Nuclear Many-Body Theories: Beyond the mean-field approaches

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A beyond-mean-field description for nuclear excitation spectra: the subtracted Second RPA (SRPA)

IPN INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY
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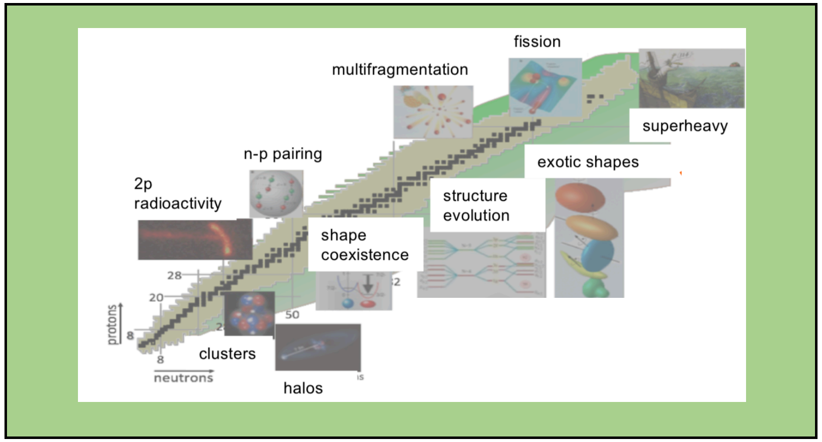
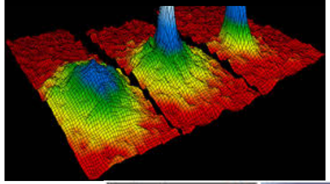
A few words on the context… nuclear energy-density- functional (EDF) theories

Many interdisciplinary links: the richness of the nuclear many-body physics

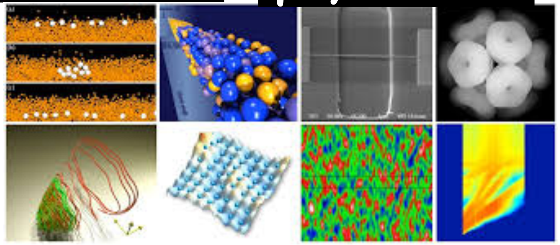


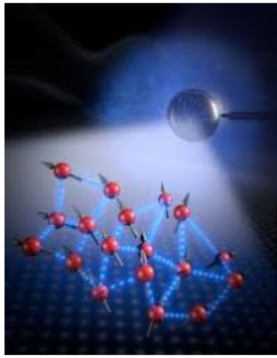
DFT
Hohenberg
-Kohn
theorem

Atomic physics



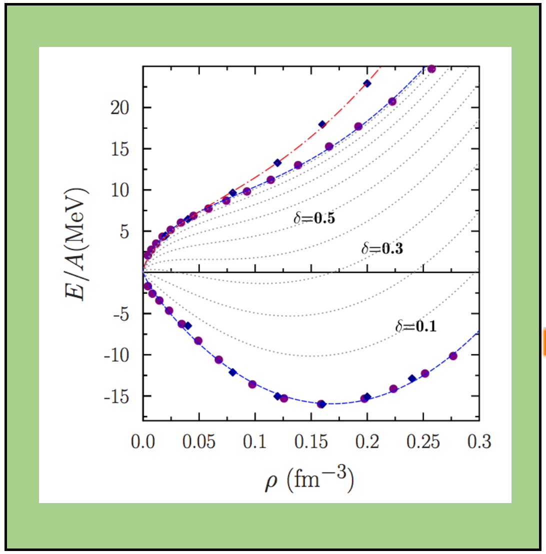
Solid-state physics



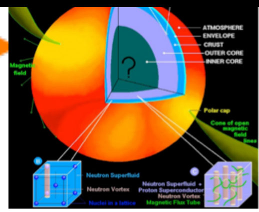


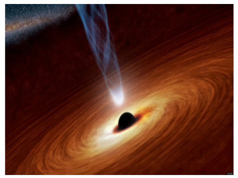
Quantum chemistry

Nuclear astrophysics



Neutron star physics

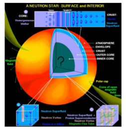


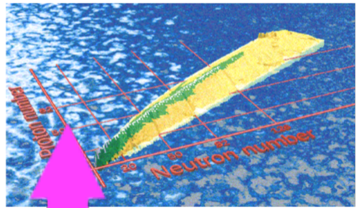


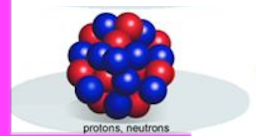
EOS
of
infinite
matter

Functionals for nuclear physics (nuclei and infinite matter). Empirical way

Nuclear phenomenology

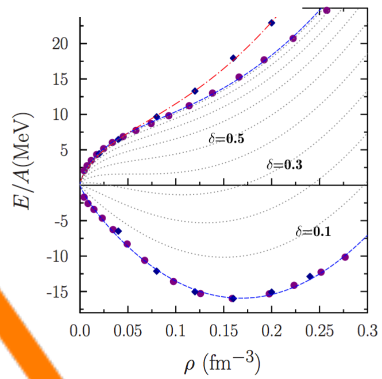






strains

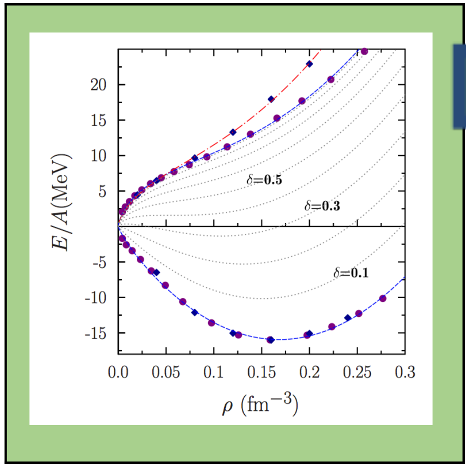
Equation of state of infinite matter

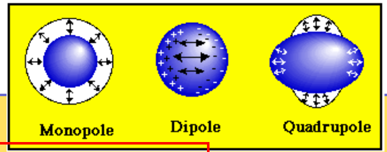


constrains/guides to construct

Functionals/interactions

Equation of state of infinite matter.
For example, links with collective modes



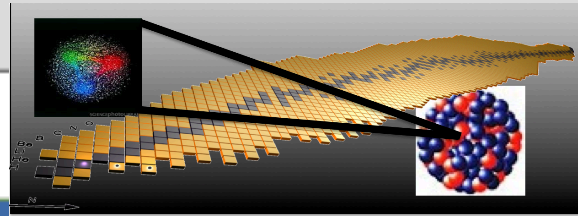


- **Incompressibility (ISGMR, ISGDR)**
- **Effective mass (axial compression modes -> ISGQR)**
- **Symmetry energy and its density dependence, neutron skin (IVGDR, dipole polarizability, pygmy dipole, IVGQR, IAS, Gamow-Teller, spin dipole, anti-analogue GDR)**

- Grasso, Effective density functionals beyond mean field, Prog. Part. Nucl. Phys. 106, 256 (2019)
- Bao-An Li, Bao-Jun Cai, Lie-wen Chen, and Jun Xu, Nucleon effective masses in neutron-rich matter, Prog. Part. Nucl. Phys. 99, 29 (2018)
- Roca-Maza and Paar, Nuclear equation of state from ground and collective excited state properties of nuclei, Prog. Part. Nucl. Phys. 101, 96 (2018)
- Garg, Colò, The compression-mode giant resonances and nuclear incompressibility, Prog. Part. Nucl. Phys. 101, 55 (2018)
- Bracco, Lanza, Tamii, Isoscalar and isovector dipole ...., Prog. Part. Nucl. Phys. 106, 360 (2019)

EDF models employ, in most cases, phenomenological effective interactions whose parameters are adjusted at the mean-field level => double counting of correlations when the mean-field approximation is overcome

Within the EDF theory: designing interactions adapted for beyond mean-field models (cancellation of double counting, regularization of divergences, …, possibly bridging with EFT/ab initio -> reducing the empirical nature)

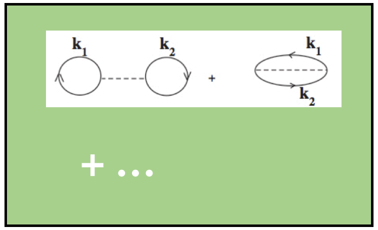


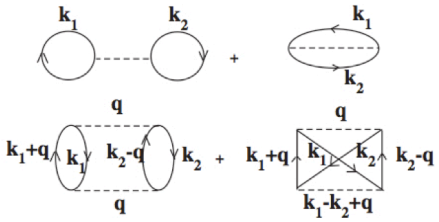
Otherwise, specific solutions exist, for example a subtraction procedure, that we have applied within the second random-phase approximation (SRPA)

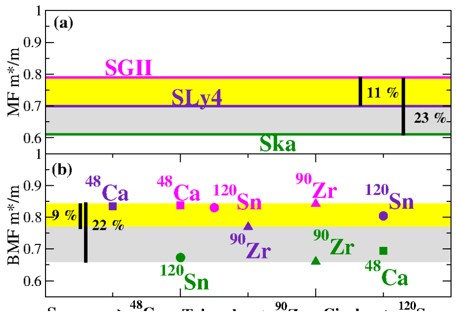
Tselyaev, PRC 75, 024306 (2007)
Tselyaev, PRC 88, 054301 (2013)

Effective density functionals (EDFs). Several directions

Beyond the mean field:







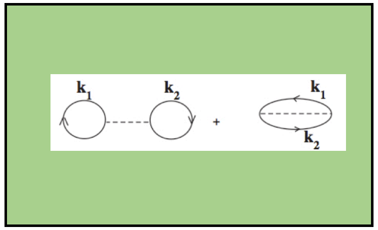
EOS of infinite matter (MBPT)

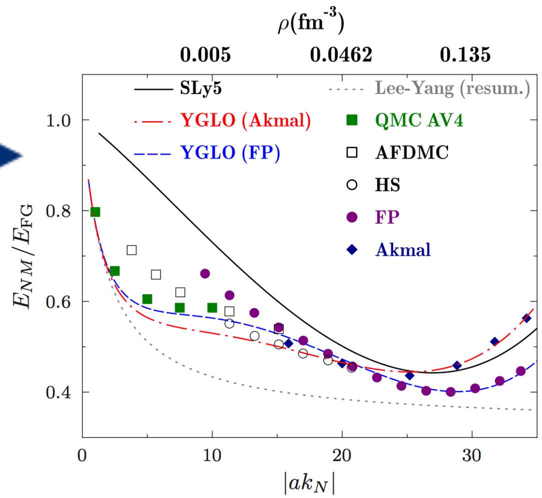
- Yang, et al., PRC 94, 034311 (2016)

For example recent estimation of BMF effects on the effective mass

- Grasso et al., PRC 98, 051303 (R) (2018)

At leading order,
bridging with ab-initio



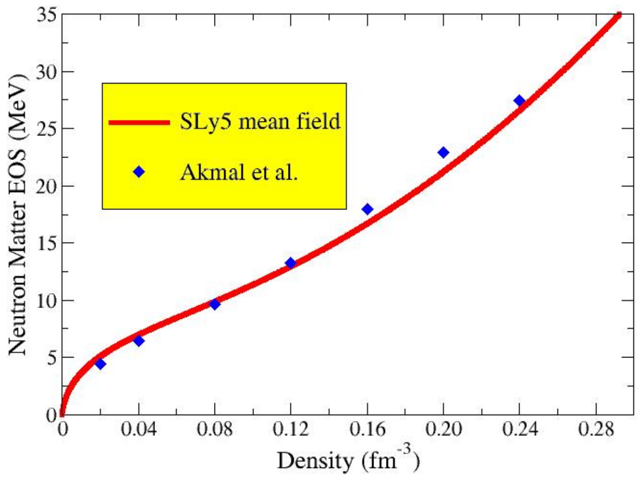


Functionals satisfying the low-density regime (less phenomenological)

- Yang et al., PRC 94, 031301 (R) (2016)
- Grasso et al., PRC 96, 054327 (2017)
- Bonnard et al., PRC 98, 034319 (2018)

Grasso,
Prog. Part. Nucl. Phys. 106, 256 (2019)

Neutron matter at 'usual' density scales. Example of Lyon-Saclay forces adjusted on the neutron EOS

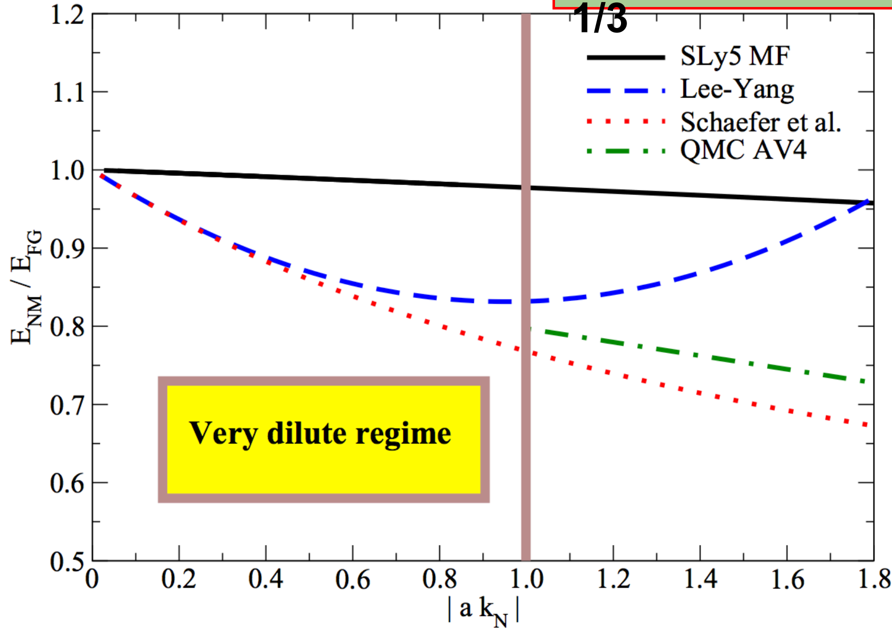


SLy5 -> Chabanat et al. NPA 627, 710 (1997); 635, 231 (1998), 643, 441 (1998)
Akmal et al. -> PRC 58, 1804 (1998)

Low-density regime

$$\frac{E}{N} = \frac{\hbar^2 k_N^2}{2m}\left[\frac{3}{5} + \frac{2}{3\pi}(k_N a) + \frac{4}{35\pi^2}(11 - 2ln2)(k_N a)^2\right]$$

kF = kN = (3π2 ρ) 1/3



Neutron matter energy divided by the free gas energy

Dealing with the large value of the neutron-neutron scattering length

Resummation techniques

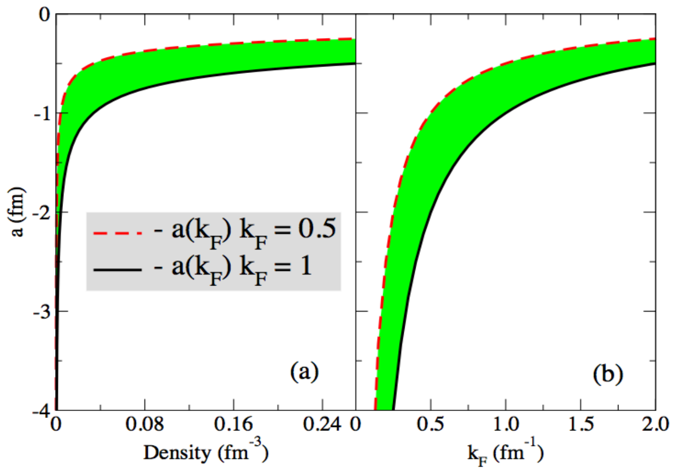
- Steele, arXiv: nucl-th/0010066v2
- Kaiser, NPA 860, 41 (2011)
- Schaefer, NPA 762, 82 (2005)

Effective field theories capable of describing systems with anomalously large scattering lengths require summing an infinite number of Feynman diagrams at leading order …

Yang et al., PRC 94, 031301 (R) (2016)

Tuning the neutron-neutron scattering length imposing

$$|ak_F| < 1.$$



Grasso et al., PRC 96, 054327 (2017)

Collaborators
(work on beyond mean-field)

- F. Catara (Catania Univ.), G. Co' (Lecce univ.), V. De Donno (Lecce Univ.), D. Gambacurta (ELI, Bucharest), J. Engel (North Carolina), O. Vasseur (IPN Orsay)

SRPA with zero-range and finite-range effective interactions and implementation with a subtraction procedure

SRPA-based models

- J. Bonnard, A. Boulet (IPN Orsay), G. Colo' (Milano Univ.), U. van Kolck (IPN Orsay), D. Lacroix, (IPN Orsay), X. Roca-Maza (Milano Univ.), J. Yang (Chalmers)

1) Designed for beyond mean field 2) Reducing the phenomenological nature (constraints from the low-density regime)

Work on density functionals

ENSAR2

JRA TheoS (Theoretical Support for Nuclear Facilities in Europe) Task: Development of suitable effective interactions in mean-field and beyond-mean-field theories

International Laboratory LIA COLL-AGAIN (France-Italy collaborations)

Outline

- <u>**Beyond RPA with the second RPA (SRPA) model**</u> **employing EDFs**

SRPA model : formally established since several decades

$$Q_\nu^\dagger = \sum_{ph} \left(X^\nu_{ph} a^\dagger_p a_h - Y^\nu_{ph} a^\dagger_h a_p \right)$$
$$+ \sum_{p<p',h<h'} \left(X^\nu_{php'h'} a^\dagger_p a_h a^\dagger_{p'} a_{h'} - Y^\nu_{php'h'} a^\dagger_h a_p a^\dagger_{h'} a_{p'} \right)$$

Excitation operators: 2p2h configurations are included, together with the RPA 1p1h configurations

- Hoshino and Arima, Phys. Rev. Lett. 37, 266 (1976)
- Knupfer and Huber, Z. Phys. A 276, 99 (1976)
- Adachi and Yoshida, Nucl. Phys. A 306, 53 (1978)
- Tohyama, Gong, Z. Phys.A 332, 269 (1989)
- Lacroix, Ayik, Chomaz, Prog. Part. Nucl. Phys. 52, 497 (2004)
- Schwesinger, Wambach, Phys. Lett. B 134, 29 (1984)
- Schwesinger, Wambach, Nucl. Phys. A 426, 253 (1984)
- Wambach, Rep. Prog. Phys. 51, 989 (1988)
- Drozdz, Nishizaki, Speth, Wambach, Phys. Rep. 197, 1 (1990)
- Nishizaki and Wambach, Phys. Lett. B 349, 7 (1995)
- Nishizaki and Wambach, Phys. Rev. C 57, 1515 (1998)

Examples of first applications for the calculation of fragmentation and spreading widths (strong cuts in the 2p2h space, Second Tamm-Dancoff, truncations and approximations in the 2p2h sector of the matrix)

In the last decade.
No approximations in 2p2h matrix elements and large 2p2h cutoff values

- Papakonstantinou and Roth, Phys. Lett. B 671, 356 (2009)
- Papakonstantinou and Roth, Phys. Rev. C 81, 024317 (2010)
- Gambacurta, Grasso, and Catara, Phys. Rev. C 81, 054312 (2010)
- Gambacurta, Grasso, and Catara, J. Phys. G 38, 035103 (2011)
- Gambacurta, Grasso, and Catara, Phys. Rev. C 84, 034301 (2011)
- Gambacurta, Grasso, De Donno, Co, and Catara, Phys. Rev. C 86, 021304(R) (2012)

Microscopic interaction (derived from Argonne V18)

Phenomen. Skyrme and Gogny interactions

SRPA model

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix}$$

Schematically: same form as RPA equations

$$\mathcal{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix},$$

$$\mathcal{X}^\nu = \begin{pmatrix} X_1^\nu \\ X_2^\nu \end{pmatrix}, \quad \mathcal{Y}^\nu = \begin{pmatrix} Y_1^\nu \\ Y_2^\nu \end{pmatrix}.$$

1 and 2:

short-hand notation for 1p1h and 2p2h

A11 and B11: standard RPA matrices

A12, A21, B12 , and B21: coupling between 1p1h and 2p2h

A22 and B22: 2p2h sector

If the interaction does not depend on the density

- B12 = B21 =B 22 = 0
- The beyond-RPA matrix elements for the matrix A are:

Coupling 1p1h with 2p2h (matrix elements of the interaction: hppp, phhh)

$$
\begin{aligned}
A_{12} &= A_{ph,p_1p_2h_1h_2} \\
&= \langle \mathrm{HF}|\big[a_h^\dagger a_p, \big[H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1}\big]\big]|\mathrm{HF}\rangle \\
&= \chi(h_1, h_2)\bar{V}_{h_1 p p_1 p_2}\delta_{hh_2} - \chi(p_1, p_2)\bar{V}_{h_1 h_1 p_1 h}\delta_{pp_2},
\end{aligned}
$$

Antisymmetrizer

Skyrme and Gogny interactions contain density-dependent terms. In RPA:

V is a density-dependent interaction in the Hamiltonian H.
It produces a functional of the density ρ, the so-called Hamiltonian density E:

$$< H >= \int dr E[\rho]$$

If h is the HF mean-field single-particle Hamiltonian, the residual interaction to be used in the computation of the matrices A and B of RPA is equal to the interaction V in the Hamiltonian plus the so-called rearrangement terms (derivation of RPA equations as small-amplitude limit of time-dependent HF equations)

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + \frac{\partial h_{ph}}{\partial \rho_{p'h'}}$$

$$\frac{\partial h_{ac}}{\partial \rho_{db}} = \frac{\partial^2 E[\rho]}{\partial \rho_{ca} \partial \rho_{db}}$$

$$B_{ph,p'h'} = \frac{\partial h_{ph}}{\partial \rho_{h'p'}}$$

SRPA with density-dependent forces (Skyrme or Gogny)

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix},$$

where:

$$\mathcal{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \mathcal{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix},$$

Inspired by the variational derivation of SRPA equations by da Providencia, Nucl. Phys. 61, 87 (1965)
<u>but with a density-dependent interaction</u>

Gambacurta, Grasso, Catara, J. Phys. G: Nucl. and Part. Phys. 38, 035103 (2011)

Residual interaction. Rearrangement terms for SRPA matrix elements when the interaction is density dependent

Examples of previous studies in beyond-RPA frameworks:

- Waroquier et al., Phys. Rep. 148, 249 (1987)

- Some matrix elements beyond standard RPA (however the procedure does not allow one to obtain the standard RPA rearrangement terms)

Adachi and Yoshida, Phys. Lett. B 81, 98 (1979)

Variational procedure to derive the SRPA equation

da Providencia Nucl. Phys. 61, 87 (1965)

$$|\Psi\rangle = e^{\hat{S}}|\Phi\rangle$$

HF state

$$\hat{S} = \sum_{ph} C_{ph}(t) a_p^\dagger a_h + \frac{1}{2} \sum_{php'h'} \hat{C}_{pp'hh'}(t) a_p^\dagger a_{p'}^\dagger a_h a_{h'} .$$

$$\hat{C}_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma\delta} - C_{\alpha\beta\delta\gamma}$$

-The coefficients C are used as variational parameters (minimization of the expectation value of the Hamiltonian)

-The coefficients C are assumed very small => expansion of the expectation values of 1- and 2-body operators truncated at second order in C

Expansion of the one-body density around the HF density □(0)

$$\rho_{\alpha\beta} = \langle\Psi|a^\dagger_\beta a_\alpha|\Psi\rangle = \langle\Phi|e^{S^\dagger} a^\dagger_\beta a_\alpha e^S|\Phi\rangle$$

$$= \langle\Phi|\left(1 + S^\dagger + \tfrac{1}{2}S^{\dagger 2} + \cdots\right)a^\dagger_\beta a_\alpha\left(1 + S + \tfrac{1}{2}S^2 + \cdots\right)|\Phi\rangle$$

$$\sim \rho^{(0)}_{\alpha\beta} + \langle\Phi|a^\dagger_\beta a_\alpha S + S^\dagger a^\dagger_\beta a_\alpha|\Phi\rangle + \langle\Phi|\tfrac{1}{2}a^\dagger_\beta a_\alpha S^2 + S^\dagger a^\dagger_\beta a_\alpha S$$

$$+ \tfrac{1}{2}S^{\dagger 2}a^\dagger_\beta a_\alpha|\Phi\rangle = \rho^{(0)}_{\alpha\beta} + \delta\rho_{\alpha\beta}$$

Linear

$$\delta\rho^{(1)}_{\alpha\beta} = \langle\Phi|a^\dagger_\beta a_\alpha S + S^\dagger a^\dagger_\beta a_\alpha|\Phi\rangle,$$

Quadratic

$$\delta\rho^{(2)}_{\alpha\beta} = \langle\Phi|\tfrac{1}{2}a^\dagger_\beta a_\alpha S^2 + S^\dagger a^\dagger_\beta a_\alpha S + \tfrac{1}{2}S^{\dagger 2}a^\dagger_\beta a_\alpha|\Phi\rangle$$

Expansion of the one-body density around the HF density

$$\delta\rho^{(1)}_{hh'} = \delta\rho^{(1)}_{pp'} = 0; \qquad \delta\rho^{(1)}_{ph} = C_{ph}; \qquad \delta\rho^{(1)}_{hp} = C^*_{ph};$$

$$\delta\rho^{(2)}_{ph} = \sum_{mi} C^*_{mi}\hat{C}_{pmhi}; \qquad \delta\rho^{(2)}_{hp} = \sum_{mi} C_{mi}\hat{C}^*_{pmhi};$$

$$\delta\rho^{(2)}_{hh'} = -\sum_{m} C^*_{mh}C_{mh'} - \frac{1}{2}\sum_{mni}\hat{C}^*_{mnih}\hat{C}_{mnih'};$$

$$\delta\rho^{(2)}_{pp'} = \sum_{i} C^*_{p'i}C_{pi} + \frac{1}{2}\sum_{mij}\hat{C}^*_{p'mij}\hat{C}_{pmij}.$$

Mean value of the Hamiltonian in the ground state:

$$\langle H \rangle = \langle \Phi | H | \Phi \rangle + \sum_{mi} (C^*_{mi} \lambda_{mi}(\rho) + C_{mi} \lambda_{im}(\rho))$$
$$+ \sum_{i<j,m<n} (\hat{C}^*_{mnij} \hat{V}_{mnij}(\rho) + \hat{C}_{mnij} \hat{V}_{ijmn}(\rho)) + F^{(2)}$$

Sum of quadratic terms

For example:

$$A_{mi,pk} = \left[\frac{\delta^2 \langle H \rangle}{\delta C^*_{mi} \delta C_{pk}} \right]_{C=C^*=0} \equiv A_{11},$$

$$A_{mi,pqkl} = \left[\frac{\delta^2 \langle H \rangle}{\delta C^*_{mi} \delta \hat{C}_{pqkl}} \right]_{C=C^*=0} \equiv A_{12},$$

Da Providencia derivation of SRPA

But our interaction is density dependent.

Expansion of the density-dependent interaction around the HF density:

$$\hat{V}_{\alpha\beta\gamma\delta}(\rho) \sim \hat{V}_{\alpha\beta\gamma\delta}(\rho^{(0)}) + \sum_{ab} \left[\frac{\delta \hat{V}_{\alpha\beta\gamma\delta}}{\delta \rho_{ab}} \right]_{\rho=\rho^{(0)}} \delta\rho_{ab} + \frac{1}{2} \sum_{abcd} \left[\frac{\delta^2 \hat{V}_{\alpha\beta\gamma\delta}}{\delta\rho_{ab}\delta\rho_{cd}} \right]_{\rho=\rho^{(0)}} \delta\rho_{ab}\delta\rho_{cd},$$

where

$$\delta\rho_{\alpha\beta} = \delta\rho^{(1)}_{\alpha\beta} + \delta\rho^{(2)}_{\alpha\beta}.$$

Adachi and Yoshida truncated at the linear term (the RPA rearrangement terms were not reproduced)

One can reproduce the RPA terms and may obtain the new ones

Drawbacks of the SRPA model (two are general –also found in metal clusters for example- and two are generated by the choice of specific interactions)

- (Too) strong shift to lower energies with respect to the RPA spectrum

General

- Instabilities (Thouless theorem)

EDFs

-

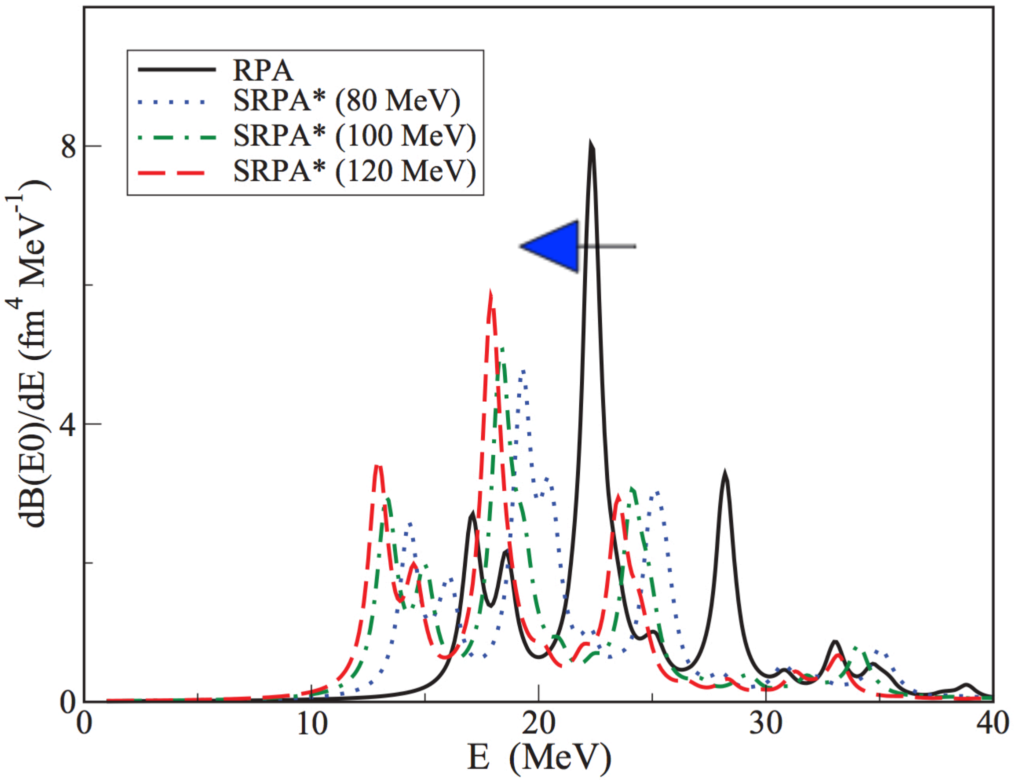
Recent studies about instabilities and double counting:

- Tselyaev, Phys. Rev. C 88, 054301 (2013)

- Papakonstantinou, Phys. Rev. C 90, 024305 (2014)

With the Gogny force (density-dependent contact term in the construction of the residual interaction) - 16O

Isoscalar monopole response. The cutoff is in 2p2h configurations (in parentheses)



Shift, double counting, and cutoff dependence

Gambacurta, Grasso, et al., Phys. Rev. C 86, 021304 (R) (2012)

EDF and double counting for extensions of RPA (Tselyaev 2013)

- **Correlations implicitly included in the functional (because of the adjustment of the parameters at mean-field level to reproduce some observables). In the spirit of DFT: 'exact' functional to be used for mean-field-type calculations**

- **Thus, this functional must produce a static RPA response function which is the 'exact' zero-energy response function.**

  The RPA static polarizability should be regarded as the 'exact' one.

- **Any modification of the response function (to go beyond the mean field) should be zero in the static limit to avoid double counting of correlations**

SRPA equations may be written as RPA-type equations with energy dependent RPA matrices

$$A_{11'}(\omega) = A_{11'} + \sum_{2,2'} A_{12}(\omega + i\eta - A_{22'})^{-1} A_{2'1'} - \sum_{2,2'} B_{12}(\omega + i\eta + A_{22'})^{-1} B_{2'1'}$$

$$B_{11'}(\omega) = B_{11'} + \sum_{2,2'} A_{12}(\omega + i\eta - A_{22'})^{-1} B_{2'1'} - \sum_{2,2'} B_{12}(\omega + i\eta + A_{22'})^{-1} A_{2'1'}$$

SRPA and RPA matrices to be diagonalized:

$$\Omega^{SRPA} \quad \begin{matrix} A_{11'}(\omega) & B_{11'}(\omega) \\ -B^*_{11'}(\omega) & -A^*_{11'}(\omega) \end{matrix} \qquad\qquad \Omega^{RPA} \quad \begin{matrix} A_{11'} & B_{11'} \\ -B^*_{11'} & -A^*_{11'} \end{matrix}$$

For cases where the interaction is density independent and A22 diagonal the expressions are simplified

$$\Omega^{SRPA} = \begin{pmatrix} A_{11'}(\omega) & B_{11'} \\ -B^{*}_{11'} & -A^{*}_{11'}(\omega) \end{pmatrix} \qquad \Omega^{RPA} = \begin{pmatrix} A_{11'} & B_{11'} \\ -B^{*}_{11'} & -A^{*}_{11'} \end{pmatrix}$$

where the energy-dependent matrix elements are

$$A_{11'}(\omega) = A_{11'} + \sum_{2} A_{12}(\omega + i\eta - A_{22'})^{-1} A_{2'1'}$$

Second-order energy-dependent self-energy insertion Σ(ω) ->
leads to a beyond mean-field model and provides the description of spreading widths and fragmentation (in addition to the single-particle Landau damping) through de coupling with 2p2h

EDF and double counting for extensions of RPA (Tselyaev)

- **Correlations implicitly included in the functional (because of the adjustment of the parameters at mean-field level to reproduce some observables). In the spirit of DFT: 'exact' functional to be used for mean-field-type calculations**

- **Thus, this functional must produce a static RPA response function which is the 'exact' zero-energy response function.**

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- **Any modification of the response function (to go beyond the mean field) should be zero in the static limit to avoid double counting of correlations**

- **It is required that the inverse energy-weighted moments are equal:**

$$\Pi^{SRPA}(0) = \Pi^{RPA}(0) = -2m_{-1}^{RPA}$$

$$\alpha^{RPA} = -\Pi(0) = 2\sum_{\nu} \frac{|<\nu|F|0>|^2}{E_\nu - E_0} = 2m_{-1}^{RPA}$$

Adachi, Lipparini, NPA 489, 445 (1988)

This is achieved by subtracting the self-energy calculated at zero energy to the energy-dependent self-energy $\Sigma(E)$ - $\Sigma(0)$ (Tselayev 2013)

Stability condition in RPA (Thouless theorem, Nucl. Phys. 21, 225 (1960), Nucl. Phys. 22, 78 (1961))

If the HF state minimizes the expectation value of the Hamiltonian -> the RPA stability matrix is positive semi-definite **(real eigenvalues and eigenvectors with positive eigenvalues have positive norm)**

Stability RPA matrix

$$S^{RPA} = M^{RPA}\Omega^{RPA} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}$$

$$M^{RPA} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This does not imply th[...]trix is also positive semi-definite.

The theorem can be extended to extensions of RPA by applying the subtraction procedure (Tselyaev 2013)

- **Double counting**

- **Instabilities (Thouless theorem)**

- **Strong shift downwards of energies (with respect to RPA) and divergences (with zero-range forces) ?**

$\Sigma(E) - \Sigma(0)$

The second-order self-energy is responsible for the divergence. The subtraction removes it.

Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

By following Tselayev 2013 ->

It is possible to rewrite the equations (after subtraction) in a non energy dependent SRPA form:

$$\mathcal{A}_F^S = \begin{pmatrix} A_{11'} + \sum_2 A_{12}(A_{22'})^{-1}A_{21'} + \sum_2 B_{12}(A_{22'})^{-1}B_{21'} & A_{12} \\ A_{21} & A_{22'} \end{pmatrix}$$

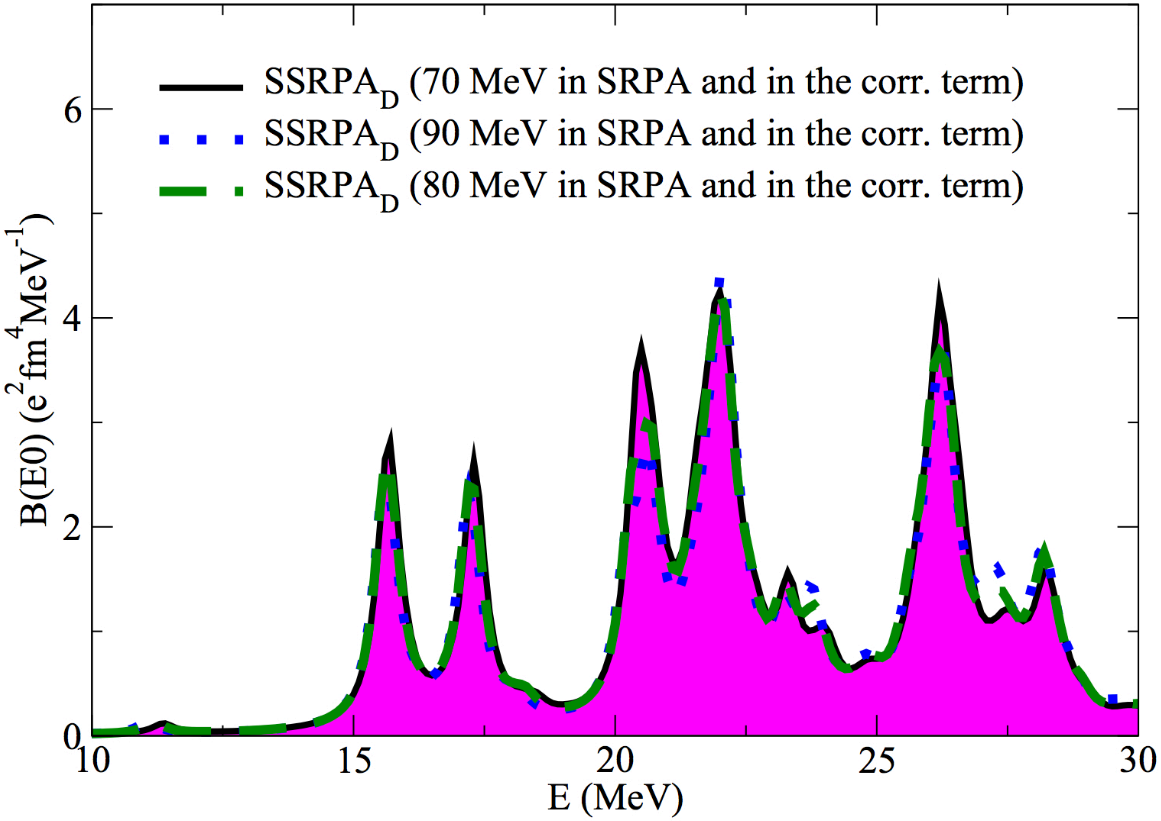
$$\mathcal{B}_F^S = \begin{pmatrix} B_{11'} + \sum_2 A_{12}(A_{22'})^{-1}B_{21'} + \sum_2 B_{12}(A_{22'})^{-1}A_{21'} & B_{12} \\ B_{21} & B_{22'} \end{pmatrix}$$

S -> subtracted
F -> full scheme (inversion of the matrix A22')

Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

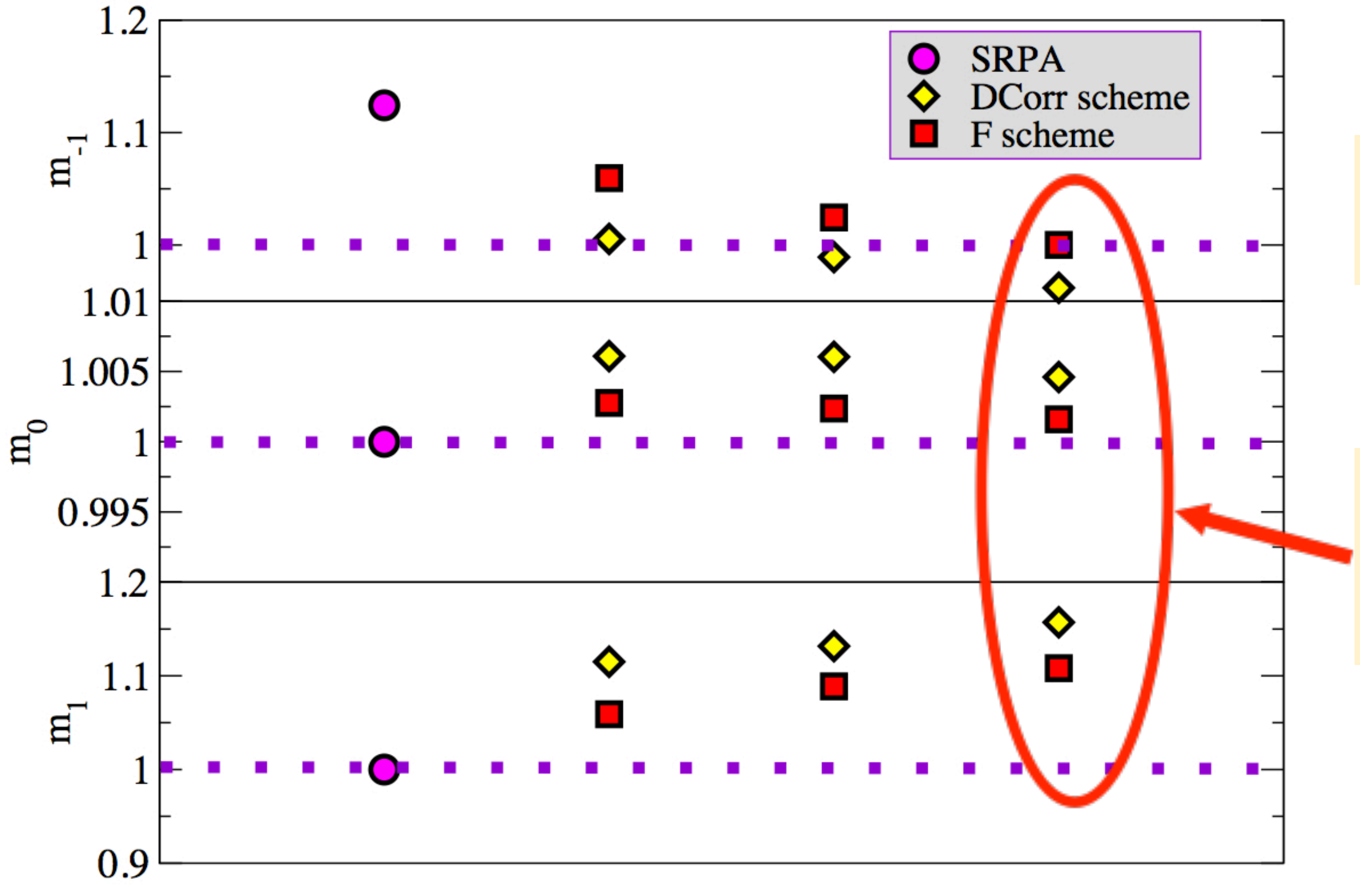
Robust prediction. No cutoff dependence ISGMR for 16O. SGII parametrization

16
O



Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

Ratios of the moments of the strength with respect to RPA



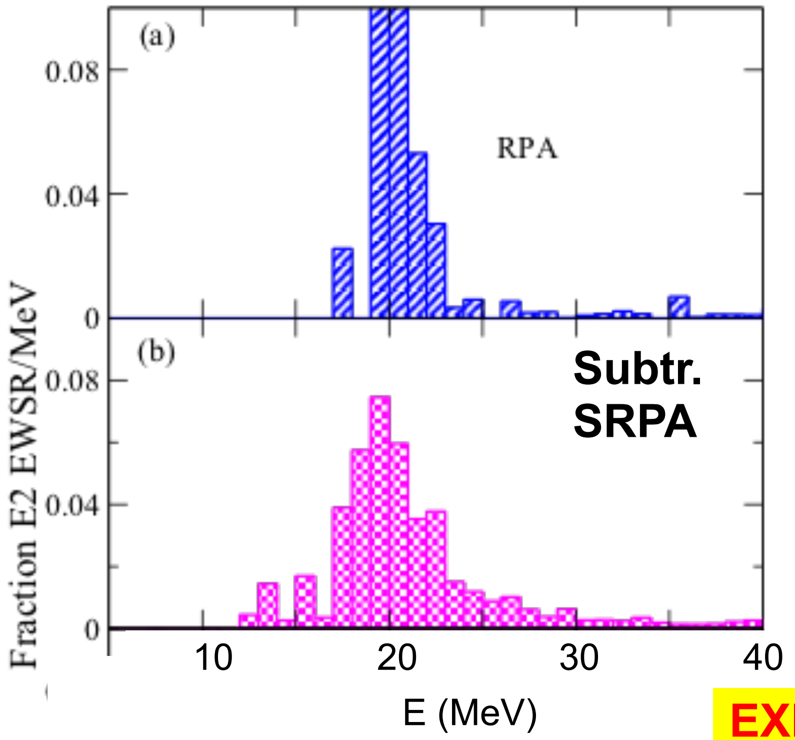
Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

Changing the cutoff in the corrective term

Same cutoff as in the 2p2h sector of the matrix

Quadrupole excitations. Spreading width (SGII)

16 O



Gambacurta, Grasso, Engel,
PRC 92, 034303 (2015)

Centroid: 20.73 MeV
Width: 2.42 MeV

Centroid: 20.21 MeV
Width: 4.05 MeV

EXP: Lui, Clark, Youngblood,
PRC 64, 064308 (2001)

Centroid: 19.76 MeV
Width: 5.11 MeV

SOME RECENT APPLICATIONS

- **Dipole excitations and dipole polarizability in 48Ca**

Gambacurta, Grasso, Vasseur, PLB 777, 163 (2018)

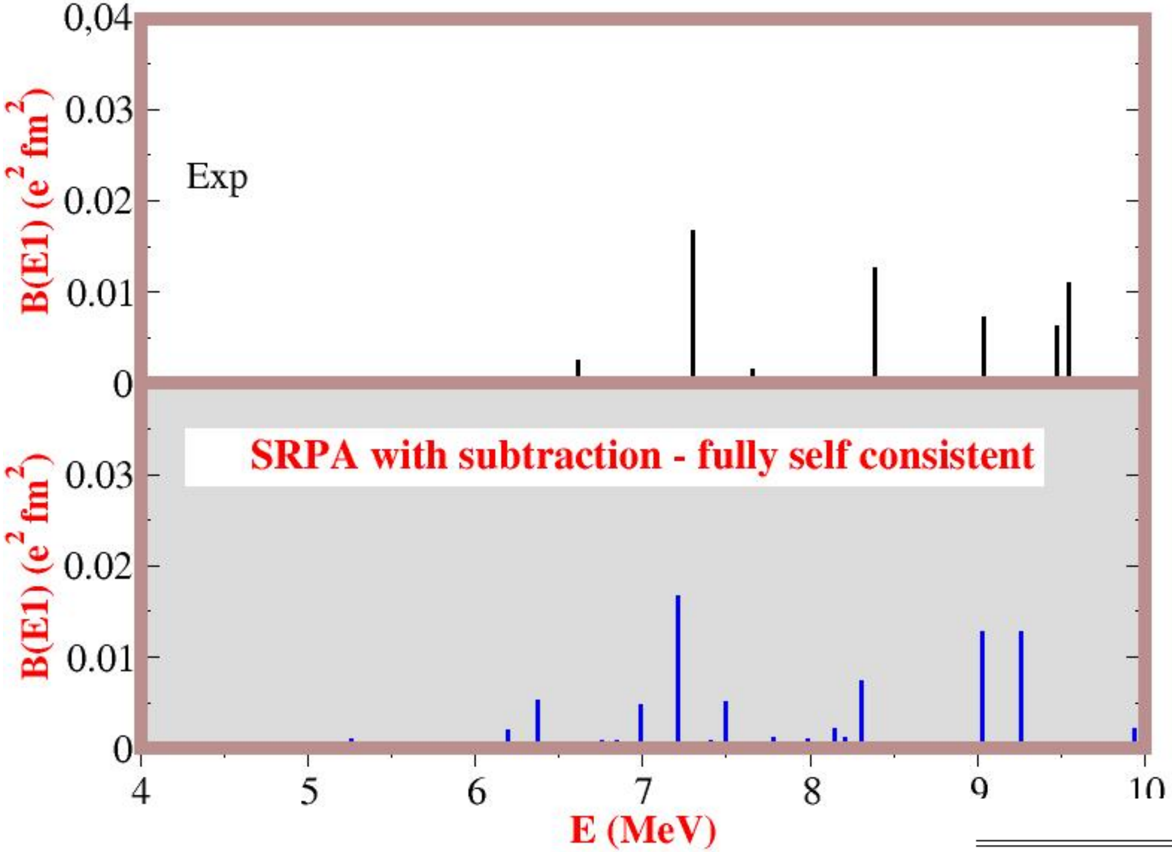
- **Systematic study of GQRs**

Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

- **Beyond-mean-field effects on effective masses**

Grasso, Gambacurta, Vasseur, PRC 98, 051303(R) (2018)

Low-lying dipole response



Dipole low-lying response in 48Ca (SGII)

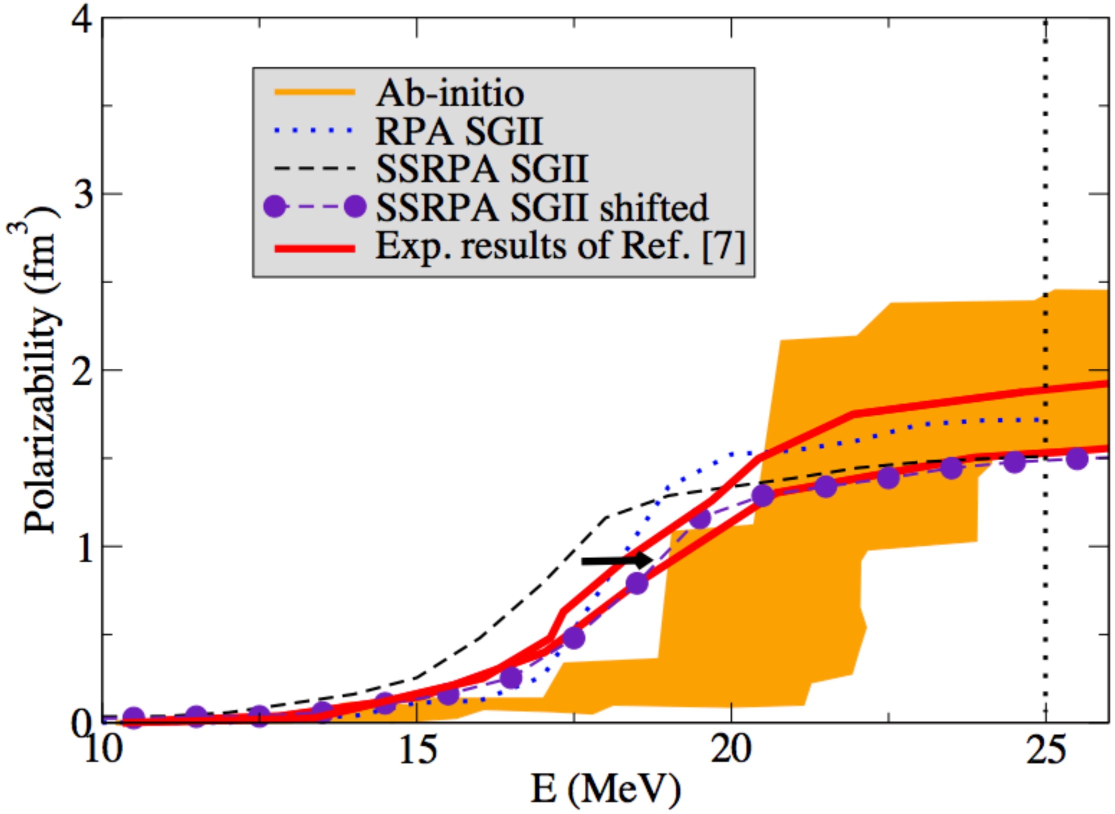
Exp: Hartmann et al., PRL 93, 192501 (2004)

(γ,γ') data at Darmstadt

	Exp	SSRPA SGII	
Centroid			
$\sum B(E1)$	0.068 $\pm$ 0.008	0.078	e2 fm2
$\sum_i E_i B_i(E1)$	0.570 $\pm$ 0.062	0.621	MeV e2 fm2

Gambacurta, Grasso, Vasseur, PLB 777, 163 (2018)

Electric dipole polarizability (important for constraining the symmetry energy -> key ingredient for predictions of neutron skin thickness, radius and proton fraction in neutron stars, …)



Gambacurta, Grasso, Vasseur, PLB 777, 163 (2018)

$$\alpha_D = \frac{8\pi}{9}\int \frac{B(E1, E_x)}{E_x} dE_x.$$

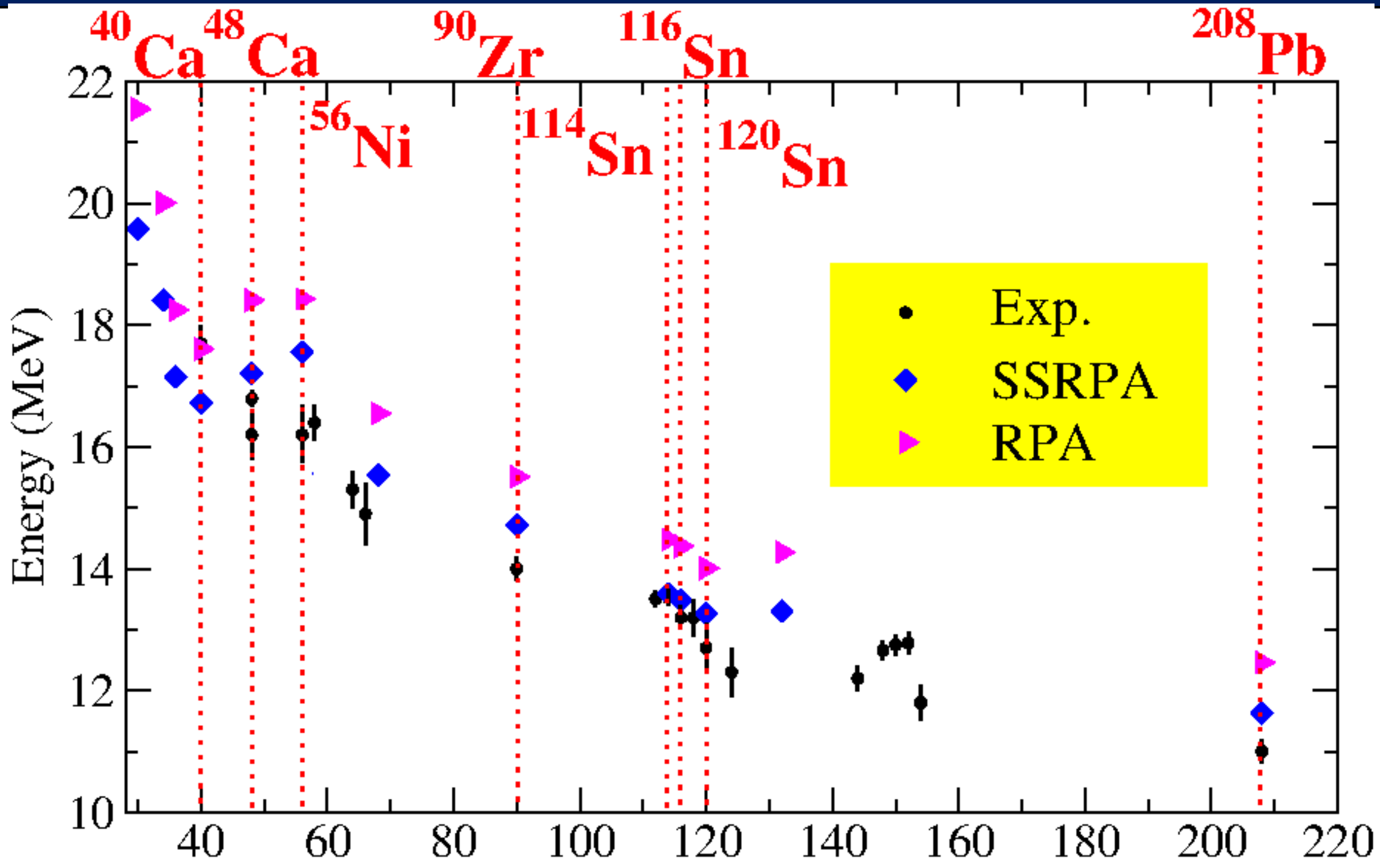
Exp: Birkhan et al., PRL 118, 252501 (2017)

(p,p') data at RCNP,Osaka

Exp centroid: 18.9 MeV
Exp width: 3.9 MeV

Shift : 1.5 MeV

Isoscalar GQRs from 30Si to 208Pb
Centroids (folding with narrow Lorentzian distributions and adjusting on it a Lorentzian distribution)

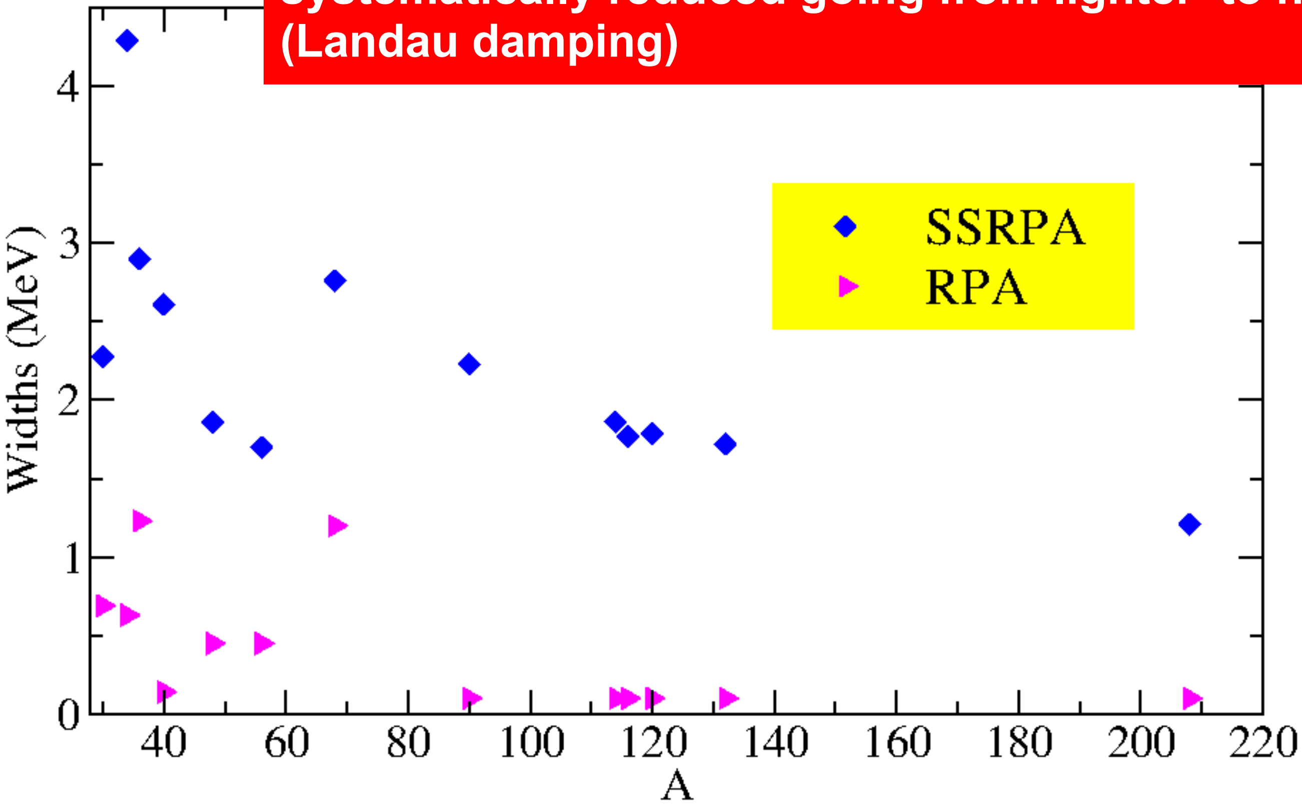


Vasseur, Gambacurta, Grasso,
PRC 98, 044313 (2018)

Globally: better agreement with the experimental data compared to RPA

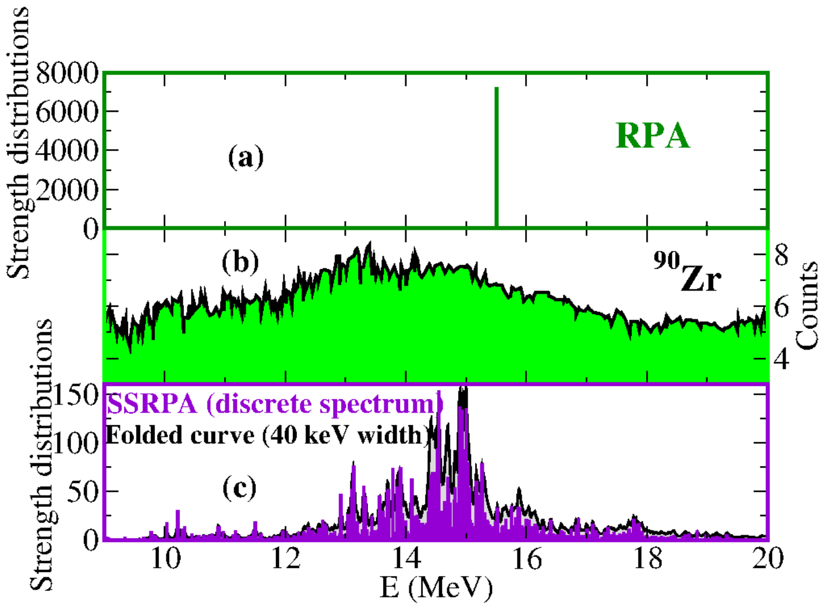
Isoscalar GQRs. Widths (folding with narrow Lorentzian distributions and adjusting on it a Lorentzian distribution)

General trend, found both in RPA and in SSRPA: the width is systematically reduced going from lighter to heavier nuclei (Landau damping)

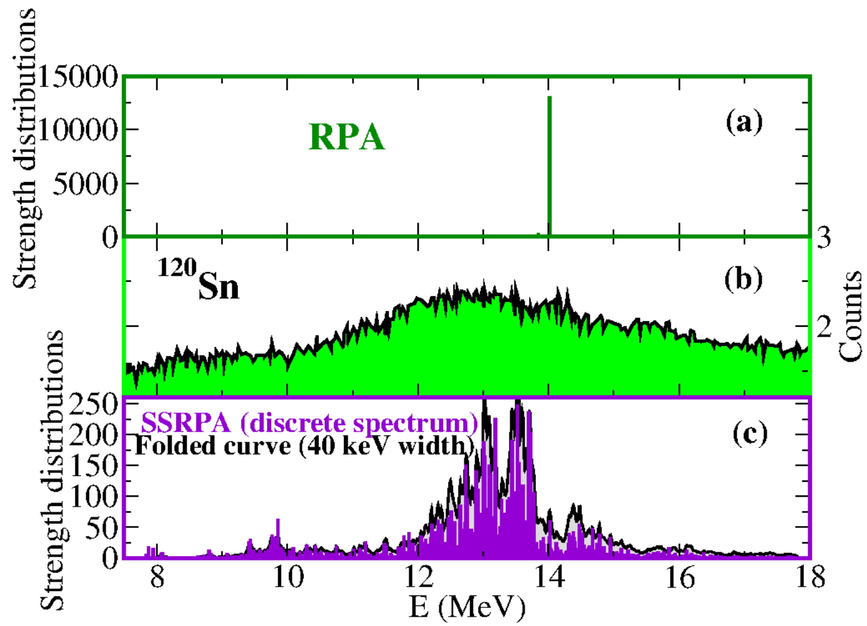


Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

High-resolution proton inelastic scattering (p,p') spectra measured at iThemba LABS



SSRPA: discrete spectra and folded spectra with a Lorentzian of width equal to 40 keV (equal to the experimental energy resolution)



Exp. data:
Shevchenko et al, PRL 93, 122501 (2004)

Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

Effective masses m*/m

- **Landau's theory of Fermi liquids: the system of interacting particles is described through quasiparticles having an effective mass m***

- **Study of m* (relevant for the properties related to the propagation of particles in a medium): broad interest in many-body physics. Impact on, for example:**

\- **Density of states in a many-body system**

\- **Specific heat of a low-temperature Fermi gas**

\- **Maximum mass of a neutron star**

\- **Energies of axial compression or breathing modes in atomic gases**

Effective masses in Fermi liquids

First dynamic measurement of the polaron effective mass

PRL **103,** 170402 (2009) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2009

Collective Oscillations of an Imbalanced Fermi Gas: Axial Compression Modes and Polaron Effective Mass

S. Nascimbène, N. Navon, K. J. Jiang, L. Tarruell,* M. Teichmann,† J. McKeever,‡ F. Chevy, and C. Salomon
Laboratoire Kastler Brossel, CNRS, UPMC, École Normale Supérieure, 24 rue Lhomond, 75231 Paris, France

The Fermi polaron is an impurity immersed in a Fermi sea (strongly imbalanced Fermi gases) *.

Based on the Landau theory of Fermi liquids, the energy spectrum of the polaron is similar to that of a free particle. Using the local-density approximation, the frequency ω* of the polaron is

$$\frac{\omega^*}{\omega} = \sqrt{\frac{1-A}{m^*/m}}.$$

ω is the frequency of the trap (harmonic oscillator), A is a dimensionless quantity that characterizes the attraction of the impurity by the other atoms

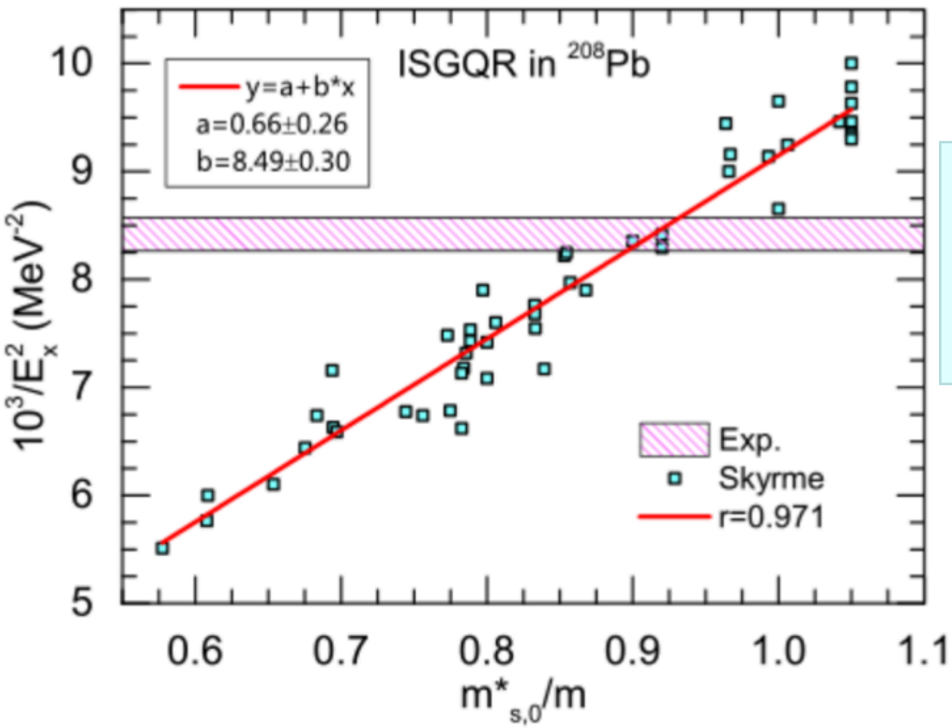
Lobo, Recati, Giorgini, Stringari, PRL 97, 200404 (2006)

* Analogous calculations for nuclear systems:
Forbes et al., PRC 89, 041301 (R) (2014)
Roggero et al., PRC 92, 054303 (2015)

Effective masses in Fermi liquids

The axial breathing mode in nuclear physics corresponds to the isoscalar GQR.

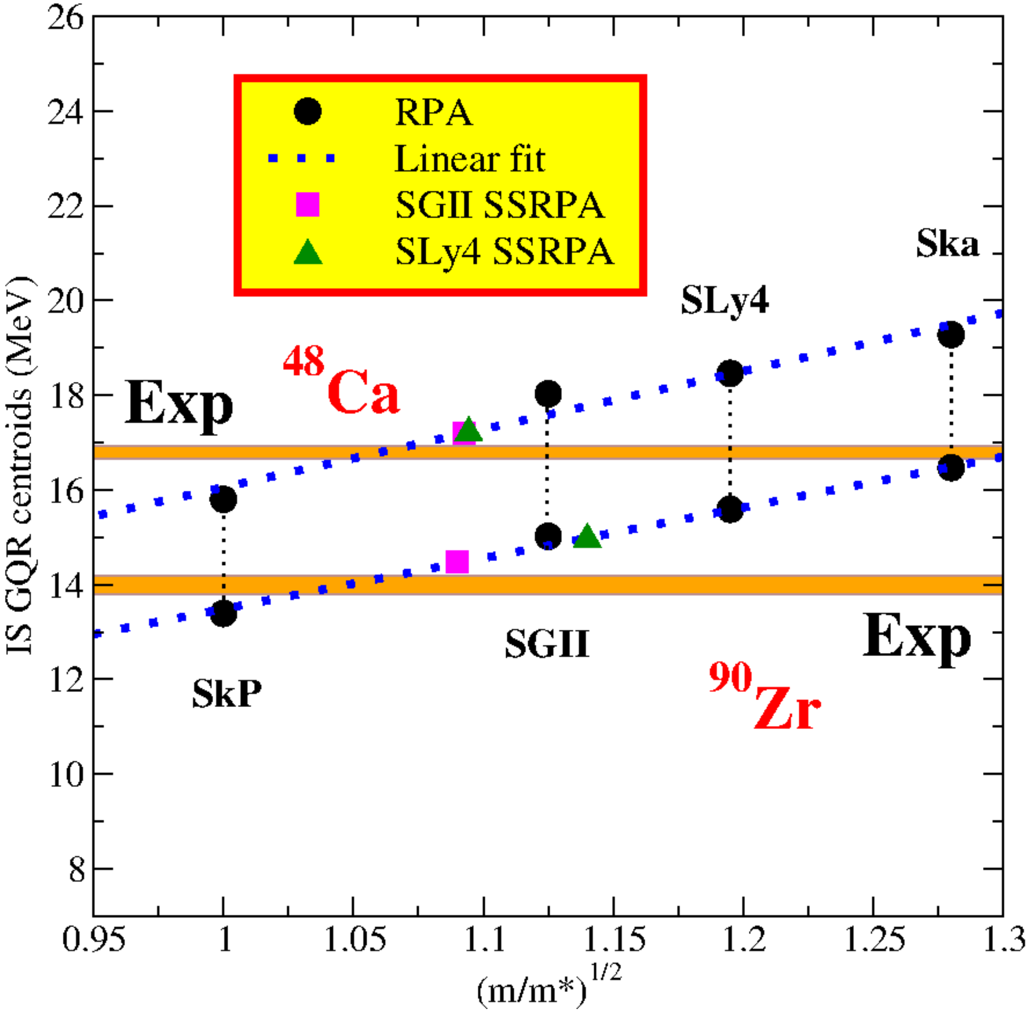
Based on the Landau theory of Fermi liquids, relation between the centroid energy of the IS GQR and (m/m*)1/2 known and used



Bao-An Li et al., Prog. Part. Nucl. Phys. 99, 29 (2018)

Blaizot, Phys. Rep. 64, 171 (1980)

Beyond-mean-field (SSRPA) effective masses in the nuclear Fermi liquid from axial breathing modes



Grasso, Gambacurta, Vasseur,
PRC 98, 051303(R) (2018)

SSRPA extraction of the effective mass

Definition of effective mass:

$$\frac{1}{m^*} = \frac{dE}{dk}\frac{1}{\hbar^2 k}$$

for a particle of energy E and momentum k, with

$$E = \frac{\hbar^2 k^2}{2m} + \Sigma_k + \Sigma_{k,E}.$$

k-mass (leading order of the Dyson equation and E-mass -> beyond mean field, energy dependence of the self-energy)

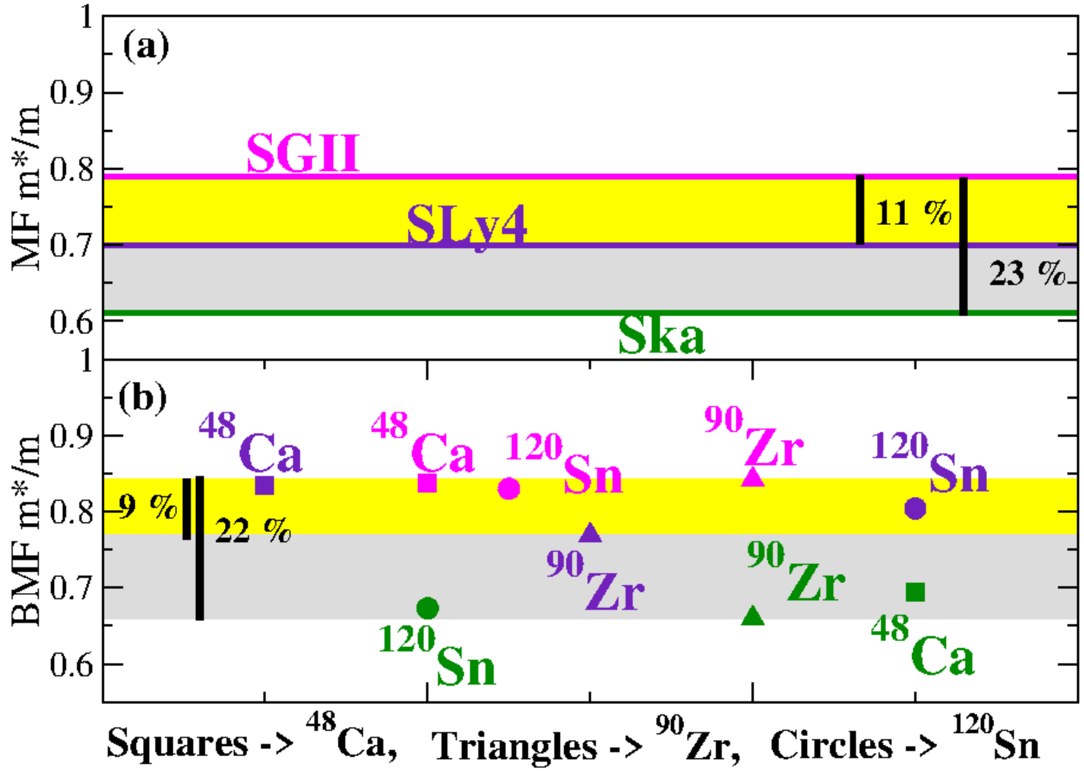
$$\frac{m^*}{m} = \left(1 - \frac{\partial \Sigma_{k,E}}{\partial E}\right) \cdot \left(1 + \frac{m}{\hbar^2 k}\frac{\partial \Sigma_k}{\partial k}\right)^{-1}$$

$$\equiv \frac{m_E^*}{m} \cdot \frac{m_k^*}{m},$$

One may extract, for each nucleus and for each interaction, an estimation of the E-mass (equal to 1 at the mean-field level).

We have found an enhancement of the E-mass between 6 and 16% with SSRPA (nucleus and interaction dependence)

Beyond-mean-field effective masses. Theoretical error



Grasso, Gambacurta, Vasseur, PRC 98, 051303(R) (2018)

Mean field -> dispersion related to the used interaction

<u>Beyond-mean-field -> in addition, nucleus dependence. However, theoretical error not larger than for the mean-field case</u>

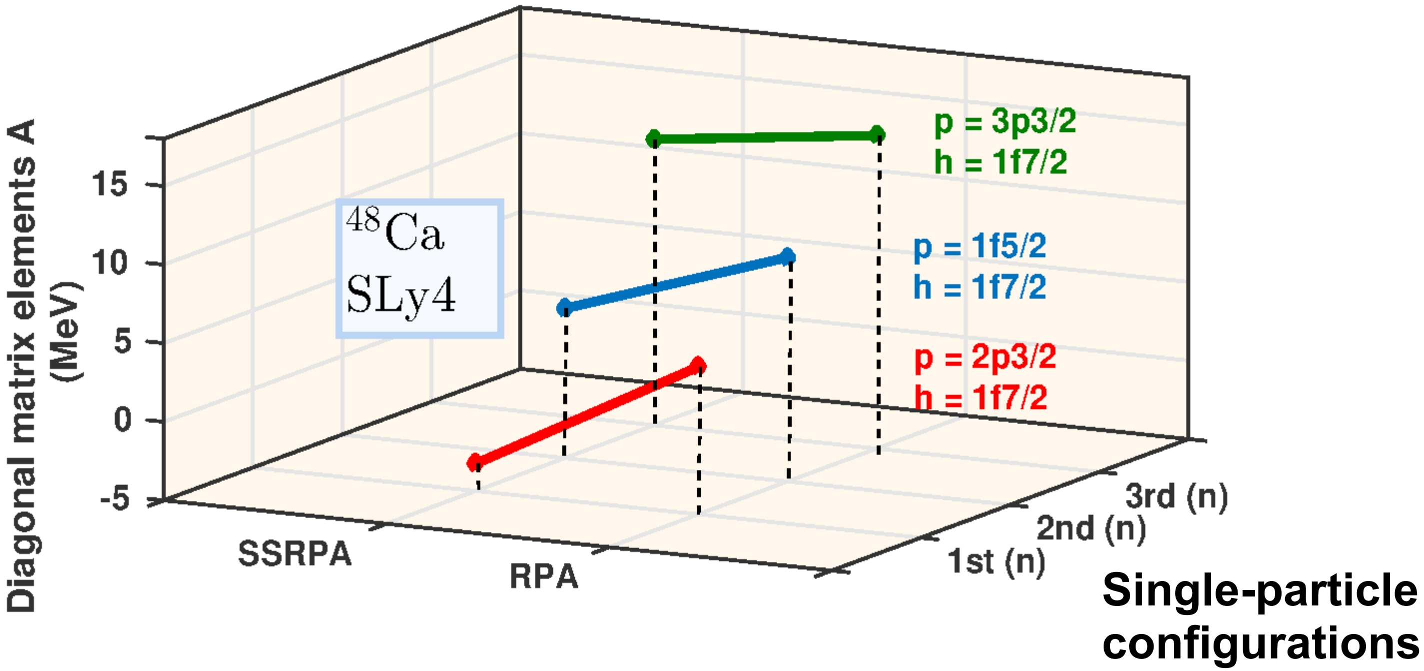
Effect on the single-particle excitation spectrum

Diagonal matrix elements of the matrix A

$$A_{1,1}^{RPA} \to A_{1,1}^{SSRPA}(E) = [\epsilon_p - \epsilon_h]_{MF} + \bar{V}_{phhp}$$
$$+ \sum_{2,2'} \frac{A_{ph,2}A_{2',ph}}{E + i\eta - A_{2,2'}} + \sum_{2,2'} \frac{A_{ph,2}A_{2',ph}}{A_{2,2'}}.$$

These matrix elements may be computed for chosen ph configurations.
For the energy-dependent self-energy, we use E=centroid of the isoscalar GQR

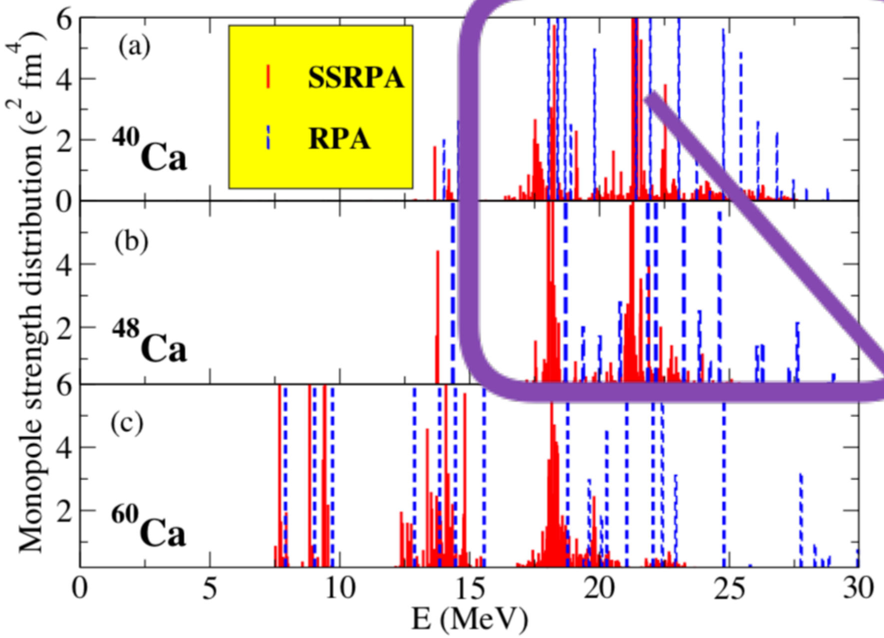
Beyond-mean-field effective masses. Effective compression of the single-particle spectrum

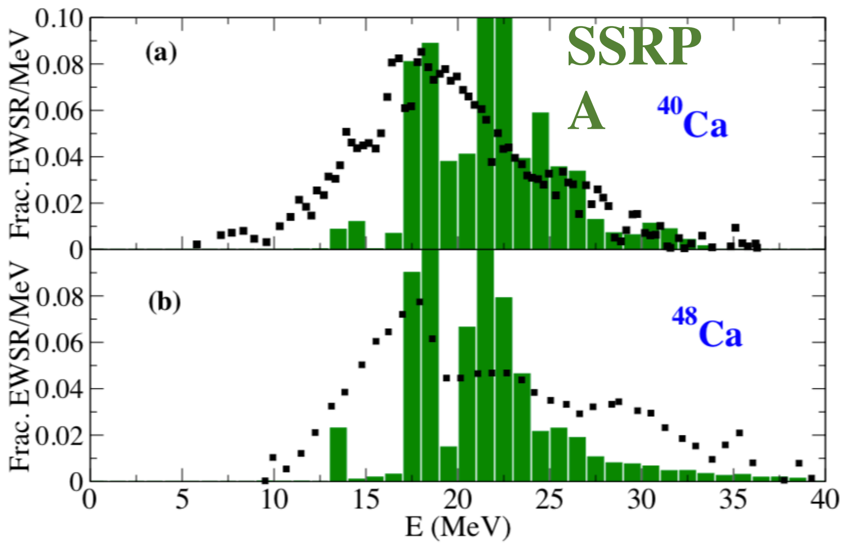


$$A^{RPA}_{1,1} \to A^{SSRPA}_{1,1}(E) = [\epsilon_p - \epsilon_h]_{MF} + \bar{V}_{phhp}$$
$$+ \sum_{2,2'} \frac{A_{ph,2}A_{2',ph}}{E + i\eta - A_{2,2'}} + \sum_{2,2'} \frac{A_{ph,2}A_{2',ph}}{A_{2,2'}}.$$

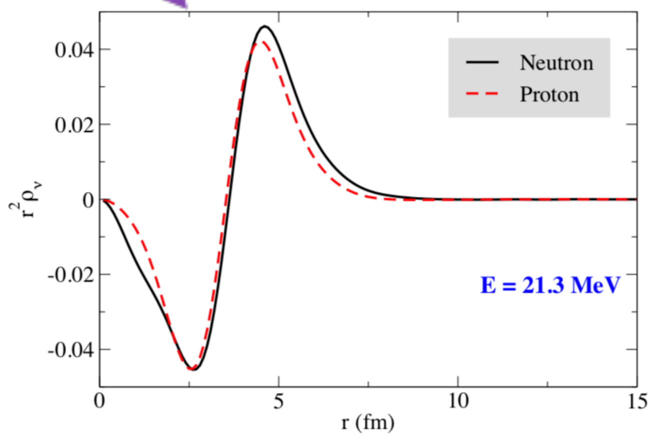
Grasso, Gambacurta, Vasseur, PRC 98, 051303(R) (2018)

Breathing modes. First check: isoscalar giant monopole in 40Ca and 48Ca





Gambacurta, Grasso, Sorlin,
in press, arXiv:1906.07977



Black points:
Exp. values: Lui et al.
PRC83, 044327 (2011)

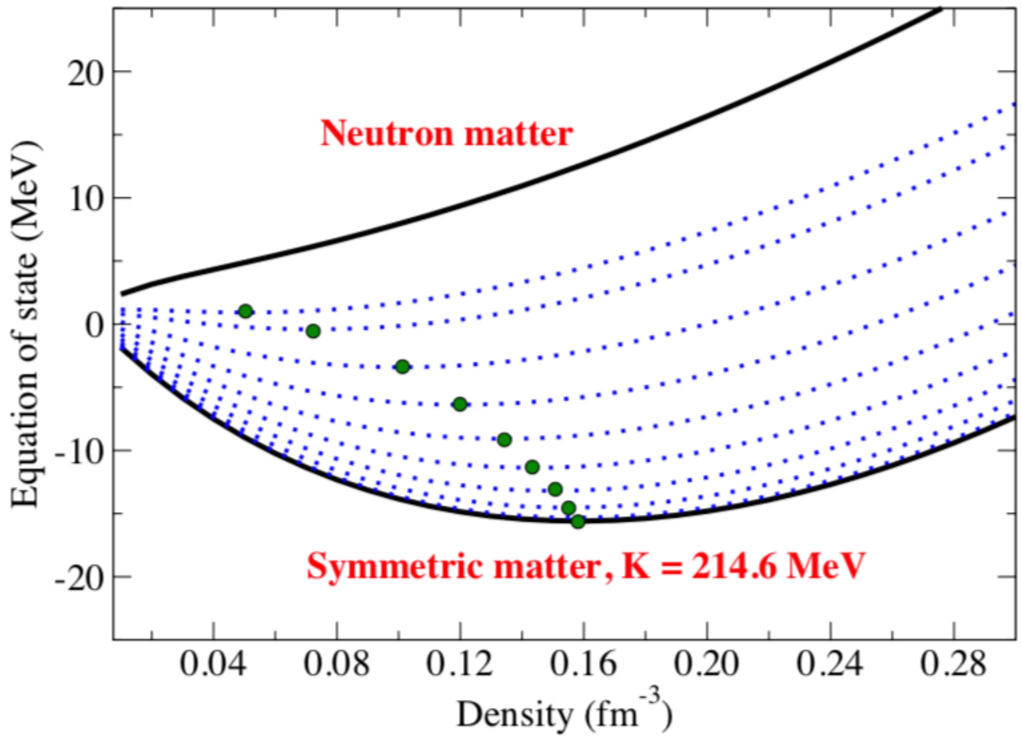
Typical transition densities for an isoscalar giant monopole resonance

Link with compressibility? Soft breathing modes in neutron-rich systems are strongly driven by neutrons

Link with asymmetric matter ?

$$K_X = 9\rho_{eq}^2 \left(\frac{\partial^2 E^X/A}{\partial \rho^2}\right)_{\rho=\rho_{eq}}$$

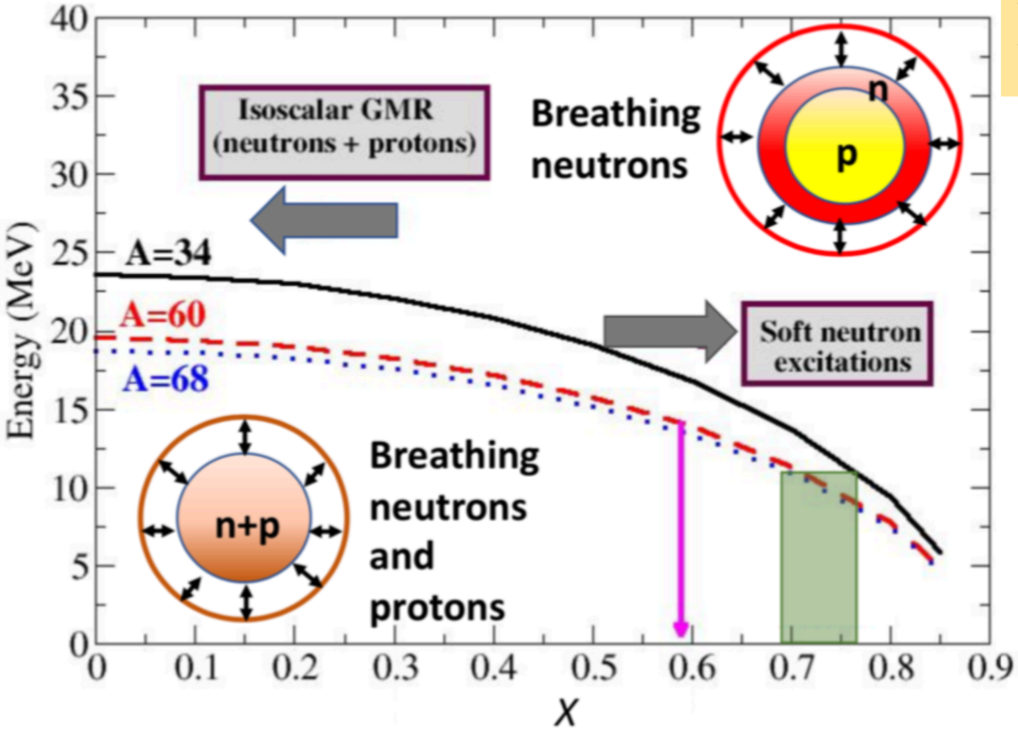
$$E(X) \sim 5.22 A^{-1/3} \sqrt{K_X}$$



X -> isospin asymmetry of asymmetric matter (and of the oscillating system)

Gambacurta, Grasso, Sorlin, in press
arXiv:1906.07977

Plotting $E(X) \sim 5.22 A^{-1/3} \sqrt{K_X}$ for a given A



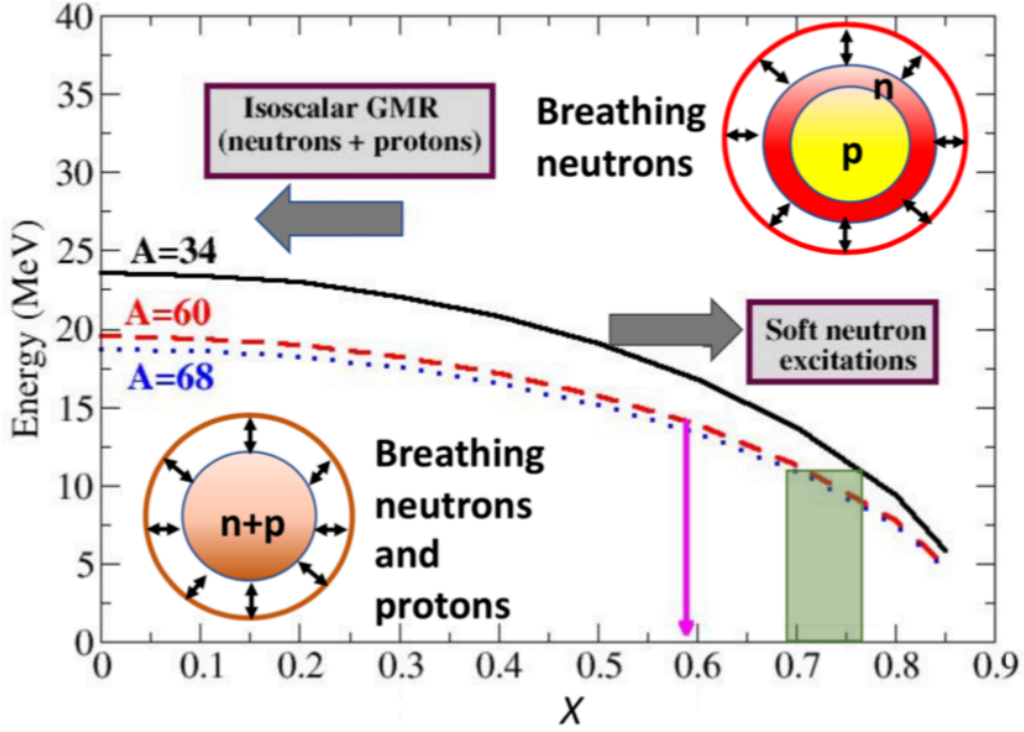
Gambacurta, Grasso, Sorlin, in press
arXiv:1906.07977

Estimating the isospin asymmetry of the oscillating system

Green area: range of isospin asymmmetry for the oscillating system involved in the excitations at 8.8 (60Ca), 11.1 (34Si), 11.0 (68Ni) MeV.

Magenta arrow: same, for the excitation at 14.1 MeV in 60Ca.

Isospin asymmetry of the oscillating system



Gambacurta, Grasso, Sorlin, in press
arXiv:1906.07977

Using the transition densities computed with SSRPA (estimation with only 1p1h contribution in the transition density)

$$X_N = 4\pi \int |\rho_\nu^n| r^2 dr$$

$$X_P = 4\pi \int |\rho_\nu^p| r^2 dr$$

$$X = \frac{X_N - X_P}{X_N + X_P}$$

Nucleus	E (MeV)	X
^{34}Si	11.07	0.73
^{68}Ni	11.02	0.78
^{60}Ca	8.8	0.84
^{60}Ca	14.1	0.73

Summary

- **Implementation of the SRPA model by a subtraction procedure:**
double counting, stability condition (correction of the shift with respect to the RPA), convergence with respect to the cutoff

- **Some recent applications:**

◆ **Dipole excitations in 48Ca (width of the GDR: slope of the dipole polarizability. Low-lying modes)**

◆ **Systematic study of GQRs (compared to RPA: centroids globally in better agreement with the experimental data; enhancement of the widths owing to the description of the spreading width)**

- **Beyond-mean-field effect on the effective mass (extraction of an enhanced effective mass produced by beyond-mean-field effects)**

◆ **Low-lying breathing modes in neutron-rich nuclei (neutron-driven modes. Link with the compressibility defined for asymmetric infinite matter)**

Two examples of effective phenomenological interactions around 10 parameters adjusted with mean-field calculations

$$V(\mathbf{r}_1,\mathbf{r}_2) =$$
$$t_0\,(1+x_0P_\sigma)\,\delta(\mathbf{r})$$
$$+\frac{1}{2}t_1\,(1+x_1P_\sigma)\left[\mathbf{P}'^2\delta(\mathbf{r})+\delta(\mathbf{r})\mathbf{P}^2\right]+t_2\,(1+x_2P_\sigma)\;\mathbf{P}'\cdot\delta(\mathbf{r})\mathbf{P}$$
$$+\frac{1}{6}t_3\,(1+x_3P_\sigma)\,[\rho\,(\mathbf{R})]^\alpha\,\delta(\mathbf{r})$$
$$+\mathrm{i}W_0\,\boldsymbol{\sigma}\cdot\left[\mathbf{P}'\times\delta(\mathbf{r})\,\mathbf{P}\right]$$

Skyrme
All terms zero range.
Density dependence

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad \mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2),\quad \mathbf{P}=\frac{1}{2\mathrm{i}}(\nabla_1-\nabla_2)\qquad \boldsymbol{\sigma}=\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2,\quad P_\sigma=\frac{1}{2}(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$$

$$V(\mathbf{r}_1,\mathbf{r}_2)=\sum_{i=1,2}[W_i+B_iP_\sigma-H_iP_\tau-M_iP_\sigma P_\tau]\,\mathrm{e}^{-r^2/\mu_i^2}$$
$$+t_3\,(1+x_3P_\sigma)\,[\rho\,(\mathbf{R})]^\alpha\,\delta(\mathbf{r})$$
$$+\mathrm{i}W_0\,\boldsymbol{\sigma}\cdot\left[\mathbf{P}'\times\delta(\mathbf{r})\,\mathbf{P}\right]$$

Gogny
Two gaussians plus
zero-range density-dependent
term and spin-orbit term