# Collective excitations and neutrino-nucleus reactions in relativistic point-coupling models APCTP, Pohang

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CENTER OF EXCELLENCE FOR THE THEORY OF QUANTUM AND COMPLEX SYSTEMS AND LIE ALGEBRA REPRESENTATION

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Simple system - Hydrogen atom

RNEDF (R)QR

Result - part 1

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s - part 2 Neutrino-nucleus reaction

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# Simple system - Hydrogen atom



## Simple system - Hydrogen atom

Dirac Hamiltonian for central field:

$$\hat{H} = c\hat{\vec{\alpha}} \cdot \hat{\vec{p}} + \beta m_0 c^2 + V(r)$$
(1)

If the field has spherical symmetry, then total angular momentum operator  $\hat{J}$  and operator  $\hat{P}$  ( $\hat{P} = \exp(i\phi)\hat{\beta}\hat{P}_0, P_0: \vec{x} \to -\vec{x}$ ) commute with  $\hat{H}$ . We assume solution:

$$\Psi_{njm} = \begin{pmatrix} \psi_{ljm}(\vec{r},t) \\ \chi_{\bar{l}jm}(\vec{r},t) \end{pmatrix}$$
(2)

Action of parity operator:

$$\hat{P}_{0}\psi_{ljm}(\vec{r},t) = \lambda_{f}\psi_{ljm}(\vec{r},t)$$
(3)

$$-\hat{P}_0\chi_{\bar{l}jm}(\vec{r},t) = -\lambda_g\chi_{\bar{l}jm}(\vec{r},t)$$
(4)

We must have  $\lambda_f = -\lambda_g$  in order to have good parity of four-spinor.

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# Simple system - Hydrogen atom

$$\hat{H}\Psi_{jm}(\vec{r},t) = -i\hbar \begin{pmatrix} 0 & \hat{\sigma} \\ \hat{\sigma} & 0 \end{pmatrix} \nabla \Psi_{jm}(\vec{r},t) + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} m_0 c^2 \Psi_{jm}(\vec{r},t)$$

$$+ V(r)\Psi_{jm}(\vec{r},t)$$
(5)

Rewritten in terms of bispinors:

$$-i\hbar c\vec{\sigma}\cdot\vec{\nabla}\chi_{\bar{l}jm} + m_0c^2\psi_{ljm} + V(r)\psi_{ljm} = E\psi_{ljm}$$
(6)

$$-i\hbar c\vec{\sigma} \cdot \vec{\nabla} \psi_{\bar{l}jm} - m_0 c^2 \chi_{ljm} + V(r) \chi_{ljm} = E \chi_{ljm}$$
<sup>(7)</sup>

For  $\psi$ 

$$\psi_{ljm} = R_{nl}(r) \sum_{m_l m_s} C^{jm}_{1/2m_s lm_l} Y_{lm_l}(\Omega) \chi_{1/2m_s}(\vec{\sigma})$$
(8)

and similar for  $\chi$ . Expansion of bispinors in spherical harmonic oscillator basis:

$$\psi_{nljm} = \sum_{n} C_n^l f_{nl}(r) \Omega_{ljm}(\hat{\Omega})$$
(9)

$$\chi_{n\bar{l}jm} = \sum_{n} C_{n}^{\bar{l}} i g_{n\bar{l}}(r) \Omega_{\bar{l}jm}(\hat{\Omega})$$
(10)

# Simple system - Hydrogen atom

Matrix elements of Dirac Hamiltonian:

$$A_{fg} = -\hbar c \int_0^\infty dr r^2 f^*_{n_a l_a}(r) \frac{\partial}{\partial r} g_{n_c \bar{l}_a}(r)$$
(11)

$$B_{fg} = -\hbar c \int_{0}^{\infty} dr r^{2} f_{n_{a} l_{a}}^{*}(r) \frac{1-\kappa}{r} g_{n_{c} \bar{l}_{a}}(r)$$
(12)

$$C_{ff}^{(S)} = \int dr r^2 f_{n_a l_a}^*(r) S(r) f_{n_c l_a}(r)$$
(13)

$$C_{ff}^{(V)} = \int dr r^2 f_{n_a l_a}^*(r) V(r) f_{n_c l_a}(r)$$
(14)

$$A_{gf} = \hbar c \int_0^\infty dr r^2 g^*_{n_a \bar{l}_a}(r) \frac{\partial}{\partial r} f_{n_c l_a}(r)$$
(15)

$$B_{gf} = \hbar c \int_{0}^{\infty} dr r^{2} g_{n_{a}\bar{l}_{a}}^{*}(r) \frac{1+\kappa}{r} f_{n_{c}l_{a}}(r)$$
(16)

$$C_{gg}^{(S)} = -\int dr r^2 g_{n_a \bar{l}_a}^*(r) S(r) g_{n_c \bar{l}_a}(r)$$
(17)

$$C_{gg}^{(V)} = \int dr r^2 g_{n_a \bar{l}_a}^*(r) V(r) g_{n_c \bar{l}_a}(r)$$
(18)

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Figure 1: Upper and lower components of spinor  $s_{1/2}$ . Dependence of binding energy of hydrogen atom on harmonic oscillator length for different numbers of major oscillator shells.  $\frac{1}{2}$  ,  $\frac{1}{2}$  ,  $\frac{1}{2}$ 

## Relativistic nuclear energy density functionals

Two types of nuclear models are usually present in relativistic energy density functionals:

- Walecka's type (nucleons are system of Dirac particles which interacts by exchange of mesons)
- Contact type (point-coupling; where meson propagator is replaced by contact interaction)



In nuclear models of Walecka's type we have meson propagator:

$$\frac{g_m}{-\Delta + m_m^2} \simeq \frac{g_m}{m_m^2} + g_m \frac{\Delta}{m_m^4}, \qquad (19)$$

Four-fermion interaction:

$$L_{4f} = -\frac{1}{2}\alpha_{S}\left(\bar{\psi}\psi\right)\left(\bar{\psi}\psi\right) - \frac{1}{2}\alpha_{V}\left(\bar{\psi}\gamma_{\mu}\psi\right)\left(\bar{\psi}\gamma^{\mu}\psi\right) \\ -\frac{1}{2}\alpha_{TV}\left(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi\right)\left(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi\right), \quad (20)$$

Derivative terms:

$$L_{der} = -\frac{1}{2} \delta_{S} \partial_{\nu} \left( \bar{\psi} \psi \right) \partial^{\nu} \left( \bar{\psi} \psi \right) - \frac{1}{2} \delta_{V} \partial_{\nu} \left( \bar{\psi} \gamma_{\mu} \psi \right) \partial^{\nu} \left( \bar{\psi} \gamma^{\mu} \psi \right) \\ - \frac{1}{2} \delta_{TV} \partial_{\nu} \left( \bar{\psi} \vec{\tau} \gamma_{\mu} \psi \right) \partial^{\nu} \left( \bar{\psi} \vec{\tau} \gamma^{\mu} \psi \right), \quad (21)$$

We can add non-linear terms:

$$L_{hot} = -\frac{1}{3}\beta_{S}\left(\bar{\psi}\psi\right)^{3} - \frac{1}{4}\gamma_{S}\left(\bar{\psi}\psi\right)^{4}, \qquad (22)$$

Dirac Lagrangian for free nucleon:

$$\mathcal{L}_{\text{free}} = \bar{\psi} \left( i \gamma_{\mu} \partial^{\mu} - m \right) \psi \tag{23}$$

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Electromagnetic part:

$$L_{em} = -eA_{\mu}\bar{\psi}\left[(1-\tau_{3})/2\right]\gamma^{\mu}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
(24)

# DD-PC1 parametrization

Lagrangian density:

$$L = \bar{\psi} (i\gamma_{\mu}\partial^{\mu} - m) \psi - \frac{1}{2} \alpha_{S} (\bar{\psi}\psi) (\bar{\psi}\psi) - \frac{1}{2} \alpha_{V} (\bar{\psi}\gamma_{\mu}\psi) \times (\bar{\psi}\gamma^{\mu}\psi) - \frac{1}{2} \alpha_{TV} (\bar{\psi}\vec{\tau}\gamma_{\mu}\psi) (\bar{\psi}\vec{\tau}\gamma^{\mu}\psi) - \frac{1}{2} \delta_{S} \partial_{\nu} (\bar{\psi}\psi) \partial^{\nu} (\bar{\psi}\psi) - e\bar{\psi}\gamma_{\mu}A^{\mu}\frac{1 - \tau_{3}}{2}\psi.$$
(25)

Coupling density dependence in case of DD-PC1 parametrization:

$$f_i[\rho] = a_i + (b_i + c_i x) \exp(-d_i x),$$
 (26)

Variation of Lagrangian with respect to  $\bar{\psi}$  we obtain one-nucleon Dirac equation:

$$\left[\gamma_{\mu}\left(i\partial^{\mu}-\Sigma^{\mu}-\Sigma^{\mu}_{rearr}\right)-\left(m+\Sigma_{S}\right)\right]\psi = 0, \qquad (27)$$

$$\Sigma^{\mu} = \Sigma^{\mu}_{V} + \vec{\tau} \cdot \vec{\Sigma}^{\mu}_{TV}, \qquad (28)$$

$$\Sigma_{S} = \alpha_{S}(\rho)\rho_{S} + \delta_{S}\Delta\rho_{S}, \qquad (29)$$

$$\Sigma_V^{\mu} = \alpha_V(\rho) j^{\mu} + e \frac{1 - \tau_3}{2} A^{\mu},$$
 (30)

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Simple system - Hydrogen atom RNEDF (R)QRPA Result - part 1 Proton-neutron (R)QRPA Results - part 2 Neutrino-nucleus reactions

$$\vec{\Sigma}^{\mu}_{TV} = \alpha_{TV}(\rho) \vec{j}^{\mu}, \qquad (32)$$

$$\Sigma^{\mu}_{rearr} = \frac{1}{2} \frac{j^{\mu}}{\rho} \left[ \frac{\partial \alpha_{5}(\rho)}{\partial \rho} \rho_{5}^{2} + \frac{\partial \alpha_{V}(\rho)}{\partial \rho} j_{\nu} j^{\nu} + \frac{\partial \alpha_{TV}(\rho)}{\partial \rho} j_{\nu} j^{\bar{\nu}} \right].$$
(33)

RPA matrix elements can be easily obtained from:

$$V_{abcd} = \frac{\delta h_{ac}}{\delta \rho_{db}} = \frac{\delta^2 E[\rho]}{\delta \rho_{ac} \delta \rho_{bd}}, \qquad (34)$$

where

$$h = -i\alpha \nabla + \beta(m+\Sigma) + \beta \gamma_{\mu} \Sigma^{\mu}$$
(35)

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# General particle-hole matrix elements

$$V_{abcd}^{(1)} = \int d^3 r_1 \int d^3 r_2 \sum_j \bar{\psi}_a(\vec{r}_1) \Gamma_{j\mu}^{(1)} O_{\tau}^{(1)} \psi_c(\vec{r}_1) D_j(\vec{r}_1, \vec{r}_2) \\ \times \bar{\psi}_b(\vec{r}_2) \Gamma_j^{(2)\mu} O_{\tau}^{(2)} \psi_d(\vec{r}_2) \quad (36)$$

$$V_{abcd}^{(2)} = \int d^3 r_1 \int d^3 r_2 \sum_j \bar{\psi}_a(\vec{r}_1) \Gamma_{j\mu}^{(1)} O_{\tau}^{(1)} \psi_c(\vec{r}_1) \frac{\partial D_j(\vec{r}_1, \vec{r}_2)}{\partial \rho(\vec{r}_1)} \\ \times \rho_j^{\mu}(\vec{r}_2) \bar{\psi}_b(\vec{r}_2) \mathbf{1}^{(2)} \psi_d(\vec{r}_2)$$
(37)

$$V_{abcd}^{(3)} = 1/2 \int d^3 r_1 \int d^3 r_2 \sum_j \bar{\psi}_a(\vec{r_1}) \mathbf{1}^{(1)} \psi_c(\vec{r_2}) \frac{\partial D_j^2(\vec{r_1}, \vec{r_2})}{\partial \rho^2(\vec{r_1})}$$

 $\times \rho_{j\mu}(\vec{r}_1) \rho_i^{\mu}(\vec{r}_2) \bar{\psi}_b(\vec{r}_2) 1^{(2)} \psi_d(\vec{r}_2)$ (38)

$$V_{abcd}^{(4)} = \int d^{3}r_{1} \int d^{3}r_{2} \sum_{j} \bar{\psi}_{\bar{s}}(\vec{r}_{1}) \mathbf{1}^{(1)} \psi_{c}(\vec{r}_{2}) \frac{\partial D_{j}(\vec{r}_{1}, \vec{r}_{2})}{\partial \rho(\vec{r}_{1})} \times \rho_{j\mu}(\vec{r}_{1}) \bar{\psi}_{b}(\vec{r}_{1}) \Gamma_{j}^{(2)\mu} O_{\tau}^{(2)} \psi_{d}(\vec{r}_{2}), \quad (39)$$

## Separable pairing matrix elements

In 2009 Y. Tian, Z. Y. Ma and P. Ring introduced separable pairing in  ${}^{1}S_{0}$  channel gap equation of symmetric nuclear matter:

$$\Delta(k) = -\int_0^\infty \frac{k'^2 dk'}{2\pi^2} \langle k | V_{sep}^{1} S_0 | k' \rangle \frac{\Delta(k')}{2E(k')}, \qquad (40)$$

where

$$\langle k|V_{sep}^{^{1}S_{0}}|k'\rangle = -Gp(k)p(k'), \qquad (41)$$

with Gaussian ansatz

$$p(k) = e^{-a^2k^2}.$$
 (42)

These two parameters a and G were fitted to density dependence of the gap at the Fermi surface in nuclear matter. After transformation the separable force from momentum to coordinate space we obtained:

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_1, \vec{r}_2) = -G\delta(\vec{R}_1 - \vec{R}_2)P(r)P(r')\frac{1 - \hat{P}^{\sigma}}{2}$$
(43)

with  $\vec{r} = 1/\sqrt{2}(\vec{r_1} - \vec{r_2})$  and  $\vec{R} = 1/\sqrt{2}(\vec{r_1} + \vec{r_2})$ . Values of G = 728 MeV fm<sup>3</sup> and  $a_0 = 0.644$  fm

# Separable pairing matrix elements

After Fourier transformation:

$$P(r) = \frac{e^{-r^2/(2a^2)}}{(4\pi a^2)^{3/2}}$$
(44)

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Due to coordinate transformation from laboratory to center of mass and relative coordinates we need to use Talmi-Moschinsky brackets:

$$|n_1l_1, n_2l_2; \lambda\mu\rangle = \sum_{NLnl} M_{n_1l_1n_2l_2}^{NLnl} |NL, nl; \lambda\mu\rangle$$
(45)

If we work in the basis of spherical harmonic oscillator:

$$I_n = \sqrt{4\pi} \int R_{nl}(r) P(r) r^2 dr = \frac{1}{2^{2/3} \pi^{3/4} b^{3/2}} \frac{(1-\alpha^2)^n}{(1+\alpha^2)^{n+3/2}} \frac{\sqrt{2n+1}}{2^n n!}, \quad (46)$$

the coupled matrix element for T = 1 pairing:

$$V_{abcd}^{JM} = G\hat{j}_{a}\hat{j}_{b}\hat{j}_{c}\hat{j}_{d}(-1)^{l_{b}+l_{d}+j_{a}+j_{c}} \left\{ \begin{array}{cc} l_{a} & j_{a} & 1/2 \\ j_{b} & l_{b} & J \end{array} \right\} \left\{ \begin{array}{cc} l_{c} & j_{c} & 1/2 \\ j_{d} & l_{d} & J \end{array} \right\} \\ \sum_{Nnn'} l_{n}l_{n'} M_{nal_{a}n_{b}l_{b}}^{NJn'0} M_{ncl_{c}n_{d}l_{d}}^{NJn'0}$$
(47)

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## Collective excitations



Figure 2: The Isoscalar monopole (left) and isovector dipole (right) strength distribution for <sup>208</sup>Pb. ISGMR centroid energy is on 14.2 (exp. 13.96  $\pm$  0.20 MeV, while for RPA IVGDR predicts the excitation energy on 13.7 (exp. 13.3 MeV).

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## Isoscalar monopole collective excitation



Figure 3: The Isoscalar monopole strength distribution for even-even nuclei A = 108 - 124 with  $\Delta A = 8.$ 

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# Proton-neutron (R)QRPA

RPA equations:

$$\begin{pmatrix} A & B \\ B* & A* \end{pmatrix} \begin{pmatrix} X^{\lambda} \\ Y^{\lambda} \end{pmatrix} = E_{\lambda} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$
(48)

$$\begin{aligned} A_{pn,p'n'} &= H^{11}_{pp'}\delta_{nn'} + H^{11}_{nn'}\delta_{pp'} + \left(u_{p}v_{n}u_{p'}v_{n'} + v_{p}u_{n}v_{p'}u_{n'}\right) \\ &\times V^{ph}_{pn'np'} + \left(u_{p}u_{n}u_{p'}u_{n'} + v_{p}v_{n}v_{p'}v_{n'}\right)V^{pp}_{pnp'n'}, \end{aligned}$$
(49)

$$B_{pn,p'n'} = (u_{p}v_{n}v_{p'}u_{n'} + v_{p}u_{n}u_{p'}v_{n'}) V_{pp'nn'}^{ph} + (u_{p}u_{n}v_{p'}v_{n'} + v_{p}v_{n}u_{p'}u_{n'}) V_{pnp'n'}^{pp}.$$
 (50)

Non-diagonal  $H^{11}$  is defined as

$$H^{11} = (u_i u_j + v_i v_j) h_{ij} + (u_i v_j - v_i u_j) \Delta_{ij}$$
(51)

# Proton-neutron (R)QRPA

Isovector-vector part of PN-(R)QRPA survives only for  $\mathsf{V}^{(1)}$  type of matrix elements. In the case of spacelike components we have:

$$V_{abcd}^{(1s)} = -\int d^3 r_1 \int d^3 r_2 \psi_a^{\dagger}(\vec{r}_1) (\vec{\tau} \gamma_0 \gamma_{\mu})^{(1)} \psi_c(\vec{r}_1) \\ \times \alpha_{TV}(\rho) \delta(\vec{r}_1 - \vec{r}_2) \psi_b^{\dagger}(\vec{r}_2) (\vec{\tau} \gamma_0 \gamma^{\mu})^{(2)} \psi_d(\vec{r}_2), \quad (52)$$

and timelike components:

$$V_{abcd}^{(1t)} = \int d^3 r_1 \int d^3 r_2 \psi_a^{\dagger}(\vec{r}_1) \vec{\tau}^{(1)} \psi_c(\vec{r}_1) \alpha_{TV}(\rho) \\ \times \delta(\vec{r}_1 - \vec{r}_2) \psi_b^{\dagger}(\vec{r}_2) \vec{\tau}^{(2)} \psi_d(\vec{r}_2).$$
(53)

## Isovector-vector matrix elements

If the radial part of wave function is given by:

$$\psi(r) = \begin{pmatrix} f(r) \\ ig(r) \end{pmatrix}$$
(54)

by coupling of angular momentum, we get coupled matrix elements for spacelike part:

$$V_{abcd}^{(1s)J} = 2\hat{J}^{-2}\sum_{L}\int drr^{2}\alpha_{TV}(f_{a}(r)g_{c}(r)\langle j_{a}||(\sigma_{S}Y_{L})_{J}||\bar{j}_{c}\rangle - g_{a}(r)f_{c}(r)\langle \bar{j}_{a}||(\sigma_{S}Y_{L})_{J}||j_{c}\rangle)(f_{b}(r)g_{d}(r)\langle \bar{j}_{d}||(\sigma_{S}Y_{L})_{J}||j_{b}\rangle - g_{b}(r)f_{d}(r)\langle j_{d}||(\sigma_{S}Y_{L})_{J}||\bar{j}_{b}\rangle), \quad (55)$$

and timelike part

$$V_{abcd}^{(1t)J} = 2J^{-2} \int dr r^2 \alpha_{TV} (f_a(r) f_c(r) + g_a(r) g_c(r)) (f_b(r) f_d(r) + g_b(r) g_d(r)) \langle j_a || Y_J || j_c \rangle \langle j_d || Y_J || j_b \rangle,$$
(56)

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# Isovector-vector matrix elements

Angular part is given by expressions:

$$\langle j_{a}||Y_{J}||j_{c}\rangle = \frac{1+(-1)^{l_{a}+l_{c}+J}}{2}\frac{\hat{j}_{a}\hat{j}_{c}\hat{J}}{\sqrt{4\pi}}(-1)^{j_{a}-1/2}\begin{pmatrix} j_{a} & J & j_{c} \\ -1/2 & 0 & 1/2 \end{pmatrix}, \quad (57)$$

and

$$\langle j_{a}||(\sigma_{5}Y_{L})_{J}||j_{c}\rangle = \frac{1+(-1)^{l_{a}+l_{c}+J}}{2} \frac{\hat{j}_{a}\hat{j}_{c}\hat{J}\hat{L}}{\sqrt{4\pi}} (-1)^{l_{a}+L} \\ \times \left[ (-1)^{l_{c}+j_{c}+1/2} \begin{pmatrix} 1 & L & J \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} j_{a} & J & j_{c} \\ -1/2 & 0 & 1/2 \end{pmatrix} - \\ \sqrt{2} \begin{pmatrix} 1 & L & J \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} j_{a} & J & j_{c} \\ 1/2 & -1 & 1/2 \end{pmatrix} \right].$$
(58)

### Isovector-pseudovector matrix elements

In order to describe the effects of pions in multipolar transitions in PN-RQRPA we use isovector-pseudovector coupling. However, we expect that the strength of pion-nucleon in nuclei coupling should be somewhat reduced by factor of g':

$$V_{PV} = -g' \left(\frac{f_{\pi}}{m_{\pi}}\right)^2 \delta(\vec{r}_1 - \vec{r}_2) \left(\gamma_0 \gamma_5 \gamma_{\mu} \vec{\tau}\right)^{(1)} \left(\gamma_0 \gamma_5 \gamma^{\mu} \vec{\tau}\right)^{(2)}, \quad (59)$$

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where  $f_{\pi}$  decay constant and  $m_{\pi}$  mass of pion. The value of g' we don't know a priori and should be deduced from experiment.

For timelike part pseudovector matrix elements look like:

$$V_{abcd}^{PV(t)J} = 2g' \hat{J}^{-2} \left(\frac{f_{\pi}}{m_{\pi}}\right)^2 \int dr r^2 (f_a(r)g_c(r) - g_a(r)f_c(r)) \\ \times (f_b(r)g_d(r) - g_b(r)f_d(r))\langle j_a||Y_J||\bar{j}_c\rangle \langle \bar{j}_d||Y_J||j_c\rangle, \quad (60)$$

Timelike matrix elements are non-zero only in case of unnatural parity transitions, like Gammow-Teller transition.

## Isovector-pseudovector matrix elements

For spacelike part isovector-pseudovector matrix elements look like:

$$V_{abcd}^{PV(s)J} = 2g'\hat{J}^{-2}\left(\frac{f_{\pi}}{m_{\pi}}\right)^{2}\sum_{L}\int drr^{2}(f_{a}(r)f_{c}(r) \times \langle j_{a}||(\sigma_{S}Y_{L})_{J}||j_{c}\rangle + g_{a}(r)g_{c}(r)\langle \bar{j}_{a}||(\sigma_{S}Y_{L})_{J}||\bar{j}_{c}\rangle)(f_{b}(r)f_{d}(r) \times \langle j_{d}||(\sigma_{S}Y_{L})_{J}||j_{b}\rangle + g_{b}(r)g_{d}(r)\langle \bar{j}_{d}||(\sigma_{S}Y_{L})_{J}||\bar{j}_{b}\rangle).$$
(61)

Spacelike isovector-pseudovector matrix elements contribute to both unnatural and natural parity transitions, but in opposite manner compared to isovector-vector matrix elements.

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# Separable pairing in the case of PN-(R)QRPA

Coupled pp-elements for PN-(R)QRPA are given by:

$$V_{abcd}^{JM} = -G\hat{j}_{a}\hat{j}_{b}\hat{j}_{c}\hat{j}_{d}\sum_{LS} \left(1 - (-1)^{S+T}\right)\hat{S}^{2}\hat{L}^{2} \left\{ \begin{array}{cc} I_{b} & 1/2 & j_{b} \\ I_{a} & 1/2 & j_{a} \\ L & S & J \end{array} \right\} \\ \left\{ \begin{array}{cc} I_{d} & 1/2 & j_{d} \\ I_{c} & 1/2 & j_{c} \\ L & S & J \end{array} \right\} \sum_{nn'} I_{n}I_{n}'M_{n_{a}I_{a}n_{b}I_{b}}^{NLn'0}M_{n_{c}I_{c}n_{d}I_{d}}^{NLn'0}$$
(62)

This is non-vanishing only for S = 0 and T = 1 or S = 1 and T = 0. Assumed form of pp-matrix elements:

$$V_{abcd}^{JM} = \langle ab | V(T=0; S=1) | cd \rangle^{JM} \times V_0 + \langle ab | V(T=1; S=0) | cd \rangle^{JM}$$
(63)

We don't know a priori the value of factor  $V_0$ . It can be somewhat reduced or enhanced compared to the case T = 1.

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### Determination of parameter g'



Figure 4: The Gammow-Teller strength distribution for <sup>208</sup>Pb for different values of parameter g', calculated with DD-PC1 interaction. The experimental value of the position of  $GT^-$  main peak in <sup>208</sup>Pb is on 19.2 MeV. This corresponds to g' = 0.51. The width of Lorentzian is 1 MeV.

# Gammow-Teller resonance



Figure 5: The Gammow-Teller strength distribution for  ${}^{48}Ca {}^{90}Zr$  and  ${}^{208}Pb$ , calculated by DD-PC1 interaction. The theoretical (experimental) values of central position GTR main peak are 11.0 (10.5) MeV for  ${}^{48}Ca$  and 16.6 (15.6) MeV for  ${}^{90}Zr$ .

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### Isobaric analog resonance



Figure 6: The isobar analog state strength distribution for  ${}^{48}Ca$ ,  ${}^{90}Zr$  and  ${}^{208}Pb$ , calculated by DD-PC1. The theoretical (experimental) values of central position GTR main peak are 7.05 (7.17) MeV for  ${}^{48}Ca$  and 11.7 (12.0 $\pm$ 0.9) MeV for  ${}^{90}Zr$  and 18.47 (18.83 $\pm$ 0.02) MeV for  ${}^{208}Pb$ .

# Isobaric analog resonance in Sn isotopes



Figure 7: PN-(R)QRPA strength distribution for even-even nuclei with A = 108 - 130, calculated by DD-PC1 with separable pairing interaction.

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Figure 8: Calculated and experimental IAR position for Sn isotopes.

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# Gammow-Teller resonance in <sup>118</sup>Sn



Figure 9: The Gammow-Teller strength distribution for <sup>118</sup>Sn for different values of parameter  $V_0$ . Notice, only T = 0 pp matrix elements are nonvanishing.

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# Standard model



Figure 10: Standard model.

## Why neutrino-nucleus reactions?

Neutrino mass hierarchy still remains an open problem due to insensitivity of previous and nowadays experiments on sign of difference between squares of neutrino masses.



Figure 11: Possible neutrino mass ordering (hierarchy). Degenerate case excluded from figure.

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Proton-neutron (R

Results - part 2 Neutrino-nucleus reactions

## Why neutrino-nucleus reactions?



Figure 12: Supernova from neutrino perspective.

## Why neutrino-nucleus reactions?

We need to choose adequate target material for detector in order to achieve good statistics and include or exclude particular neutrino species in a specific energy range.



Figure 13: The inclusive  $\nu_e(\bar{\nu}_e)$  cross sections with <sup>12</sup> C, <sup>56</sup>Fe and <sup>208</sup>Pb as a function of  $\nu_e(\bar{\nu}_e)$  energy (charged current reactions only). (see refs. D. Vale, N. Paar AIP Conference Proceedings 1681 (1), 050011; N. Paar, T. Marketin, D. Vale, D. Vretenar International Journal of Modern Physics E 24 (09), 1541004.

# Why neutrino-nucleus reactions?



Figure 14: Example of multipole decomposition of the flux averaged cross sections for  $\nu_e - {}^{208}Pb$  charged current reaction. see ref. D Vale, T Rauscher, N Paar Journal of Cosmology and Astroparticle Physics 2016 (02), 007

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### Why neutrino-nucleus reactions?



FIG. 6: The ratios  $r_{acc}$  of the number of the detector events induced in mineral oil (CH<sub>2</sub>), water (H<sub>2</sub>O), and <sup>208</sup>Pb for the incoming (anti)neutrino fluxes of the accretion phase in normal (NH) and inverted (IH) mass hierarchy: (a)  $r_{tere, p}^{ree, p, Pb} [10^{-1}]$ (b)  $r_{tere, p, n}^{ree, p, Pb} (c) <math>r_{ete, r, tot, n}^{ree, p, rbe} [10^{-1}]$ . The three columns correspond to (i)  $\langle E_{\nu_{\alpha}}^{e} \rangle = 8$  MeV,  $\langle E_{\nu_{\alpha}}^{0} \rangle = 13$  MeV; (ii)  $\langle E_{\nu_{\alpha}}^{e} \rangle = 10$  MeV,  $\langle E_{\nu_{\alpha}}^{0} \rangle = 15$  MeV; and (iii)  $\langle E_{\nu_{\alpha}}^{0} \rangle = 12$  MeV,  $\langle E_{\nu_{\alpha}}^{0} \rangle = 19$  MeV. A1, A2, A3 denote the combinations of the luminosity ratios for the accretion phase given in Tab. []

Proton-neutron (R)

## Why neutrino-nucleus reactions?



FIG. 7: The ratios  $r_{cool}$  of the number of the detector events induced in mineral oil (CH<sub>2</sub>), water (H<sub>2</sub>O), and <sup>208</sup>Pb for the incoming (anti)neutrino fluxes of the cooling phase in normal (NH) and inverted (IH) mass hierarchy for three configurations of initial  $\nu$  average energies. The same notation as in Fig. <sup>6</sup> applies. C1, C2, C3 denote the combinations of the luminosity ratios for the cooling phase given in Tab. <sup>[1]</sup>

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