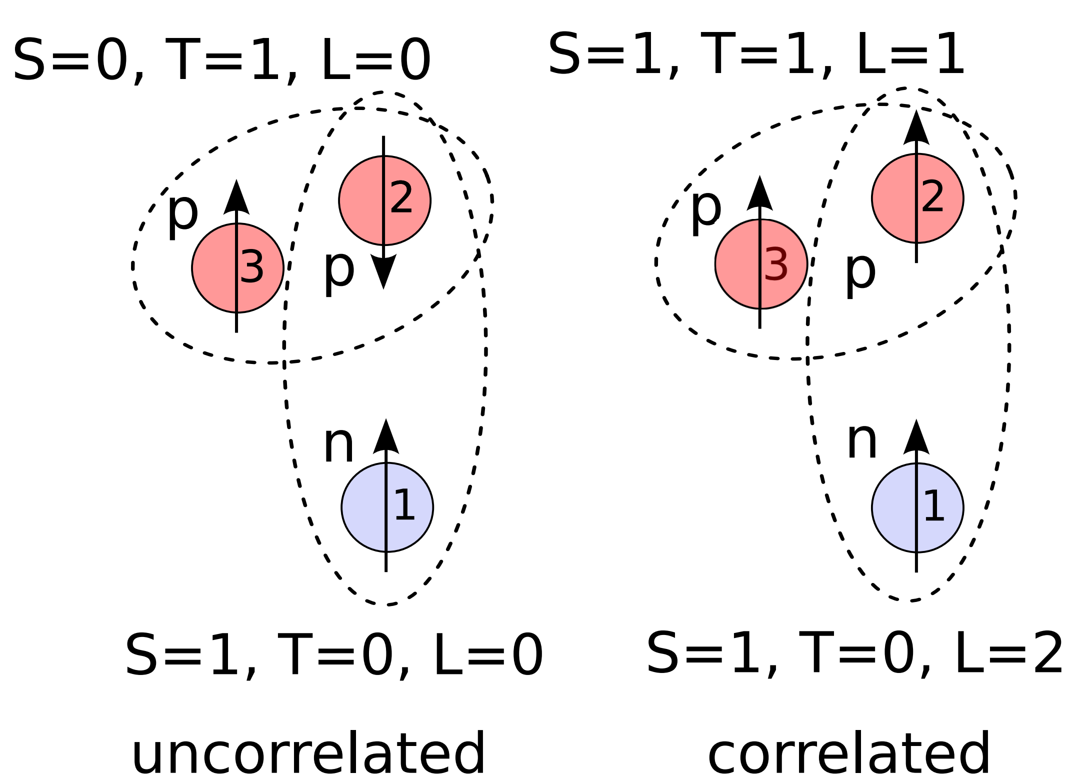
- strong tensor breaks pair {2,3} with (ST)=(01) and aligns spin of proton {2} to get pair {1,2} in (ST)=(10)
- pair {2,3} is left in (ST)=(11)
- energy paid by moving pair from (ST)=(01) channel to (ST)=(11), but more energy gained by pair in (ST)=(10) channel
- 3-body correlations induced by the 2-body tensor force

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Many-Body Correlations show up in 2-Body Density

number of pairs in ST channels

(ST) L	(10) even	(01) even	(11) odd	(00) odd
d	1	-	-	-
t	1.490	1.361	0.139	0.010
h	1.489	1.361	0.139	0.011
α	2.992	2.572	0.428	0.008
α*	2.966	2.714	0.286	0.034



★ Similar 3-body correlations in ^{3}H, ^{3}He, less pronounced in ^{4}He* (^{3}H+p,^{3}He+n cluster structure)

Feldmeier, Horiuchi, Neff, Suzuki, Phys. Rev. C **84**, 054003 (2011)

Why does Nuclear Shell Model work?

★ Apparent problem:

- Nuclear shell model with nucleons moving independently in mean-field works quite well (Goeppert-Mayer, Jensen Nobel price)
- Slater determinant |Φ> can not describe the short range correlations we just saw.
- <Φ|V_{NN}| Φ> is positive and large, should be negative for self-bound system!

Why does Nuclear Shell Model work?

- Independent of nucleus or density two-body correlations are much alike for $r < 1$ fm
- when two nucleons come closer than 1 fm their pairwise interactions dominates

correlation distance	D_{corr}	1 fm
mean distance at saturation	d_{mean}	1.8 fm
diameter of proton	$2\ R_{proton}$	1.6 fm

- probability to find 3rd nucleon in correlation volume is small
  $(D_{corr}/2)^3 \times \rho_0 = 0.125\ \mathrm{fm}^3 \times 0.16\ \mathrm{fm}^{-3} = 0.08$
  With respect to SRC nucleons form a dilute system,
  SRC essentially of 2-body nature

★ Idea: Universal Similarity transformation for pairs k,l
schematic: $\Psi'(r_{kl})\ V_{NN}(r_{kl})\ \Psi(r_{kl}) \rightarrow \varphi'(r_{kl})\ V_{eff}(r_{kl})\ \varphi(r_{kl})$ for $r_{kl} < 1$ fm
$V_{eff}(r_{kl})$ shell model interaction
$\varphi(r_{kl})$ shell model states

Summary - 1

- NN-interaction causes tensor and central-repulsive short range correlations (SRC)
- For S=1, T=0 proton-neutron pairs align their distance vector ***r*** and spin ***S*** (tensor) (like regular bar magnets)
- For all S,T channels very strong repulsion for $0 < r < 0.5$ fm (central)
- One-to-one correspondence between NN potential and 2-body SRC correlations (like a cast and its molding form)
- For **r < 1** fm two-body SRC are much alike, independent of nucleus or density
- For **r < 1** fm their pairwise interactions dominates

- 1-body $\boldsymbol{n(k_1)}$: 2-body SRC give raise to high momentum tails
- 1-body $\rho(\boldsymbol{r_1})$ insensitive to 2-body SRC
- 2-body $\boldsymbol{n^{rel}(k=(k_1-k_2)/2)}$ and $\rho^{rel}(\boldsymbol{r=r_1-r_2})$ reveal details of SRC

SRG=Short Range Correlations

Short-range correlations in nuclei using No-Core Shell Model and SRG

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

Unitary Transformations

- Many-body problem very hard to solve for bare interaction
- Unitary trafo of bare → soft Hamiltonian, evolution parameter α
- $\hat{U}_\alpha$ imprints correlations of $|\Psi>$ into mean-field like state $|\Phi_\alpha>$
- Equivalent description, pre-diagonalization

$$\hat{H}|\Psi> = (\hat{T} + \hat{V}_{NN} + \hat{V}_{NN})|\Psi> = E|\Psi>$$

$$\hat{H}_\alpha = \hat{U}_\alpha^\dagger \hat{H} \hat{U}_\alpha \quad , \quad \hat{U}_\alpha^\dagger = \hat{U}_\alpha^{-1}$$

$$|\Psi> = \hat{U}_\alpha |\Phi_\alpha>$$

$$<\Psi'|\hat{H}|\Psi> = <\Phi'_\alpha|\hat{H}_\alpha|\Phi_\alpha>$$

$$<\Psi'|\hat{B}|\Psi> = <\Phi'_\alpha|\hat{B}_\alpha|\Phi_\alpha>$$

★ Goal: find $\hat{U}_\alpha$ such that $|\Phi_\alpha>$ looses high momentum components with evolving α

Similarity Renormalization Group

- SRG provides a family of similarity transformations depending on a flow parameter α
- Evolve Hamiltonian and unitary transformation matrix (in momentum space)

$$\frac{d\hat{H}_\alpha}{d\alpha} = [\hat{\eta}_\alpha, \hat{H}_\alpha]_- \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- generator for the evolution

$$\hat{\eta}_\alpha = (2\mu)^2 \, [\hat{T}, \hat{H}_\alpha]_-$$

- Intrinsic kinetic energy as meta-generator (other choices possible, but that does the job)

α=0:
bare interaction
fully correlated wave function

↓

α large:
soft interaction, many-body forces
mean-field like wave function

✱ soft Hamiltonian $\hat{H}_\alpha$ is now a A-body operator !

$$\hat{H}_\alpha = \hat{T} + \hat{V}_\alpha^{[2]} + \hat{V}_\alpha^{[3]} + \hat{V}_\alpha^{[4]} + \cdots + \hat{V}_\alpha^{[A]}$$

Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)
Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

Similarity Renormalization Group

★ Two-body approximation for many-body calculations used in following calculations

- Evolution is done only on the **2-body level**
  α-dependence can be used to investigate the role of missing higher-order contributions

$$\frac{d\hat{H}_\alpha}{d\alpha} = [\hat{\eta}_\alpha, \hat{H}_\alpha]_- \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\,\hat{U}_\alpha\,\hat{\eta}_\alpha$$

$$\hat{\eta}_\alpha = (2\mu)^2\;[\,\hat{T}, \hat{H}_\alpha]_-$$

- 1-body observables

$$\hat{B}_\alpha = \hat{U}^\dagger_\alpha\,\hat{B}\;\hat{U}_\alpha = \hat{B} + \hat{B}^{[2]}_\alpha$$

- 2-body observables

$$\hat{C}_\alpha = \hat{U}^\dagger_\alpha\,\hat{C}\;\hat{U}_\alpha = \hat{C}^{[2]}_\alpha$$

$$\hat{H}_\alpha = \hat{U}^\dagger_\alpha\,\hat{H}\;\hat{U}_\alpha = \hat{T} + \hat{V}^{[2]}_\alpha$$

- Hamiltonian evolution can nowadays be done on the 3-body level

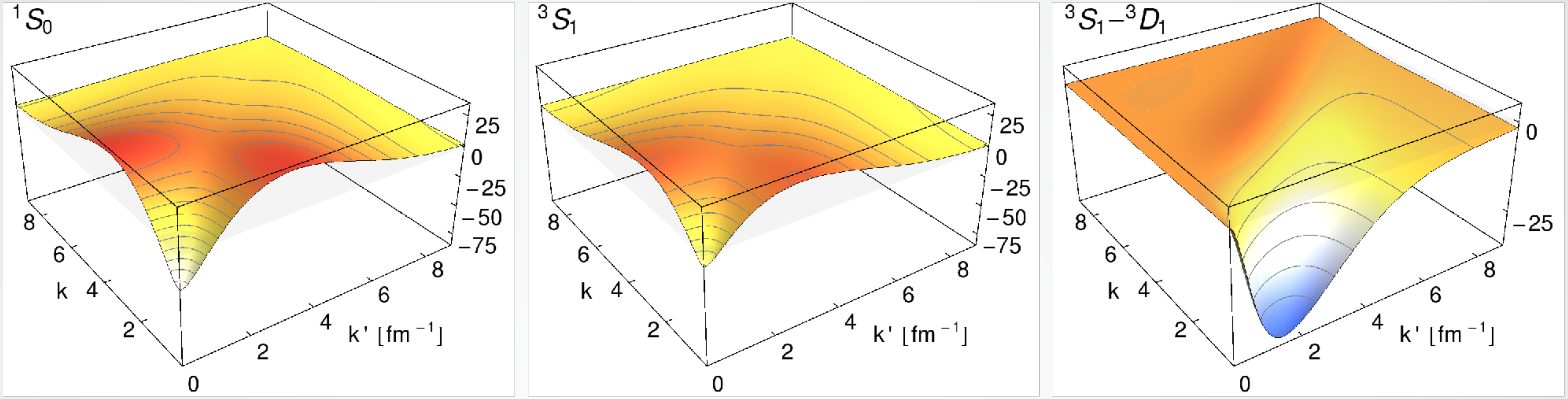
(Jurgenson, Roth, Hebeler, . . .)

Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)

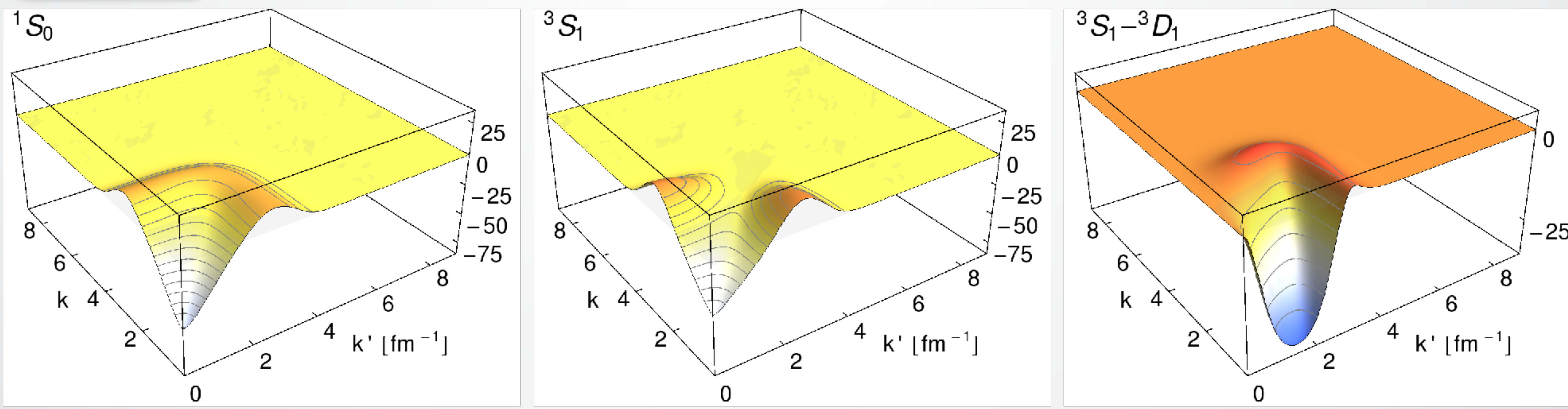
Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

Similarity Renormalization Group

AV8′



N3LO

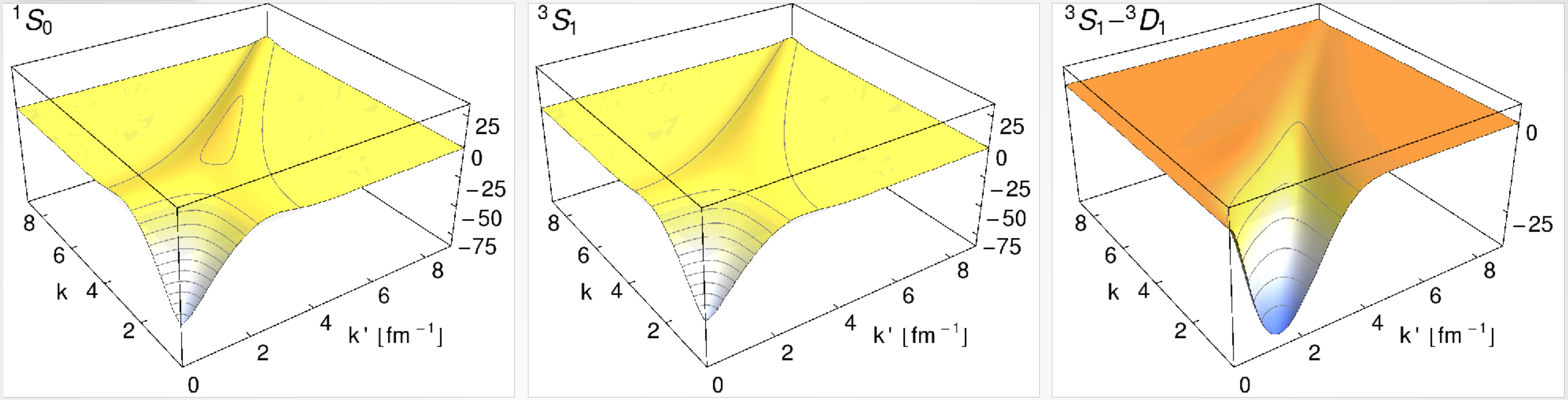


α=0.00 fm⁴

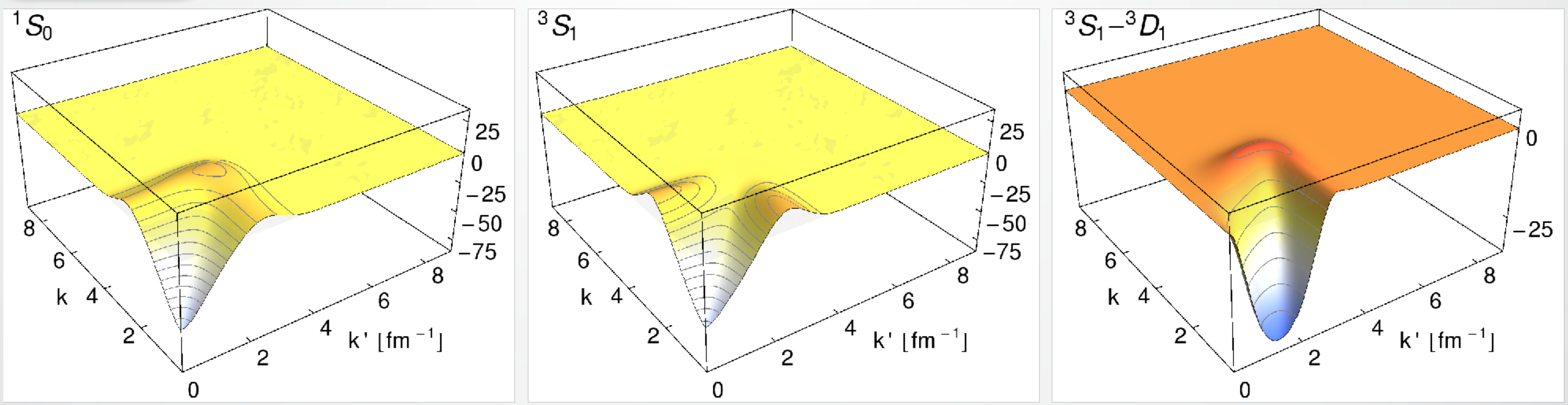
$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

Similarity Renormalization Group

AV8′



N3LO

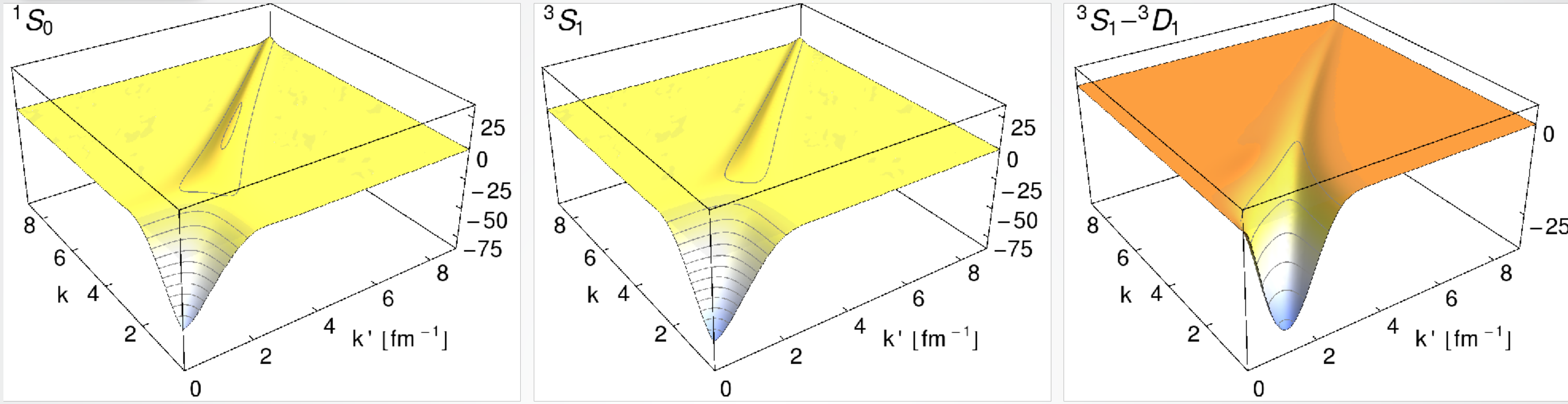


$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

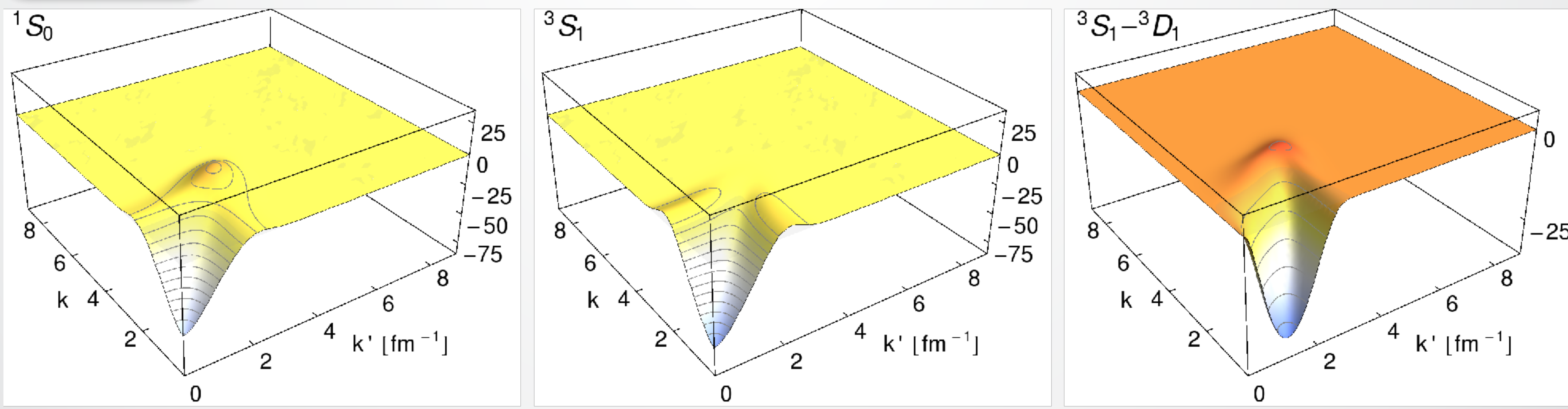
α=0.01 fm⁴

Similarity Renormalization Group

AV8′



N3LO

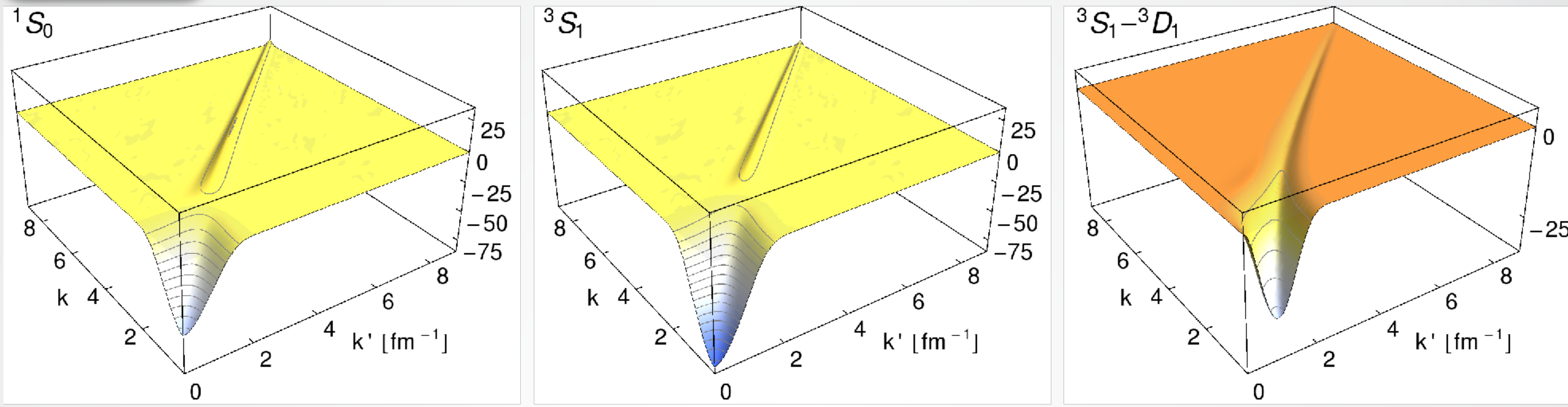


$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

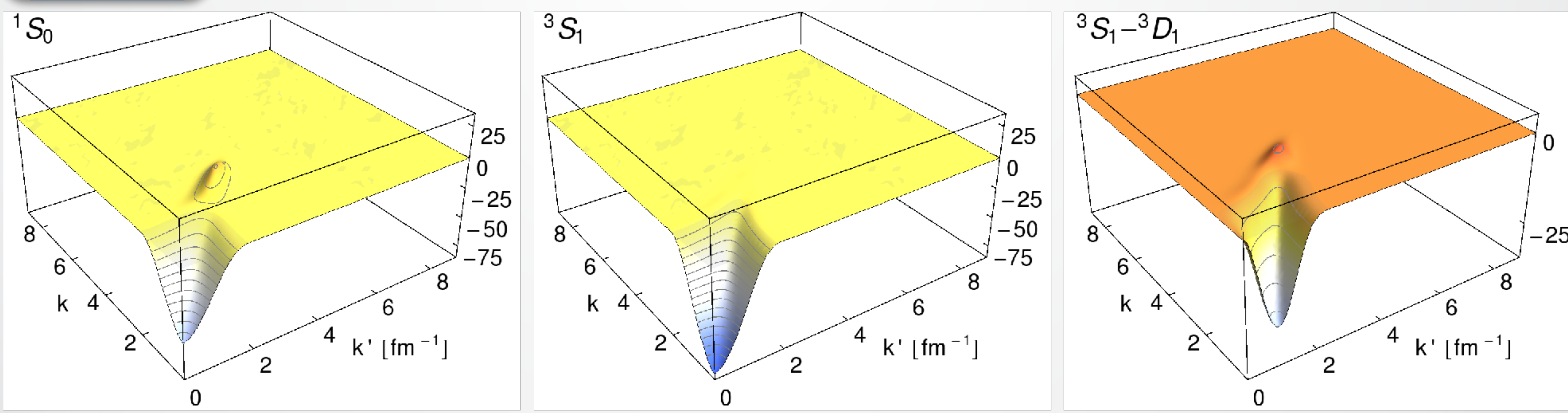
$\alpha = 0.04\ \mathrm{fm}^4$

Similarity Renormalization Group

AV8’



N3LO



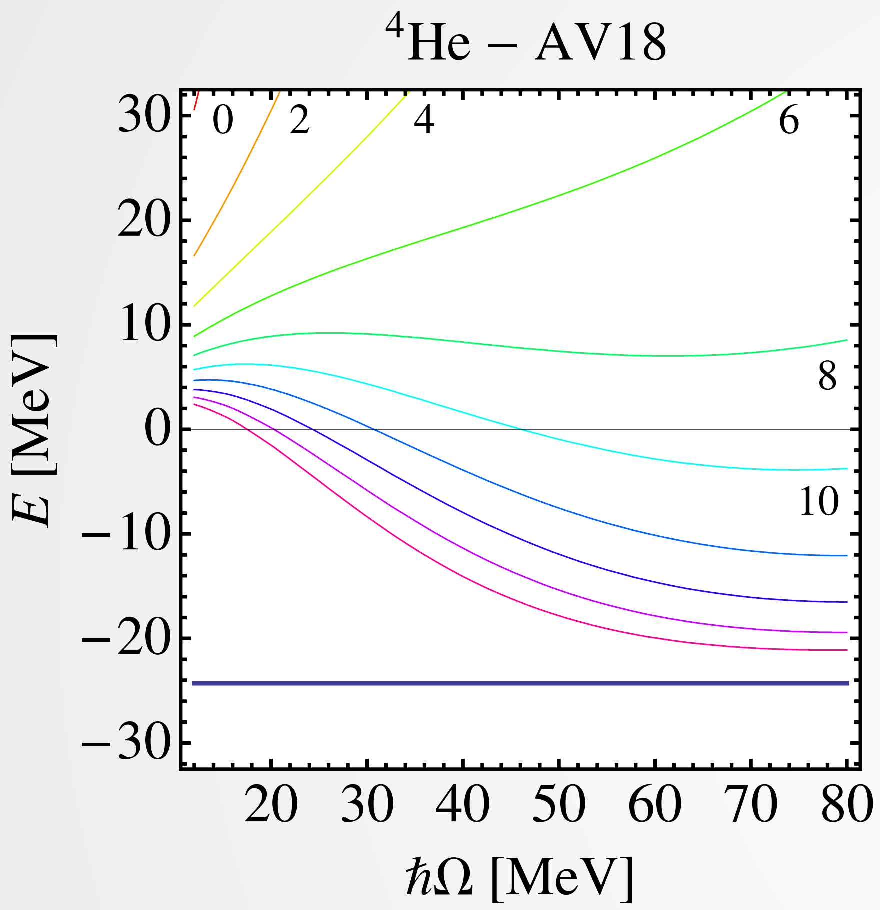
$$V_{(LL'S)J}(k,k') = \langle k(LS)J | \hat{V} | k'(L'S)J \rangle$$

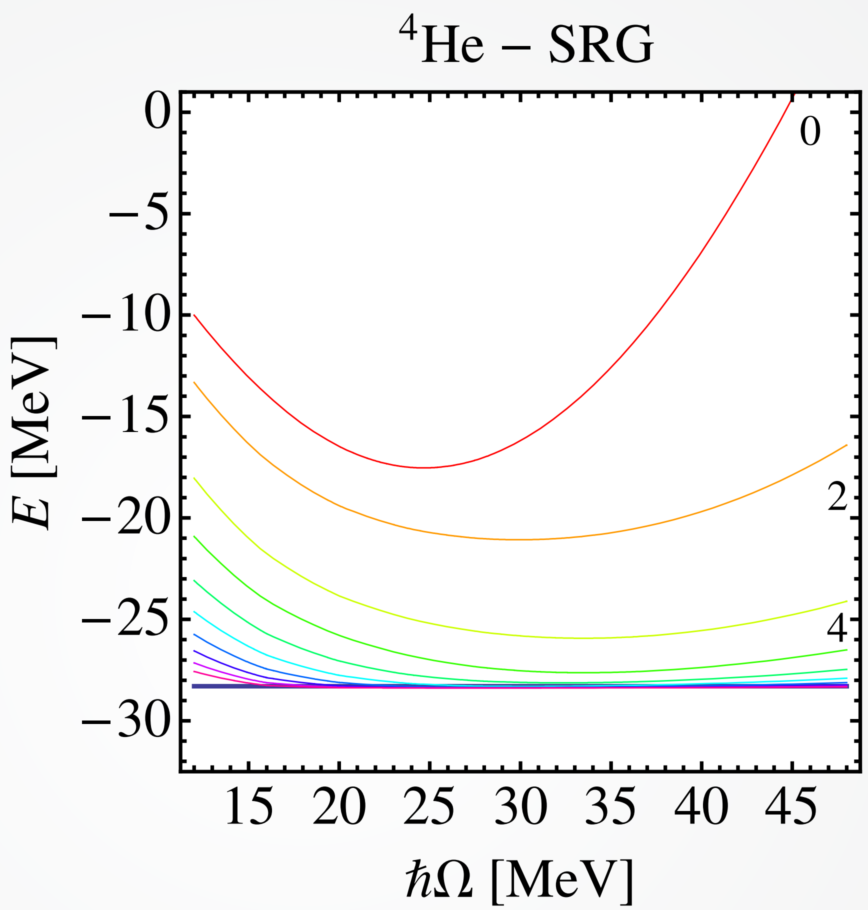
α=0.20 fm⁴

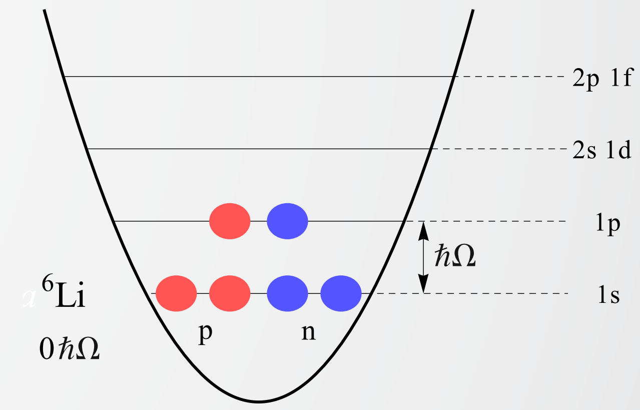
Convergence in No-Core Shell Model

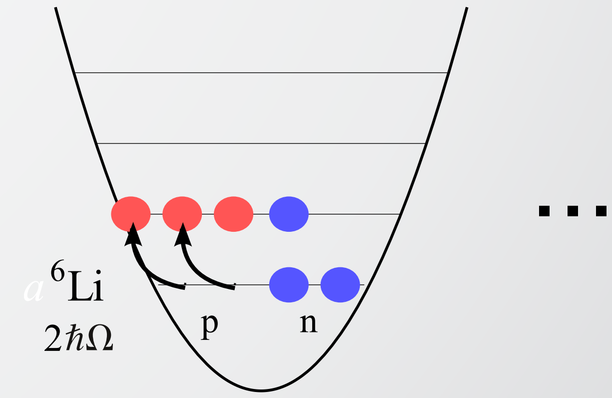
bare interaction

SRG (α=0.03 fm⁴)









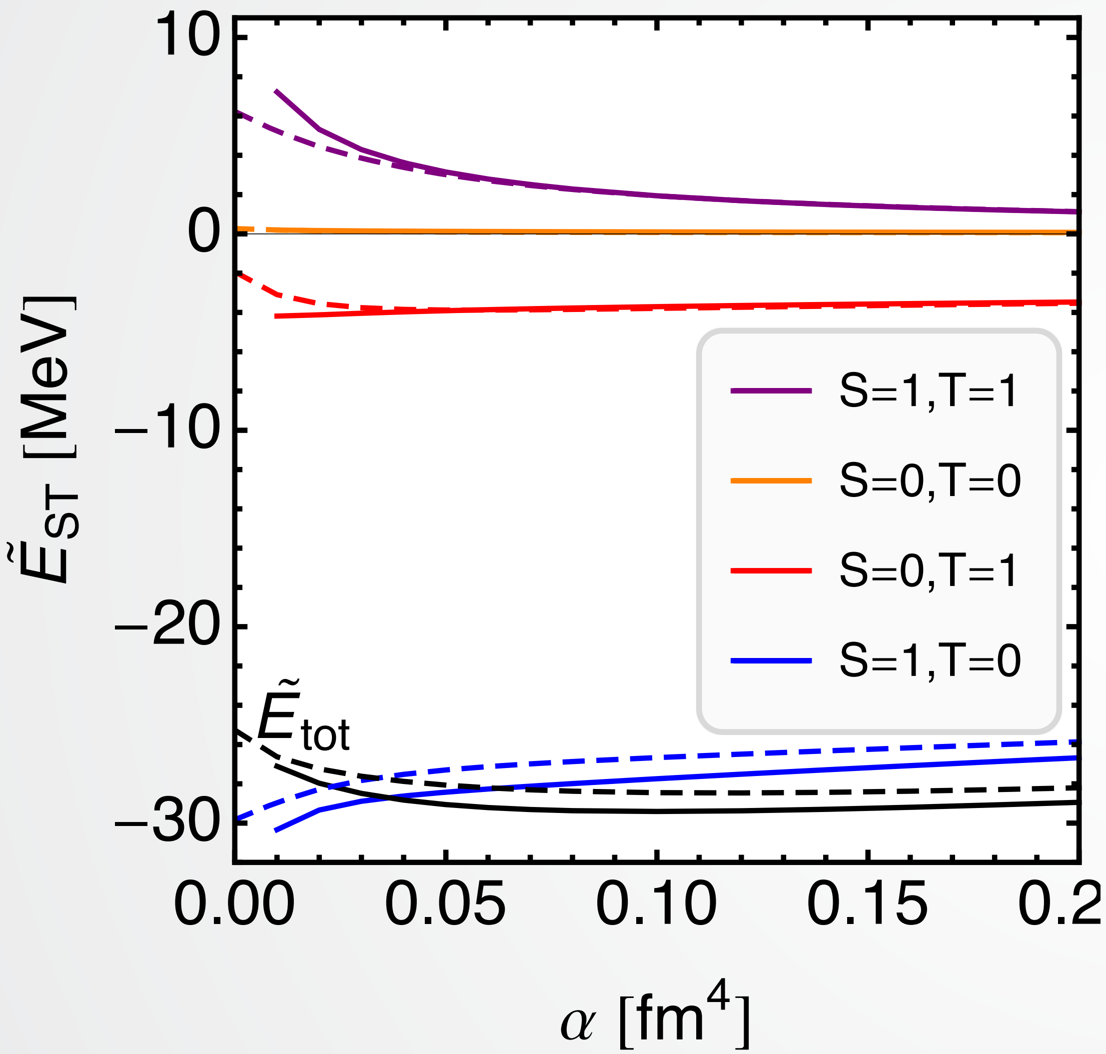
No-Core Shell Model (NCSM)

- Diagonalization of Hamiltonian in harmonic oscillator basis
- $N\ \hbar\Omega$ configuration: N oscillator quanta above 0 $\hbar\Omega$ configuration
- Model space sizes grow rapidly with A and N_{max}

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

Contributions to the binding energy

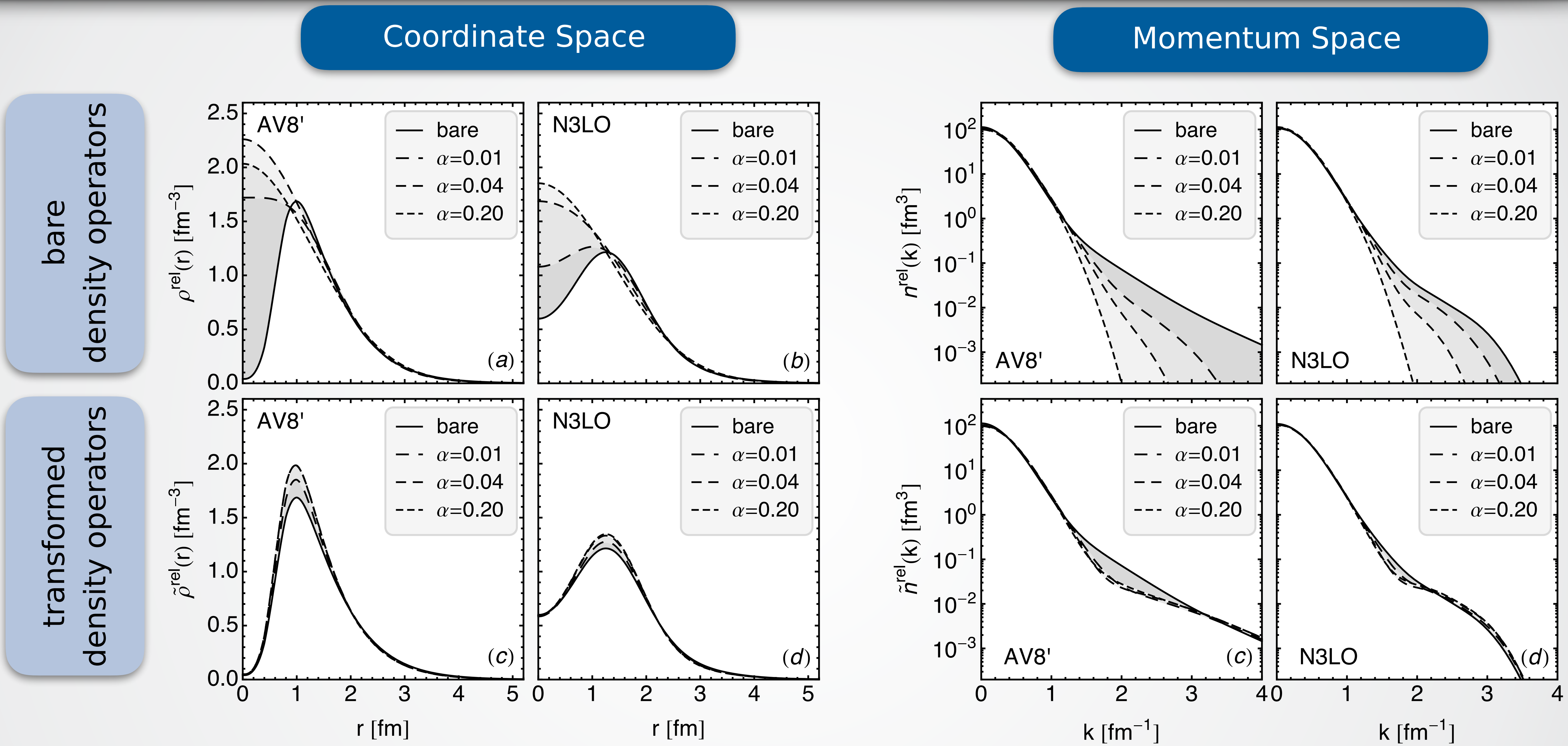
solid: AV8’, dashed: N3LO



- Energy depends slightly on flow parameter — indicates missing three-body terms in effective Hamiltonian
- Binding energy dominated by (ST)=(10) channel, contribution from tensor part of effective Hamiltonian decreases with flow parameter
- Sizeable repulsive contribution from odd (ST)=(11) channel related to many-body correlations — decreases with flow parameter

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

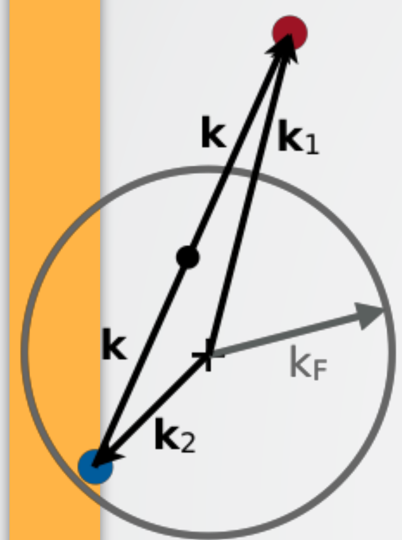
^{4}He: $\rho^{rel}(r)$ and $n^{rel}(k)$

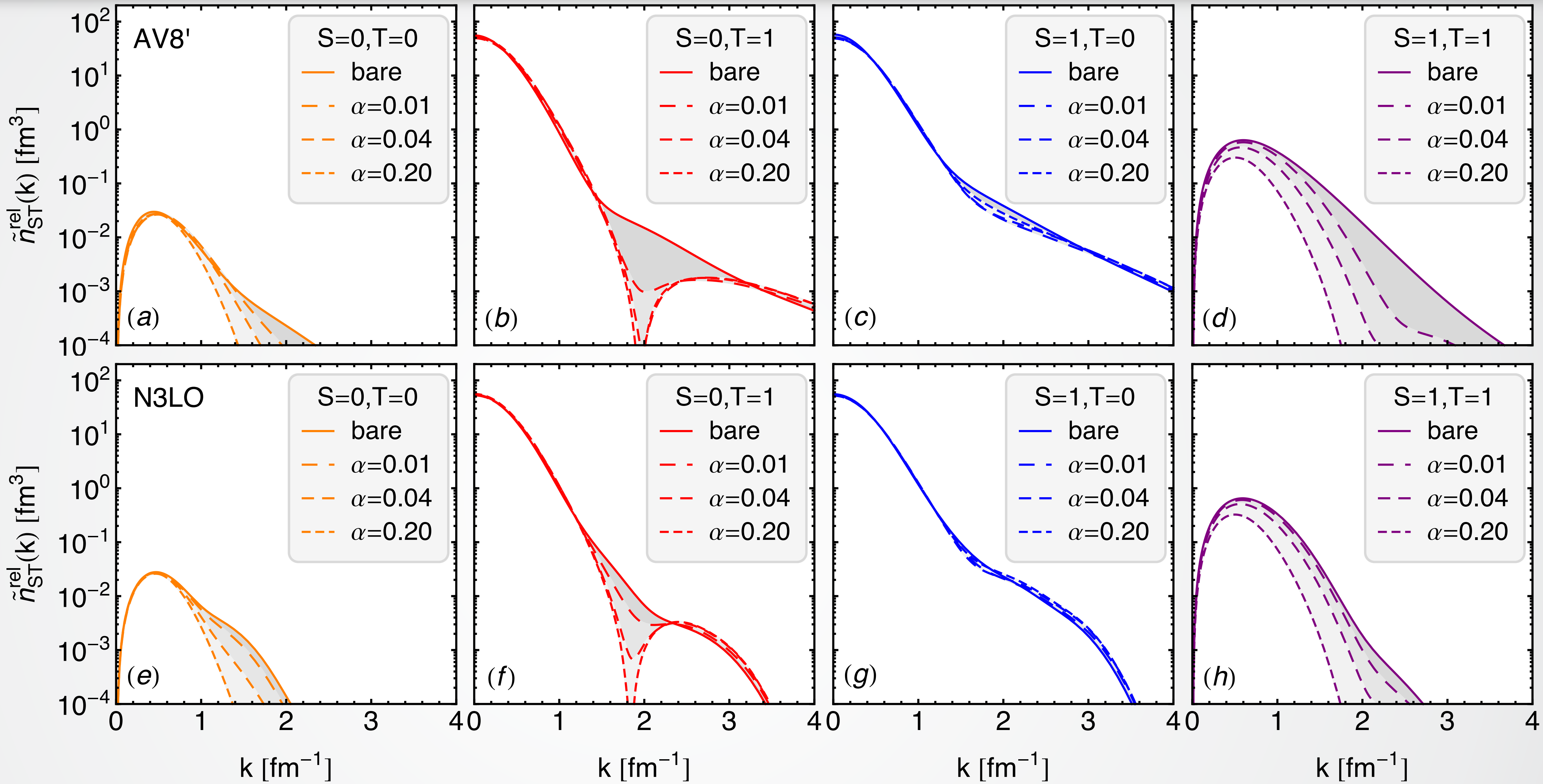


- SRG softens interaction - suppression at short distances and high-momentum components removed in wave function
- these features are recovered with SRG transformed density operators
- small but noticeable dependence on flow parameter α

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He: $n^{rel}_{ST}(k)$

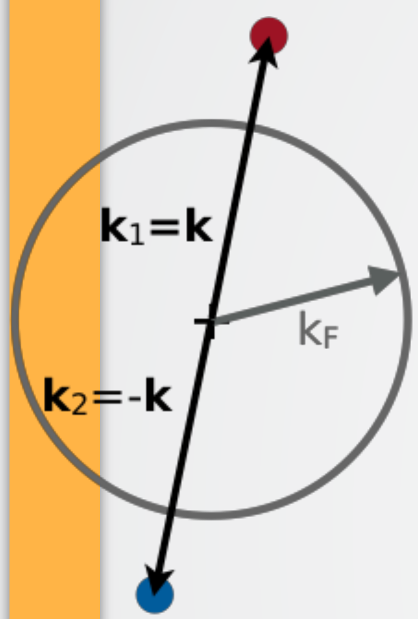


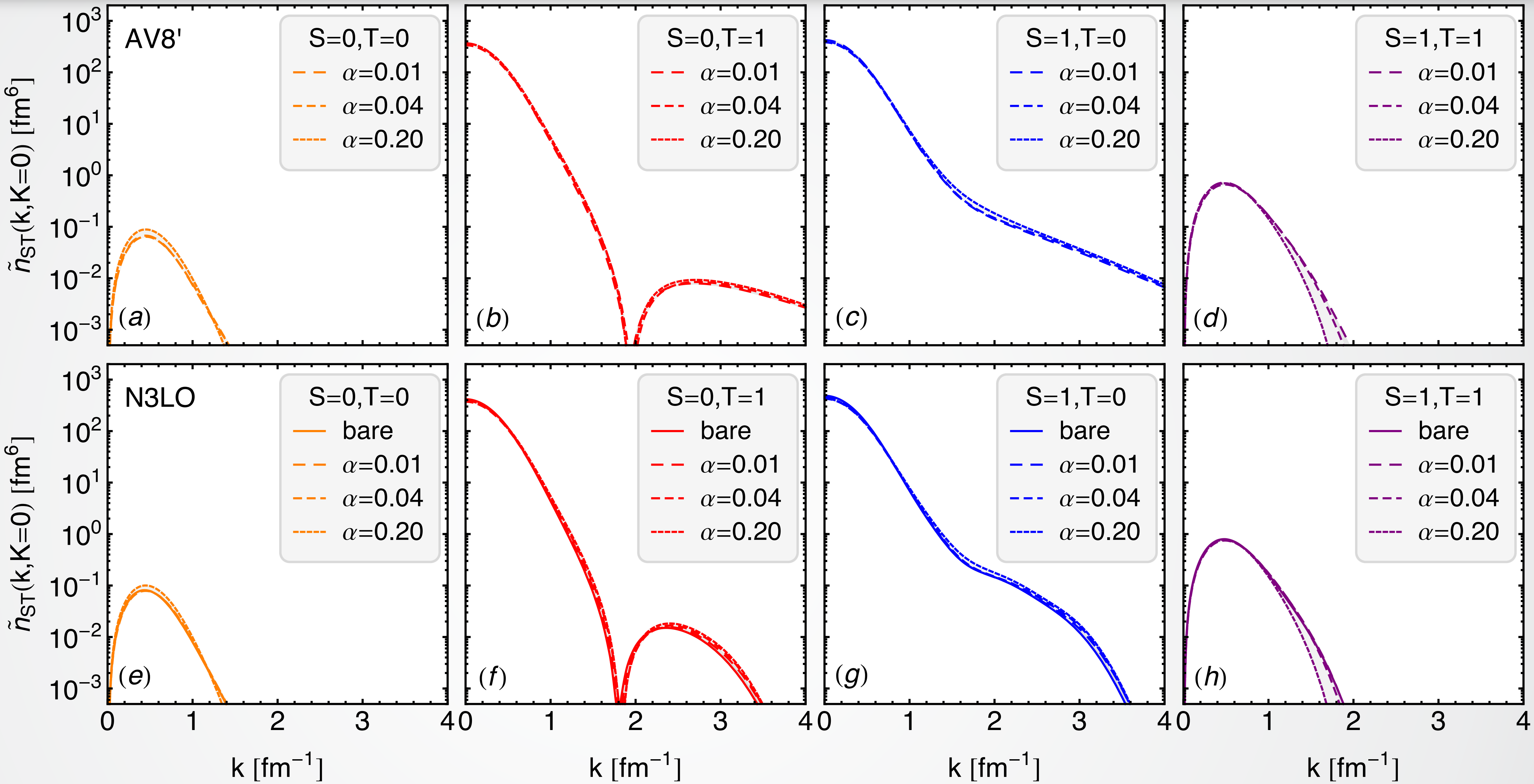


- high-momentum components much stronger in (ST)=(10) channel
- flow dependence is weak in (ST)=(10) channel
- flow dependence is strong in (ST)=(01) and (11) channels, especially for momenta above Fermi momentum — signal of many-body correlations

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He: n_{ST}(k, K=0)





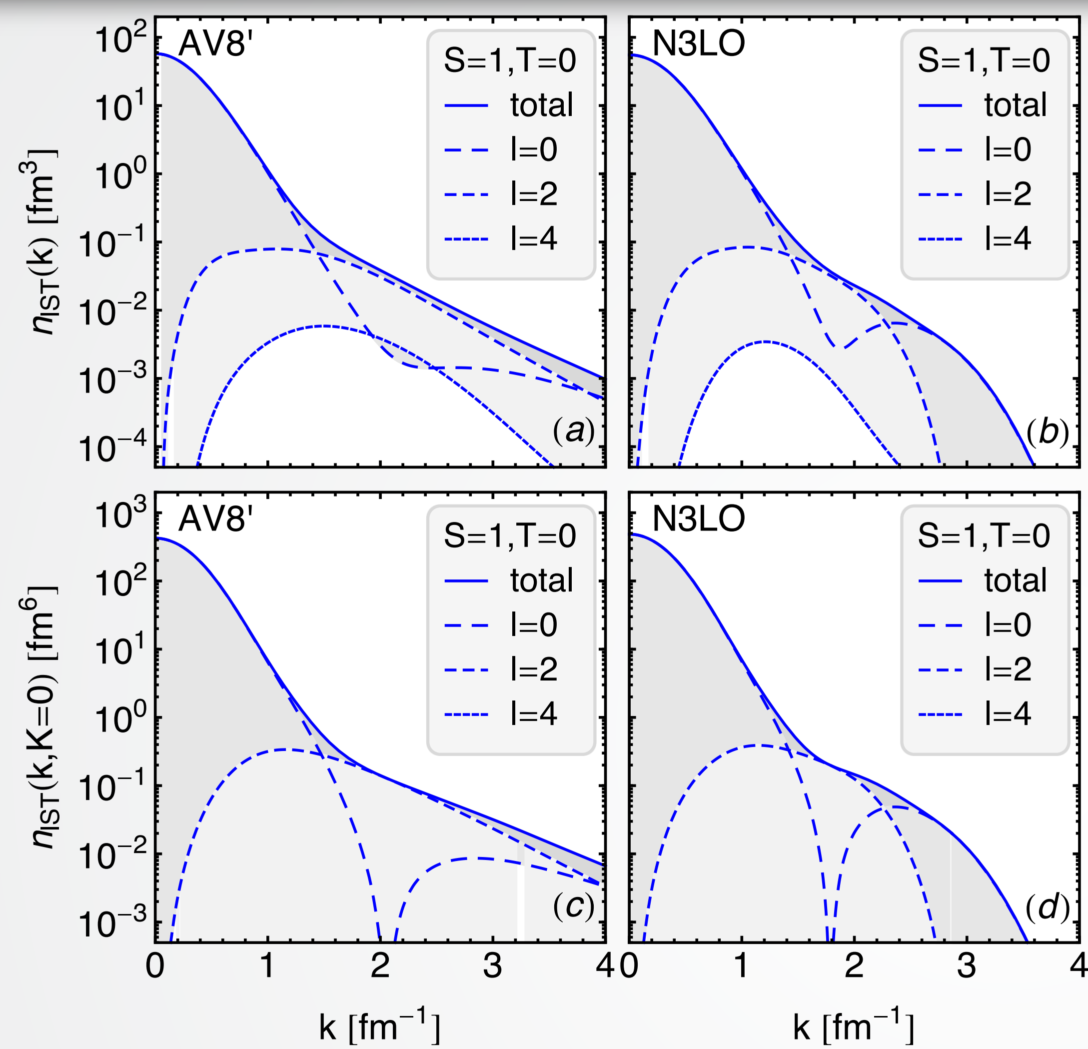
- Relative momentum distributions for K=0 pairs show a very weak dependence on flow parameter and therefore on many-body correlations — ideal to study two-body correlations
- Momentum distribution vanishes for relative momenta around 1.8 fm^{-1} in the (ST)=(01) channel

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

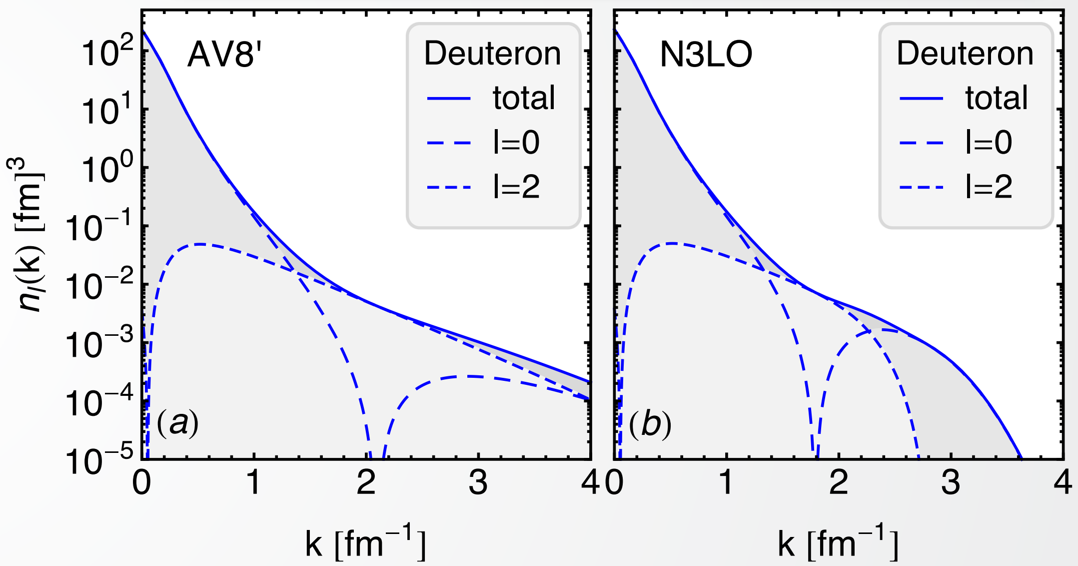
^{4}He: Tensor Correlations

all pairs

K=0 pairs



Deuteron



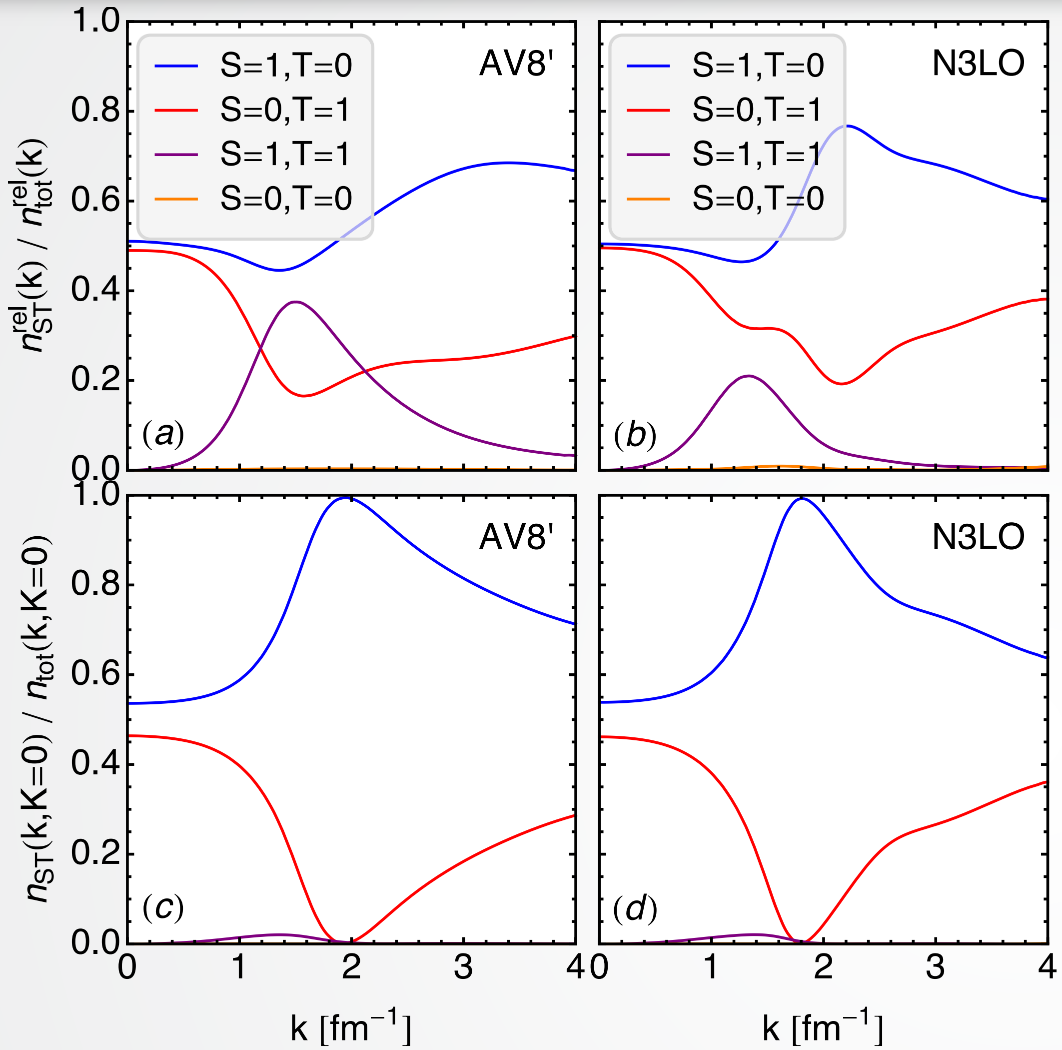
- In (ST)=(10) channel momentum distributions above Fermi momentum dominated by pairs with orbital angular momentum L=2
- For K=0 pairs only L=0,2 relevant, for all pairs also higher orbital angular momenta contribute
- The ^{4}He K=0 momentum distributions in (ST)=(10) channel above 1.5 fm^{-1} look like Deuteron momentum distributions

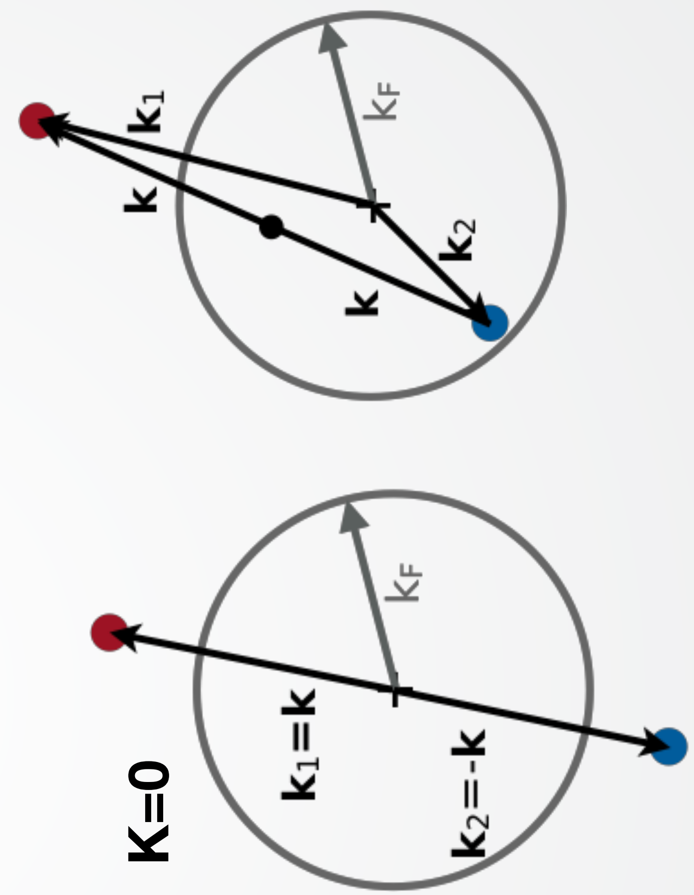
Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He: Relative Probabilities

all pairs

K=0 pairs



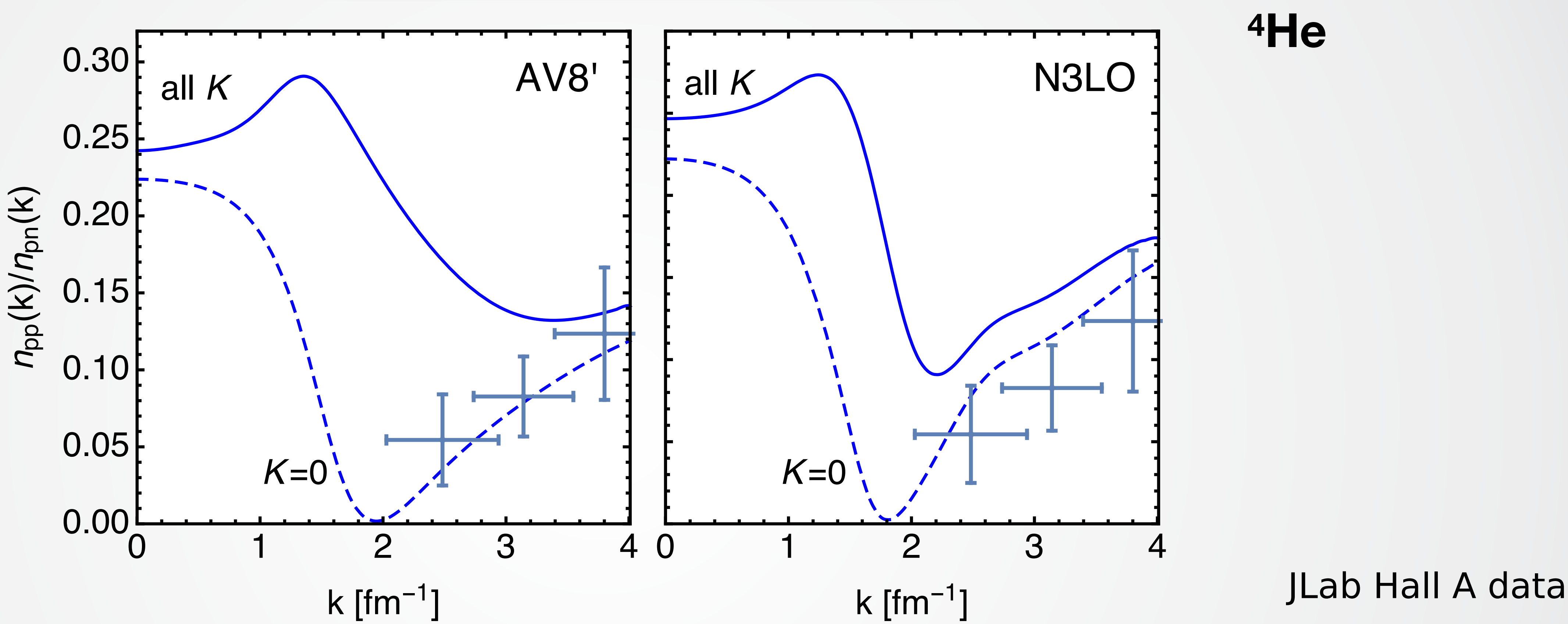


- Relative probabilities for K=0 pairs similar for AV8’ and N3LO interactions
- For $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2=0$ contribution from S=0,T=1 pairs goes to zero for **k** about 1.8 fm^{-1}
- This is not the case if we look at all pairs, here many-body correlations generate many pairs in the (ST)=(11) channel

Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He: Relative Probabilities

as ratio of **pp/pn** pairs

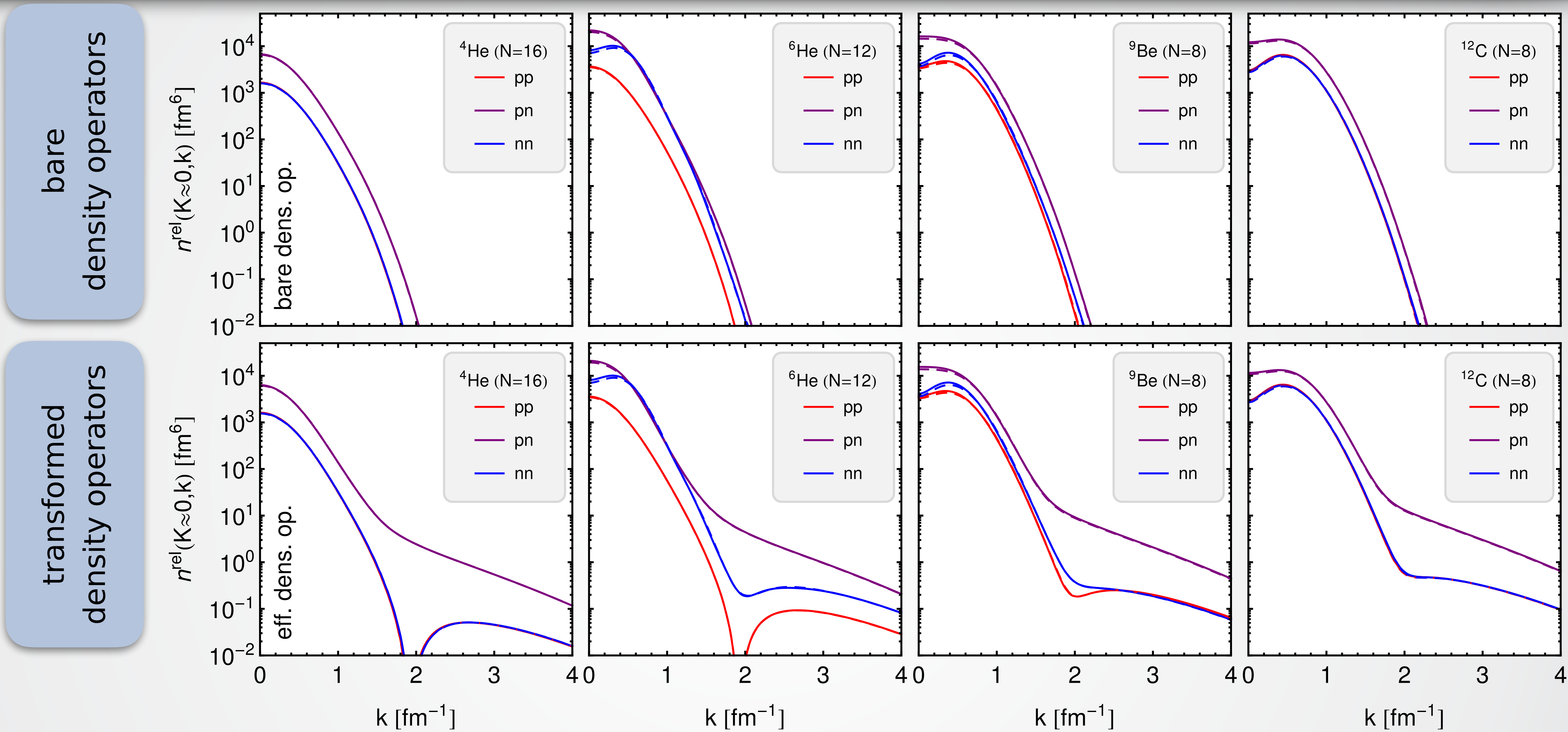


Korover *et al.*, Phys. Rev. Lett. **113**, 022501 (2014)

- For **K=0** pairs ratio of pp/pn pairs goes to zero for relative momenta **k** of about 1.8 fm^{-1}
- This is not the case if we look at all pairs, here many-body correlations generate many pairs in the (ST)=(11) channel
- AV8’ in good agreement with JLab data

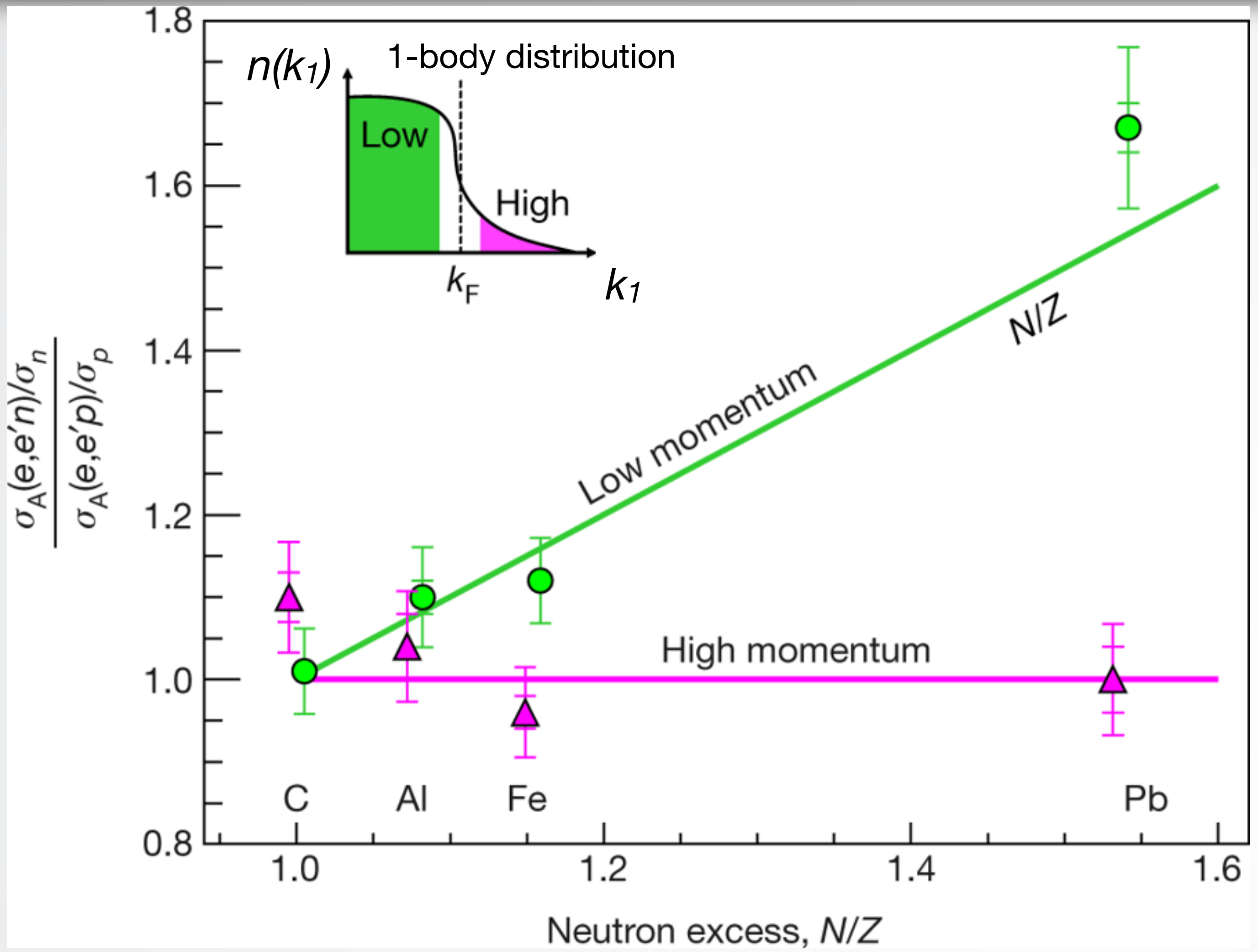
Neff, Feldmeier, Horiuchi, Phys. Rev. C **92**, 024003 (2015)

^{4}He, ^{6}He, ^{9}Be, ^{12}C: n^{rel}(k, K=0)



- Momentum distributions obtained in NCSM are well converged for larger flow parameters
- high-momentum **pn** (and total) momentum distributions very similar for all nuclei
- *p*-shell nucleons fill up the node around 1.8 fm^{-1} for **pp/pn** pairs

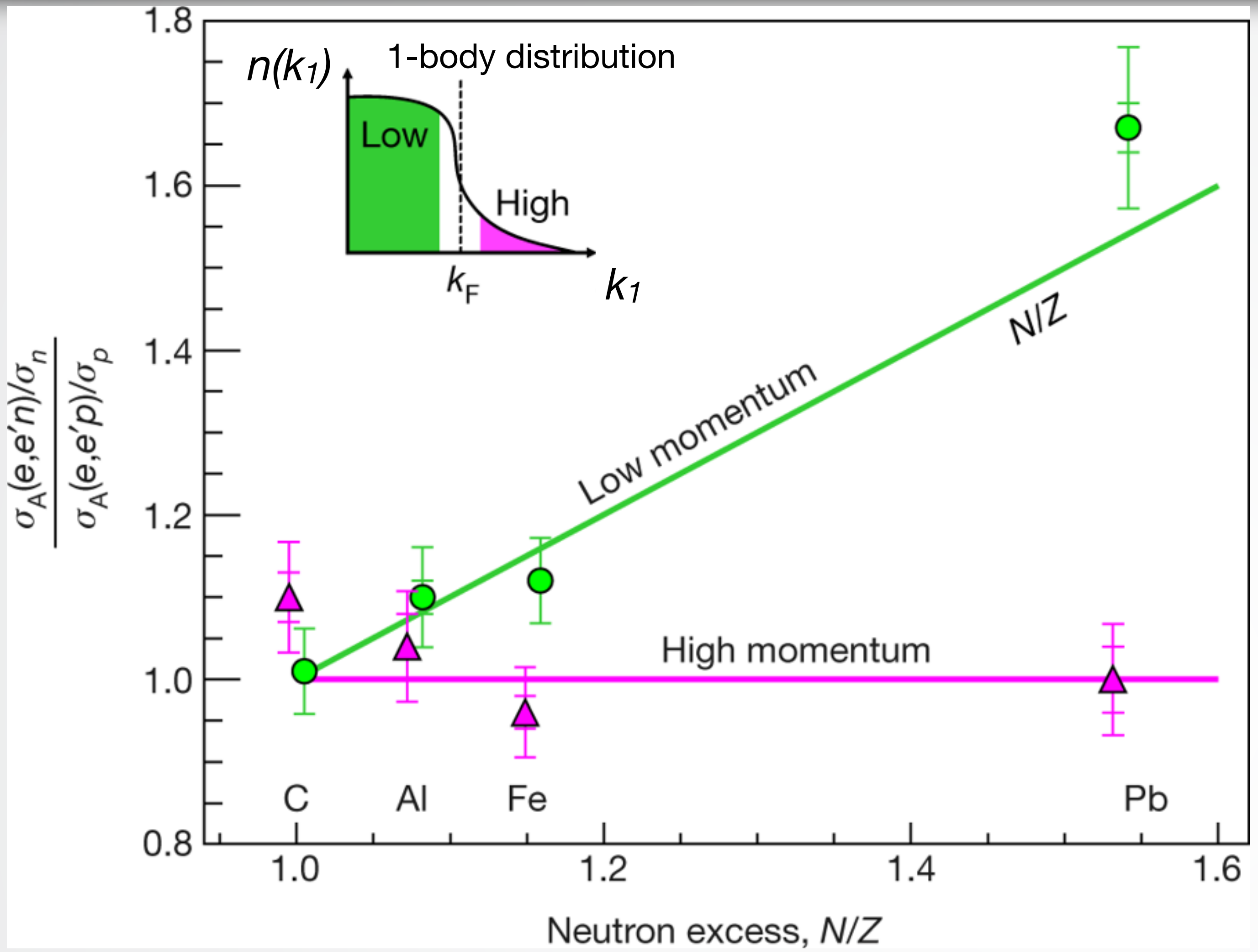
Signs of Correlation already in One-Body Momentum Distribution



- Ratio of knocked out **n** to **p** with low $\mathbf{k_1}$ proportional to *N/Z,* as expected
- But at high momenta $\mathbf{k_1}$ as many **n** as **p,** ▶ 2-body correlations show up in 1-body distribution

CLAS collaboration, Nature 560, 617,(2018)

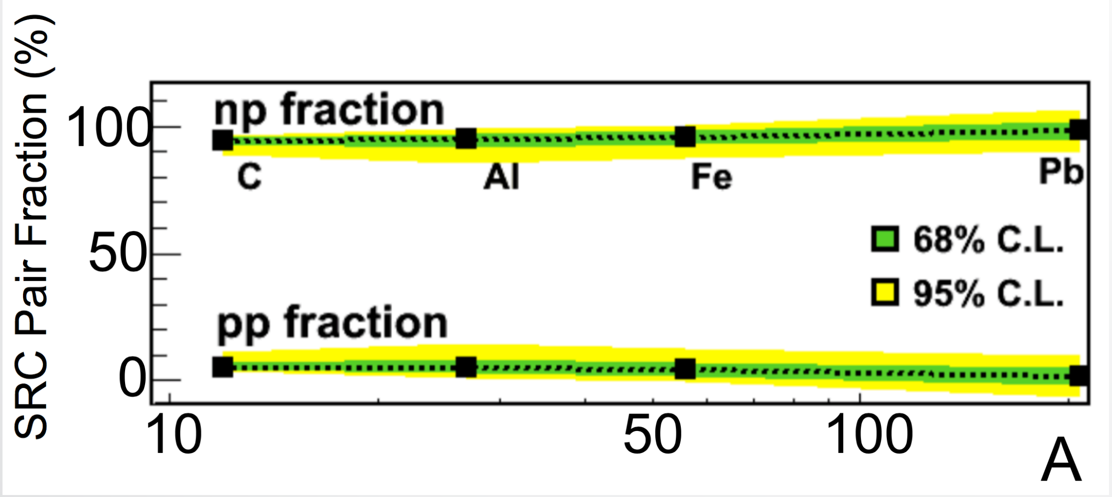
Signs of Correlation already in One-Body Momentum Distribution



- Ratio of knocked out **n** to **p** with low $\mathbf{k_1}$ proportional to *N/Z,* as expected
- But at high momenta $\mathbf{k_1}$ as many **n** as **p,**
  ▶ 2-body correlations show up in 1-body distribution

CLAS collaboration, Nature 560, 617,(2018)

High rel. momentum **np** and **pp** pairs in nuclei



- **K≈0** back to back **np** pairs with rel. mom. $\mathbf{k>2\ fm^{-1}}$ are predominant in all nuclei
- no dependence on N/Z

Shell Model works

- By unitary trafo $\mathbf{U}_\alpha$ of $\mathbf{H}$ → effective $\mathbf{H}_\alpha$ and SM wave functions $|\Phi_\alpha>$ without SRC
- Universality of SRC below r<1fm and low saturation density → one $\mathbf{U}_\alpha$ for all nuclei
- Energies are same because of unitarity
- Usual observables are 1-body and long ranged, R_{ms} radii, electromagnetic transitions $\mathbf{B}_\alpha = \mathbf{U}_\alpha^{-1}\, \mathbf{B}\, \mathbf{U}_\alpha \approx \mathbf{B}$
- But measured one-body momentum distributions show high momentum tails, not possible with $|\Phi_\alpha>$
- Measured two-body correlations in momentum space clearly exhibit SRC, in particular tensor type

- Message: observables **B** blind to SRC can be described in SM by naively using **B**
- observables that see SCR can not be described in SM, but SRC can be recovered by transforming the operator $\mathbf{B} \to \mathbf{B}_\alpha$

Summary

A) NN-Interactions & Short Range Correlations (SRC)

- Nucleons are complex many-body systems
  interaction approximated by 2- and 3-body forces
  analogue to van-der-Waals pot. between atoms, but depend on ***S***, ***T*** and ***p***, besides ***r***
- Pion exchange dominates at large distance, source for tensor interaction
- mainly responsible for correlations above $\mathbf{k_F}$ and higher (SCR)
- strong central repulsion (SRC)
- NN interaction imprints corresponding correlations into many-body state, universal for r_{ik}< 1 fm
- shell model (independent particles in mean-field, no high momenta)?

B) Similarity Transformation of Hamiltonian and Observables

- SRC can not be represented in mean-field basis of shell model
- way out: similarity transformation of operators, soften $\mathbf{H} \rightarrow \mathbf{H}_\alpha = \mathbf{U}_\alpha^{-1}\, \mathbf{H}\, \mathbf{U}_\alpha$ **,**
- drawback: $\mathbf{H}_\alpha$ contains induced many-body forces,
  approximation: neglect induced 4-body and higher-body terms
- do many-body calculations with $\mathbf{H}_\alpha$ in Hilbert-space spanned by Slater determinants (shell model with configuration mixing)
- long-range observables (radius, BE2-transitions, spatial densities) are very little influenced by SRC
- when needed, retrieve SRC with $\mathbf{B}_\alpha = \mathbf{U}_\alpha^{-1}\, \mathbf{B}\, \mathbf{U}_\alpha$ (momentum distributions, knock out of protons by high momentum electrons)
- shell model with configuration mixing works because of universality, same unitary transformation in all nuclei, same effective soft $\mathbf{H}_\alpha$

Thank You for Surviving 2 Hours

Many thanks to my collaborators
Wataru Horiuchi
Thomas Neff
Dennis Weber

and many thanks to all the people discussing the subject with us
Yasuyuki Suzuki
Robert Roth
Heiko Hergert

.
.
.

The Wigner Function of the Deuteron

A phase-space picture of short-range correlations

$$W_{M_S,M_S'}(\mathbf{r},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s \left\langle \mathbf{r}+\tfrac{1}{2}\mathbf{s}; SM_S \middle| \hat{\rho} \middle| \mathbf{r}-\tfrac{1}{2}\mathbf{s}; SM_S' \right\rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$

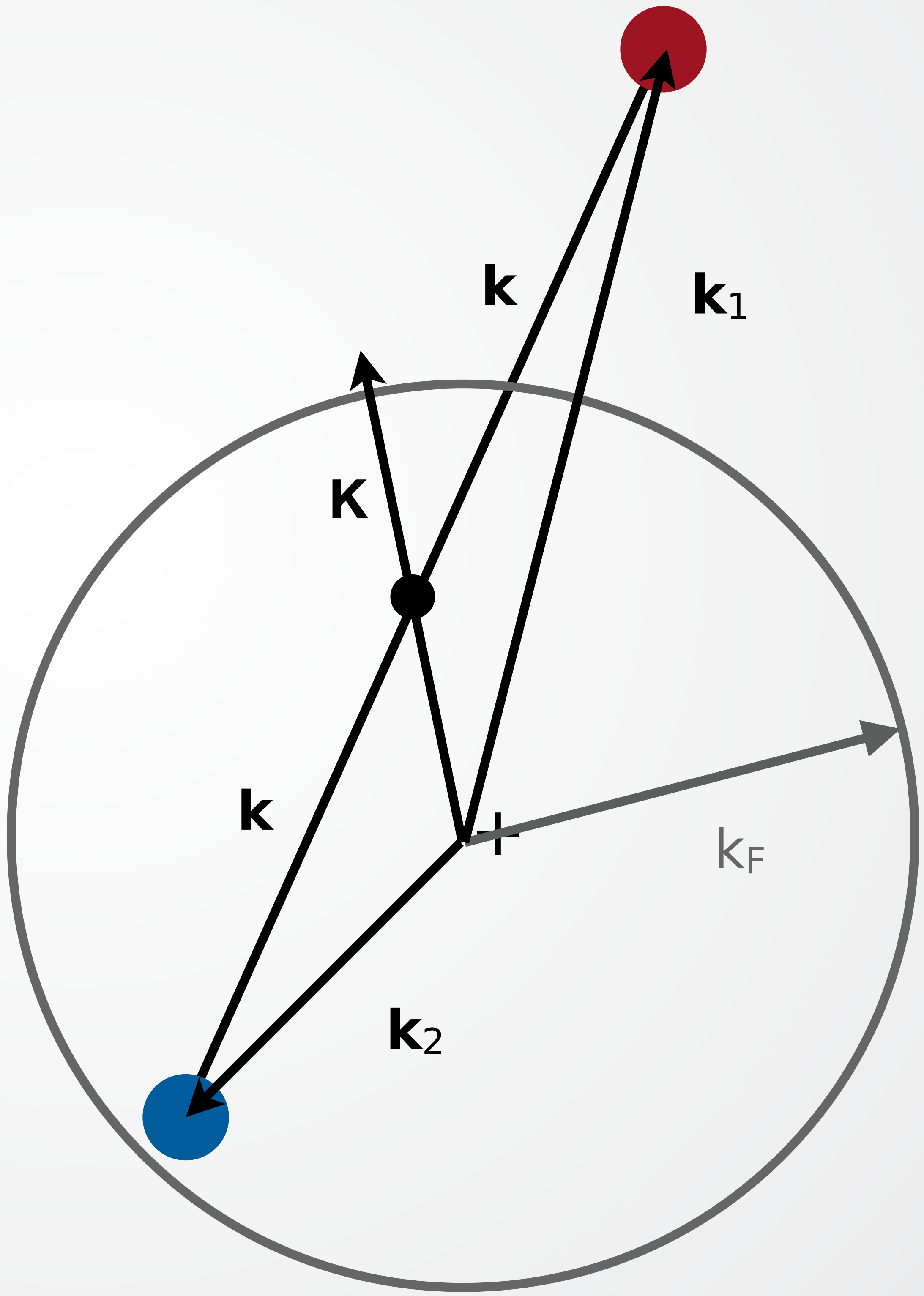
$$\hat{\rho} = \frac{1}{3}\sum_M \left|\Psi; 1M\right\rangle\left\langle\Psi; 1M\right|$$

- Coordinate & momentum space densities

$$\rho_{M_S}(\mathbf{r}) = \left\langle \mathbf{r}; SM_S \middle| \hat{\rho} \middle| \mathbf{r}; SM_S \right\rangle = \int d^3p\, W_{M_S,M_S}(\mathbf{r},\mathbf{p})$$

$$n_{M_S}(\mathbf{p}) = \left\langle \mathbf{p}; SM_S \middle| \hat{\rho} \middle| \mathbf{p}; SM_S \right\rangle = \int d^3r\, W_{M_S,M_S}(\mathbf{r},\mathbf{p})$$

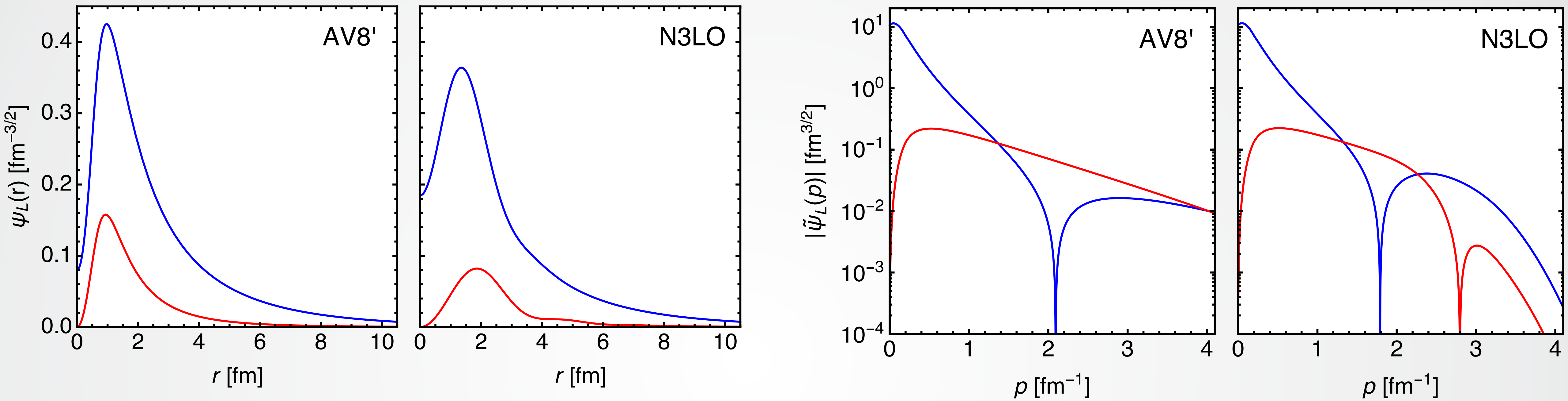
Neff, Feldmeier, arXiv:1610.04066



k
k1
K
k
kF
k2

Deuteron Wave Functions

Coordinate Space | **Momentum Space**



- Suppression of the wave function at short distances due to repulsion
- *D*-wave admixture due to tensor force
- *D*-wave dominates high-momentum region around 2 fm^{-1}

- Short-range repulsion stronger for AV8’, 500 MeV cut-off in N3LO reflected in momentum space wave function
- N3LO wave function shows “kinks" at large distances — artefact of sudden cut-off

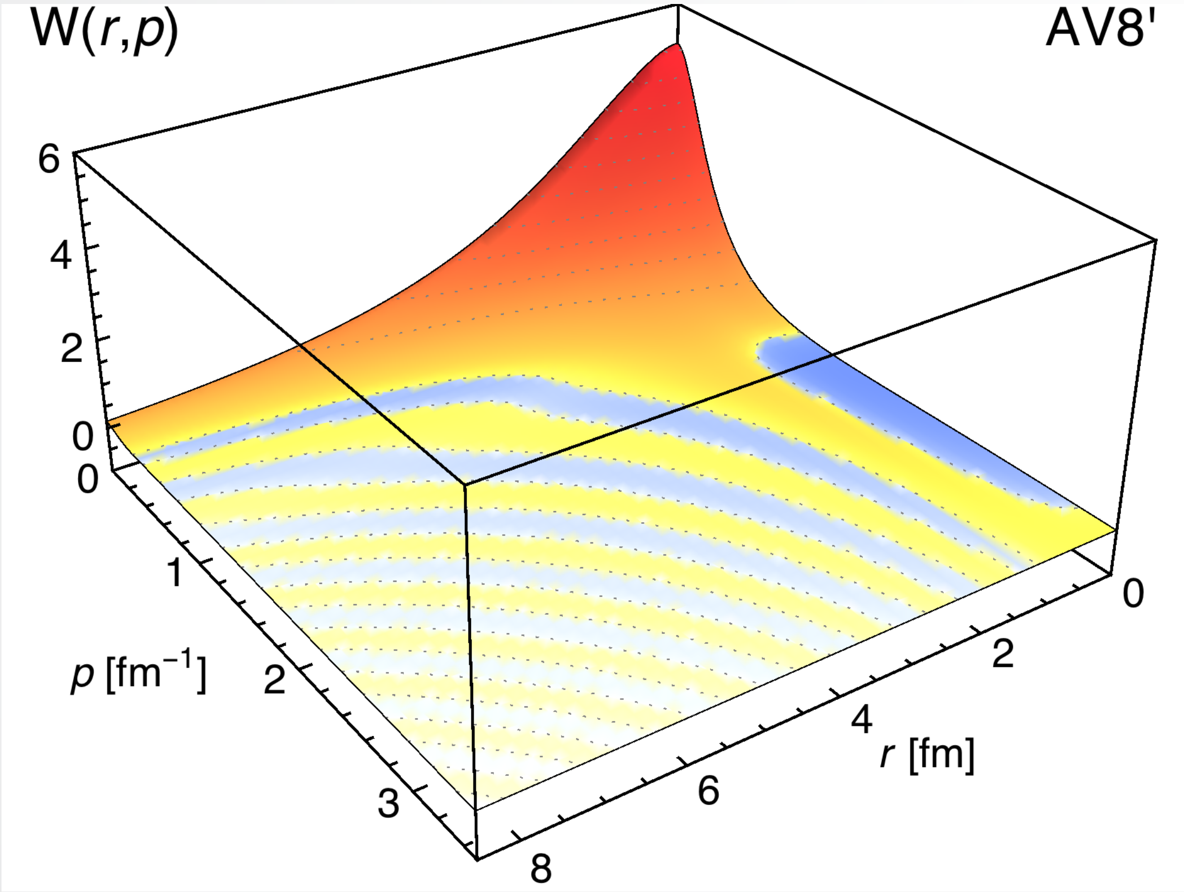
Wigner Function of the Deuteron

$$W(\mathbf{r},\mathbf{p}) = \frac{1}{(2\pi)^3}\int d^3s\,\langle \mathbf{r}+\tfrac{1}{2}\mathbf{s}|\hat{\rho}|\mathbf{r}-\tfrac{1}{2}\mathbf{s}\rangle e^{-i\mathbf{p}\cdot\mathbf{s}}$$
$$= \frac{1}{(2\pi)^3}\int d^3s\,\Psi(\mathbf{r}+\tfrac{1}{2}\mathbf{s})\Psi(\mathbf{r}-\tfrac{1}{2}\mathbf{s})^{\star} e^{-i\mathbf{p}\cdot\mathbf{s}}$$

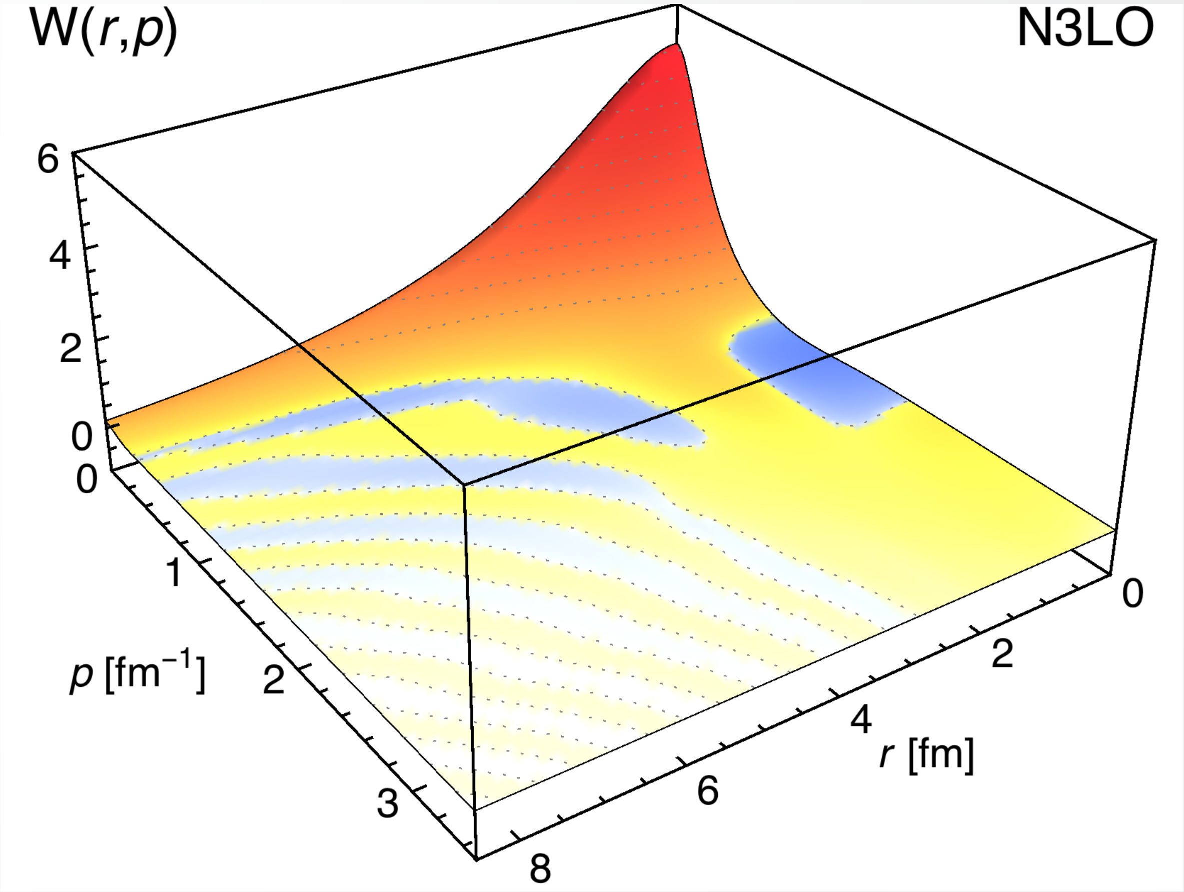
- Integrate over angles

$$W(r,p) = \int d\Omega_r \int d\Omega_p\, W(\mathbf{r},\mathbf{p})$$

AV8′

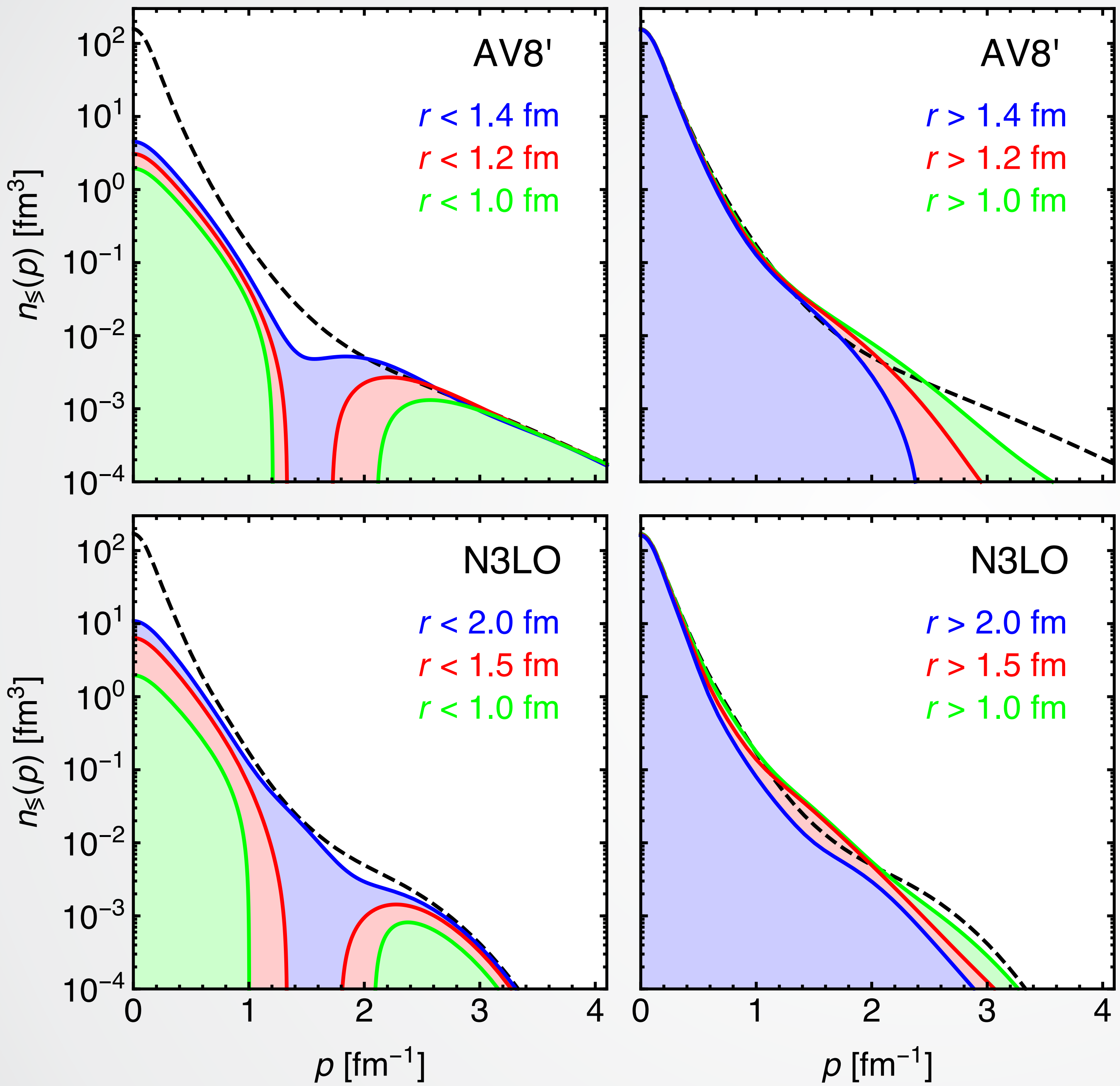


N3LO



- Wigner function not suppressed at small distances r
- short-range physics is encoded in high-momentum region

(Partial) Momentum Distributions

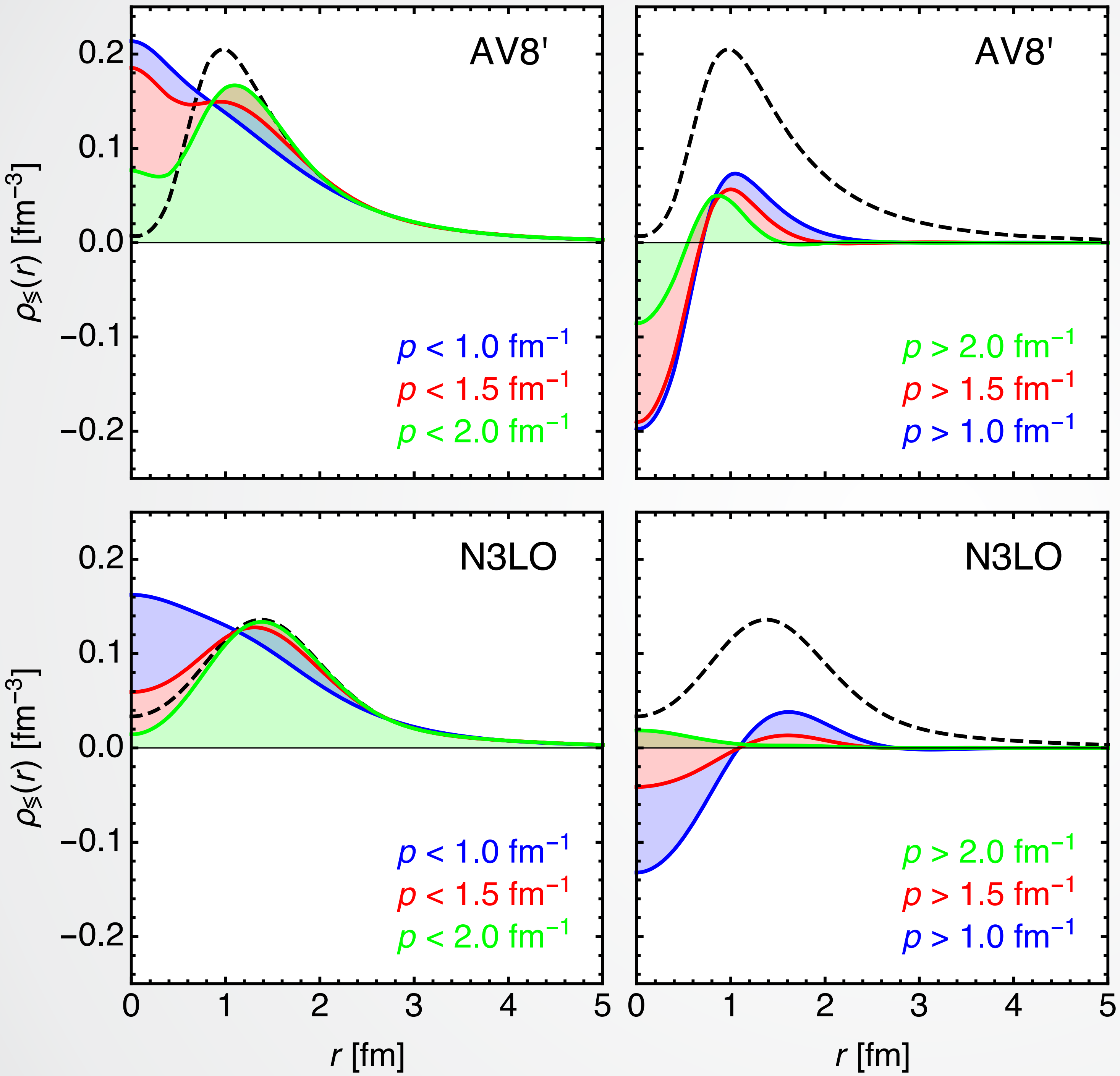


$$n_{\lessgtr}(p) = \int_{r \lessgtr r_{sep}} dr\, r^2 W(r,p)$$

- Integrate Wigner function over small or large distance regions
- not an observable but provides intuition

- small distance pairs determine high momentum part of momentum distribution
- large distance pairs give momentum distributions in low momentum region

(Partial) Coordinate Space Distributions



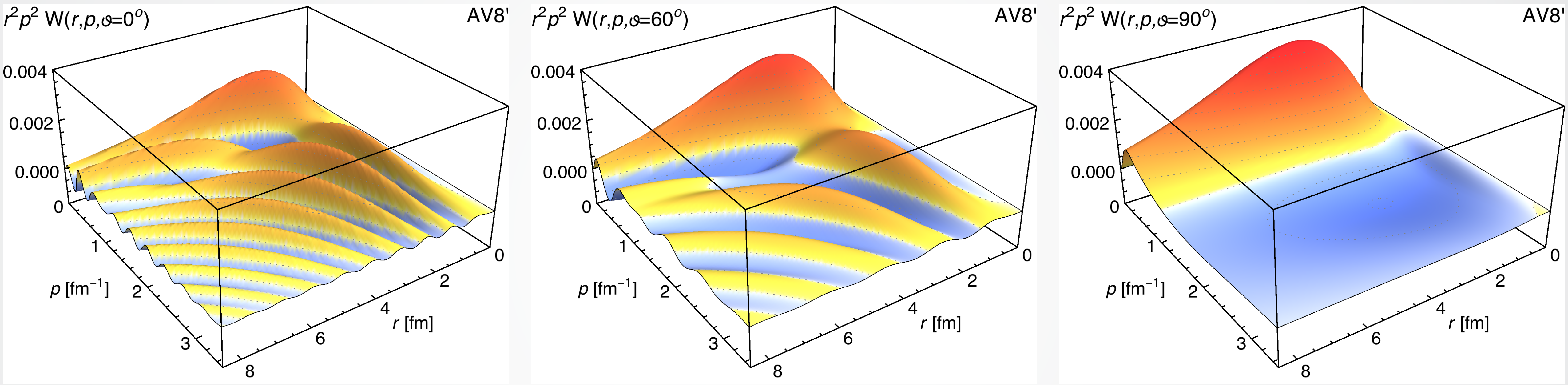
$$\rho_{\lessgtr}(r) = \int_{p \lessgtr p_{sep}} dp\, p^2 W(r,p)$$

- Integrate Wigner function over regions of low and high momenta
- density at large distances given by low-momentum pairs
- correlation hole at small distances is created by interference of low- and high-momentum pairs

Orientation dependence

$$W(\mathbf{r},\mathbf{p}) = W(r,p,\cos\vartheta)$$

AV8′



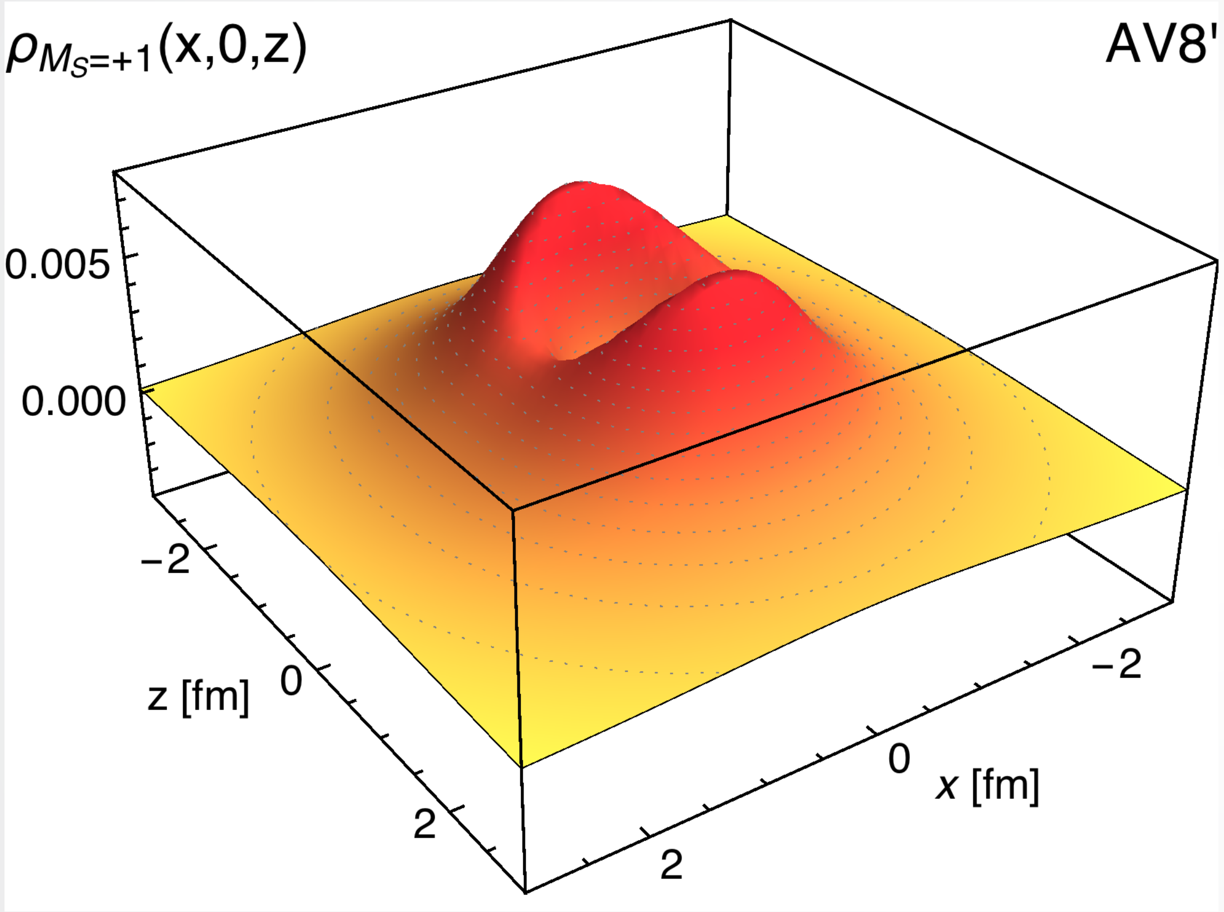
r and **p** parallel

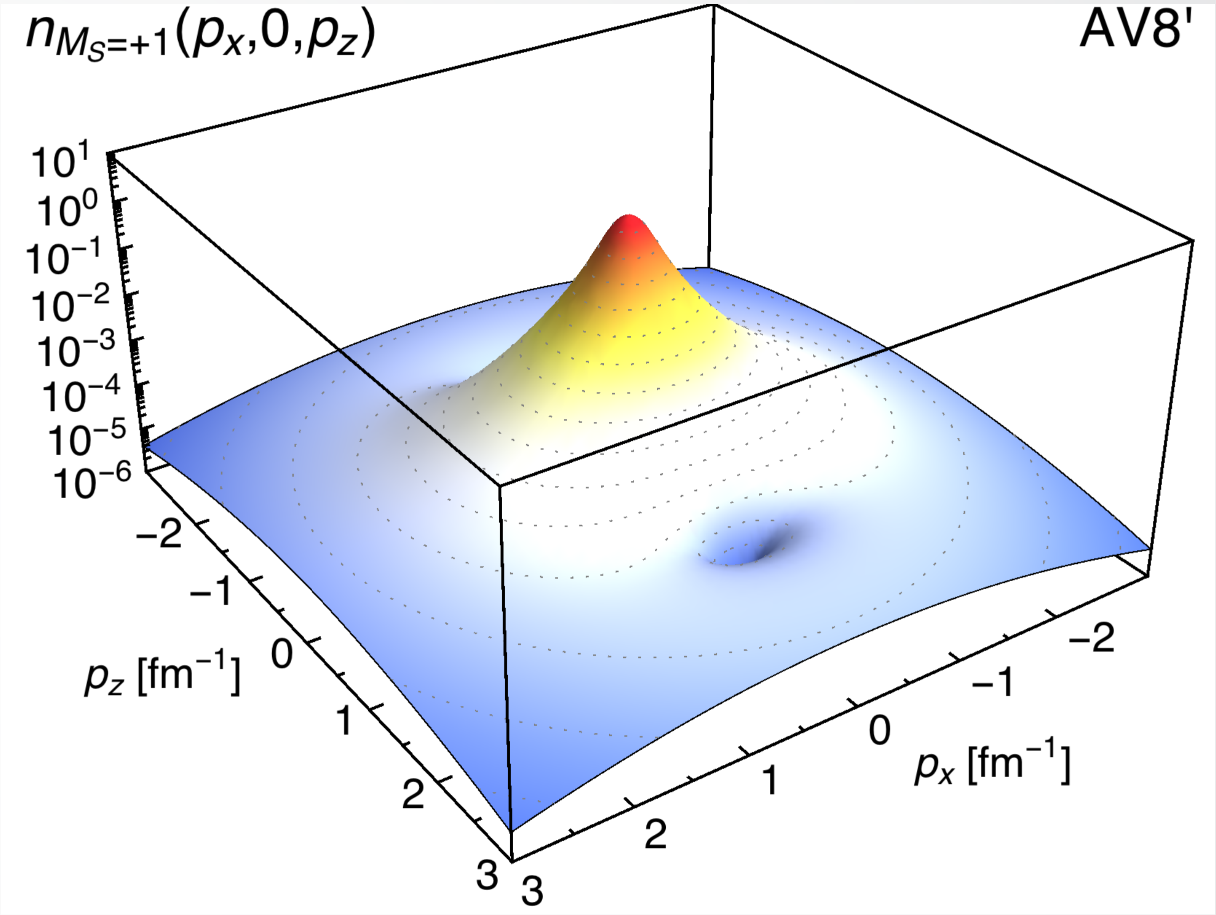
r and **p** perpendicular

- oscillations reflect uncertainty principle for non-commuting observables
- three-dimensional problem, small angles correspond to small impact parameters, angles around 90° to circular motion around the core
- highest probability for angles around 90°

Spin dependence

$$\rho_{M_S}(\mathbf{r}) = \langle \mathbf{r}; SM_S | \hat{\rho} | \mathbf{r}; SM_S \rangle = \int d^3p \, W_{M_S,M_S}(\mathbf{r},\mathbf{p}) \qquad n_{M_S}(\mathbf{p}) = \langle \mathbf{p}; SM_S | \hat{\rho} | \mathbf{p}; SM_S \rangle = \int d^3r \, W_{M_S,M_S}(\mathbf{r},\mathbf{p})$$





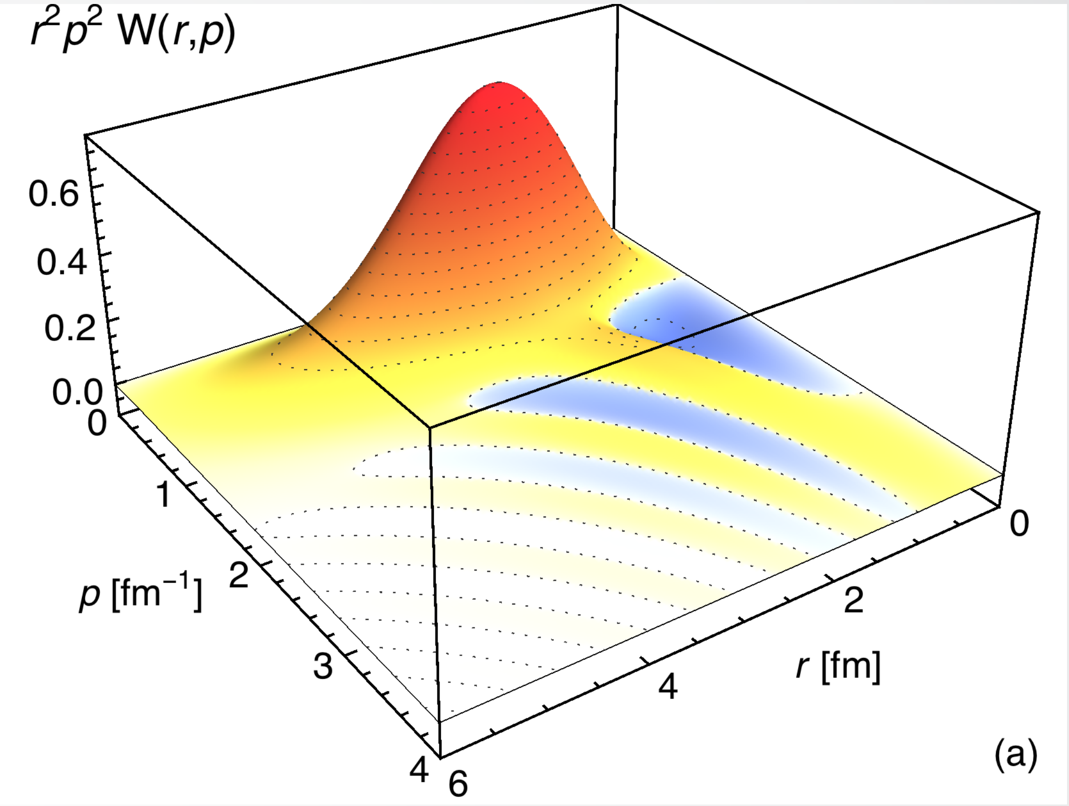
- density and momentum distributions depend on orientation of the spin due to tensor force
- dumbbell (M_S=±1) and donut (M_S=0) shapes in coordinate space
- dip in momentum distribution for momenta parallel to spin orientation
- tensor correlations strongest in mid-momentum region (1.5 fm^{-1} ≲ p ≲ 2.5 fm^{-1})

Wigner function of two-Gaussian toy model

$$\psi(\mathbf{r}) = \alpha_1 \psi_1(\mathbf{r}) + \alpha_2 \psi_2(\mathbf{r})$$

$$\psi_i(\mathbf{r}) = \frac{1}{(\pi a_i)^{3/4}} \exp\left\{-\frac{\mathbf{r}^2}{2a_i}\right\}$$

$$W(\mathbf{r},\mathbf{p}) = W_{11}(\mathbf{r},\mathbf{p}) + W_{12}(\mathbf{r},\mathbf{p}) + W_{22}(\mathbf{r},\mathbf{p})$$



interference term