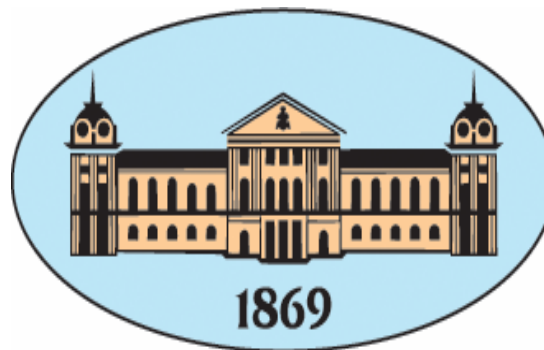
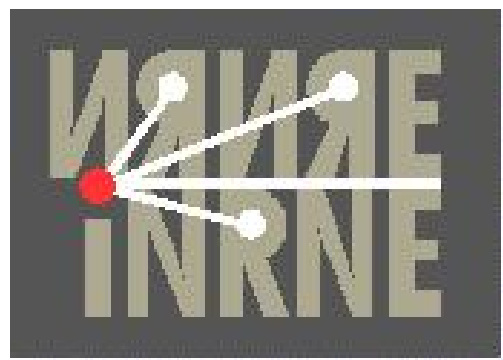


Examples of Nucleon Correlations Effects on Nuclear Structure and Reactions

Anton N. Antonov

*Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia 1784, Bulgaria*



**APCTP Focus Program in Nuclear Physics 2019,
Pohang, Republic of Korea**

Some Examples:

- Nucleon momentum distributions
- Spectral functions
- Natural orbitals
- Overlap functions (one- and two- body)
- (p,d) , $(e,e'p)$, $(e,e'pp)$ reactions
- Exotic nuclei (structure)
- Exotic nuclei (processes)
- Superscaling in electron- and neutrino(antineutrino)–nuclei scattering
- Information on the nucleon momentum distribution from the scaling function

Theoretical Correlation Methods Used:

- The Coherent Density Fluctuation Model (CDFM) [Sofia, 1979-till now]

based on the delta-function approximation for the overlap and energy kernels of the Generator Coordinate Method

- The Generator Coordinate Method
- The Jastrow Correlation Method
- The Natural Orbital and Overlap Functions Representations
- The Nuclear Density Functional Theory

...and others

Coherent Density Fluctuation Model (CDFM)

- A.N. Antonov, I.Zh. Petkov, V. Nikolaev, P.E. Hodgson (1979, 1980, 1982, 1985, 1988, 1993, ...)

$$\rho(\mathbf{r}, \mathbf{r}') = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_x(\mathbf{r}, \mathbf{r}') \quad (1)$$

$$\rho_x(\mathbf{r}, \mathbf{r}') = 3\rho_0(x) \frac{j_1(k_F(x)|\mathbf{r} - \mathbf{r}'|)}{(k_F(x)|\mathbf{r} - \mathbf{r}'|)} \Theta\left(x - \frac{|\mathbf{r} + \mathbf{r}'|}{2}\right) \quad (2)$$

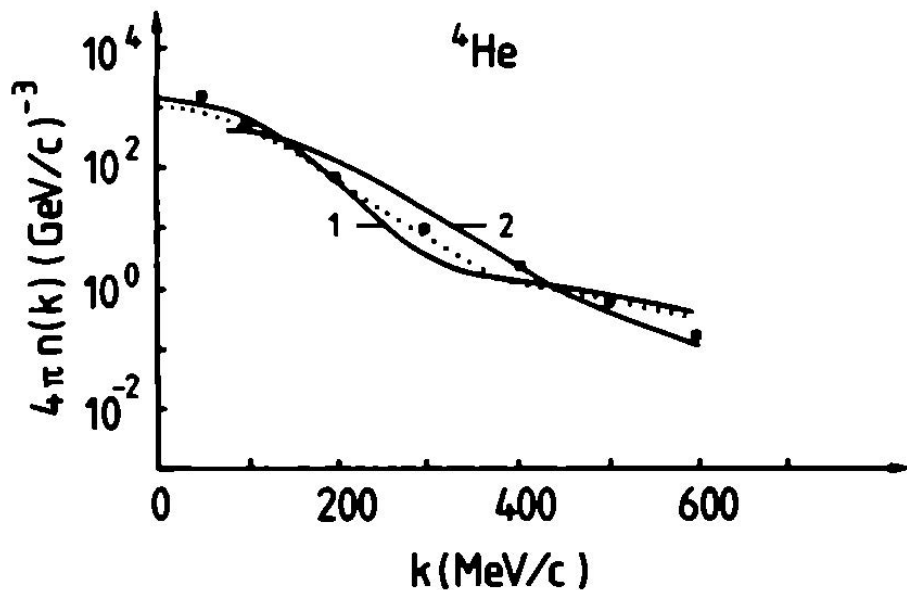
$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\beta}{x}; \quad \rho_0(x) = \frac{3A}{4\pi x^3} \quad (3)$$

$$\beta = \left(\frac{9\pi A}{8}\right)^{1/3} \simeq 1.52A^{1/3} \quad (4)$$

$$\rho(\mathbf{r}) = \int_0^\infty dx |\mathcal{F}(x)|^2 \rho_0(x) \Theta(x - |\mathbf{r}|) \quad (5)$$

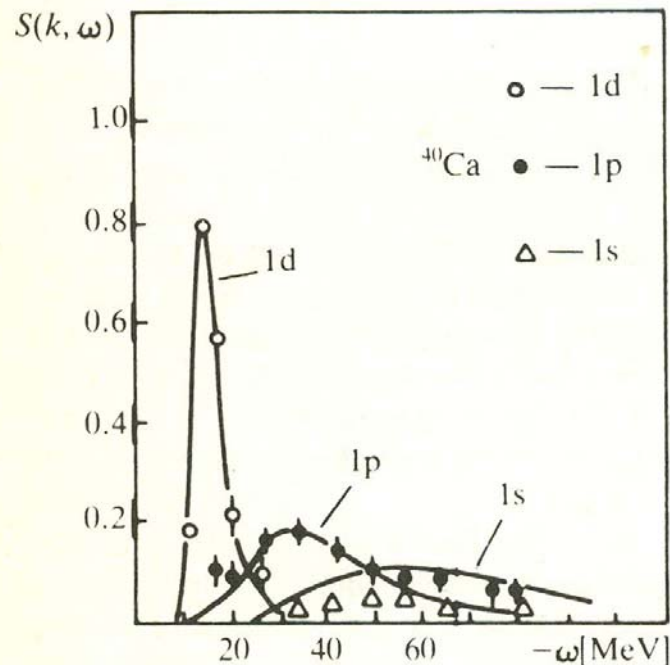
$$|\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)} \left. \frac{d\rho(r)}{dr} \right|_{r=x}; \quad \left(\frac{d\rho}{dr} \leq 0\right); \quad \int_0^\infty dx |\mathcal{F}(x)|^2 = 1 \quad (6)$$

$$n(\mathbf{k}) = \int_0^\infty dx |\mathcal{F}(x)|^2 \frac{4}{3}\pi x^3 \Theta(k_F(x) - |\mathbf{k}|) \quad (7)$$



Nucleon momentum distribution for ${}^4\text{He}$: the black squares are the exp. data, the exp (S)-method (dotted line), the correlation method of Akaishi (curve 1) and the CDFM (curve 2).
 Normalization: $\int n(\mathbf{k})d\mathbf{k}=1$

Bulg. J. Phys. **13**, 110 (1986)



Spectral functions for ${}^{40}\text{Ca}$ in CDFM

Z. Phys. A **304**, 239 (1982)

Natural Orbitals

- Löwdin (1955)

$$\rho(\mathbf{r}, \mathbf{r}') = \sum_{\alpha} N_{\alpha} \psi_{\alpha}^{*}(\mathbf{r}) \psi_{\alpha}(\mathbf{r}') \quad (1)$$

$$0 \leq N_{\alpha} \leq 1, \quad \sum_{\alpha} N_{\alpha} = A \quad (2)$$

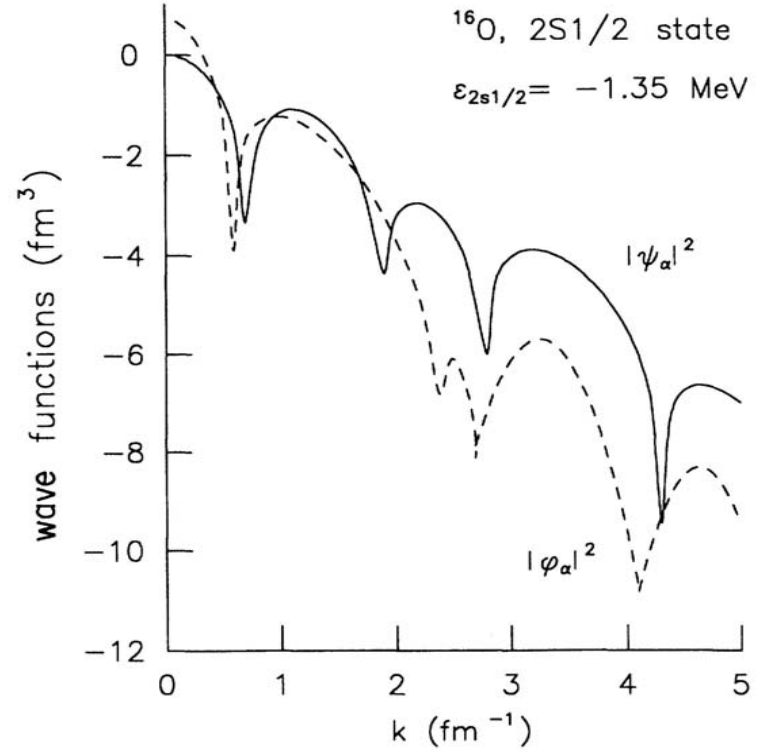
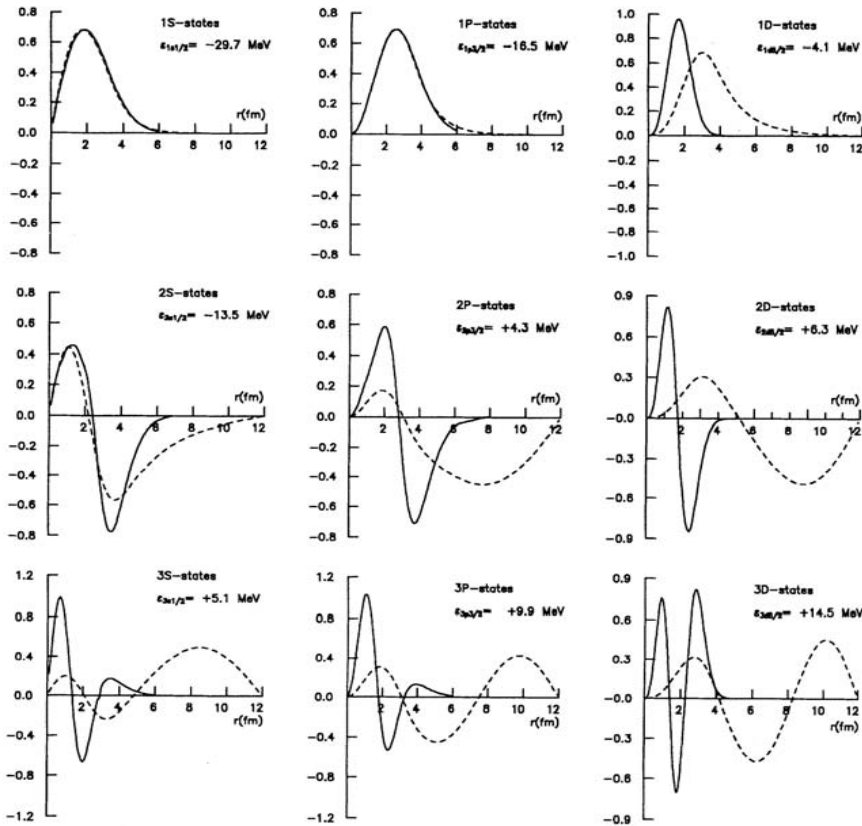
$$\rho(\mathbf{r}) = \sum_{\alpha} N_{\alpha} |\psi_{\alpha}(\mathbf{r})|^2 \quad (3)$$

$$n(\mathbf{k}) = \sum_{\alpha} N_{\alpha} |\psi_{\alpha}(\mathbf{k})|^2 \quad (4)$$

$\{\psi_{\alpha}(\mathbf{r})\}$: complete orthonormal set

$$\int \rho(\mathbf{r}, \mathbf{r}') \psi_{\alpha}(\mathbf{r}') d\mathbf{r}' = N_{\alpha} \psi_{\alpha}(\mathbf{r}) \quad (5)$$

$$\int \rho(\mathbf{k}, \mathbf{k}') \psi_{\alpha}(\mathbf{k}') d\mathbf{k}' = N_{\alpha} \psi_{\alpha}(\mathbf{k}) \quad (6)$$



Phys. Rev. C **48**, 74 (1993)

Overlap Functions

- One-body overlap functions

$$\phi_\alpha(\mathbf{r}) = \langle \Psi_\alpha^{(A-1)} | a(\mathbf{r}) | \Psi^{(A)} \rangle \quad (1)$$

Spectroscopic factor:

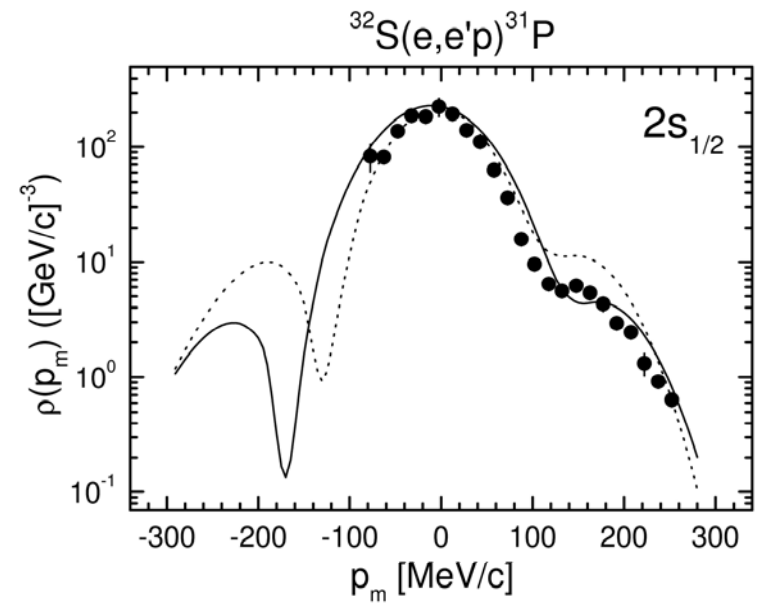
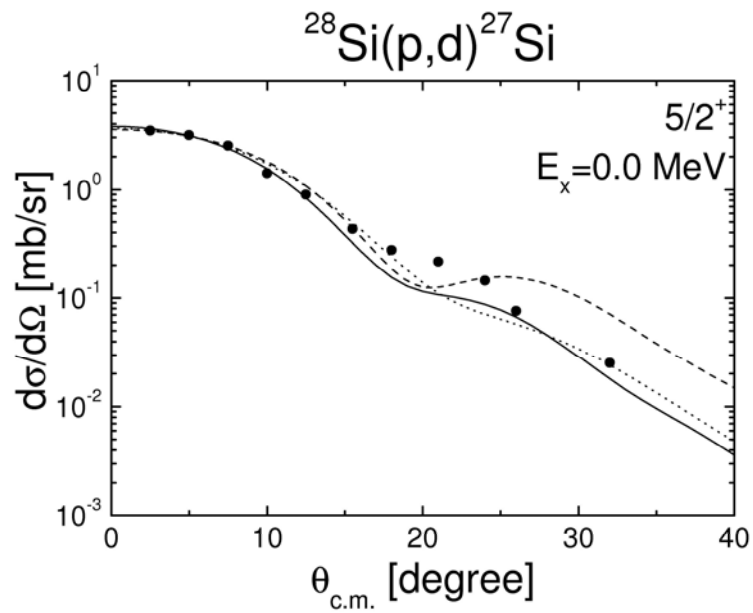
$$S_\alpha = \langle \phi_\alpha | \phi_\alpha \rangle \quad (2)$$

$$\tilde{\phi}_\alpha(\mathbf{r}) = S_\alpha^{-1/2} \phi_\alpha(\mathbf{r}) \quad (3)$$

$$\rho(\mathbf{r}, \mathbf{r}') = \sum_\alpha \phi_\alpha^*(\mathbf{r}) \phi_\alpha(\mathbf{r}') = \sum_\alpha S_\alpha \tilde{\phi}_\alpha^*(\mathbf{r}) \tilde{\phi}_\alpha(\mathbf{r}') \quad (4)$$

D. Van Neck *et al.*, Phys. Lett. B **314**, 255 (1993):

$$\phi_{n_0 l j}(r) = \frac{\rho_{l j}(r, a)}{C_{n_0 l j} \exp(-k_{n_0 l j} a) / a} \quad (5)$$



Phys. Rev. C **66**, 064308 (2002)

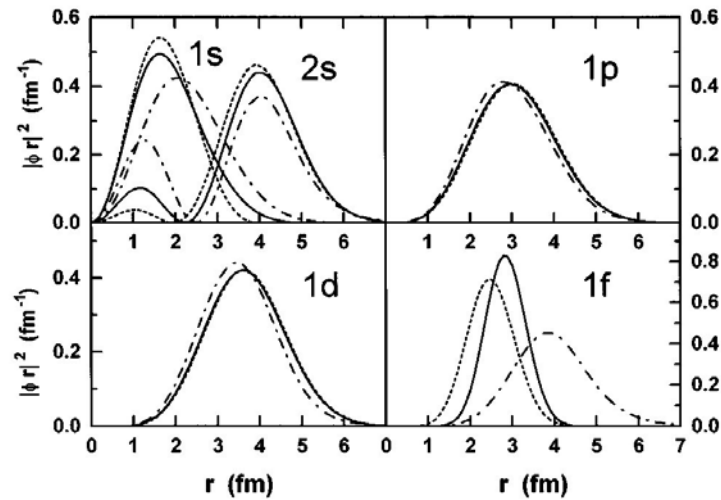


FIG. 1. Overlap functions (solid line), self-consistent Hartree-Fock single-particle wave functions (dot-dashed line), and natural orbitals (dashed line) for the nucleus ^{40}Ca .

Phys. Rev. C **53**, 1254 (1996)

$$\Phi_{\alpha_0 J S L L_R}(r, R) = \frac{\rho_{J S L L_R}^{(2)}(r, R; a, b)}{\Phi_{\alpha_0 J S L L_R}(a, b)} \quad (25)$$

$$= \frac{\rho_{J S L L_R}^{(2)}(r, R; a, b)}{N \exp\{-k \sqrt{[b^2 + (1/4)a^2]}\} [b^2 + (1/4)a^2]^{-5/2}} \quad (26)$$

Phys. Rev. C **59**, 722 (1999)

Removal of 1S_0 and 3P_1 (pp) pairs from $^{16}\text{O}(e, e'pp)^{14}\text{C}_{\text{g.s.}}$

Partial waves: $2S+1l_L$; $\vec{L} = \vec{l} + \vec{L}_R$

Phys. Rev. C **68**, 014617 (2003)

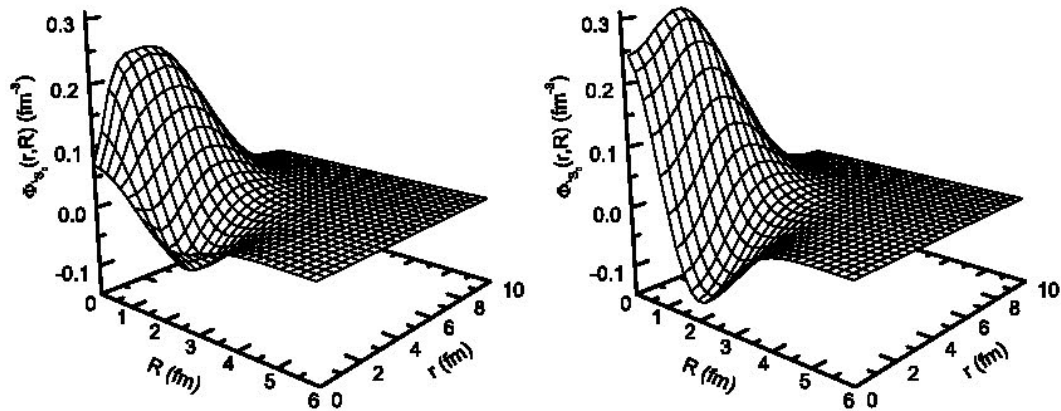


FIG. 1. The 1S_0 two-proton overlap functions for the nucleus ^{16}O leading to the 0^+ ground state of ^{14}C extracted from the JCM (left) and uncorrelated (right) two-body density matrices.

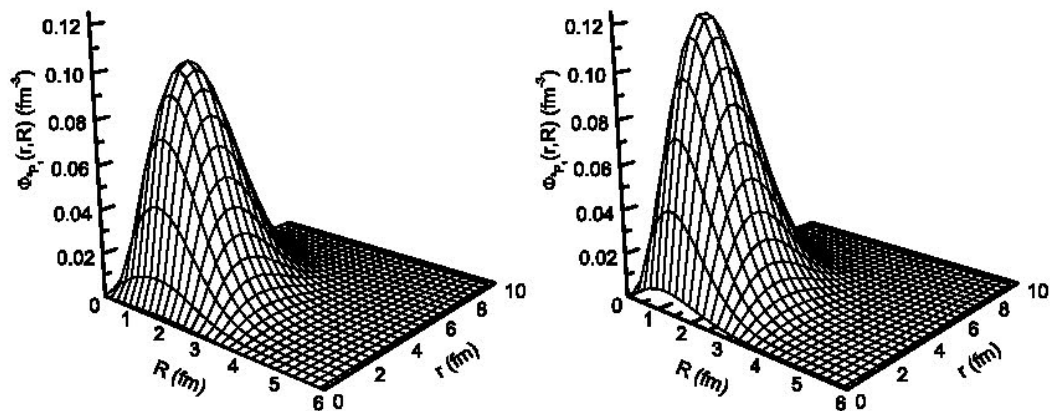
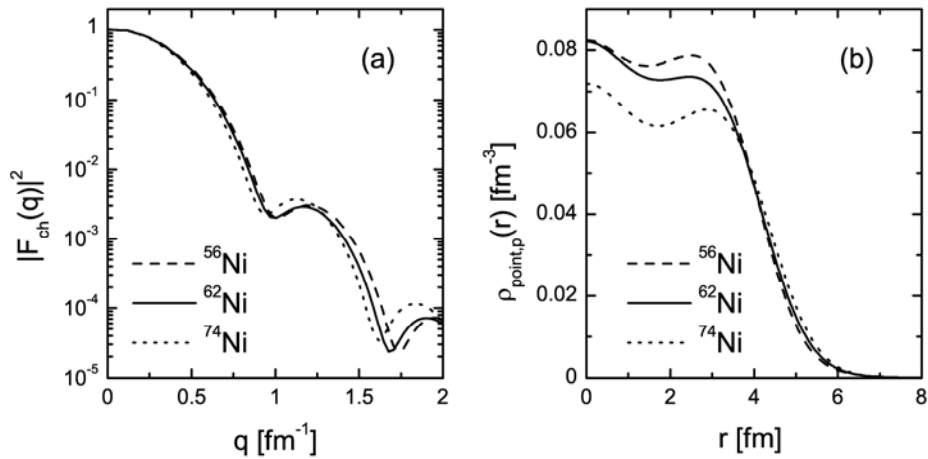
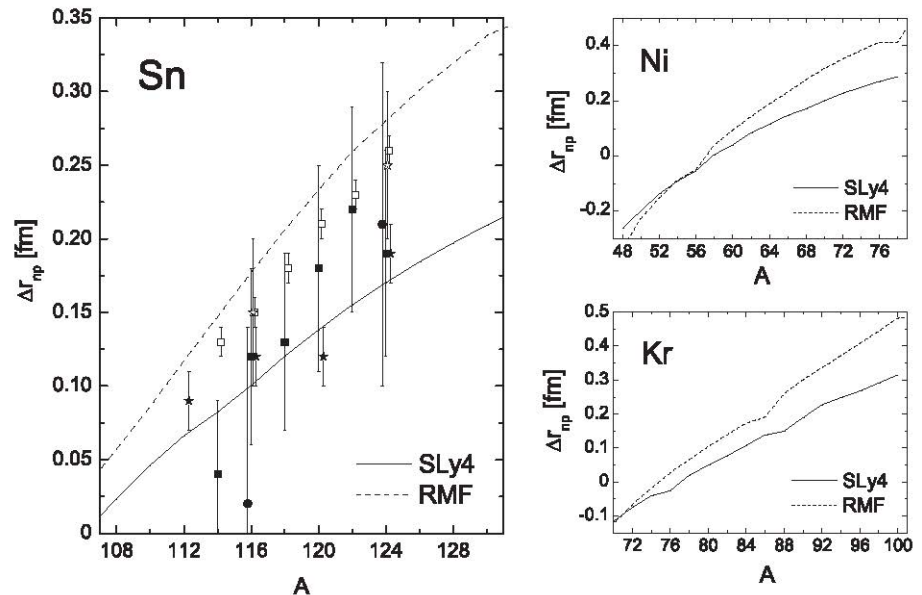


FIG. 2. The 3P_1 two-proton overlap functions for the nucleus ^{16}O leading to the 0^+ ground state of ^{14}C extracted from the JCM (left) and uncorrelated (right) two-body density matrices.

Exotic Nuclei (Structure)



Phys. Rev C **72**, 044307 (2005)



Phys. Rev. C **76**, 044322 (2007)

Exotic Nuclei (processes)

Microscopic optical potential; elastic scattering; breakup reactions

$$U_{opt}(r) = N_R V^F(r) + i N_I W^H(r) \quad (1)$$

1. **Direct and exchange parts of the real OP (ReOP)**

Folding:

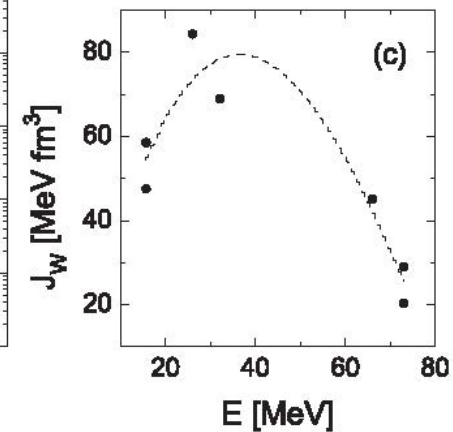
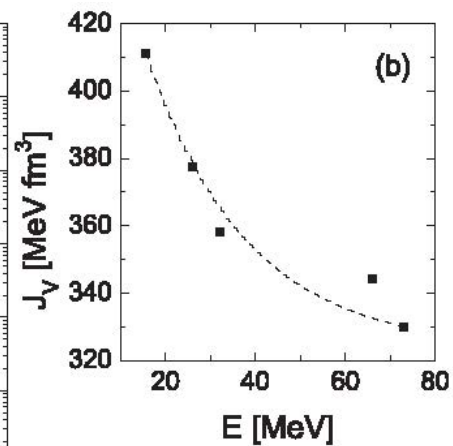
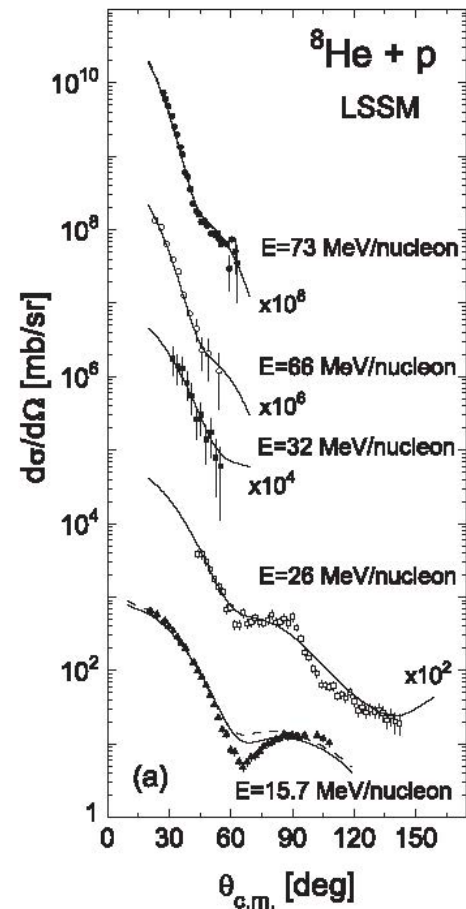
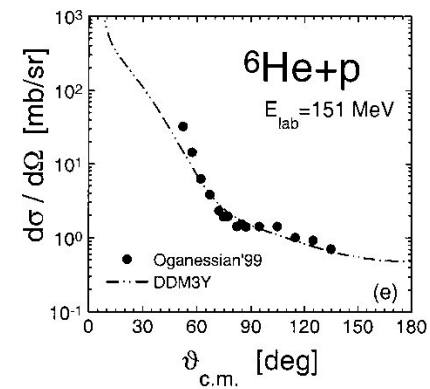
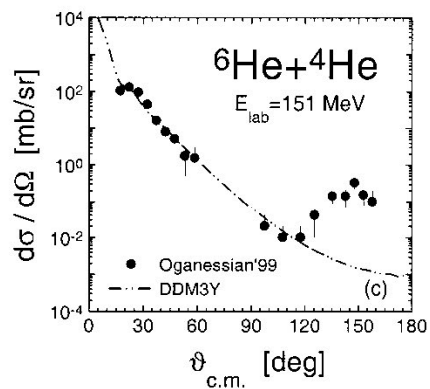
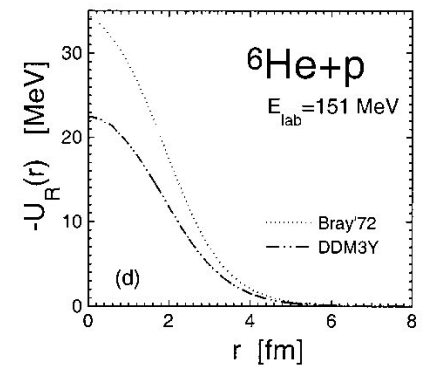
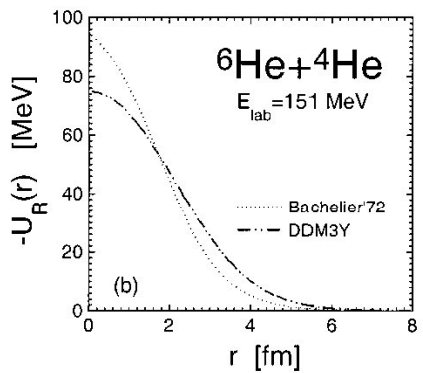
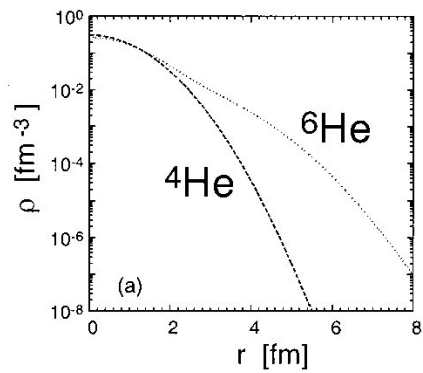
$$V^F(r) = V^D(r) + V^{EX}(r) \quad (2)$$

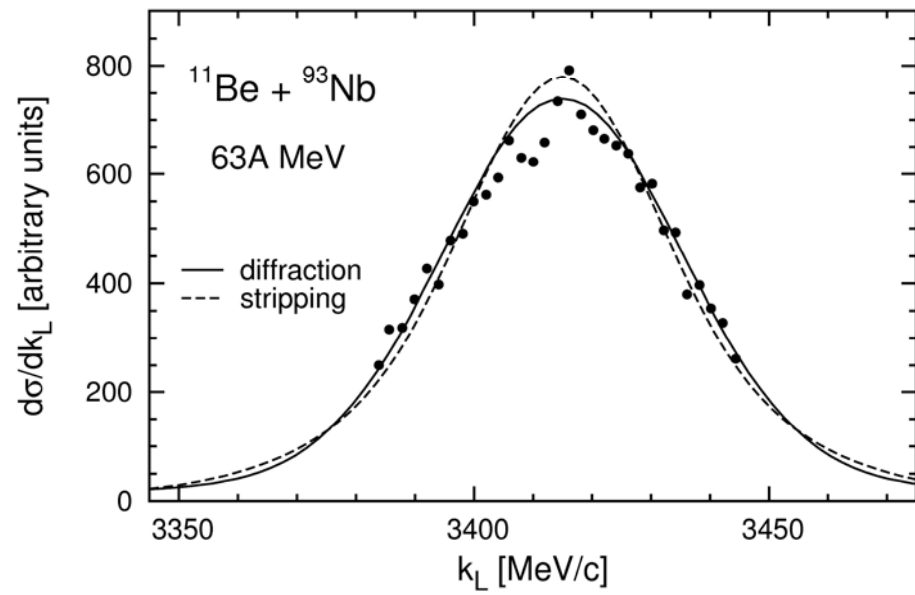
$$V_{IS}^D, V_{IV}^D, V_{IS}^{EX}, V_{IV}^{EX}$$

$v_{(00)(01)}^D(\rho, E), v_{(00)(01)}^{EX}(\rho, E)$ – M3Y effective interactions

2. **Imaginary part of the OP (ImOP) within the high-energy approximation**

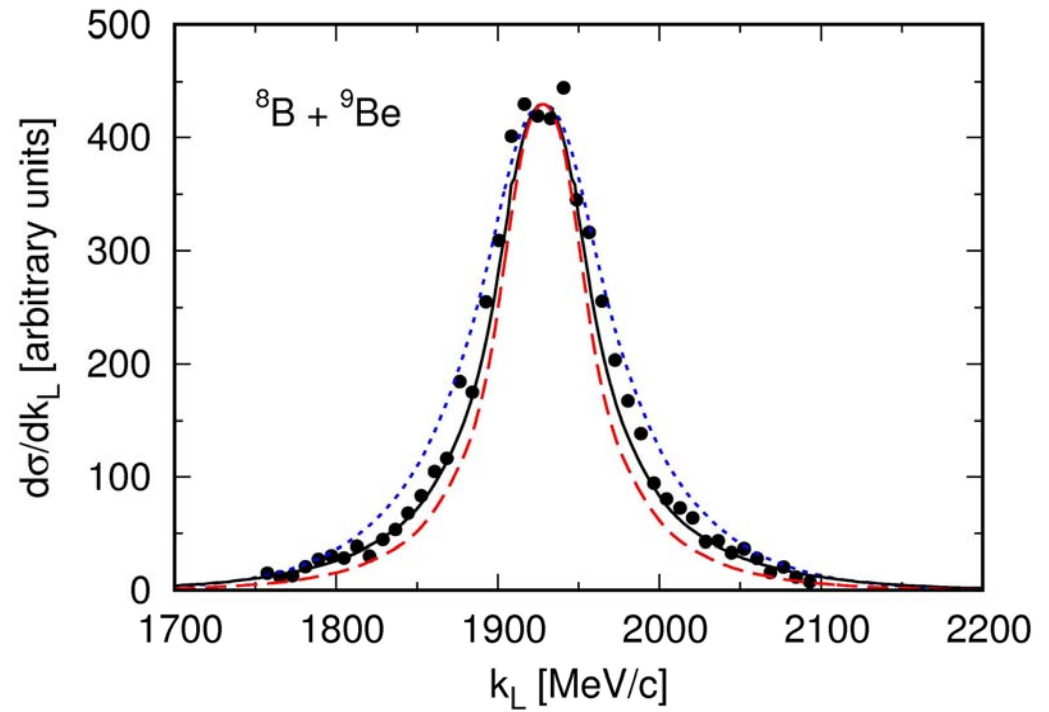
$$W^H(r) = -\frac{1}{2\pi^2} \frac{E}{k} \bar{\sigma}_{NN} \int_0^\infty j_0(qr) \rho_p(q) \rho_t(q) f_{NN}(q) q^2 dq \quad (3)$$





Phys. Rev. C **91**, 034606 (2015)

Eur. Phys J A **53**, 31 (2017)



Superscaling in Electron- and Neutrino- Nuclei Scattering

PWIA; $(e, e'N)$:

$$\left[\frac{d\sigma}{d\epsilon' d\Omega' dp_N d\Omega_N} \right]_{(e, e'N)}^{PWIA} = K \sigma^{eN}(q, \omega; p, \mathcal{E}, \phi_N) S(p, \mathcal{E}) \quad (1)$$

$$F(q, \omega) \cong \frac{[d\sigma/d\epsilon' d\Omega']_{(e, e')}}{\bar{\sigma}^{eN}(q, \omega; p = |y|, \mathcal{E} = 0)} \quad (2)$$

RFG:

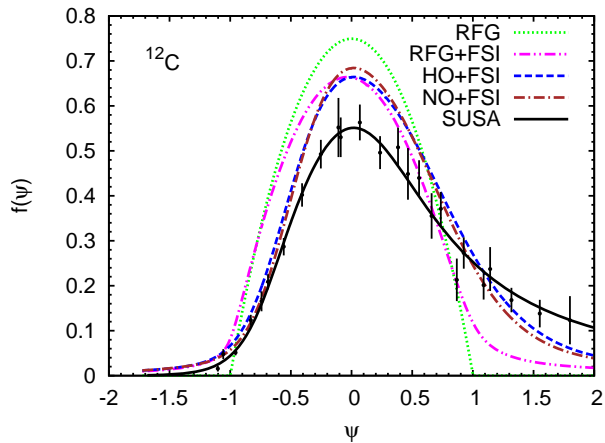
$$f_{\text{RFG}}(\psi') \simeq \frac{3}{4} (1 - \psi'^2) \theta(1 - \psi'^2) \quad (3)$$

$$S(p, \mathcal{E}) = \sum_i 2(2j_i + 1) n_i(p) L_{\Gamma_i}(\mathcal{E} - \mathcal{E}_i); \quad (4)$$

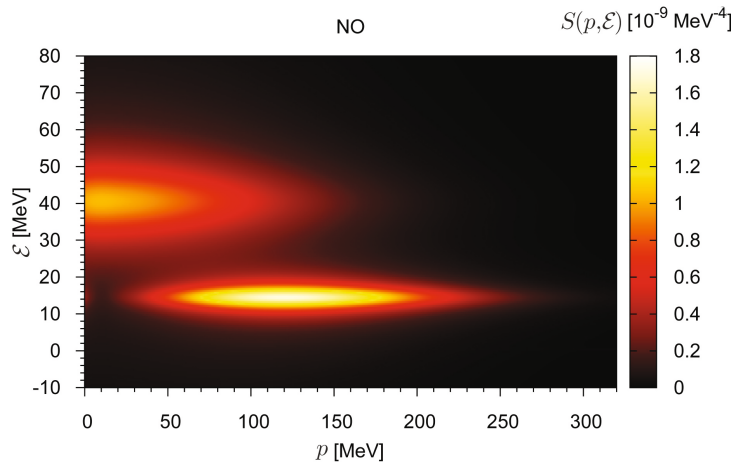
$$L_{\Gamma_i}(\mathcal{E} - \mathcal{E}_i) = \frac{1}{\pi} \frac{\Gamma_i/2}{(\mathcal{E} - \mathcal{E}_i)^2 + (\Gamma_i/2)^2}; \quad (5)$$

$$(\Gamma_{1p} = 6 \text{ MeV and } \Gamma_{1s} = 20 \text{ MeV})$$

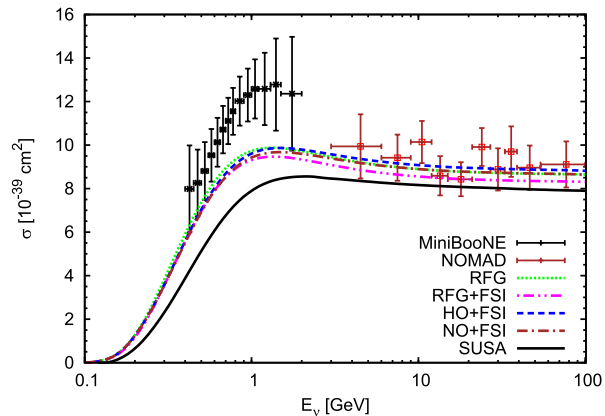
$$\rho(\mathbf{r}, \mathbf{r}') = \sum_{\alpha} N_{\alpha} \varphi_{\alpha}^*(\mathbf{r}) \varphi_{\alpha}(\mathbf{r}'); \quad [0 \leq N_{\alpha} \leq 1; \sum_{\alpha} N_{\alpha} = A]; \quad (6)$$



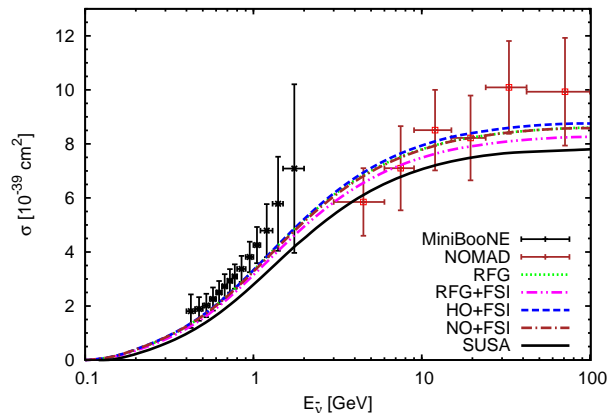
Phys. Rev. C **89**, 014607 (2014)



Phys. Rev. C **91**, 034607 (2015)



Phys. Rev. C **89**, 014607 (2014)



Phys. Rev. C **89**, 014607 (2014)

Information on the nucleon momentum distributions from the scaling function

– Amado, Woloshyn (1976–77):

$$n(k) \xrightarrow{k \rightarrow \infty} \left[\frac{\widetilde{V}_{NN}(k)}{k^2} \right]^2 \quad (1)$$

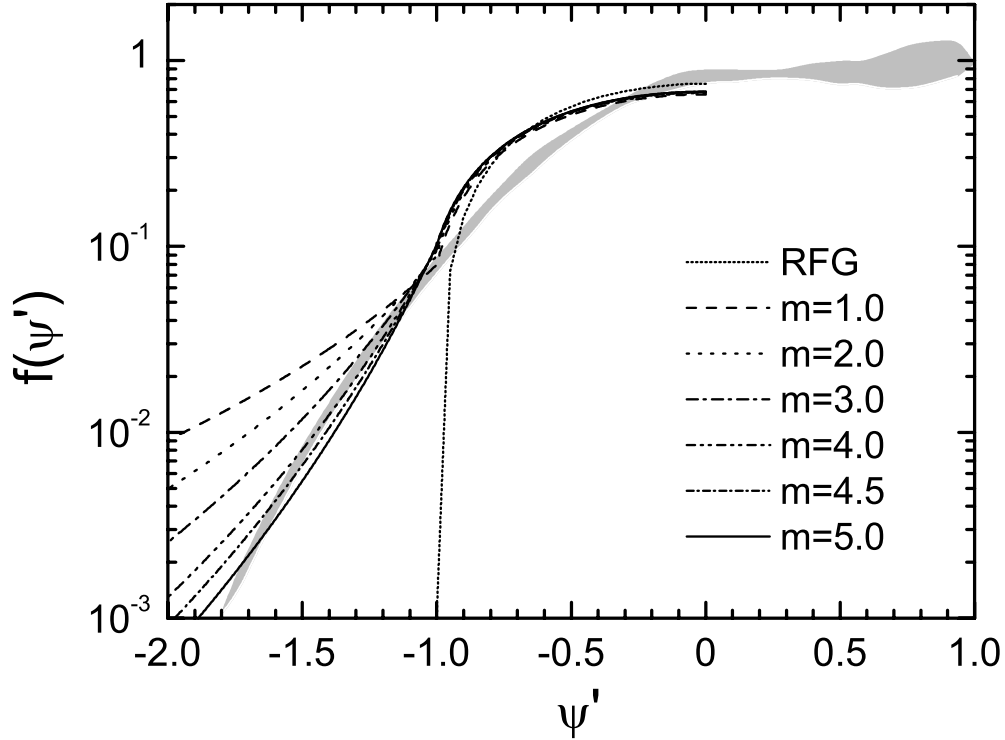
(unknown if k or k/A must be large)

$$f(\psi') = 0.12 \left(\frac{1+m}{2+m} \right) \frac{1}{|\psi'|^{2+m}} \quad (2)$$

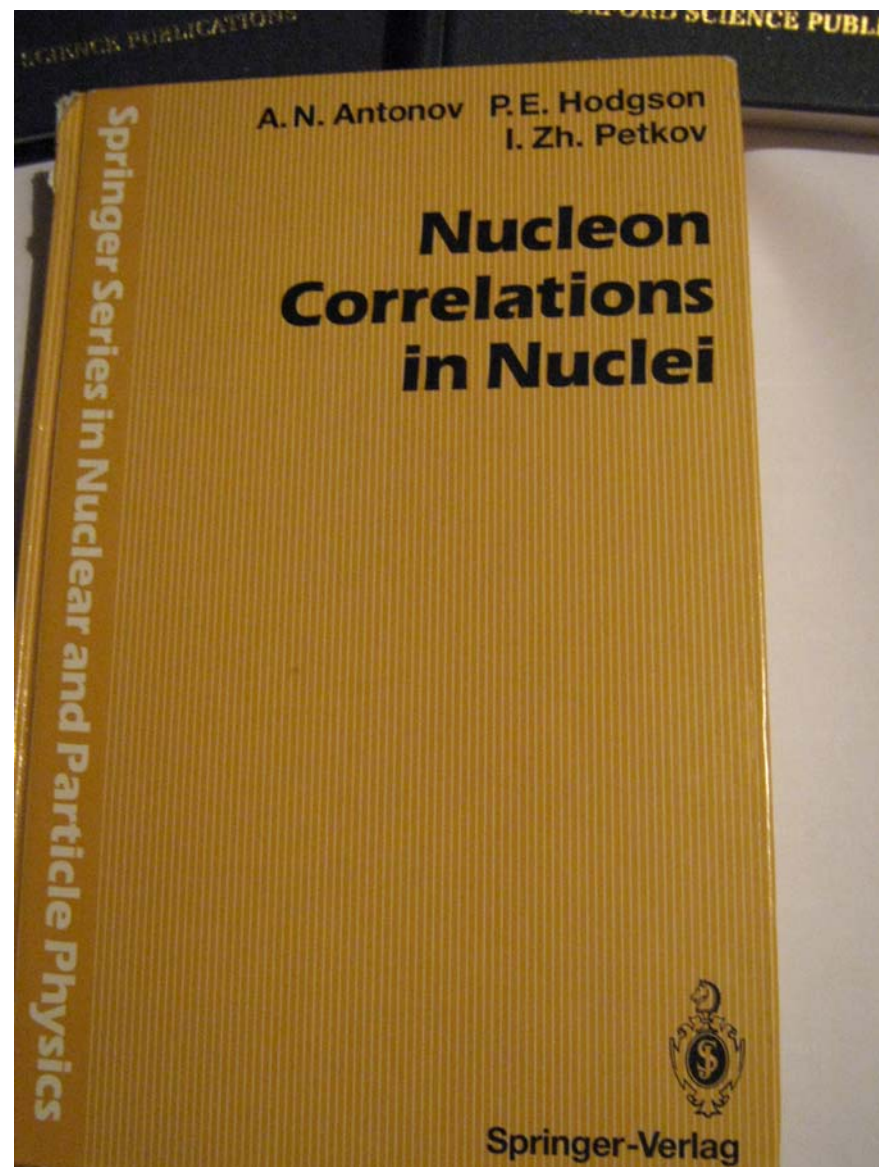
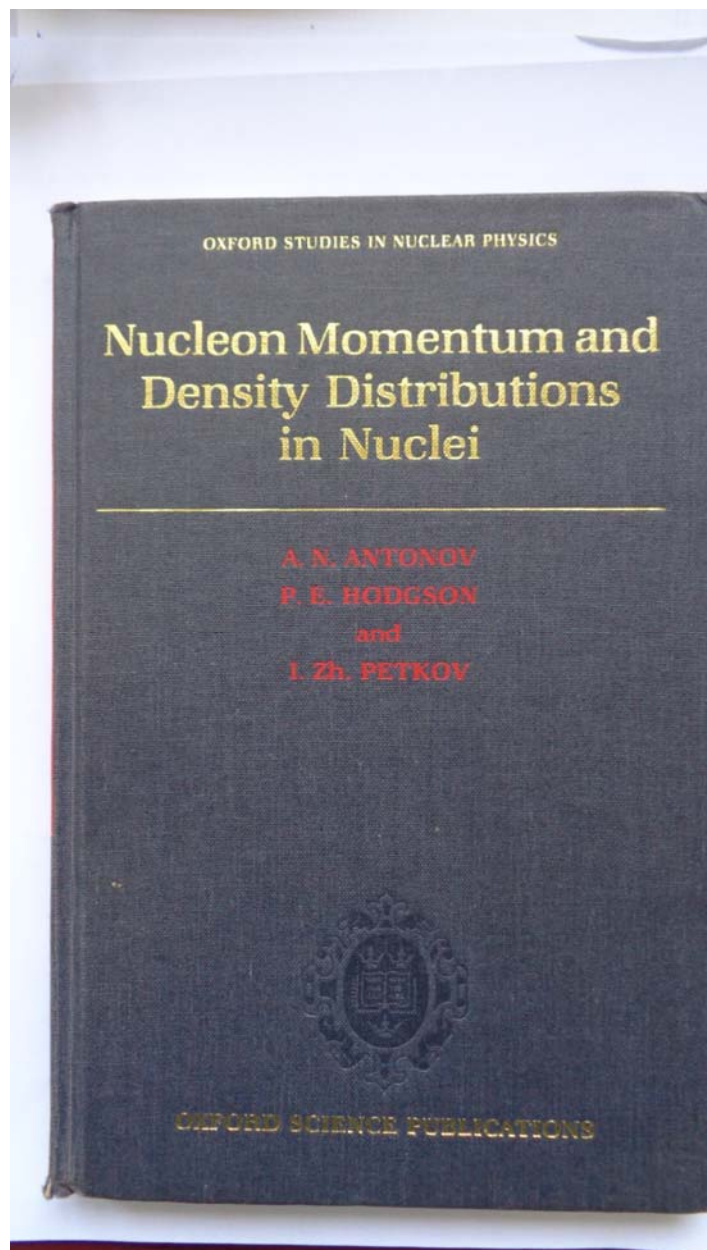
$$n(k) \sim \frac{1}{k^{4+m}}; \quad \text{Results: } m \simeq 4.5 \quad (3)$$

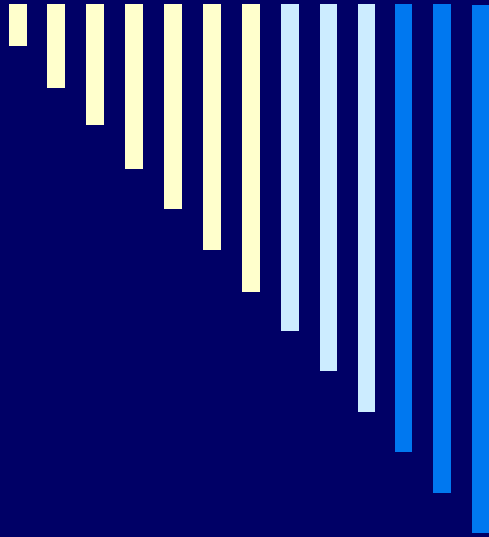
$$\text{For } m = 4 \quad V_{NN}(r) \sim \frac{1}{r} \quad (\text{at } r \rightarrow 0)$$

$$\text{For } m = 5 \quad V_{NN}(r) \sim \frac{1}{r^{1/2}} \quad (\text{at } r \rightarrow 0)$$



$f(\psi')$ [$n(k) \sim 1/k^{4+m}$], ($m = 1 \dots 5$); Phys. Rev. C **75**, 034319 (2007)





Thank you!