Quasiparticle-vibration coupling with ground-state correlations: application to the charge-exchange response of nuclei

Caroline Robin

Institute for Nuclear Theory, University of Washington, Seattle, WA, USA
JINA-CEE, Michigan State University, East Lansing, MI, USA

in collaboration with Elena Litvinova

Western Michigan University, Kalamazoo, MI, USA

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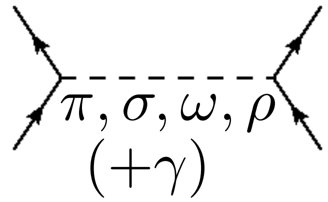
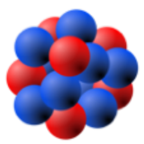
Relativistic Nuclear Field Theory: overview

mesons

$m_{\pi,\sigma,\omega,\rho}$~140-800 MeV



quantum hadrodynamics: relativistic nucleons & effective mesons

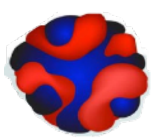


nucleons

S_n ~ 10 MeV

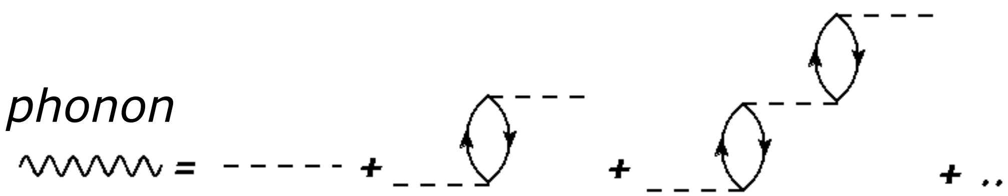
mean-field + superfluidity

self-consistent extensions of the Relativistic Mean-Field via Green function techniques



collective vibrations (phonons) ~few MeV

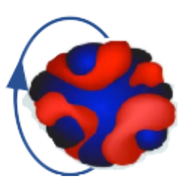
phonon



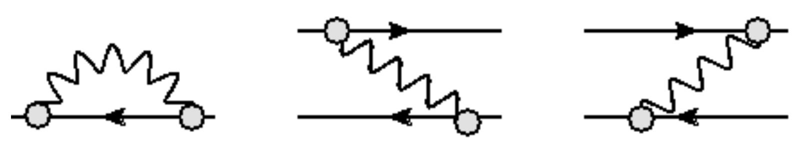
1p-1h

Random Phase Approximation

successive corrections in the single-particle motion and effective interaction

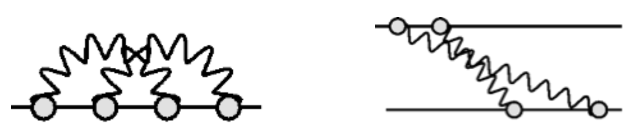


nucleons & phonons



2p-2h

Particle-vibration coupling, Nuclear field Theory, Time Blocking



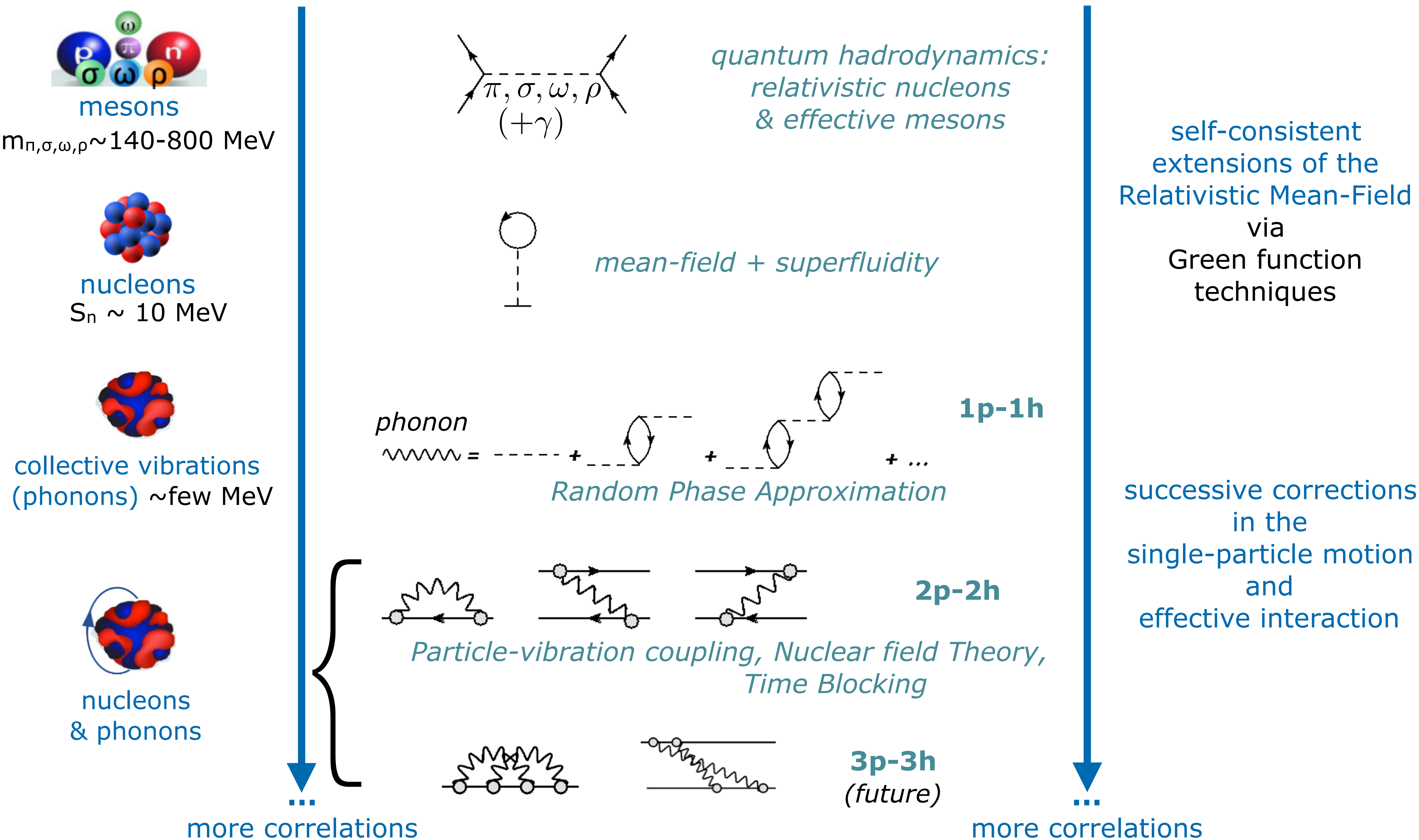
3p-3h

(future)

... more correlations

... more correlations

Relativistic Nuclear Field Theory: overview



Include complex configurations of nucleons step by step to:

- Keep the advantages of mean-field methods (large valence space, applicability up to (super)heavy nuclei)
- Ultimately achieve a highly-precise description of nuclear phenomena (similar to shell models)

Outline

- ★ Relativistic Nuclear Field Theory: formalism in the resonant approximation (reminder)
- ★ Application to charge-exchange modes: Gamow-Teller (GT) transitions, beta-decay half-lives and the quenching problem
- ★ Recent development: Ground-state correlations from the quasiparticle-vibration coupling
  - Effect on GT transitions: importance in the GT^+ channel, interplay with proton-neutron pairing
- ★ Application to 2νββ decay: preliminary results for ^{48}Ca, and some ideas for describing double-beta decay in the Green's function formalism
- ★ Conclusion, perspectives

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RNFT: Formalism

RNFT is based on Green's function techniques which enable the calculation of many nuclear properties

★ One-nucleon propagator: $G(1,2) = -i\langle 0|\mathcal{T}\left(\psi(1)\bar{\psi}(2)\right)|0\rangle$

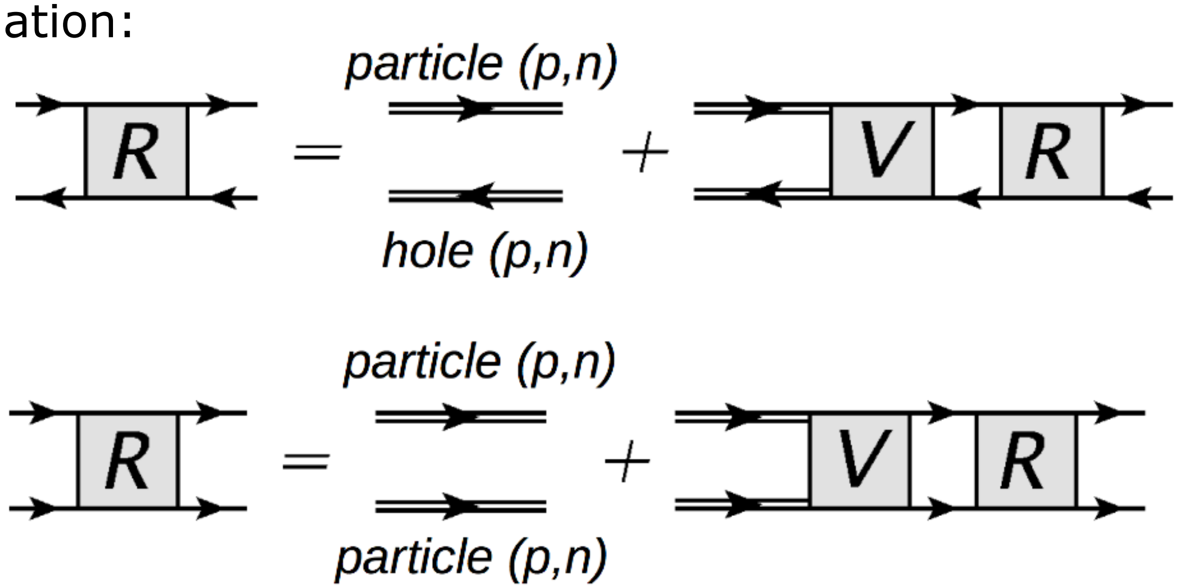
Dyson equation:

$\Sigma =$
Nucleon self-energy

- Single nucleons: fragmentation of single-particle states…
- Many-body ground state: binding energy, one-body density…

★ Two-nucleon propagator (response): $R(14,23) = \langle 0|\mathcal{T}\left(\psi(1)\psi(4)\bar{\psi}(3)\bar{\psi}(2)\right)|0\rangle + G(1,2)G(4,3)$

Bethe-Salpeter equation:



$V = i\frac{\delta\Sigma}{\delta G}$
Effective Interaction induced by the nuclear medium

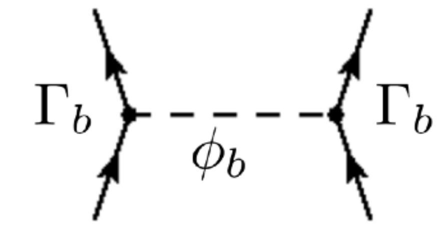
- Excited states of $(N,Z), (N\pm1, Z\mp1), (N\pm1, Z\pm1), (N\pm2, Z), (N, Z\pm2)$ nuclei, transition densities, two-body densities

From the QHD Lagrangian to the relativistic mean field

★ Nucleus = system of **relativistic** nucleons interacting via meson (+ photon) exchange

governed by an effective Lagrangian of Quantum Hadrodynamics

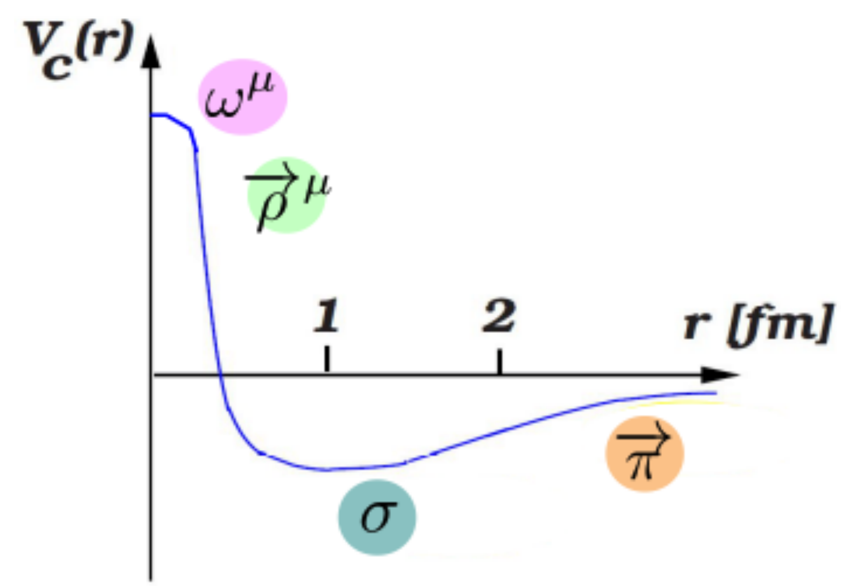
$$\mathcal{L}_{eff} = \mathcal{L}_{nucleons} + \mathcal{L}_{mesons} + \mathcal{L}_{interaction}$$



$$\phi_b = (\vec{\pi},\ \sigma,\ \omega^\mu,\ \vec{\rho}^{\,\mu},\ A^\mu)$$

$\vec{\pi}$: $(J^\pi = 0^-, T = 1)$; σ: $(0^+, 0)$; ω^μ: $(1^-, 0)$; $\vec{\rho}^{\,\mu}$: $(1^-, 1)$

$$\begin{aligned}
\mathcal{L}_{eff} = {} & \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{g_2}{3}\sigma^3 - \frac{g_3}{4}\sigma^4 \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^{\,\mu}\vec{\rho}_\mu \\
& + \frac{1}{2}\partial^\mu\vec{\pi}\partial_\mu\vec{\pi} - m_\pi^2\vec{\pi}^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\
& - \bar{\psi}\left(\Gamma_\sigma\sigma + \Gamma_\omega\omega_\mu + \vec{\Gamma}_\rho^\mu\vec{\rho}_\mu + \vec{\Gamma}_\pi\vec{\pi} + \Gamma_e^\mu A_\mu\right)\psi
\end{aligned}$$



From the QHD Lagrangian to the relativistic mean field

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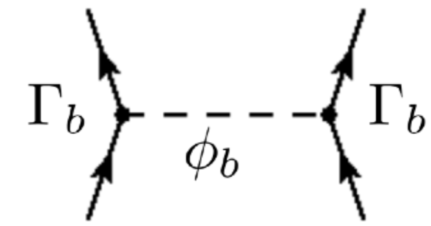

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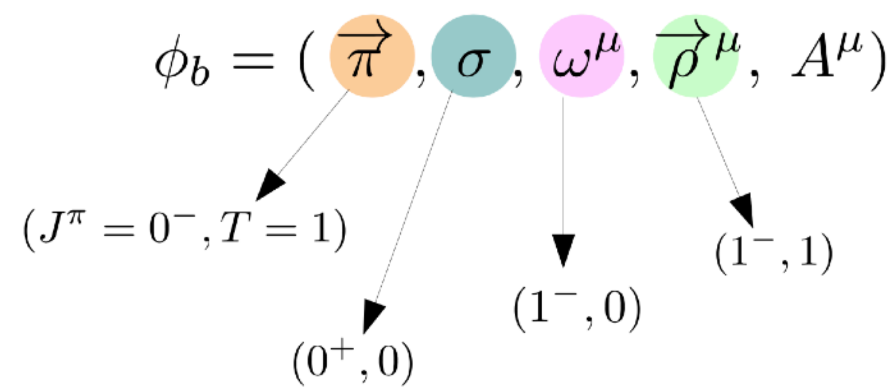
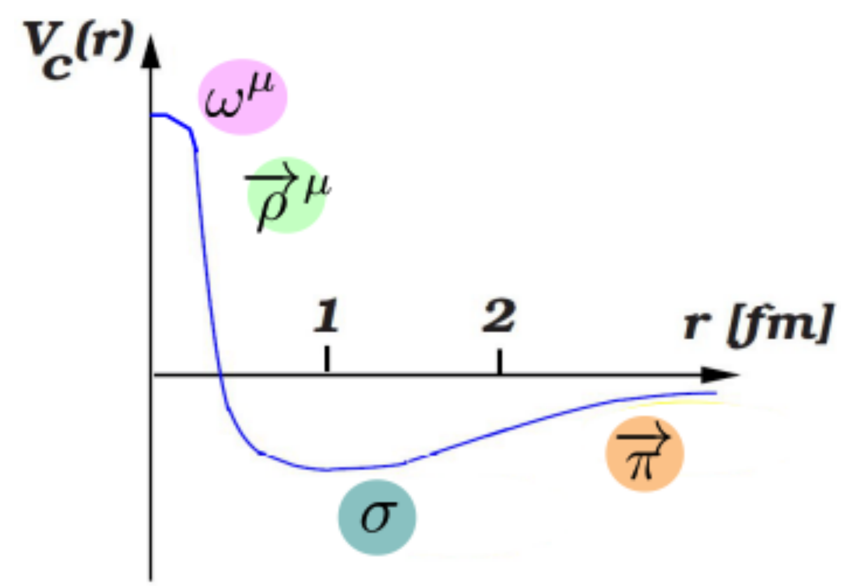


$$\begin{aligned}\mathcal{L}_{eff} = &\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{g_2}{3}\sigma^3 - \frac{g_3}{4}\sigma^4 \\ &-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu \\ &+\frac{1}{2}\partial^\mu\vec{\pi}\partial_\mu\vec{\pi} - m_\pi^2\vec{\pi}^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &-\bar{\psi}\left(\Gamma_\sigma\sigma + \Gamma_\omega\omega_\mu + \vec{\Gamma}_\rho^\mu\vec{\rho}_\mu + \vec{\Gamma}_\pi\vec{\pi} + \Gamma_e^\mu A_\mu\right)\psi\end{aligned}$$



$\Longrightarrow$ When both nucleons and mesons are quantized the equations of motion

$$(i\gamma_\mu\partial^\mu - m - \sum_b \Gamma_b\phi_b)\psi = 0 \qquad \text{(Nucleons)}$$

$$(\Box + m_b^2)\phi_b = \mp\bar{\psi}\Gamma_b\psi \qquad \text{(Mesons)}$$

are too complicated to solve…

From the QHD Lagrangian to the relativistic mean field

★ 1st-order approximation: ***Mean-field approximation***

$\Leftrightarrow$ the mesons and photon are treated as classical fields: $\phi_b \to \langle \phi_b \rangle$

$\Rightarrow$ The pion does not contribute in the ground state (would break parity)

$\Rightarrow$ Equations of motion

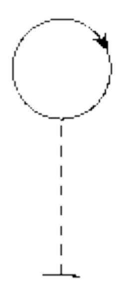
$$(i\gamma_\mu \partial^\mu - m - \widetilde{\Sigma}_{RMF})\psi = 0 \quad \text{(Dirac for nucleons)}$$

$$\left(-\Delta + m_b^2\right)\langle \phi_b \rangle = \mp \langle \bar{\psi} \Gamma_b \psi \rangle \quad \text{(KG for mesons)}$$

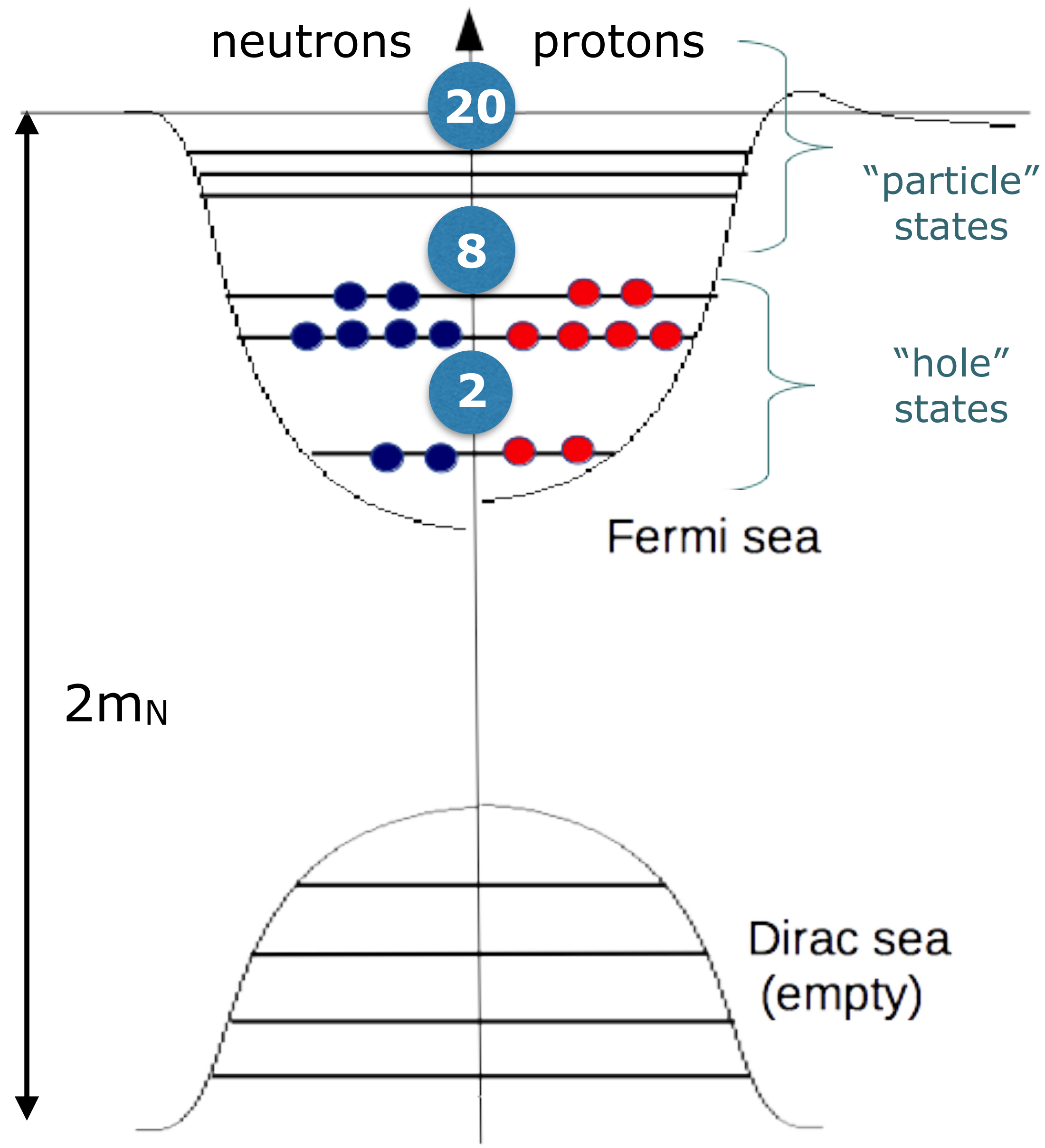
describe independent nucleons in classical meson fields

→ Self-energy (static):

$$\widetilde{\Sigma}_{RMF} = \sum_b \Gamma_b \langle \phi_b \rangle =$$



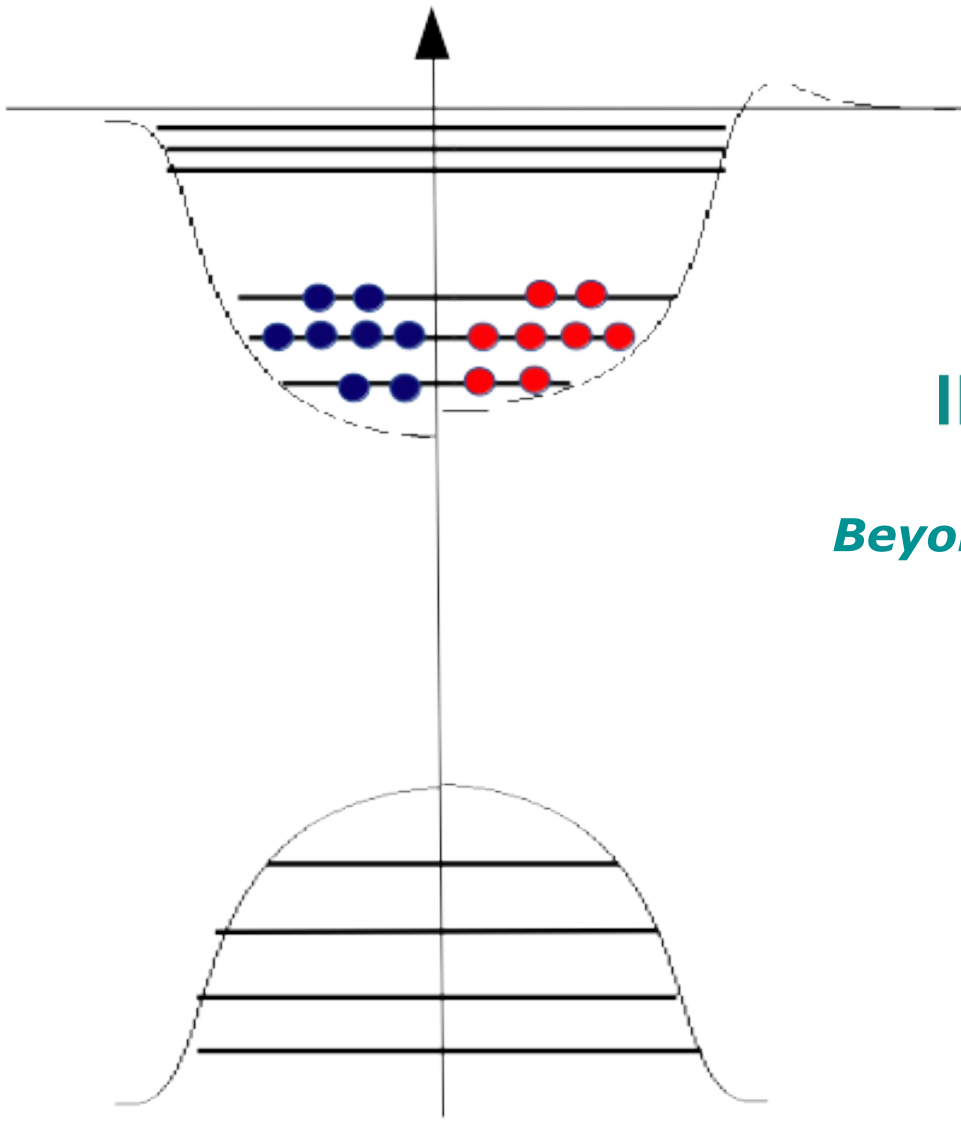
(Hartree)



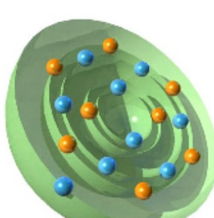
"No-sea approximation"

Going beyond mean field: nucleons coupled to vibrations

1st order approximation:
= relativistic mean field
=independent nucleons



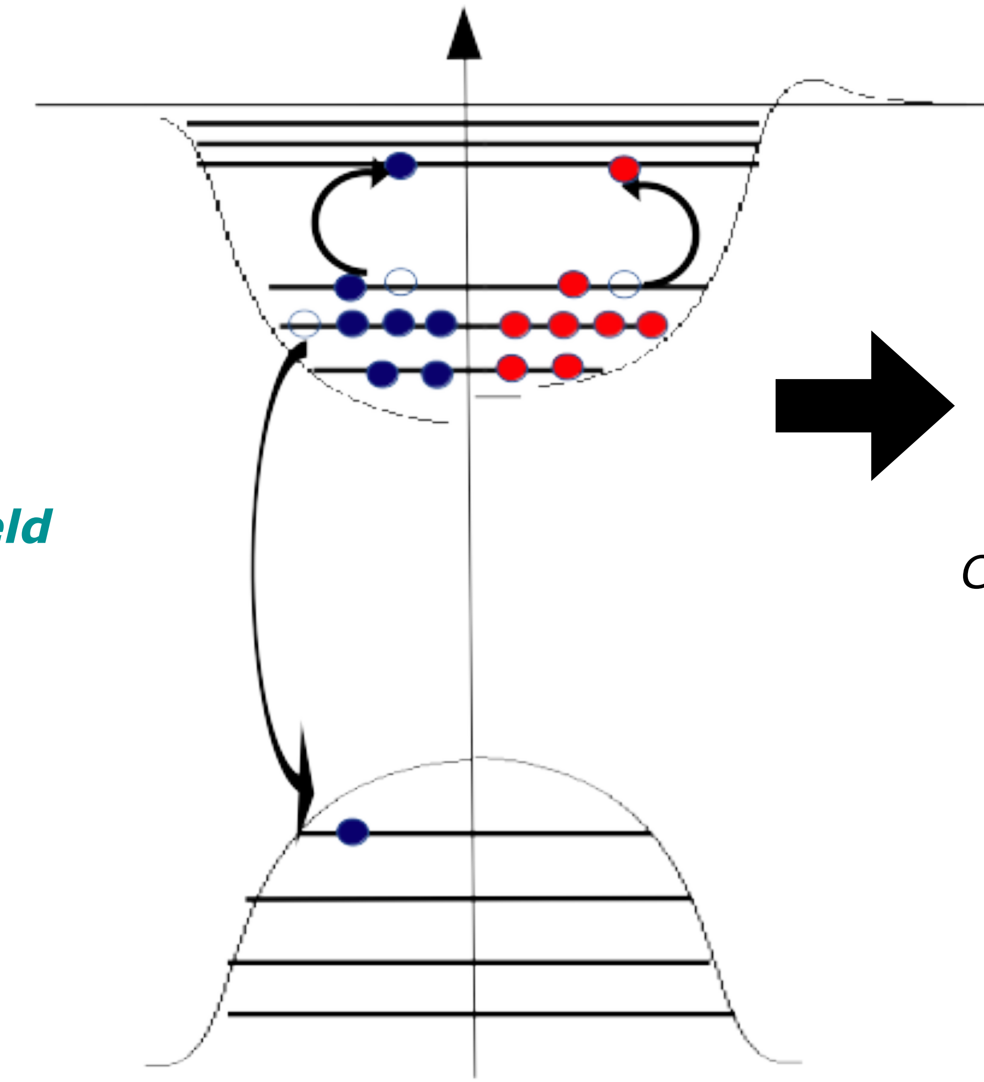
static

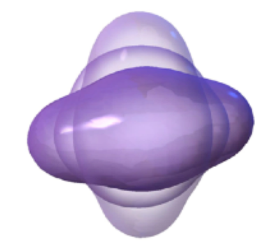


Beyond mean field

Correlations

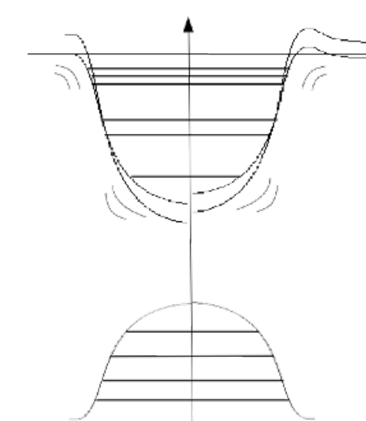
Particle-vibration coupling





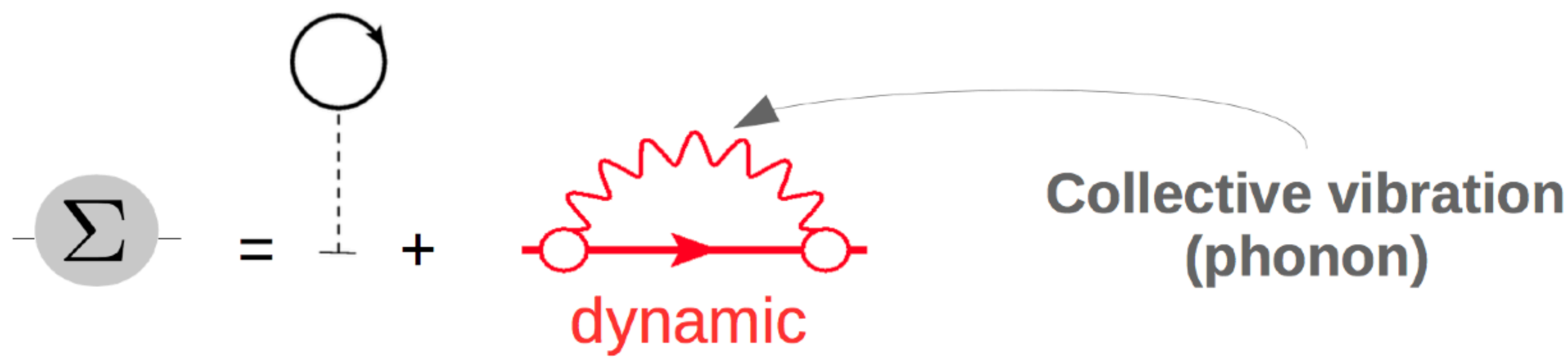
Collective vibration (phonon)

dynamic

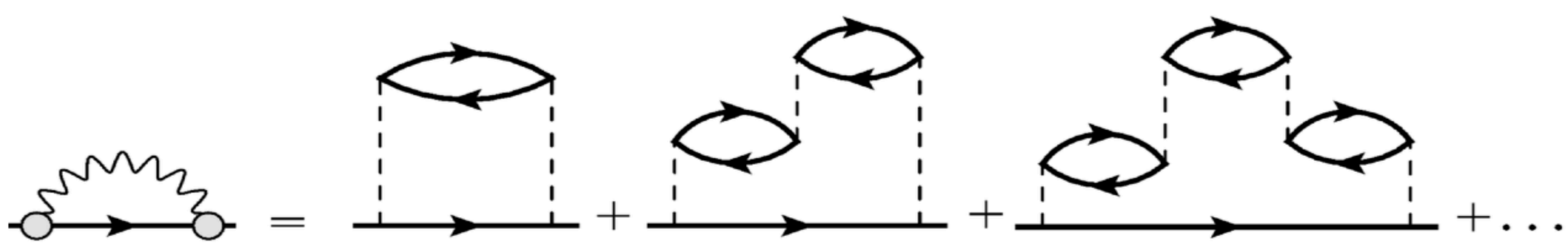


Going beyond mean field: nucleons coupled to vibrations

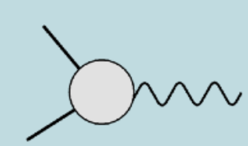
★ Particle-vibration coupling in the nucleonic self-energy:



$$\Sigma^{e}_{kl}(\varepsilon) = \sum_{\mu,n} \frac{\gamma^{\mu(\sigma_n)}_{kn}\,\gamma^{\mu(\sigma_n)*}_{ln}}{\varepsilon - \varepsilon_n - \sigma_n(\Omega^{\mu} - i\eta)},$$

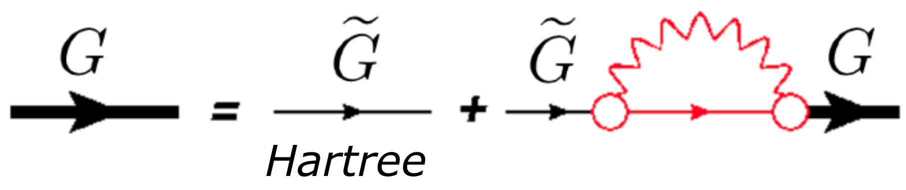


= ∞ serie of p-h excitations

- ↠ Allows a non-perturbative treatment of the NN interaction
- ↠ New expansion parameter = PVC vertex 

Going beyond mean field: nucleons coupled to vibrations

↠ Single-particle propagator:

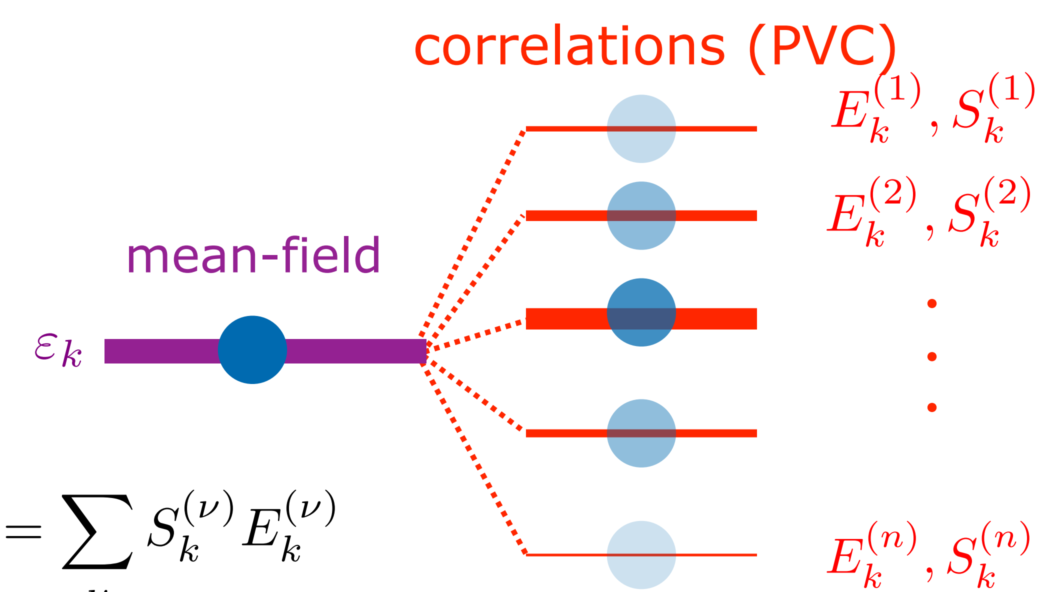


$$G(E) = \left(E - h - \Sigma^{(e)}(E)\right)^{-1}$$

Introduces new poles

$$E_k^{(\nu)} = \varepsilon_k + \Sigma_k^{(e)}(E_k^{(\nu)})$$

↠ Fragmentation of single-particle states:



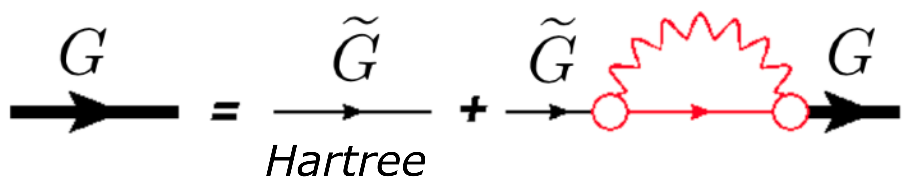
$$\varepsilon_k = \sum_\nu S_k^{(\nu)} E_k^{(\nu)}$$

$$\sum_\nu S_k^{(\nu)} = 1$$

↠ *no more well defined nucleons on shells: each single-particle states is fragmented (fractional occupation numbers)*

Going beyond mean field: nucleons coupled to vibrations

↠ Single-particle propagator:



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Introduces new poles

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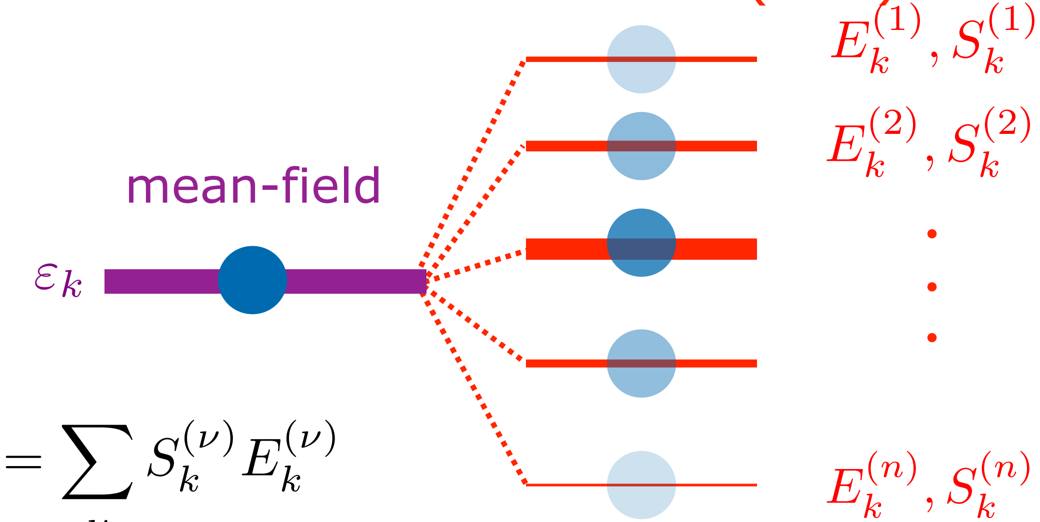
↠ Fragmentation of single-particle states:

correlations (PVC)



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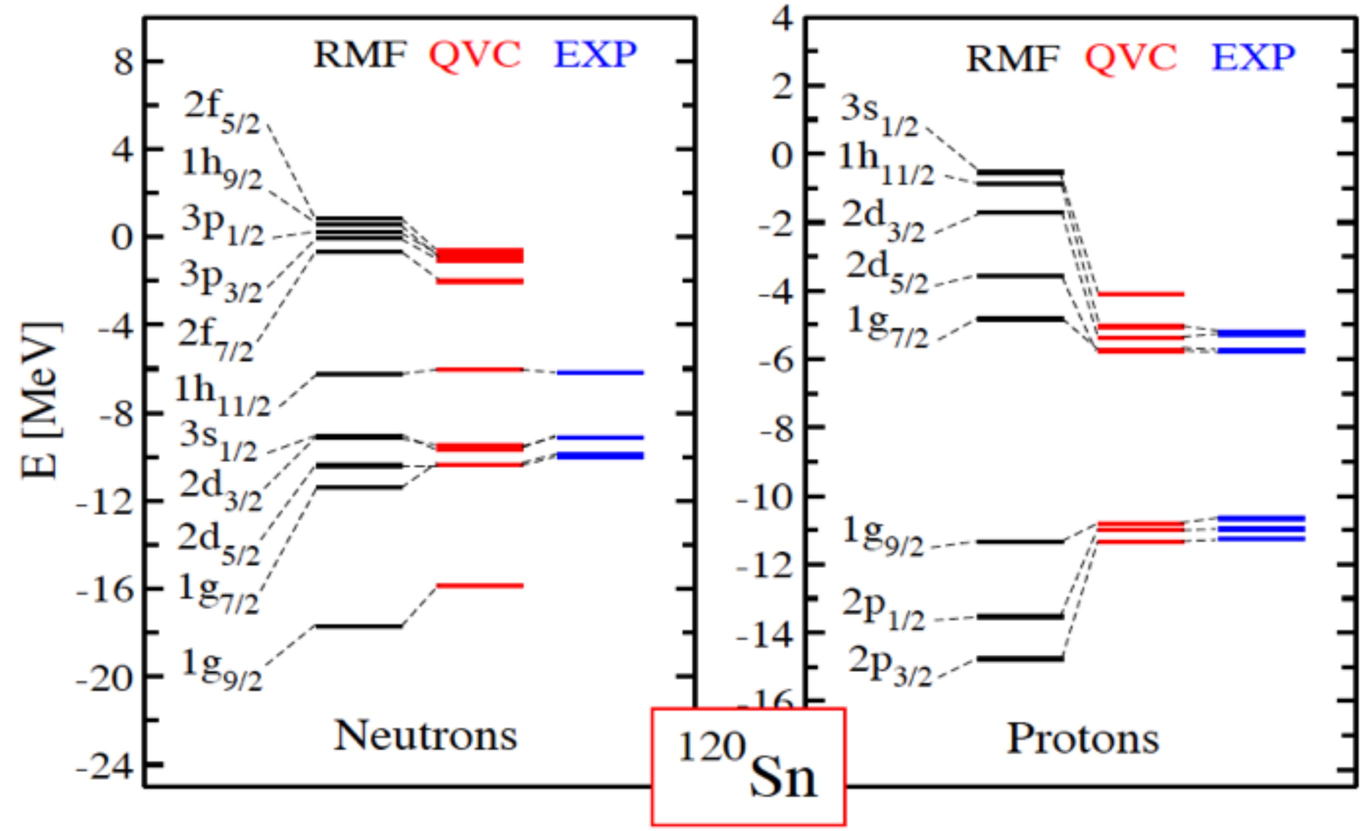
→ Example: Dominant level:

Coupling to T=0 phonons ($J^\pi = 2^+, 3^-, 4^+, 5^-, 6^+$)



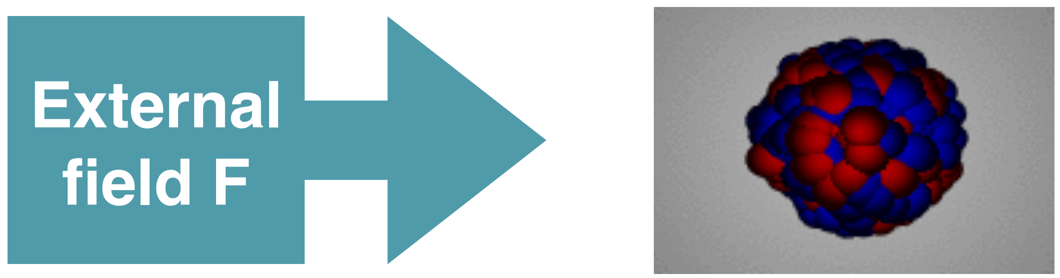
E. Litvinova, PRC 85, 021303(R) (2012)

Spectroscopic factors in ^{120}Sn:

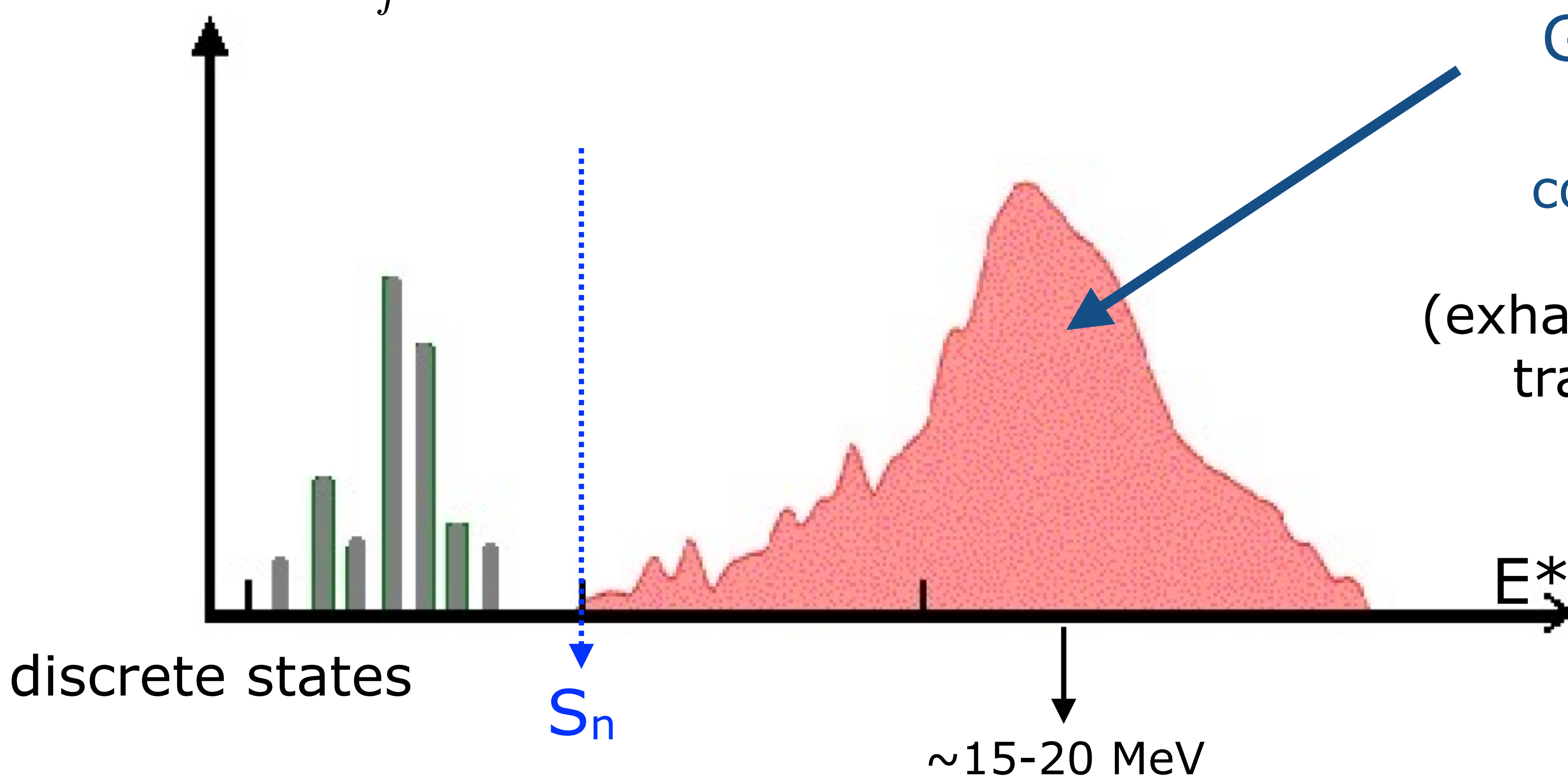
(n|j) ν	S^{th}	"S^{exp}"
$2d_{5/2}$	0.32	0.43
$1g_{7/2}$	0.40	0.60
$2d_{3/2}$	0.53	0.45
$3s_{1/2}$	0.43	0.32
$1h_{11/2}$	0.58	0.49
$2f_{7/2}$	0.31	0.35
$3p_{3/2}$	0.58	0.54

Model dependence

★ Response of the nucleus to an external field:



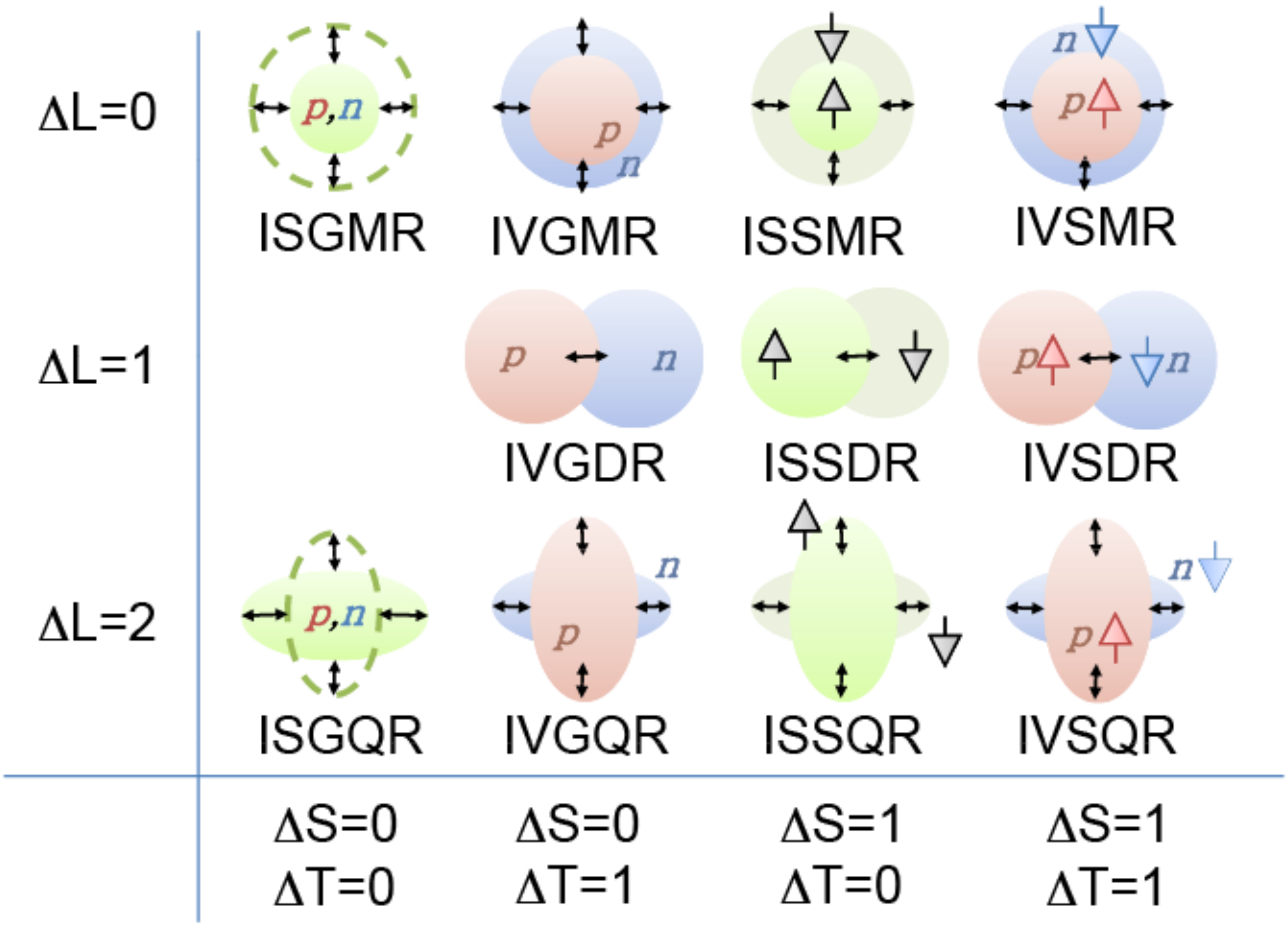
transition strength $\sum_f |\langle \Psi_f | \hat{F} | \Psi_i \rangle|^2 \delta(E - E_f + E_i)$



Giant resonance
=
coherent excitations
of all nucleons
(exhausts >50% of the total
transition probability)

Nuclear response in RNFT

According to the way it is probed, the nucleus can exhibit many types of response:



Classification of giant resonances

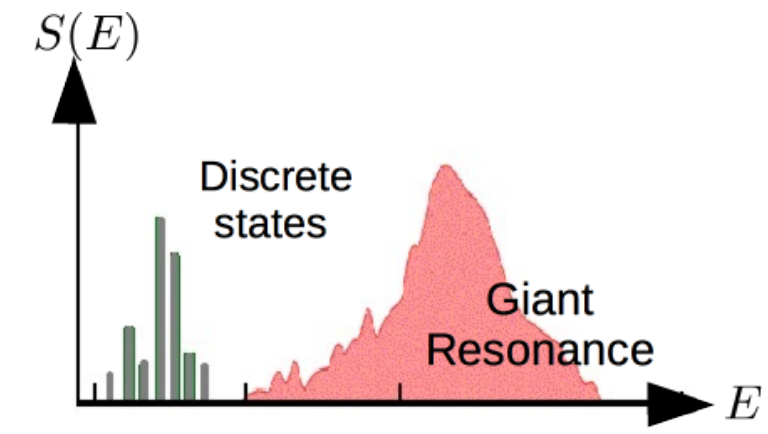
Nuclear response in RNFT

★ *Transition strength distribution:*

$$S(E) = \sum_f |\langle \Psi_f | \hat{F} | \Psi_i \rangle|^2 \delta(E - E_f + E_i)$$

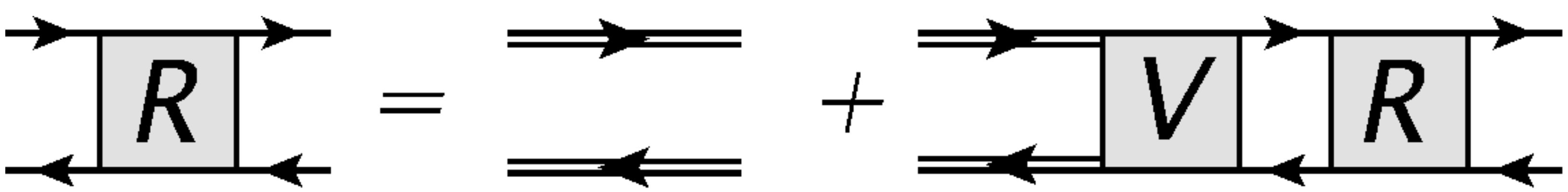
$$= -\frac{1}{\pi} \lim_{\Delta \to 0^+} \text{Im} \, \langle \Psi_i | \hat{F}^\dagger R(E + i\Delta) \hat{F} | \Psi_i \rangle$$

External field



Response function *=*
propagator of two correlated nucleons in the p-h channel

→ *Solution of the Bethe-Salpeter equation:*

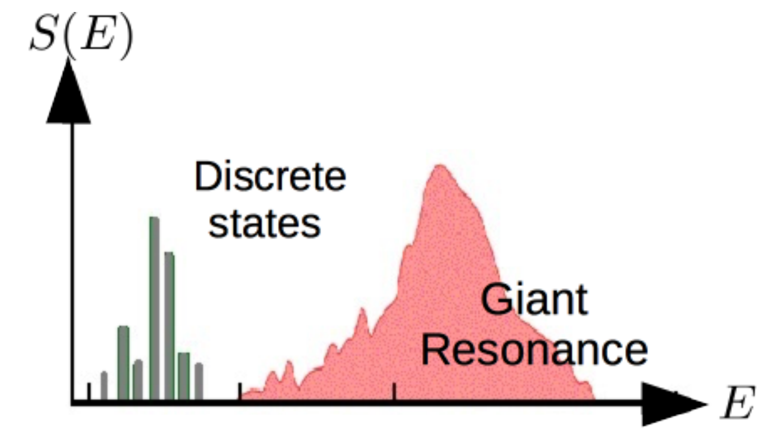


$$V = i \frac{\delta \Sigma}{\delta G}$$

Effective Interaction induced by the medium

Nuclear response in RNFT

★ *Transition strength distribution:*



External field

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$$= -\frac{1}{\pi} \lim_{\Delta \to 0^+} \text{Im} \, \langle \Psi_i | \hat{F}^\dagger R(E + i\Delta) \hat{F} | \Psi_i \rangle$$

Response function =
propagator of two correlated nucleons in the p-h channel

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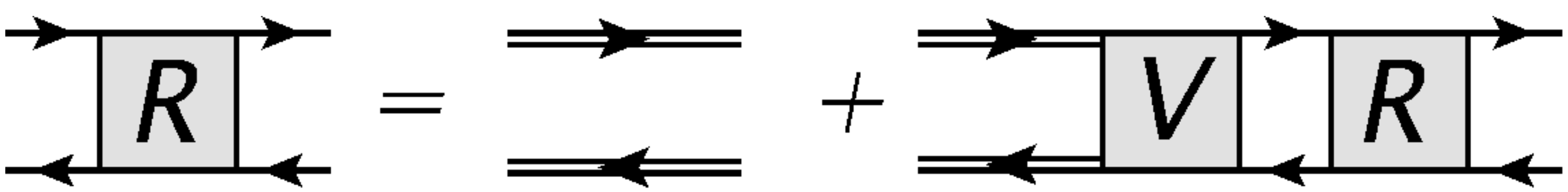
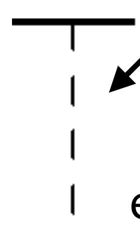


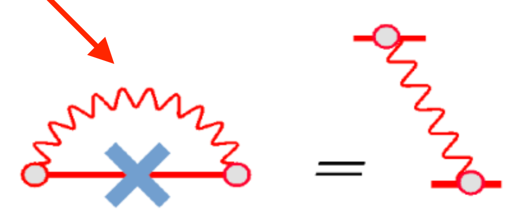
$$V = i \frac{\delta \Sigma}{\delta G}$$

Effective Interaction induced by the medium

$$V(E) = i \frac{\delta \widetilde{\Sigma}_{RMF}}{\delta G} + i \frac{\delta \Sigma^{(e)}(E)}{\delta G}$$



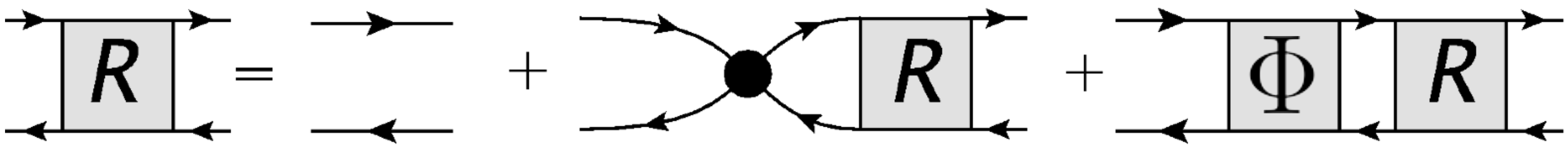
Meson exchange



Energy-dependent phonon exchange

Nuclear response in RNFT

⇒ Bethe-Salpeter equation:



Nuclear response in RNFT

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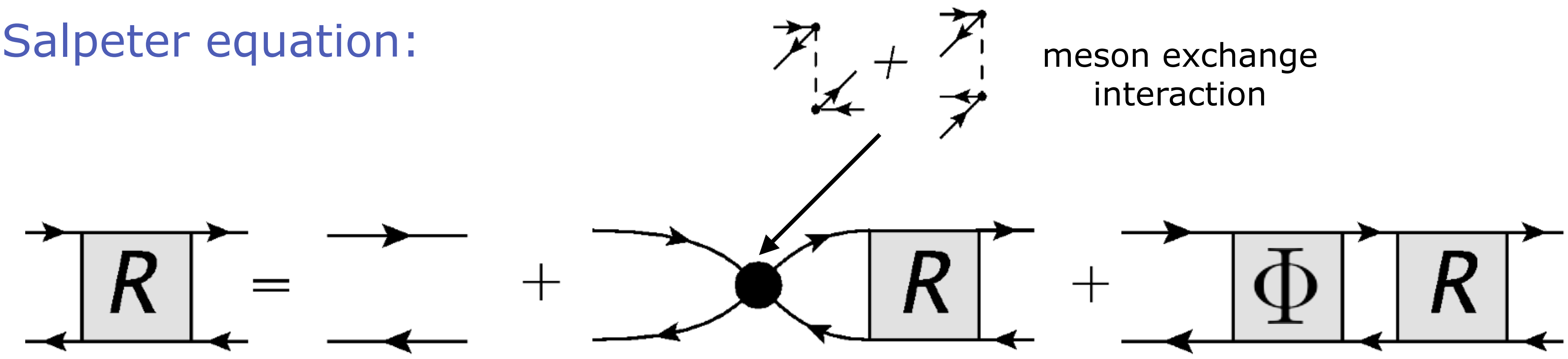

Nuclear response in RNFT

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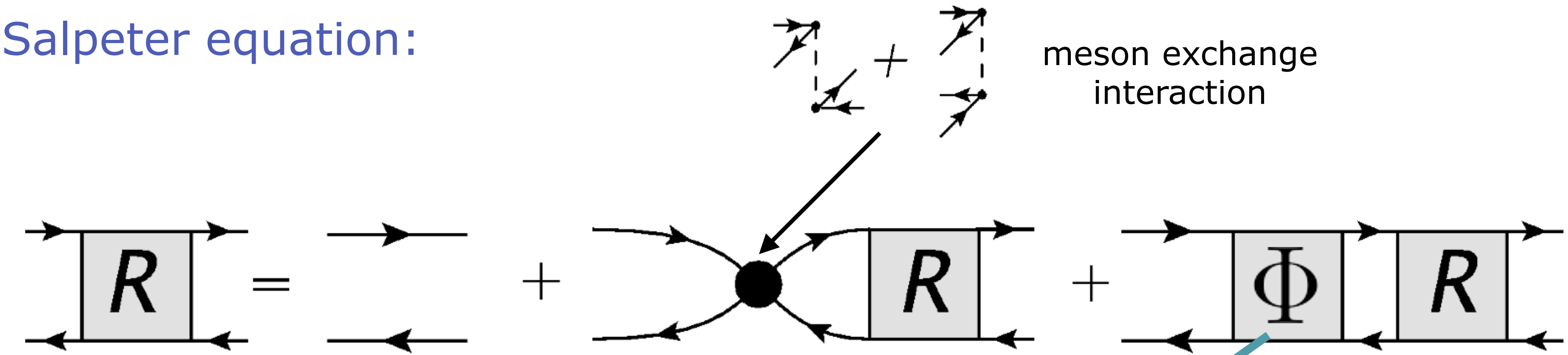
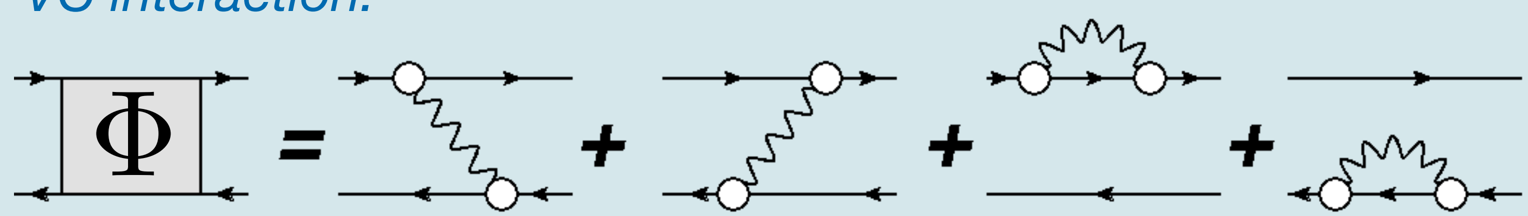
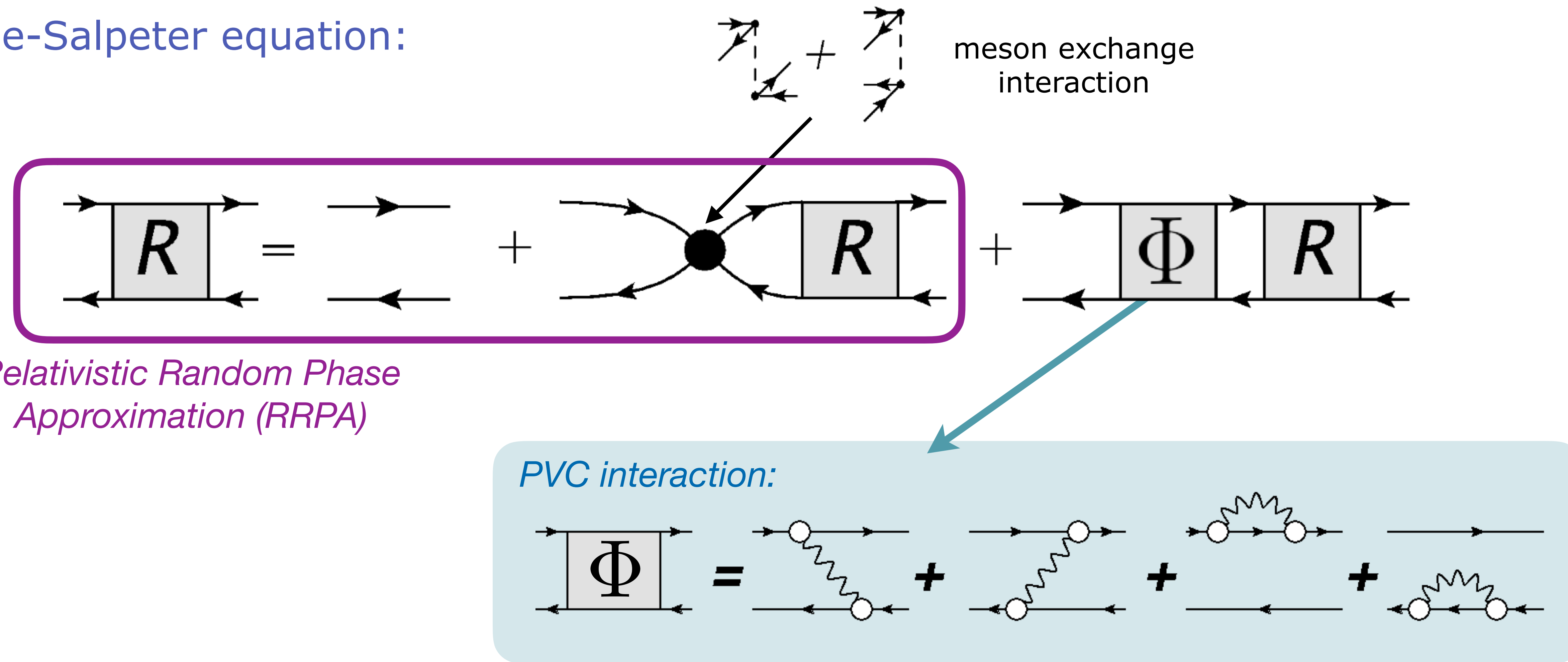


PVC interaction:

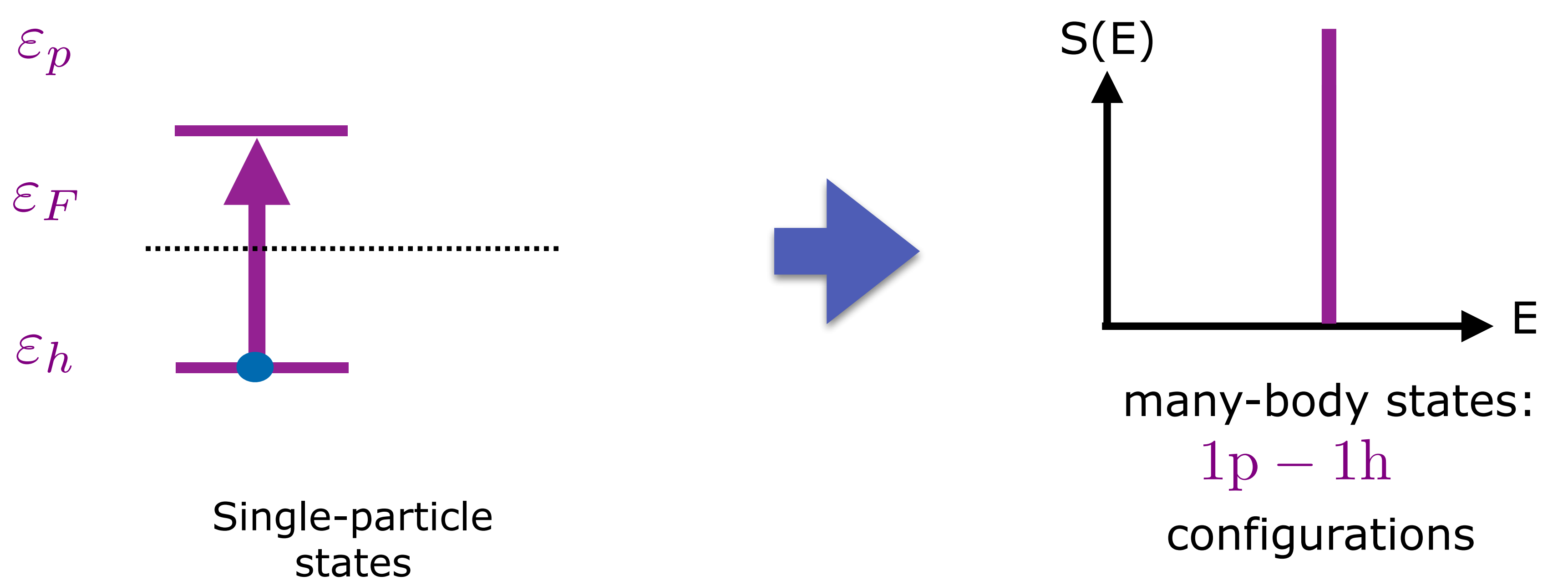


Nuclear response in RNFT

⇒ Bethe-Salpeter equation:

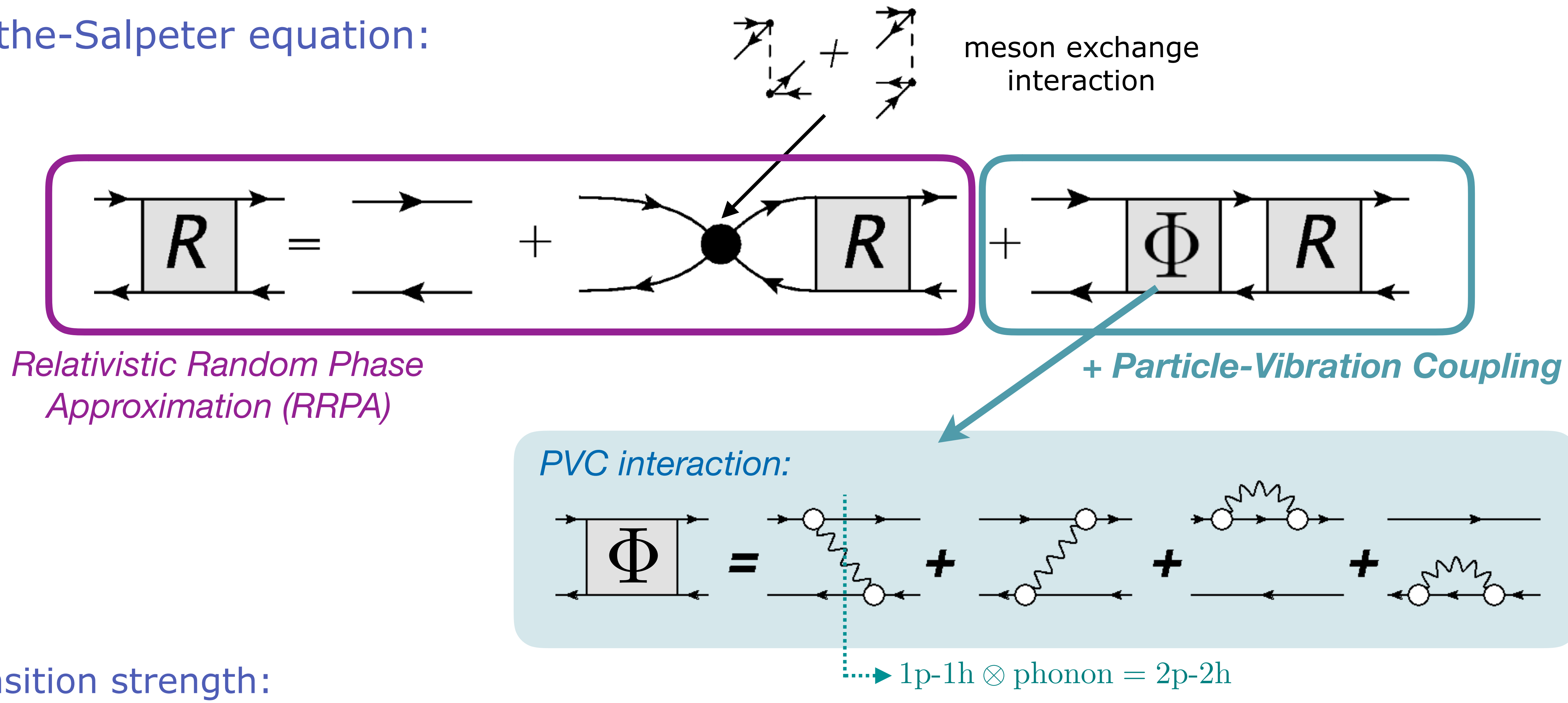


⇒ Transition strength:

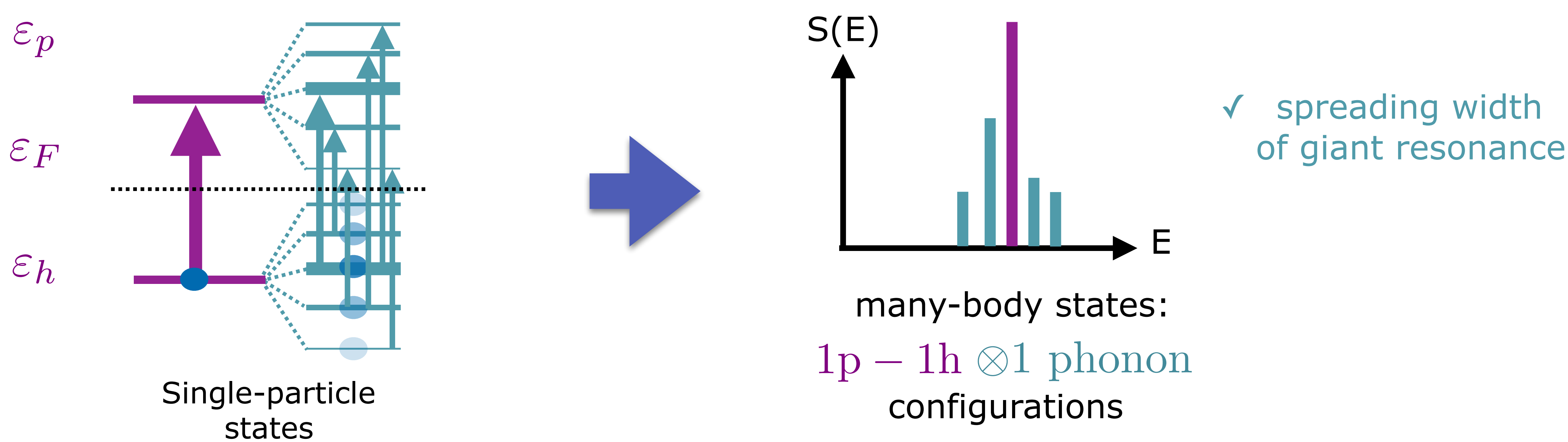


Nuclear response in RNFT

⇒ Bethe-Salpeter equation:



⇒ Transition strength:

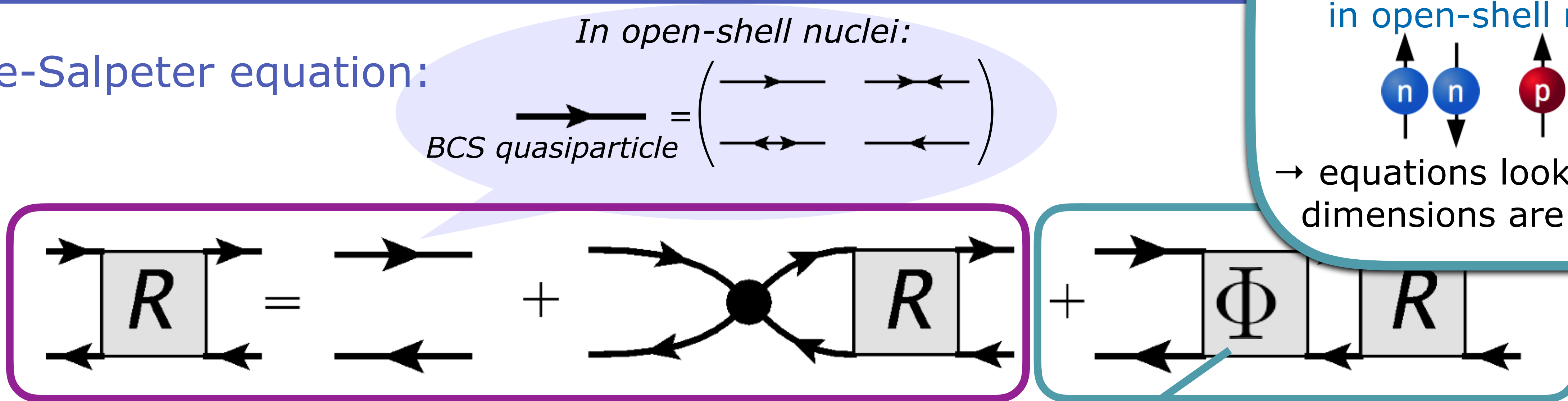


Nuclear response in RNFT

\+ superfluid pairing correlations in open-shell nuclei!

→ equations look the same, dimensions are doubled

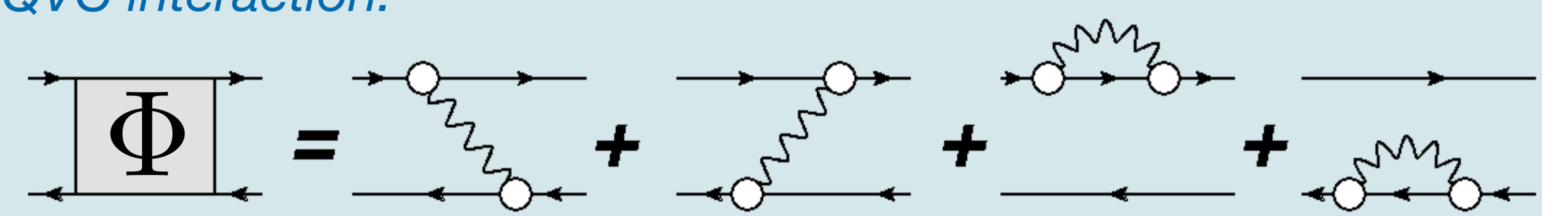
⇒ Bethe-Salpeter equation:



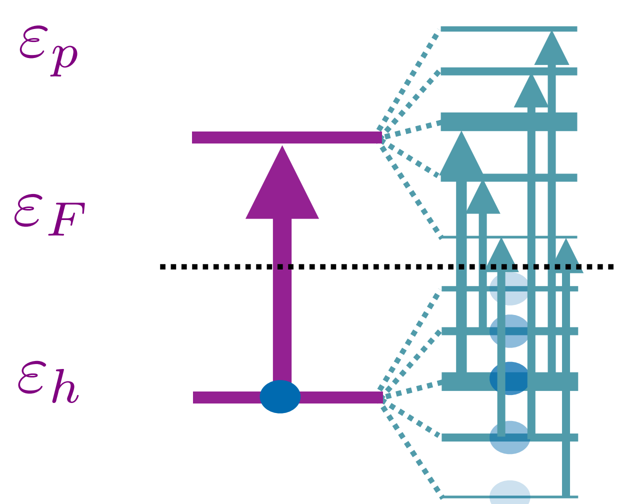
Relativistic Quasiparticle Random Phase Approximation (RQRPA)

+ QuasiParticle-Vibration Coupling

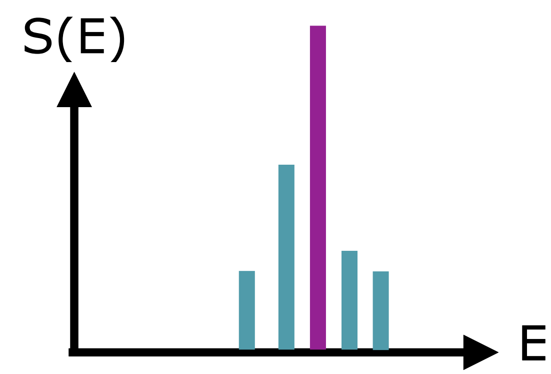
QVC interaction:



⇒ Transition strength:



Single-particle states



many-body states:

$2\ \mathrm{qp} \otimes 1\ \mathrm{phonon}$

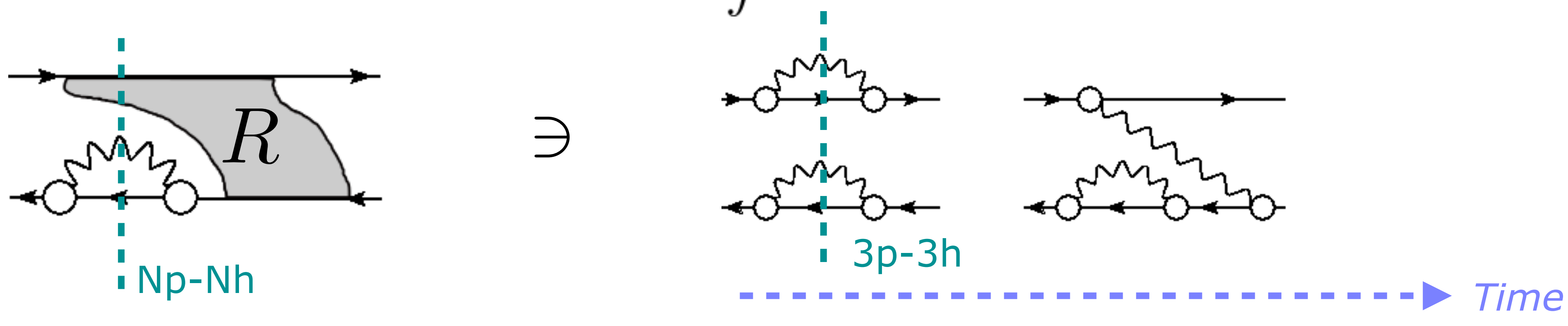
configurations

✓ spreading width of giant resonance

Nuclear response in RNFT

★ ***Problem:*** Integration over all intermediate times ⇒ complicated BSE

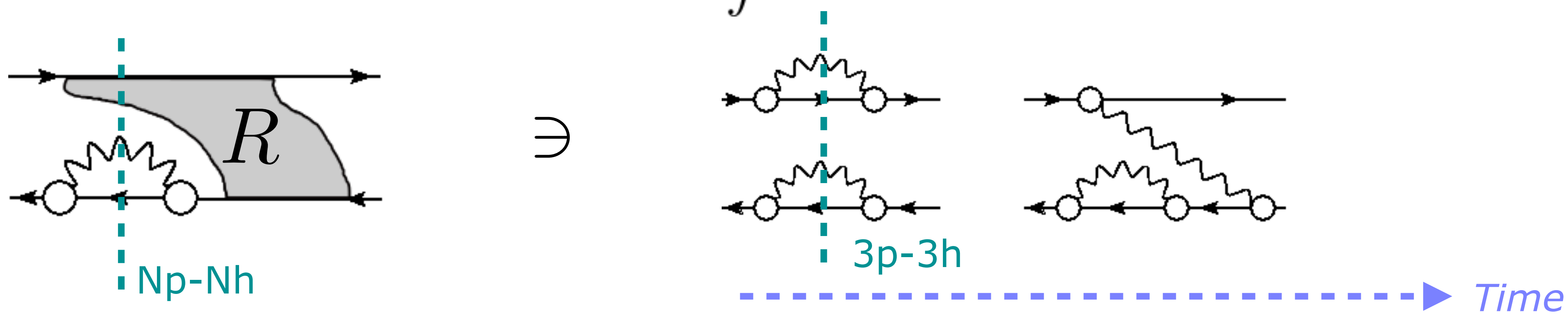
$$R(\omega,\varepsilon) = G(\omega+\varepsilon)G(\varepsilon) - iG(\omega+\varepsilon)G(\varepsilon)\int d\varepsilon_1 d\varepsilon_2 V(\varepsilon,\varepsilon_2,\omega)R(\varepsilon_2,\varepsilon_1,\omega)$$



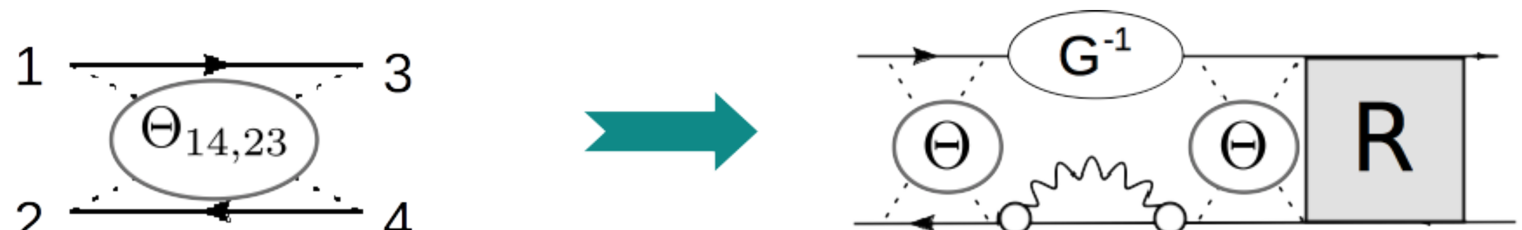
Nuclear response in RNFT

★ ***Problem:*** Integration over all intermediate times ⇒ complicated BSE

$$R(\omega,\varepsilon) = G(\omega+\varepsilon)G(\varepsilon) - iG(\omega+\varepsilon)G(\varepsilon)\int d\varepsilon_1 d\varepsilon_2 V(\varepsilon,\varepsilon_2,\omega)R(\varepsilon_2,\varepsilon_1,\omega)$$



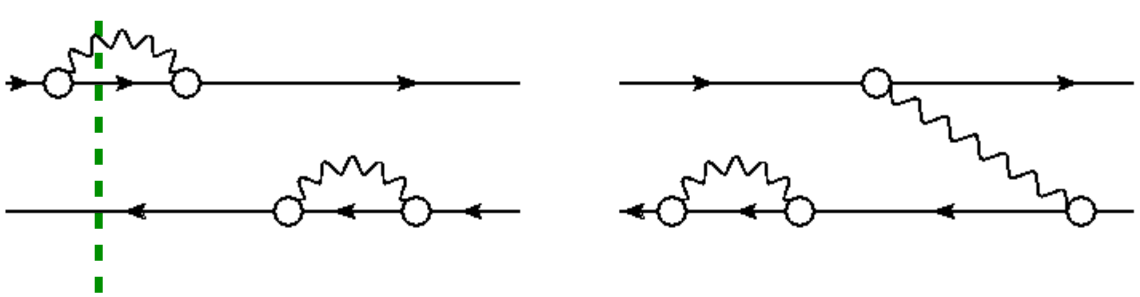
★ ***Solution:*** Time-Blocking Approximation (TBA) [V.I. Tselyaev, Yad. Fiz. 50,1252 (1989)]



$$\Theta_{14,23} = \theta(t_4 - t_1)\theta(t_3 - t_2)$$

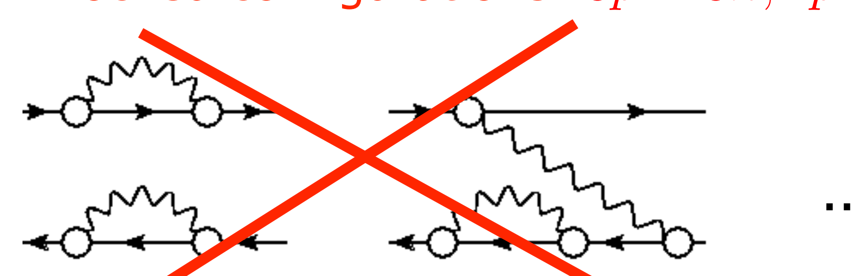
$$R(\omega) = R^0(\omega) - iR^0(\omega)(\tilde{V} + \Phi(\omega))R(\omega)$$

→ Allowed configurations:



$1p - 1h \otimes$ phonon i.e. 2p-2h

→ Blocked configurations: $3p - 3h, 4p - 4h...$



.. but can be included in a next step
(under development - see E. Litvinova's talk)

Outline

★ Relativistic Nuclear Field Theory: formalism in the resonant approximation (reminder)

★ Application to charge-exchange modes: Gamow-Teller (GT) transitions, beta-decay half-lives and the quenching problem

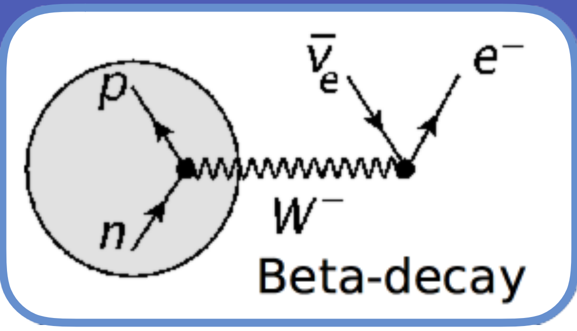
★ Recent development: Ground-state correlations from the quasiparticle-vibration coupling

- Effect on GT transitions: importance in the GT^+ channel, interplay with proton-neutron pairing

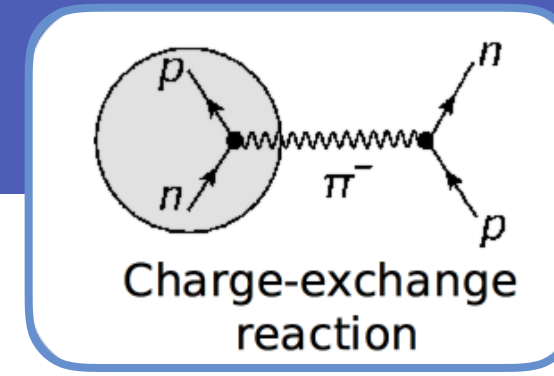
★ Application to $2\nu\beta\beta$ decay: preliminary results for ^{48}Ca, and some ideas for describing double-beta decay in the Green's function formalism

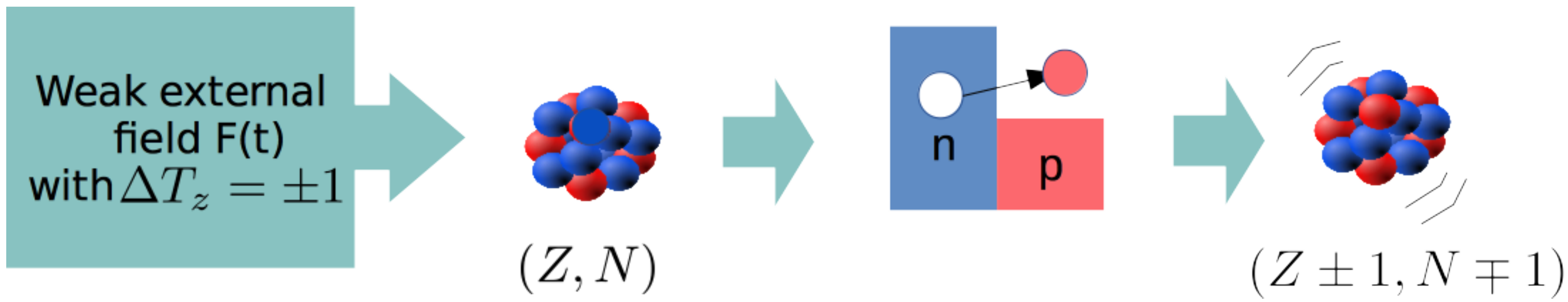
★ Conclusion, perspectives

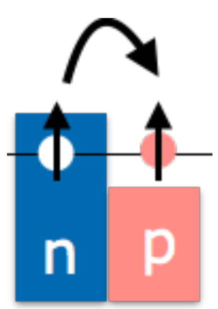
Nuclear charge-exchange modes



Response of the nucleus to an external field involving a change of the isospin projection:

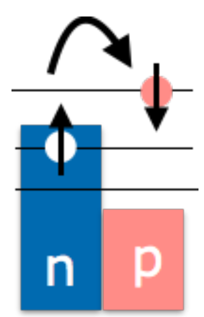






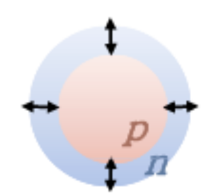
Fermi

$$F_F = \sum_n \tau_\pm^{(n)}$$



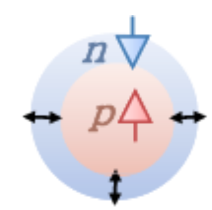
Gamow-Teller

$$F_{GT} = \sum_n \boldsymbol{\sigma}^i_{(n)} \tau_\pm^{(n)}$$



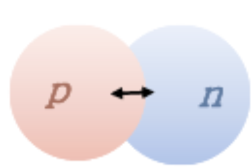
IVGMR

$$F_M = \sum_n r^2_{(n)} \tau_\pm^{(n)}$$



IVSMR

$$F_{SM} = \sum_n r^2_{(n)} \boldsymbol{\sigma}^i_{(n)} \tau_\pm^{(n)}$$

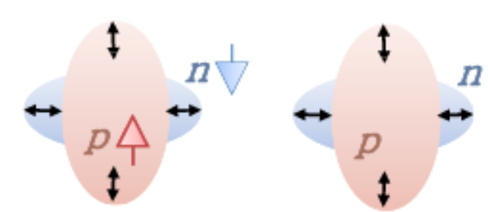


IVGDR

$$F_D = \sum_n r_{(n)} Y_1^{(n)} \tau_\pm^{(n)}$$

IVSDR

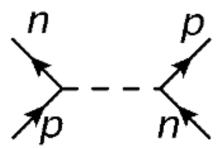
$$F^\lambda_{SD} = \sum_n r_{(n)} [\boldsymbol{\sigma}^i_{(n)} \otimes Y_1^{(n)}]_\lambda \tau_\pm^{(n)}$$

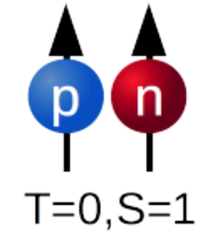


IVSQR IVGQR

...

Nuclear charge-exchange modes

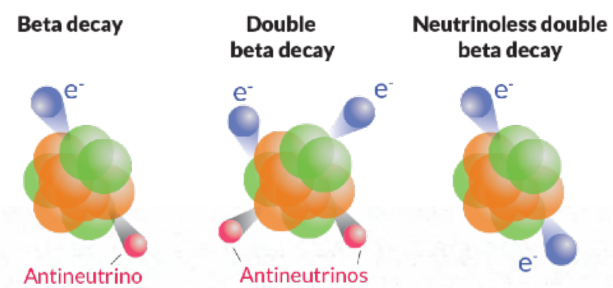




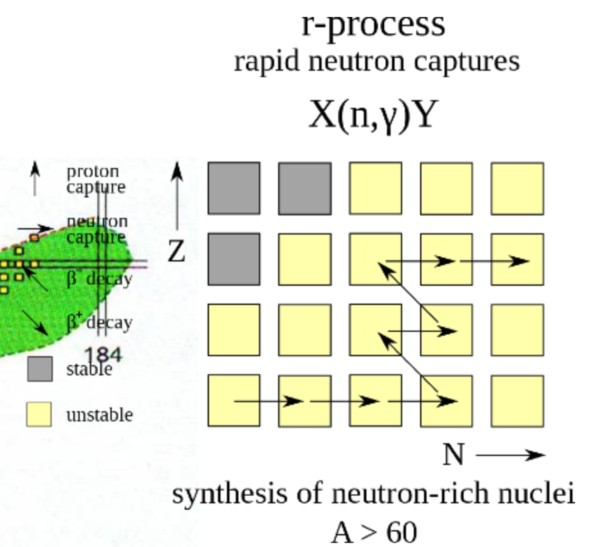
★ The study of nuclear isospin-transfer excitations has many applications in

→ **Nuclear physics:** constraints on the (S,T) channels of the nuclear interaction...

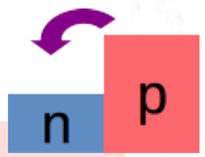
→ **Particle physics:** nature of neutrinos, BSM physics



→ **Astrophysics:**



They determine the rates of many weak processes occurring in stellar environments...

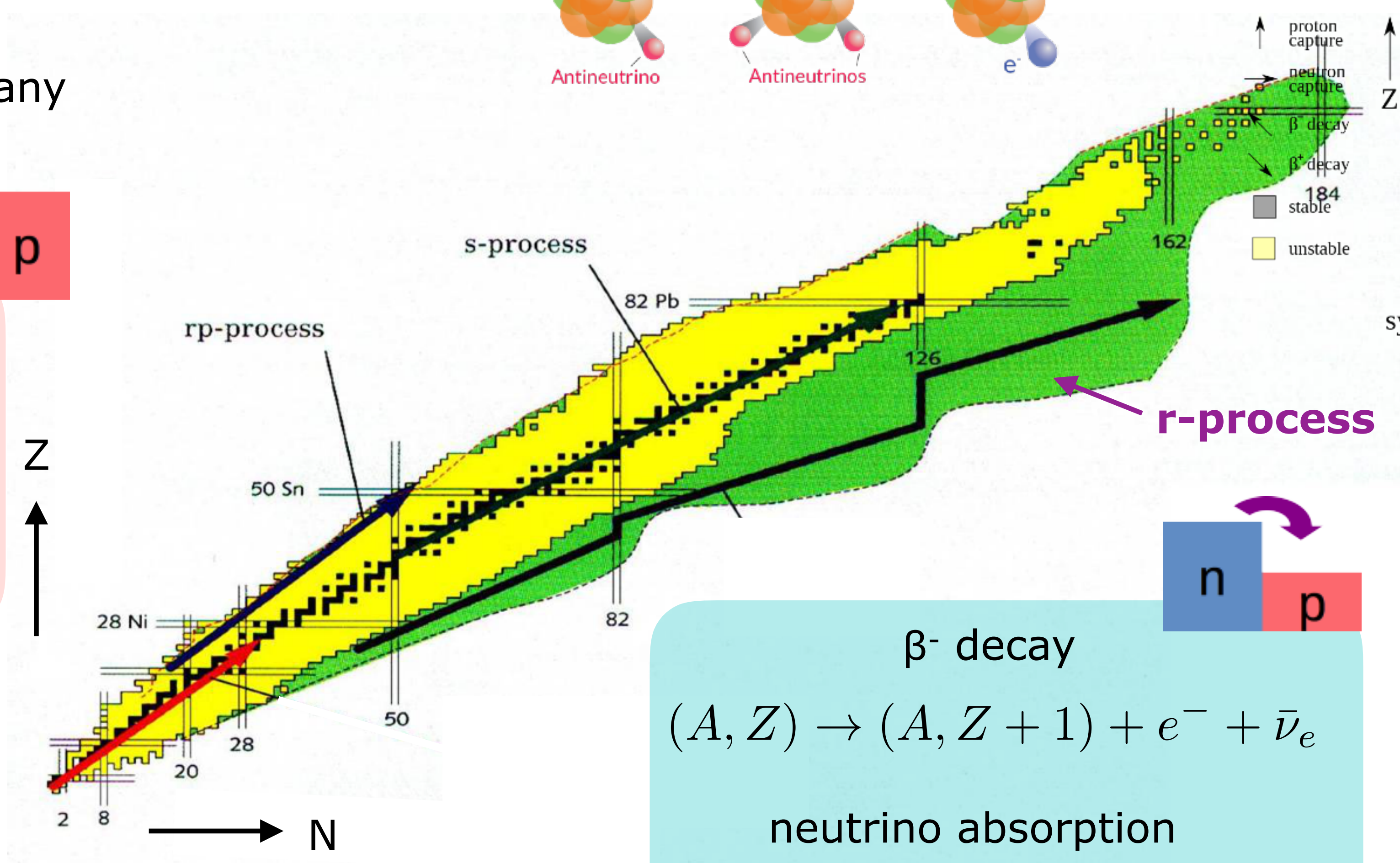


β⁺ decay

$$(A,Z) \rightarrow (A,Z-1) + e^{+} + \nu_e$$

electron capture

$$(A,Z) + e^{-} \rightarrow (A,Z-1) + \nu_e$$

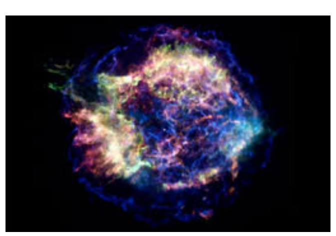


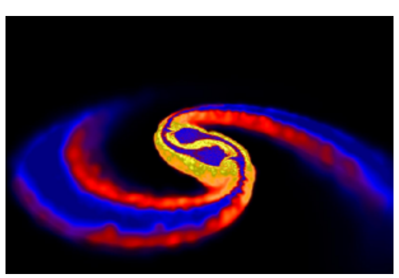
β⁻ decay

$$(A,Z) \rightarrow (A,Z+1) + e^{-} + \bar{\nu}_e$$

neutrino absorption

$$(A,Z) + \nu_e \rightarrow (A,Z+1) + e^{-}$$

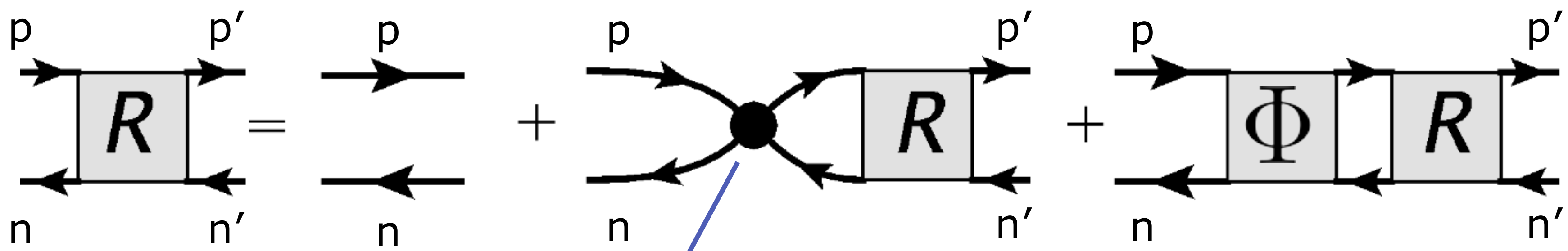




Astrophysical modeling requires properties of thousands of nuclei far from stability

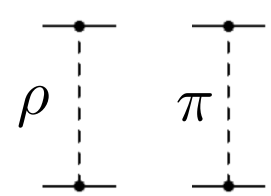
⇒ Need precise and predictive information from theory

Response theory for charge-exchange modes

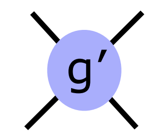


Isovector static interaction

$$\widetilde{V} = \widetilde{V}_{\rho} + \widetilde{V}_{\pi} + \widetilde{V}_{\delta\pi}$$

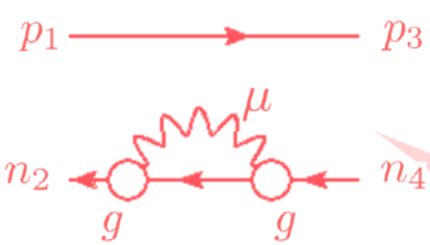


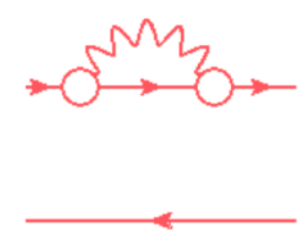
Landau-Migdal contact term



g′=0.6

(derived value of 1/3 can be adopted in approaches with the Fock term [H. Liang et al. PRL 101, 122502 (2008)])

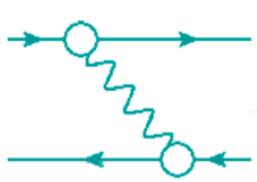


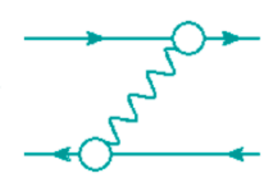


$$\Phi^{\eta}_{p_1 n_4, n_2 p_3}(\omega) =$$

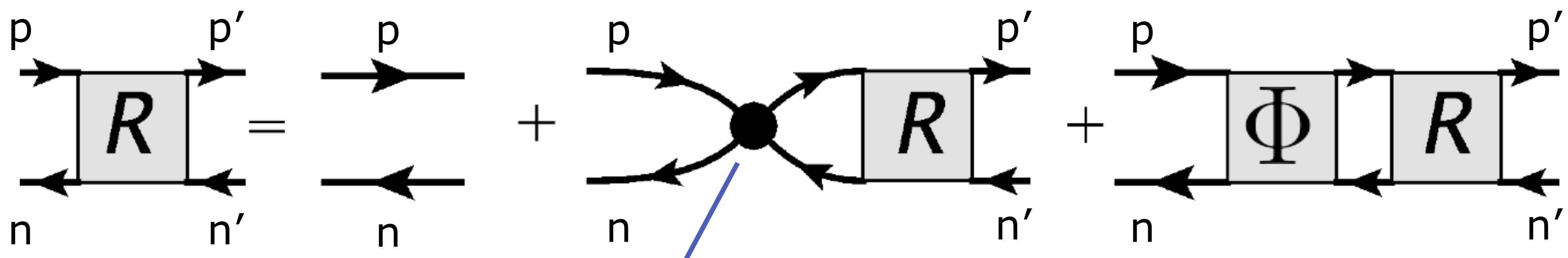
$$\sum_{\mu\xi} \delta_{\eta\xi} \left[\delta_{p_1 p_3} \sum_{n_6} \frac{g^{\eta;-\xi}_{\mu;n_6 n_2} g^{\eta;-\xi *}_{\mu;n_6 n_4}}{\eta\omega - E_{p_1} - E_{n_6} - \Omega_\mu} + \delta_{n_2 n_4} \sum_{p_5} \frac{g^{\eta;\xi}_{\mu;p_1 p_5} g^{\eta;\xi *}_{\mu;p_3 p_5}}{\eta\omega - E_{p_5} - E_{n_2} - \Omega_\mu} \right.$$

$$\left. - \frac{g^{\eta;\xi}_{\mu;p_1 p_3} g^{\eta;-\xi *}_{\mu;n_2 n_4}}{\eta\omega - E_{p_3} - E_{n_2} - \Omega_\mu} - \frac{g^{\eta;\xi *}_{\mu;p_1 p_3} g^{\eta;-\xi}_{\mu;n_4 n_2}}{\eta\omega - E_{p_1} - E_{n_4} - \Omega_\mu} \right]$$





Response theory for charge-exchange modes



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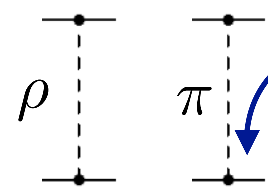
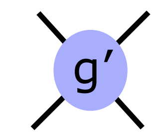
$$\widetilde{V} = \widetilde{V}_\rho + \widetilde{V}_\pi + \widetilde{V}_{\delta\pi}$$



Landau-Migdal contact term

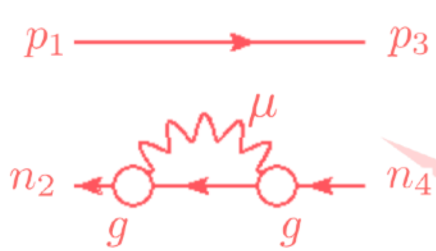


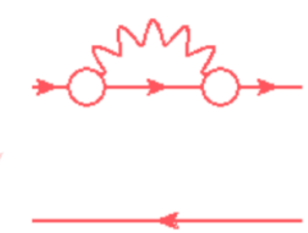
g′=0.6

(derived value of 1/3 can be adopted in approaches with the Fock term [H. Liang et al. PRL 101, 122502 (2008)])

free-space coupling constant

➡ ***no subtraction*** *for unnatural-parity modes*

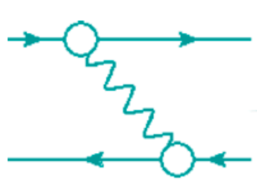


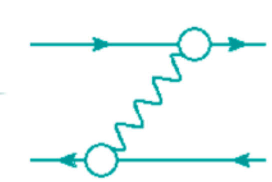


$$\Phi^{\eta}_{p_1 n_4, n_2 p_3}(\omega) =$$

$$\sum_{\mu\xi} \delta_{\eta\xi} \left[\delta_{p_1 p_3} \sum_{n_6} \frac{g^{\eta;-\xi}_{\mu;n_6 n_2} g^{\eta;-\xi *}_{\mu;n_6 n_4}}{\eta\omega - E_{p_1} - E_{n_6} - \Omega_\mu} + \delta_{n_2 n_4} \sum_{p_5} \frac{g^{\eta;\xi}_{\mu;p_1 p_5} g^{\eta;\xi *}_{\mu;p_3 p_5}}{\eta\omega - E_{p_5} - E_{n_2} - \Omega_\mu} \right.$$

$$\left. - \frac{g^{\eta;\xi}_{\mu;p_1 p_3} g^{\eta;-\xi *}_{\mu;n_2 n_4}}{\eta\omega - E_{p_3} - E_{n_2} - \Omega_\mu} - \frac{g^{\eta;\xi *}_{\mu;p_1 p_3} g^{\eta;-\xi}_{\mu;n_4 n_2}}{\eta\omega - E_{p_1} - E_{n_4} - \Omega_\mu} \right]$$



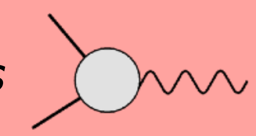


Numerical algorithm

Inputs:
Meson parameters (<10) (NL3)
\+ monopole pairing force (1-2)

Relativistic mean-field with superfluid pairing

RQRPA calculation of the phonons

⇒ coupling vertices 

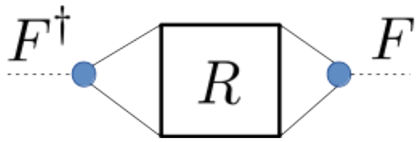
Phonons are selected according to their J^π, T, and energies

Typically: phonons with $J^\pi=2^+ \rightarrow 6^+$, E^*_{max} = 20-30 MeV

Solve BSE with quasiparticle-vibration coupling in RQTBA

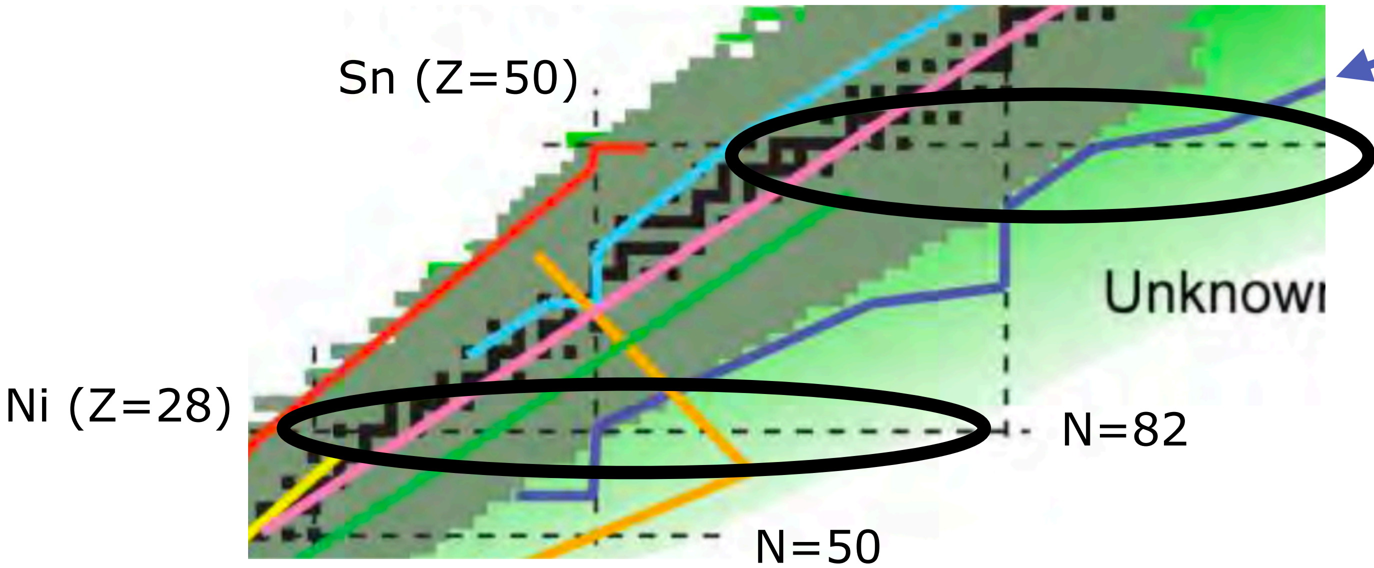
for the response function
with given quantum numbers

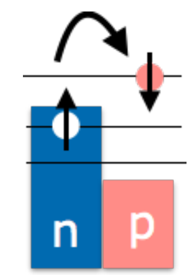
Ex: Gamow-Teller: $J^\pi=1^+$



Transition strength distribution $$S(E) = -\frac{1}{\pi} \lim_{\Delta\to 0^+} \mathrm{Im}\, \langle \Psi_i | \hat{F}^\dagger R(E+i\Delta) \hat{F} | \Psi_i \rangle$$

Gamow-Teller and beta decay of nuclei near the r-process path

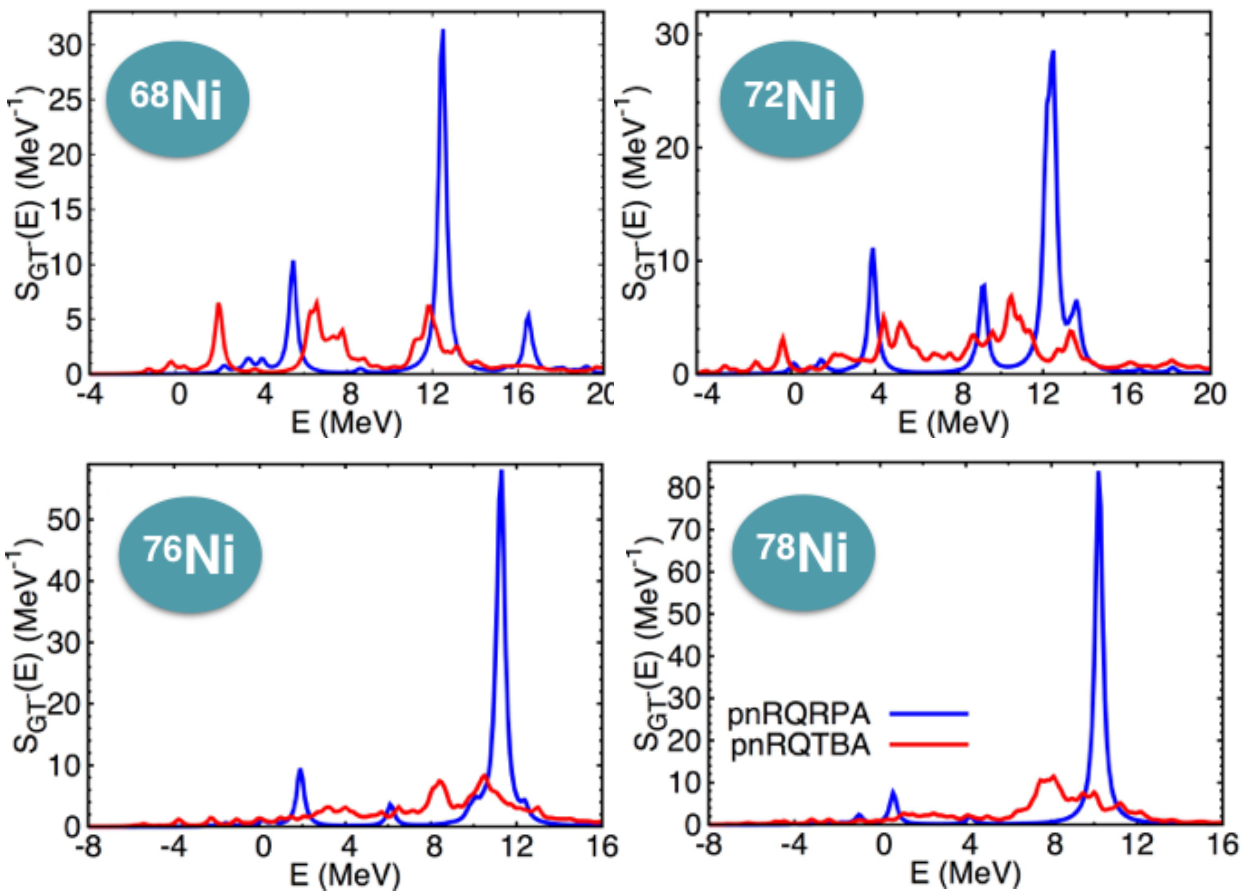


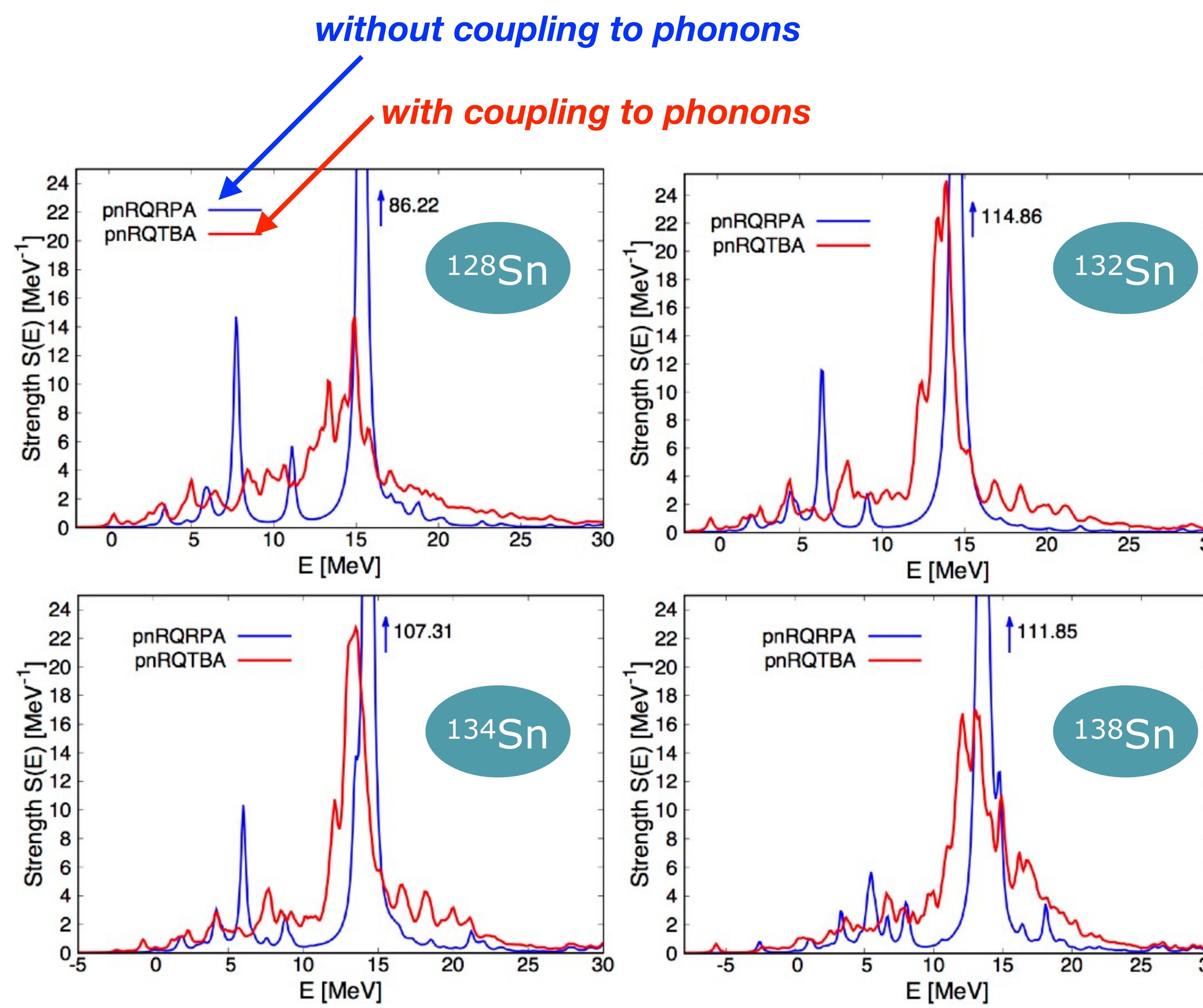


✦ ***Gamow-Teller mode:***

$$F_{GT^-} = \sum_{n=1}^{A} \begin{pmatrix} \sigma^i_{(n)} & 0 \\ 0 & \sigma^i_{(n)} \end{pmatrix} \tau_-^{(n)}$$

$$\Delta T_z = -1 \qquad \Delta S = 1 \qquad \Delta L = 0$$

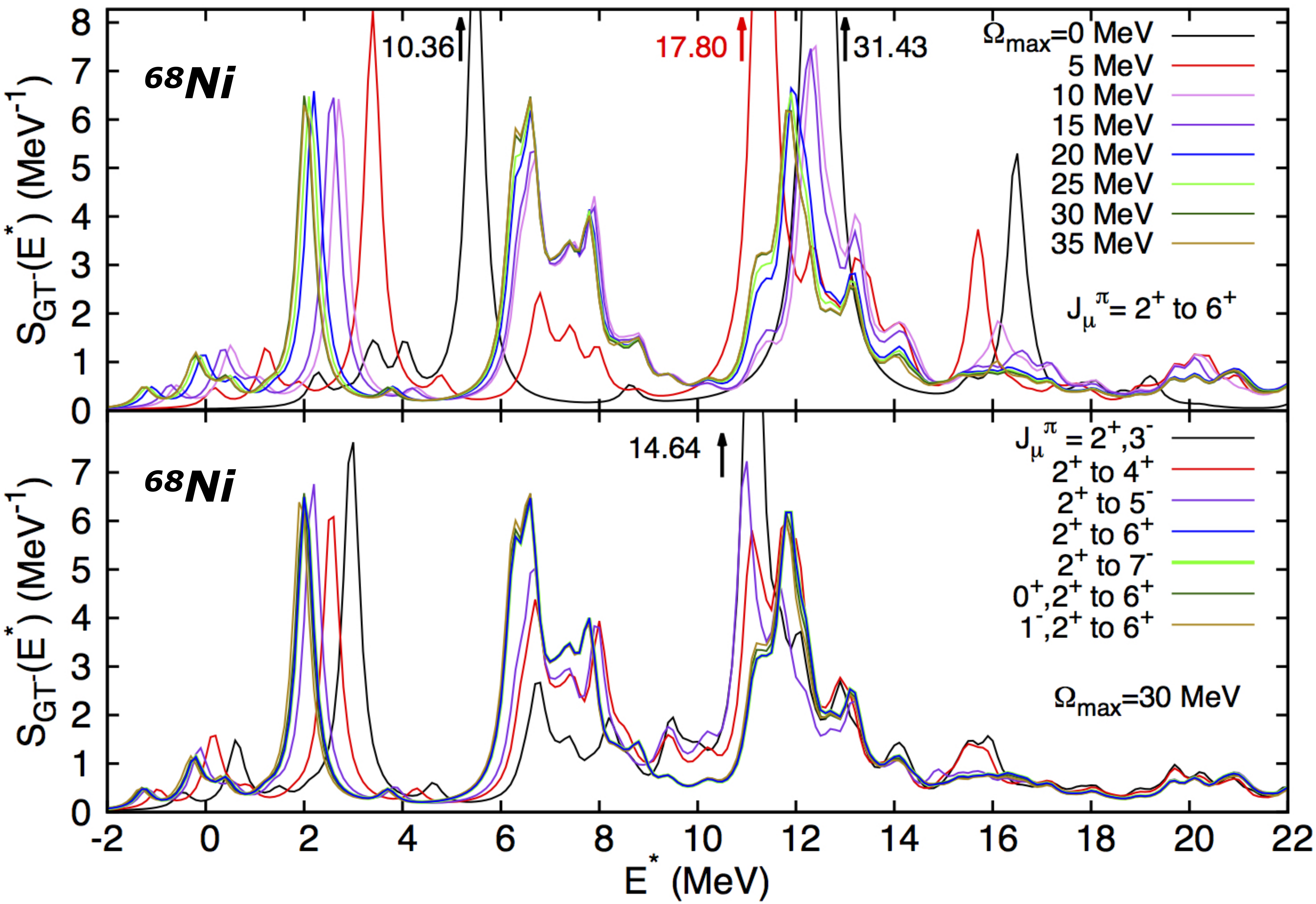




C. R. and E. Litvinova EPJA 52, 205 (2016) & AIP Conf. Proc. 1912, 020014 (2017).

Gamow-Teller and beta decay of nuclei near the r-process path

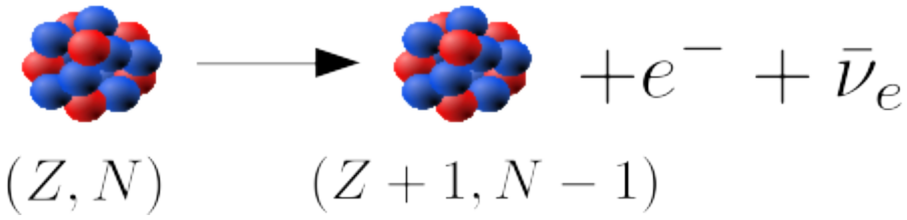
Convergence of the strength according to the phonon spectrum (natural-parity neutral phonons only):

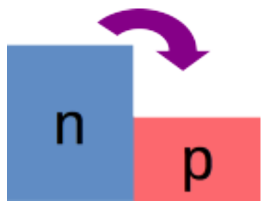


C. Robin and E. Litvinova EPJA 52, 205 (2016).

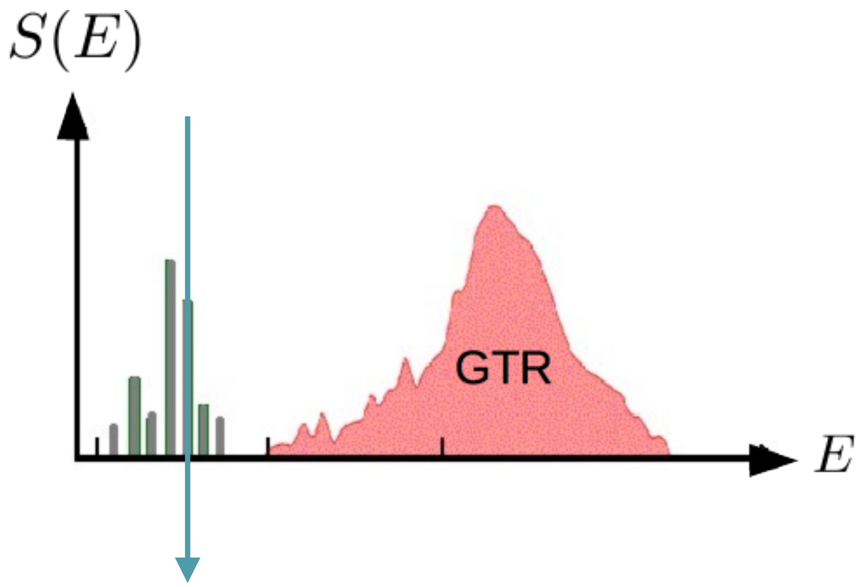
Gamow-Teller and beta decay of nuclei near the r-process path

✦ **β⁻ -decay:**





In the allowed approximation, it is determined by the low-lying GT strength:



$Q_\beta = M_{at}(Z,n) - M_{at}(Z+1,N-1)$

→ beta-decay half-lives:

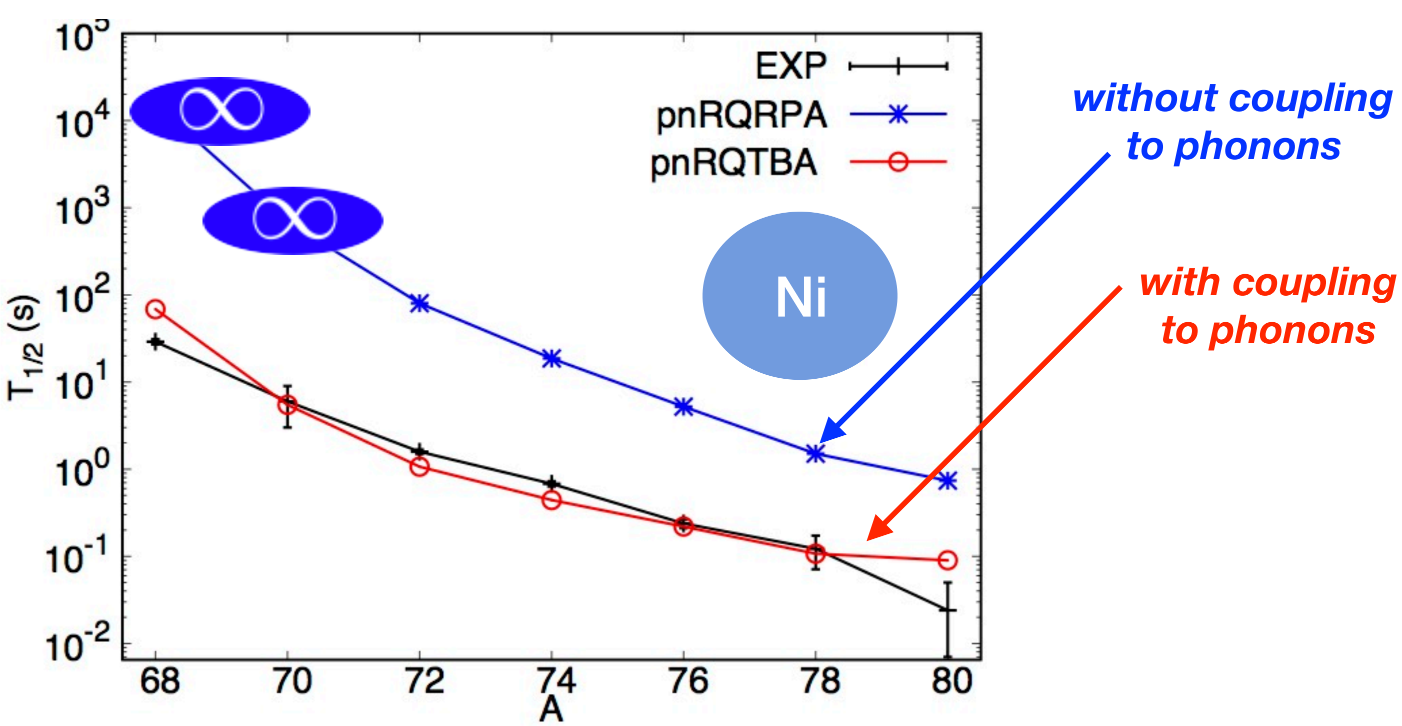
$$\frac{1}{T_{1/2}} = \frac{g_a^2}{D} \int^{Q_\beta} f(Z, Q_\beta - E) S(E) dE$$

Leptonic phase space

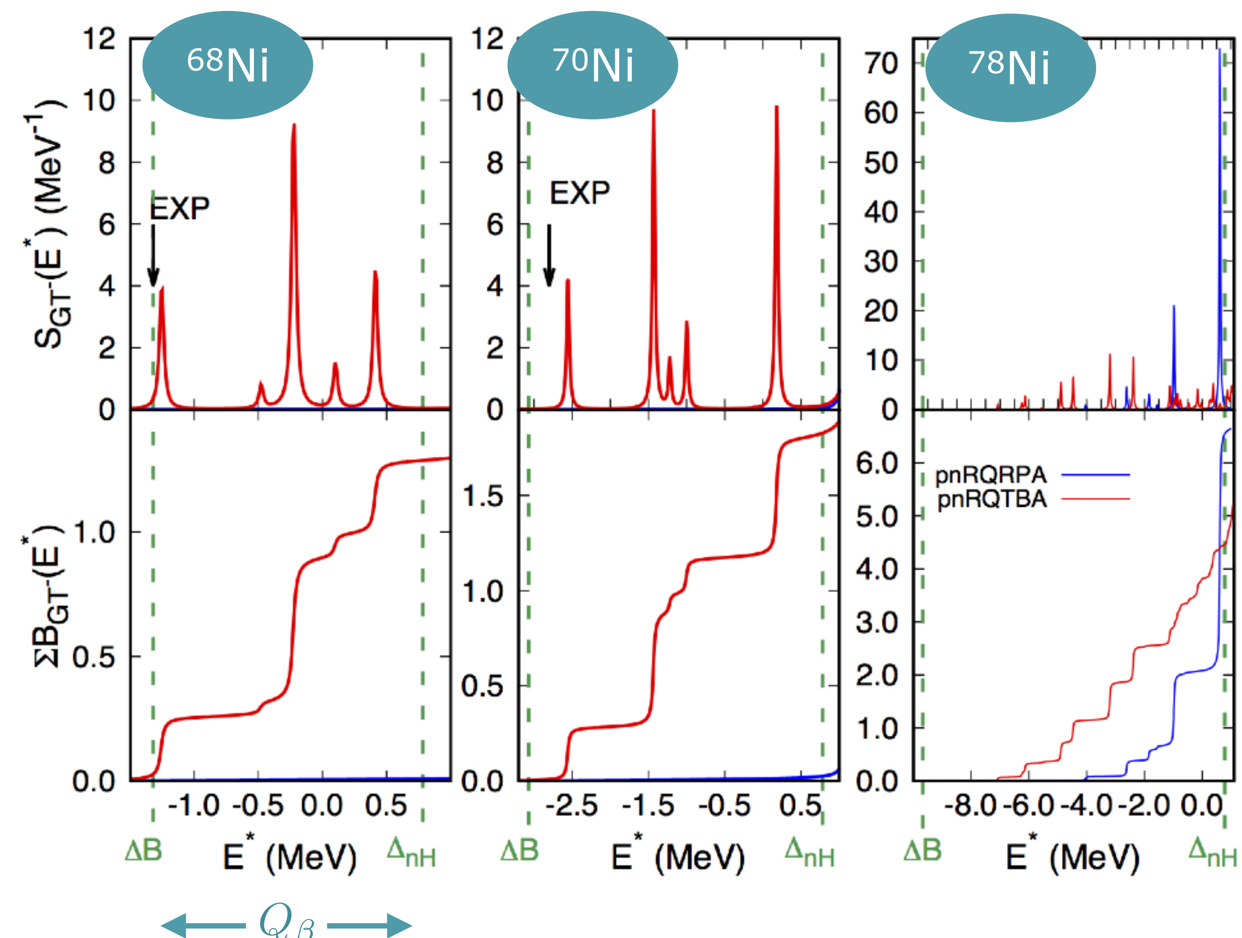
Gamow-Teller and beta decay of nuclei near the r-process path

✦ ***Beta-decay half-lives:***

$$(Z,N) \rightarrow (Z+1,N-1) + e^- + \bar{\nu}_e$$

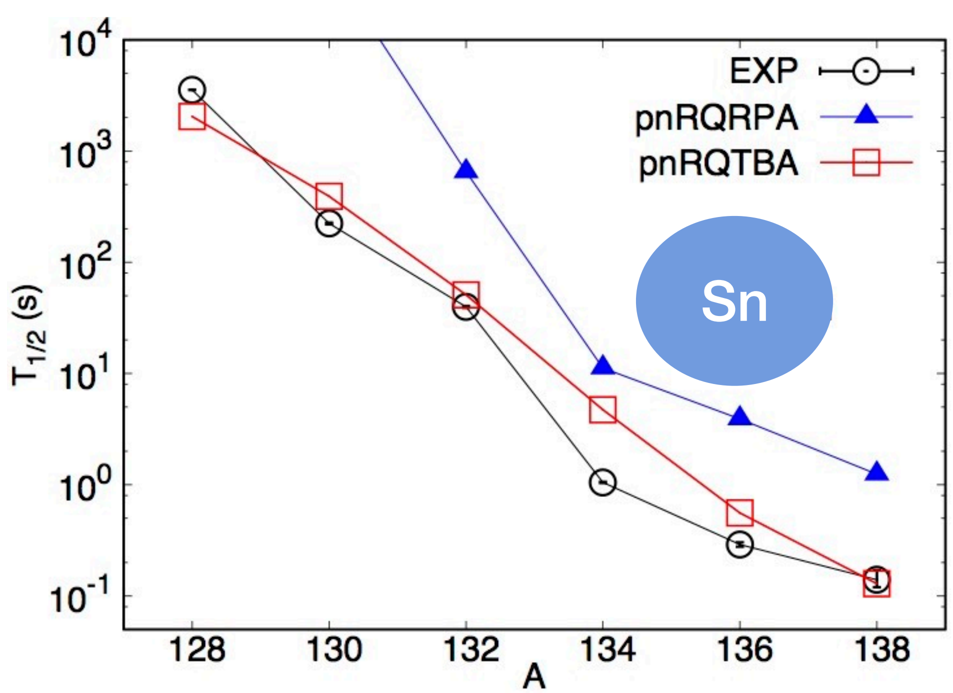


✦ ***Low-energy strength:***



with g_A=1.27

No new parameter introduced



- ✴ In the lighter isotopes: appearance of strength in the Q_β window due to QVC → finite lifetime
- ✴ ^{78}Ni: more strength with RQRPA but located at higher energies → smaller lifetime with QVC due to phase space factor

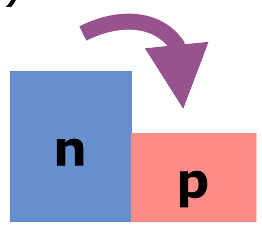
C. R. and E. Litvinova EPJA 52, 205 (2016) & AIP Conf. Proc.1912, 020014 (2017).

Quenching of the Gamow-Teller strength

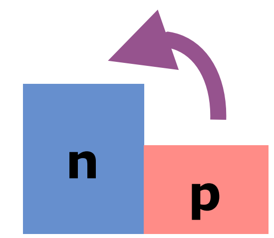
✦ *Missing strength / quenching problem:*

The observed GT strength (~up to the GR region) in nuclei is ~40-50% less than the model independent Ikeda sum rule: $S_- - S_+ = 3(N-Z)$

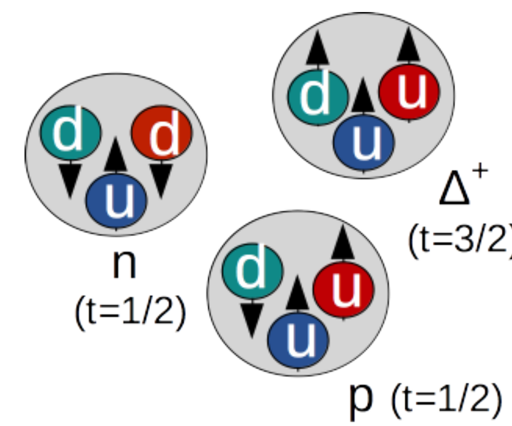
$$S_- = \sum B(GT^-)$$



$$S_+ = \sum B(GT^+)$$



⇒ some strength is pushed at high energies → possible mechanisms?



★ Coupling of 1p1h to the Δ Baryon (believed to be small? - not done here)

★ Two-body currents (not considered here)

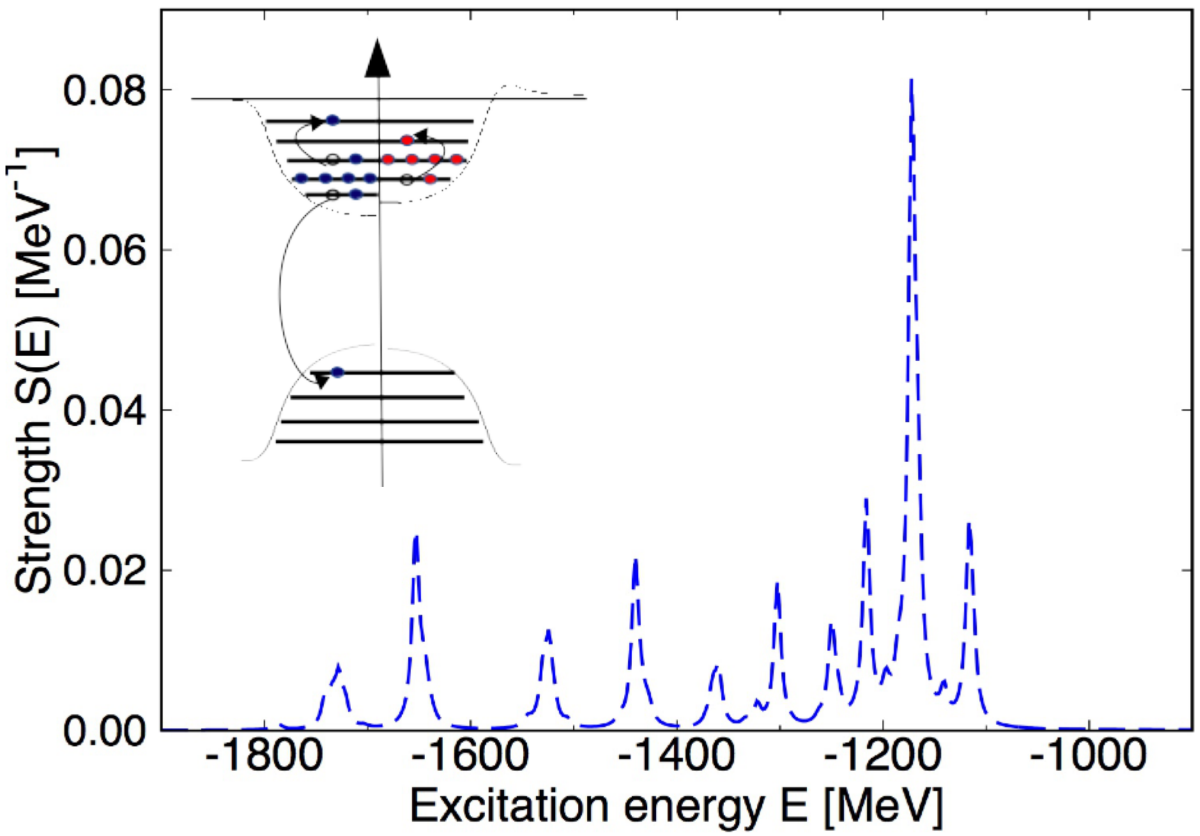
★ Coupling of 1p1h to higher-order configurations (2p2h, 3p3h etc…)

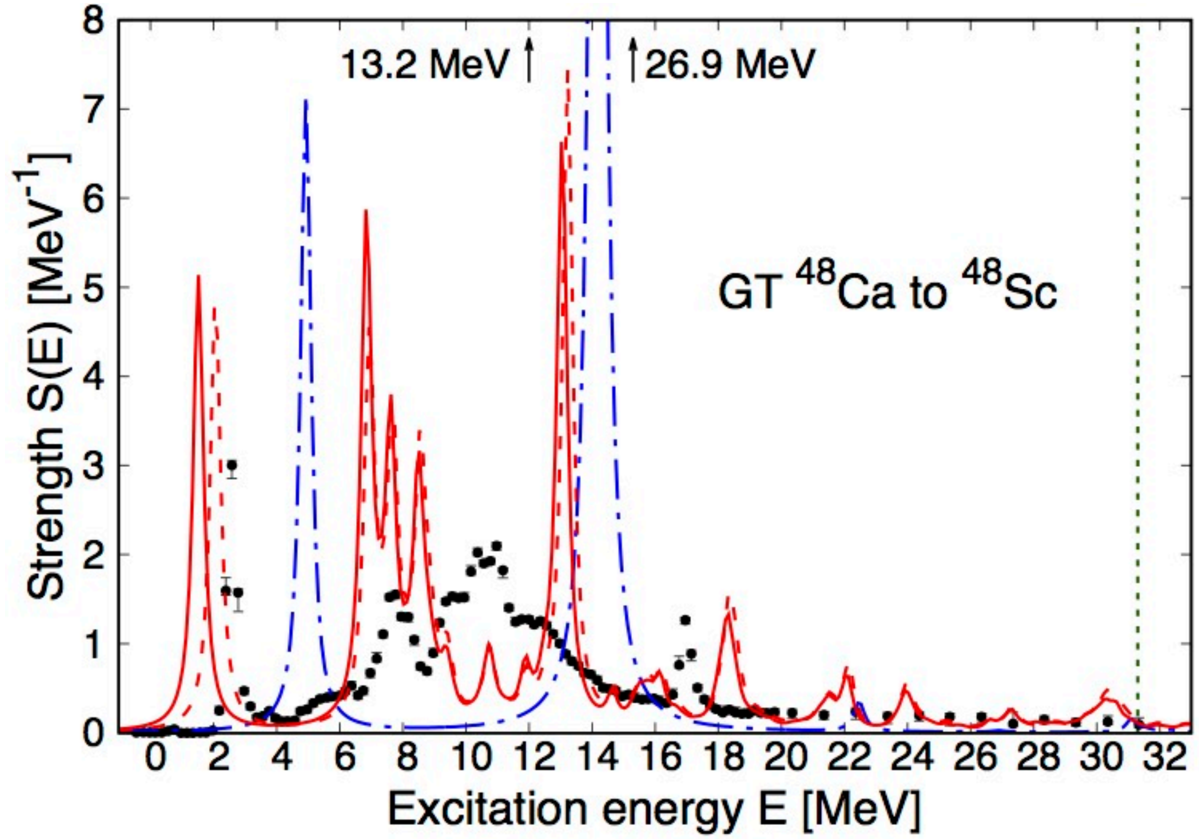
➡ Important to include complex configurations in large model spaces

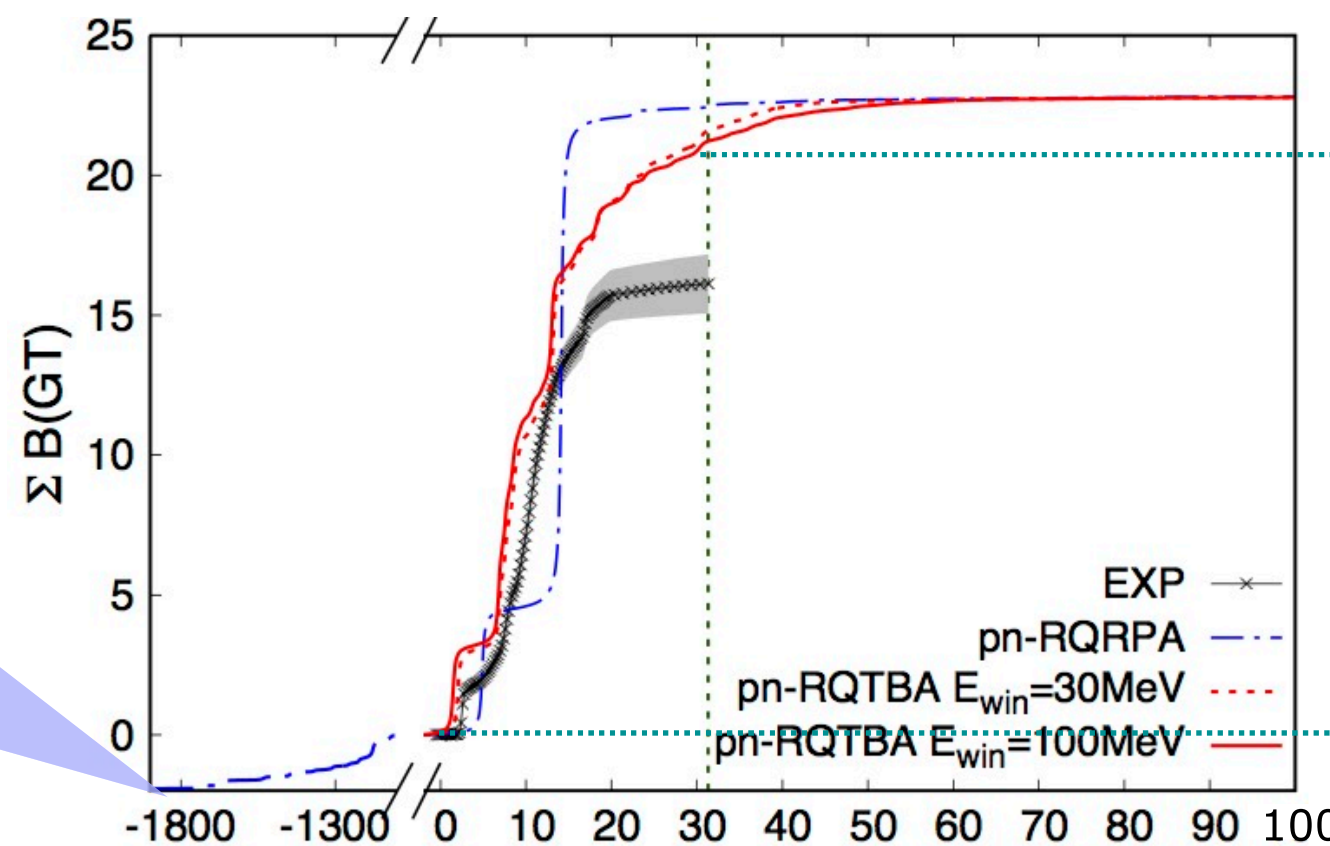
At present in RNFT+TBA:

✓ 2p-2h (4qp) configurations
✓ in a “large” energy window (from 30 up to ~100 MeV in mid-mass or doubly magic nuclei)

Quenching of the Gamow-Teller strength







\+ transitions from the Fermi sea to the Dirac sea (~8%)

[Kurasawa et al. PRL ***91****, 062501 (2003)]*

Up to 30 MeV: ~91% of the total GT⁻ strength

versus 98% in RQRPA

→ the strength is naturally "quenched" due to complex configurations

but not enough…
EXP: ~71%

C.R. & E. Litvinova, PoS INPC2016 (2017) 020 (arXiv:1702.00044)

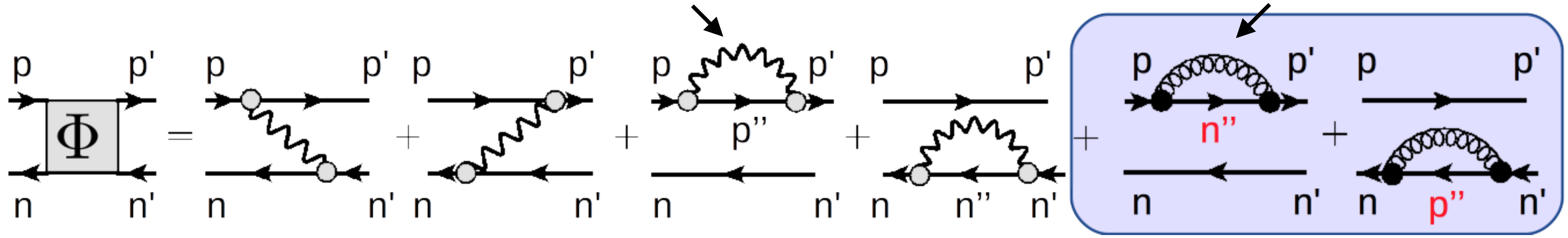
EXP: K. Yako et al., PRL 103, 012503 (2009)

Quenching of the Gamow-Teller strength

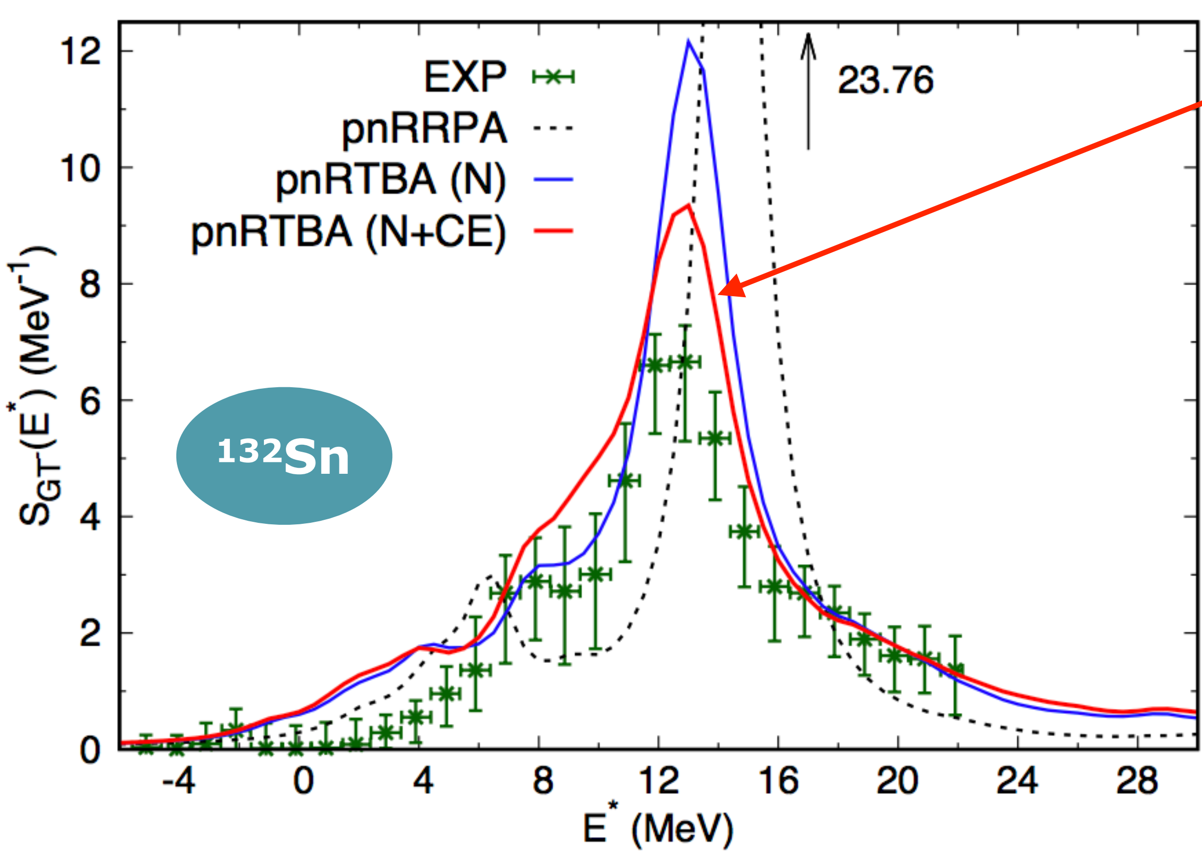
✦ ***The case of ^{132}Sn:***

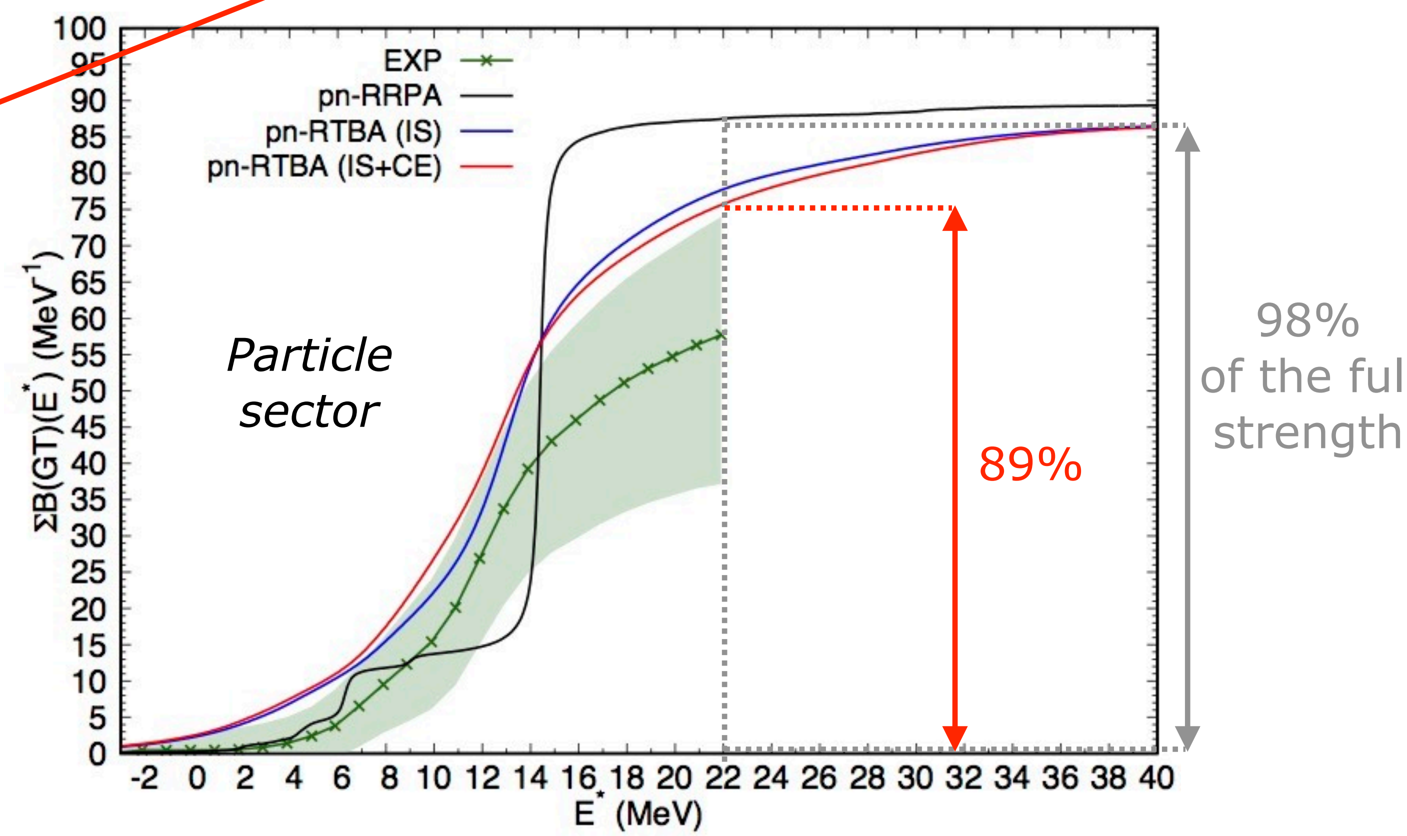
Neutral (T=0) phonons

Charge-exchange (T_z=+/- 1) phonons



New development: inclusion of the coupling of nucleons to charge-exchange phonons





→ *GTR reduced by ~25% by the inclusion of the new phonons*

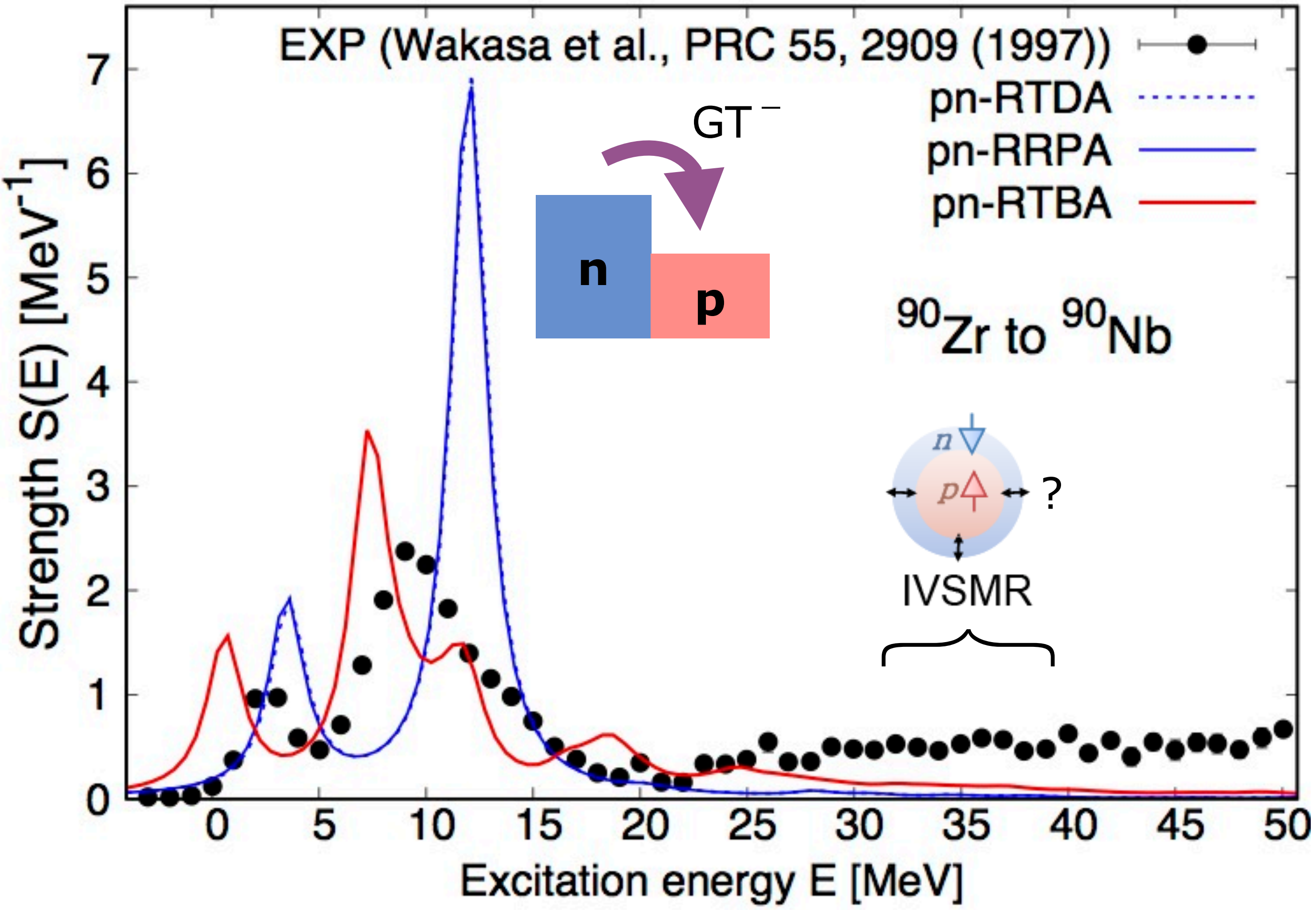
→ *Shoulder structure is enhanced*

→ **At 22 MeV: strength naturally quenched by ~9 % due to the complex configurations**

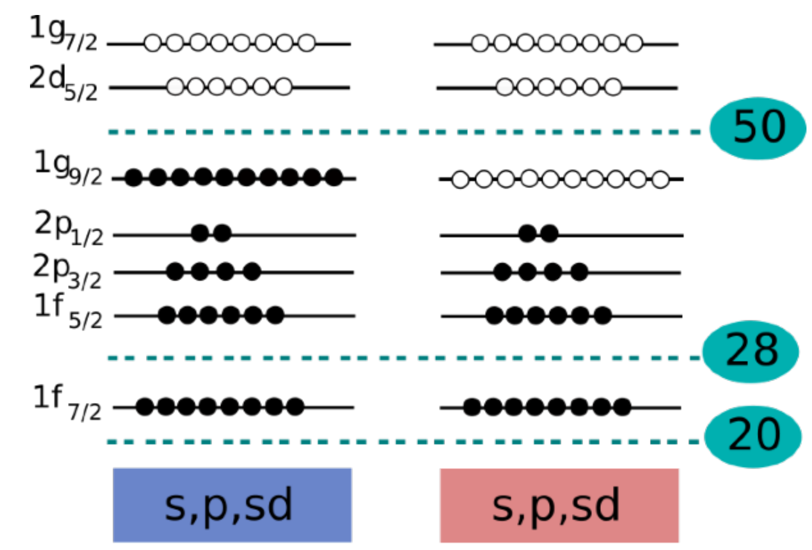
C. R. and E. Litvinova Phys.Rev. C98, 051301 (2018). EXP: J. Yasuda et al., Phys. Rev. Lett. 121, 132501 (2018).

Gamow-Teller in the GT+ channel

^{90}Zr treated here as doubly magic (no pairing)



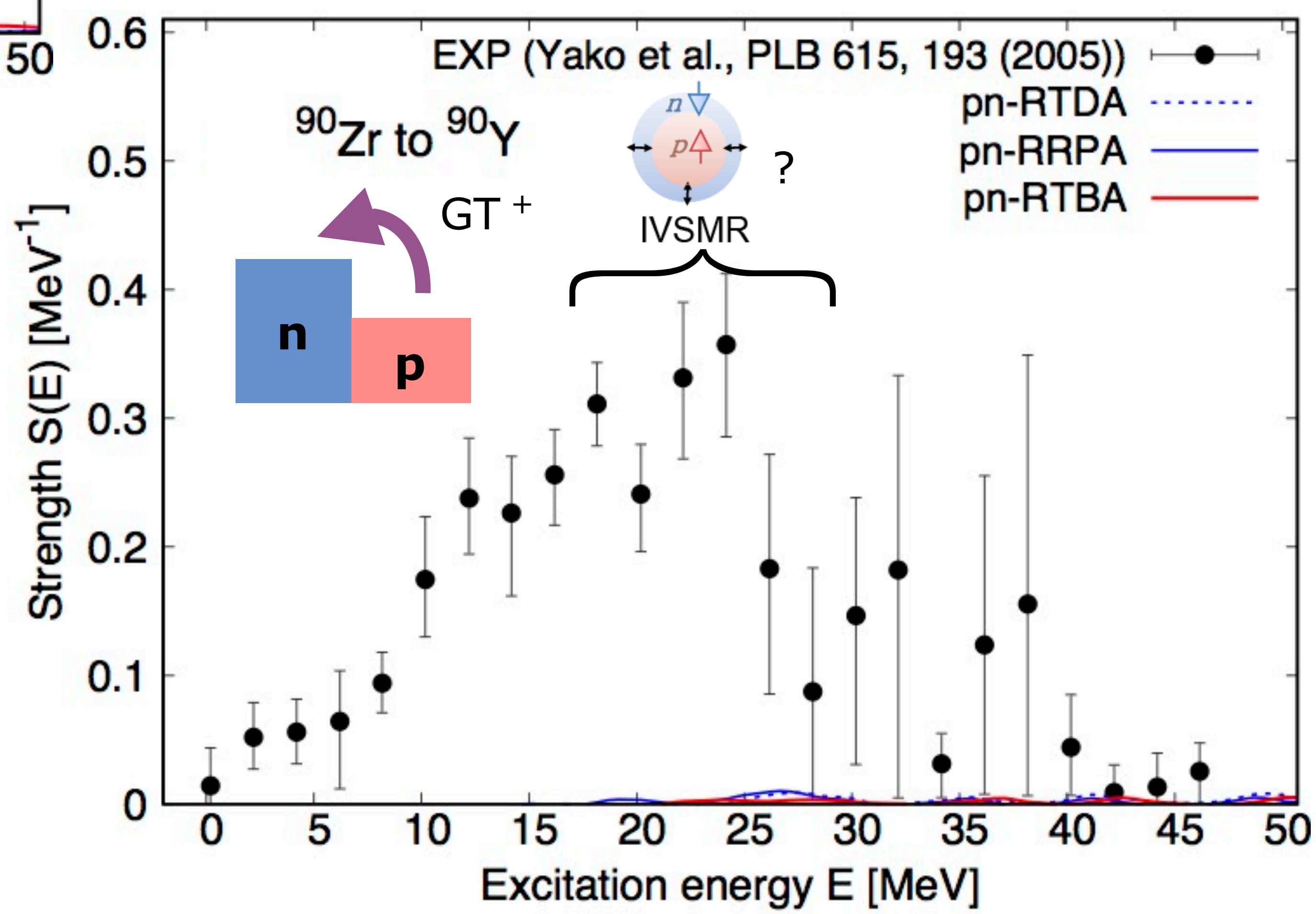
Schematic single-particle spectrum:



In a pure independent-particle picture, the transitions in the GT+ channel are blocked by the Pauli principle, due to the neutron excess.

Ground-state correlations (GSC) can smear the occupancies of the single-particle shells and unlock GT+ transitions.

- *GSC of RRPA have a negligible effect here.*
- *GSC induced by the particle-vibration coupling?*



Outline

★ Relativistic Nuclear Field Theory: formalism in the resonant approximation (reminder)

★ Application to charge-exchange modes: Gamow-Teller (GT) transitions, beta-decay half-lives and the quenching problem

★ Recent development: Ground-state correlations from the quasiparticle-vibration coupling

- Effect on GT transitions: importance in the GT^+ channel, interplay with proton-neutron pairing

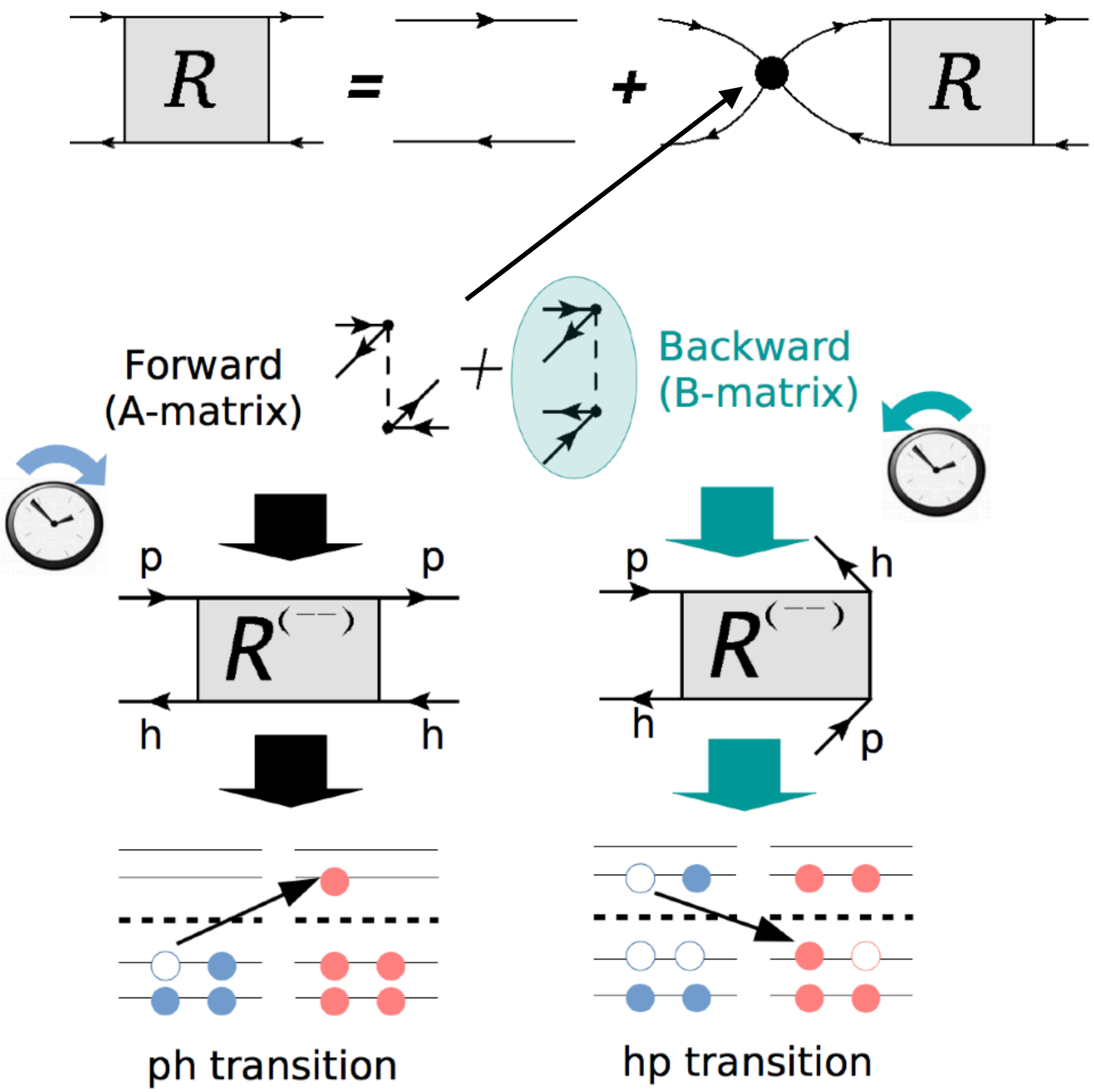
★ Application to $2\nu\beta\beta$ decay: preliminary results for ^{48}Ca, and some ideas for describing double-beta decay in the Green's function formalism

★ Conclusion, perspectives

Ground-state correlations in RQTBA

In the Green's functions formalism ground-state correlations (GSC) are generated by the so-called "backward-going diagrams":

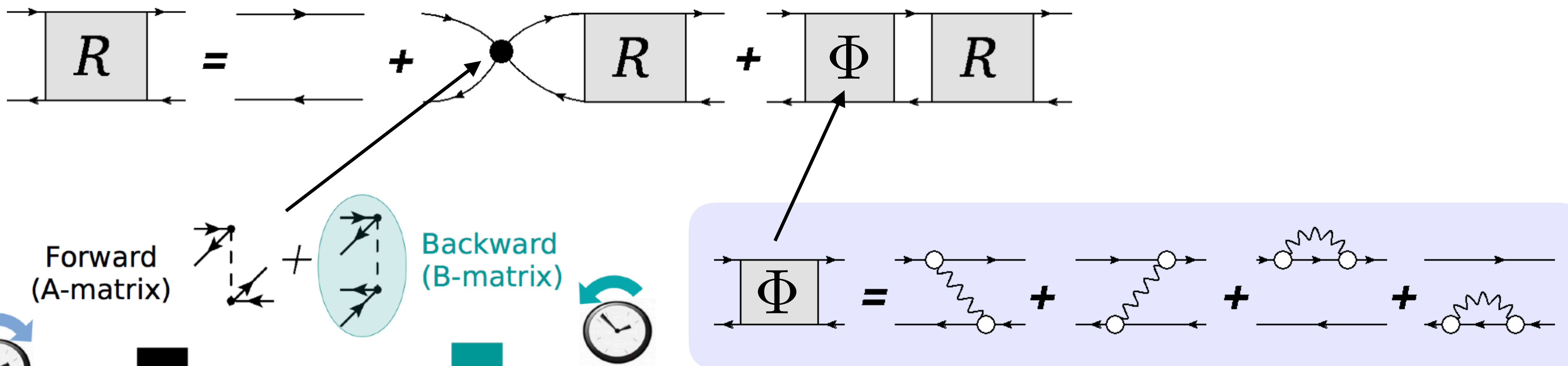
✦ ***In R(Q)RPA:***

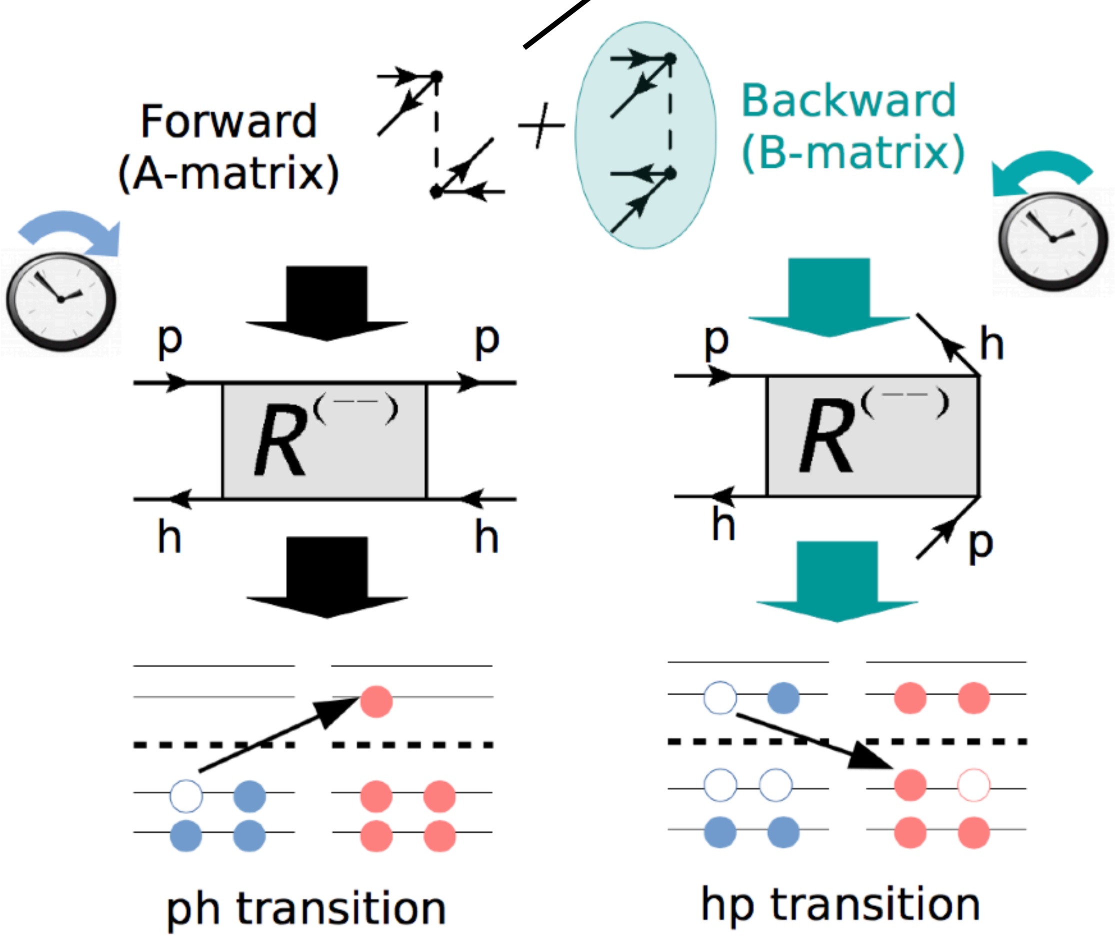


Ground-state correlations in RQTBA

In the Green's functions formalism ground-state correlations (GSC) are generated by the so-called "backward-going diagrams":

✦ ***Up to now in R(Q)TBA:***





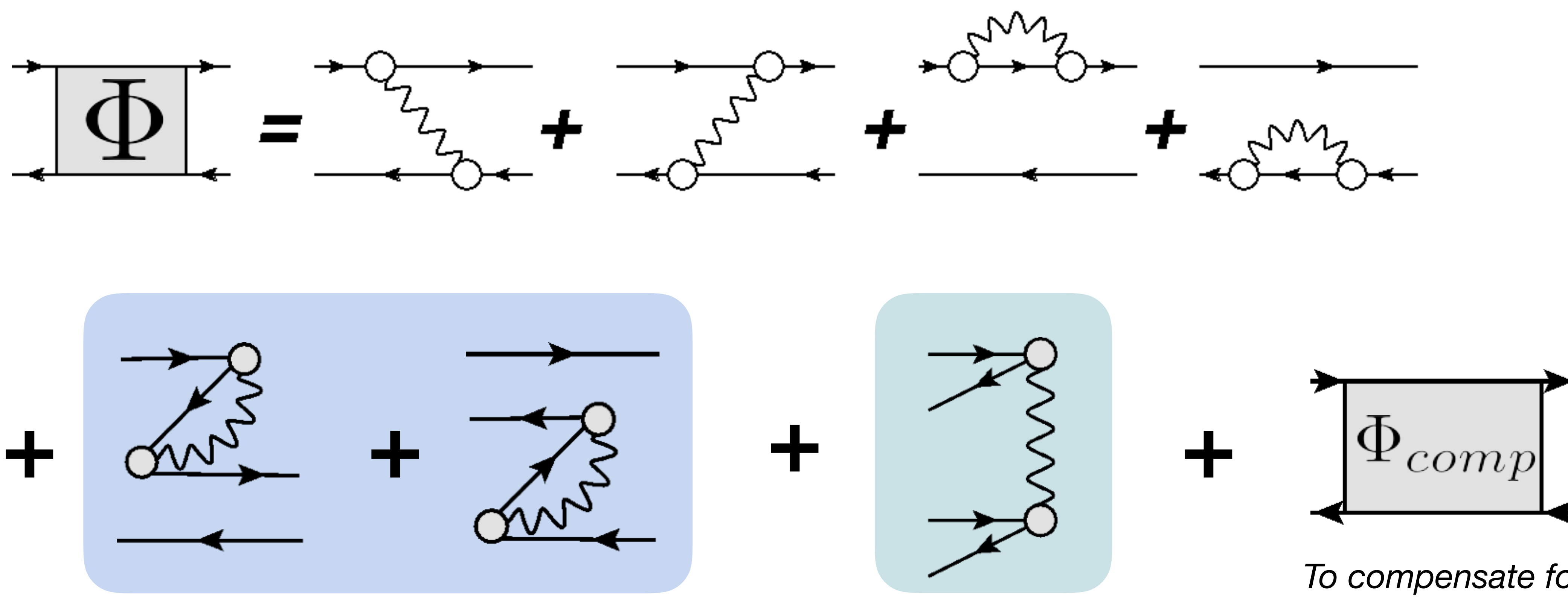
Only forward-going diagrams

⇉ no GSC induced by (quasi)vibration coupling

Ground-state correlations in RQTBA

★ When GSC induced by QVC are included in the TBA, the component $R^{(--)}$ of the response are modified by the following diagrams:

Time



Add to the A-matrix
Dynamic, but do not introduce new poles

Adds to the B-matrix
Static

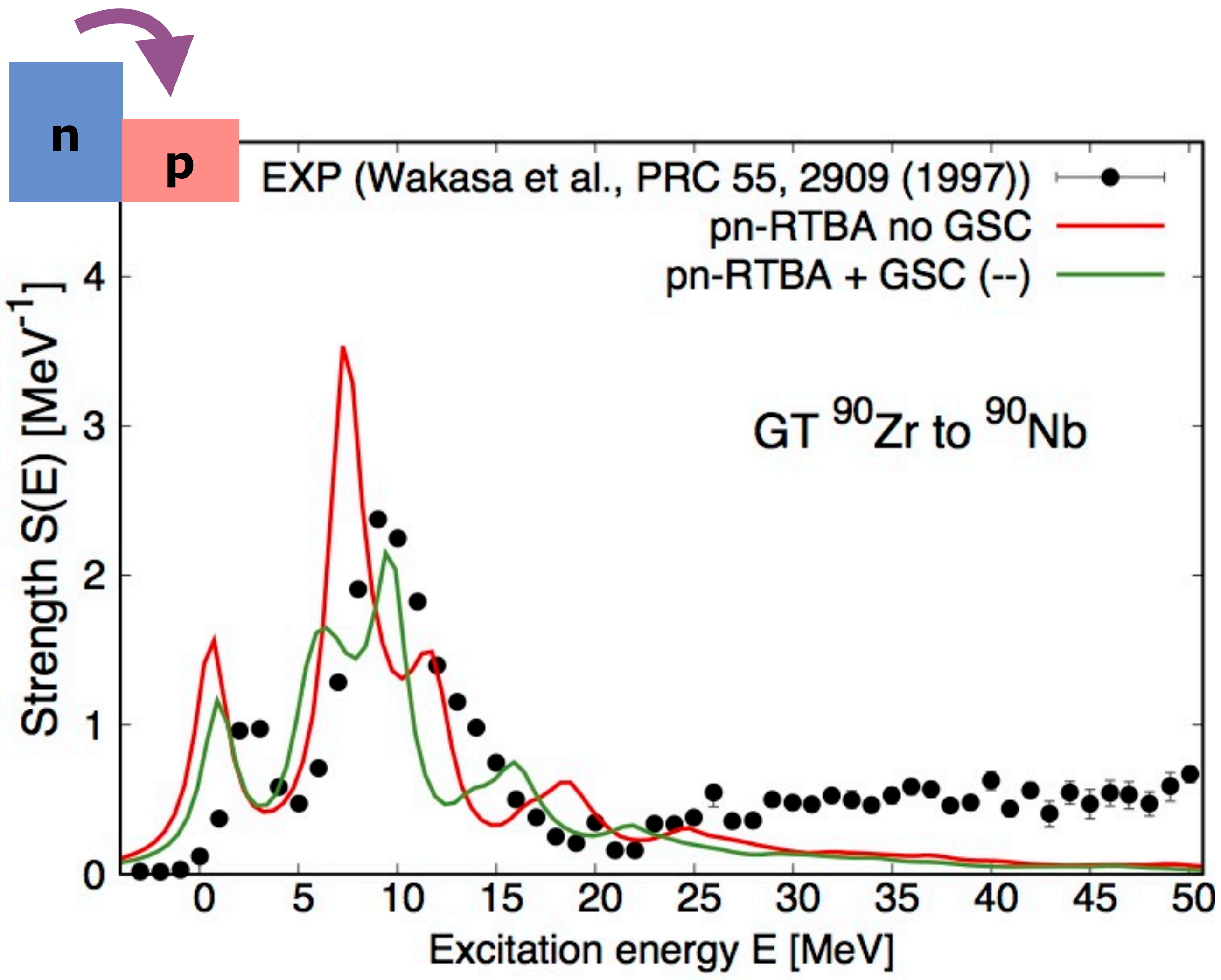
To compensate for double-counting of double self-energy insertions

➡ No new states: these diagrams only shift the previous R(Q)TBA poles

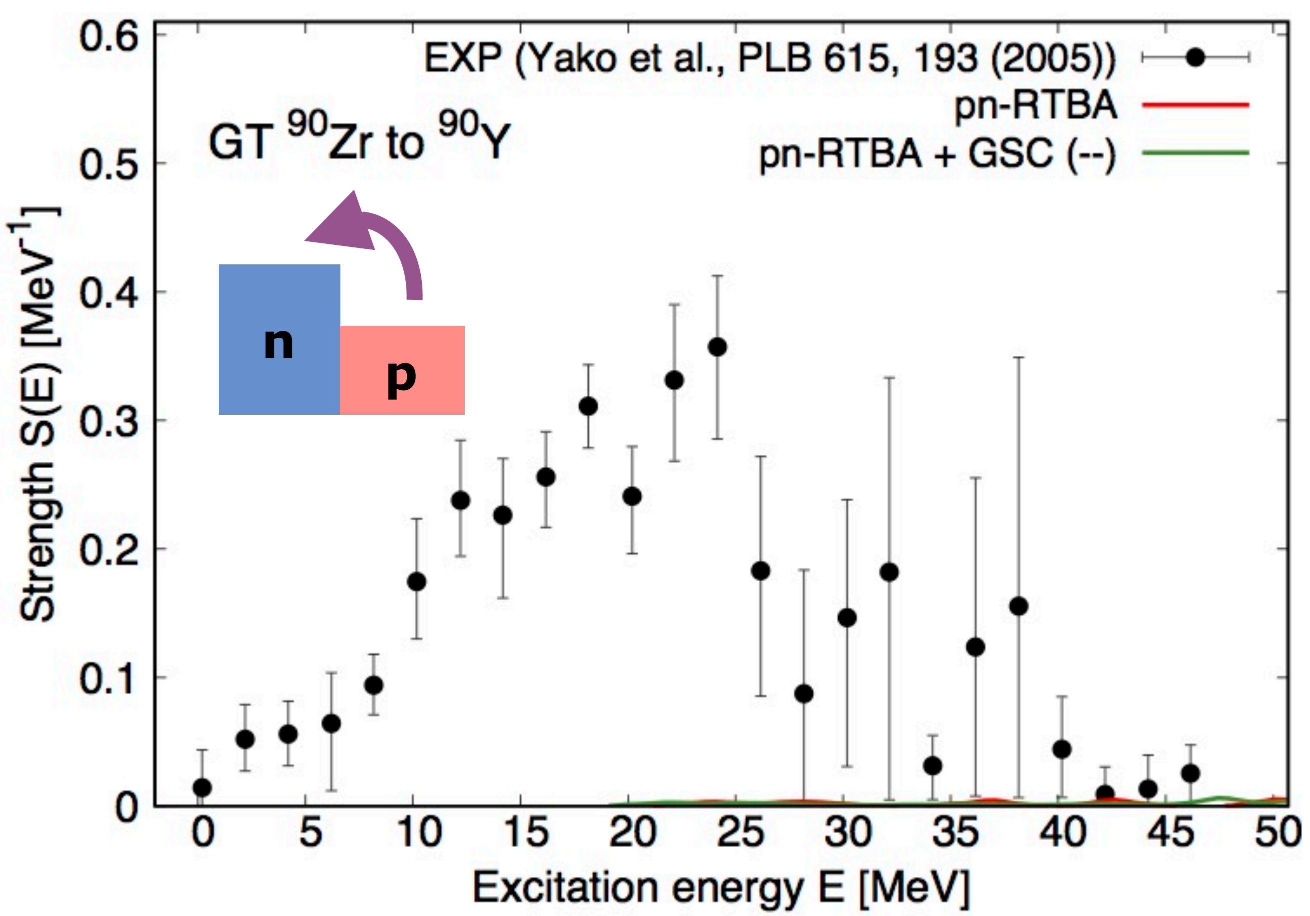
S.P. Kamerdzhiev, G.Ya. Tertychny, V.I. Tselyaev, Fiz. Elem. Chastits At. Yadra 28, 333–390 (1997)

Ground-state correlations in RQTBA

→ *Results in ^{90}Zr:*

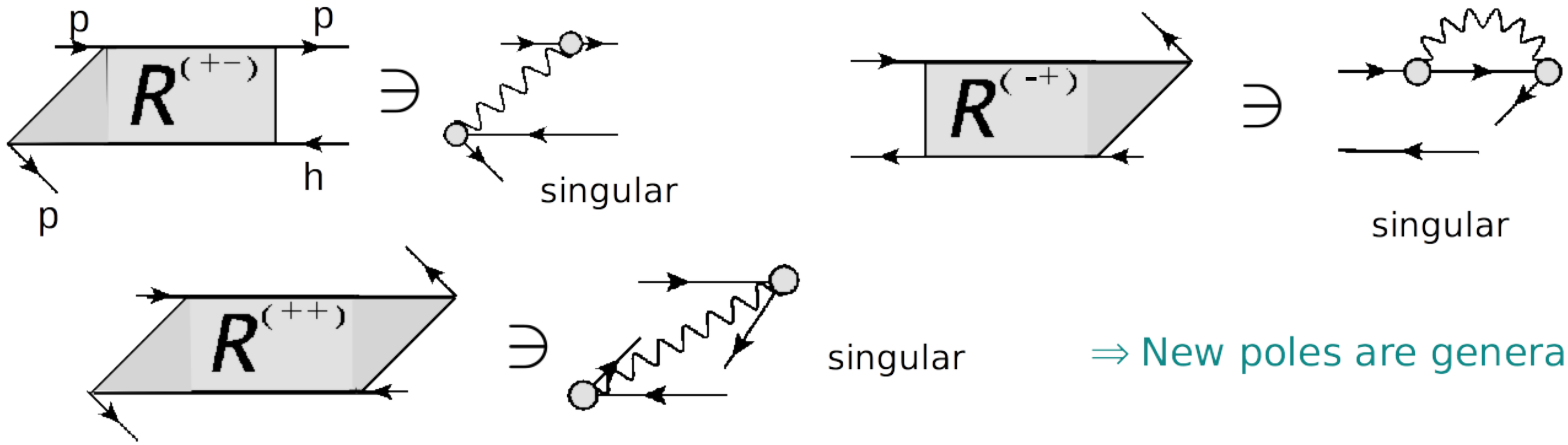


- quenching of the GT- branch
- still no effect in the GT+ branch



Ground-state correlations in RQTBA

★ Additionally, new components of the response appear:

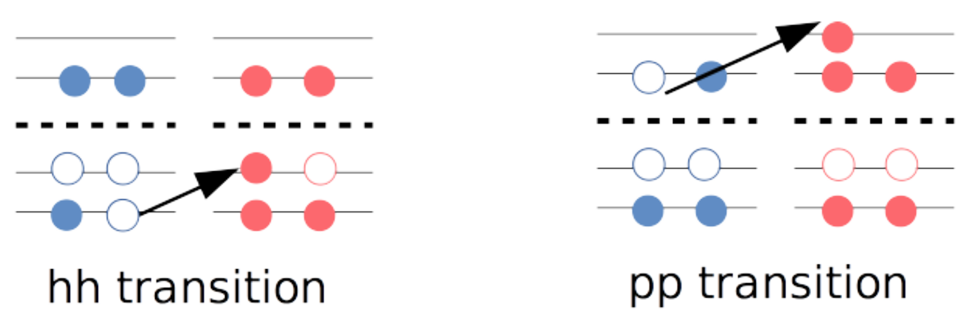


⇒ New poles are generated

✳ These components are related to $R^{(--)}$ through:

$$R(\omega) = \left(1 + Q^{(+-)}(\omega)\right) R^{(--)}(\omega) \left(1 + Q^{(-+)}(\omega)\right) + P^{(++)}(\omega)$$

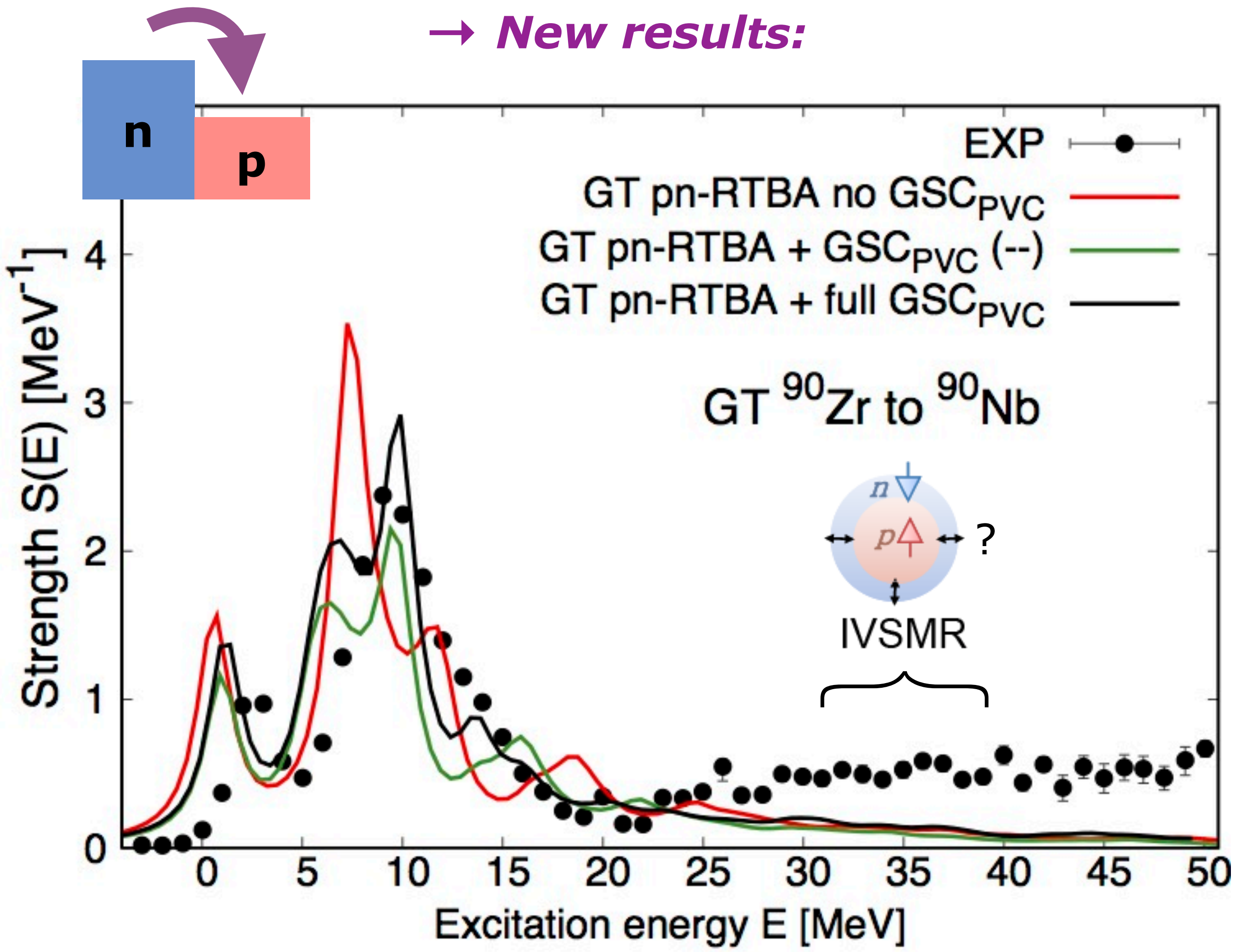
✳ They induce new types of transitions:



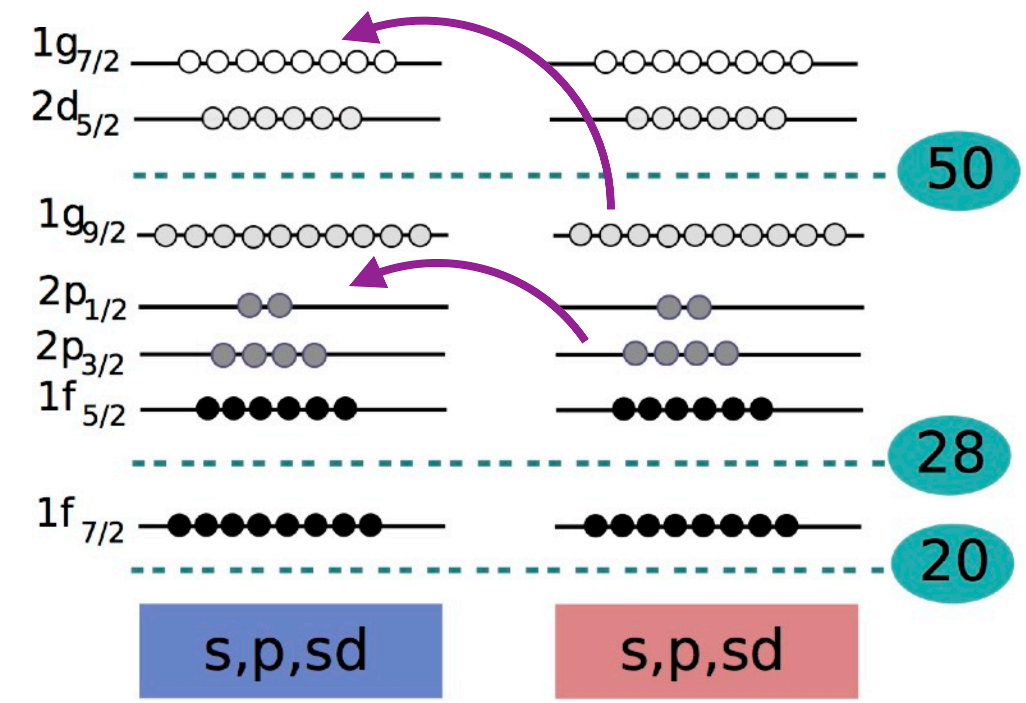
S.P. Kamerdzhiev, G.Ya. Tertychny, V.I. Tselyaev, Fiz. Elem. Chastits At. Yadra 28, 333–390 (1997)

Ground-state correlations in RQTBA

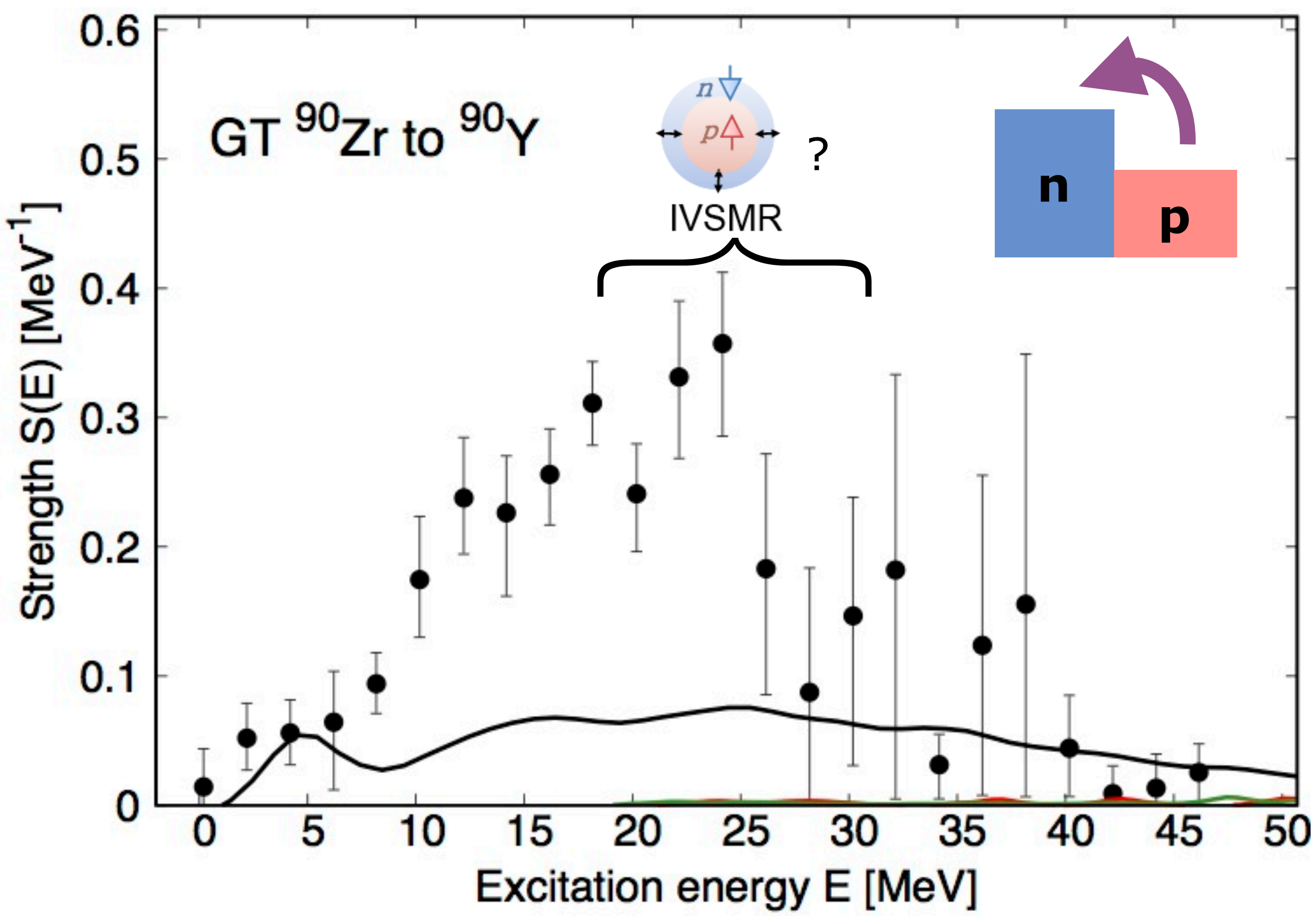
→ ***New results:***



some transitions unlocked by the GSC



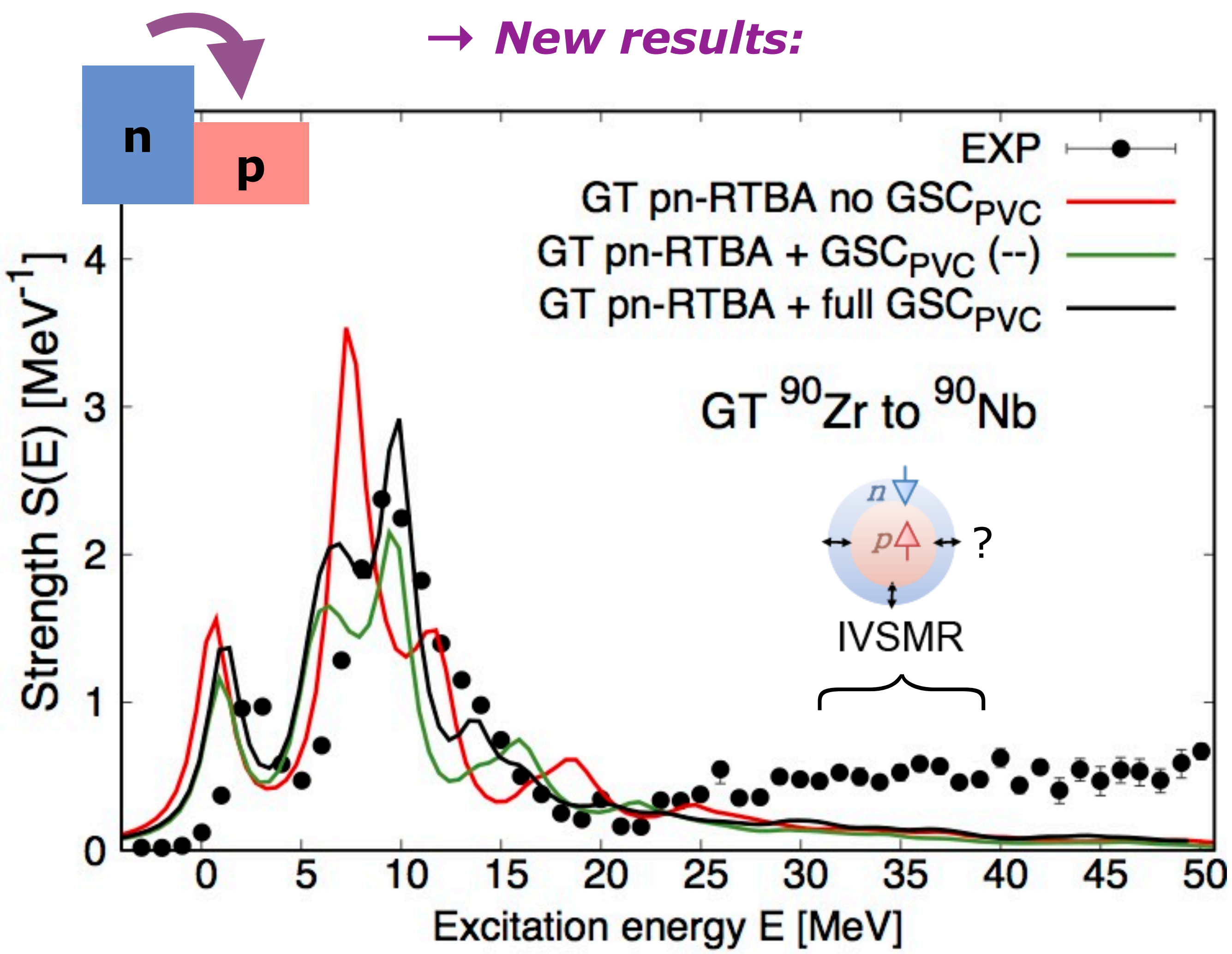
↠ ***Small effect in the GT$^-$ channel, large effect in the GT$^+$ channel!***



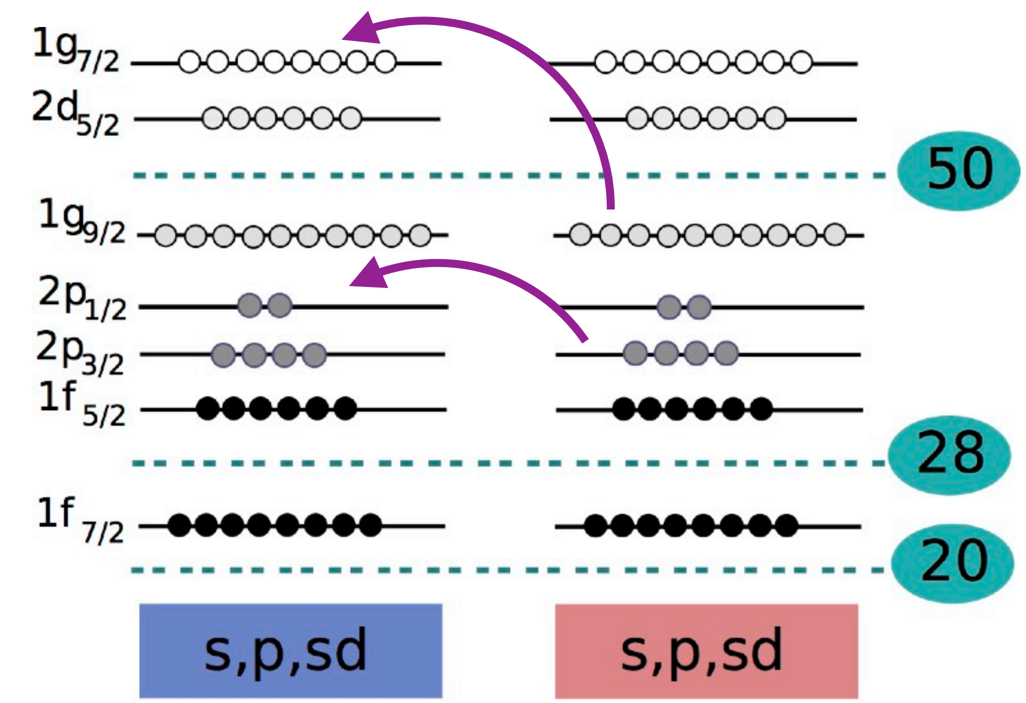
C.R and E. Litvinova, arXiv:1903.09182 [nucl-th].

Ground-state correlations in RQTBA

→ New results:



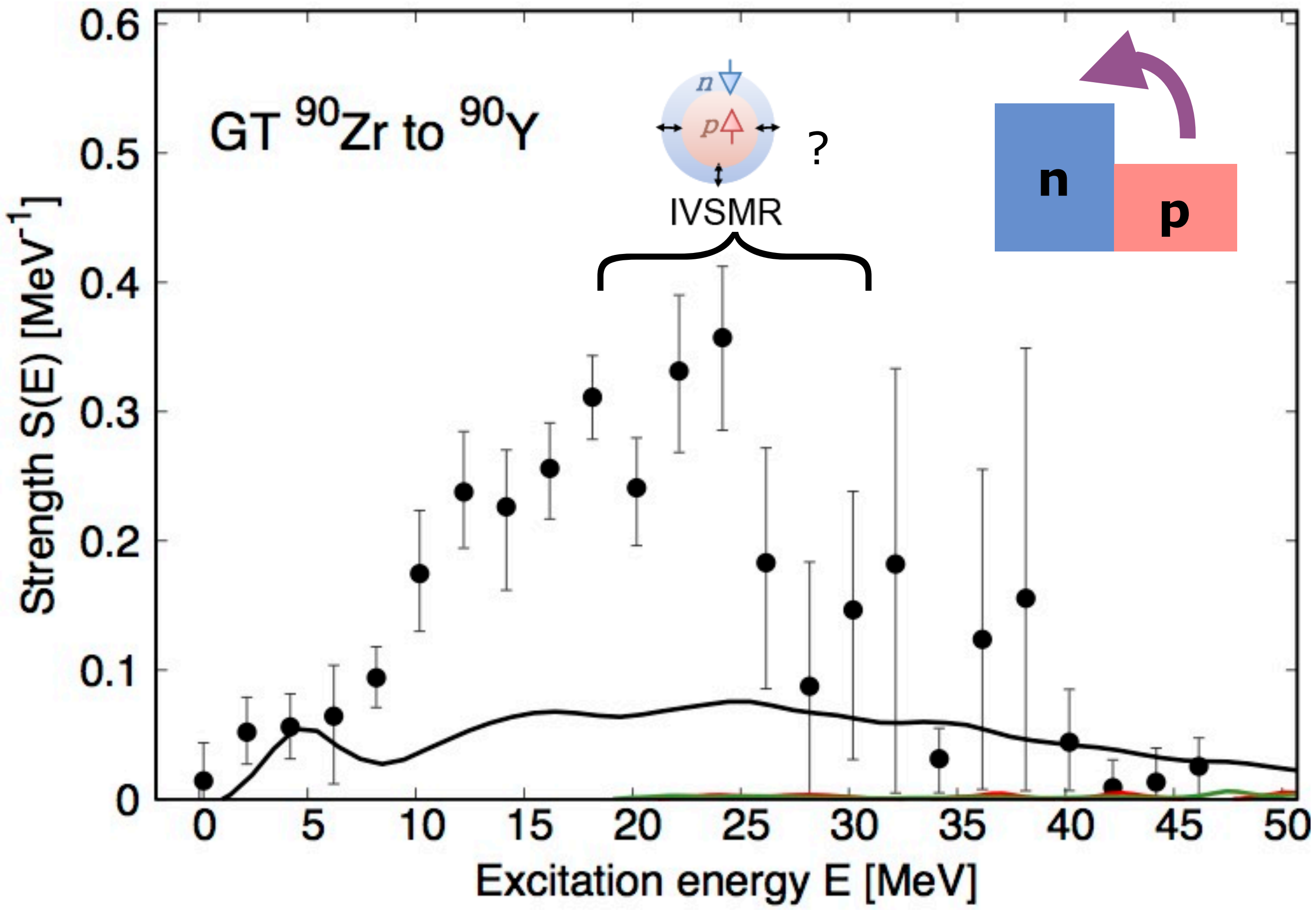
some transitions unlocked by the GSC



↠ Small effect in the GT$^-$ channel, large effect in the GT$^+$ channel!

But, the theoretical total strength still underestimates experiment at large excitation energy…

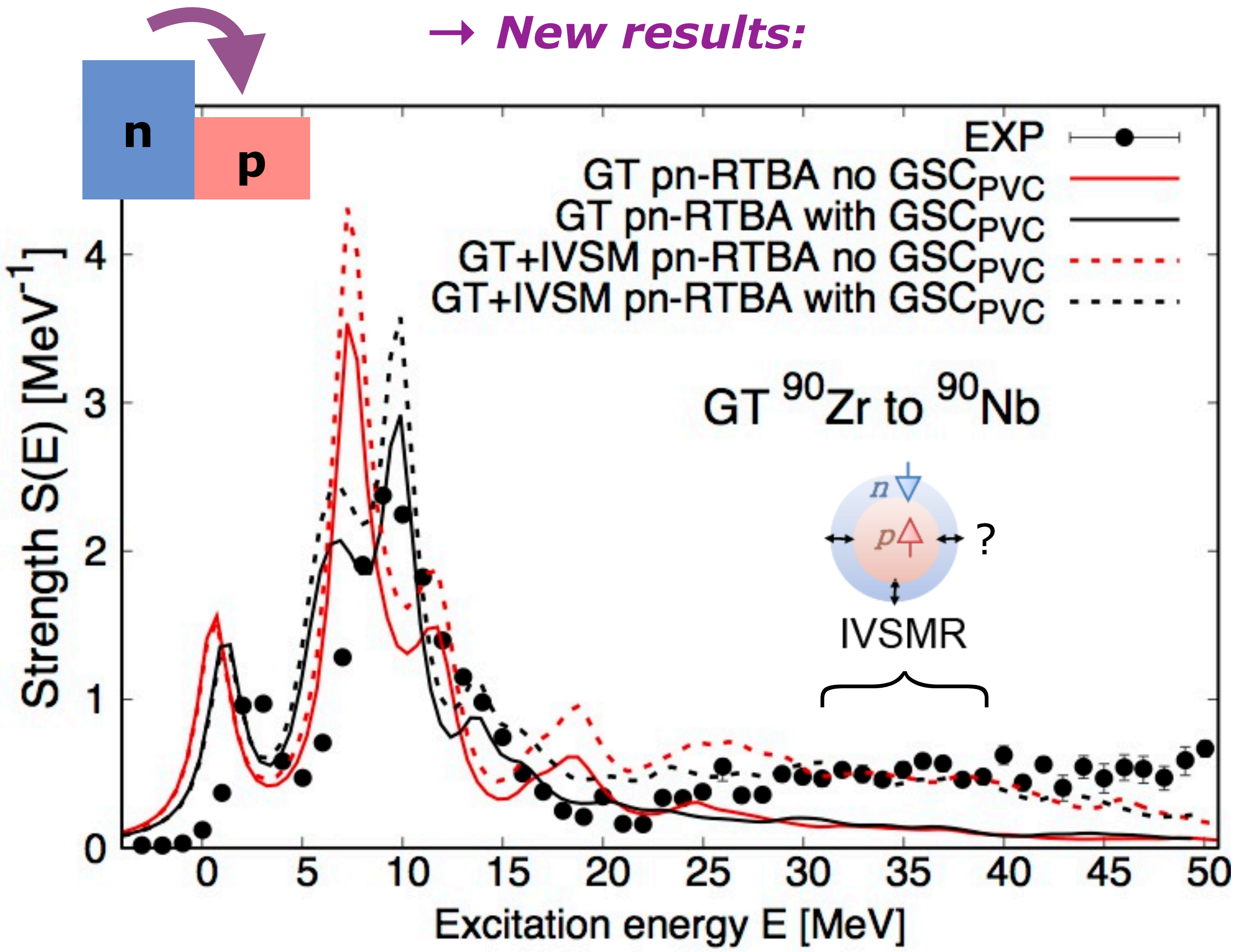
→ contribution of the IVSM ?



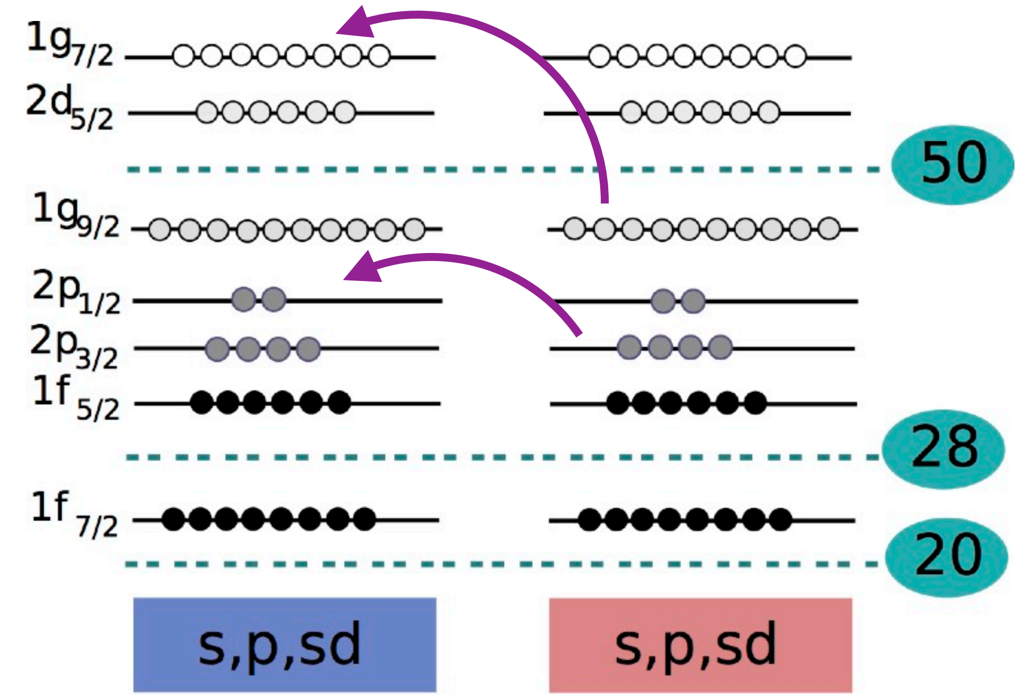
C.R and E. Litvinova, arXiv:1903.09182 [nucl-th].

Ground-state correlations in RQTBA

→ *New results:*



some transitions unlocked by the GSC

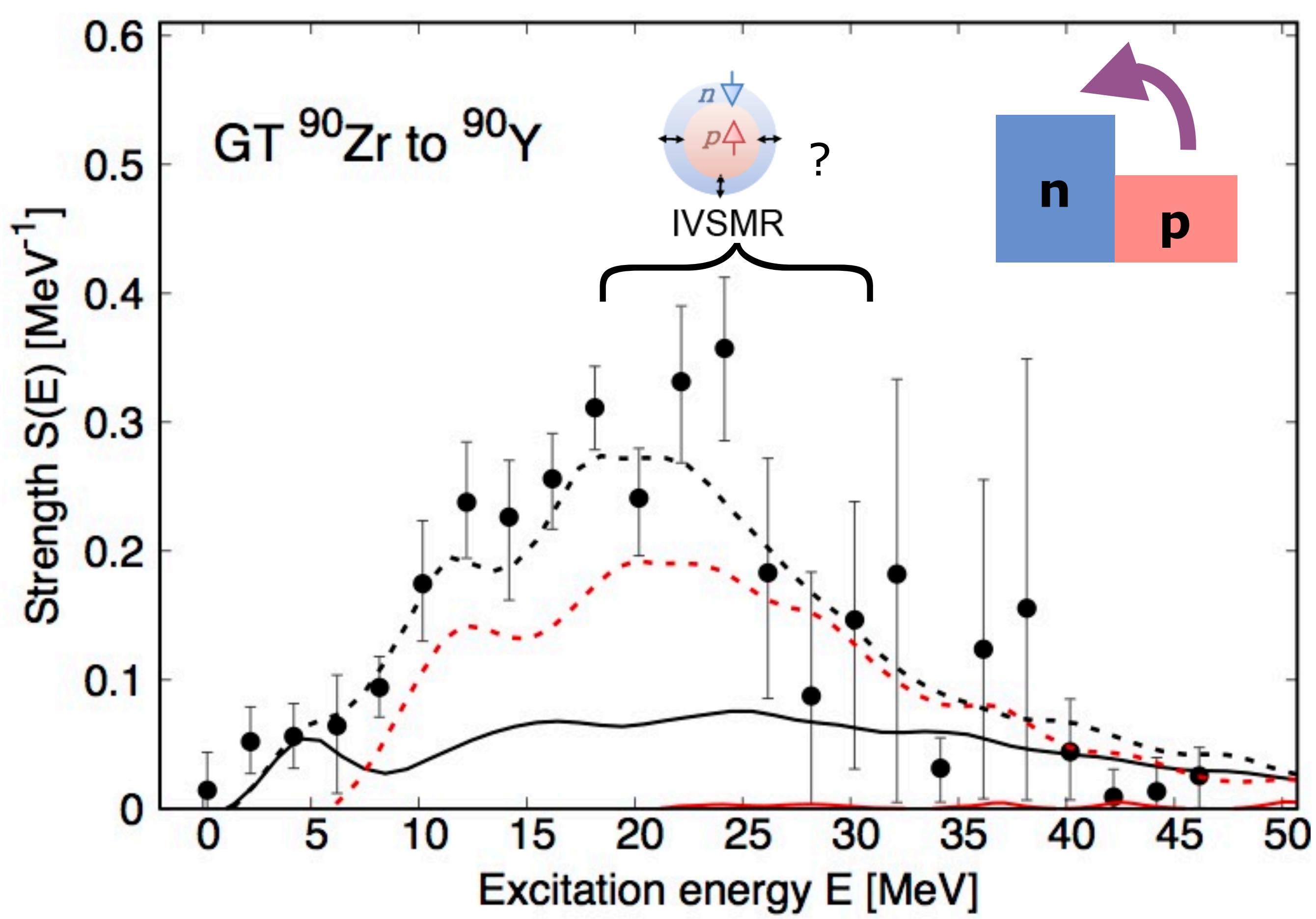


→ Response to the GT+IVSM operator:

$$\hat{F}_{\pm} = (1 + \alpha r^2)\hat{\boldsymbol{\Sigma}}\hat{\tau}_{\pm}$$

adjusted to reproduce our low-energy GT peak

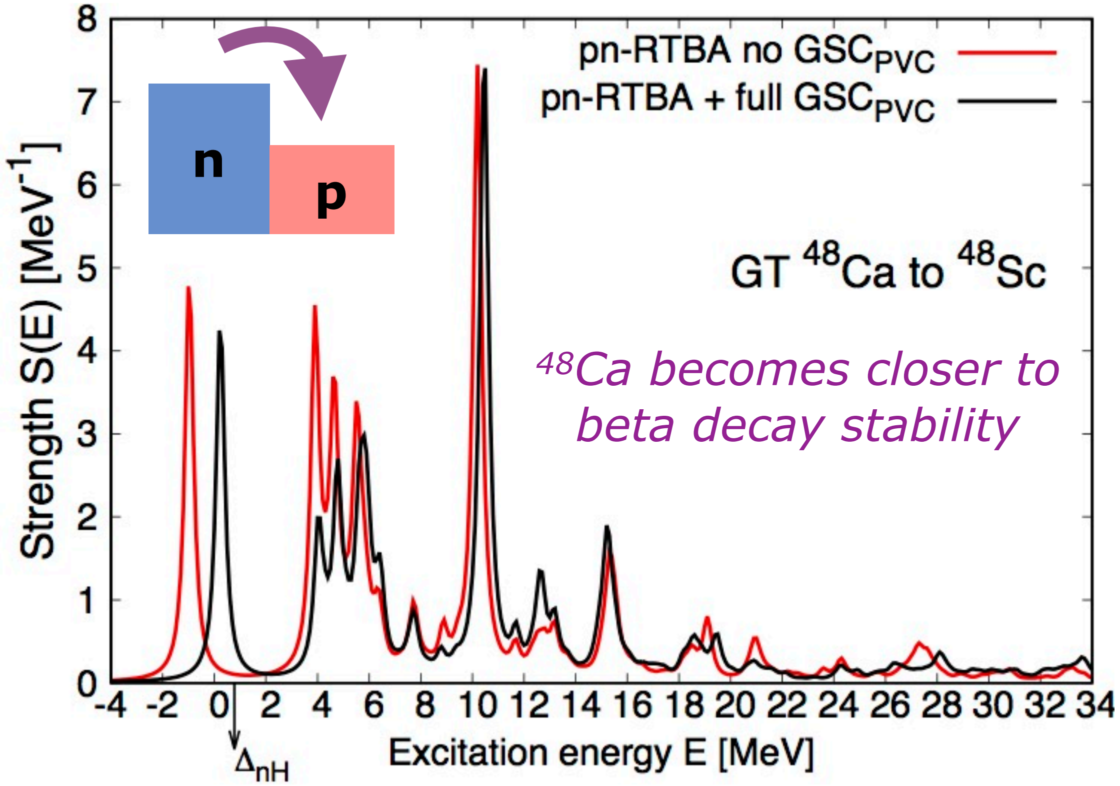
Procedure adopted from: J. Terasaki, Phys. Rev. C 97, 034304 (2018)

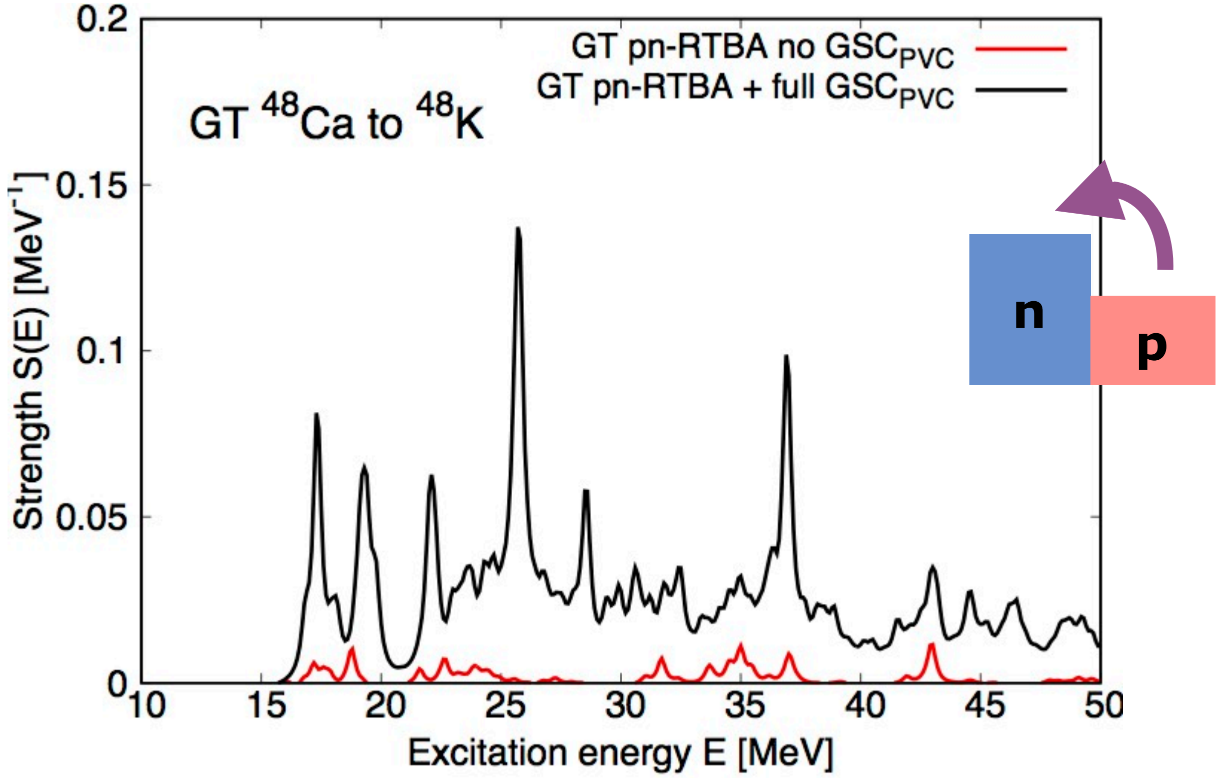


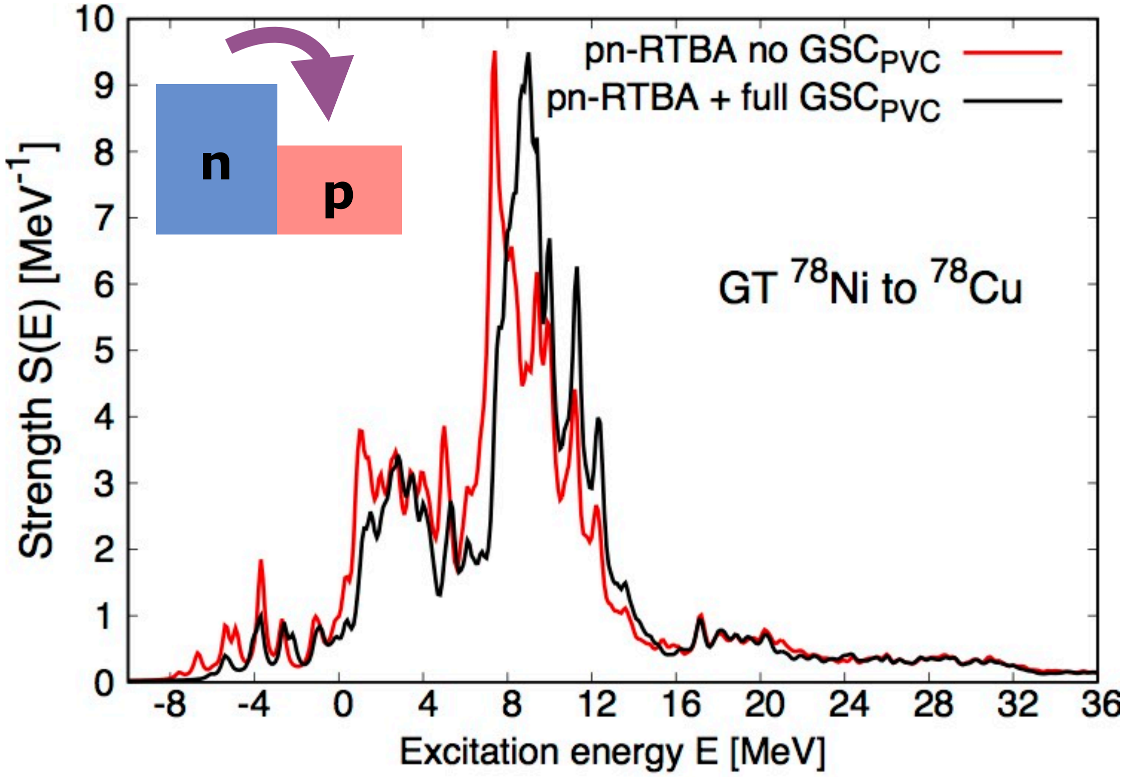
C.R and E. Litvinova, arXiv:1903.09182 [nucl-th].

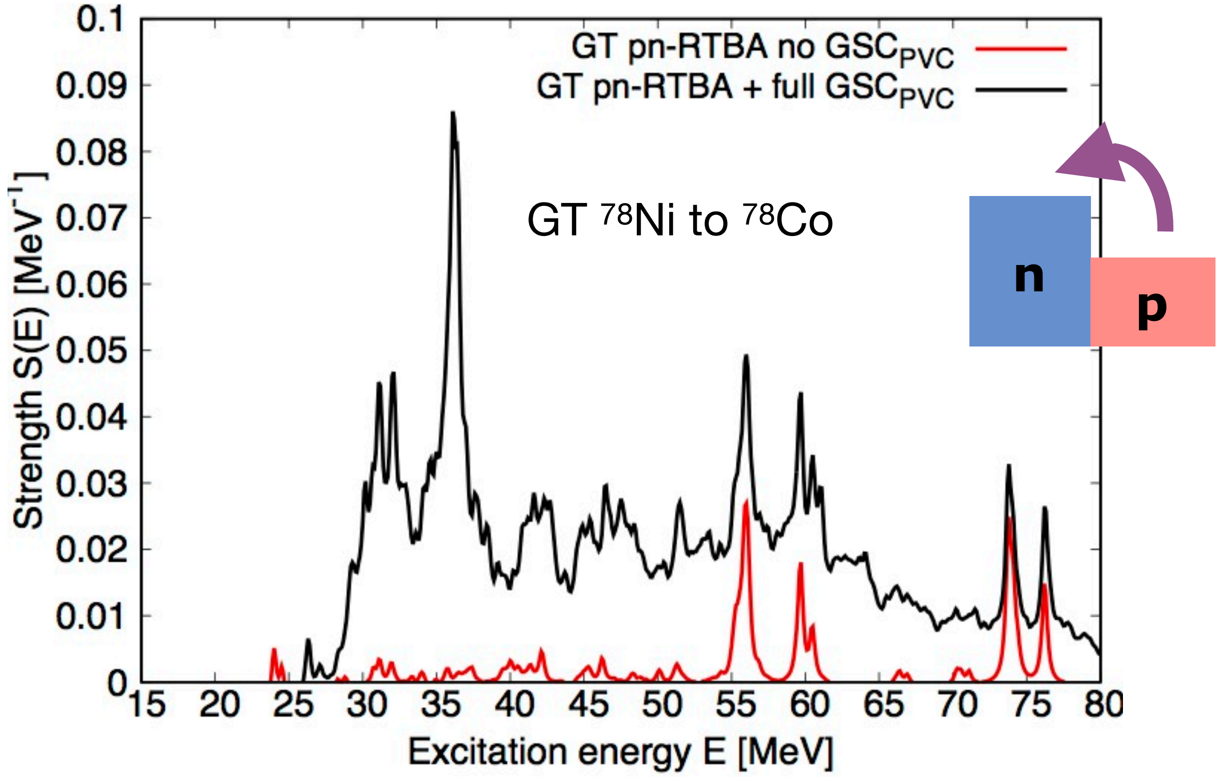
Ground-state correlations in RQTBA

→ *Some other doubly-magic N>Z nuclei:*







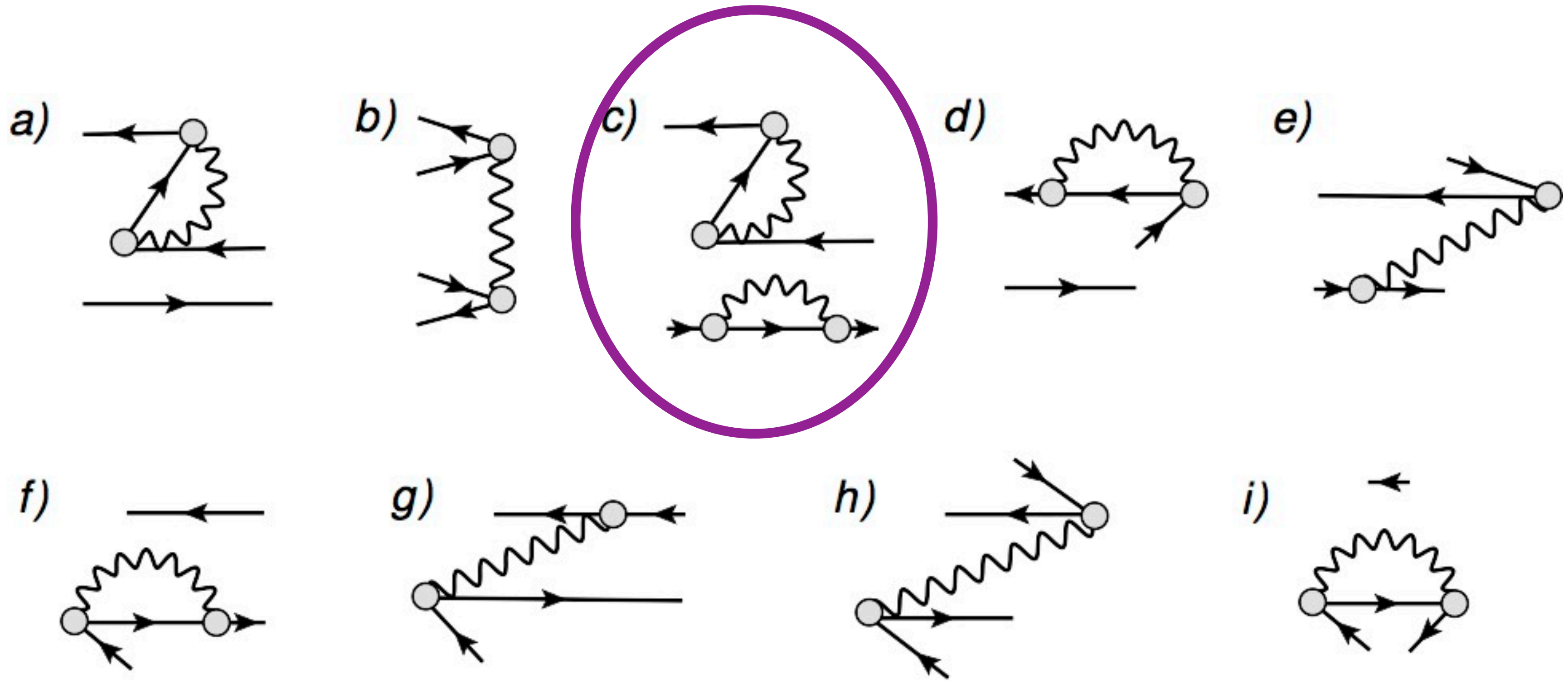


Ground-state correlations in RQTBA

Drawback: The new ground-state correlations induce a breaking of the Ikeda sum rule.

For ex: ^{90}Zr: 7%, ^{48}Ca: 6%, ^{78}Ni: 3.5% of breaking

It was previously realized by V.I. Tselyaev in [*Tselyaev, Phys. Rev. C 75, 024306 (2007)*] that this breaking is due to diagrams of 4th order in the QVC vertex, that are consistent with the Time-Blocking Appproximation:

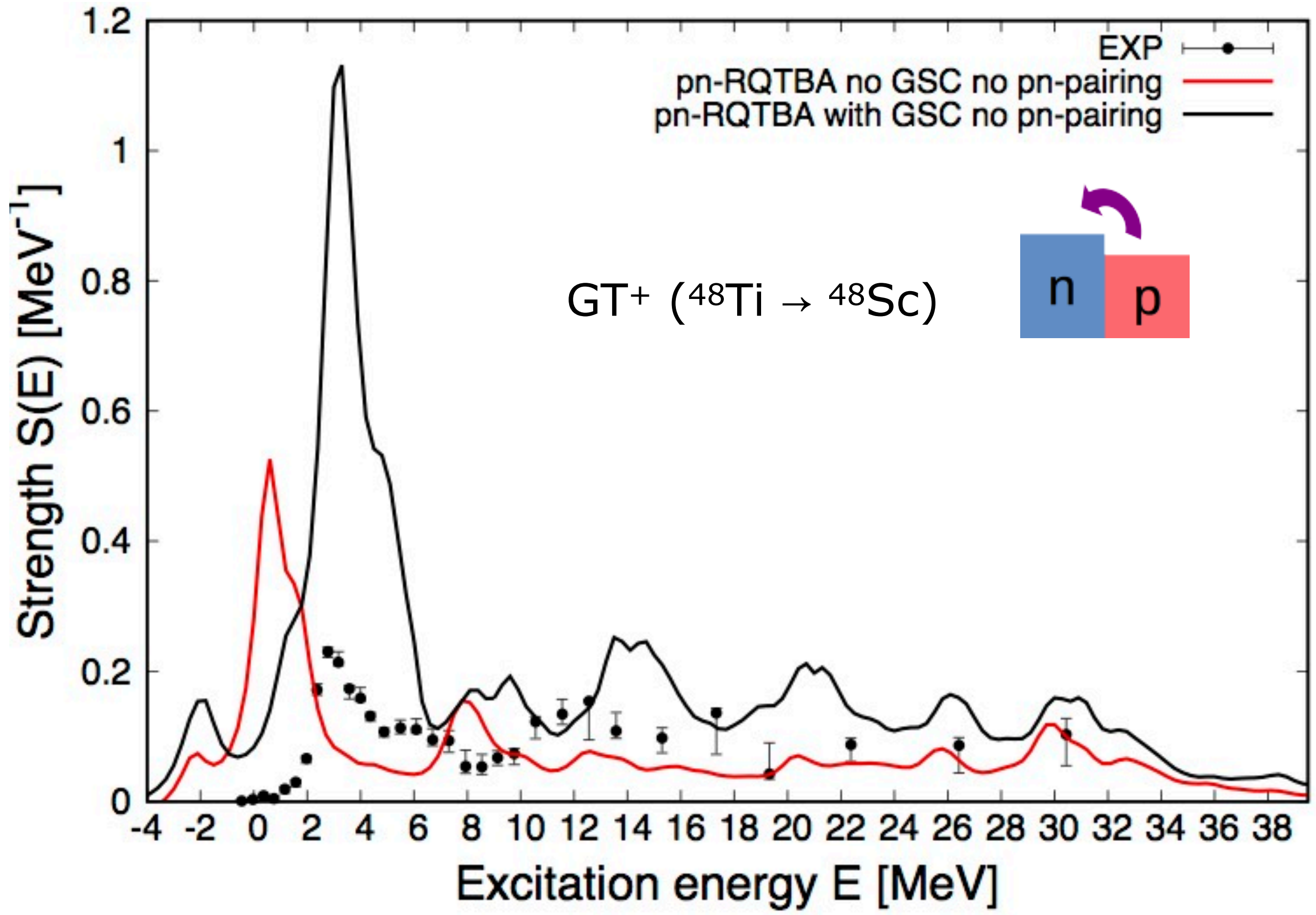


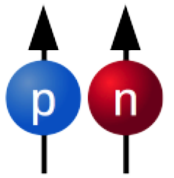
A procedure to eliminate such diagrams is proposed in that reference, but has not been implemented here.

Proton-neutron pairing from the meson-exchange interaction

✦ The case of an open-shell nucleus: ^{48}Ti

- When including GSC induced by QVC, pn-RQTBA largely overestimates the experimental strength:





T=0, S=1

- However, so far we have not considered the (static) proton-neutron pairing interaction…

…and here N-Z = 4 ➡ the proton-neutron pairing interaction must be important in ^{48}Ti

EXP: K. Yako et al., PRL 103, 012503 (2009)

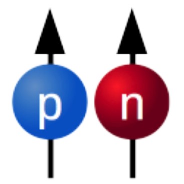
Proton-neutron pairing from the meson-exchange interaction

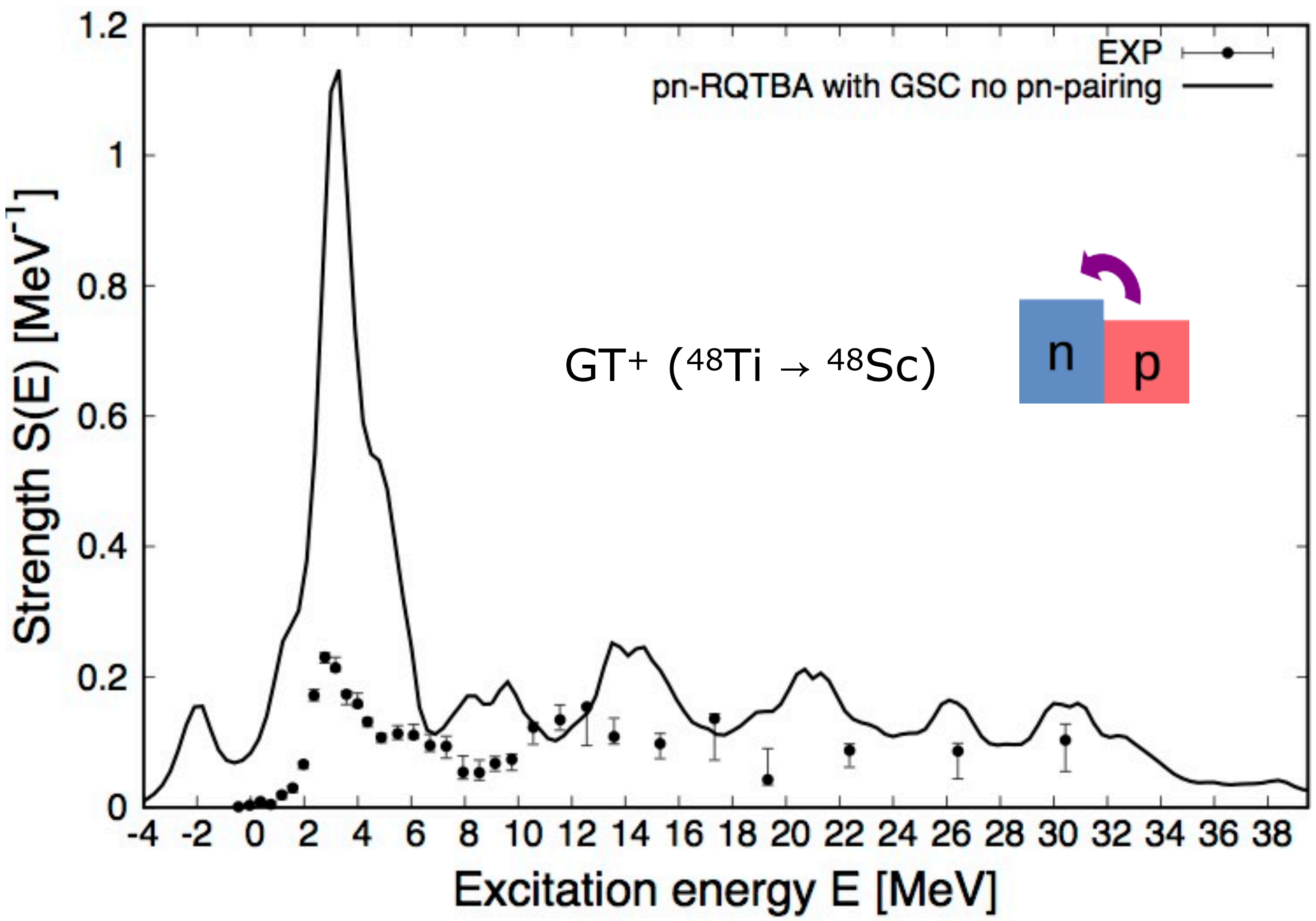
✦ The case of an open-shell nucleus: ^{48}Ti

- Usually in pn Relativistic QRPA, one uses non-relativistic pn pairing interaction of the Gogny-type:

$$V_{12} = -V_0 \sum_{j=1}^{2} g_j e^{-r_{12}^2/\mu_j^2} \hat{\Pi}_{S=1,T=0}$$

and vary the parameter V_0





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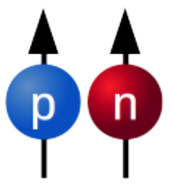
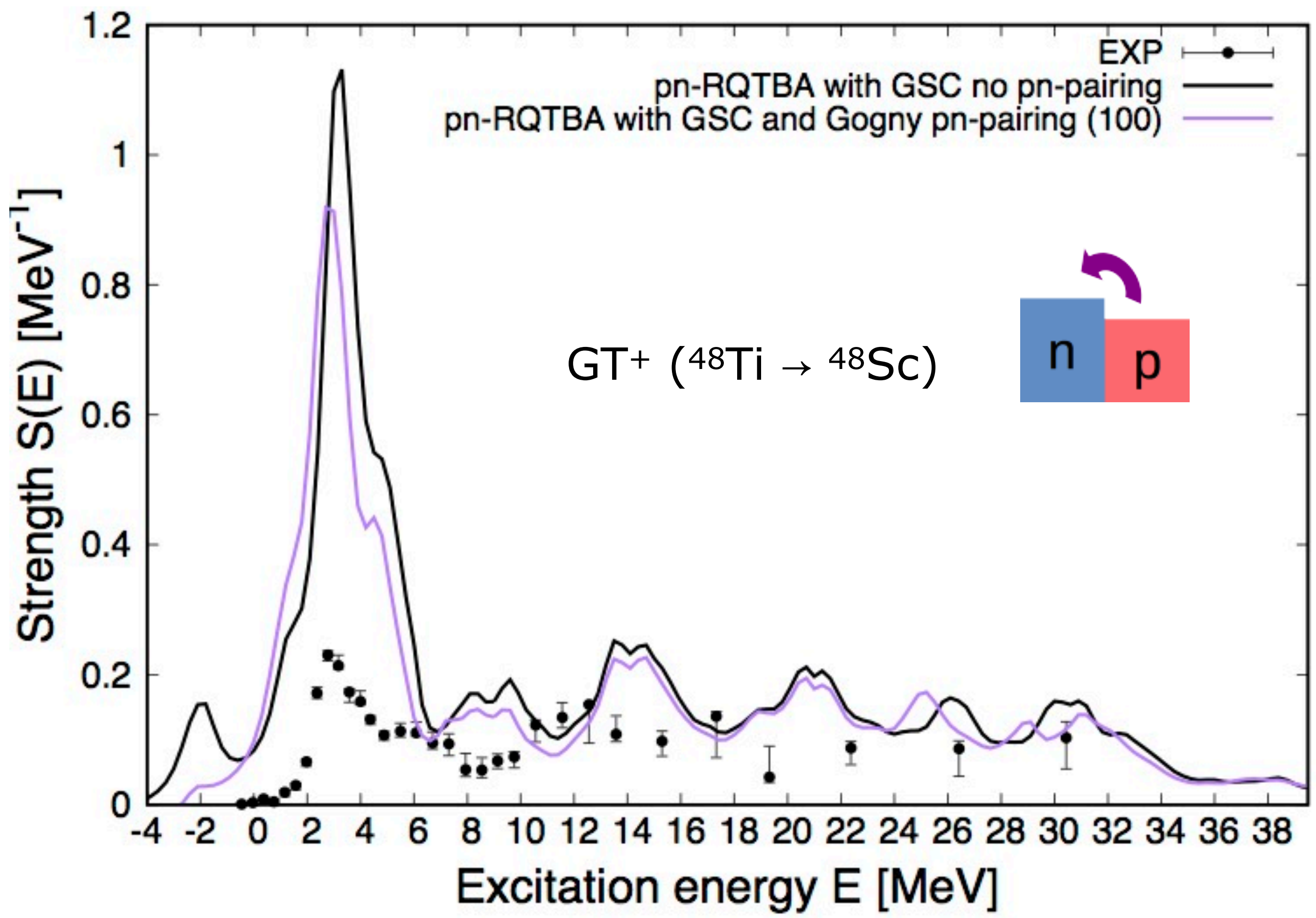


T=0, S=1

and vary the parameter V_0



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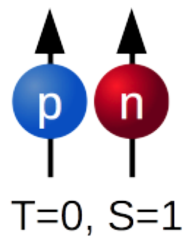

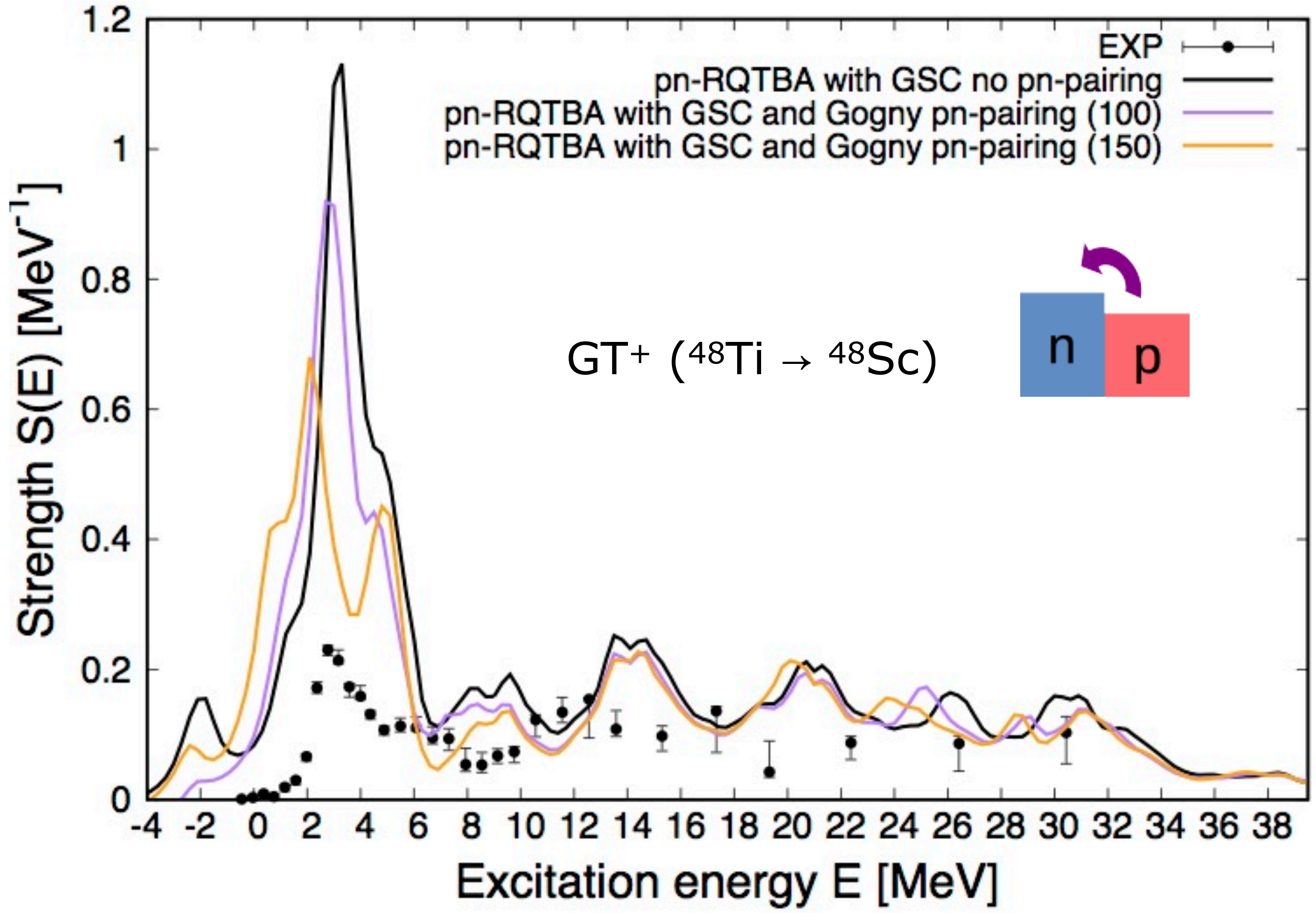

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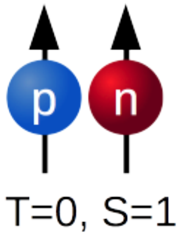
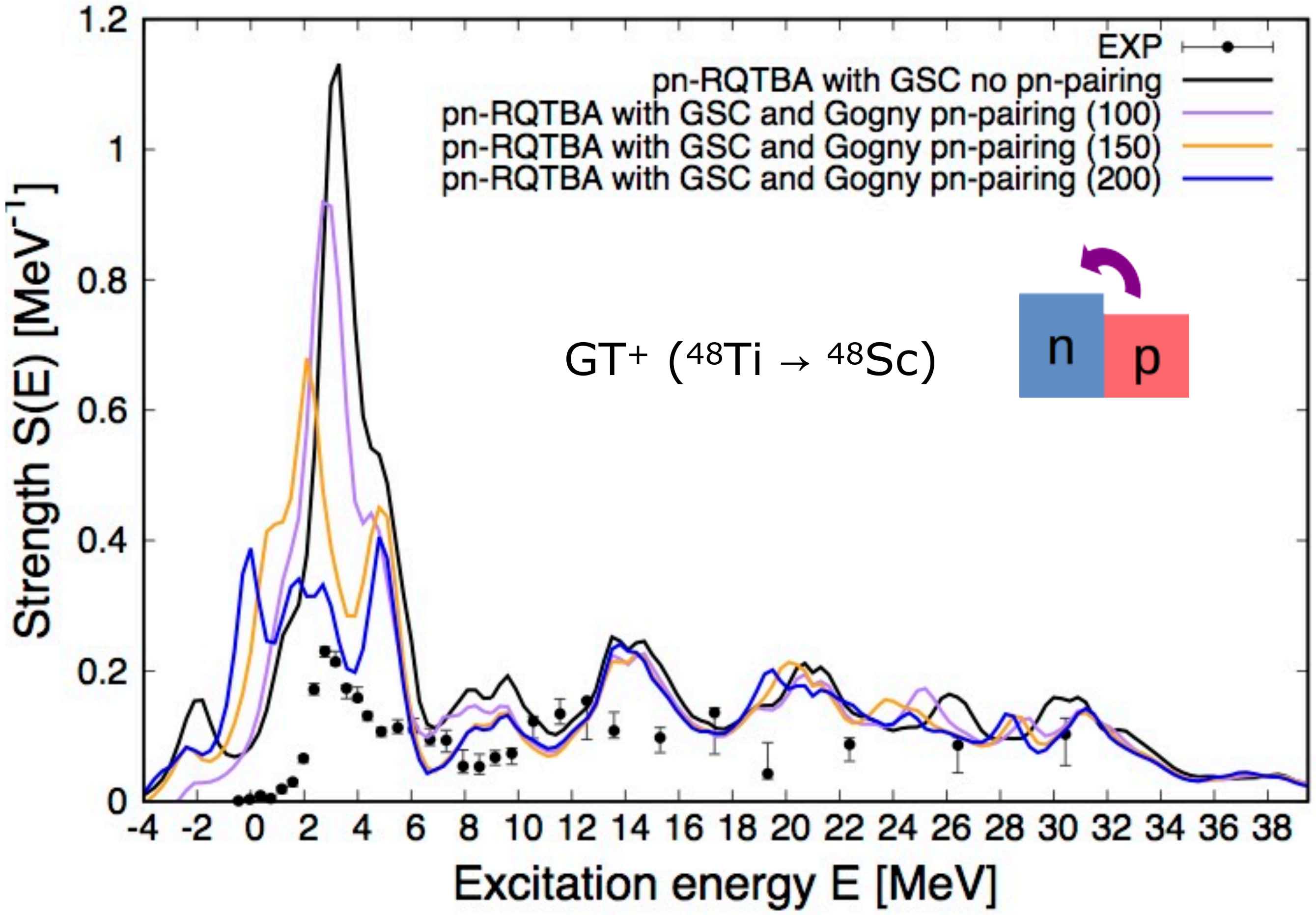


and vary the parameter V_0

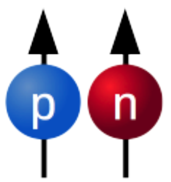


Proton-neutron pairing from the meson-exchange interaction

✦ The case of an open-shell nucleus: ^{48}Ti

▶ Here we use the same meson-exchange interaction in the pn particle-hole and pn particle-particle channel, via the "*Pandya*" transformation:

$$v^{(pp)J}_{(1243)} = \sum_{\lambda} (2\lambda+1)(-)^{J+j_3+j_4} \begin{Bmatrix} j_1 & j_2 & J \\ j_3 & j_4 & \lambda \end{Bmatrix} v^{(ph)\lambda}_{(1243)}$$



T=0, S=1

Proton-neutron pairing from the meson-exchange interaction

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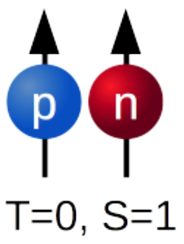
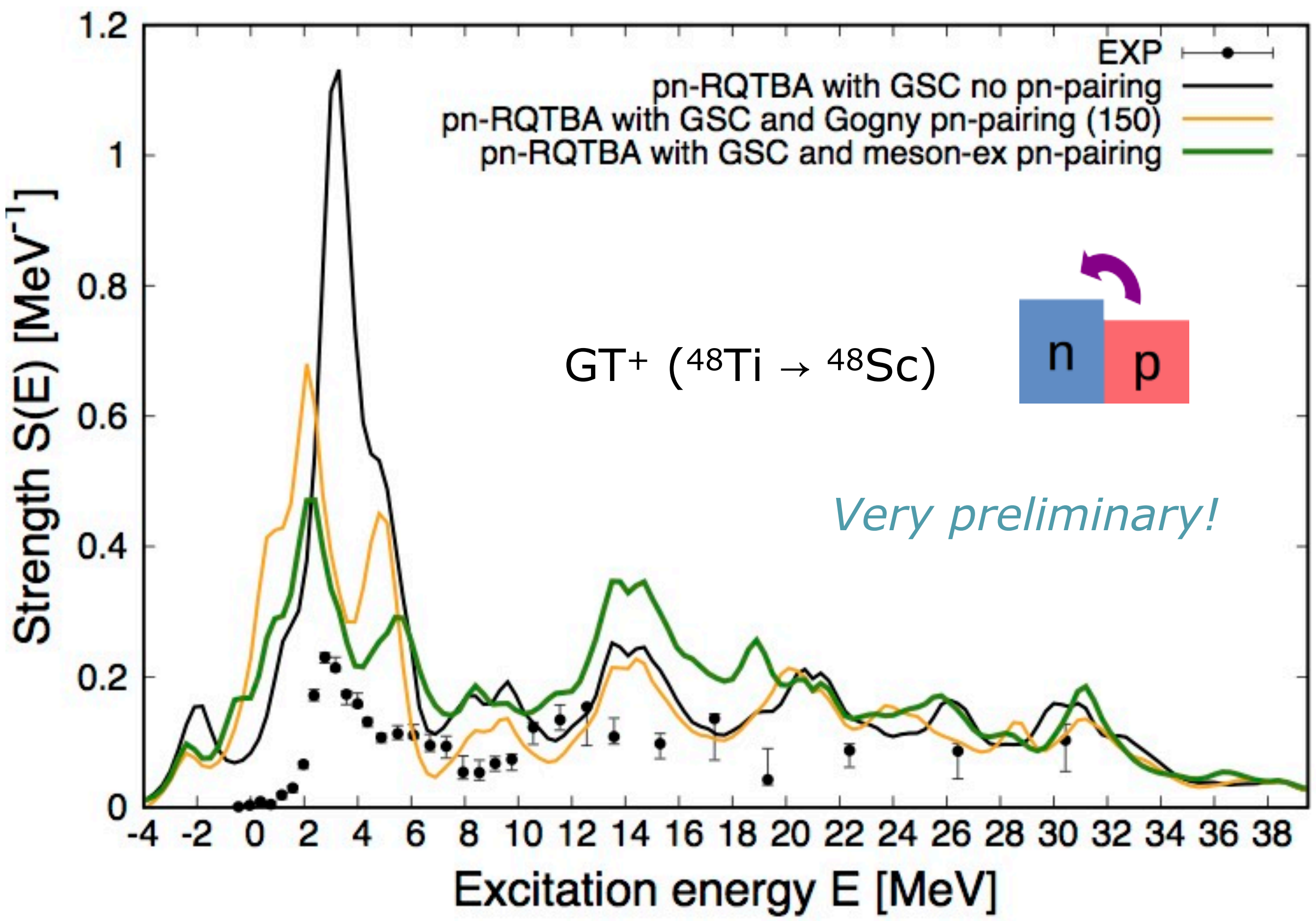




▶ Complex GSC and pn-pairing seem to partly compensate each other

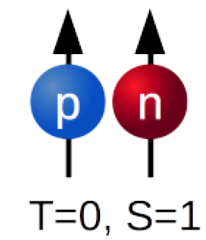
▶ Meson-exchange pn-pairing gives similar effect as Gogny-type pn-pairing with V_0=150 MeV

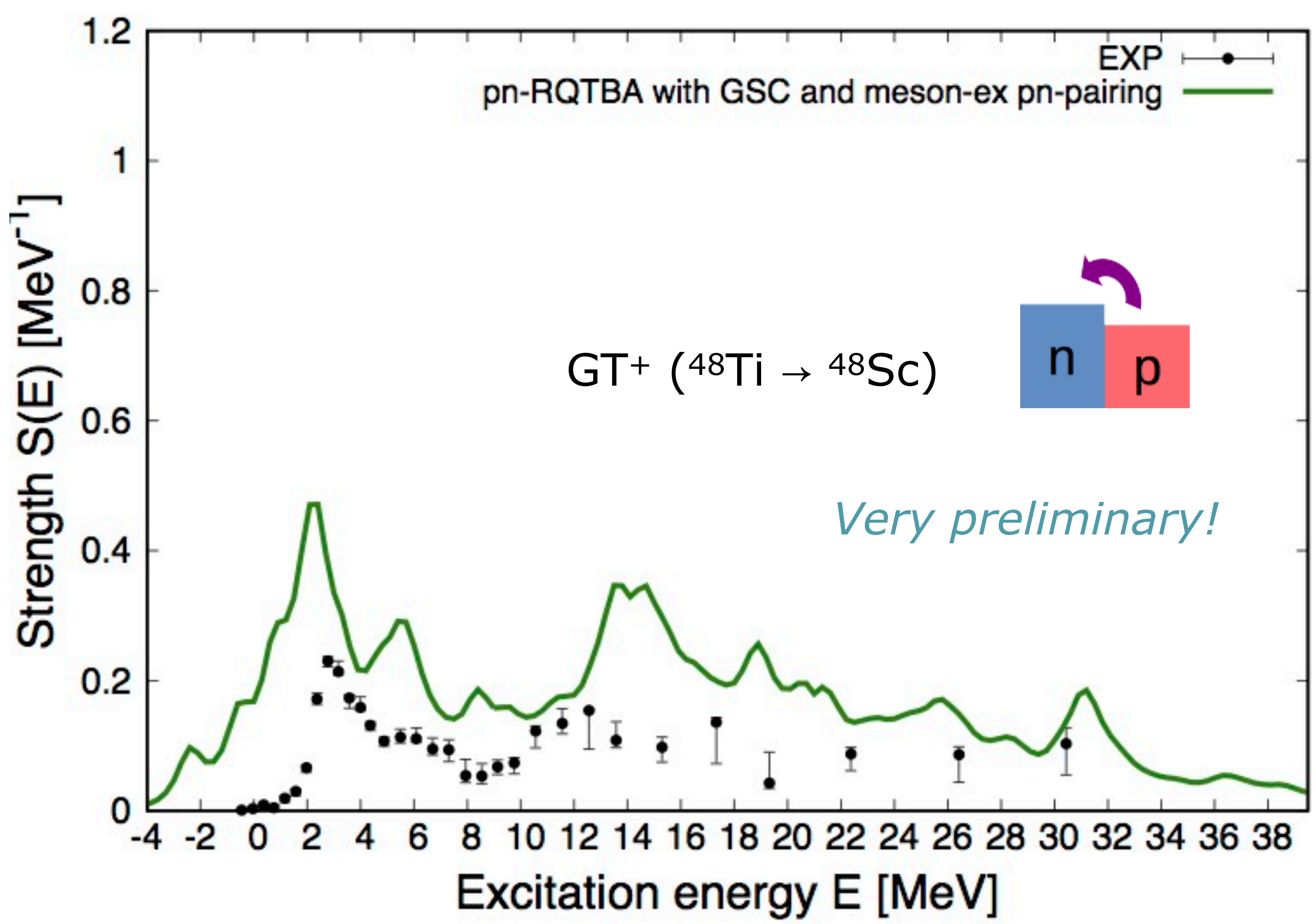
Proton-neutron pairing from the meson-exchange interaction

✦ The case of an open-shell nucleus: ^{48}Ti

▶ Here we use the same meson-exchange interaction in the pn particle-hole and pn particle-particle channel, via the "*Pandya*" transformation:

$$v^{(pp)J}_{(1243)} = \sum_{\lambda} (2\lambda+1)(-)^{J+j_3+j_4} \begin{Bmatrix} j_1 & j_2 & J \\ j_3 & j_4 & \lambda \end{Bmatrix} v^{(ph)\lambda}_{(1243)}$$

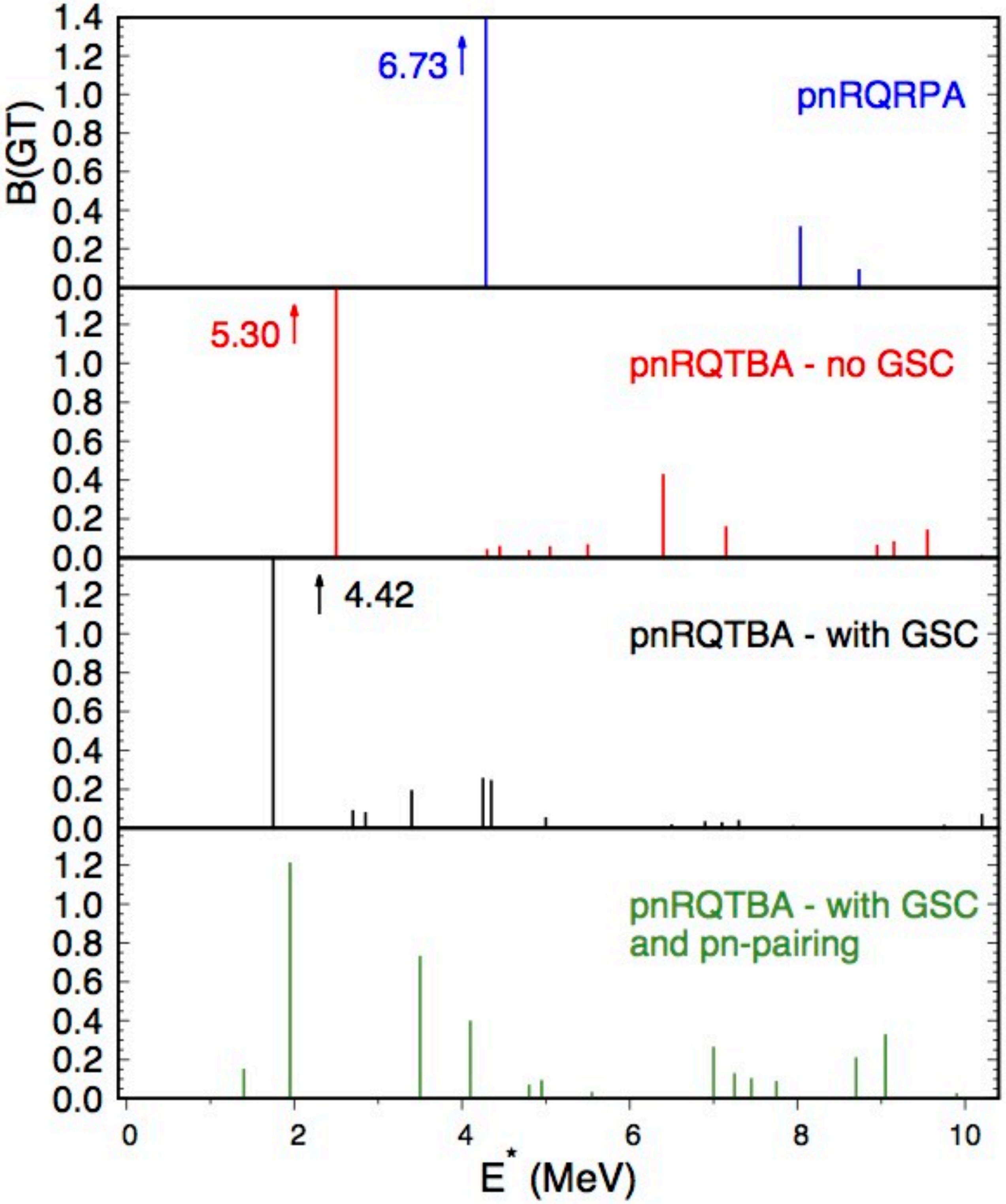




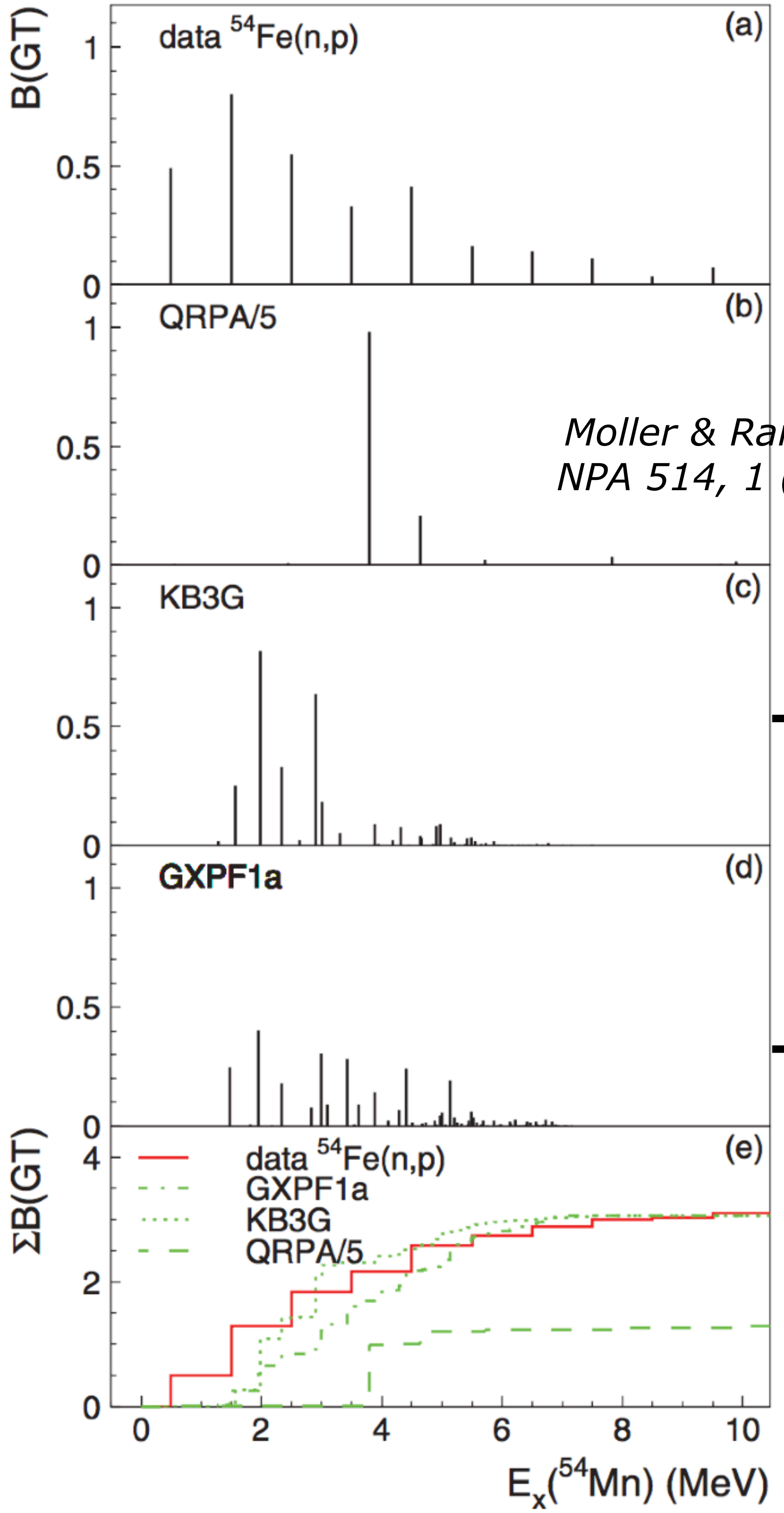
- ▶ Complex GSC and pn-pairing seem to partly compensate each other
- ▶ Meson-exchange pn-pairing gives similar effect as Gogny-type pn-pairing with V_0=150 MeV

Ground-state correlations and pn pairing

★ **GT⁺ strength in ⁵⁴Fe (Z=26, N=28):**



A.L. Cole, T.S. Anderson, R.G.T. Zegers et al., PRC 86, 015809 (2012):



Moller & Randrup, NPA 514, 1 (1990)

Shell-Model

GT⁺ strength and electron-capture rates

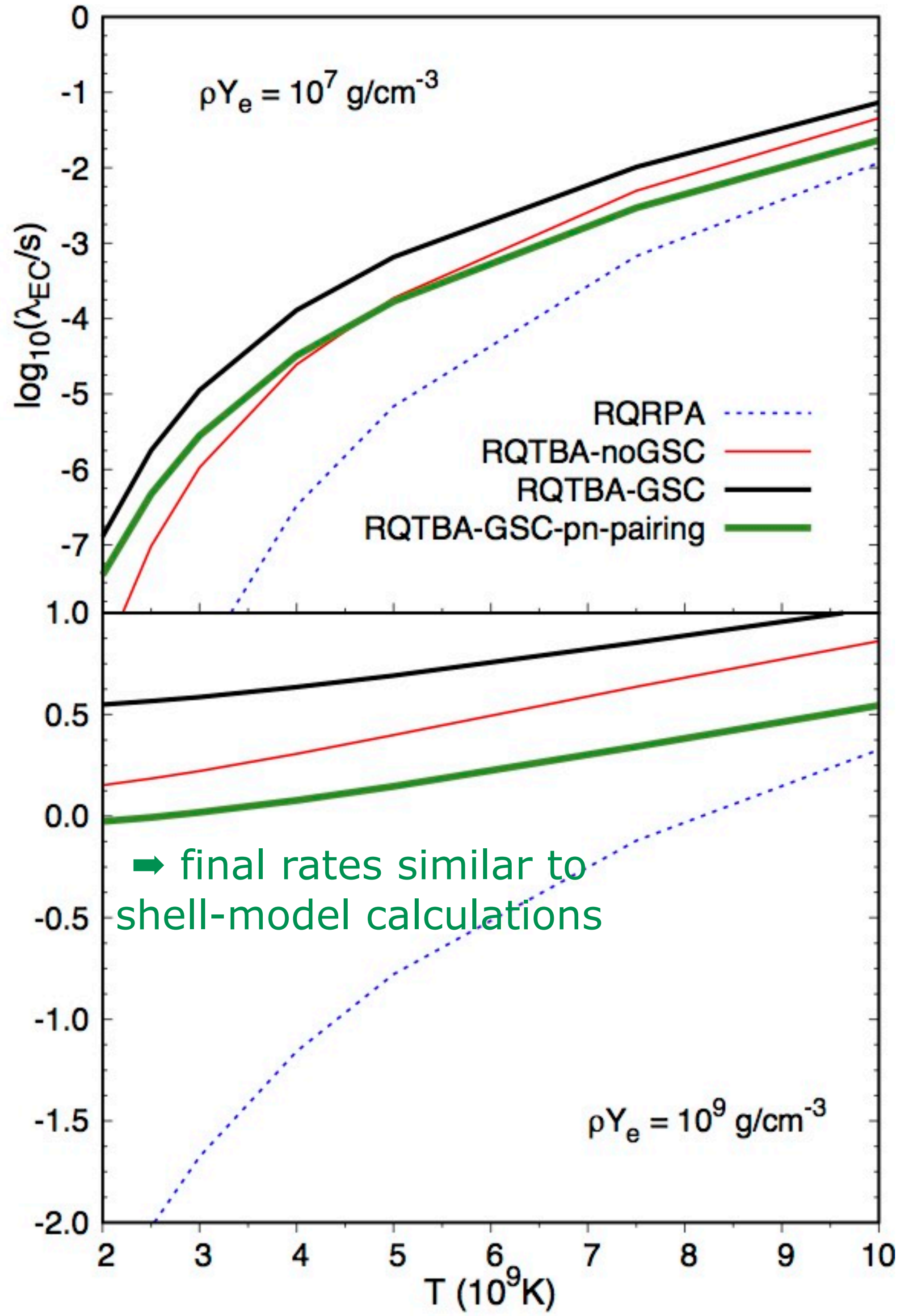
preliminary

➡ EC rates in ^{54}Fe:

$$\lambda_{EC} = \ln 2 \sum_j f_j(T, \rho, U_F) B(GT)_j$$

Q_{EC} = -1.2 MeV

Electron-capture rates calculated with the code written by S. Gupta, courtesy of R.G.T Zegers



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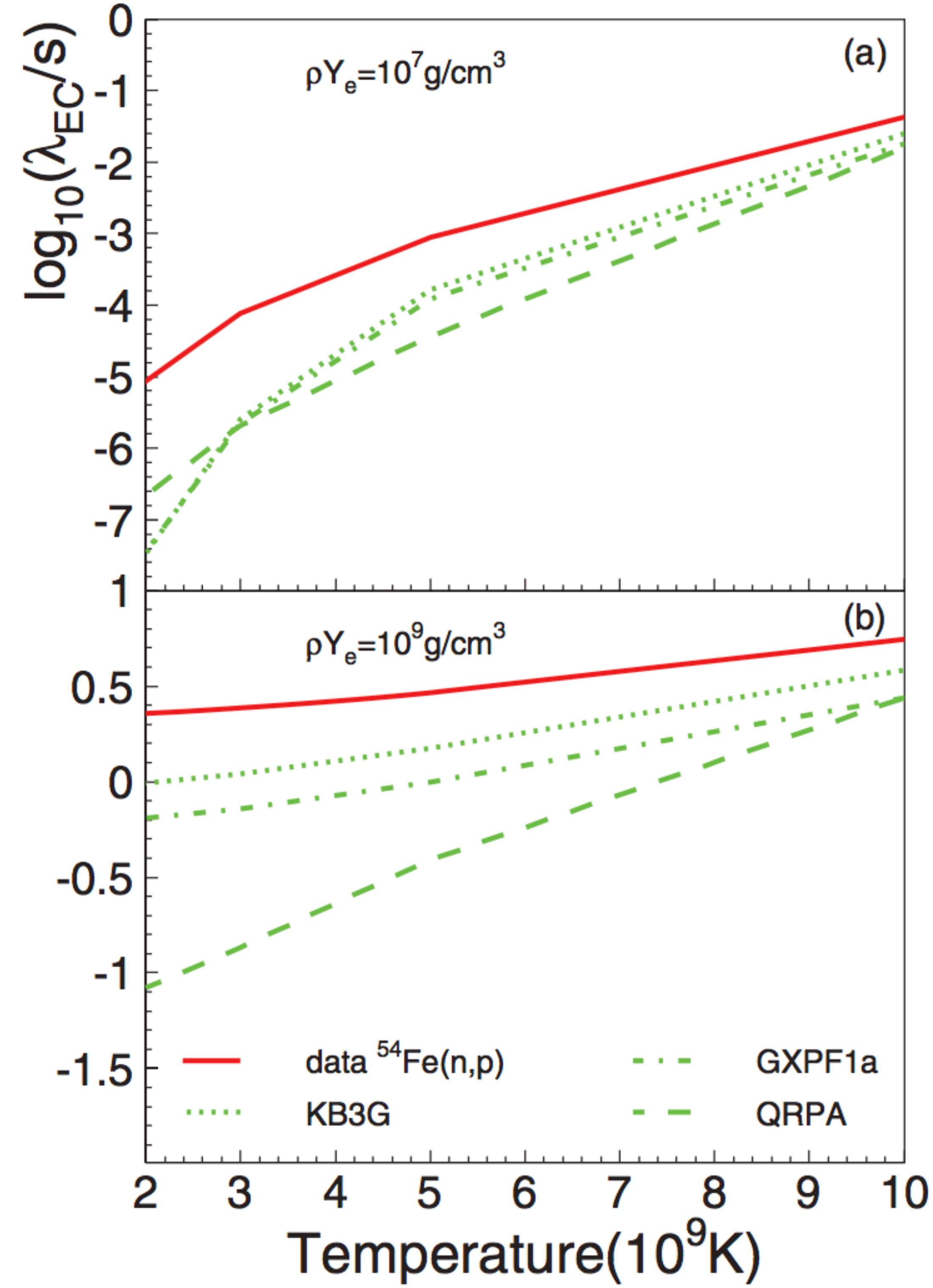
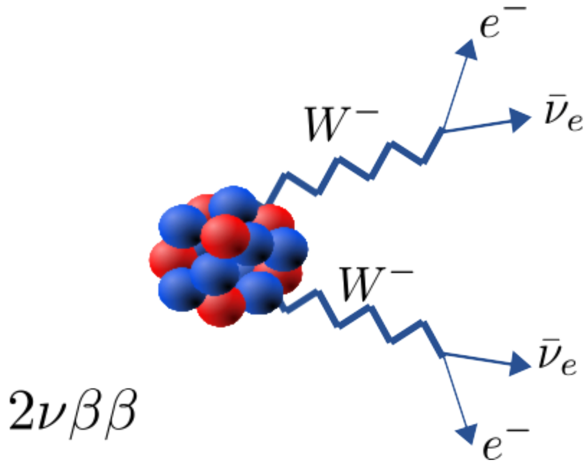


Outline

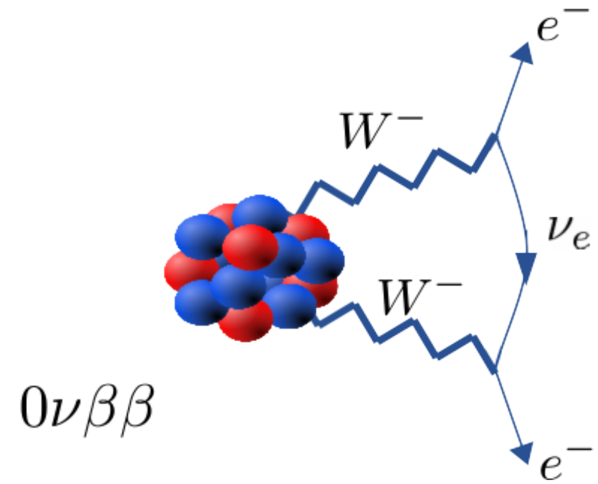
- ★ Relativistic Nuclear Field Theory: formalism in the resonant approximation (reminder)
- ★ Application to charge-exchange modes: Gamow-Teller (GT) transitions, beta-decay half-lives and the quenching problem
- ★ Recent development: Ground-state correlations from the quasiparticle-vibration coupling
  - Effect on GT transitions: importance in the GT^+ channel, interplay with proton-neutron pairing
- ★ Application to $2\nu\beta\beta$ decay: preliminary results for ^{48}Ca, and some ideas for describing double-beta decay in the Green's function formalism
- ★ Conclusion, perspectives

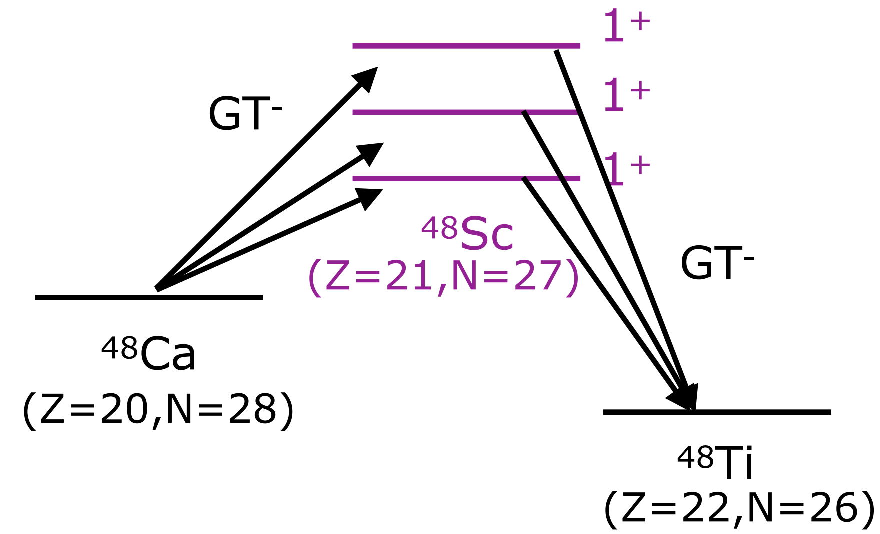
Application to two-neutrino double-beta decay

observed

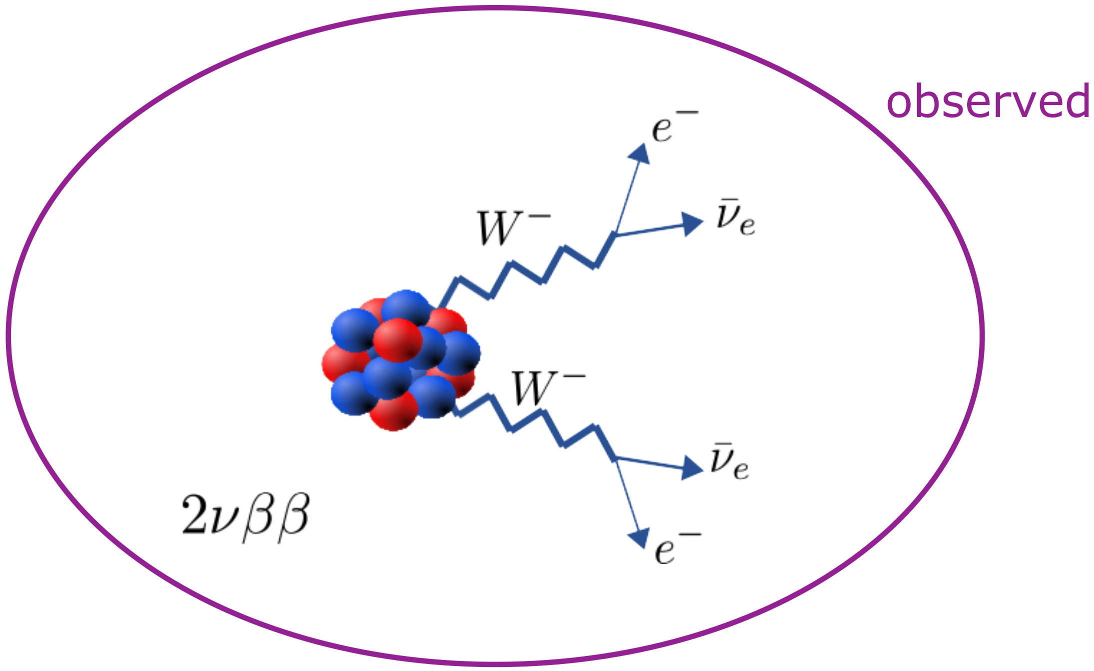


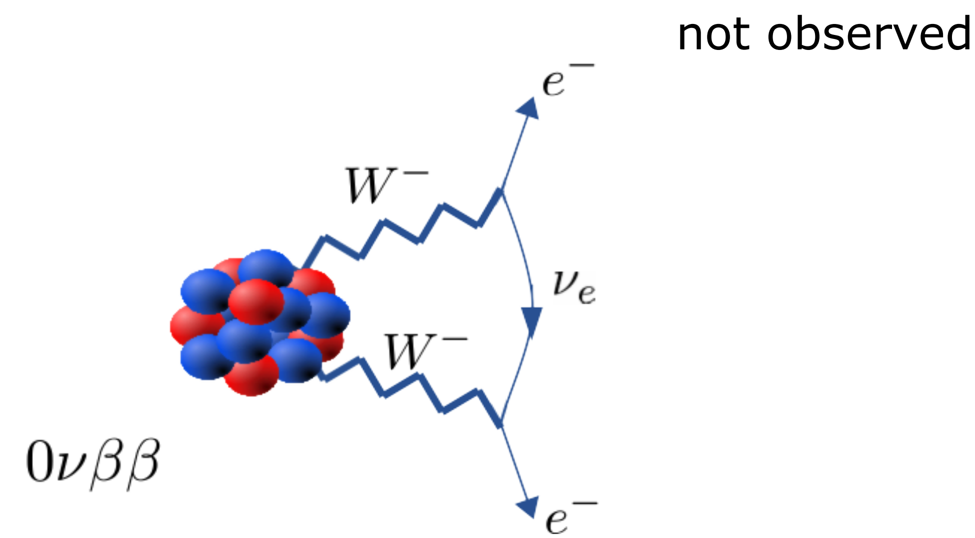
not observed

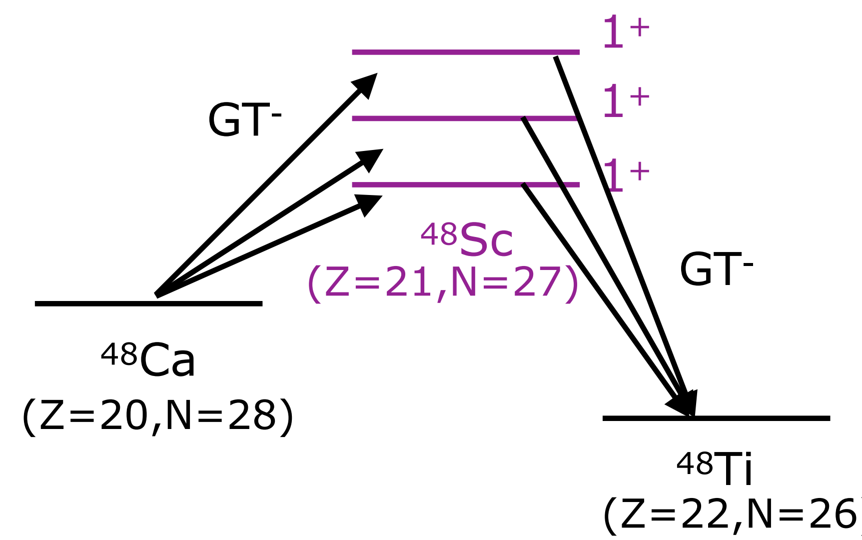




Application to two-neutrino double-beta decay







★ Two-neutrino double-beta decay matrix element:

$$M_{2\nu\beta\beta} \simeq \sum_N \frac{\langle 0_f|\hat{O}_{GT^-}|N\rangle\langle N|\hat{O}_{GT^-}|0_i\rangle}{(\Omega^f_N + \Omega^i_N)/2}$$

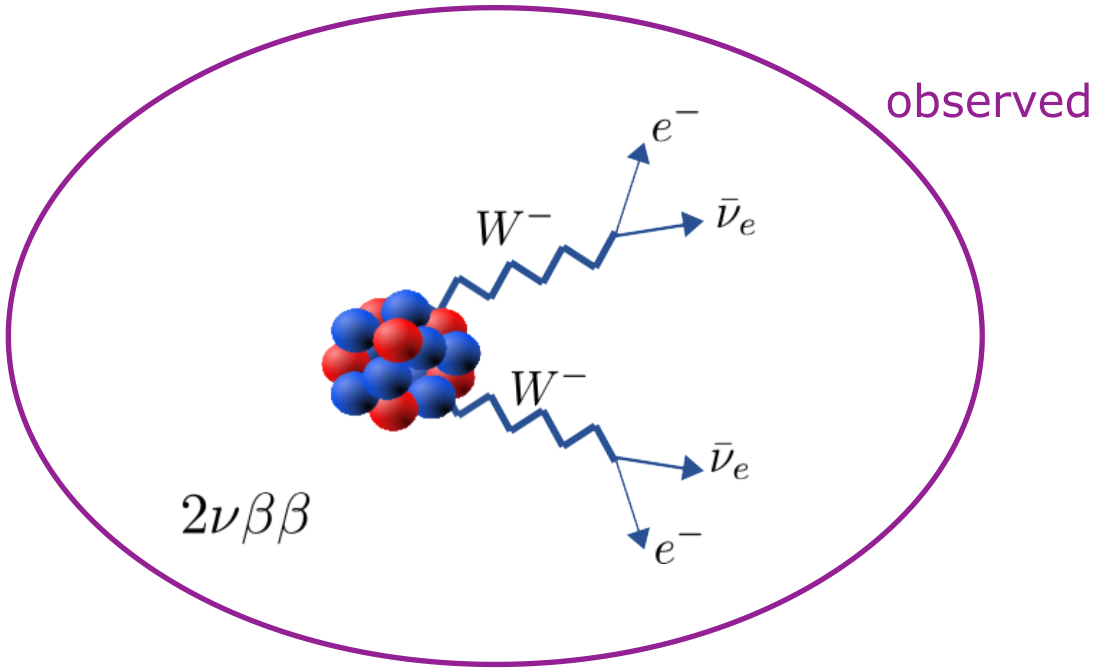
?

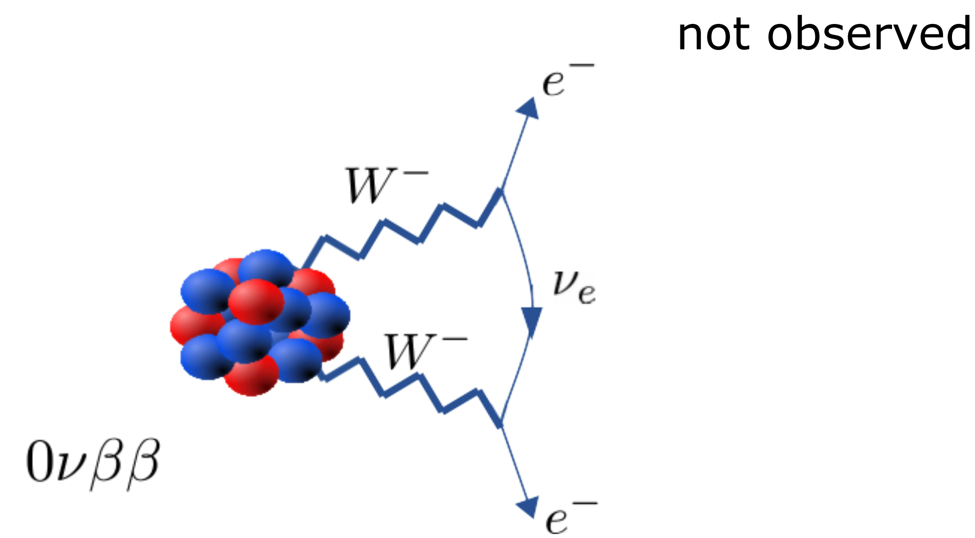
$$\simeq \sum_{MN} \frac{\langle 0_f|\hat{O}_{GT^-}|M\rangle\langle M|N\rangle\langle N|\hat{O}_{GT^-}|0_i\rangle}{(\Omega^f_M + \Omega^i_N)/2}$$

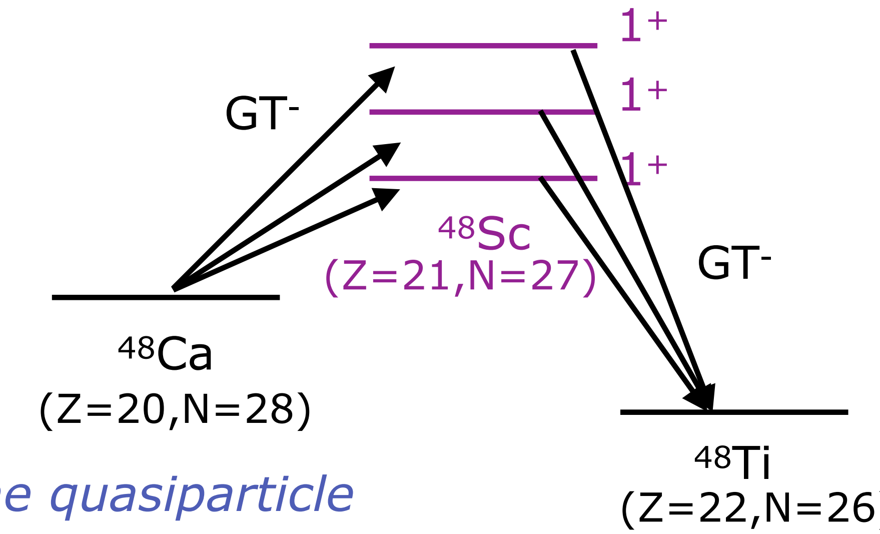
$$\Omega^{i/f}_N = E_N - E^{I/f}_0$$

In collaboration with J. Engel, UNC

Application to two-neutrino double-beta decay







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$$M_{2\nu\beta\beta} \simeq \sum_N \frac{\langle 0_f | \hat{O}_{GT^-} | N \rangle \langle N | \hat{O}_{GT^-} | 0_i \rangle}{(\Omega_N^f + \Omega_N^i)/2}$$

Calculated in the quasiparticle Tamm-Dancoff approximation (RQRPA without B matrix)

$$\simeq \sum_{MN} \frac{\langle 0_f | \hat{O}_{GT^-} | M \rangle \langle M | N \rangle \langle N | \hat{O}_{GT^-} | 0_i \rangle}{(\Omega_M^f + \Omega_N^i)/2}$$

$$\Omega_N^{i/f} = E_N - E_0^{I/f}$$

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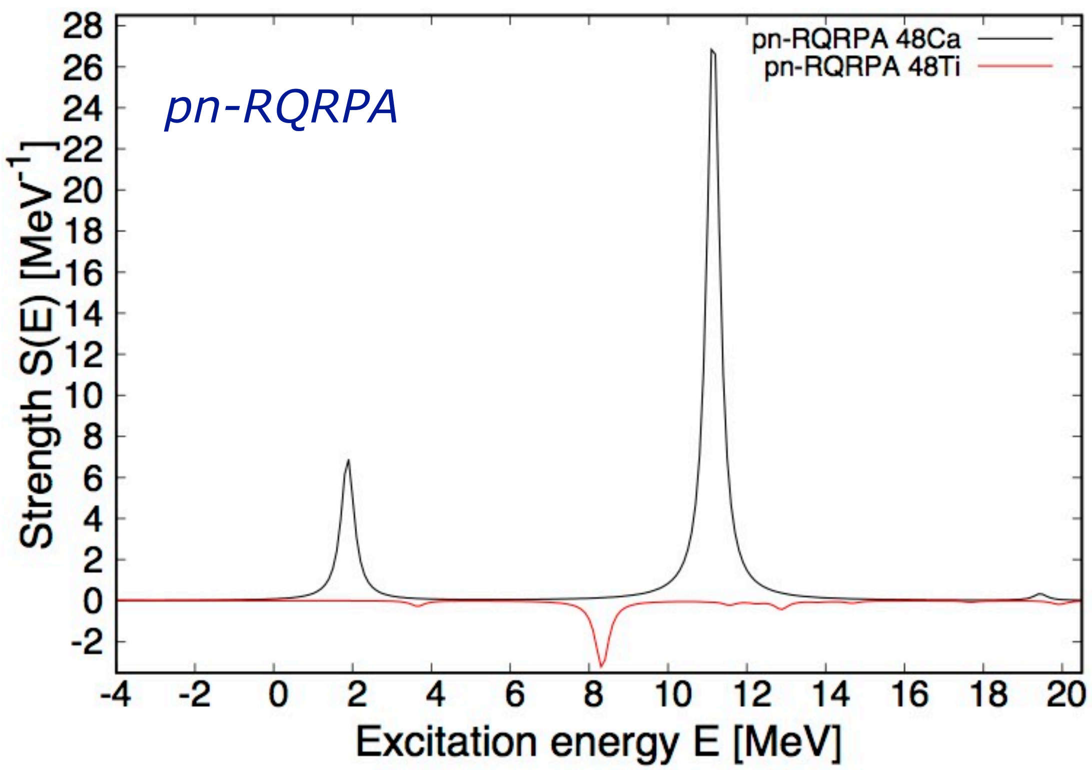
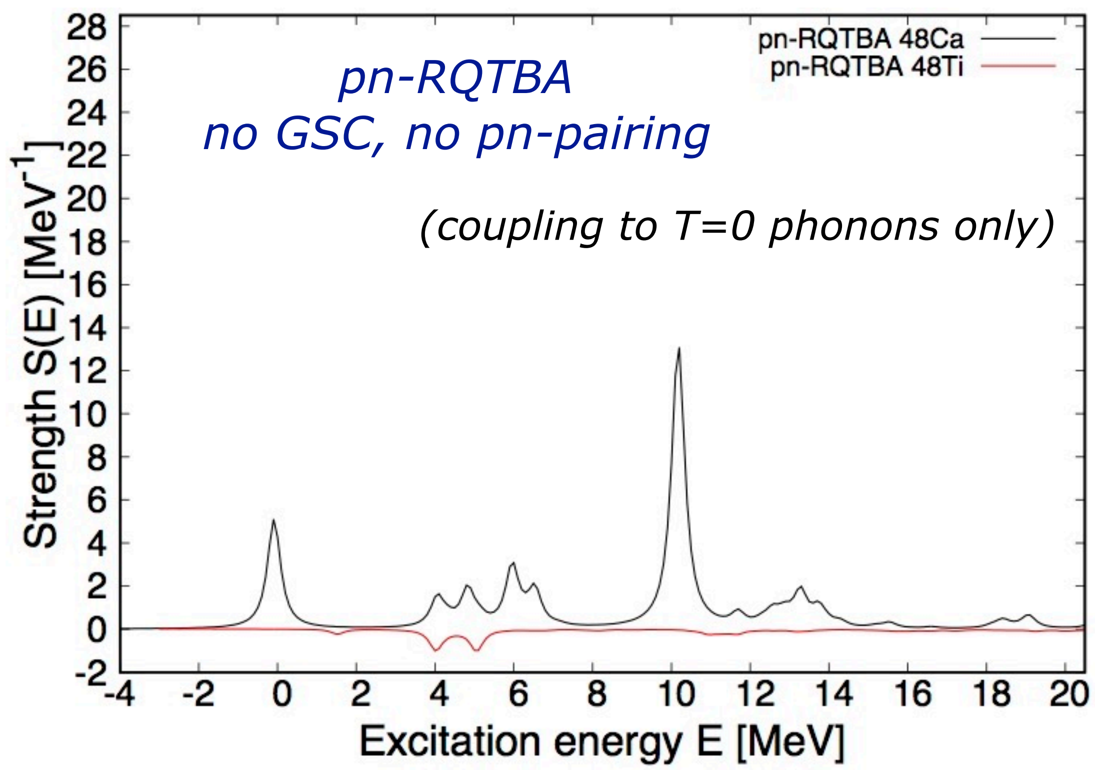
Application to two-neutrino double-beta decay

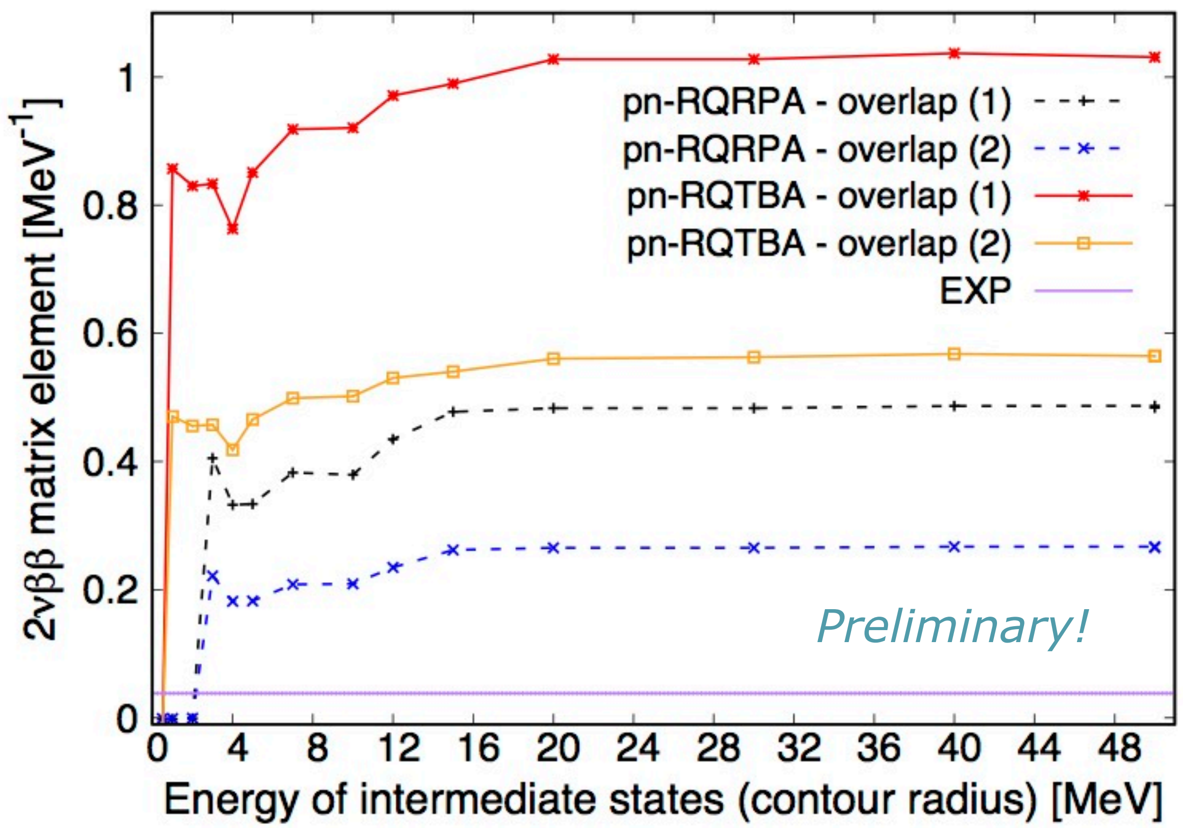




Overlap (1): full RQTDA
Mustonen and Engel,
PRC 87, 064302 (2013)

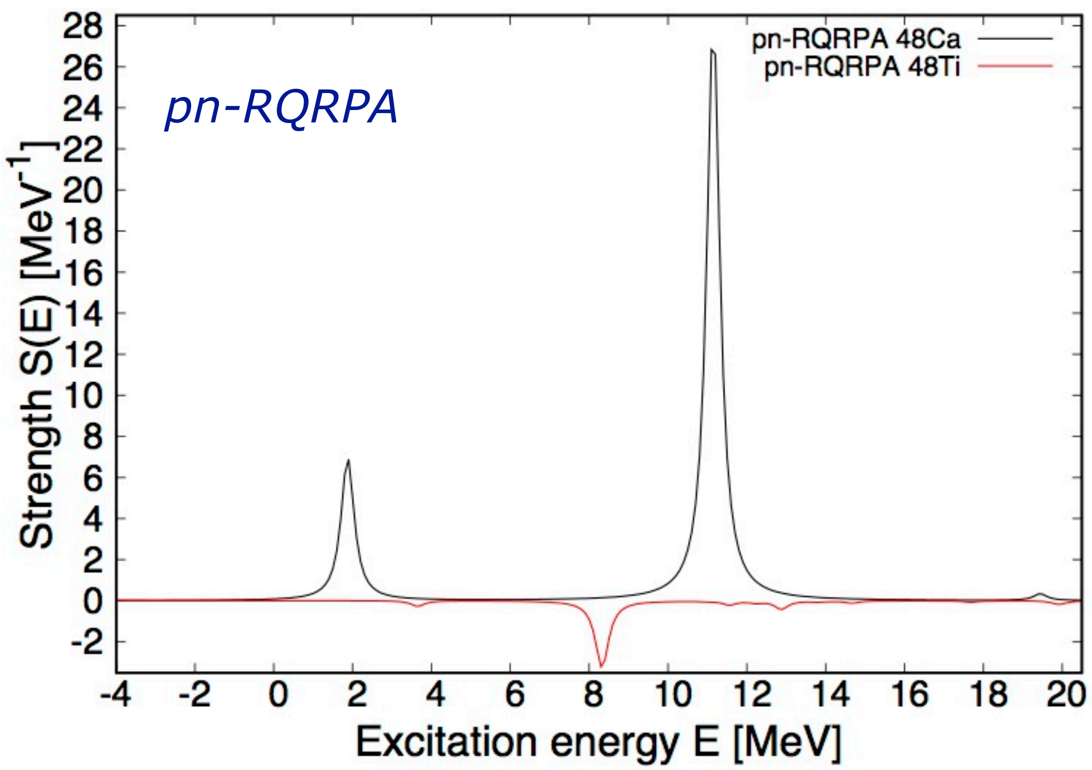
Overlap (2): Simkovic et al.,
NPA 733, 321 (2004)

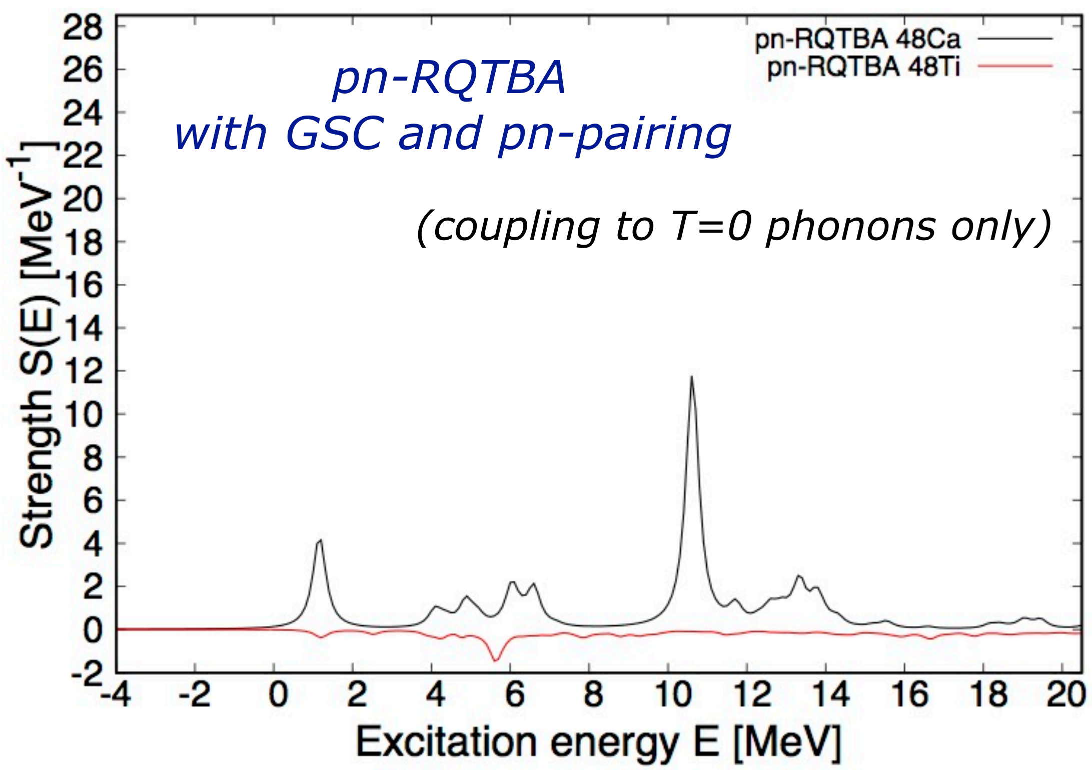
Unquenched



- ~84% of the ME comes from the first GT state due to the energy denominator

Application to two-neutrino double-beta decay





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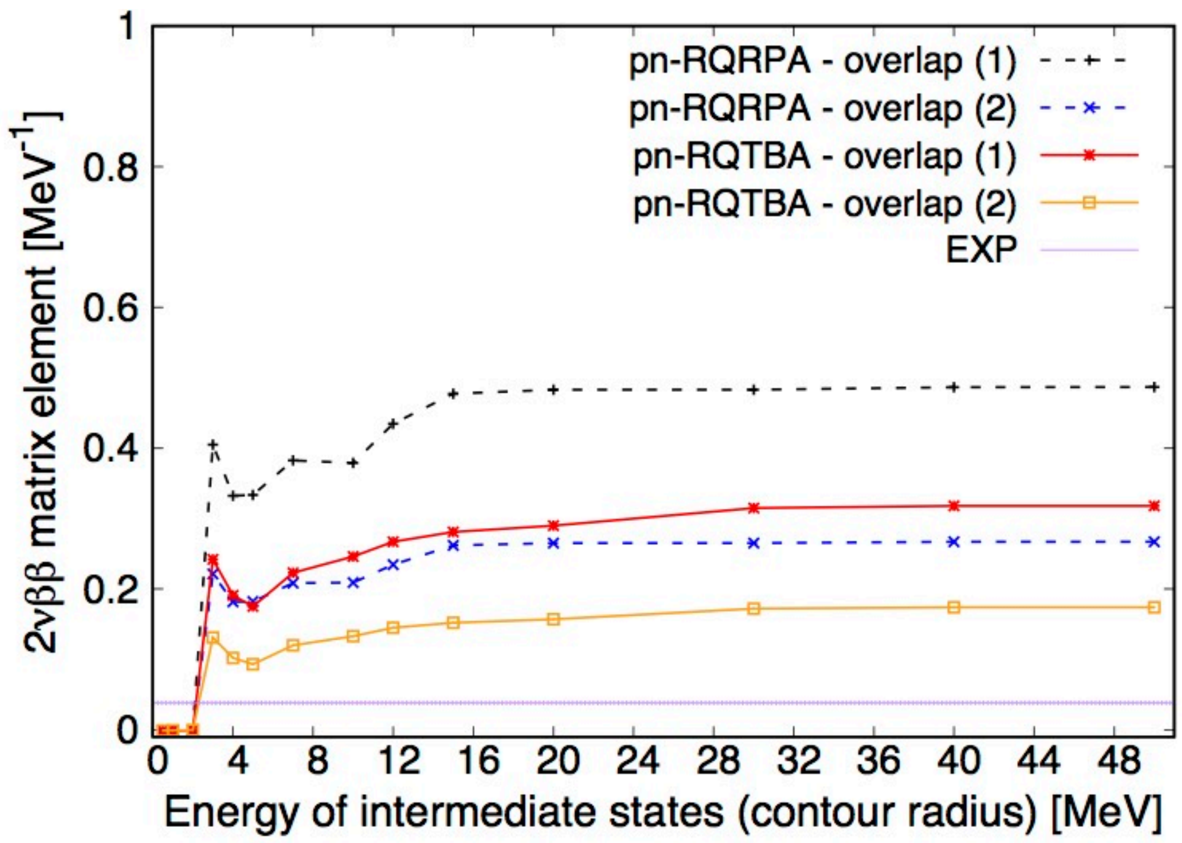
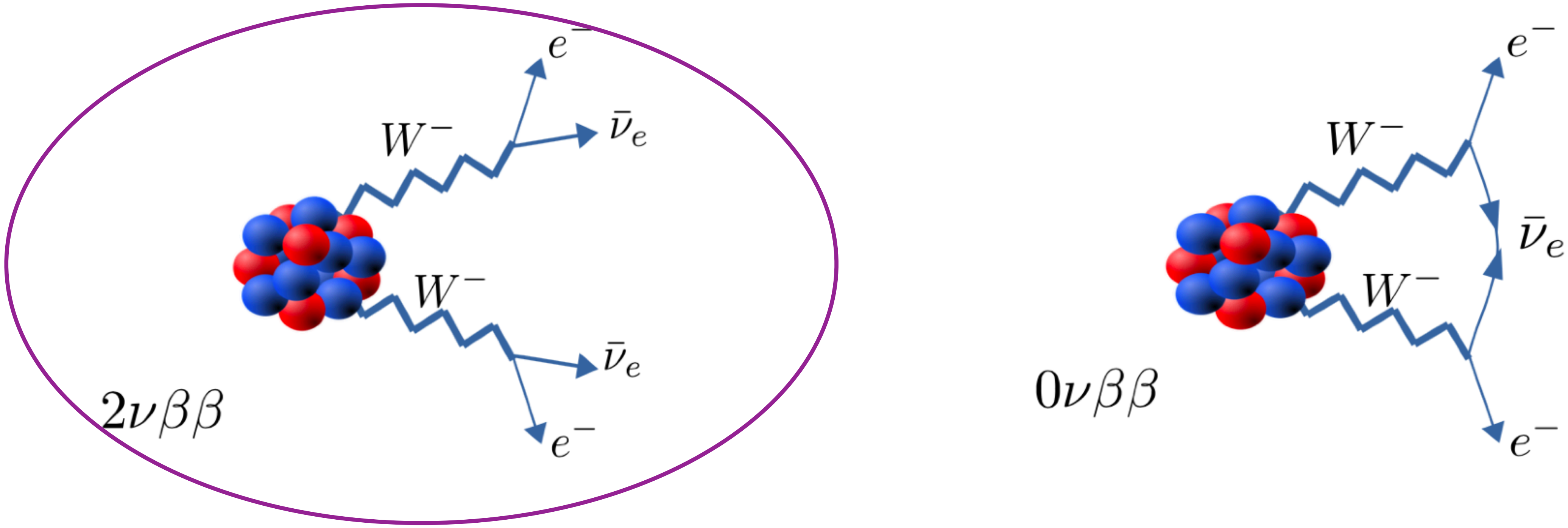


- Improvement but still overestimates the experiment by factor ~ 10
- Need to consider two-body currents
- Need better approximation for the overlap…

… Or find a way to calculate double-beta decay in the GF formalism… see next:

Double-beta decay in the GF formalism: some ideas



★Two-neutrino double-beta decay amplitude:

$$A_{i\to f}^{2\nu\beta\beta} = -\frac{1}{2}\int d^4x_1 d^4x_2$$

$$\times \langle \Psi_f; (\mathbf{p_1}, s_1); (\mathbf{p_2}, s_2); (\mathbf{q_1}, \sigma_1); (\mathbf{q_2}, \sigma_2) | \mathcal{T}\left(\mathcal{H}_{weak}(x_1)\mathcal{H}_{weak}(x_2)\right) | \Psi_i \rangle$$

(N-2,Z+2) e^- $\bar{\nu}_e$ (N,Z)

$$\mathcal{H}_{weak}(x) = \frac{G_F}{\sqrt{2}} J_\mu(x) L^{\dagger\mu}(x)$$

Double-beta decay in the GF formalism: some ideas

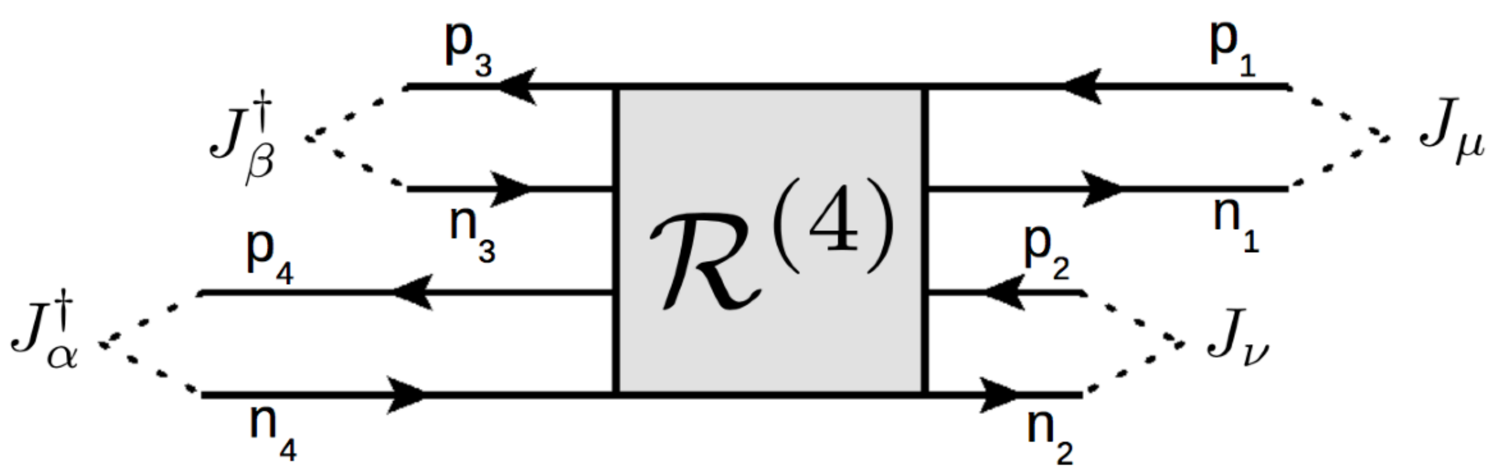
[…] → Inclusive probability for double-beta decay (after summation over final states):

$$P^{(2\nu\beta\beta)} \sim G_F^4 \int d^3p_1\, d^3p_2\, d^3q_1\, d^3q_2\, dx_1^0\, dx_2^0\, dy_1^0\, dy_2^0$$
$$\times e^{i(p_1^0+q_1^0)(x_1^0-y_1^0)} e^{i(p_2^0+q_2^0)(x_2^0-y_2^0)}$$
$$\times \mathcal{W}_{\alpha\beta\mu\nu}(x_1^0, x_2^0, y_1^0, y_2^0)\mathcal{L}^{\alpha\beta\mu\nu}(p_1, p_2, q_1, q_2)$$

Hadronic tensor

Leptonic tensor

$$\mathcal{W}_{\alpha\beta\mu\nu}(x_1^0, x_2^0, y_1^0, y_2^0) = \sum_{p_1 \ldots p_4, n_1 \ldots n_4} \langle n_4 | J_\alpha^\dagger | p_4 \rangle \langle n_3 | J_\beta^\dagger | p_3 \rangle$$
$$\times \mathcal{R}^{(4)}_{n_4 p_4, n_3 p_3, p_1 n_1, p_2 n_2}(y_2^0, y_1^0, x_1^0, x_2^0)\ \langle p_1 | J_\mu | n_1 \rangle \langle p_2 | J_\nu | n_2 \rangle$$



Double-beta decay in the GF formalism: some ideas

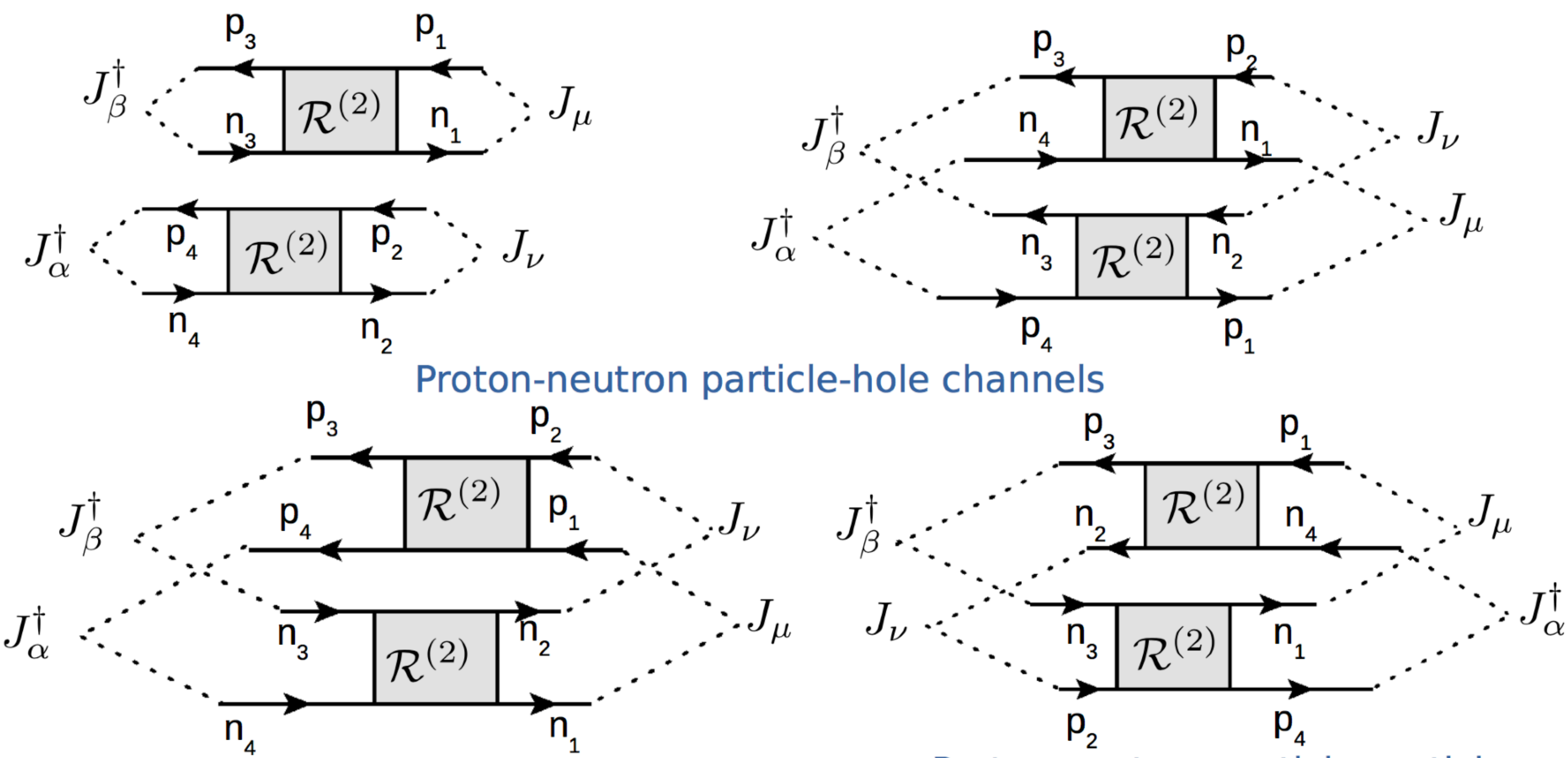
- Decomposition of the four-nucleon Green's function:

$$\mathcal{R}^{(4)} = \sum \mathcal{R}^{(2)}\mathcal{R}^{(2)} + \mathcal{R}^{(3)C}\mathcal{R}^{(1)} + \mathcal{R}^{(4)C}$$

Double-beta decay in the GF formalism: some ideas

- Decomposition of the four-nucleon Green's function:
  - → Possible approximation: neglect pure three- and four-body correlations

$$\mathcal{R}^{(4)} = \sum \mathcal{R}^{(2)}\mathcal{R}^{(2)} + \cancel{\mathcal{R}^{(3)C}}\mathcal{R}^{(1)} + \cancel{\mathcal{R}^{(4)C}}$$



Proton-neutron particle-hole channels

Neutral particle-particle

Proton-neutron particle-particle

Outline

★ Relativistic Nuclear Field Theory: formalism in the resonant approximation (reminder)

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★ Conclusion, perspectives

Conclusion, perspectives

→ ***Conclusions:***

- ★ The RNFT is a powerful framework for the microscopic description of mid-mass to heavy nuclei, which can account for complex configurations of nucleons in a large single-particle space
- ★ Encouraging in the charge-exchange channel → description of both low-energy strength and overall distribution to higher excitation energy.
- ★ QVC-induced GSC are important in the description of (n,p) transitions in N>Z nuclei and (p,n) transitions in N<Z nuclei
- ★ Encouraging preliminary results from meson-exchange proton-neutron pairing interaction.

Conclusion, perspectives

→ ***Perspectives:***

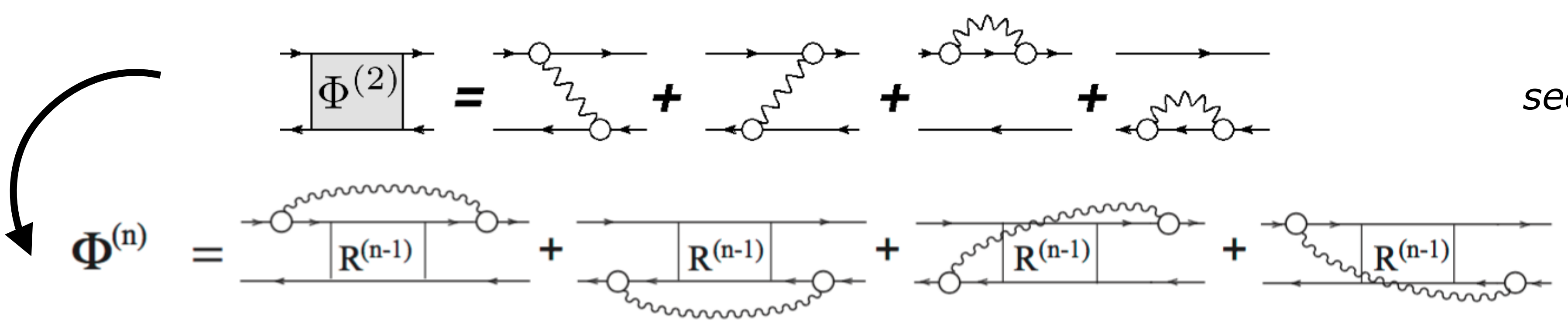
★ Applications to Astrophysics:

- Calculation of consistent (n,γ) and β-decay rates for r-process nucleosynthesis studies *(with N. Vassh and R. Surman @ Notre Dame University)*.
- Extension to finite temperature *(Elena Litvinova's talk)* → stellar electron-capture rates

★ Applications to Fundamental Physics: 2νββ and 0νββ decay, neutrino scattering at intermediate energies

★ Long-term extensions of the formalism:

- Inclusion of higher-order configurations (beyond 2p2h) to resolve finer details of nuclear phenomena



see E. Litvinova's talk for 3p-3h

- extension to odd-even/odd-odd and deformed nuclei,
- develop a relativistic theory of isovector pairing,
- inclusion of the Fock term *(with H. Liang @ RIKEN)* ➡ subtraction in the pn channel (at which energy)?
- inclusion of two-body currents and Δ resonance,
- towards an "ab-initio" theory with bare meson exchange…

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★ Long-term ...

- Inclus ... nomena

... a, Phys.Rev. C 91, ...4332 (2015).

$\Phi^{(n)} =$

Thank you!

This work is supported by
INT US-DOE Grant DE-FG02-00ER41132 and
JINA-CEE US-NSF Grant PHY-1430152

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