



***Nuclear response beyond one-loop approximation:
from zero to finite temperature***

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APCTP Focus Program 2019, Pohang, South Korea, July 1-10, 2019



Introduction

- **Motivation:** to build a consistent and predictive approach to describe the entire nuclear chart, numerically executable and useful for applications.
- **Challenges:** the nuclear hierarchy problem, complexity of NN-interaction.
- **Accurate non-perturbative solutions:** Relativistic Nuclear Field Theory (RNFT). Emerged as a synthesis of Landau-Migdal Fermi-liquid theory, Copenhagen-Milano NFT and Quantum Hadrodynamics (QHD); now put in the context of a systematic equation of motion (EOM) formalism and linked to ab-initio interaction.
- **n-body correlation functions:** complete characteristics of strongly-coupled many-body systems. Define all dynamical and geometrical properties of nuclear and condensed matter systems, quantum chemistry, **various QFT's**.
- **Nuclear 1-body and 2-body correlation functions = observable nuclear shell structure and response to major neutral and charge-exchange probes:** giant EM resonances, Gamow-Teller, spin dipole etc. (neutron capture, gamma and beta decays, pair transfer, ...).
New: correlated 3p3h configurations have been included up to high excitation energies in medium-mass nuclei.
- **Nuclear response at finite temperature:** thermal RNFT for transitions between nuclear excited states.
- **Conclusions and perspectives.**

Hierarchy problem and connection to fundamental physics

- Nuclear scales: Hierarchy problem

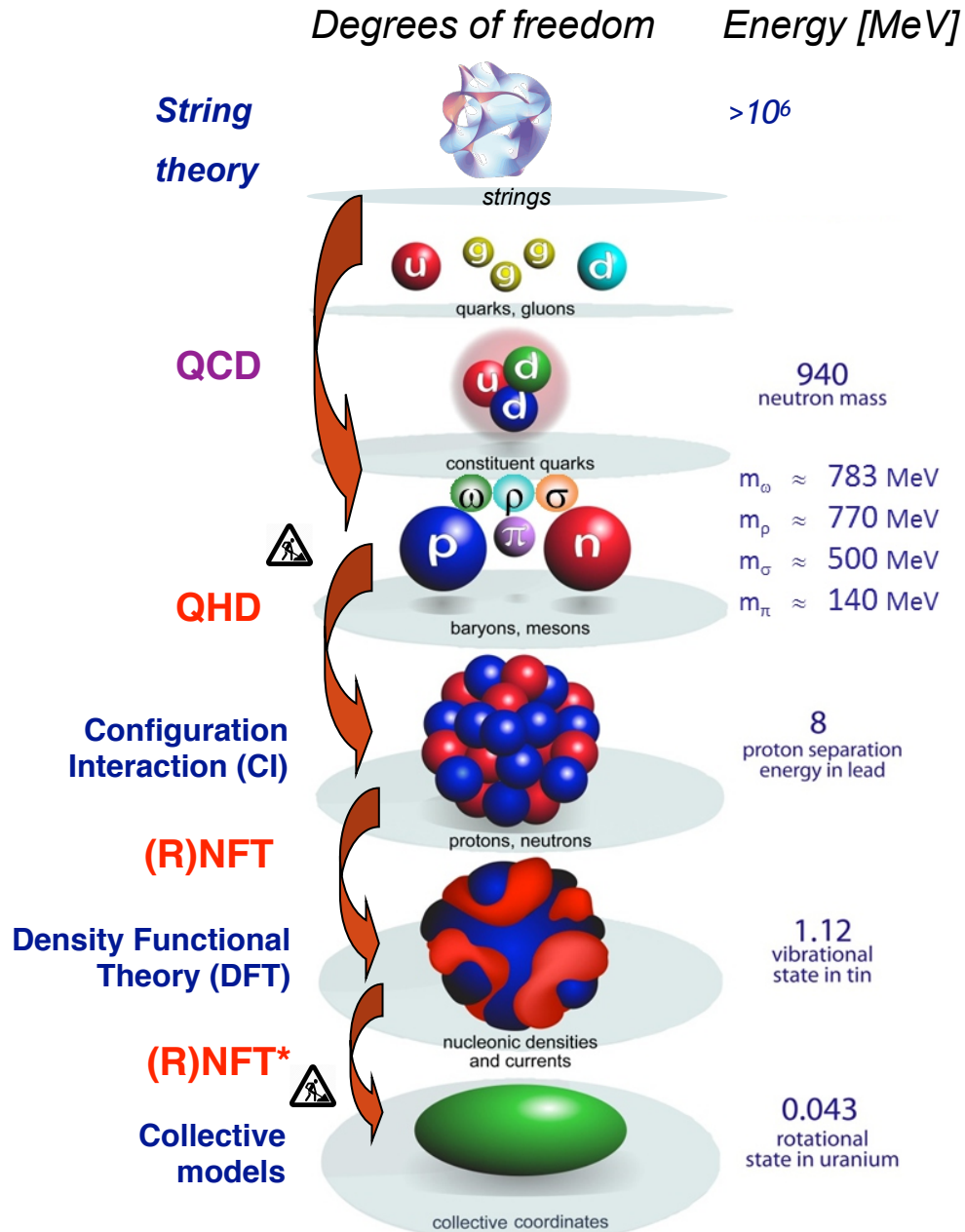
$$H = K + V$$

center of mass internal DOF's:
 next energy scale

QFT:
"interaction"

String theory:
merging strings

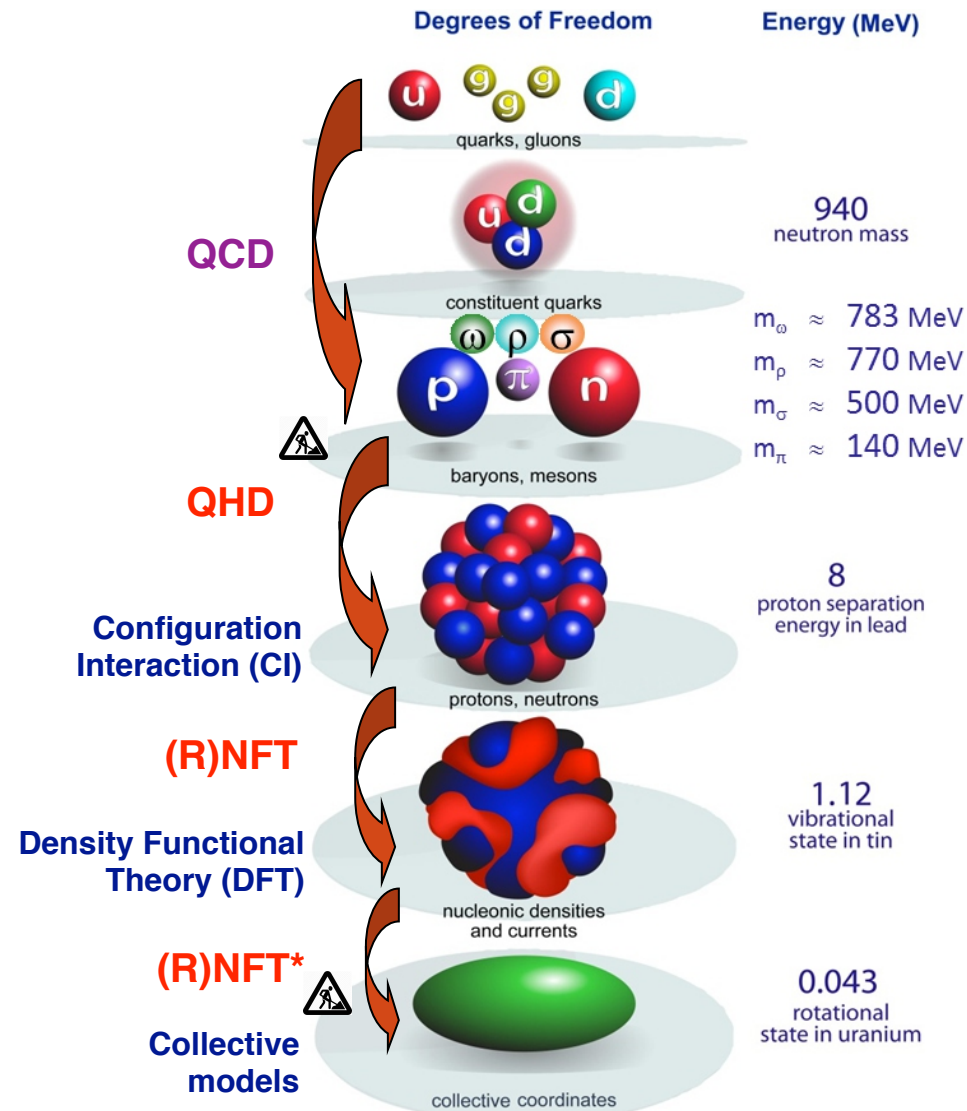
- Physics on a certain energy scale is defined by the next (higher) energy scale(s)
- Effective theories: separation of scales, energy-independent "interactions" => infrared behavior is lost
- Should we instead elaborate on more accurate methods for connecting scales?



Relativistic Nuclear Field Theory (RNFT):

Nuclear scales

- RNFT: combines “*ab initio*”, DFT and CI
- Connects scales from Quantum Hadrodynamics (QHD) to emergent collective phenomena
- Lagrangian for mesons and nucleons constrained by QCD
- Lorentz covariance: ~5-10% accuracy at the excitation energy of interest (grows with energy)
- Spin-orbit and tensor “forces” are naturally included
- Fewer parameters; hidden correlations minimized (4-10 universal parameters)
- Natural extensions to the inclusion of the delta isobar, to finite temperature, high excitation energies and densities (FAIR and FRIB upgrade)
- Non-perturbative self-consistent response theory with high-order NN-correlations



A strongly-correlated many body system: single-fermion propagator, particle-hole propagator and related observables

$$H = \sum_{12} t_{12} \psi_1^\dagger \psi_2 + \frac{1}{4} \sum_{1234} \bar{v}_{1234} \psi_1^\dagger \psi_2^\dagger \psi_4 \psi_3$$

Hamiltonian, non-relativistic
or relativistic, extendable to 3-body etc.

$$G_{11'}(t - t') = -i \langle T \psi(1) \psi^\dagger(1') \rangle$$

$$1 = \{\xi_1, t\}$$

Single-particle propagator

$$G_{11'}(\varepsilon) = \sum_n \frac{\eta_1^n \eta_{1'}^{n*}}{\varepsilon - (E_n^{(N+1)} - E_0^{(N)}) + i\delta} + \sum_m \frac{\eta_1^{m*} \eta_{1'}^m}{\varepsilon + (E_m^{(N-1)} - E_0^{(N)}) - i\delta}$$

Fourier transform:
Spectral expansion
(Lehmann)

$$\eta_1^n = \langle 0 | \psi_1 | n^{(N+1)} \rangle,$$

$$\eta_1^m = \langle m^{(N-1)} | \psi_1 | 0 \rangle$$

Residues - spectroscopic
(occupation) factors

Poles - single-particle energies

Ground state of N
particles

(Excited) state
of $(N+1)$ particles

$$R_{12,1'2'}(t - t') = -i \langle T (\psi_1^\dagger \psi_2)(t) (\psi_{2'}^\dagger \psi_{1'})(t') \rangle$$

Particle-hole response function

$$R_{12,1'2'}(\omega) = \sum_\nu \left[\frac{\rho_{21}^\nu \rho_{2'1'}^{\nu*}}{\omega - \omega_\nu + i\delta} - \frac{\rho_{12}^{\nu*} \rho_{1'2'}}{\omega + \omega_\nu - i\delta} \right]$$

Fourier transform: Spectral expansion

Excitation
energies

$$\rho_{12}^\nu = \langle 0 | \psi_2^\dagger \psi_1 | \nu \rangle$$

Residues - transition densities

Poles - excitation energies

Diagrammatic conventions

One-fermion propagator:

$$G_{11'}(t - t') = -i \langle T \psi(1) \psi^\dagger(1') \rangle = \text{Diagram: } \begin{array}{c} 1 \quad \quad 1' \\ \longleftarrow \quad \longleftarrow \end{array}$$

Two-fermion propagator:

$$G(12, 1'2') = (-i)^2 \langle T \psi(1) \psi(2) \psi^\dagger(2') \psi^\dagger(1') \rangle = \text{Diagram: } \begin{array}{c} 1 \quad \quad 1' \\ \longleftarrow \quad \longleftarrow \\ | \quad | \\ G^{(2)} \\ | \quad | \\ 2 \quad \quad 2' \\ \longleftarrow \quad \longleftarrow \end{array}$$

Three-fermion propagator:

$$G(123, 1'2'3') = (-i)^3 \langle T \psi(1) \psi(2) \psi(3) \psi^\dagger(3') \psi^\dagger(2') \psi^\dagger(1') \rangle = \text{Diagram: } \begin{array}{c} 1 \quad \quad 1' \\ \longleftarrow \quad \longleftarrow \\ | \quad | \quad | \\ G^{(3)} \\ | \quad | \quad | \\ 2 \quad \quad 2' \\ \longleftarrow \quad \longleftarrow \\ 3 \quad \quad 3' \\ \longleftarrow \quad \longleftarrow \end{array}$$

Two-fermion interaction:

$$\bar{v}_{1234} = \text{Diagram: } \begin{array}{c} 3 \quad \quad 2 \\ \longleftarrow \quad \longleftarrow \\ | \quad | \\ \bar{v} \\ | \quad | \\ 1 \quad \quad 4 \\ \longrightarrow \quad \longrightarrow \end{array}$$

Hartree-Fock self-energy:

$$\sum_{jl} \bar{v}_{1j1'l} \rho_{lj} = \text{Diagram: } \begin{array}{c} \text{Loop with } j \text{ and } l \\ | \\ \bar{v} \\ | \\ 1 \quad \quad 1' \\ \longrightarrow \quad \longrightarrow \end{array}$$

Dynamical self-energy:

$$\sum_{234} \sum_{2'3'4'} \bar{v}_{1234} G(432', 23'4') \bar{v}_{4'3'2'1'}$$

$$= \text{Diagram: } \begin{array}{c} 3 \quad \quad 3' \\ \longrightarrow \quad \longrightarrow \\ | \quad | \\ \bar{v} \quad \bar{v} \\ | \quad | \\ 2 \quad \quad 2' \\ \longleftarrow \quad \longleftarrow \\ | \quad | \\ G^{(3)} \\ | \quad | \\ 4 \quad \quad 4' \\ \longleftarrow \quad \longleftarrow \\ 1 \quad \quad 1' \\ \longrightarrow \quad \longrightarrow \end{array}$$

Exact equations of motion (EOM) for pairwise interactions: one-body problem

Differentiation d/dt and d/dt'
and
Fourier transformation
lead to the Dyson equation:

$$G(\omega) = G^{(0)}(\omega) + G^{(0)}(\omega)\Sigma(\omega)G(\omega)$$

Free propagator

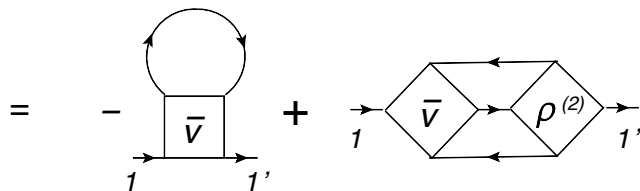
Irreducible kernel:

$$\Sigma(\omega) = \Sigma^{(0)} + \Sigma^{(r)}(\omega)$$

Self-energy, **exact**

instantaneous term (Hartree-Fock incl. "tadpole")

$$\Sigma_{11'}^{(0)} = -\delta(t-t')\langle [[\psi_1, V], \psi_{1'}^\dagger] \rangle$$

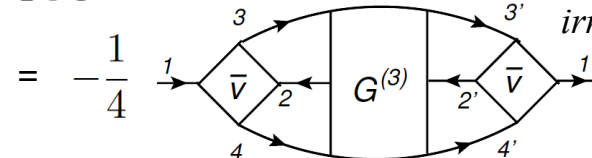


Mean field

t -dependent (retarded & advanced) term

$$\Sigma_{11'}^{(r)}(t-t') = -i\langle T[\psi_1, V](t)[V, \psi_{1'}^\dagger](t') \rangle^{irr}$$

$$= -\frac{1}{4} \sum_{234} \sum_{2'3'4'} \bar{v}_{1234} G^{irr}(432', 23'4') \bar{v}_{4'3'2'1'}$$



Dynamical self-energy

$$\rho_{ij} = -i \lim_{t=t'-0} G_{ij}(t-t')$$

$$\rho_{ijkl}^{(2)} = - \lim_{t=t'-0} G_{kijl}^{(2)}(t-t')$$

depends on the dynamical term
and goes beyond one-body problem

EOM method:

S. Adachi and P. Schuck, NPA496, 485 (1989).

J. Dukelsky, G. Roepke, and P. Schuck, NPA 625, 14 (1995).

P. Schuck and M. Tohyama, PRB 93, 165117 (2016).
 etc.

Cluster expansion of the 3-fermion propagator

2-fermion GF:

$$G(12, 1'2') = (-i)^2 \langle T \psi(1) \psi(2) \psi^\dagger(2') \psi^\dagger(1') \rangle$$

3-fermion GF:

$$G(123, 1'2'3') = (-i)^3 \langle T \psi(1) \psi(2) \psi(3) \psi^\dagger(3') \psi^\dagger(2') \psi^\dagger(1') \rangle,$$

n-fermion GF:

$$G(12\dots n, 1'2' \dots n') = (-i)^n \langle T \psi(1) \psi(2) \dots \psi(n) \psi^\dagger(n') \dots \psi^\dagger(2') \psi^\dagger(1') \rangle.$$

**Cluster expansion
(up to correlated 2p-2h):**

(N. Vinh Mau, Lecture Notes, 1979; P. Ring & P. Schuck, 1980)

$$G(432', 23'4') \stackrel{irr}{=} G(4, 4')G(32', 23') + G(3, 3')G(42', 24') + G(2', 2)G(43, 3'4') \\ - G(4, 3')G(32', 24') - G(3, 4')G(42', 23') - 2G^{(0)}(432', 23'4')$$

Uncorrelated term:

$$G^{(0)}(432', 23'4') = -G(4, 4')G(3, 3')G(2', 2) + G(4, 3')G(3, 4')G(2', 2) + G(4, 2)G(3, 3')G(2', 4') \\ + G(4, 4')G(3, 2)G(2', 3') - G(4, 2)G(3, 4')G(2', 3') - G(4, 3')G(3, 2)G(2', 4')$$

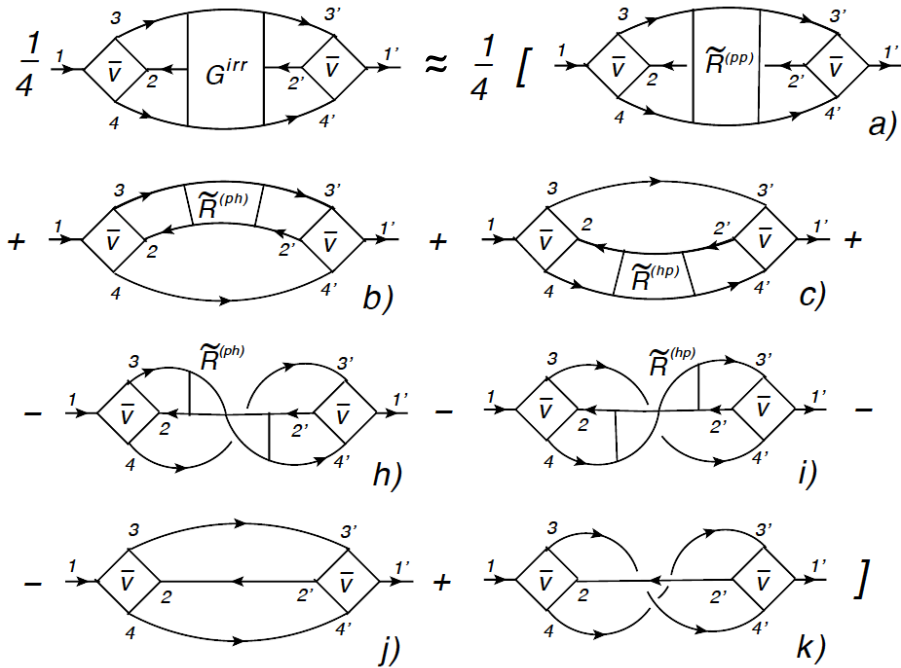
$$R(12', 21') = G(12', 21') - G(1, 2)G(2', 1'),$$

Response function (pp)

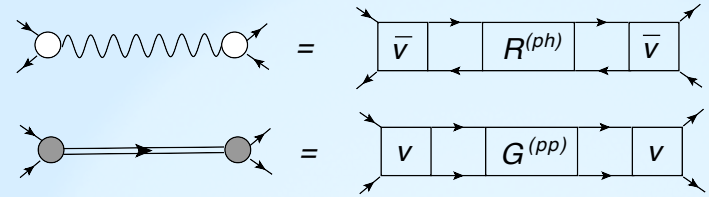
$$\tilde{R}(12', 21') = G(12', 21') - (G(1, 2)G(2', 1') - G(1, 1')G(2', 2))$$

Fully correlated part (pp)

(Exact) mapping to the (quasi)particle-vibration coupling (QVC, PVC)



ph correlator: coupling to normal phonons (vibrations)



pp correlator: coupling to pairing phonons (vibrations)

Model-independent mapping to the QVC:

$$\sum_{343'4'} \bar{v}_{1234} R_{34,3'4'}(\omega) \bar{v}_{3'4'1'2'} = \sum_{m, \sigma = \pm 1} g_{12}^{m(\sigma)} D_m^{(\sigma)}(\omega) g_{1'2'}^{m(\sigma)*}$$

“phonon” vertex:

$$g_{12}^m = \sum_{34} \bar{v}_{1234} \rho_{43}^m$$

“phonon” propagator:

$$R_{12,1'2'}(\omega) = \sum_m \left[\frac{\rho_{21}^m \rho_{2'1'}^{m*}}{\omega - \Omega_m + i\delta} - \frac{\rho_{12}^{m*} \rho_{1'2'}}{\omega + \Omega_m - i\delta} \right]$$

$$D_m^{(\sigma)}(\omega) = \frac{\sigma}{\omega - \sigma(\Omega_m - i\delta)}$$

Exact equation of motion (EOM) for the particle-hole response

Particle-hole response
(correlation function):

$$R_{12,1'2'}^{(ph)}(t-t') = -i \langle T(\psi_1^\dagger \psi_2)(t)(\psi_2^\dagger \psi_{1'})(t') \rangle$$

spectra of excitations,
masses, decays, ...

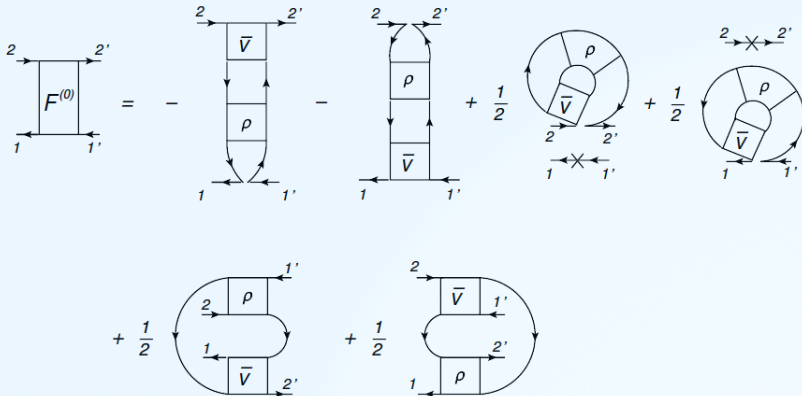
EOM:

$$R(\omega) = R^{(0)}(\omega) + R^{(0)}(\omega)F(\omega)R(\omega) \quad (**) \quad F(t-t') = F^{(0)}\delta(t-t') + F^{(r)}(t-t')$$

Free propagator

Irreducible kernel (exact):

instantaneous term ("bosonic" mean field):

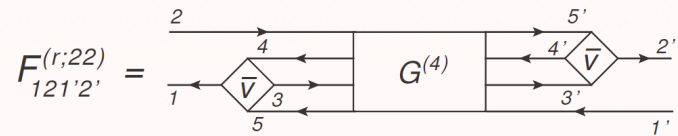
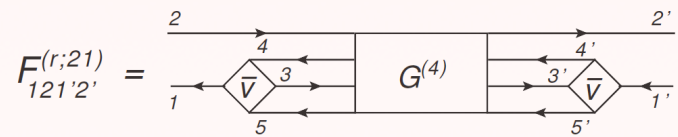
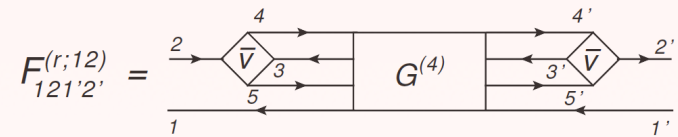
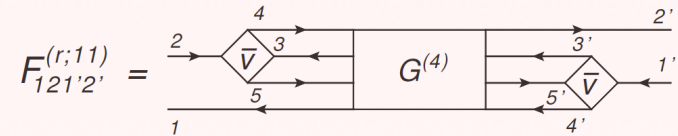


Mean field $F^{(0)}$, where $\rho_{12,1'2'} = \lim_{t \rightarrow t'-0} R(12, 1'2')$

contains the full solution of (**) including the dynamical term!

t-dependent (dynamical) term:

$$F_{12,1'2'}^{(r)}(t-t') = F_{12,1'2'}^{(r;11)}(t-t') + F_{12,1'2'}^{(r;12)}(t-t') + F_{12,1'2'}^{(r;21)}(t-t') + F_{12,1'2'}^{(r;22)}(t-t')$$



EOM:

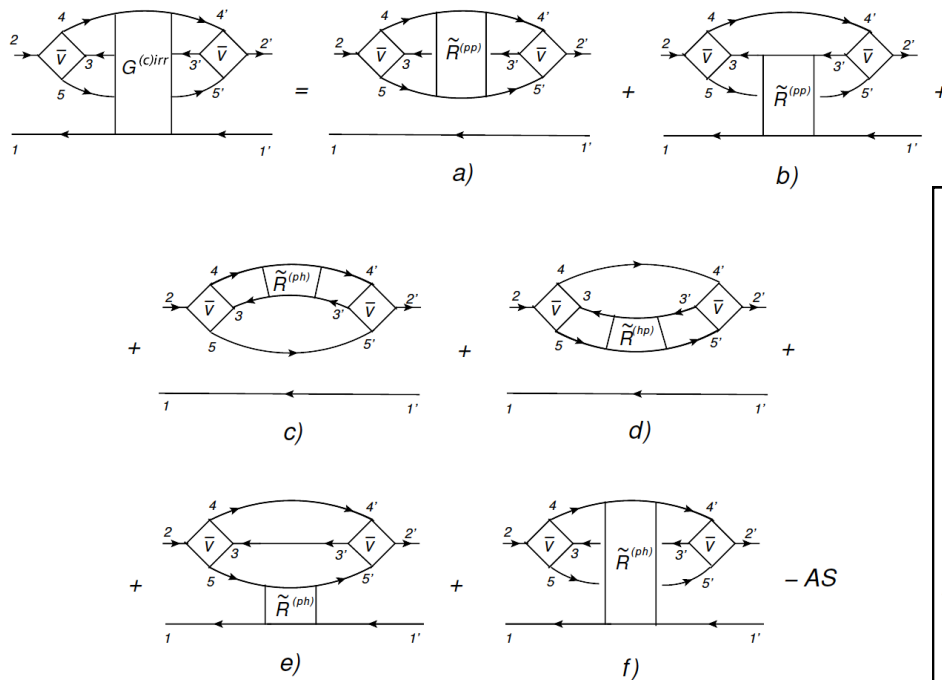
- S. Adachi and P. Schuck, NPA496, 485 (1989).
- J. Dukelsky, G. Roepke, and P. Schuck, NPA 625, 14 (1995).
- P. Schuck and M. Tohyama, PRB 93, 165117 (2016). Etc.

Expansion of the dynamics kernel $F(r;12)_{irr}$: truncation at the 2-body level

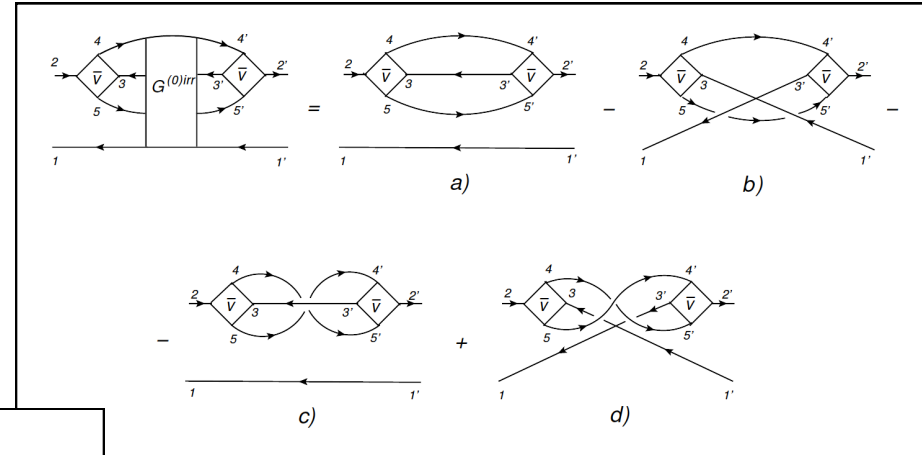
P. Schuck, S. Adachi, M. Tohyama et al.:
 Irreducible part of $G^{(4)}$ is decomposed
 into uncorrelated, singly-correlated and
 doubly-correlated terms (*cluster expansion*):

$$G^{irr}(543'1', 5'4'31) = G^{(0)irr}(543'1', 5'4'31) + G^{(c)irr}(543'1', 5'4'31) + G^{(cc)irr}(543'1', 5'4'31)$$

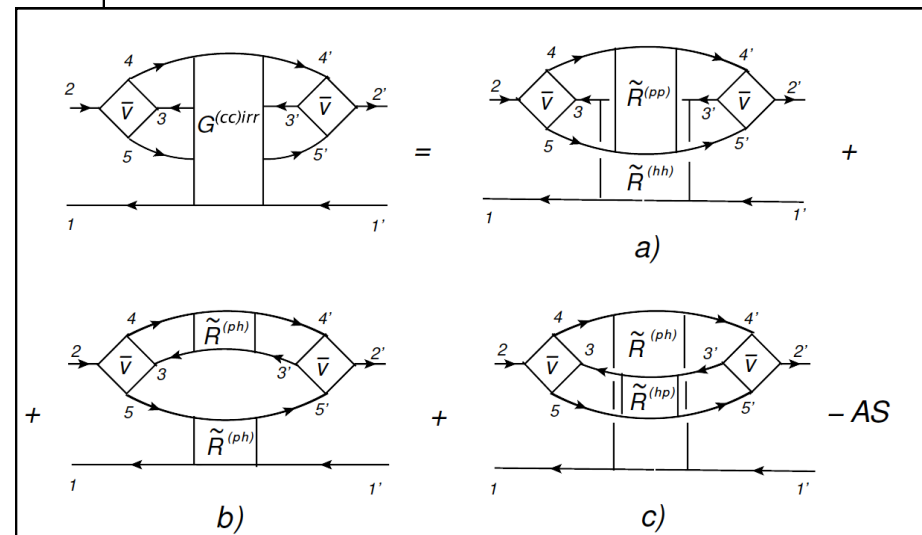
(ii) *Singly-correlated terms, up to phases (PVC, QVC, ...)*:



(i) *Uncorrelated terms ("Second RPA")*:

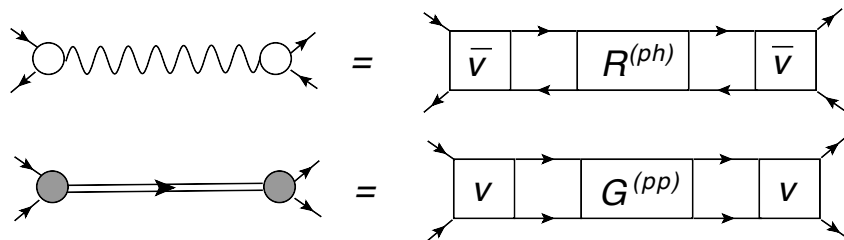


(iii) *Doubly-correlated terms, up to phases (generalized QVC)*:



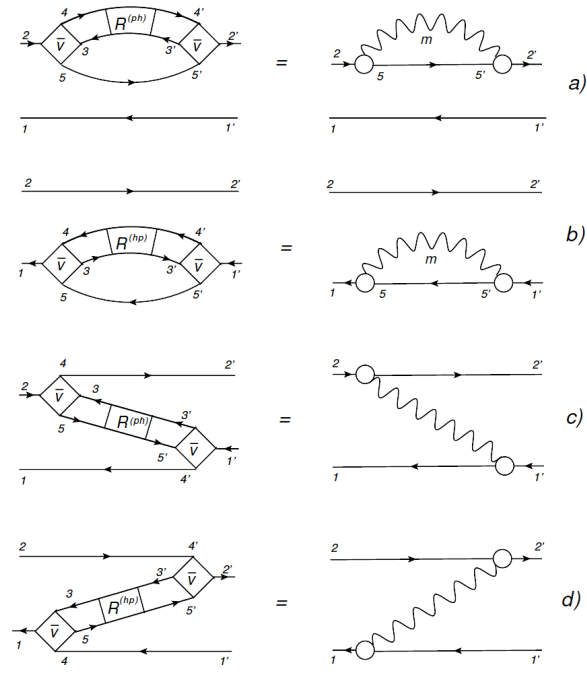
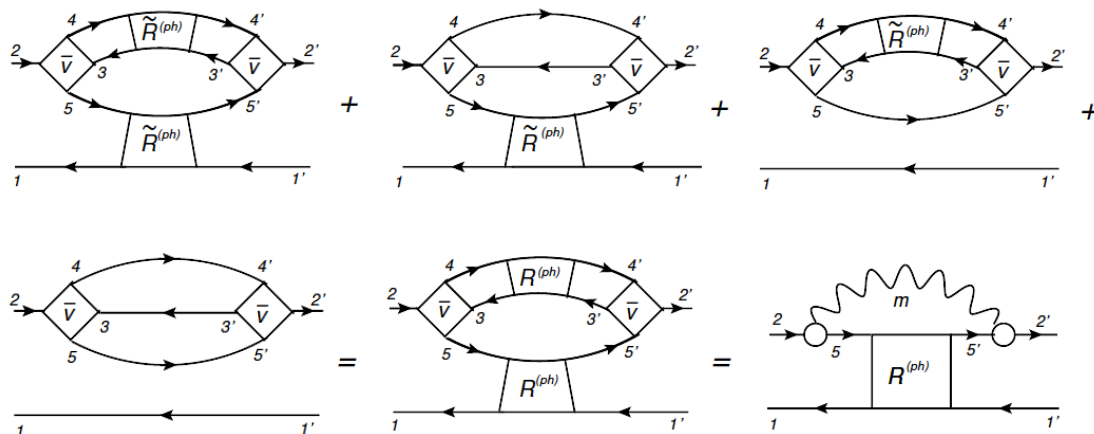
Mapping to the (quasi)particle-vibration coupling

Model-independent mapping to the QVC:



Original QVC, (Relativistic) Nuclear Field Theory,
 (Relativistic) Quasiparticle Time Blocking
 Approximation (RQTBA): singly-correlated terms

Generalized QVC (with time blocking) meets EOM: ALL correlated terms (E.L. PRC91, 034332 (2015))



Self-consistent closed system of equations:

$$\hat{R}(\omega) = \hat{R}^{(0)}(\omega) + \hat{R}^{(0)}(\omega)W[\hat{R}(\omega)]\hat{R}(\omega)$$

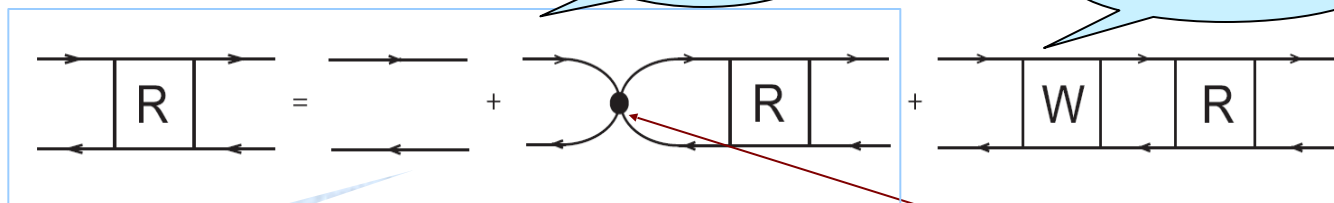
All channels are coupled in $W[\hat{R}(\omega)]$:

$$\hat{R} = \left\{ R^{(ph)}, R^{(hp)}, R^{(pp)}, R^{(hh)} \right\}$$

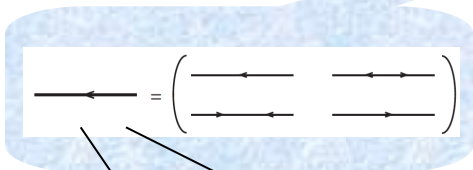
E.L., P. Schuck, in progress

Nuclear field theory for the particle-hole response function

Bethe-Salpeter Equation (BSE)
4-times response:



E. L., P. Ring, and V. Tselyaev,
Phys. Rev. C 78, 014312 (2008)

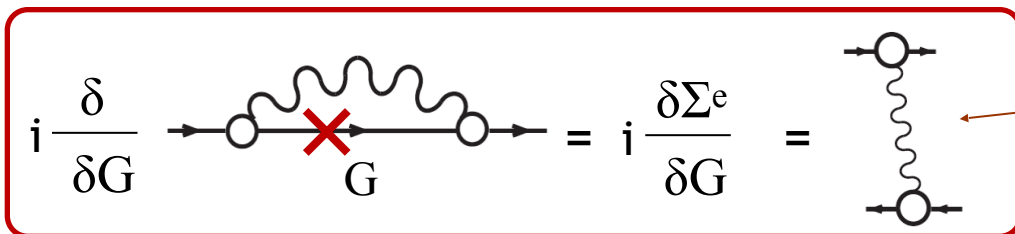
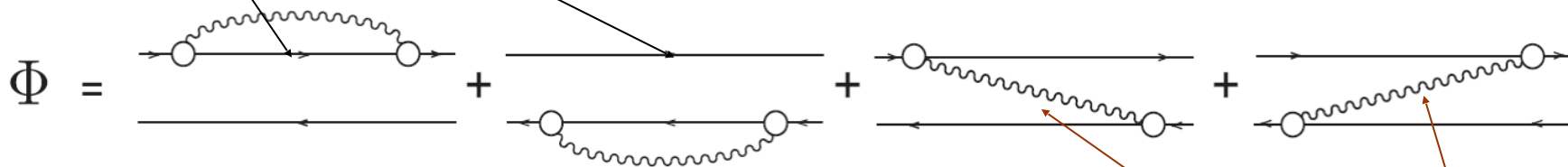


$$R(\omega) = A(\omega) + A(\omega) [V + W(\omega)] R(\omega)$$

$$V = \frac{\delta \Sigma^{\text{MF}}}{\delta \rho}$$

$$W(\omega) = \Phi(\omega) - \Phi(0)$$

Self-consistency

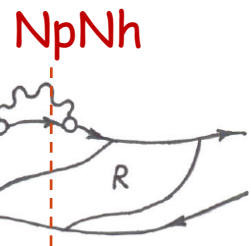
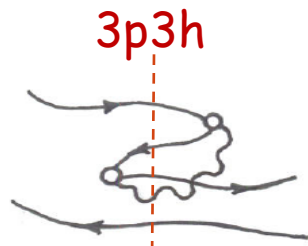


$$U^e = i \frac{\delta \Sigma^e}{\delta G}$$

Consistency on 2p2h-level

Time blocking approximation

Problem:
Singular kernel



Perturbative schemes:

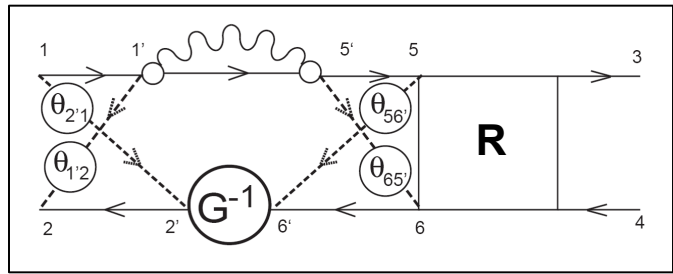
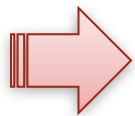
Unphysical result:
negative cross sections



Solution:
Time-projection operator:

$$\delta_{\sigma_1 - \sigma_2} \theta(\sigma_1 t_{2'1}) = 1 \rightarrow \theta_{2'1} \rightarrow 2'$$

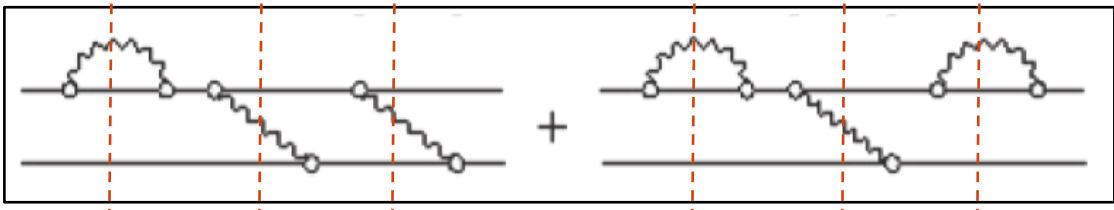
$$\delta_{\sigma_2 - \sigma_1} \theta(\sigma_1 t_{1'2}) = 2 \leftarrow \theta_{1'2} \leftarrow 1'$$



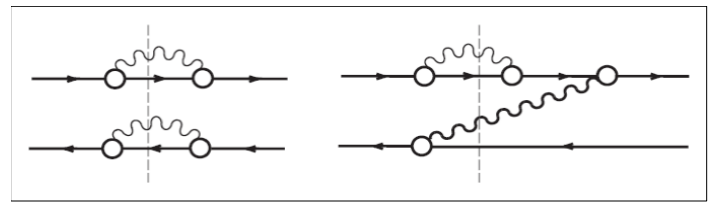
Partially fixed

V.I. Tselyaev,
Yad. Fiz. 50,1252 (1989)

Allowed terms: 1p1h, 2p2h



Blocked terms: 3p3h, 4p4h, ...



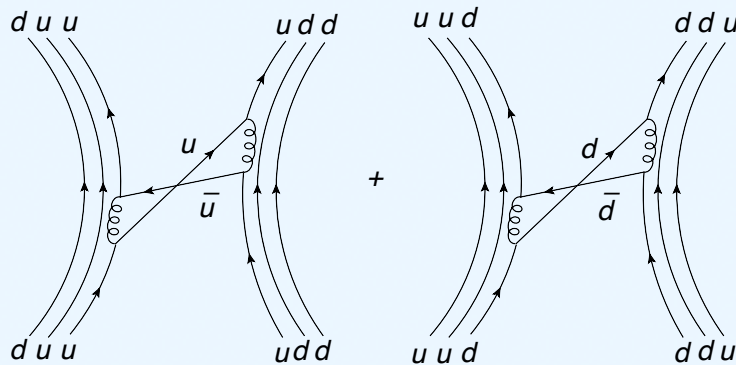
- Time**
- Separation of the integrations in the BSE kernel
 - R has a simple-pole structure (spectral representation)
 - »» Strength function is positive definite!
 - The kernel is equivalent to the EOM kernel for the two-point correlation function

Included on the next step
(based on the EOM)

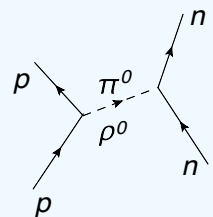
The underlying NN-interaction: meson exchange (ME)

Neutral mesons σ , ω , π , ρ ...

QCD



QHD

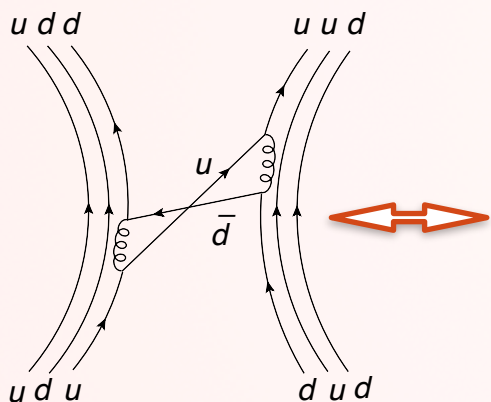


• The full many-body scheme has not been (yet) executed neither for the bare meson-exchange (ME) interaction nor for any other bare interaction.

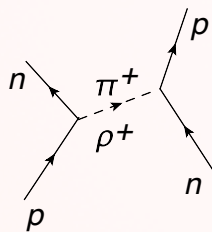
• A good starting point - the use of **effective ME interactions** adjusted to nuclear bulk properties on the mean-field level (J. Walecka, M. Serot, ..., P. Ring) and to supplement the **many-body correlation theory with proper subtraction techniques** (V. Tselyaev), in the covariant framework.

Charged mesons: π , ρ ,...

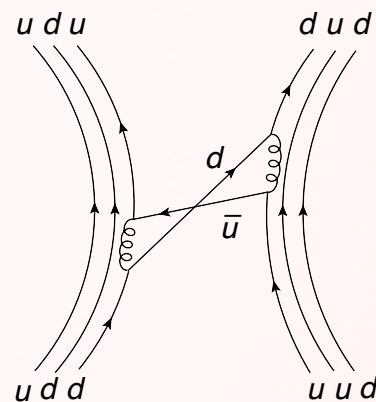
QCD



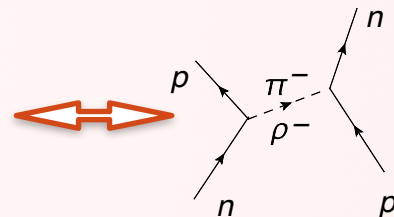
QHD



QCD

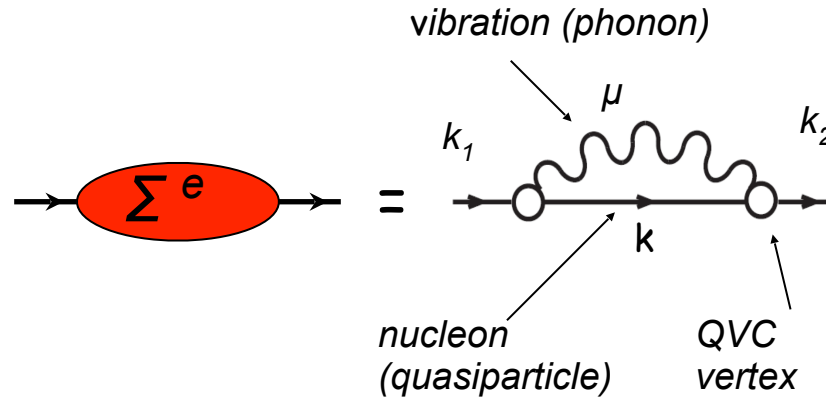


QHD



Beyond mean field: quasiparticle-vibration coupling (QVC)

Additional "potential"
 = "self-energy" =
 = "mass operator"
 with energy dependence



One-body propagator G : Dyson equation for Gor'kov Green function

$$\begin{array}{c}
 \begin{array}{c} k \quad k' \\ \text{---} \end{array} = \begin{array}{c} k \quad k' \\ \text{---} \end{array} + \begin{array}{c} k \quad k_1 \quad k_2 \quad k' \\ \text{---} \end{array} \\
 G(\epsilon) = G_0(\epsilon) + G_0(\epsilon) [\Sigma^{RHF} + \Sigma^e(\epsilon)] G(\epsilon)
 \end{array}$$

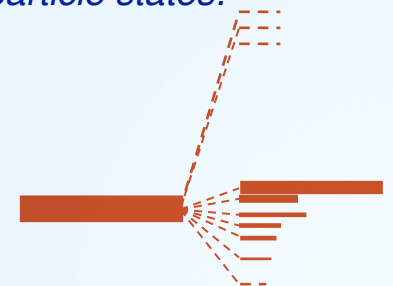
Dynamical self-energy:

$$\Sigma_{k_1 k_2}^{(e)\eta_1 \eta_2}(\epsilon) = \sum_{\eta=\pm 1} \sum_{k, \mu} \frac{\gamma_{\mu; k_1 k}^{\eta; \eta_1 \eta} \gamma_{\mu; k_2 k}^{\eta; \eta_2 \eta^*}}{\epsilon - \eta(E_k + \Omega_\mu - i\delta)}$$

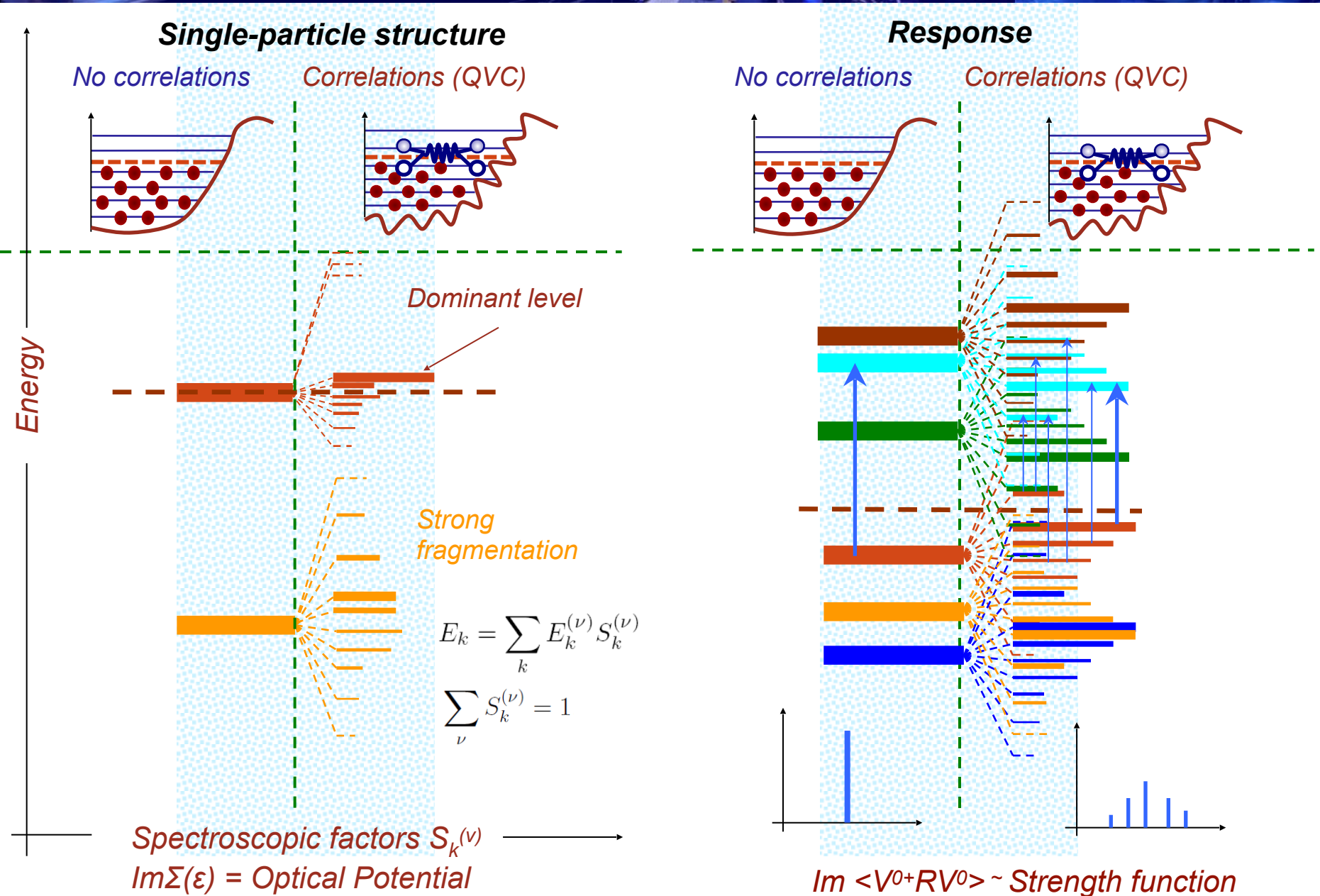
$$\eta = \pm 1$$

forward / backward

Single-particle states:

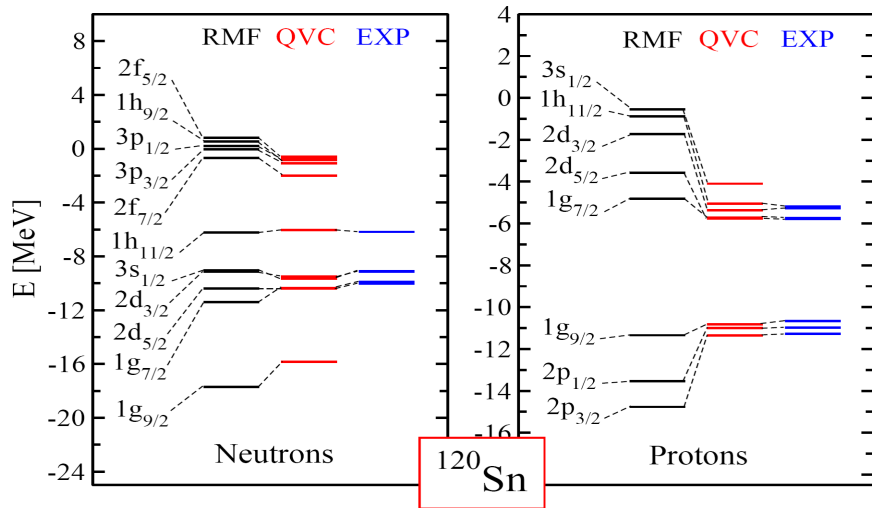


Fragmentation of single-particle states and particle-hole excitations due to the t -dependent interaction (correlations)



(Quasi)particle-vibration coupling (QVC, PVC): Pairing correlations of the superfluid type + coupling to phonons

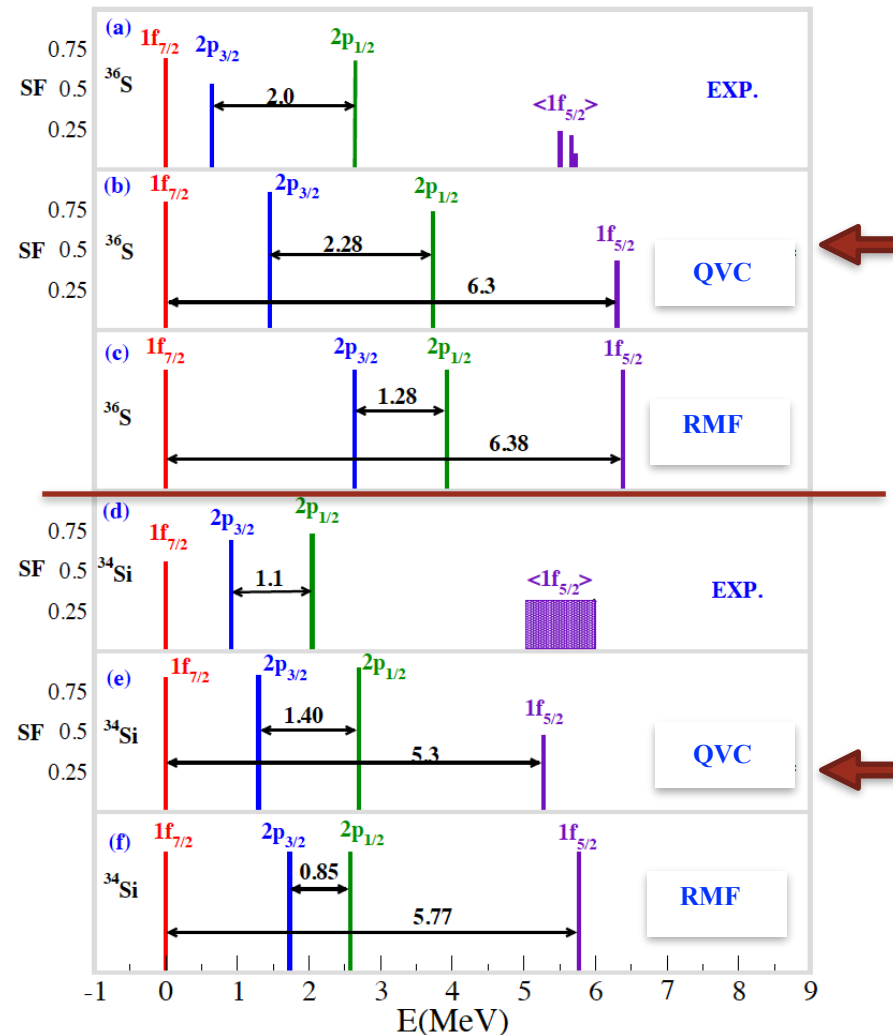
Dominant states and spectroscopic factors in ^{120}Sn :



(nlj) v	S^{th}	S^{exp}
$2d_{5/2}$	0.32	0.43
$1g_{7/2}$	0.40	0.60
$2d_{3/2}$	0.53	0.45
$3s_{1/2}$	0.43	0.32
$1h_{11/2}$	0.58	0.49
$2f_{7/2}$	0.31	0.35
$3p_{3/2}$	0.58	0.54

E. L., P. Ring, PRC 73, 044328 (2006)
E.L., PRC 85, 021303(R) (2012)

Spin-orbit splittings in ^{36}S
vs a bubble nucleus ^{34}Si ; neutron states:



Exp: Burgunder et al., PRL 112, 042502 (2014)
Th: K. Karakatsanis et al., PRC 95, 034901 (2017)

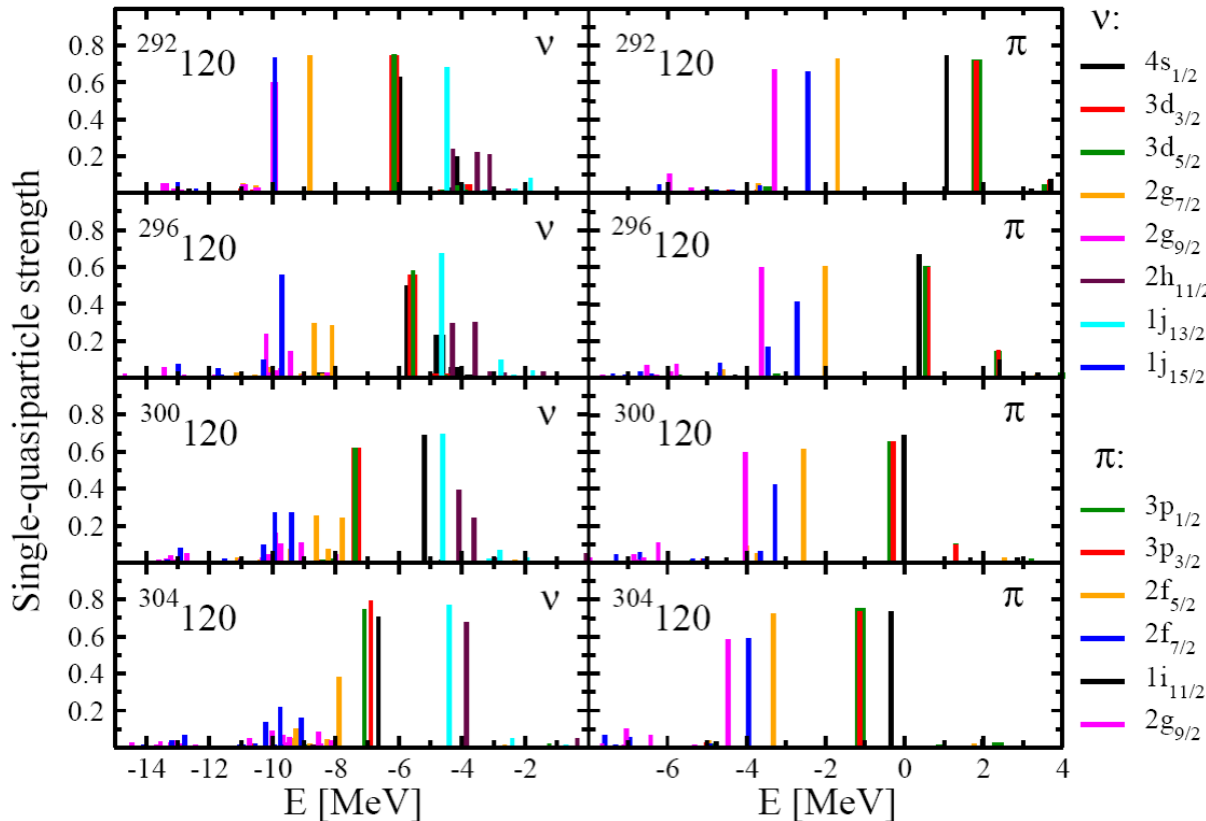
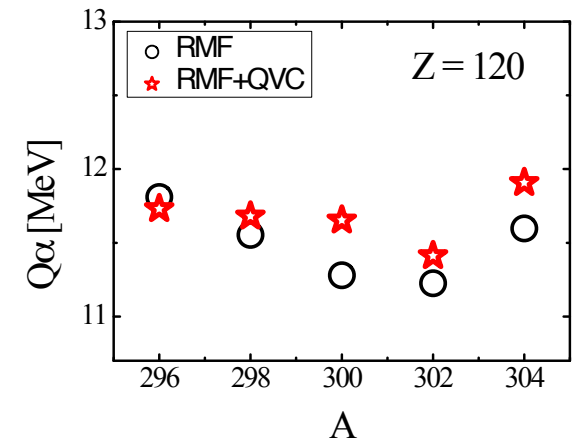
Shell evolution in superheavy $Z = 120$ isotopes

1. Relativistic Mean Field: *spherical minima*
2. π : collapse of pairing, *clear shell gap*
3. ν : survival of *pairing coexisting with the shell gap*
4. Very *soft* nuclei: large amount of low-lying collective vibrational modes (~ 100 phonons below 15 MeV)

Vibration corrections to binding energy (RQRPA)

$$E_{VC} = - \sum_{\mu} \Omega_{\mu} \sum_{k_1 k_2} |Y_{k_1 k_2}^{\mu}|^2$$

Vibration corrections to α -decay Q -values



Vibrational corrections:
 • Impact on the shell gaps
 • Smearing out the shell effects

Shell stabilization & vibration stabilization/destabilization (?)

E.L., PRC 85, 021303(R) (2012)

Response function in the neutral channel (leading approximation in QVC): relativistic quasiparticle time blocking approximation (RQTBA)

Response

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega)W(\omega)R(\omega)$$

Interaction kernel

$$W(\omega) = \underbrace{V_\sigma + V_\omega + V_\rho + V_e}_{\text{Static: RQRPA}} + \underbrace{\Phi(\omega) - \Phi(0)}_{\text{Subtraction to avoid double counting (if CDFT-based)}}$$

Static:
RQRPA

$$\left\{ \begin{aligned} v_\sigma(1, 2) &= -g_\sigma^2 \gamma_1^0 D_\sigma(1, 2) \gamma_2^0 \\ v_\omega(1, 2) &= +g_\omega^2 (\gamma^0 \gamma_\mu)_1 D_\omega^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu)_2 \\ v_\rho^V(1, 2) &= +g_\rho^2 (\gamma^0 \gamma_\mu \vec{\tau})_1 \vec{\tau}_1 \cdot \vec{\tau}_2 D_\rho^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu \vec{\tau})_2 \end{aligned} \right.$$

Subtraction
to avoid double
counting (if CDFT-based)

Dynamic
(retardation):

Quasiparticle-
vibration
coupling

in time blocking
approximation

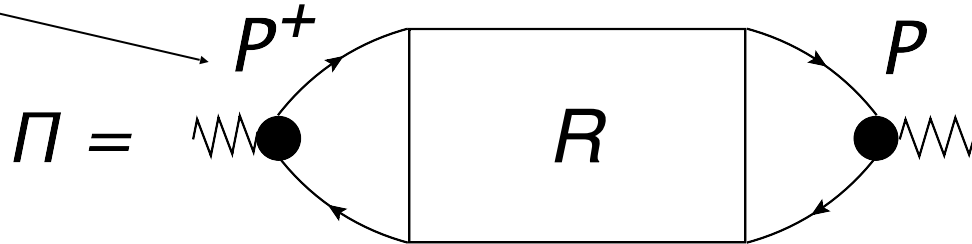
$$\begin{aligned} \Phi_{k_1 k_4, k_2 k_3}^\eta(\omega) &= \\ &= \sum_{\mu\xi} \delta_{\eta\xi} \left[\delta_{k_1 k_3} \sum_{k_6} \frac{\gamma_{\mu; k_6 k_2}^{\eta; -\xi} \gamma_{\mu; k_6 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_1} - E_{k_6} - \Omega_\mu} + \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma_{\mu; k_1 k_5}^{\eta; \xi} \gamma_{\mu; k_3 k_5}^{\eta; \xi*}}{\eta\omega - E_{k_5} - E_{k_2} - \Omega_\mu} \right. \\ &\quad \left. - \left(\frac{\gamma_{\mu; k_1 k_3}^{\eta; \xi} \gamma_{\mu; k_2 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_3} - E_{k_2} - \Omega_\mu} + \frac{\gamma_{\mu; k_3 k_1}^{\eta; \xi*} \gamma_{\mu; k_4 k_2}^{\eta; -\xi}}{\eta\omega - E_{k_1} - E_{k_4} - \Omega_\mu} \right) \right] \end{aligned}$$

Response to an external field: strength function

Nuclear Polarizability:

$$\Pi_{PP}(\omega) = P^\dagger R(\omega) P := \sum_{k_1 k_2 k_3 k_4} P_{k_1 k_2}^* R_{k_1 k_4, k_2 k_3}(\omega) P_{k_3 k_4}$$

External
field



Strength function:

$$S(E) = -\frac{1}{\pi} \lim_{\Delta \rightarrow +0} \text{Im} \Pi_{PP}(E + i\Delta)$$

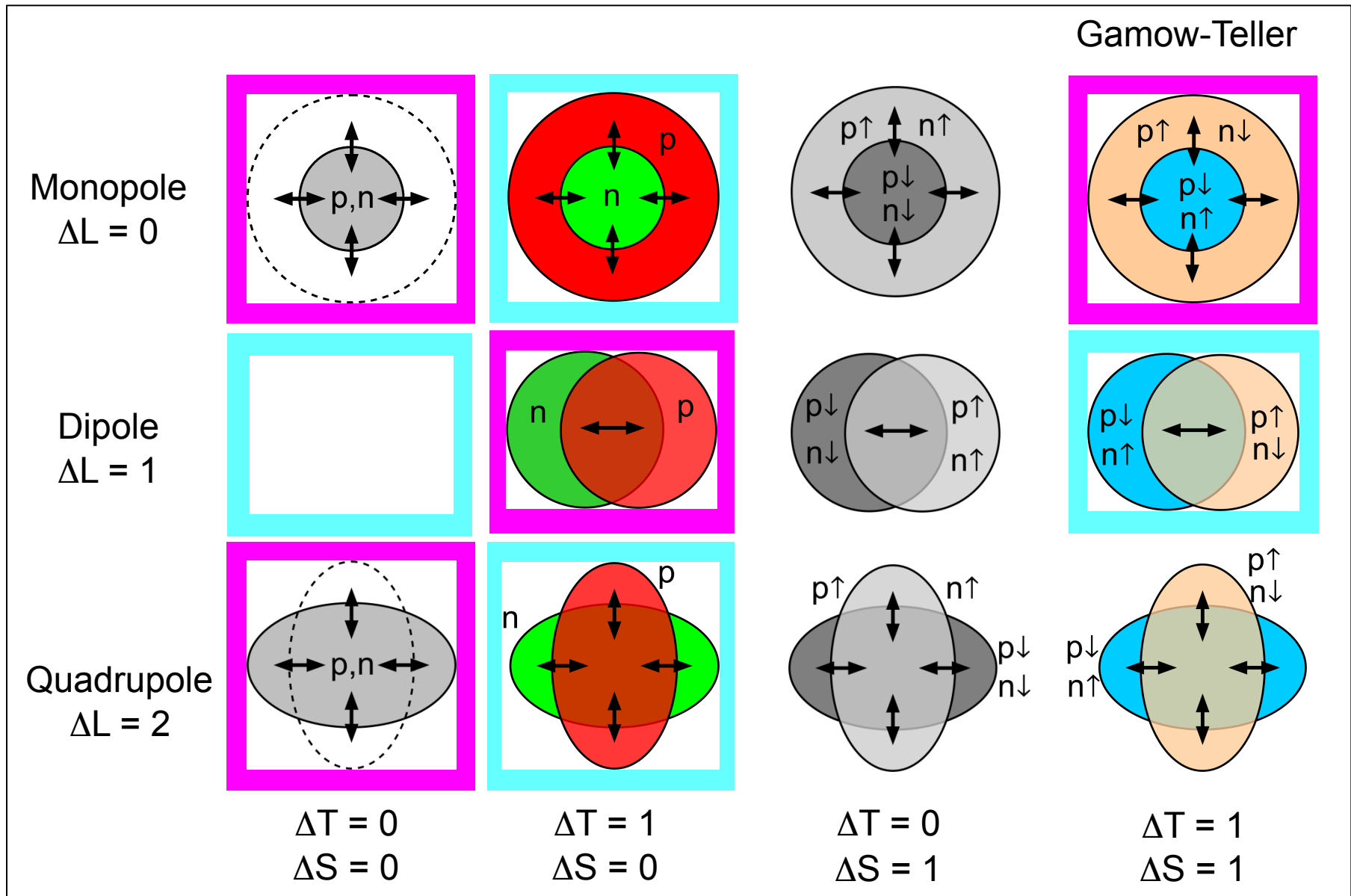
Transition density:

$$\rho_{12}^\nu = \langle 0 | \psi_2^\dagger \psi_1 | \nu \rangle$$

Response function:

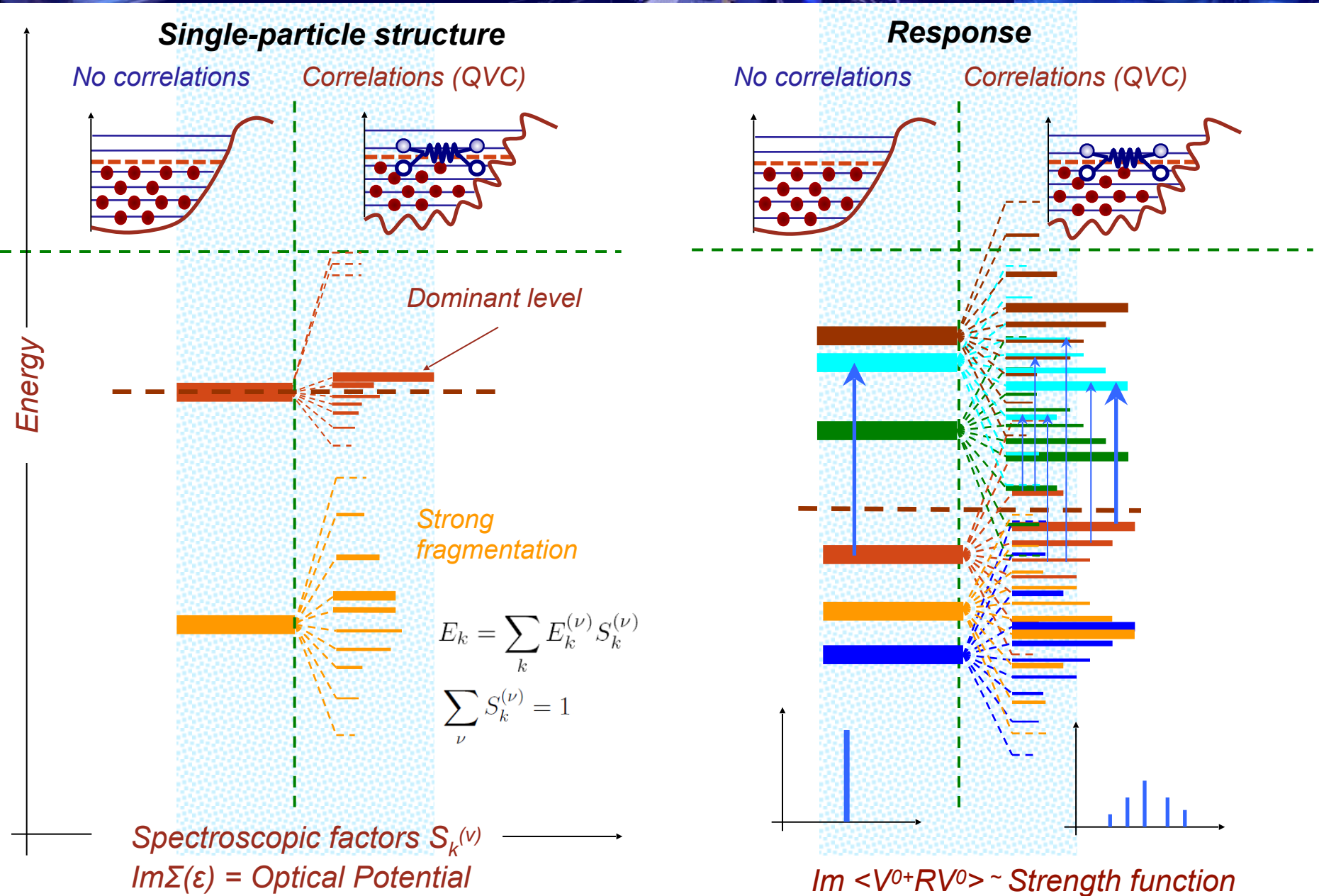
$$R_{12,1'2'}(\omega) = \sum_\nu \left[\frac{\rho_{21}^\nu \rho_{2'1'}^{\nu*}}{\omega - \omega_\nu + i\delta} - \frac{\rho_{12}^{\nu*} \rho_{1'2'}^\nu}{\omega + \omega_\nu - i\delta} \right]$$

Nuclear excitation modes



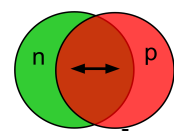
* M. N. Harakeh and A. van der Woude: *Giant Resonances*

Fragmentation of single-particle states and particle-hole excitations due to the t -dependent interaction (correlations)

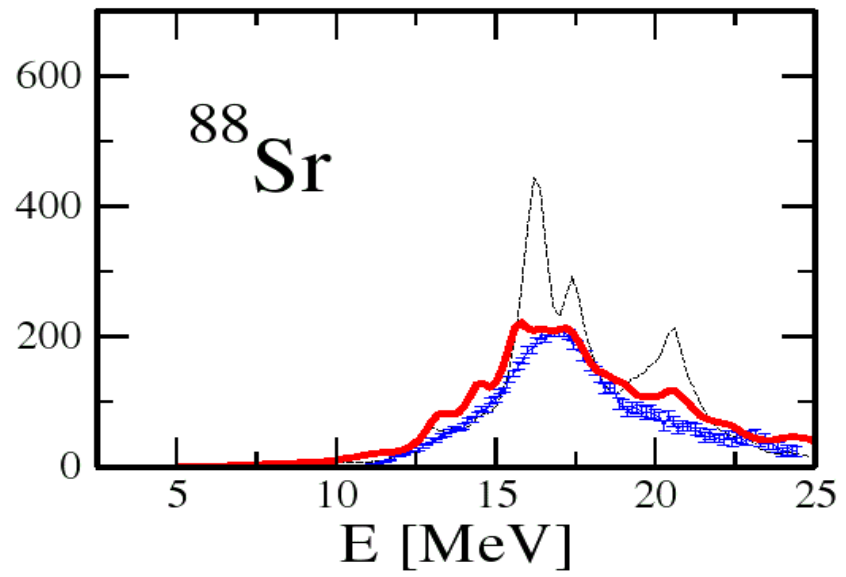
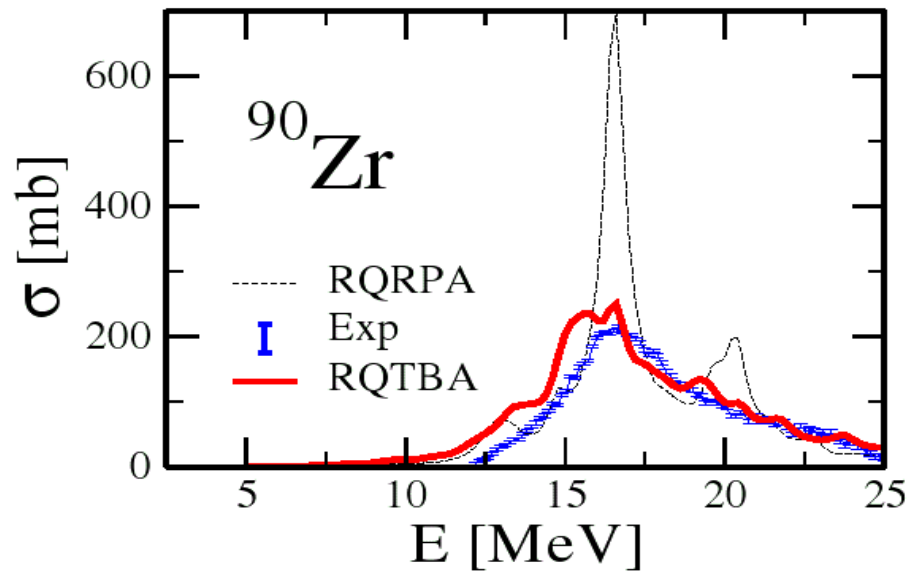
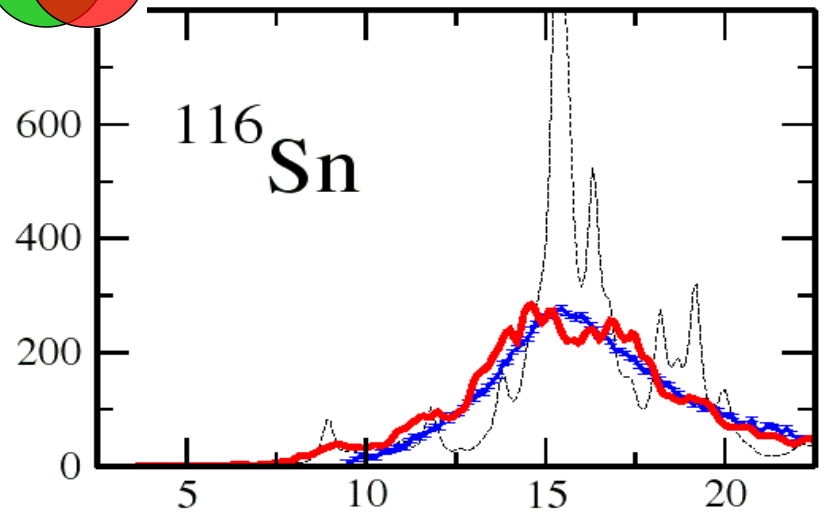
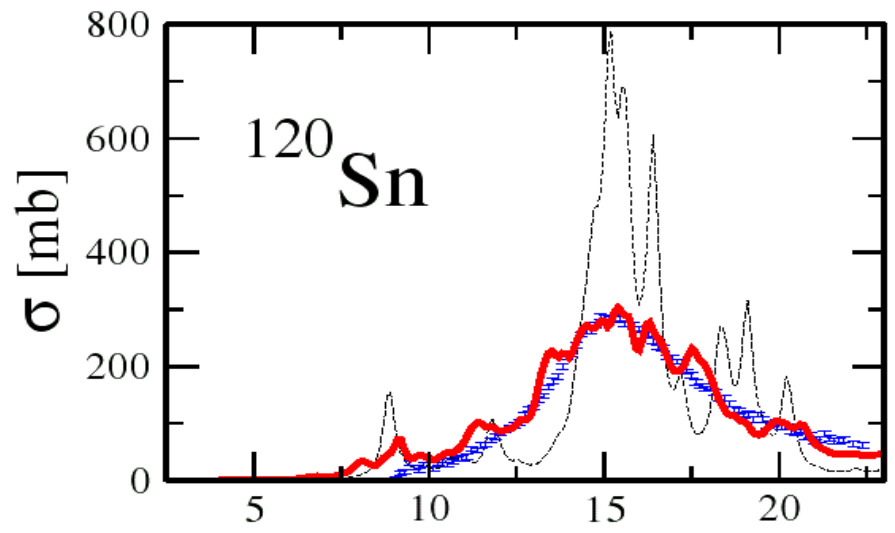


Dipole response of medium-mass and heavy nuclei within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)

Test case: E1 (IVGDR) in stable nuclei

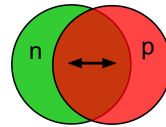
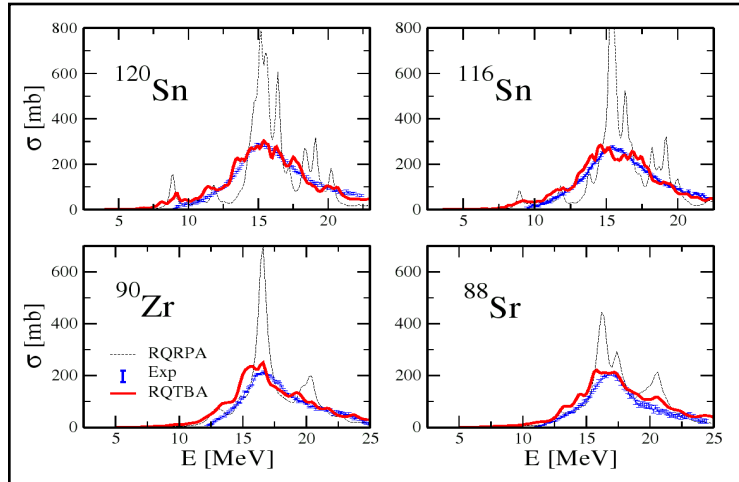


$$P_{1M} = \frac{eN}{A} \sum_{i=1}^Z r_i Y_{1M}(\Omega_i) - \frac{eZ}{A} \sum_{i=1}^N r_i Y_{1M}(\Omega_i)$$



Response of medium-mass and heavy nuclei within Relativistic (Quasiparticle) Time Blocking Approximation (R(Q)TBA)

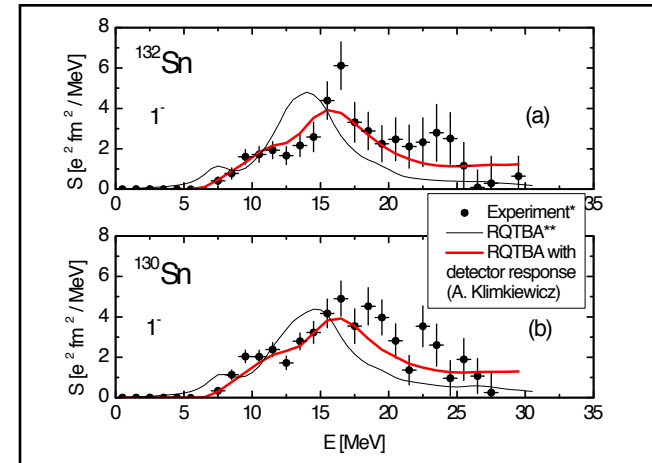
Giant dipole resonance (GDR) in stable nuclei



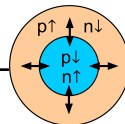
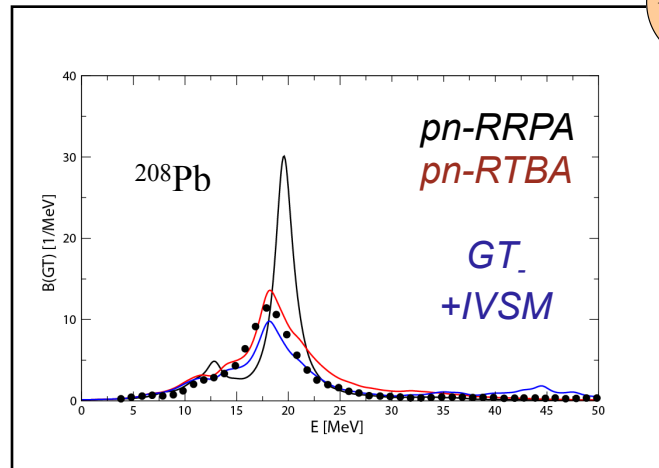
**E. L., P. Ring, and V. Tselyaev et al.,
Phys. Rev. C 78, 014312 (2008).
Phys. Rev. C 79, 054312 (2009).
Phys. Rev. Lett. 105, 022502 (2010).
Phys. Rev. C 88, 044320 (2013).

J. Endres, E. Litvinova, D. Savran et al.,
Phys. Rev. Lett. 105, 212503 (2010).

GDR in neutron-rich Sn



Gamow-Teller resonance

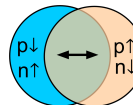


E.L., B.A. Brown, D.-L. Fang, T. Marketin, R.G.T. Zegers,
Phys. Lett. B 730, 307 (2014).

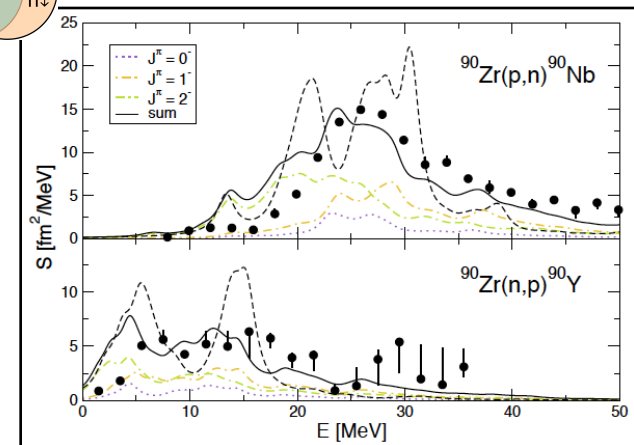
T. Marketin, E.L., D. Vretenar,
P. Ring,
Phys. Lett. B 706, 477 (2012).

C. Robin and E. Litvinova,
Eur. Phys. J. A 52, 205 (2016).

E. Litvinova, C. Robin, and I.A. Egorova,
Phys. Lett. B 776, 72 (2018).



Spin-dipole resonance

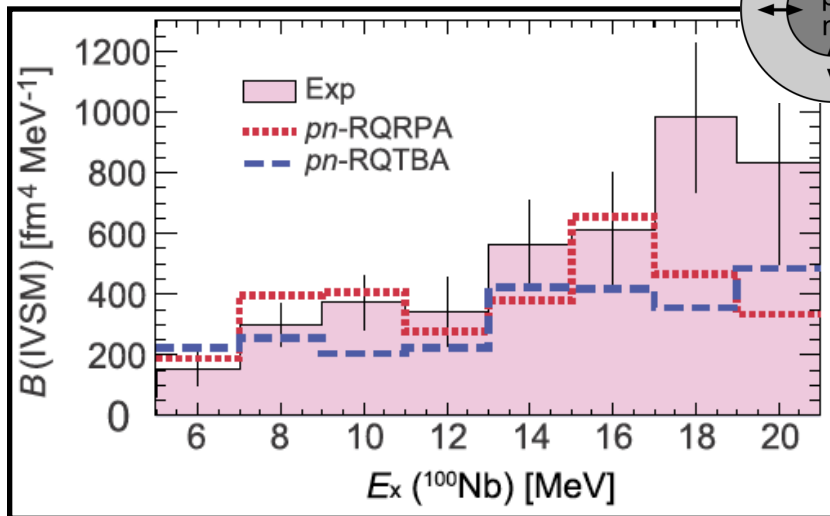


The dynamical part of the interaction kernel (quasiparticle-vibration coupling) brings a significant overall improvement to the description of both high-frequency and low-lying strengths.

Exotic spin-isospin excitations

Recent measurements at MSU

$^{100}\text{Mo} (t, ^3\text{He}) ^{100}\text{Nb}$



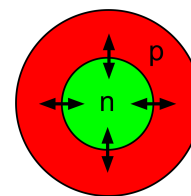
Isovector spin monopole resonance

K. Miki, R.G.T. Zegers, ..., E.L., ..., C. Robin et al.,
Phys. Lett. B 769, 339 (2017)

Recent developments on the spin-isospin response:

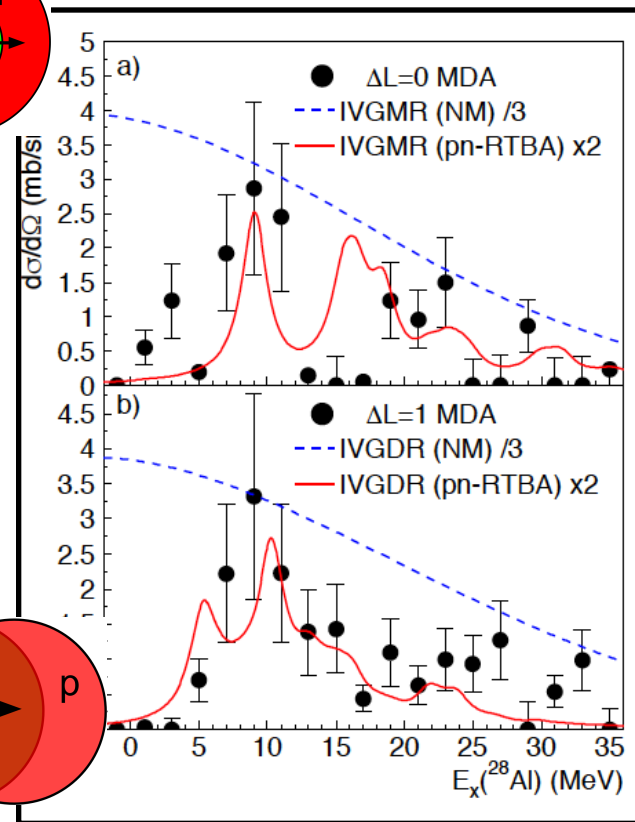
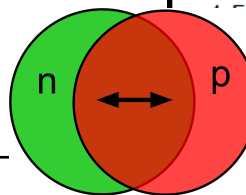
- Superfluid pairing included (C. Robin)
- Coupling to charge-exchange phonons (C. Robin)
- Beta decay: well described, quenching: explained (C. Robin, E.L.)
- QVC-induced ground state correlations (C. Robin)
- Meson-exchange pn-pairing (C. Robin, E.L.)
- 3p3h-configurations (E.L.)
- Finite temperature (E.L., H. Wibowo, C. Robin), see below

$^{28}\text{Si} (^{10}\text{Be}, ^{10}\text{B}) ^{28}\text{Al}$



*Isovector
monopole*

*Isovector
dipole*



M. Scott, R.G.T. Zegers, ...,
E.L., ..., C. Robin et al.,
Phys. Rev. Lett. 118, 172501 (2017)

Why proton-neutron pairing?

- **In a complete theory** we need all channels in both $T=1$ and $T=0$ domains:

$$\hat{R} = \left\{ R^{(ph)}, R^{(hp)}, R^{(pp)}, R^{(hh)} \right\}$$

$$\hat{R}(\omega) = \hat{R}^{(0)}(\omega) + \hat{R}^{(0)}(\omega)W[\hat{R}(\omega)]\hat{R}(\omega)$$

- **Experiment:**
 - odd-odd $N = Z$, $A < 40$ nuclei: ($T=0$, $J>0$) ground states,
 - odd-odd $N=Z$, $A > 40$, nuclei: ($T=1$, $J=0$) ground states (except ^{58}Cu)
- **Theory:**
 - Possibility of $T=0$ pairing condensate in heavy $N \sim Z$ nuclei as a consequence of the attractive proton-neutron interaction in the 3S_1 channel
 - Influence of the dynamical correlations on pn -pairing
 - Influence of pn -pairing (treated with a free strength parameter) on GTR in QRPA
- **Recent Review:** ($T=1$, $J=0$) pairing is more likely than ($T=0$, $J=1$) between $A=40$ and $A=100$

J. Engel, et al., PLB 389, 211 (1996).
K. Langanke and G. Martínez-Pinedo,
Fifty Years of Nuclear BCS, Ch. 12, p. 154 (2013).
W. Satula and R. Wyss, PLB 393, 1 (1997).
A. L. Goodman, PRC 60, 014311 (1999).
G. F. Bertsch and Y. Luo, PRC 81, 064320 (2010).
A. Gezerlis et al., PRL106, 252502 (2011).
K. Yoshida, PRC 90, 031303 (2014).

S. S. Zhang et al., PRC 93, 044329 (2016) (NM).
F. J. W. Hahne et al., Ann. Phys. 104, 251 (1977).

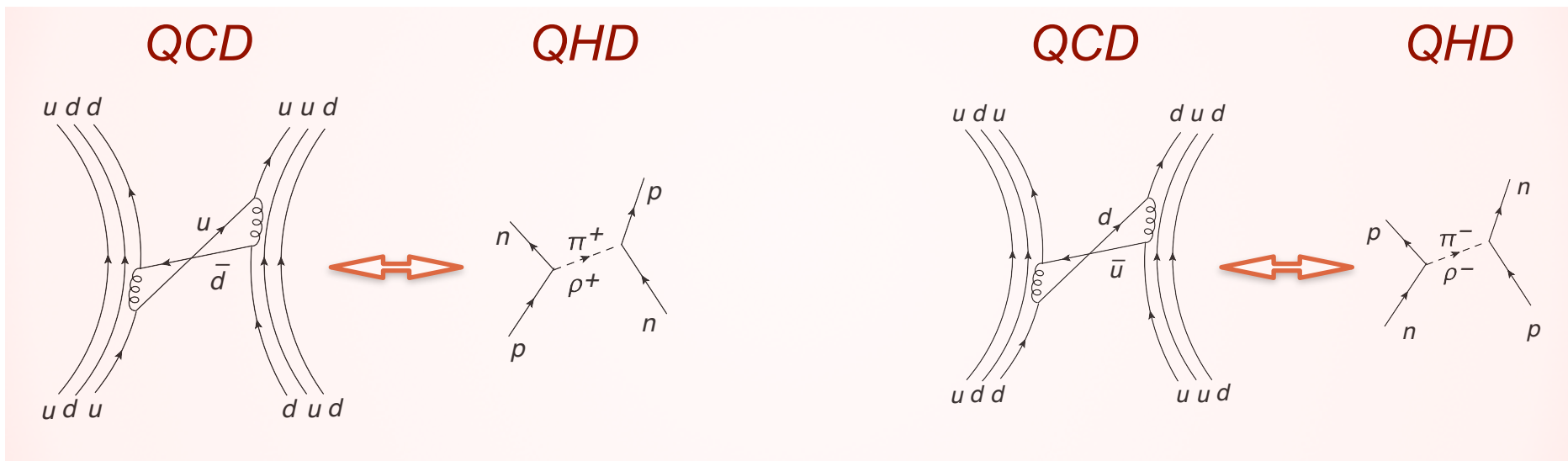
T. Nikšić et al., PRC 71, 014308 (2005).
J. Engel et al., PRC 60, 014302 (1999).

S. Frauendorf and A. Macchiavelli,
Progr. Part. Nucl. Phys. 78, 24 (2014).

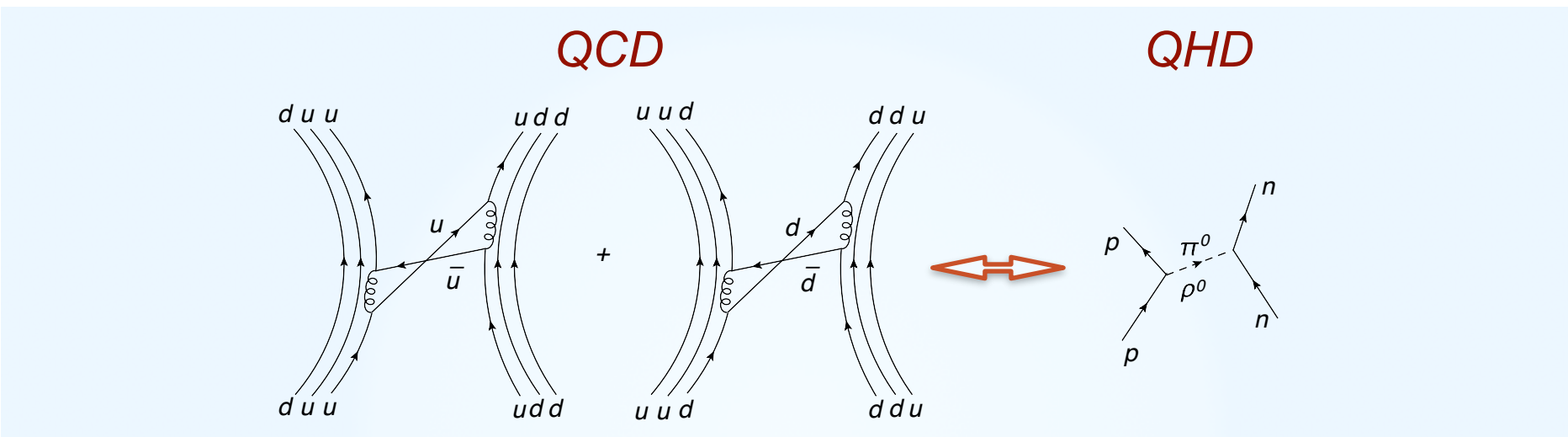
• **Open questions:** What are the mechanisms underlying the pn -pairing? Can we constrain them?

The underlying mechanism of pn-pairing: isovector meson-exchange interaction

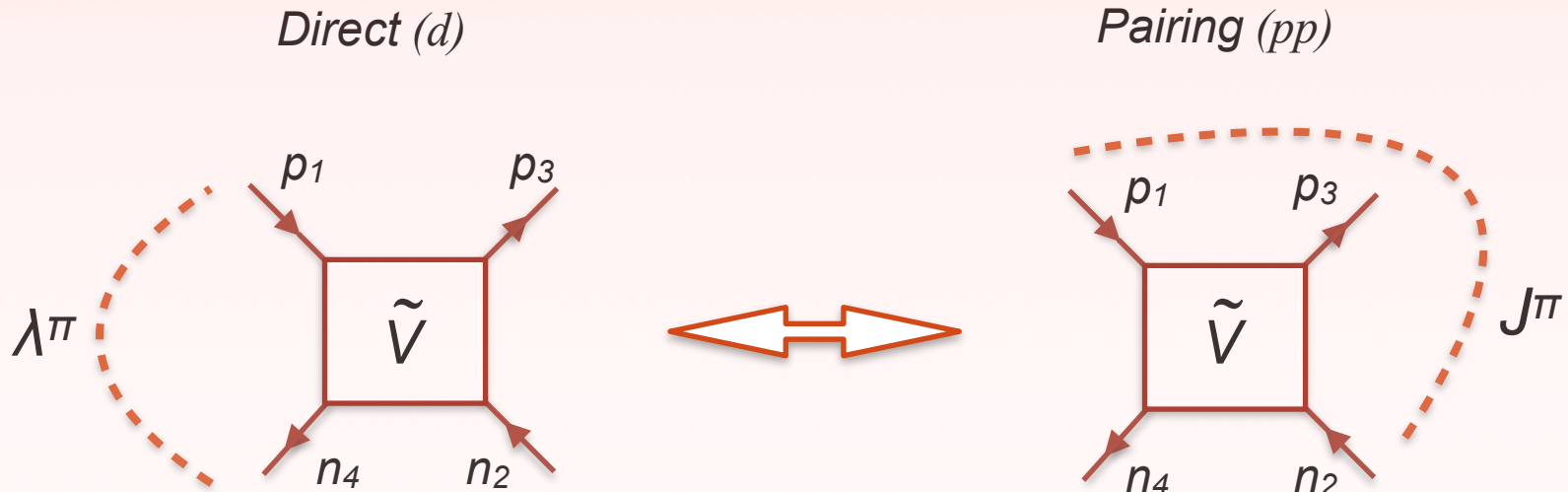
Charged mesons:



Neutral mesons:



From direct to pairing channel



Recoupling

$$\langle p_1 n_2 || \tilde{V}^{(pp)} || p_3 n_4 \rangle^J = \sum_{\lambda} (2\lambda + 1) (-1)^{j_3 + j_4 + J} \times \\ \times \left\{ \begin{matrix} j_1 & j_2 & J \\ j_3 & j_4 & \lambda \end{matrix} \right\} \langle p_1 n_2 || \tilde{V}^{(d)} || p_3 n_4 \rangle^{\lambda}$$

- G.E. Brown, T.T.S. Kuo, J.W. Holt, and S.Lee, *The NN-interaction and the Nuclear Many-body Problem* (2010)
- M. Serra, *PhD Thesis, TUM* (2001)
- M. Serra, P. Ring, *The Nuclear Many-Body Problem 2001*, p. 169

Isospin transfer response function: proton-neutron particle-particle relativistic time blocking approximation (pn-pp-RTBA)

Response

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega) \bar{W}(\omega) R(\omega)$$

Interaction kernel

$$\bar{W}(\omega) = \underbrace{V_\rho + V_\pi + V_{\delta\pi}}_{\text{free-space coupling}} + \underbrace{\Phi(\omega)}_{\text{fixed strength: ab initio if the Fock term is present}} - \underbrace{\Phi(0)}_{\text{subtraction to avoid double counting of } \rho \text{ (if CDFT-based)}}$$

Static:
R(Q)RPA

$$\left\{ \begin{aligned} V_\rho(1, 2) &= g_\rho^2 \vec{\tau}_1 \vec{\tau}_2 (\beta \gamma^\mu)_1 (\beta \gamma_\mu)_2 D_\rho(\mathbf{r}_1, \mathbf{r}_2) \\ V_\pi(1, 2) &= - \left(\frac{f_\pi}{m_\pi} \right)^2 \vec{\tau}_1 \vec{\tau}_2 (\boldsymbol{\Sigma}_1 \nabla_1) (\boldsymbol{\Sigma}_2 \nabla_2) D_\pi(\mathbf{r}_1, \mathbf{r}_2), \\ V_{\delta\pi}(1, 2) &= g' \left(\frac{f_\pi}{m_\pi} \right)^2 \vec{\tau}_1 \vec{\tau}_2 \boldsymbol{\Sigma}_1 \boldsymbol{\Sigma}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) \end{aligned} \right.$$

Subtraction
to avoid double
counting of ρ
(if CDFT-based)

free-space
coupling

fixed strength:
ab initio
if the Fock term
is present

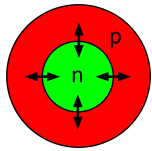
Dynamic
(retardation),
2-nd order:

quasiparticle-
vibration
coupling

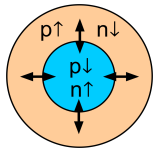
in time blocking
approximation

$$\begin{aligned} \Phi_{p_1 n_2, p_3 n_4}^\eta(\omega) &= \\ &= \eta \sum_\mu \left[\delta_{p_1 p_3} \sum_{n_6} \frac{\gamma_{\mu; n_2 n_6}^\eta \gamma_{\mu; n_4 n_6}^{\eta*}}{\omega - \varepsilon_{p_1} - \varepsilon_{n_6} - \eta \Omega_\mu} + \delta_{n_2 n_4} \sum_{p_5} \frac{\gamma_{\mu; p_1 p_5}^\eta \gamma_{\mu; p_3 p_5}^{\eta*}}{\omega - \varepsilon_{p_5} - \varepsilon_{n_2} - \eta \Omega_\mu} \right. \\ &\quad \left. - \left(\frac{\gamma_{\mu; p_1 p_3}^\eta \gamma_{\mu; n_4 n_2}^{\eta*}}{\omega - \varepsilon_{p_3} - \varepsilon_{n_2} - \eta \Omega_\mu} + \frac{\gamma_{\mu; p_3 p_1}^{\eta*} \gamma_{\mu; n_2 n_4}^\eta}{\omega - \varepsilon_{p_1} - \varepsilon_{n_4} - \eta \Omega_\mu} \right) \right] \end{aligned}$$

Response in proton-neutron particle-particle (deuteron transfer) channel: quest for deuteron condensate and pn-pairing



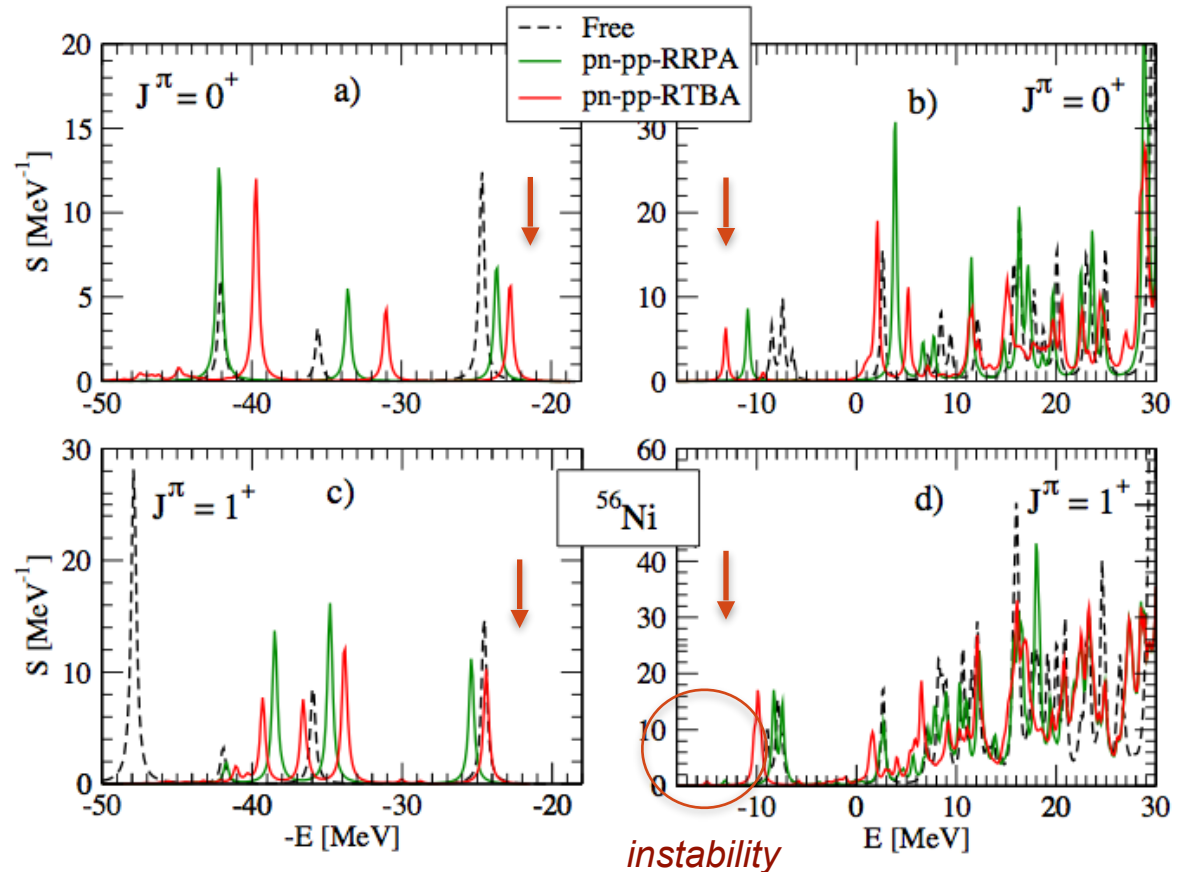
$$J^\pi = 0^+$$



$$J^\pi = 1^+$$

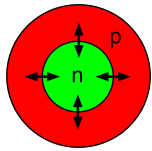
- The pairing interaction in the proton-neutron channel is a delicate interplay of the ρ -meson and π exchanges, and the exchange by core vibrations
- In the odd-odd $N=Z$ nuclei around closed shells the lowest 0^+ states are accurately described

^{54}Co ← ^{56}Ni → ^{58}Cu

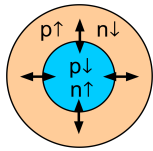


- The lowest 1^+ solutions in the addition channel become unstable, which may indicate the onset of the triplet deuteron pairing
 - The particle-vibration coupling provides an overall attractive interaction and, thus, reinforces the pairing
- E.L., C. Robin, I. Egorova, *Phys. Lett. B* 776, 72 (2018)

Response in proton-neutron particle-particle (deuteron transfer) channel: quest for deuteron condensate and pn-pairing



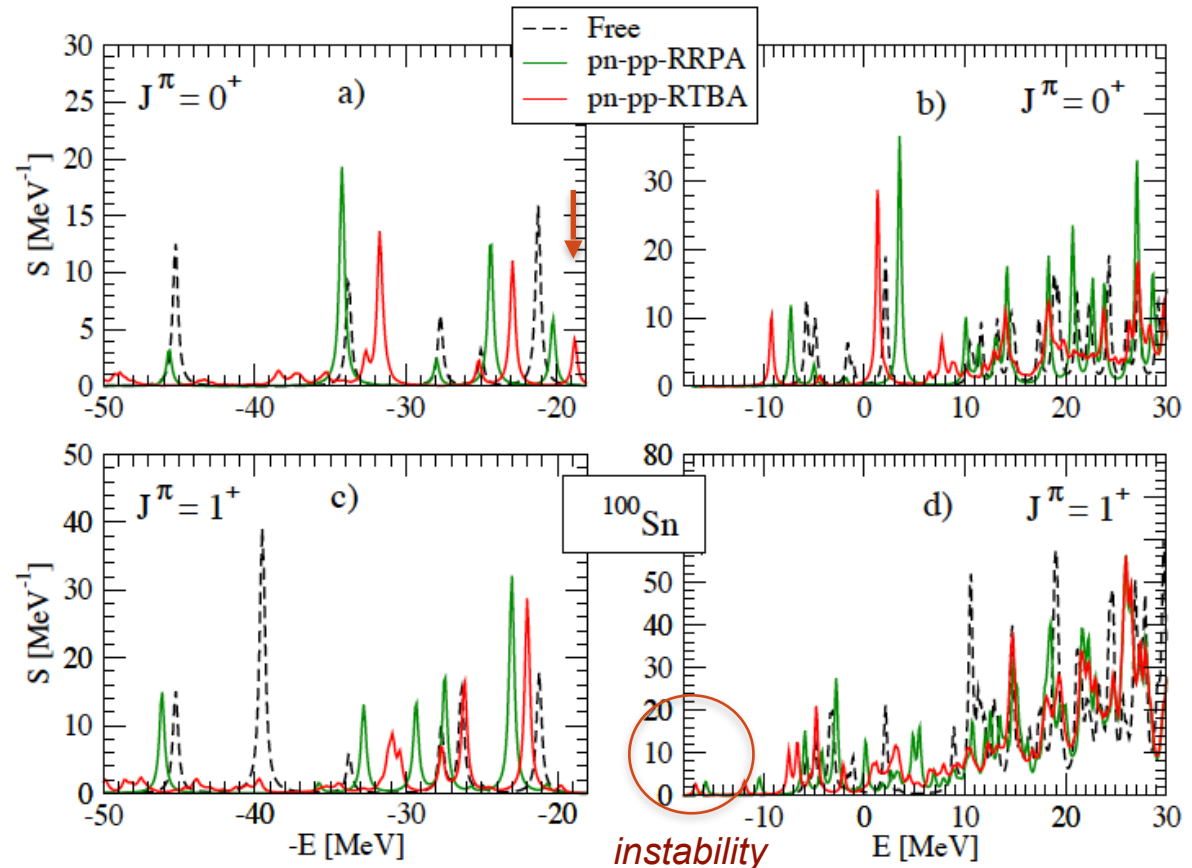
$$J^\pi = 0^+$$



$$J^\pi = 1^+$$

- The pairing interaction in the proton-neutron channel is a delicate interplay of the ρ -meson and π exchanges, and the exchange by core vibrations
- In the odd-odd $N=Z$ nuclei around closed shells the lowest 0^+ states are accurately described

^{98}In ← ^{100}Sn → ^{102}Sb



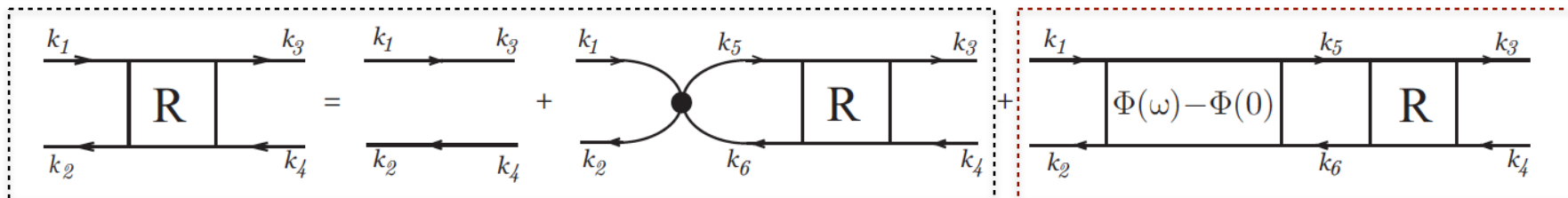
- The lowest 1^+ solutions in the addition channel become unstable, which may indicate the onset of the triplet deuteron pairing
- The particle-vibration coupling provides an overall attractive interaction and, thus, reinforces the pairing

In progress...

E.L., C. Robin, I. Egorova, *Phys. Lett. B* 776, 72 (2018)

Higher orders: toward a "complete" theory

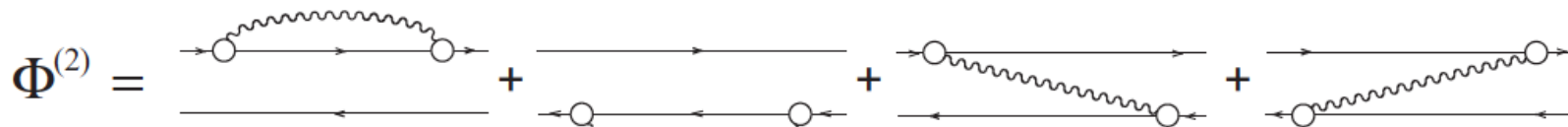
Bethe-Salpeter (Dyson) equation for the ph-response:



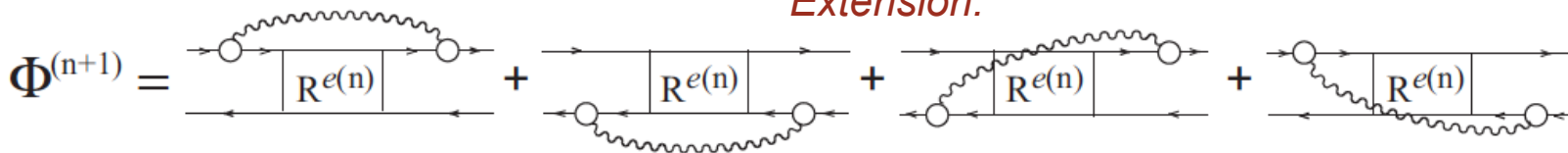
(Q)RPA

"Conventional" Nuclear Field Theory and Time Blocking Approximation:

(Q)PVC

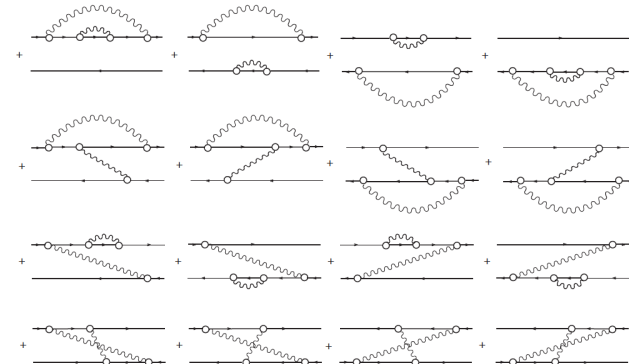
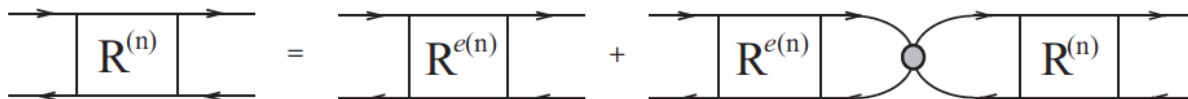
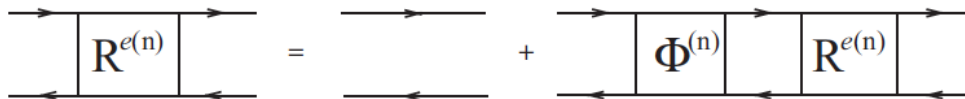


Extension:



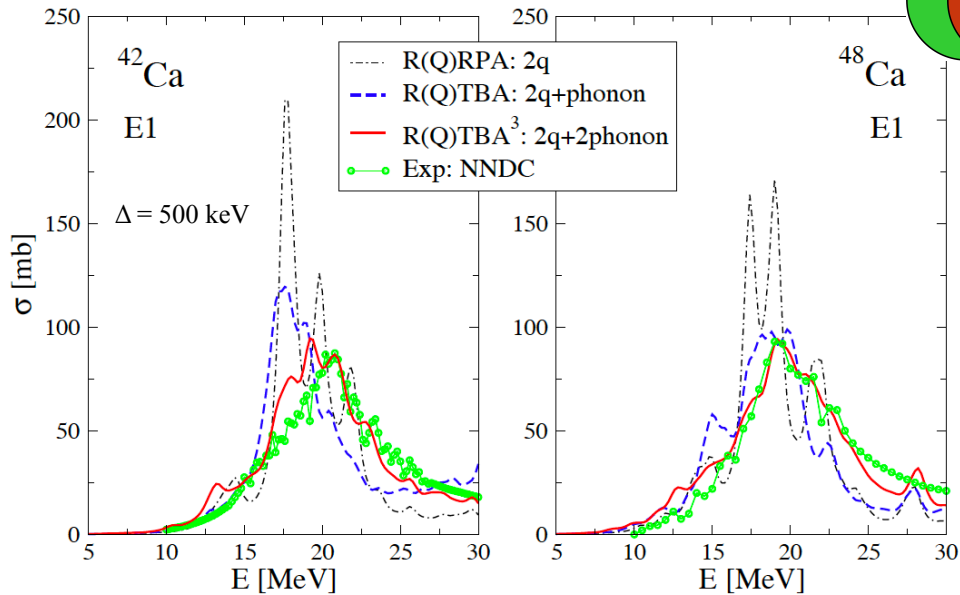
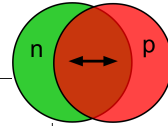
$n = 3$: 3p3h (3-body effects)

n -th order correlated propagator:

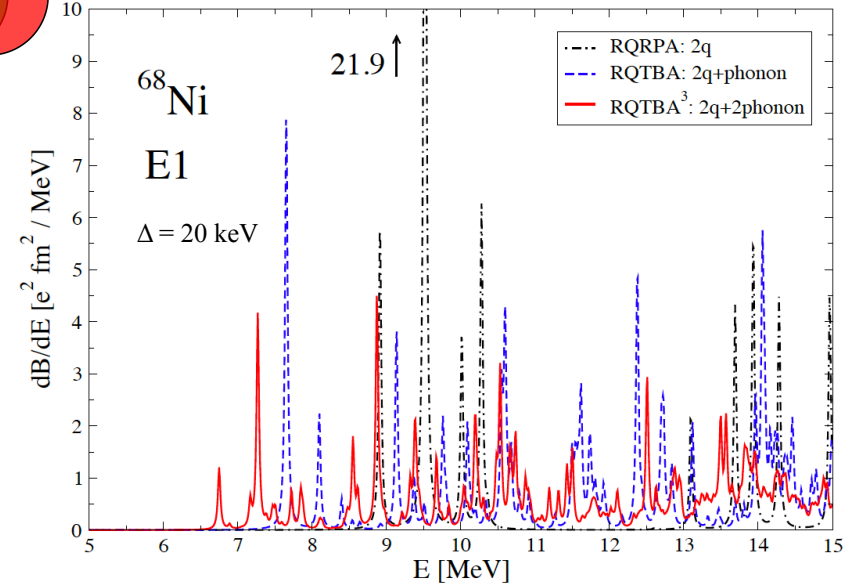


RQTBA³ with correlated 3p3h configurations: 2q+2phonon (preliminary results)

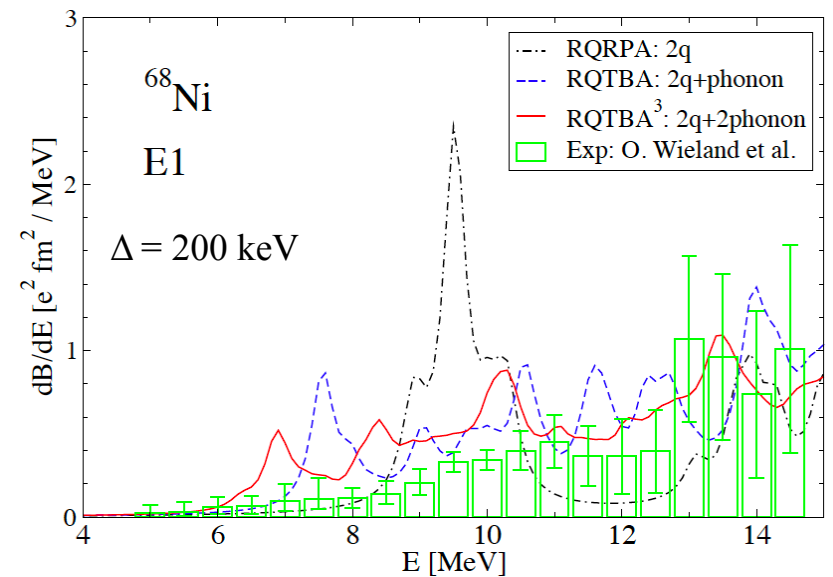
Giant Dipole Resonance in Ca isotopes



Pygmy Dipole Resonance in ⁶⁸Ni



- The new complex configurations 2q+2phonon included for the first time enforce fragmentation and spreading toward higher and lower energies, thus, modifying both giant and pygmy dipole resonances;
- Exp. Data: V.A. Erokhova et al., Bull. Rus. Acad. Phys. 67, 1636 (2003); O. Wieland et al., Phys. Rev. C 98, 064313;
- RQTBA³ demonstrates an overall systematic improvement of the description of nuclear excited states heading toward spectroscopic accuracy without strong limitations on masses and excitation energies.



Finite temperature Giant dipole resonance: data from HIC

- J.J. Gaardhøje, C. Ellegaard, B. Herskind, S.G. Steadman, *Phys. Rev. Lett.* 53, 148 (1984).
- J.J. Gaardhøje, C. Ellegaard, B. Herskind, et al., *Phys. Rev. Lett.* 56, 1783 (1986).
- D.R. Chakrabarty, S. Sen, M. Thoennessen et al., *Phys. Rev. C* 36, 1886 (1987).
- A. Bracco, J.J. Gaardhøje, A.M. Bruce et al., *Phys. Rev. Lett.* 62, 2080 (1989).
- G. Enders, F.D. Berg, K. Hagel, et al., *Phys. Rev. Lett.* 69, 249 (1992).
- H.J. Hofmann, J.C. Bacelar, M.N. Harakeh, et al., *Nucl. Phys. A* 571, 301 (1994).
- E. Ramakrishnan, T. Baumann, A. Azhari et al., *Phys. Rev. Lett.* 76, 2025 (1996).
- P. Heckman, D. Bazin, J.R. Beene, Y. Blumenfeld, et al., *Phys. Lett. B* 555, 43 (2003).
- F. Camera, A. Bracco, V. Nanal, et al., *Phys. Lett. B* 560, 155 (2003).
- M. Thoennessen, *Nucl. Phys. A* 731, 131 (2004).
- O. Wieland et al., *Phys. Rev. Lett.* 97, 012501 (2006).
- **A (relatively) recent review:**
D. Santonocito and Y. Blumenfeld, *Eur. Phys. J. A* 30, 183 (2006).

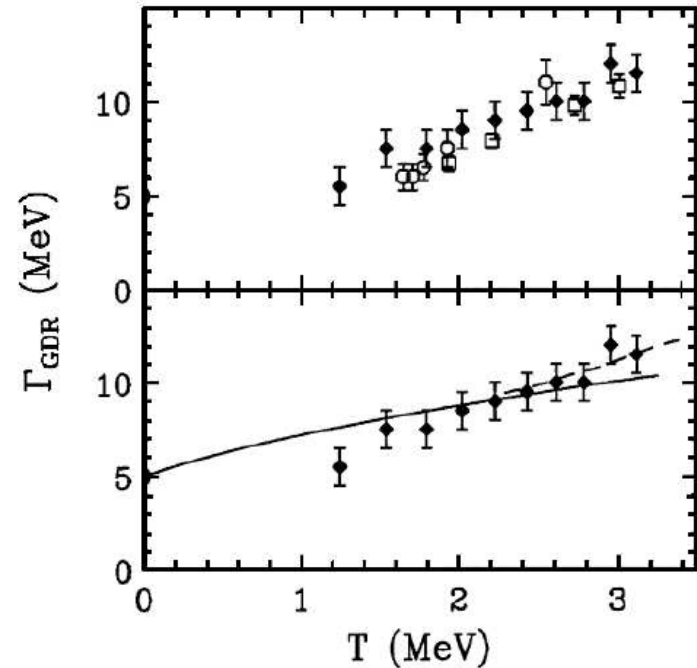


Fig. 4. Comparison of the GDR width extracted from 50A MeV α -particle inelastic-scattering experiment (full symbols) on ^{120}Sn [29] and from fusion reaction data (open symbols) on $^{108-112}\text{Sn}$ nuclei [23–25]. The lower part shows the comparison of the α inelastic-scattering experiment results with adiabatic coupling calculations [32] shown as a full line. The dashed line includes the contribution to the width due to particle evaporation width [35].

General observations:

- **Broadening of the GDR with temperature**
- **Disappearance of the GDR at $T \sim 5$ MeV**

Low-energy strength functions at finite T

Experimental data from transfer reactions :

• Spherical mid-mass nuclei

A. Schiller, M. Thoennessen, *Atomic Data and Nuclear Data Tables* 93, 549 (2007).
A. Voinov et al., *Phys. Rev. Lett.* 93, 142504.
M. Guttormsen et al., *Phys. Rev. C* 71, 044307 (2005).
A. C. Larsen et al., *Phys. Rev. C* 76, 044303 (2007).
E. Algin et al., *Phys. Rev. C* 78, 054321 (2008).
M. Wiedeking et al., *Phys. Rev. Lett.* 108, 162503.
A.C. Larsen, S. Goriely, *Phys. Rev. C* 82, 014318 (2010).

• Tin nuclei (no enhancement)

H.K. Toft et al., *Phys. Rev. C* 81, 064311 (2010).
H.K. Toft et al., *Phys. Rev. C* 83, 044320 (2011).

• Heavier masses

B. V. Kheswa et al., *Phys. Lett. B* 744, 268 (2015).

• Rare-earth nuclei:

A. Simon et al., *Phys. Rev. C* 93, 034303 (2016).

Theory:

• Shell-model

R. Schwengner, S. Frauendorf, and A. C. Larsen, *Phys. Rev. Lett.* 111, 232504 (2013).
B. A. Brown and A. C. Larsen, *Phys. Rev. Lett.* 113, 252502 (2014).
K. Sieja, *Phys. Rev. Lett.* 119, 052502 (2017).

• Finite-temperature QFT calculations (FT-CQRPA)

E. Khan, N. Van Giai, M. Grasso, *Nucl. Phys. A* 731, 311 (2004).
E. Litvinova and N. Belov, *Phys. Rev. C* 88, 031302(R) (2013).

• Microscopic calculations agree on the enhancement of $M1$ strength at $E_\gamma < 1$ MeV and $E1$ strength at higher energies. Important implications for the r -process.

Microscopic and phenomenological finite-temperature approaches

• *Finite-Temperature Green function formalism*

*T. Matsubara, Prog. Theor. Phys. 14, 351 (1955).
A.A. Abrikosov, L.P. Gor'kov, and I.E. Dzyaloshinski,
Methods of Quantum Field Theory in Statistical Physics*

• *Finite-Temperature Hartree-Fock, Hartree-Fock-Bogolyubov and random phase approximations*

*A.L. Goodman, Nucl. Phys. A352, 30 (1981).
P. Ring et al., Nucl. Phys. A419, 261 (1983).
H.M. Sommermann, Ann. Phys. 151, 163 (1983).
Y.F. Niu et al., Phys. Lett. B 681, 315 (2009).*

• *Continuum RPA and QRPA at finite temperature*

*J. Bar-Touv, Phys. Rev. C 32, 1369 (1985).
V.A. Rodin and M.G. Urin, PEPAN 31, 975 (2000).
E.V. Litvinova, S.P. Kamerdzhiev, and V.I. Tselyaev,
Phys. At. Nucl. 66, 558 (2003).
E. Khan, N. Van Giai, M. Grasso,
Nucl. Phys. A731, 311 (2004).
E. Litvinova and N. Belov,
Phys. Rev. C88, 031302(R) (2013).*

• *Finite-Temperature approaches beyond RPA*

*P.F. Bortignon et al., Nucl. Phys. A460, 149 (1985).
D. Lacroix et al., PRC 58, 2154 (1998).*

• *Theory of thermal shape fluctuations*

*W. E. Ormand et al., Nucl. Phys. A 519, 61 (1990).
W.E. Ormand et al., Phys. Rev. Lett. 77, 607 (1996).
D. Kusnezov et al., Phys. Rev. Lett. 81, 542 (1998).*

• *FT-RPA, FT-CRPA and FT-QRPA seem to be understood, however, microscopic calculations beyond one-loop approximations are still very limited and their results are not assessed systematically.*

• *Open questions: What are the microscopic mechanisms of the GMR's broadening with temperature? What happens to the soft modes and to the low-lying strength at $T>0$?*

Nucleus in the thermal equilibrium: a compound state

$$\Omega(\lambda, T) = E - \lambda N - TS$$

Grand thermodynamical potential to be minimized with the Covariant Energy Density Functional (NL3, P. Ring et al.)

$$E[\mathcal{R}, \phi] = \text{Tr}[(\vec{\alpha}\vec{p} + \beta m)\mathcal{R}] + \sum_m \left\{ \text{Tr}[(\beta \Gamma_m \phi_m)\mathcal{R}] \mp \int d^3r \left[\frac{1}{2}(\vec{\nabla}\phi_m)^2 + U(\phi_m) \right] \right\}$$

$$S = -k\text{Tr}(\mathcal{R}\ln\mathcal{R})$$

Entropy (maximized)

$$N = \text{Tr}(\mathcal{R}\mathcal{N})$$

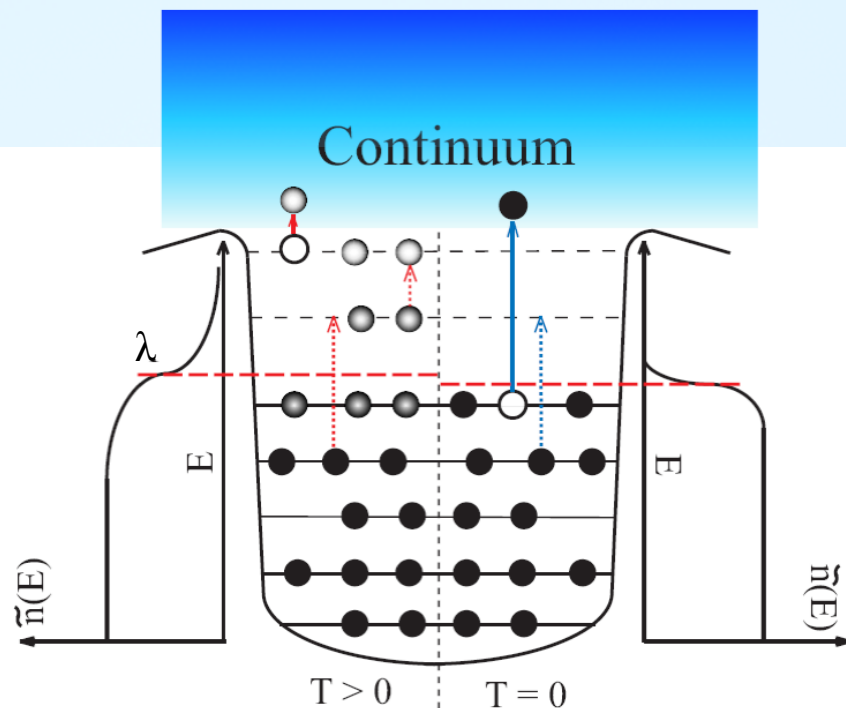
Particle number

$$\mathcal{R} = \frac{e^{-(\mathcal{H}-\lambda\mathcal{N})/kT}}{\text{Tr}\left[e^{-(\mathcal{H}-\lambda\mathcal{N})/kT}\right]}$$

Density matrix

$$\mathcal{H} = \frac{\delta E[\mathcal{R}]}{\delta \mathcal{R}}$$

Single-particle Hamiltonian



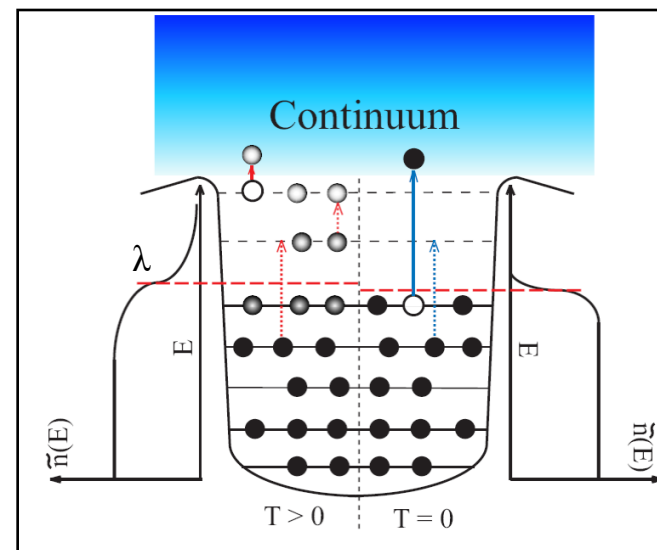
Nucleus in the thermal equilibrium: a compound state

Fractional occupancies and thermal unblocking:

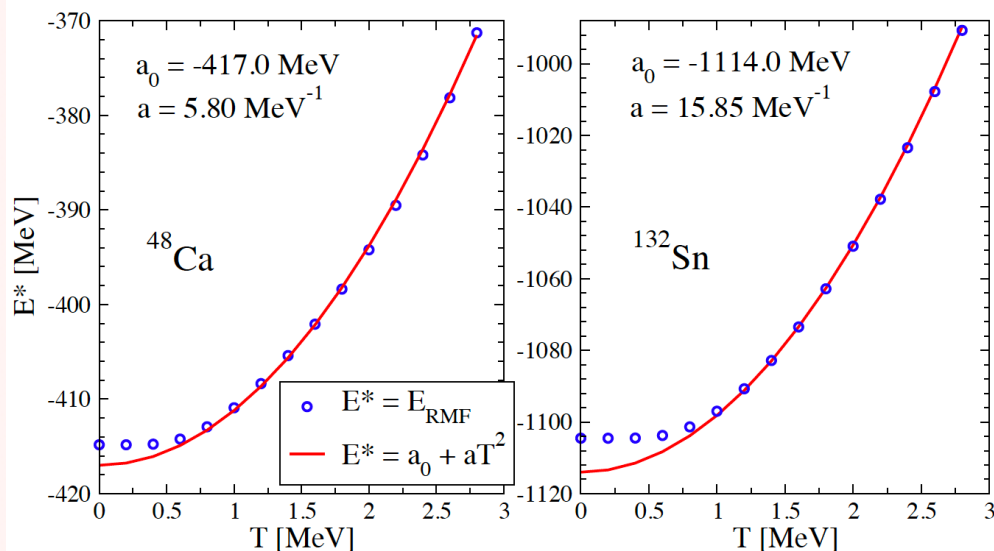
$$n_i(T) = n(\varepsilon_i, T) = \frac{1}{1 + \exp\{\varepsilon_i/T\}} \quad \text{Fermions}$$

$$N(\Omega_\mu, T) = \frac{1}{\exp\{\Omega_\mu/T\} - 1} \quad \text{Bosons}$$

$\varepsilon_i = \tilde{\varepsilon}_i - \lambda$



RMF excitation energies vs temperature Calculations of H. Wibowo (WMU):



Parabolic fit of the RMF $E^*(T)$ gives the level density parameters a_{RMF} close to those of the empirical Fermi gas model

Matsubara Green function formalism for $T > 0$

Mean-field single-fermion propagator in t -representation:

$$\tilde{\mathcal{G}}(1, 2) = \tilde{\mathcal{G}}_{12}(\tau) = -\sigma \delta_{12} \theta(\sigma \tau) n(-\sigma \varepsilon_1) e^{-\varepsilon_1 \tau}$$

$$\sigma = \text{sign}(\tau)$$

To be compared to $T=0$ case:

$$1 = \{\xi_1, t_1\}$$

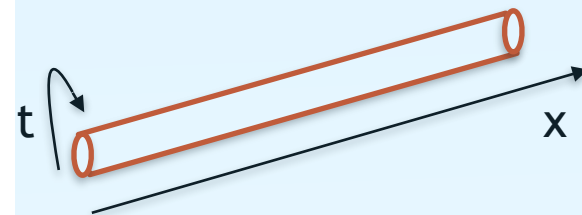
$$\tau = t_1 - t_2$$

$$\tilde{\mathcal{G}}(1, 2) = -i \sigma_1 \delta_{12} \theta(\sigma_1 \tau) e^{-i \varepsilon_1 \tau}, \quad \sigma_1 = \text{sign}(\varepsilon_1)$$

Finite temperature:
imaginary time technique

$$\tau \rightarrow -i\tau$$

$2/T$ periodic:
time as a closed loop



Fourier transform to the imaginary discrete energy variable:

$$\tilde{\mathcal{G}}_{12}(i\xi_l) = \frac{1}{2} \int_{-1/T}^{1/T} d\tau \tilde{\mathcal{G}}_{12}(\tau) e^{i\xi_l \tau} = \frac{\delta_{12}}{i\xi_l - \varepsilon_1}, \quad \xi_l = (2l + 1)\pi T$$

Dyson equation for the single-fermion propagator:

$$\mathcal{G}(1, 2) = \tilde{\mathcal{G}}(1, 2) + \sum_{1'2'} \tilde{\mathcal{G}}(1, 1') \Sigma^e(1'2') \mathcal{G}(2', 2)$$

Bethe-Salpeter equation for the nuclear particle-hole response

Bethe-Salpeter equation (BSE) for the 4-times response function (more general):

$$\mathcal{R}(14, 23) = \mathcal{G}(1, 3)\mathcal{G}(4, 2) - i \sum_{5678} \mathcal{G}(1, 5)\mathcal{G}(6, 2)V(58, 67)\mathcal{R}(74, 83)$$

BSE in terms of free one-fermion propagator:

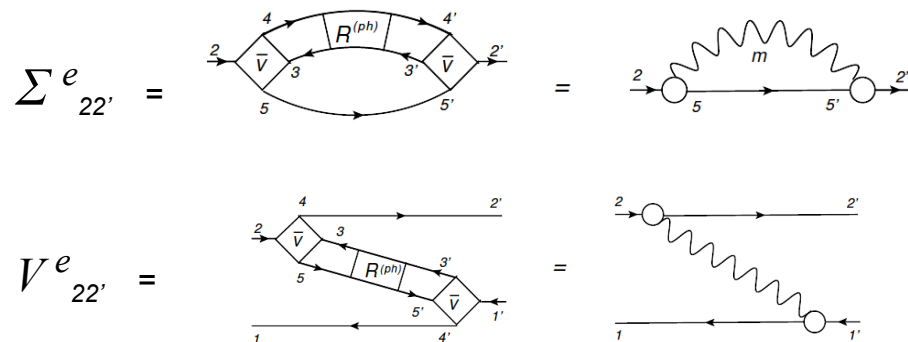
$$\mathcal{R}(14, 23) = \tilde{\mathcal{G}}(1, 3)\tilde{\mathcal{G}}(4, 2) - i \sum_{5678} \tilde{\mathcal{G}}(1, 5)\tilde{\mathcal{G}}(6, 2)\mathcal{W}(58, 67)\mathcal{R}(74, 83)$$

Free response: $\tilde{\mathcal{R}}^{(0)}(14, 23) = \tilde{\mathcal{G}}(1, 3)\tilde{\mathcal{G}}(4, 2)$

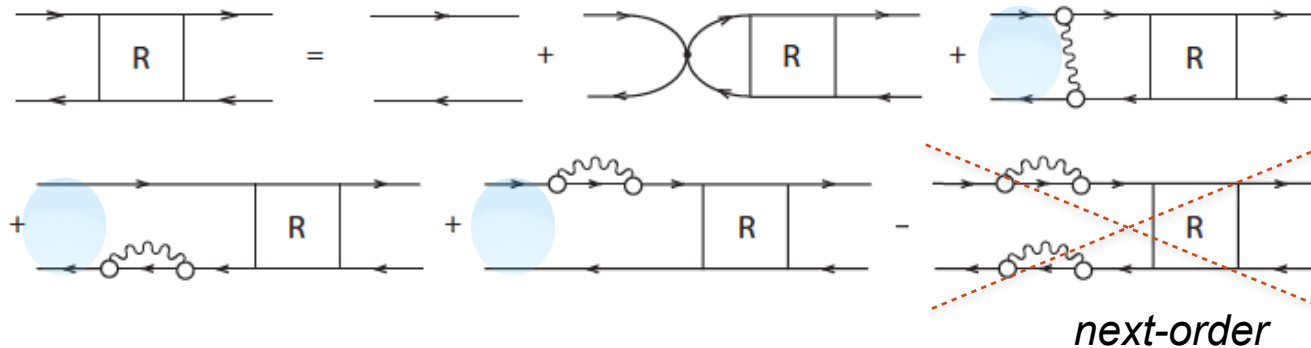
Interaction kernel:

$$\mathcal{W}(14, 23) = \tilde{V}(14, 23) + V^e(14, 23) + i\tilde{\mathcal{G}}^{-1}(1, 3)\Sigma^e(4, 2) + i\Sigma^e(1, 3)\tilde{\mathcal{G}}^{-1}(4, 2) - i\Sigma^e(1, 3)\Sigma^e(4, 2)$$

Leading-approximation self-energy and induced interaction:



Time blocking method at $T=0$



Formally,
the same BSE
at $T=0$ and $T>0$

$$R = \tilde{R}^0 - i \tilde{R}^0 W R$$

$$\tilde{R}^0(14, 23) = \tilde{G}(1, 3)\tilde{G}(4, 2) \rightarrow \tilde{D}^0(14, 23) = \Theta(14, 23)\tilde{G}(1, 3)\tilde{G}(4, 2)$$

Time- projection
operator:

$$\Theta(14, 23) = \delta_{\sigma_1, -\sigma_2} \theta(\sigma_1 t_{14}) \theta(\sigma_1 t_{23})$$

V.I. Tselyaev,
Yad. Fiz. 50,1252 (1989)

$$\tilde{R}_{14,23}^0(\omega, \varepsilon, \varepsilon') = 2\pi \delta_{13} \delta_{24} \delta(\varepsilon - \varepsilon') \tilde{G}_1(\varepsilon + \omega) \tilde{G}_2(\varepsilon)$$

Non-separable

$$\tilde{D}_{14,23}^0(\omega, \varepsilon, \varepsilon') = i \delta_{\sigma_1, -\sigma_2} \delta_{13} \delta_{24} \sigma_1 (\omega - \varepsilon_{12} + i \sigma_1 \delta) \tilde{G}_1(\varepsilon + \omega) \tilde{G}_2(\varepsilon) \tilde{G}_3(\varepsilon' + \omega) \tilde{G}_4(\varepsilon')$$

Separable


Time blocking method at $T > 0$

How to transform the BSE at $T > 0$?

Free two-fermion propagator:

$$\tilde{\mathcal{R}}^0(14, 23) = \tilde{\mathcal{G}}(1, 3)\tilde{\mathcal{G}}(4, 2)$$

Fourier transform to the imaginary discrete energy variables:


$$\tilde{\mathcal{R}}_{14,23}^0(i\omega_n, i\xi_l, i\xi_{l'}) = \frac{\delta_{13}\delta_{24}\delta_{ll'}}{T(i\xi_l - \varepsilon_2)(i\omega_n + i\xi_l - \varepsilon_1)} = \frac{\delta_{13}\delta_{24}\delta_{ll'}}{T}\mathcal{G}_1(i\omega_n + i\xi_l)\mathcal{G}_2(i\xi_l)$$

• Which projection operator can bring $\tilde{\mathcal{R}}_{14,23}^0(i\omega_n, i\xi_l, i\xi_{l'})$ to a symmetric form at $T > 0$?

• The operator $\Theta(14, 23) = \delta_{\sigma_1, -\sigma_2}\theta(\sigma_1 t_{14})\theta(\sigma_1 t_{23})$ used at $T=0$ can not...

• We have found that the operator

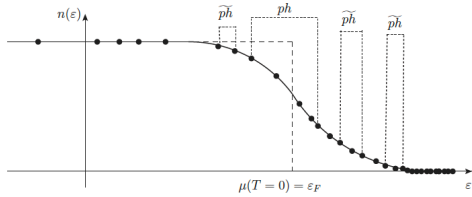
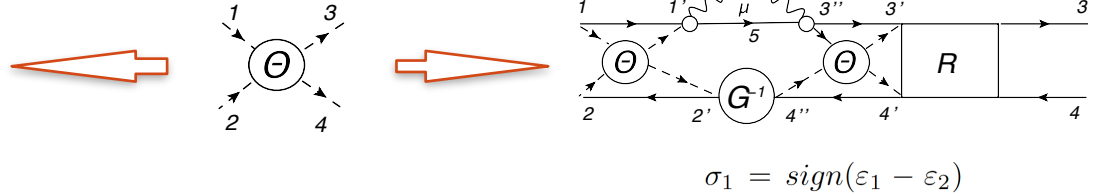
$$\Theta(14, 23; T) = \delta_{\sigma_1, -\sigma_2} \left[n(\sigma_1 \varepsilon_2, T)\theta(\sigma_1 t_{12}) + n(-\sigma_1 \varepsilon_1, T)\theta(-\sigma_1 t_{12}) \right] \theta(\sigma_1 t_{14})\theta(\sigma_1 t_{23})$$

$$\lim_{T \rightarrow 0} \theta(12, T) = \lim_{T \rightarrow 0} \left[n(\sigma_1 \varepsilon_2, T)\theta(\sigma_1 t_{12}) + n(-\sigma_1 \varepsilon_1, T)\theta(-\sigma_1 t_{12}) \right] = 1 \quad \text{at } \sigma_1 = -\sigma_2$$

can do this

Time blocking (diagram ordering) at $T > 0$: $2q + \text{phonon}$ case

$$\Theta(14, 23; T) = \delta_{\sigma_1, -\sigma_2} \theta(\sigma_1 t_{14}) \theta(\sigma_1 t_{23}) \times \left[n(\sigma_1 \varepsilon_2, T) \theta(\sigma_1 t_{12}) + n(-\sigma_1 \varepsilon_1, T) \theta(-\sigma_1 t_{12}) \right]$$



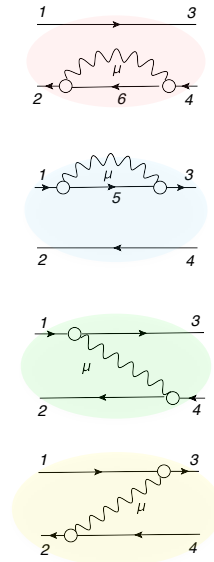
“Soft” time blocking at $T > 0$ leads to a single-frequency variable equation for the response function

$$\begin{aligned} \mathcal{R}_{14,23}(\omega, T) &= \tilde{\mathcal{R}}_{14,23}^0(\omega, T) + \\ &+ \sum_{1'2'3'4'} \tilde{\mathcal{R}}_{12',21'}^0(\omega, T) [\tilde{V}_{1'4',2'3'}(T) + \delta\Phi_{1'4',2'3'}(\omega, T)] \mathcal{R}_{3'4',4'3}(\omega, T) \\ \delta\Phi_{1'4',2'3'}(\omega, T) &= \Phi_{1'4',2'3'}(\omega, T) - \Phi_{1'4',2'3'}(0, T) \end{aligned}$$

$T > 0$:

$$\begin{aligned} \Phi_{14,23}^{(ph)}(\omega, T) &= \frac{1}{n_{43}(T)} \sum_{\mu} \eta_{\mu} \left[\delta_{13} \sum_6 \gamma_{\mu;62}^{\eta_{\mu}} \gamma_{\mu;64}^{\eta_{\mu}*} \times \right. \\ &\times \frac{(N(\eta_{\mu}\Omega_{\mu}) + n_6(T))(n(\varepsilon_6 - \eta_{\mu}\Omega_{\mu}, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_6 - \eta_{\mu}\Omega_{\mu}} + \\ &+ \delta_{24} \sum_5 \gamma_{\mu;15}^{\eta_{\mu}} \gamma_{\mu;35}^{\eta_{\mu}*} \times \\ &\times \frac{(N(\eta_{\mu}\Omega_{\mu}) + n_2(T))(n(\varepsilon_2 - \eta_{\mu}\Omega_{\mu}, T) - n_5(T))}{\omega - \varepsilon_5 + \varepsilon_2 - \eta_{\mu}\Omega_{\mu}} - \\ &- \gamma_{\mu;13}^{\eta_{\mu}} \gamma_{\mu;24}^{\eta_{\mu}*} \times \\ &\times \frac{(N(\eta_{\mu}\Omega_{\mu}) + n_2(T))(n(\varepsilon_2 - \eta_{\mu}\Omega_{\mu}, T) - n_3(T))}{\omega - \varepsilon_3 + \varepsilon_2 - \eta_{\mu}\Omega_{\mu}} - \\ &- \gamma_{\mu;31}^{\eta_{\mu}*} \gamma_{\mu;42}^{\eta_{\mu}} \times \\ &\left. \times \frac{(N(\eta_{\mu}\Omega_{\mu}) + n_4(T))(n(\varepsilon_4 - \eta_{\mu}\Omega_{\mu}, T) - n_1(T))}{\omega - \varepsilon_1 + \varepsilon_4 - \eta_{\mu}\Omega_{\mu}} \right], \end{aligned}$$

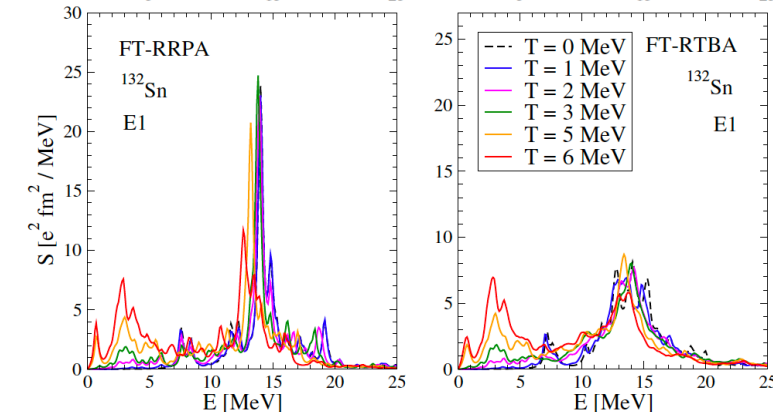
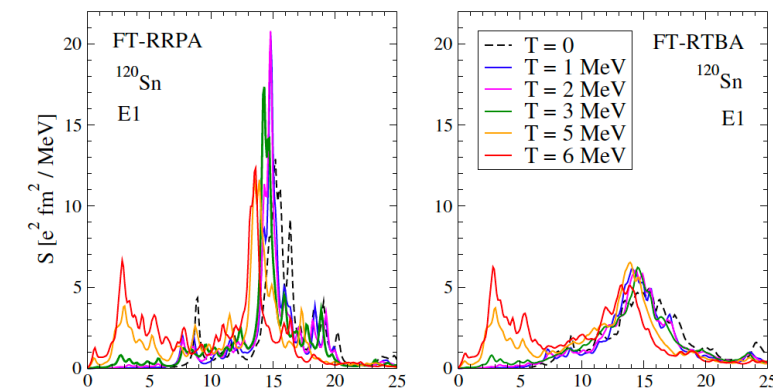
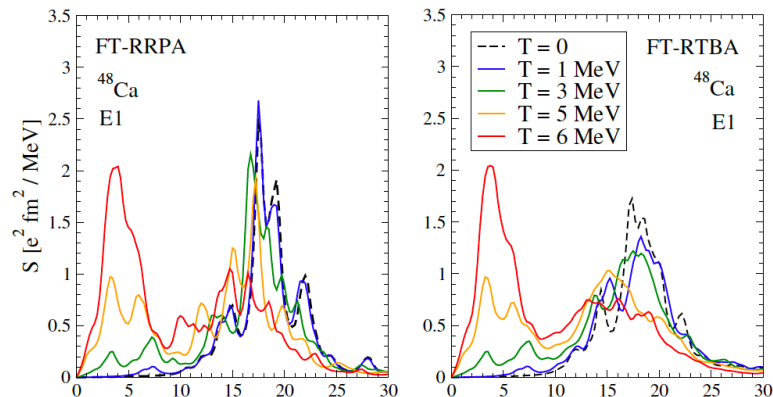
Dynamical kernel:



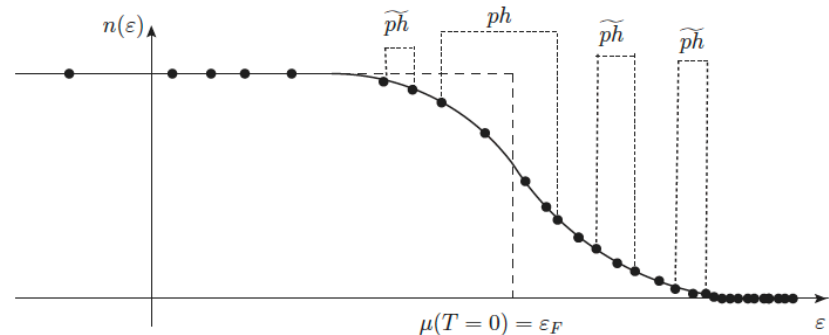
$T = 0$:

$$\begin{aligned} \Phi_{14,23}^{(ph,ph)}(\omega) &= \sum_{\mu} \times \\ &\times \left[\delta_{13} \sum_6 \frac{\gamma_{62}^{\mu} \gamma_{64}^{\mu*}}{\omega - \varepsilon_1 + \varepsilon_6 - \Omega_{\mu}} + \right. \\ &+ \delta_{24} \sum_5 \frac{\gamma_{15}^{\mu} \gamma_{35}^{\mu*}}{\omega - \varepsilon_5 + \varepsilon_2 - \Omega_{\mu}} - \\ &- \frac{\gamma_{13}^{\mu} \gamma_{24}^{\mu*}}{\omega - \varepsilon_3 + \varepsilon_2 - \Omega_{\mu}} - \\ &\left. - \frac{\gamma_{31}^{\mu*} \gamma_{42}^{\mu}}{\omega - \varepsilon_1 + \varepsilon_4 - \Omega_{\mu}} \right] \end{aligned}$$

Giant Dipole Resonance in ^{48}Ca and $^{120,132}\text{Sn}$ at $T > 0$



Thermal unblocking:



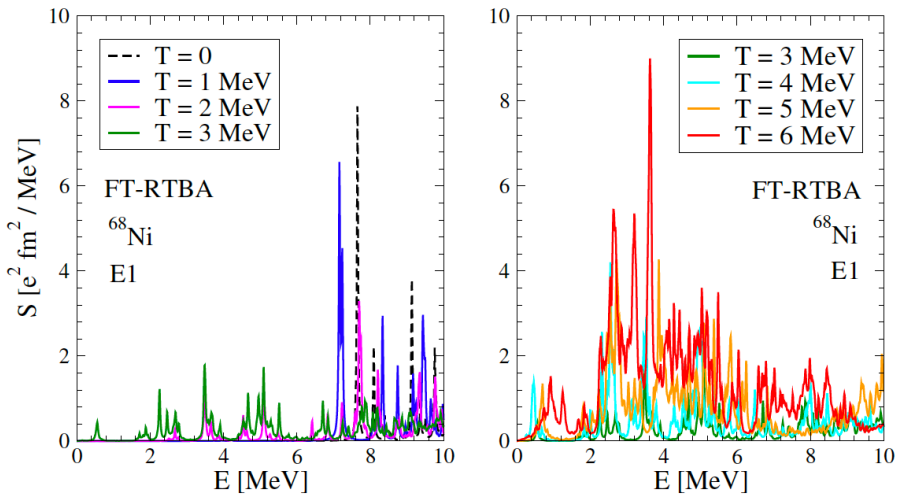
Uncorrelated propagator:

$$\tilde{R}_{14,23}^0(\omega) = \delta_{13}\delta_{24} \frac{n_2 - n_1}{\omega - \epsilon_1 + \epsilon_2}$$

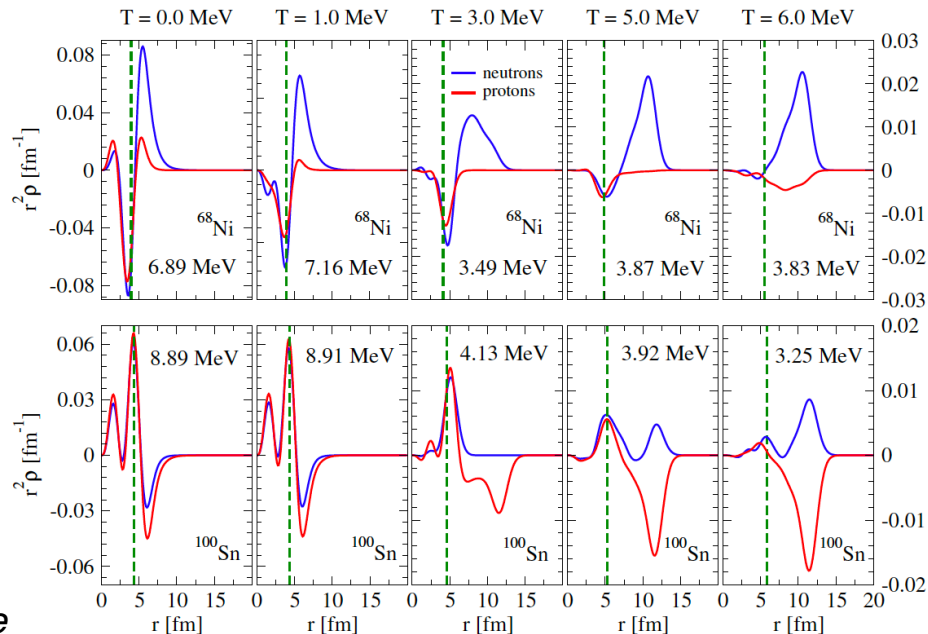
- New transitions due to the thermal unblocking effects
 - More collective and non-collective modes contribute to the PVC self-energy (~400 modes at $T=5-6$ MeV)
 - Broadening of the resulting GDR spectrum
 - Development of the low-energy part => a feedback to GDR
- The spurious translation mode is properly decoupled as the mean field is modified consistently
 - The role of the new terms in the Φ amplitude increases with temperature
 - A very little fragmentation of the low-energy peak (cancellations in the Φ amplitude, possibly due to the absence of GSC/PVC)

Evolution of the pygmy dipole resonance (PDR) at $T > 0$

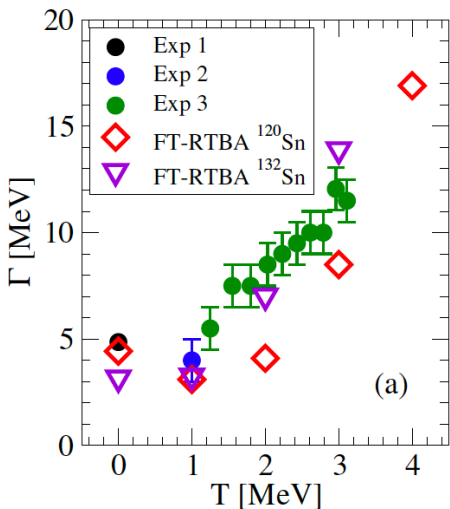
Low-energy strength distribution in ^{68}Ni



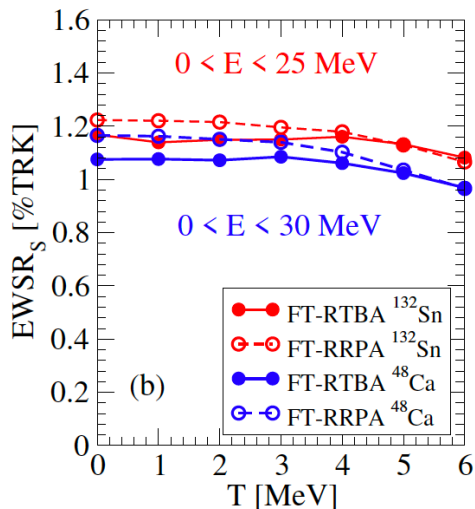
Transition density for the low-energy peak in ^{68}Ni , ^{100}Sn



GDR's width



Energy-weighted sum rule



- ☛ The low-energy peak (PDR) gains the strength from the GDR with the temperature growth: $EWSR \sim \text{const}$
- ☛ The total width $\Gamma \sim T^2$ (as in the Landau theory); shape fluctuations are missing for $T \sim 2-3$ MeV
- ☛ The PDR develops a new type of collectivity originated from the thermal unblocking
- ☛ The same happens with other low-lying modes ($2+$, $3-$, ...) \Rightarrow strong PVC \Rightarrow "destruction" of the GDR at high temperatures

E.L., H. Wibowo, *Phys. Rev. Lett.* 121, 082501 (2018).
 H. Wibowo, E.L., *arXiv:1810.01456*, *Phys. Rev. C* (2019).

The role of the exponential factor: low-energy strength

$$S(E, T) = -\frac{1}{\pi} \lim_{\Delta \rightarrow +0} \text{Im} \langle V^{0\dagger} \mathcal{R}(E + i\Delta, T) V^0 \rangle$$

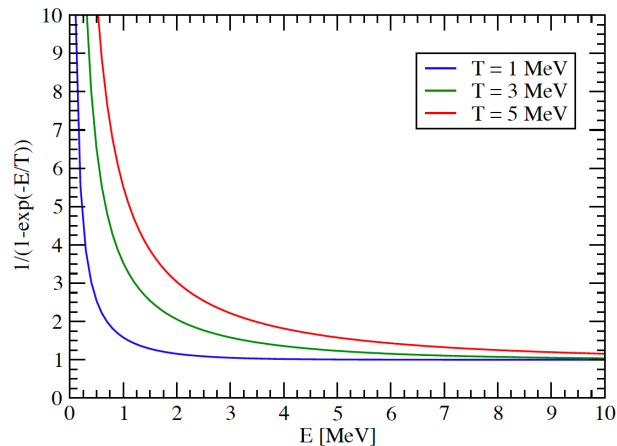
The final strength function at $T > 0$:

$$\tilde{S}(E) = \frac{1}{1 - e^{-E/T}} S(E)$$

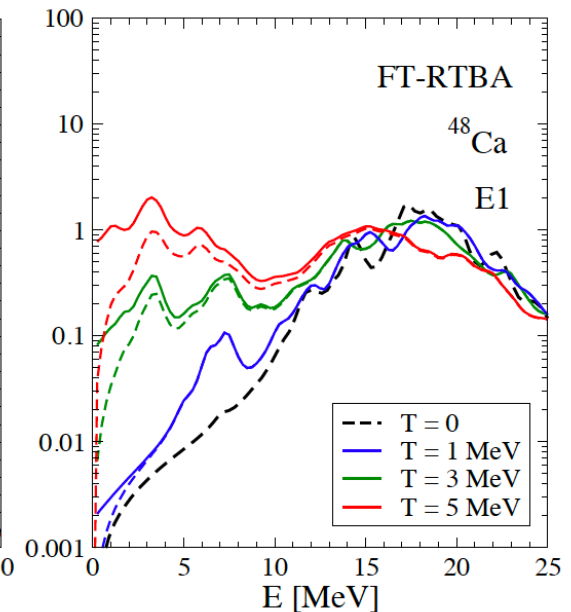
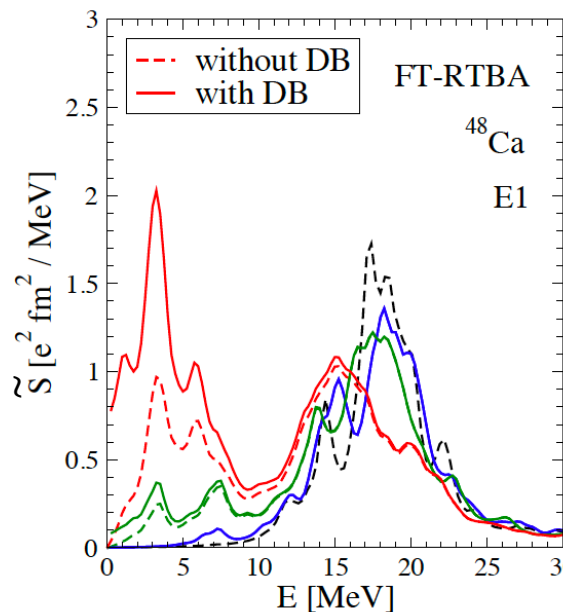
Averaging over the initial state energies, Detailed balance at $T > 0$

$$\lim_{E \rightarrow 0} S(E, T) = 0$$

The exponential factor:



Dipole strength: absorption at $T > 0$:



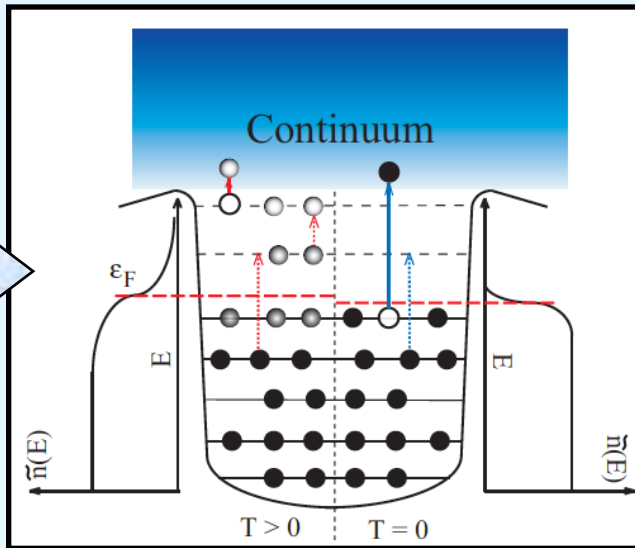
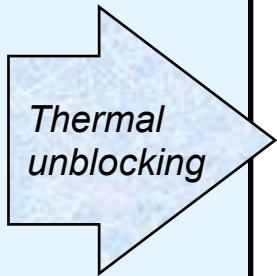
- The exponential factor brings an additional enhancement in $E < T$ energy region and provides the finite zero-energy limit of the strength (regardless its spin-parity)

Continuum effects

• Theory including the continuum, QVC and superfluid pairing at $T=0$:
 E.L., V.I. Tselyaev, *PRC* 75, 054318 (2007):

Continuum is mostly important for light nuclei

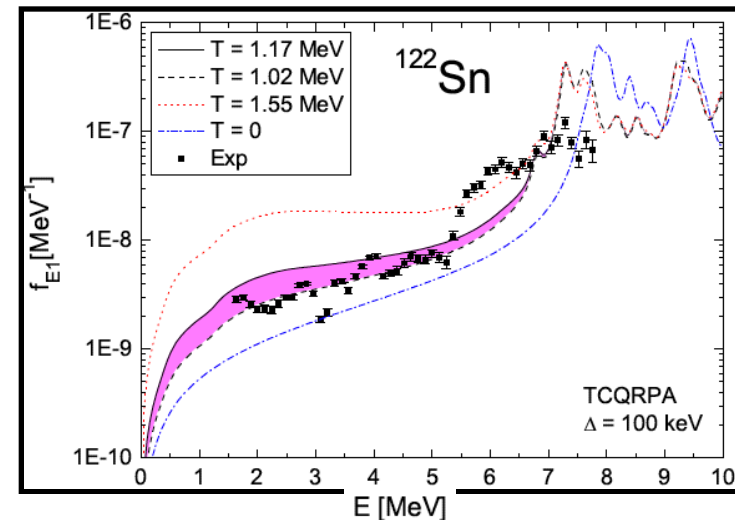
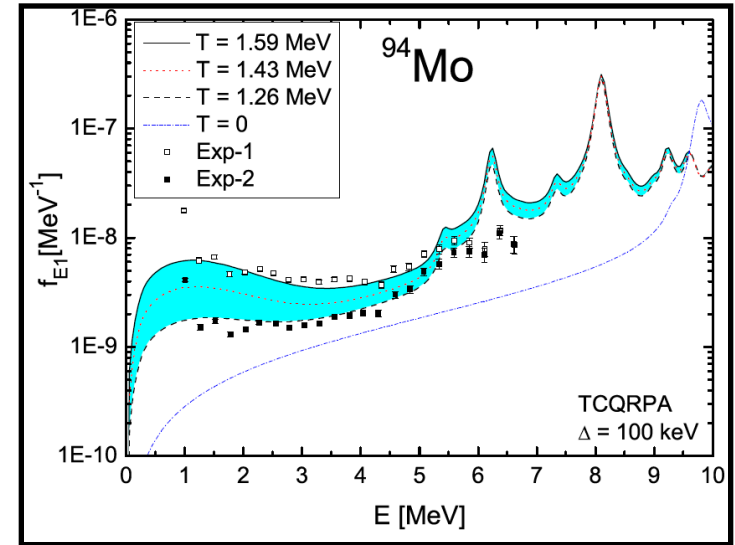
• However, the continuum effects become important also in heavy systems *at finite temperatures: excited compound nuclei (CN), for instance, after the thermal neutron capture*



Theory: E. Litvinova, N. Belov, *PRC* 88, 031302(R)(2013)

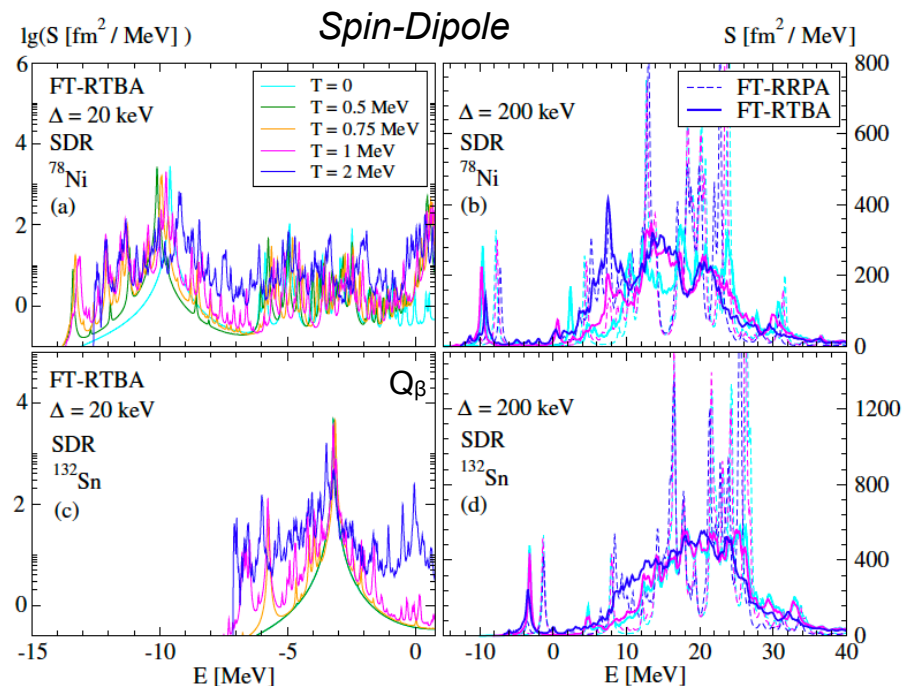
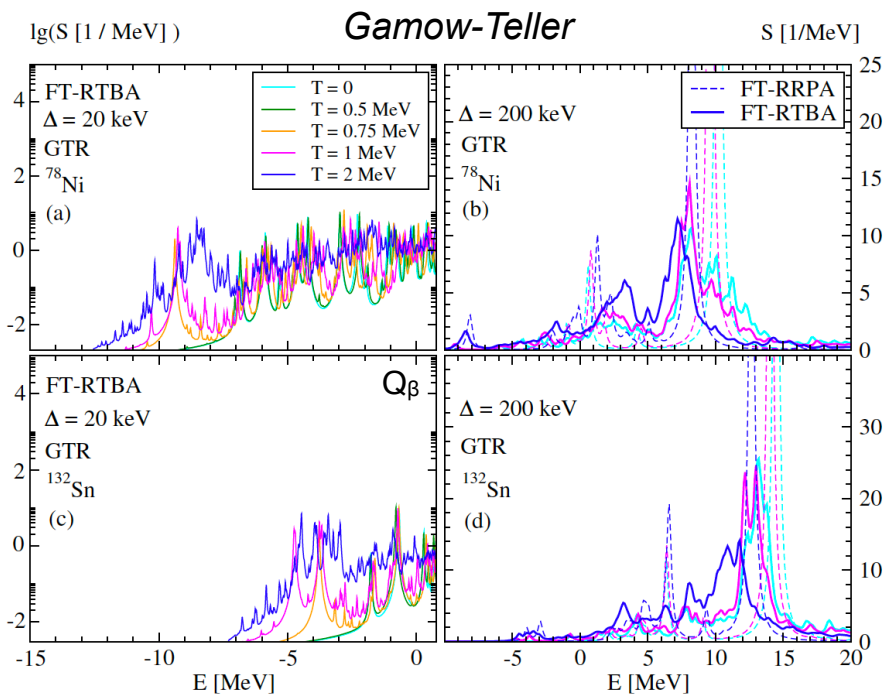
Exp.: Oslo data M. Guttormsen et al., *PRC* 71, 044307 (2005),
 S. Goriely et al., *PRC* 78, 064307 (2008)

Low-energy limit of the radiative dipole strength functions



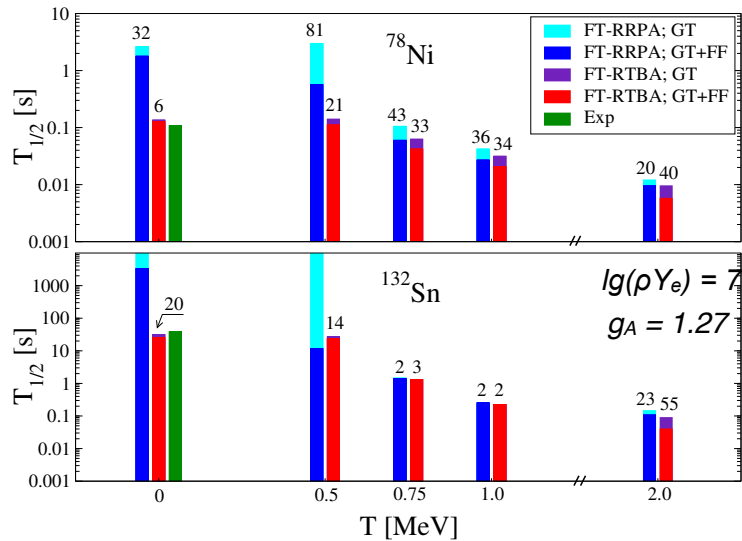
FT continuum + QVC: work in progress

Gamow-Teller and Spin Dipole Resonances at $T > 0$: ^{78}Ni and ^{132}Sn



- The thermally unblocked transitions enhance both the GTR and the SDR strengths within the Q_β window. This causes the decrease of the $T_{1/2}$ with temperature.
- The contribution from SDR-like (first-forbidden transitions) increases with temperature.
- At the typical r -process temperatures $T \sim 0.2-0.3 \text{ MeV}$ the thermal unblocking is still suppressed by the large shell gaps, however, the effect should be stronger in open-shell nuclei.

Beta decay half-lives in a hot stellar environment





Outlook

Summary:

- *Relativistic NFT offers a powerful framework for a consistent treatment of nuclear many-body correlations.*
- *The non-perturbative response theory based on QHD is available now for a large class of nuclear excited states in even-even and odd-odd nuclei. It is shown to be systematically improvable and heading toward a high-precision solution of the nuclear many-body problem.*
- *The response theory beyond one-loop approximation is generalized to finite temperature and applied to neutral and charge-exchange response of medium-heavy nuclei.*

Current and future developments:

- *An approach to nuclear response including both continuum and PVC at finite temperature, for both neutral and charge-exchange excitations;*
- *Deformed nuclei: correlations vs shapes;*
- *Inclusion of the superfluid pairing at $T > 0$ to extend the application range (r-process);*
- *Inclusion of 3p3h and higher configurations for spin-isospin response;*
- *Applications to neutron stars and other QFT cases;*
- *Toward an “ab initio” description: realization of the approach based on the bare relativistic meson-exchange potential (Bonn etc.).*

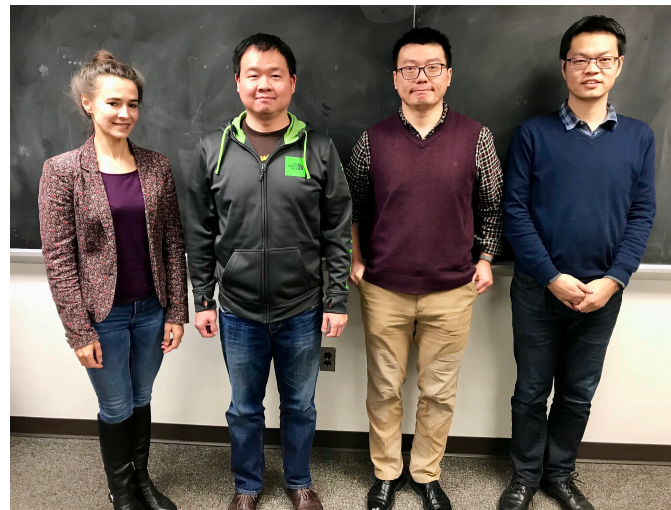
Many thanks for collaboration and support:

Herlik Wibowo (WMU)
Caroline Robin (INT Seattle)
Peter Schuck (IPN Orsay)
Peter Ring (TU München)
Yinu Zhang (WMU)
Tamara Niksic (U Zagreb)
Tomislav Marketin (U Zagreb)
Dario Vretenar (U Zagreb)
Remco Zegers (NSCL MSU)
Vladimir Zelevinsky (NSCL MSU)
Jian Li (Jilin U)
And others...

WMU Nuclear Theory group in 2017



...in fall 2018



Funding:

US-NSF
PHY-1404343
(2014-2018)

NSF CAREER
PHY-1654379
(2017-2022)