Effects of the tensor force on the properties of finite nuclei within Skyrme energy density functional

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Part I. Effect of tensor force on the properties of excited states in finite nuclei

1.Introduction

The Skyrme energy density functional(SEDF) is applied successfully to describe the properties of finite nuclei after the pioneer work of Vautherin and Brink.

The parameters of SEDF are fitted by using the bulk properties of infinite nuclear matter and finite nuclei, but most of them lose the tensor part.

Following the work of Otsuka, there are many works contribute to describle successfully the evolution of single-particle states based on Skyrme force with tensor.

Here we extend our calculations to excited states, and discuss the effects of tensor on the properties of excited states, we will foucs on the non-spin flip 2+ as well as on spin flip and charge exchange states.

2 Formula
Skyrme Force

$$
\begin{aligned}
V(\vec{r}_1,\vec{r}_2) = & \; t_0(1+x_0P_\sigma)\delta(\vec{r}) && \textit{central term} \\
& +\tfrac{1}{2}t_1(1+x_1P_\sigma)[k'^2\delta(\vec{r})+\delta(\vec{r})k^2] \\
& +t_2(1+x_2P_\sigma)k'\delta(\vec{r})k && \textit{nonlocal term} \\
& +\tfrac{1}{6}t_3(1+x_3P_\sigma)[\rho(\vec{R})]^\alpha\delta(\vec{r}) && \textit{density dependent term} \\
& +iW_0\sigma\cdot[k'\times\delta(\vec{r})k] && \textit{spin-orbit term}
\end{aligned}
$$

Skyrme Energy Density Functional：

$$H = K + H_0 + H_3 + H_{eff} + H_{fin} + H_{so} + H_{coul} + H_{sg}$$

E. Chabanat et al.， NPA627(1997)710
NPA635(1998)231
NPA643(1998)441

Tensor Force:

$$V_{tensor} = \frac{T}{2}\{[(\sigma_1 \cdot k')(\sigma_2 \cdot k') - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k'^2]\delta(r_1 - r_2)$$

$$+\delta(r_1 - r_2)[(\sigma_1 \cdot k)(\sigma_2 \cdot k) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k^2]\}$$

$$+\frac{U}{2}[(\sigma_1 \cdot k')\delta(r_1 - r_2)(\sigma_2 \cdot k) + (\sigma_2 \cdot k')\delta(r_1 - r_2)(\sigma_1 \cdot k) - \frac{2}{3}(\sigma_1 \cdot \sigma_2)\delta(r_1 - r_2)(k' \cdot k)]$$

The energy density founctional for centeral exchange and tensor part:

$$H_{tensor} = \frac{5}{24}(T + U)J_n J_p + \frac{5}{24}U(J_n^2 + J_p^2)$$

$$H_{sg} = -\frac{1}{16}(t_1 x_1 + t_2 x_2)J^2 + \frac{1}{16}(t_1 - t_2)[J_p^2 + J_n^2]$$

$J_q(r)$ **is the spin density**

$$\Delta H = \frac{1}{2}\alpha(J_n^2 + J_p^2) + \beta J_n J_p$$

$$\alpha = \alpha_c + \alpha_T, \beta = \beta_c + \beta_T$$

$$\alpha_c = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2); \beta_c = -\frac{1}{8}(t_1 x_1 + t_2 x_2)$$

$$\alpha_T = \frac{5}{12}U; \beta_T = \frac{5}{24}(T + U)$$

$J_q(r)$ **is defined as:**

$$J_q(r) = \frac{1}{4\pi r^3}\sum_i (2j_i+1)[j_i(j_i+1)-l_i(l_i+1)-\frac{3}{4}]R_i^2(r)$$

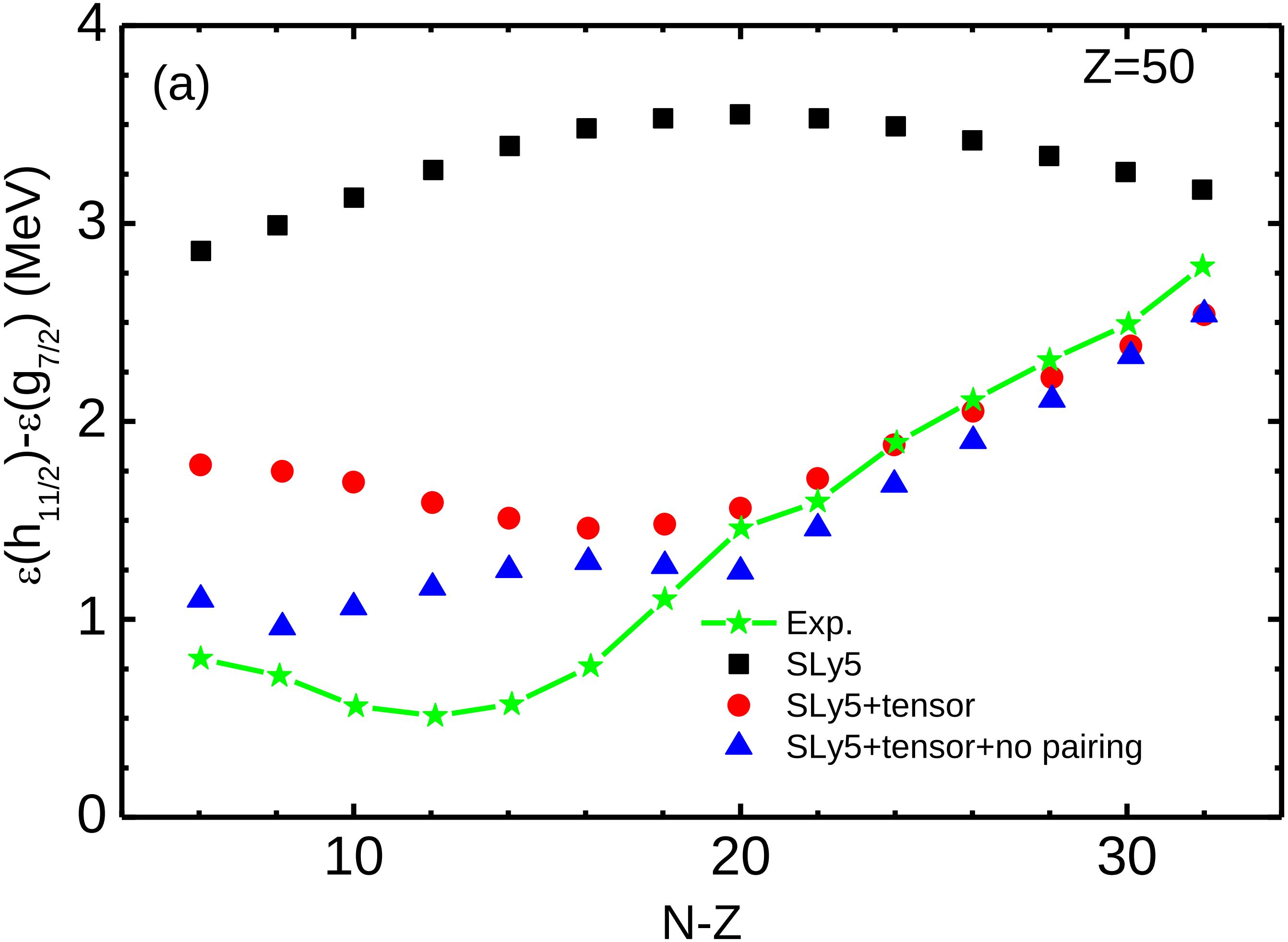
$$J_q(r) \sim 2l(l+1) > 0 \quad for \quad j = l+\tfrac{1}{2}$$

$$J_q(r) \sim -2l(l+1) < 0 \quad for \quad j = l-\tfrac{1}{2}$$

For spin-saturated nuclear $J_q(r) = 0$

The total spin-orbit potential:

$$U_{s.o.}^{(q)} = \frac{W_0}{2r}\left(2\frac{d\rho_q}{dr}+\frac{d\rho_{q'}}{dr}\right)+\left(\alpha\frac{J_q}{r}+\beta\frac{J_{q'}}{r}\right)$$



SLy5

G. Colo, et. al. PLB646(2007)227

$\alpha_c = 80.2 MeVfm^5$

$\beta_c = -48.9 MeVfm^5$

$\alpha_T = -170 MeVfm^5$

$\beta_T = 100 MeVfm^5$

$\alpha = -89.8 MeVfm^5$

$\beta = 51.1 MeVfm^5$

$T = 888.0 MeVfm^5$

$U = -408.0 MeVfm^5$

$$U^p_{s.o.} = \frac{W_0}{2r}\left(2\frac{d\rho_p}{dr} + \frac{d\rho_n}{dr}\right) + \left(\alpha\frac{J_p}{r} + \beta\frac{J_n}{r}\right)$$

^{120}Sn neutron spin-saturated state

$J_n=0$

$1g_{9/2},\quad J_p > 0, \alpha = -89.8 MeVfm^5 < 0 \quad U^p_{s.o.}\uparrow \quad \varepsilon_{h11/2}\downarrow \quad \varepsilon_{g7/2}\uparrow$

For Excited states

RPA equation:

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^{\nu} \\ Y^{\nu} \end{pmatrix} = E_{\nu} \begin{pmatrix} X^{\nu} \\ -Y^{\nu} \end{pmatrix}$$

$$A_{\min j} = (\varepsilon_m - \varepsilon_i)\delta_{ij}\delta_{mn} + \langle mj | V_{res} | in \rangle$$

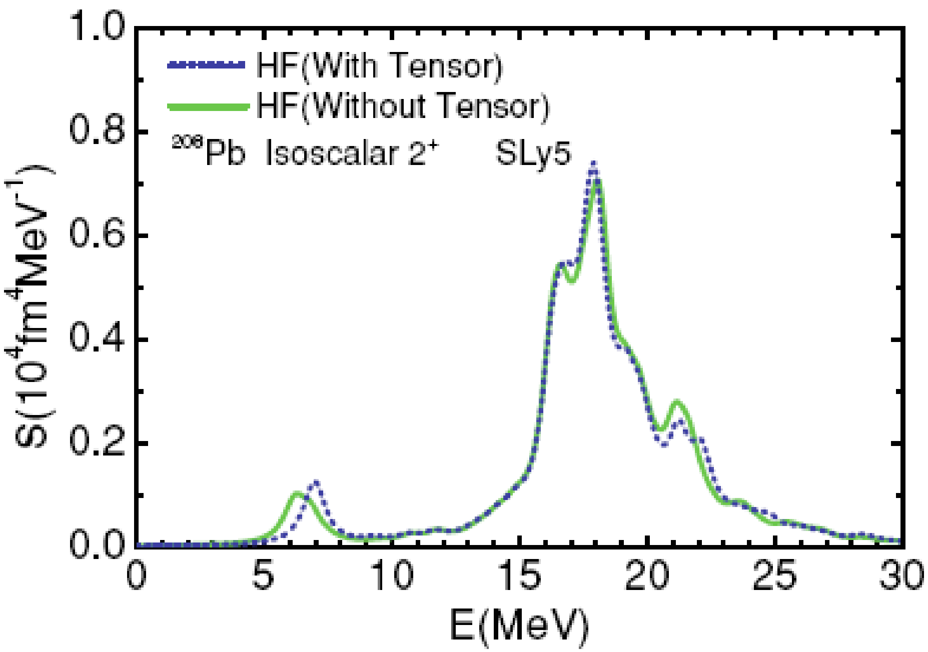
$$= (\varepsilon_m - \varepsilon_i)\delta_{ij}\delta_{mn} + V_{mjin}$$

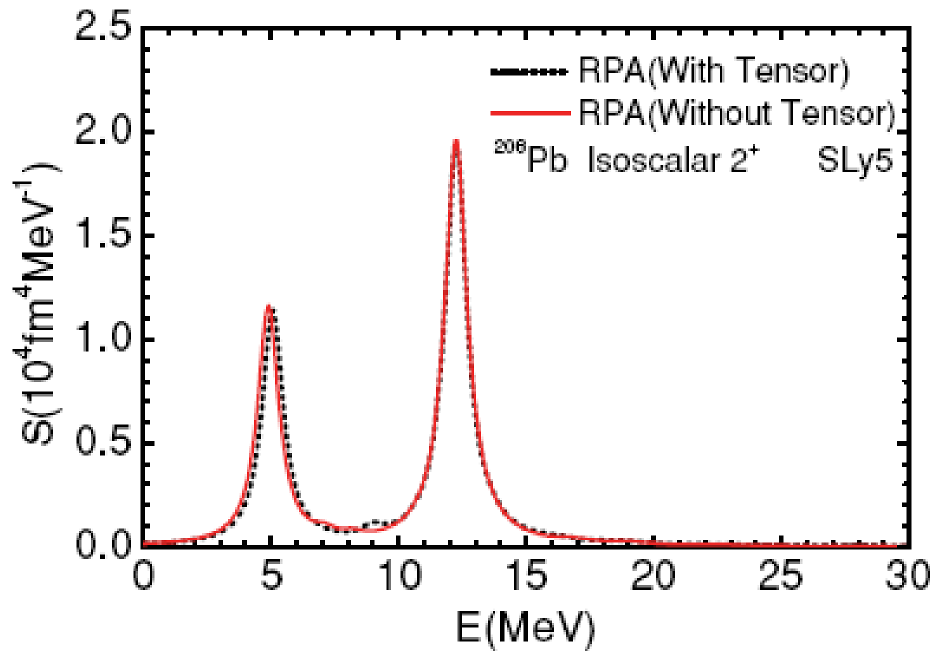
$$B_{\min j} = \langle mn | V_{res} | ij \rangle = V_{mnij}$$

V_{mjin} includes: central part of the Skyrme force, spin-orbit, tensor force

Cao L. G. et.al.,PRC 80, 064304(2009)

ISGQR: Cao L. G. et.al.,PRC 80, 064304(2009)





$(2f_{7/2}1h_{11/2}^{-1})_{\pi}$

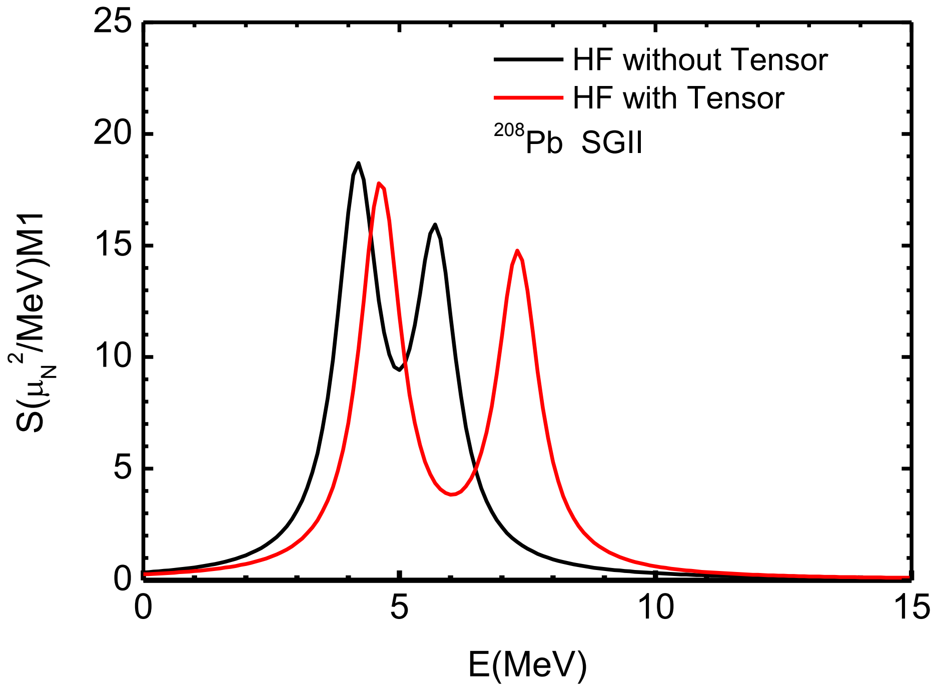
$(2g_{9/2}1i_{13/2}^{-1})_{\upsilon}$

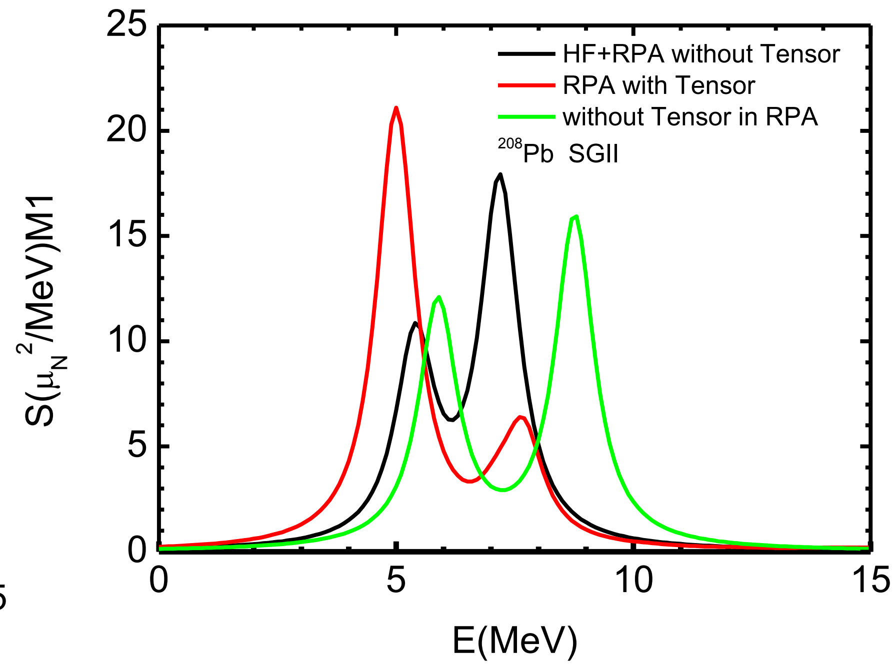
$\Delta E_{RPA} \approx \Delta E_{HF} + \langle V_{tensor} \rangle$

$\Delta E_{HF} = 0.60 MeV$

$\Delta E_{RPA} = 0.17 MeV$

$\langle V_{tensor} \rangle = -0.43 MeV$



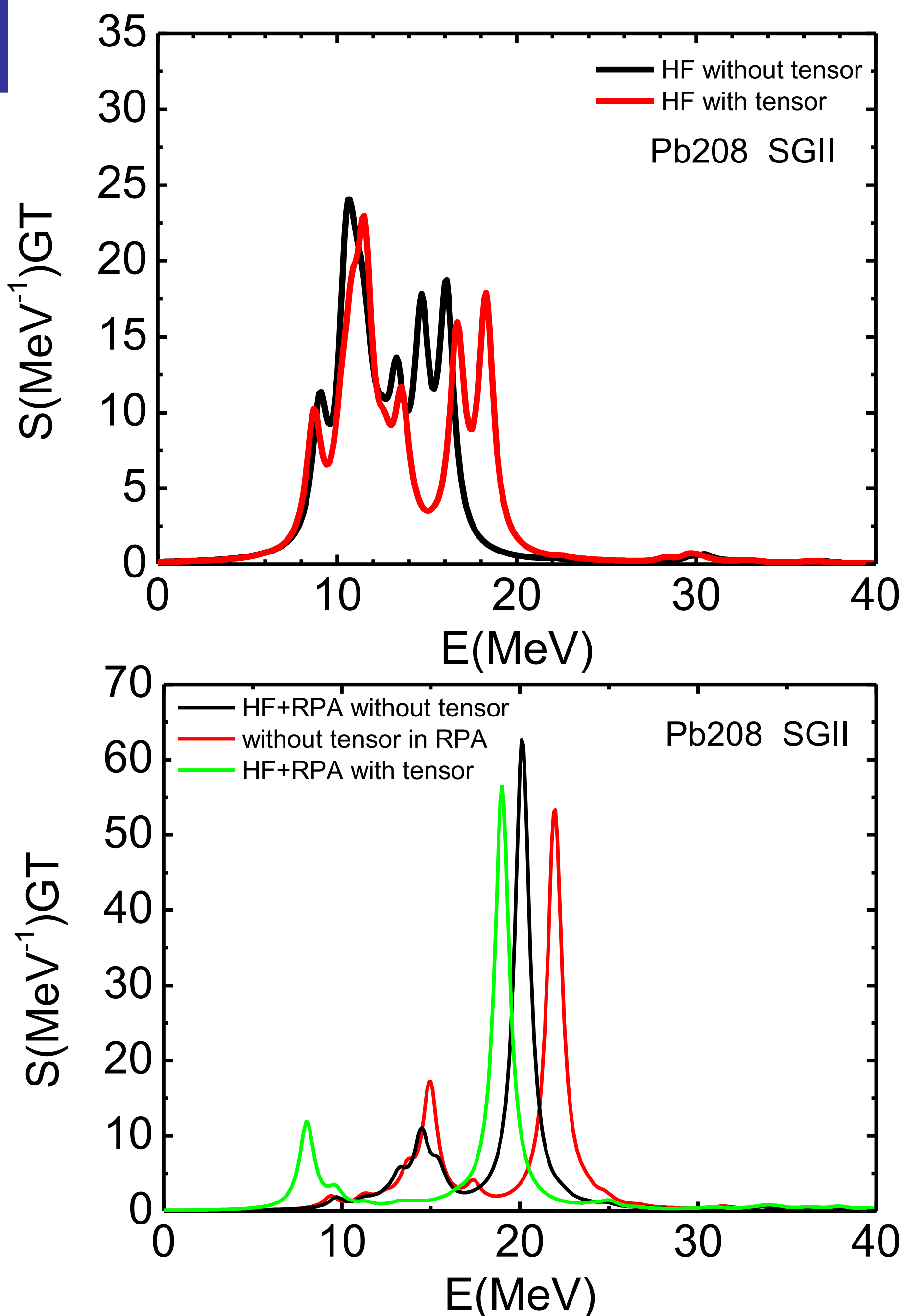


$$\Delta E_{RPA} \approx \Delta E_{HF} + \langle V_{tensor} \rangle$$

M1:

proton $h_{11/2} \rightarrow h_{9/2}$	*neutron* $i_{13/2} \rightarrow i_{11/2}$
$4.63MeV \rightarrow 4.18MeV$	$7.31MeV \rightarrow 5.71MeV$
$\Delta E_{HF} = 0.45MeV$	$\Delta E_{HF} = 1.60MeV$
$5.00MeV \rightarrow 5.40MeV$	$7.68MeV \rightarrow 7.18MeV$
$\Delta E_{RPA} = -0.40MeV$	$\Delta E_{RPA} = 0.50MeV$
$\langle V_{tensor} \rangle = -0.85MeV$	$\langle V_{tensor} \rangle = -1.10MeV$

Gamow-Teller:



Summary I

1. **We have shown the effects of tensor on properties of finite nuclei both in ground states and excited states.**

2. **The isotope dependence of energy splitting of proton states (ε(h11/2) − ε(g7/2)) in Sn isotopes is well reproduced by a parameter set of tensor interactions.**

3. **Self-consistent HF+RPA calculations are performed for non-spin flip 2+ as well as for spin flip M1 and charge-exchange GT excitations for ^{208}Pb. We found that the the effects of tensor on non-spin flip giant resonance is very small, tensor has weak effect on low-lying states, but the effect is mainly from mean field level. For spin-flip case, the tensor gives large effects, the ph residual interactions change drastically the properties of the spin-flip states.**

Part II. Study on the properties of single-particle states within beyond mean field theory

Motivation of this work

self-consistent mean field theory or the nuclear energy density functional theory
starts from an effective NN interactions
(properties of nuclear matter and finite nuclei)

such as: zero-range Skyrme force
finite-range Gogny force
relativistic Lagrangian

very successful → bulk properties
Binding energy(0.581MeV), Radii(0.03fm)
S. Goriely, N. Chamel, et. al., PRL 102(2009)152503
stable nuclei and exotic nuclei

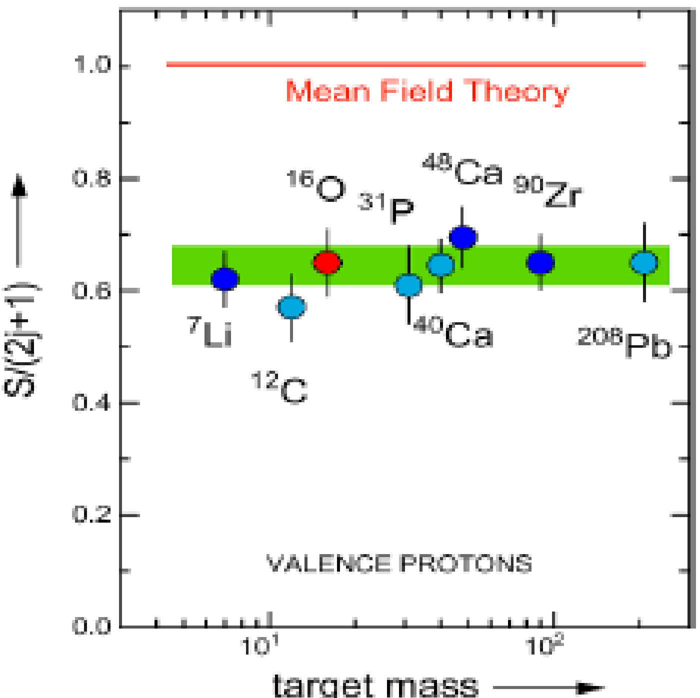
Giant resonances(monopole, dipole, Pygmy dipole…)

limitations of NEDFT

- Widths of GRs.
- Single-particle states and their spectroscopic factors

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{exp}} = S^2 \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{DWBA}}$$

NIKHEF NPA 553, 297c (1993)



Particle-vibration coupling (PVC) method for nuclei

2nd order PT:

$\varepsilon + \langle\Sigma(\varepsilon)\rangle$

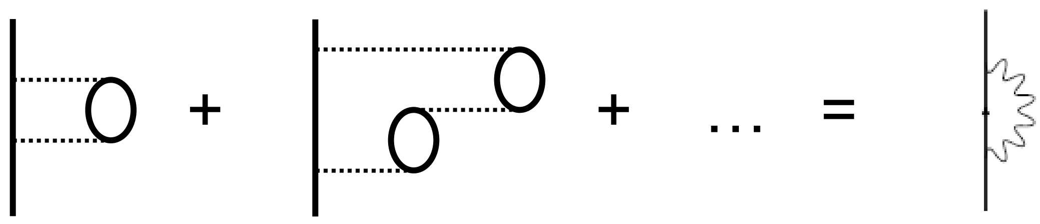
In the Dyson equation

$$G(\omega) = G_0(\omega) + G_0(\omega)\Sigma(\omega)G(\omega)$$

we assume the self-energy is given by the coupling with RPA vibrations

$$\Sigma(\vec{r},\vec{r}';\omega) = \int d_3r_1 d_3r_2 \; v(\vec{r},\vec{r}_1)\Pi^{(\mathrm{RPA})}(\vec{r}_1,\vec{r}_2;\omega)v(\vec{r}_2,\vec{r}')$$

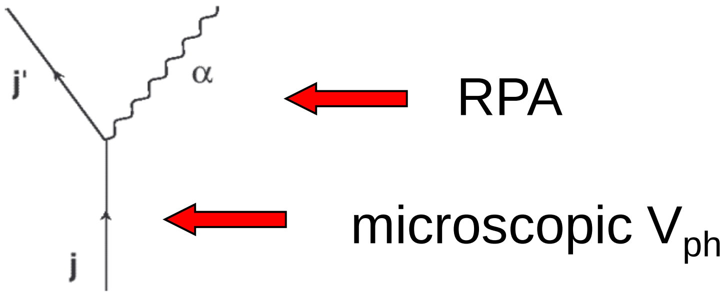
In a diagrammatic way



Particle-vibration coupling

Density vibrations are the most prominent feature of the low-lying spectrum of spherical systems

•Pioneering Skyrme calculation by V. Bernard and N. Van Giai in the 80s (neglect of the velocity-dependent part of V_{eff} in the PVC vertex).



- Microscopic calculations are now feasible. One starts from Hartree or Hartree-Fock with V_{eff} and add PVC on top of it. All is calculated using the same Hamiltonian or EDF consistently.

Microscopic HF plus RPA

Skyrme effective force

attraction

$$\hat{v}_{\rm Sk}(\boldsymbol{r}_{12}) = \underline{t_0(1+x_0\hat{P}_\sigma)\delta(\boldsymbol{r}_{12})} + \frac{1}{2}t_1(1+x_1\hat{P}_\sigma)(\hat{\boldsymbol{k}}^{\dagger 2}\delta(\boldsymbol{r}_{12}) + \delta(\boldsymbol{r}_{12})\hat{\boldsymbol{k}}^2)$$

$$+ t_2(1+x_2\hat{P}_\sigma)\hat{\boldsymbol{k}}^\dagger\cdot\delta(\boldsymbol{r}_{12})\hat{\boldsymbol{k}} + \underline{\frac{1}{6}t_3(1+x_3\hat{P}_\sigma)\delta(\boldsymbol{r}_{12})\rho^\alpha\left(\frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\right)}$$

$$+ iW_0(\hat{\sigma}_1+\hat{\sigma}_2)\cdot\hat{\boldsymbol{k}}^\dagger\times\delta(\boldsymbol{r}_{12})\hat{\boldsymbol{k}}.$$

short-range repulsion

- After generating the HF mean-field, one is left with a residual force V_{res}.
- The residual force acts as a restoring force, and sustains collective oscillations (like GRs). Its effect is included in the linear response theory = RPA.

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega\begin{pmatrix} X \\ Y \end{pmatrix}$$

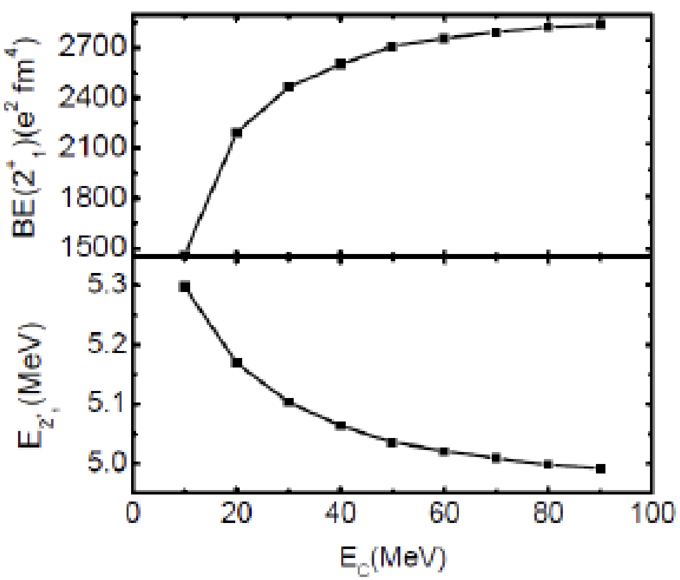
Our fully self-consistent implementation

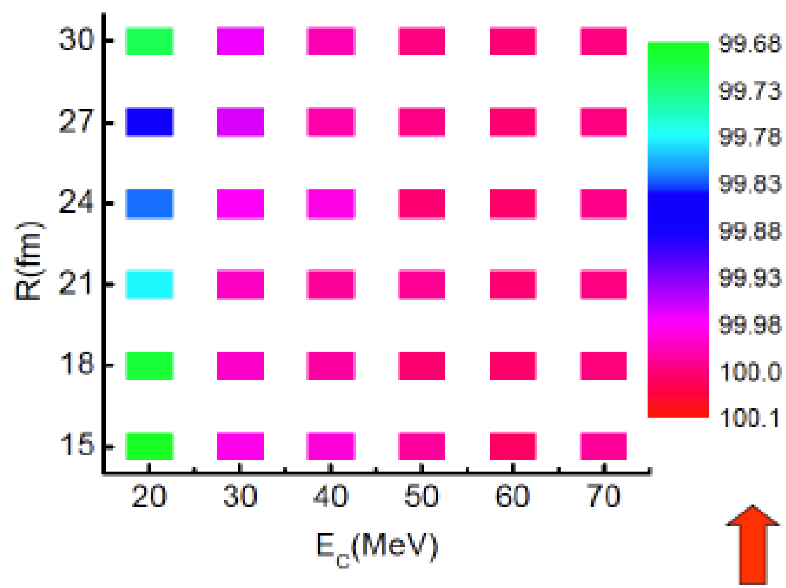
The continuum is discretized. The basis must be large due to the zero-range character of the force. Parameters: R, E_C.

The energy-weighted sum rule should be equal to the double-commutator value: well fulfilled !

$$m_1(\hat{O}) = \sum_\nu E_\nu |\langle \nu|\hat{O}|\tilde{0}\rangle|^2 = \frac{1}{2}\langle 0|[\hat{O},[H,\hat{O}]]|0\rangle$$

^{208}Pb - SGII





G. Colò, L. Cao, N. Van Giai, L. Capelli Comp. Phys. Comm. 184, 142 (2013).

Percentages $m_1(RPA)/m_1(DC)$ [%]

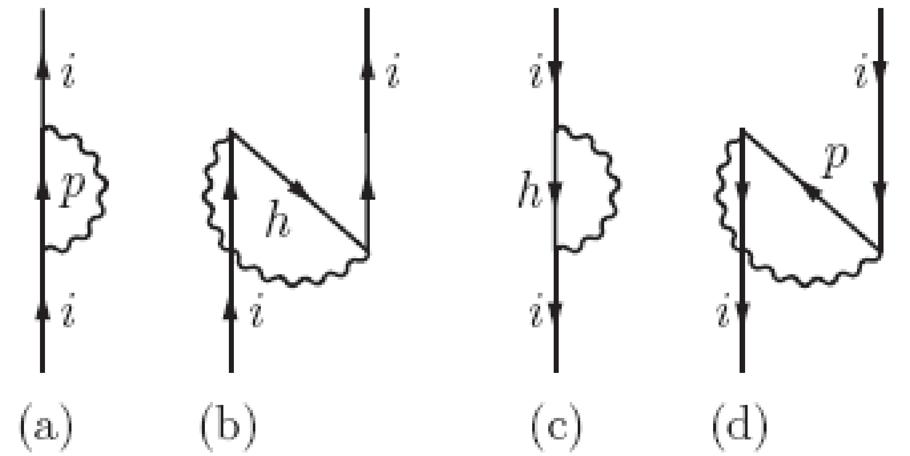


FIG. 1. The four diagrams associated with the single-nucleon self-energy. See the text for details.

$$\Sigma_i(\omega) = \frac{1}{2j_i+1}\left(\sum_{nL,\ p>F}\frac{|\langle i||V||p,nL\rangle|^2}{\omega-\varepsilon_p-\omega_{nL}+i\eta} + \sum_{nL,\ h<F}\frac{|\langle i||V||h,nL\rangle|^2}{\omega-\varepsilon_h+\omega_{nL}-i\eta}\right),$$

A consistent study within the Skyrme framework

We have implemented a version of PVC in which the treatment of the coupling is exact, namely we do not wish to make any approximation in the vertex.

$$\langle i||V||j, nL\rangle = \sqrt{2L+1} \sum_{\rm ph} X_{\rm ph}^{nL} V_L(ihjp) + (-)^{L+j_h-j_p} Y_{\rm ph}^{nL} V_L(ipjh)$$

The whole phonon wavefunction is considered, and all the terms of the Skyrme force enter the p-h matrix elements

$$V_L(ihjp) = \sum_{{\rm all}\ m} (-)^{j_j-m_j+j_h-m_h} \langle j_i m_i j_j m_j | LM\rangle \langle j_p m_p j_h m_h | LM\rangle \langle j_i m_i, j_h m_h | V | j_j m_j, j_p m_p\rangle$$

Cao LG, et.al., Phys.Rev. C89, 044314 (2014)

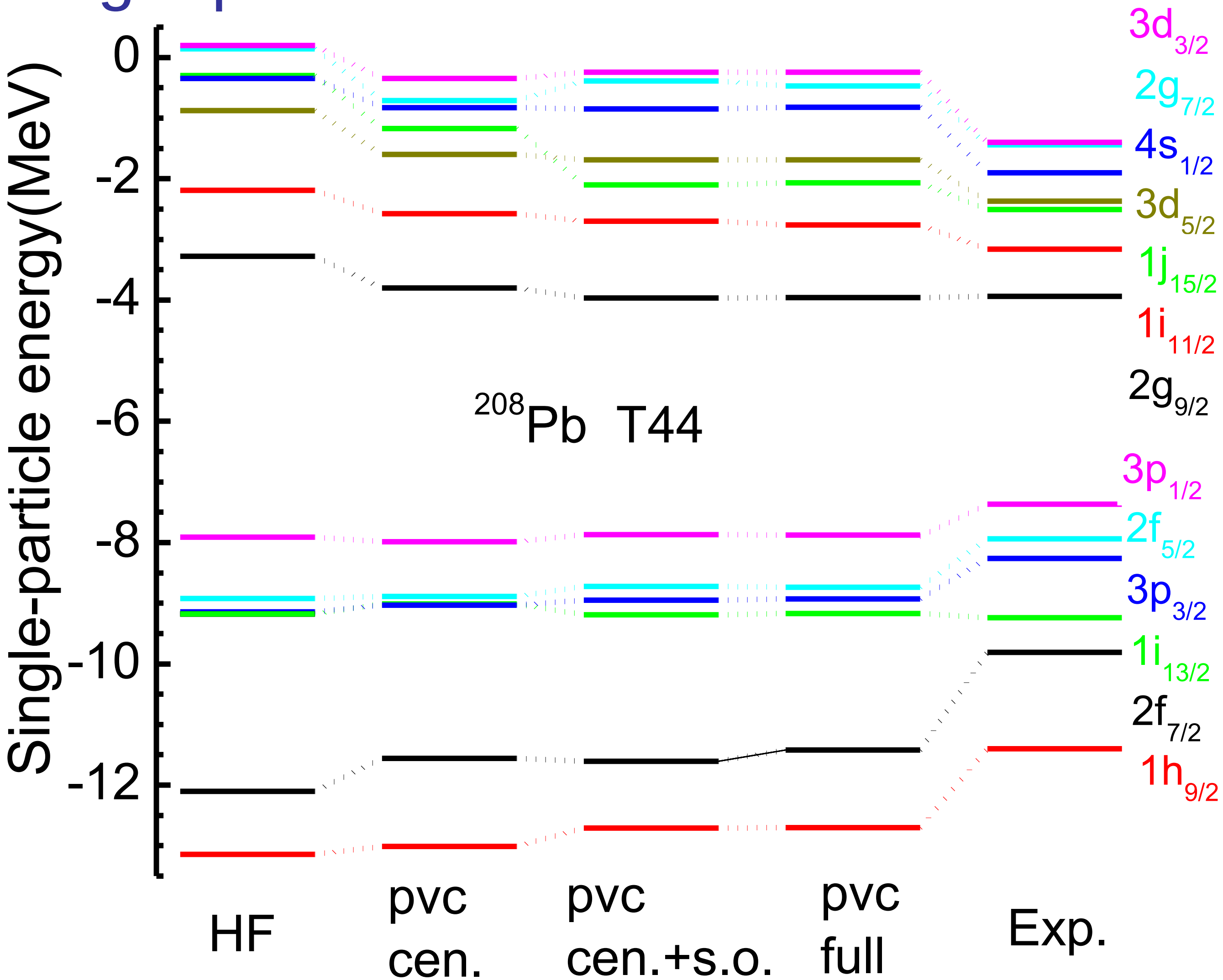
TABLE I. Energies and reduced transition probabilities of the low-lying states in ^{40}Ca and ^{208}Pb obtained by HF+RPA with SLy5 and T44 parameter sets. The values in parentheses are the results obtained without the contribution of the tensor force. The experimental data are from Ref. [33].

	J^π	Theory				Experiment	
		SLy5		T44			
		Energy (MeV)	$B(EL, 0 \to L)$ (e^2 fm^{2L})	Energy (MeV)	$B(EL, 0 \to L)$ (e^2 fm^{2L})	Energy (MeV)	$B(EL, 0 \to L)$ (e^2 fm^{2L})
^{40}Ca	3^-	3.225(3.822)	$0.884(1.285) \times 10^4$	1.366(1.508)	$0.852(1.280) \times 10^4$	3.74	1.18×10^4
^{208}Pb	2^+	5.155(4.934)	$3.065(2.858) \times 10^3$	4.549(5.105)	$2.478(2.785) \times 10^3$	4.09	4.09×10^3
	3^-	3.585(3.671)	$4.928(6.374) \times 10^5$	3.337(3.629)	$5.739(5.523) \times 10^5$	2.61	6.21×10^5
	4^+	5.760(5.417)	$1.395(1.256) \times 10^7$	4.655(5.684)	$0.782(1.382) \times 10^7$	4.32	1.29×10^7
	5^-	4.022(4.560)	$2.881(4.898) \times 10^8$	3.977(4.092)	$3.796(2.443) \times 10^8$	3.19	4.62×10^8
		4.507(5.589)	$0.748(1.642) \times 10^8$	4.532(5.021)	$0.345(1.929) \times 10^8$	3.71	3.30×10^8

Results and discussion

I. Single-particle states

Cao LG, et.al., Phys.Rev. C89, 044314 (2014)



The values of σ are 1.421, 1.002, 0.907, 0.873 for T44 .

II. Effective mass

Cao LG, et.al., Phys.Rev. C89, 044314 (2014)

$$\frac{m^*}{m} = \frac{\widetilde{m}}{m} \times \frac{\overline{m}}{m}$$

$$\frac{\widetilde{m}}{m} = \left(1 + \frac{m}{\hbar^2 k}\frac{\partial M}{\partial k}\right)^{-1}$$

$$\frac{\overline{m}}{m} = \left(1 - \frac{\partial M}{\partial E}\right)$$

TABLE V: The calculated effective mass for neutron states in ^{208}Pb in various approximations. The results are obtained by using SLy5 and T44 parameter sets.

	HF	pvc central		pvc central+S.O.		pvc full	
	m_k^*	m_e^*	m^*	m_e^*	m^*	m_e^*	m^*
SLy5	0.839	1.156	0.968	1.198	1.002	1.229	1.028
T44	0.841	1.157	0.973	1.200	1.009	1.235	1.038

III. Spectroscopic factor

$$S_\alpha^\lambda = \left(1 - \frac{\partial \Sigma_\alpha}{\partial \varepsilon}\right)^{-1}_{\varepsilon=\varepsilon_\alpha^\lambda}.$$

TABLE IV: The energies and spectroscopic factors of the single-particle states in ^{208}Pb in various approximations. The results are obtained by using SLy5 and T44 parameter sets. The experimental data are taken from Ref.[31, 32].

		HF	pvc central		pvc central+S.O.		pvc full			Spectroscopic factors	
		$\varepsilon^{(0)}$	$\Delta\varepsilon_i$	ε_i	$\Delta\varepsilon_i$	ε_i	$\Delta\varepsilon_i$	ε_i	ε_i^{exp}	S_i^{th}	S_i^{exp}
T44	$3d_{3/2}$	0.20	-0.55	-0.35	-0.44	-0.24	-0.44	-0.24	-1.40	0.895	1.09
	$2g_{7/2}$	0.14	-0.85	-0.71	-0.53	-0.39	-0.61	-0.47	-1.44	0.832	1.05
	$4s_{1/2}$	-0.35	-0.48	-0.83	-0.50	-0.85	-0.47	-0.82	-1.90	0.896	0.98
	$3d_{5/2}$	-0.88	-0.72	-1.60	-0.81	-1.69	-0.81	-1.69	-2.37	0.855	0.98
	$1j_{15/2}$	-0.30	-0.87	-1.17	-1.80	-2.10	-1.77	-2.07	-2.51	0.583	0.58
	$1i_{11/2}$	-2.19	-0.39	-2.58	-0.51	-2.70	-0.57	-2.76	-3.16	0.884	0.86
	$2g_{9/2}$	-3.28	-0.52	-3.80	-0.69	-3.97	-0.68	-3.96	-3.94	0.877	0.83
	$3p_{1/2}$	-7.91	-0.08	-7.99	0.04	-7.87	0.03	-7.88	-7.37	0.905	0.90
	$2f_{5/2}$	-8.92	0.03	-8.89	0.19	-8.72	0.18	-8.74	-7.94	0.888	0.60
	$3p_{3/2}$	-9.14	0.11	-9.03	0.19	-8.95	0.21	-8.93	-8.26	0.844	0.88
	$1i_{13/2}$	-9.18	0.17	-9.01	-0.01	-9.19	0.01	-9.17	-9.24	0.903	0.91
	$2f_{7/2}$	-12.10	0.54	-11.56	0.49	-11.61	0.68	-11.42	-9.81	0.580	0.95
	$1h_{9/2}$	-13.14	0.13	-13.01	0.43	-12.71	0.44	-12.70	-11.40	0.831	0.98

Cao LG, et.al., Phys.Rev. C89, 044314 (2014)

Summary II

•Microscopic particle-vibration coupling calculations are now available - based on the self-consistent use of nonrelativistic functionals.

• Results for single-particle states are improved compared to mean-field results.

•The effective mass is enchanced by particle-vibration copuling calculations, so the level density is improved around the Fermi surface which is improtant for astrophysics study. The spectroscopic factor is quched by about 20%.

Thanks!