RPA approaches and effective interactions

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A rendering of the future RAON complex, under construction in Daejeon

Overview

- Introduction: the general issue
- Introduction: the specifics
  - Transformed NN interactions via UCOM, SRG
  - "Self-consistent" Second RPA
- Applications of transformed NN interactions in (S)RPA
  - Giant resonances: centroids
  - Giant resonances: fine structure
- Some comments on...
  - Three-nucleon interactions
  - Ground-state correlations
- Outlook

see also: Hans Feldmeier

see also: Marcella Grasso

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The (non-relativistic) nuclear many-body problem

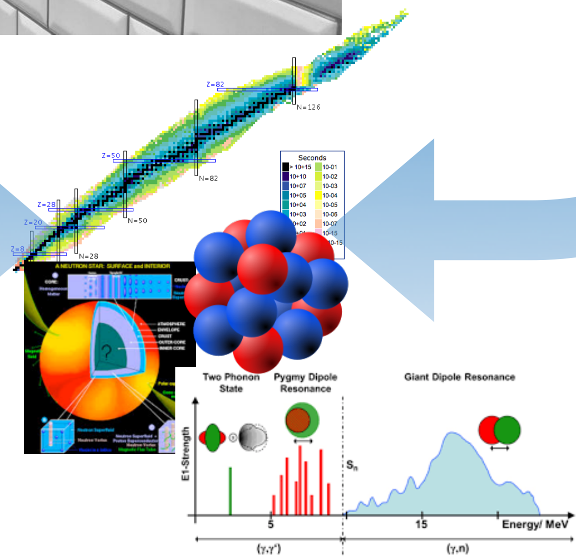
❖ **A** interacting nucleons

NN(N) interactions

Schroedinger equation

Effective/transformed Hamiltonian

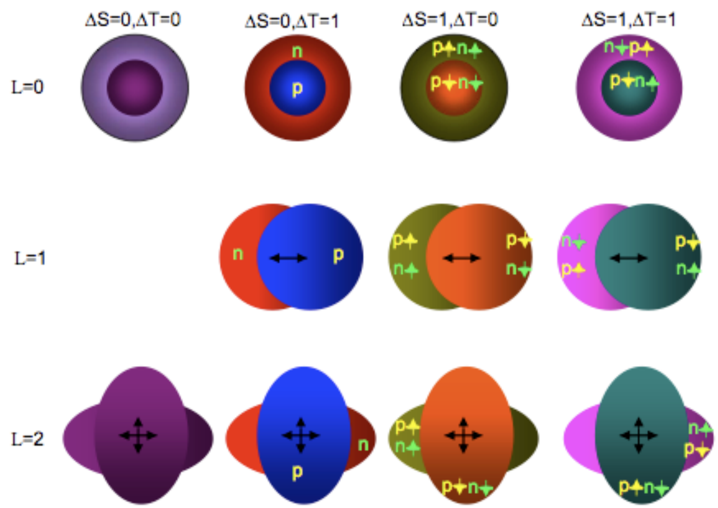
Quantum many-body method



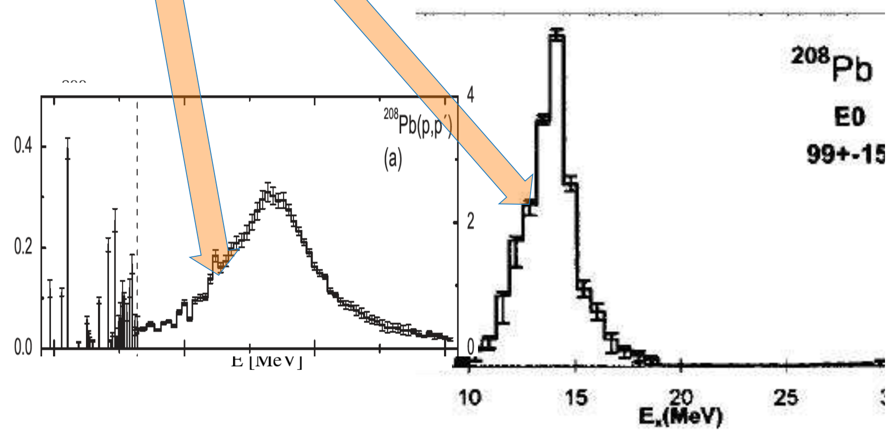
Normal modes of vibration

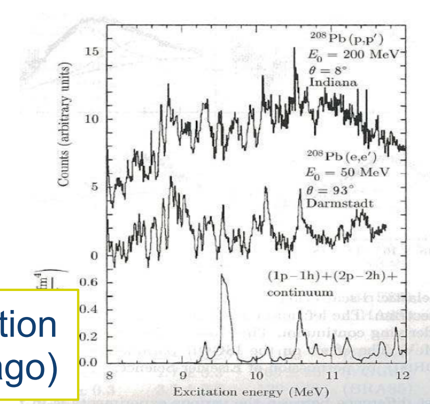
Major part of the EWSR

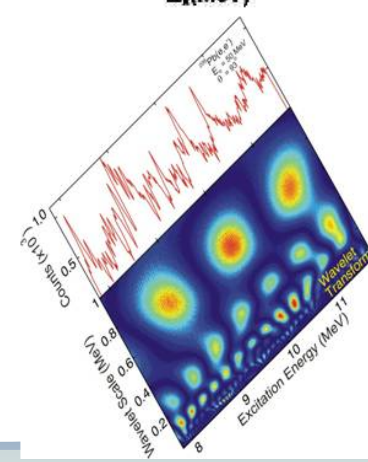
The simplified picture



The reality







SRPA explored to explain attenuation (~30yrs ago)

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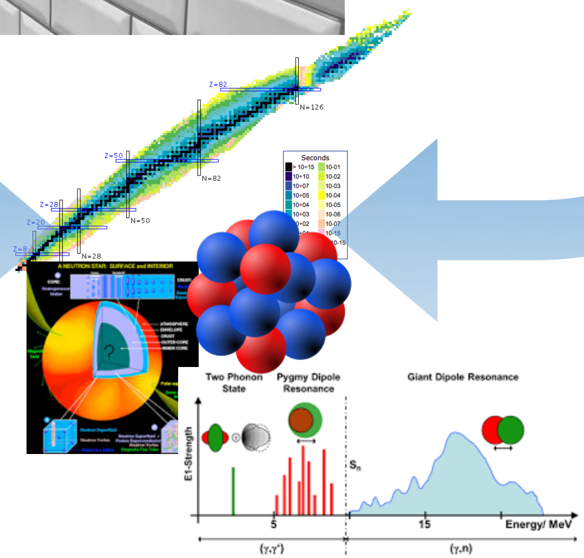
Schroedinger equation

Effective/transformed Hamiltonian

Quantum many-body method



The (non-relativistic) nuclear many-body problem

- **A** interacting nucleons

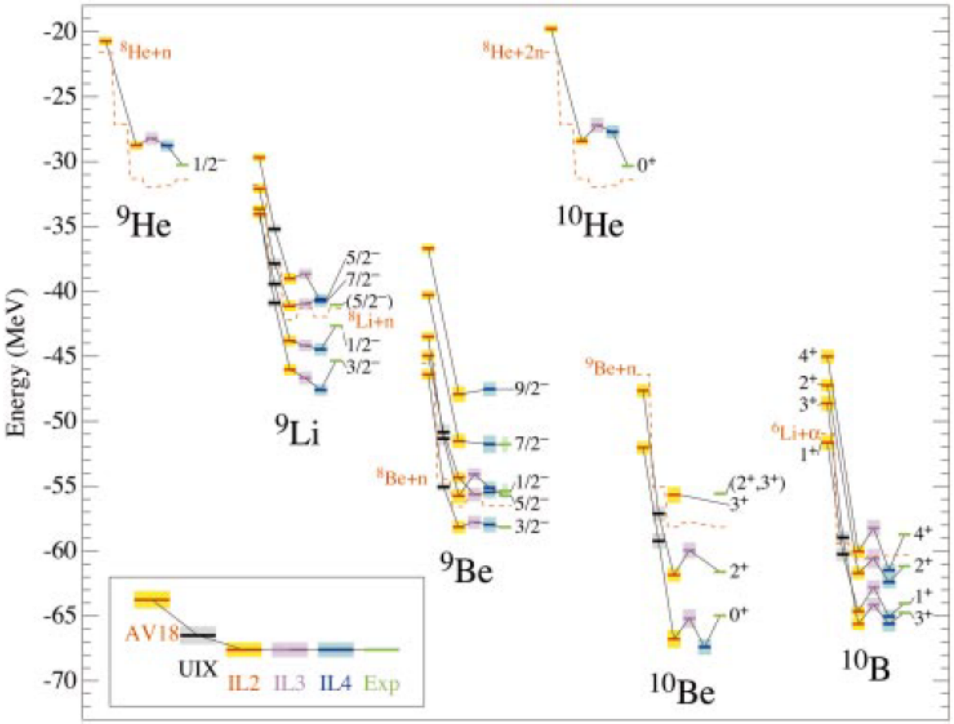
NN(N) interactions

chiral or directly fitted to few-nucleon data

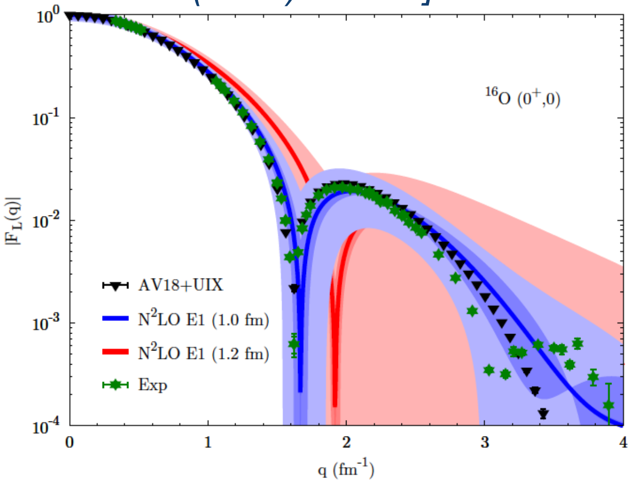
Schroedinger equation

Quantum many-body method

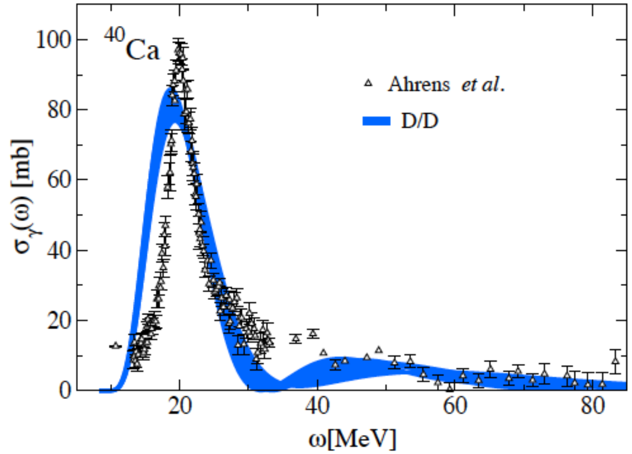
QMC [Pieper et al., PRC66(2002)044310]



QMC [Lonardoni et al., PRC97(2018)044318]



CCM+LIT [Simonis et al, arXiv: 1905.02055]



The (non-relativistic) nuclear many-body problem

❖ **A** interacting nucleons

NN(N) interactions

Schroedinger equation

Hartree-Fock, RPA
("Mean field")

standard DFT

... where a couple of MeV discrepancy in the total mass of heavy nuclei is seen as a failure!

The (non-relativistic) nuclear many-body problem

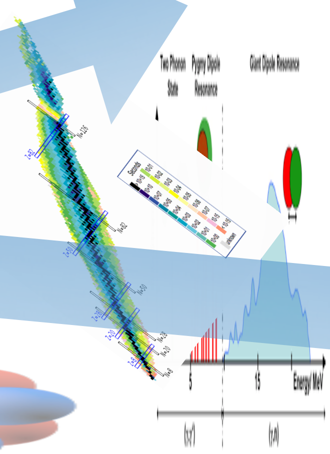
❖ **A** interacting nucleons

NN(N) interactions

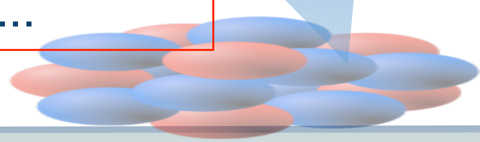
Schroedinger equation

Effective/transformed
Hamiltonian

Quantum many-body
method



double counting,
undercounting,
divergencies,
...



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Transformed interactions via UCOM, SRG

See review by Roth,Neff,Feldmeier, PPNP65(2010)50

❖ Unitary correlation operator method (UCOM)

$$\langle \Psi | H | \Psi' \rangle = \langle \Phi | C^{\dagger} H C | \Phi' \rangle = \langle \Phi | C^{-1} H C | \Phi' \rangle = \langle \Phi | \widehat{H} | \Phi' \rangle$$

❖ Similarity renormalization group (SRG)

$$\frac{d}{d\alpha}\tilde{H}_\alpha = [\eta_\alpha, \tilde{H}_\alpha] \qquad \frac{d}{d\alpha}U_\alpha = -U_\alpha \eta_\alpha$$

❖ Starting Hamiltonian:

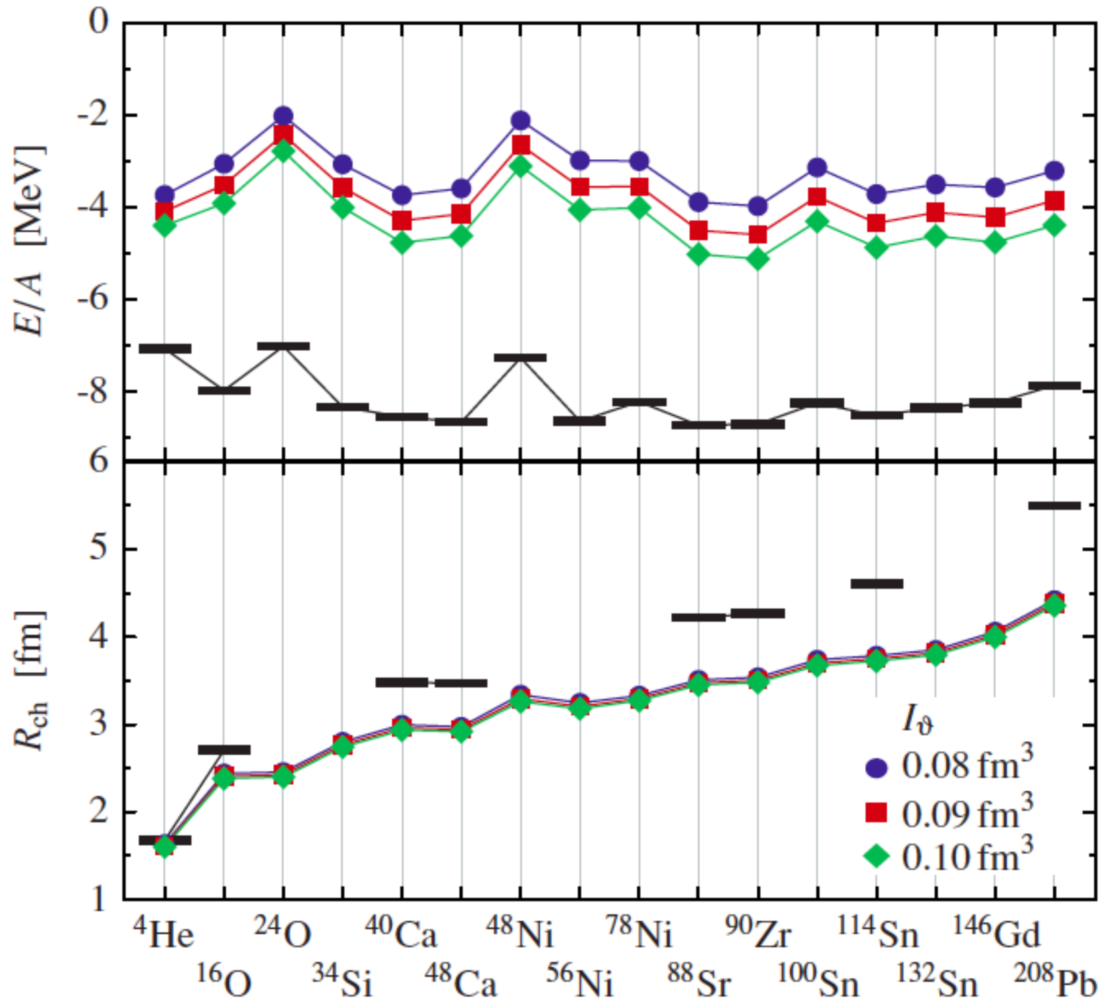
- Precisely fit phenomenological (e.g., Argonne V18)
- From chiral EFT

❖ Prediagonalized Hamiltonian: to be used within tractable Hilbert spaces – How tractable?

Hartree-Fock

❖ AV18+UCOM

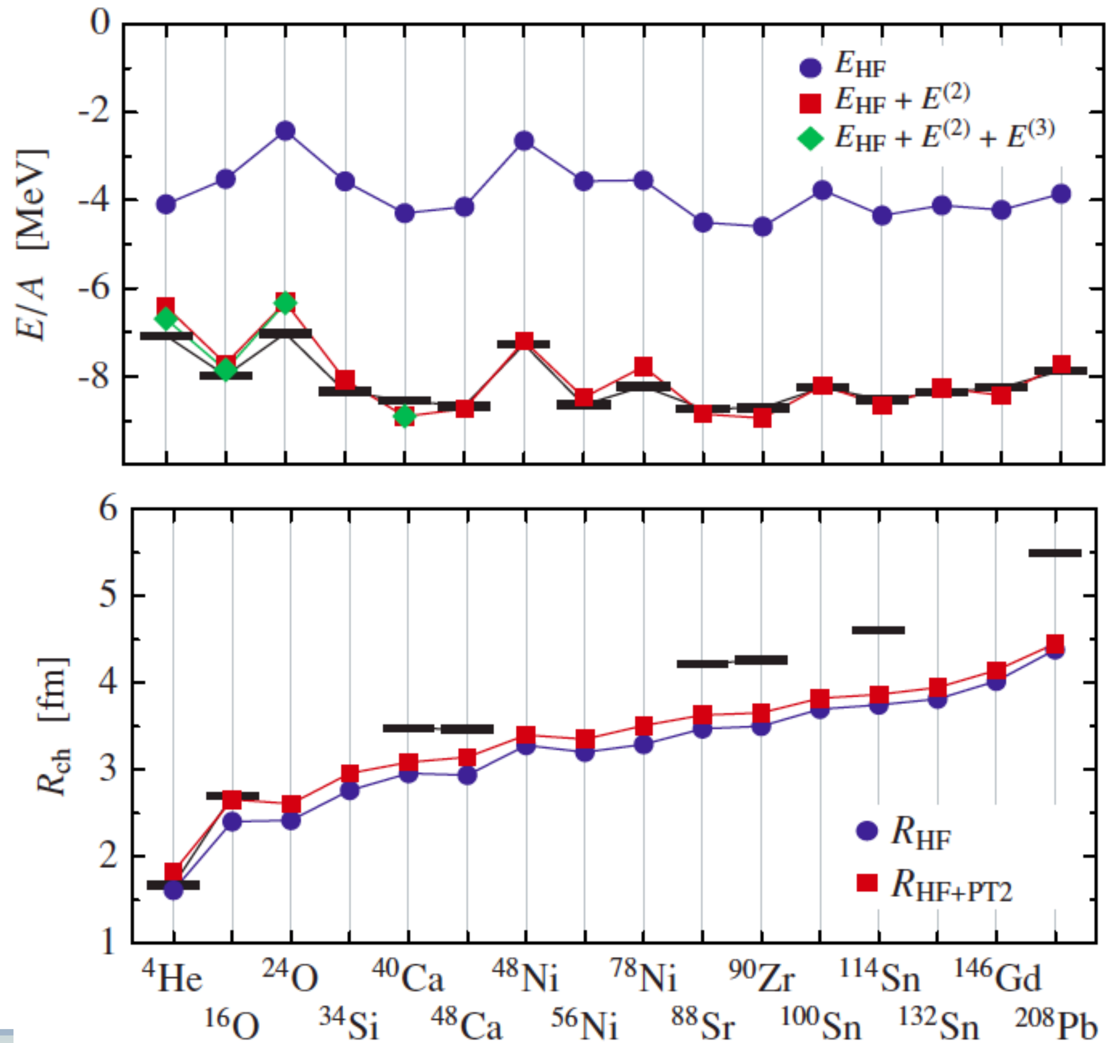
[Roth,PP, et al., PRC73(06)044312]



Corrections from perturbation theory

❖ AV18+UCOM

[Roth,PP, et al., PRC73(06)044312]



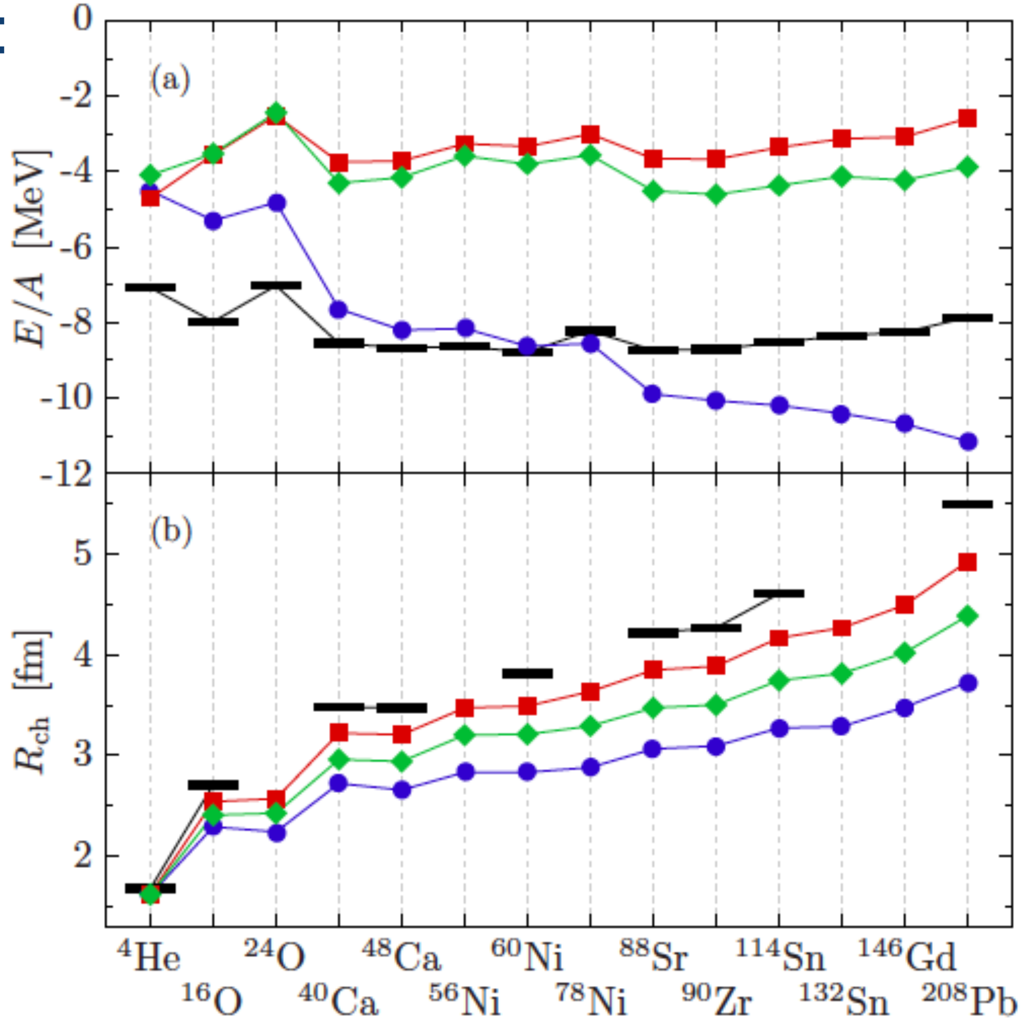
Hartree-Fock

❖ AV18 transformed via:

original UCOM

UCOM-SRG

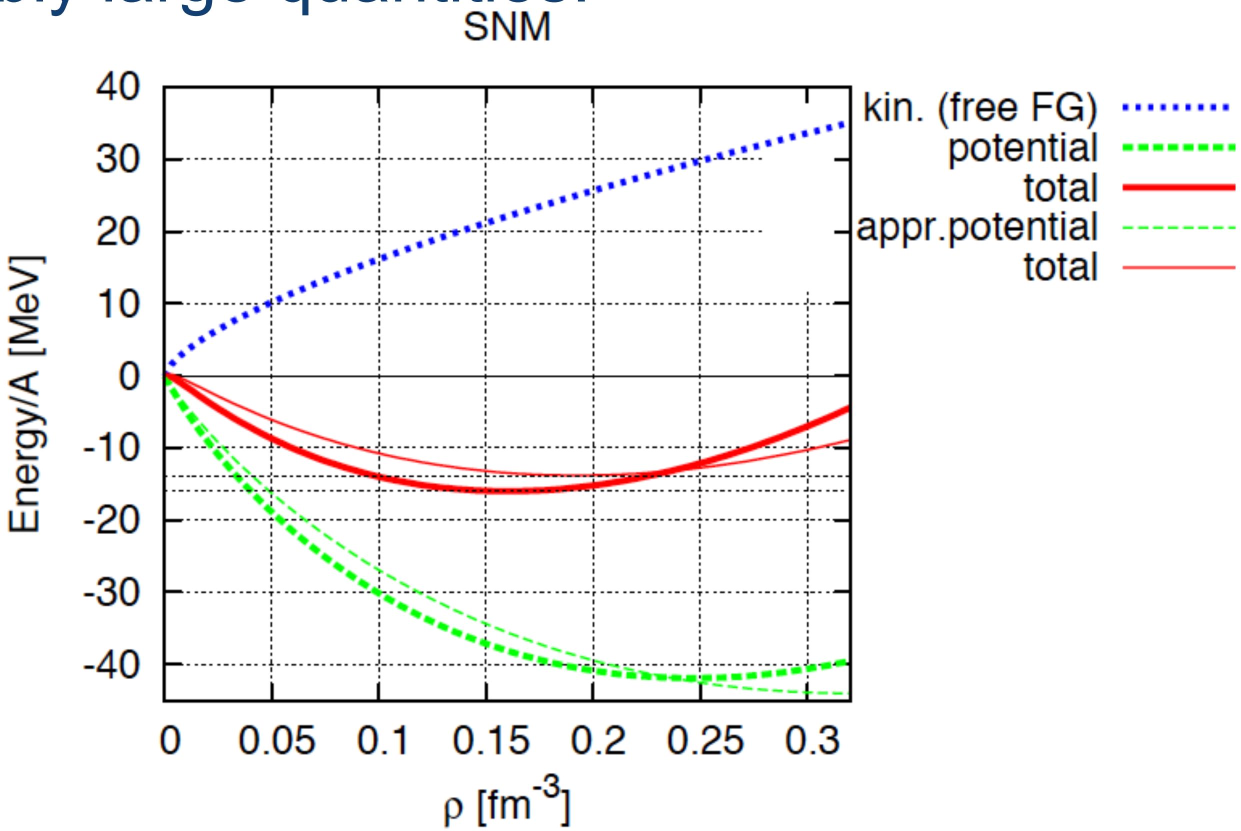
SRG



[Roth et al.,PRC77(2008)064003]

The saturation point is a delicate thing

❖ The total energy is the difference of two comparably large quantities:



❖ Small residual 3N effects can play a critical role

Standard RPA

- Vibration creation operator:

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^\nu O_{ph}^\dagger - \sum_{ph} Y_{ph}^\nu O_{ph} \quad ; \quad Q_\nu|\mathrm{RPA}\rangle = 0 \quad ; \quad Q_\nu^\dagger|\mathrm{RPA}\rangle = |\nu\rangle$$

- Standard RPA - the RPA vacuum is approximated by the HF ground state:

$$\langle \mathrm{RPA}|\ldots|\mathrm{RPA}\rangle \rightarrow \langle \mathrm{HF}|\ldots|\mathrm{HF}\rangle \quad ; \quad O_{ph}^\dagger \rightarrow a_p^\dagger a_h$$

- RPA equations in $ph-$space:

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \hbar\omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

$$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H_{hp',ph'} \quad ; \quad B_{ph,p'h'} = H_{hh',pp'} \quad ;$$

For some in-medium Hamiltonian H

☞ Self-consistent HF+RPA: spurious state and sum rules

- Equivalent to small-amplitude Time-Dependent Hartree-Fock
- Linear Response Theory

Second RPA

- Vibration creation operator: Includes $2p2h$ configurations

$$Q_\nu^\dagger = \sum_{ph} X^\nu_{ph} O^\dagger_{ph} - \sum_{ph} Y^\nu_{ph} O_{ph} + \sum_{p_1h_1p_2h_2} \mathcal{X}^\nu_{p_1h_1p_2h_2} O^\dagger_{p_1h_1p_2h_2}$$
$$- \sum_{p_1h_1p_2h_2} \mathcal{Y}^\nu_{p_1h_1p_2h_2} O_{p_1h_1p_2h_2}$$

- The SRPA vacuum is approximated by the HF ground state:

$$\langle \mathrm{SRPA}|\ldots|\mathrm{SRPA}\rangle \rightarrow \langle \mathrm{HF}|\ldots|\mathrm{HF}\rangle$$

- SRPA equations in $ph \oplus 2p2h-$space:

$$\left(\begin{array}{cc|cc} A & \mathcal{A}_{12} & B & 0 \\ \mathcal{A}_{21} & \mathcal{A}_{22} & 0 & 0 \\ \hline -B^* & 0 & -A^* & -\mathcal{A}^*_{12} \\ 0 & 0 & -\mathcal{A}^*_{21} & -\mathcal{A}^*_{22} \end{array}\right) \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right) = \hbar\omega_\nu \left(\begin{array}{c} X^\nu \\ \mathcal{X}^\nu \\ \hline Y^\nu \\ \mathcal{Y}^\nu \end{array}\right)$$

$$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H_{hp',ph'} \;\; ; \;\; B_{ph,p'h'} = H_{hh',pp'}$$

For some in-medium Hamiltonian H

$\mathcal{A}_{12}$: interactions between ph and $2p2h$ states

$\mathcal{A}_{22}$: $\delta_{p_1p'_1}\delta_{h_1h'_1}\delta_{p_1p'_1}\delta_{h_1h'_1}(e_{p_1} + e_{p_2} - e_{h_1} - e_{h_2})$ + interactions among $2p2h$ states

Drożdż, Nishizaki, Speth, Wambach; Yannouleas; and others, early '90s

Second RPA

- Vibration creation operator: Includes $2p2h$ configurations

$$Q^\dagger_\nu = \sum_{ph} X^\nu_{ph} O^\dagger_{ph} - \sum_{ph} Y^\nu_{ph} O_{ph} + \sum_{p_1h_1p_2h_2} \mathcal{X}^\nu_{p_1h_1p_2h_2} O^\dagger_{p_1h_1p_2h_2}$$
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$$\langle \mathrm{SRPA}|\ldots|\mathrm{SRPA}\rangle \to \langle \mathrm{HF}|\ldots|\mathrm{HF}\rangle$$

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$A_{ph,p'h'} = \delta_{pp'}\delta_{hh'}(e_p - e_h) + H$

$\mathcal{A}_{12}$: interactions between ph

$\mathcal{A}_{22}$: $\delta_{p_1p'_1}\delta_{h_1h'_1}\delta_{p_1p'_1}\delta_{h_1h'_1}(e_{p_1} + $

linear response with collision term

Drożdż, Nishizaki, Speth, Wambach; Yannouleas; and others, early '90s

Formal properties of RPA...

[D.J.Rowe, Rev.Mod.Phys.40(1968)153]
[D.J.Thouless, Nucl.Phys.22(1961)78]

- Solutions appear in adjoint pairs
- If the stability condition is satisfied,

$$\langle 0|[F^\dagger,[H,F]]|0\rangle = \sum_{ab} F_a^* M_{ab} F_b \geq 0$$

solutions are real

- The EWSR is preserved $\left(\sum_n (E_n - E_0)|\langle 0|V|n\rangle|^2 = \int d^3r \rho_0(\vec{r}) \frac{(\vec{\nabla}V)^2}{2m} \right)$

$$\sum_{n:N_n=1} |\langle 0|O|n\rangle|^2 E_{n0} = \frac{1}{2}\langle 0|[O,H,O]|0\rangle$$

- ... and therefore spurious transitions have zero energy (restoration of symmetries)

|HF) = vacuum of ph states ➔ stability of RPA for physical states

... are not identical in SRPA

[PP, PRC90(2014)024305]

- Solutions still appear in adjoint pairs
- However, the stability condition is violated and spurious states appear at non-zero energy...
- Note that the rhs is still zero:

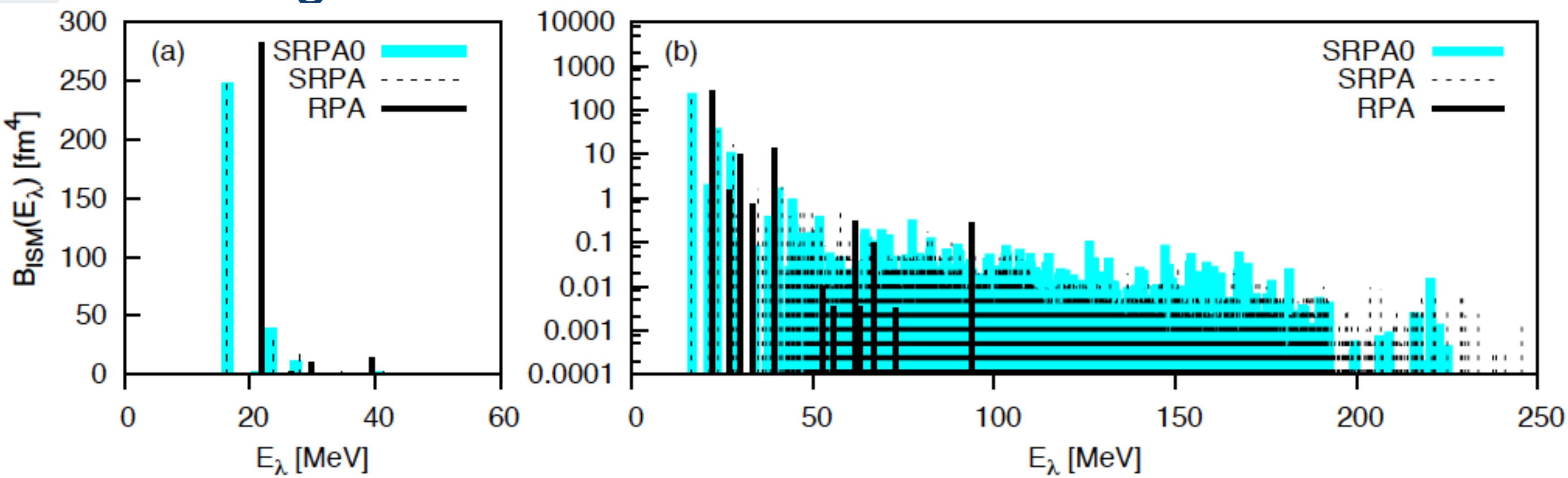
$$\sum_{n:N_n=1} |\langle 0|O|n\rangle|^2 E_{n0} = \frac{1}{2}\langle 0|[O,H,O]|0\rangle$$

- Does this mean that the EWSR is not conserved?
- **The EWSR is conserved**: the difference is that there are positive-energy states with negative norm / negative-energy states with positive norm
- Has been verified numerically

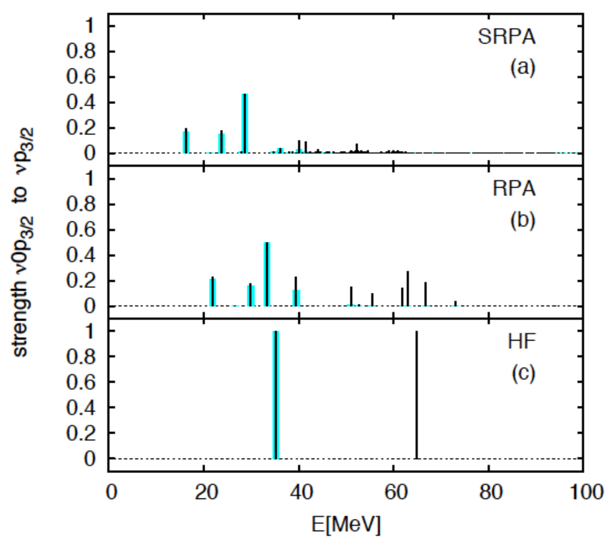
RPA vs SRPA: a numerical demonstration

first "self-concistent" SRPA calculations: [PP &Roth, PRC81(2010)024317]

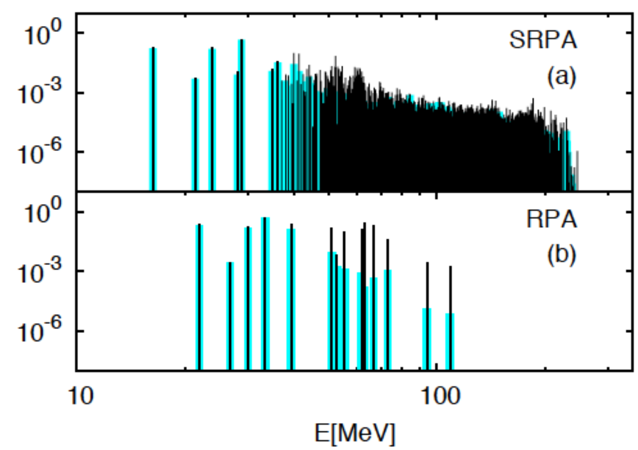
❖ Strength distribution



❖ ph states:



log scale:



^{16}O, 7 shells

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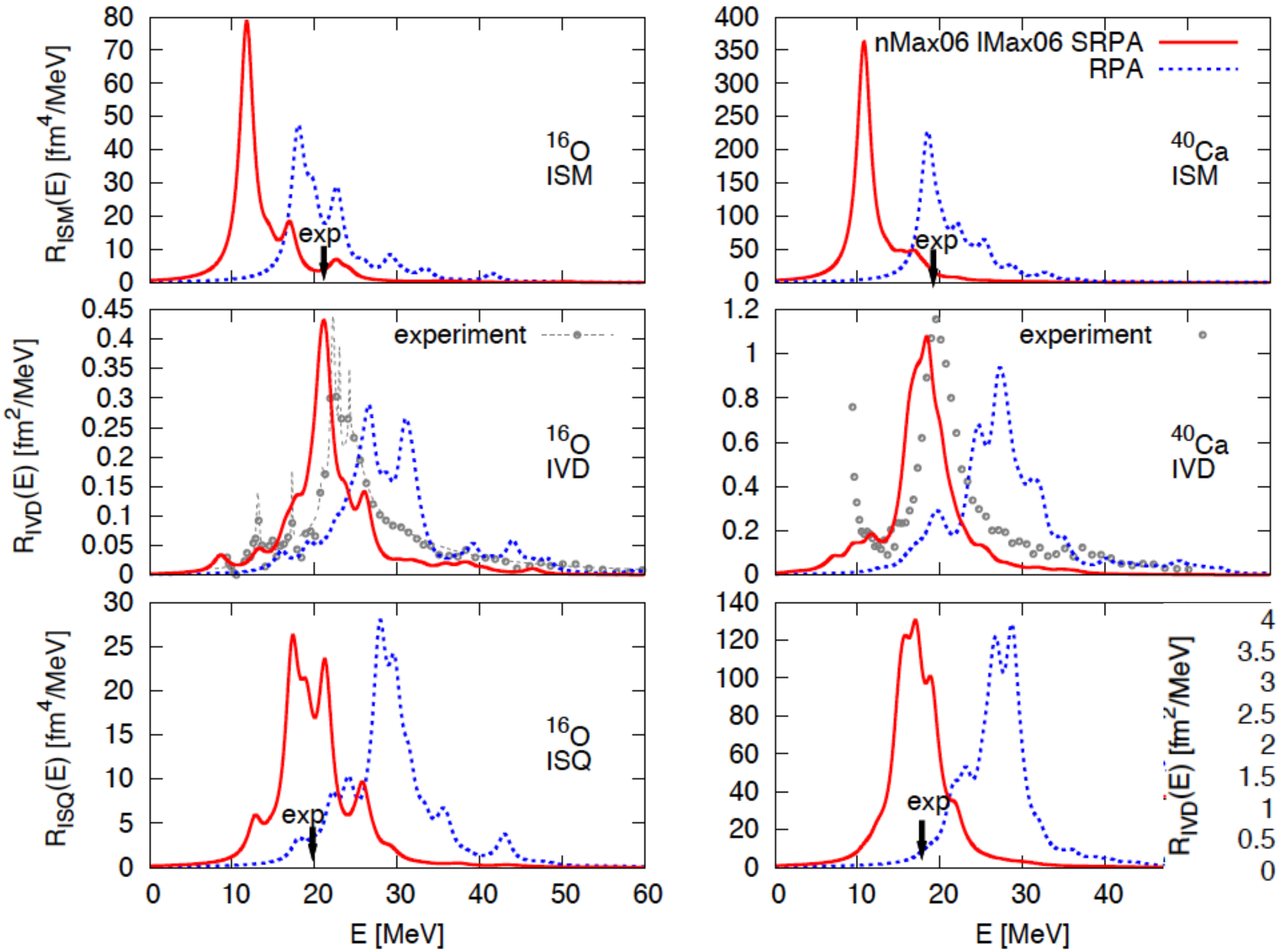
RAON

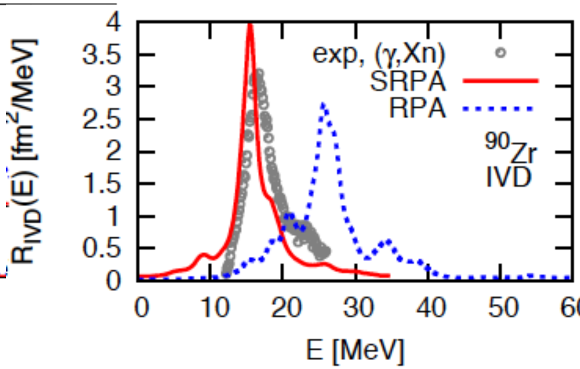
- RPA vs SRPA with UCOM

[PP &Roth, PLB671(2009)356]



Residual 3N effects?

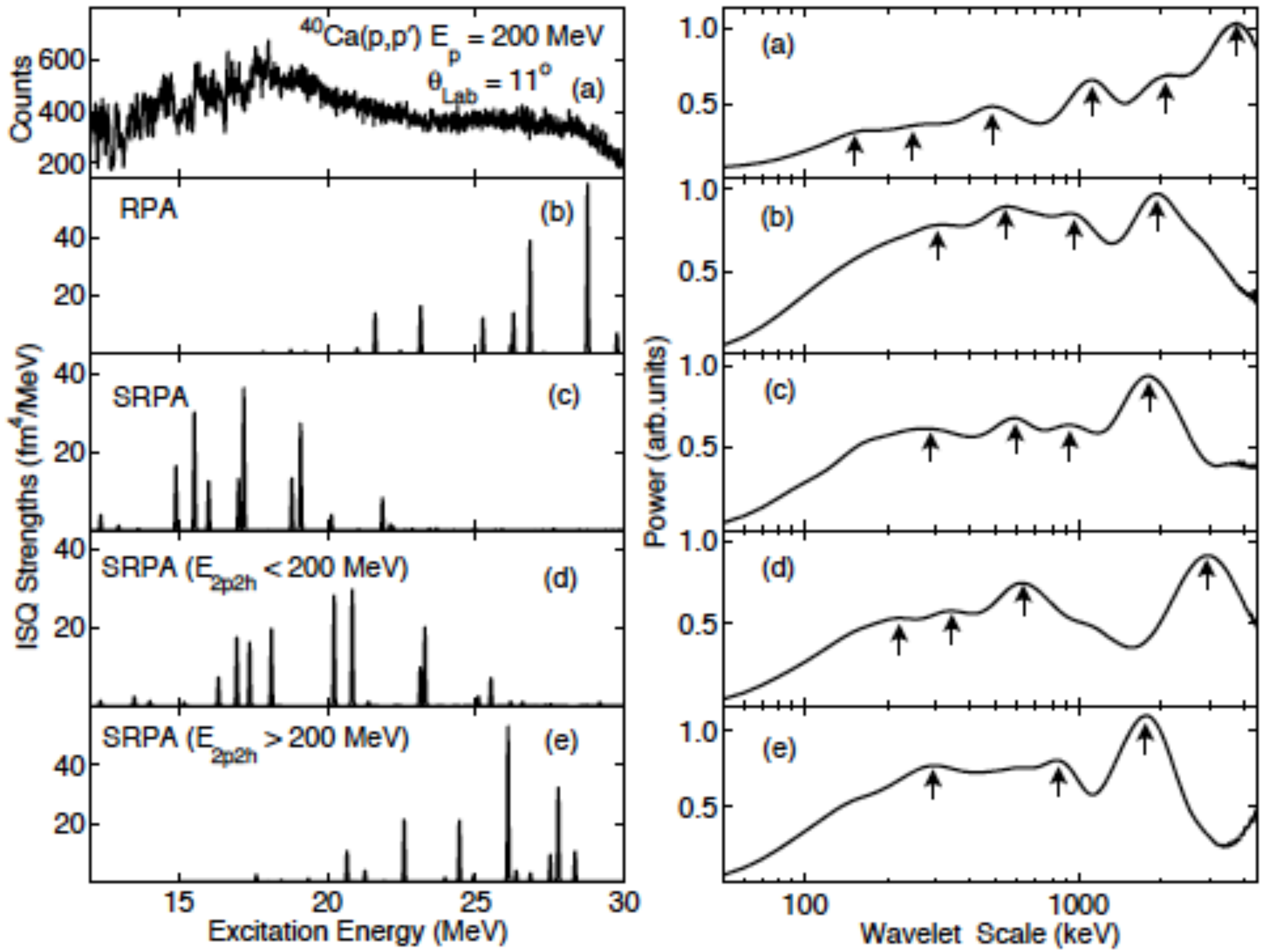
Dressing of orbitals



^{40}Ca (p,p') and UCOM-SRPA

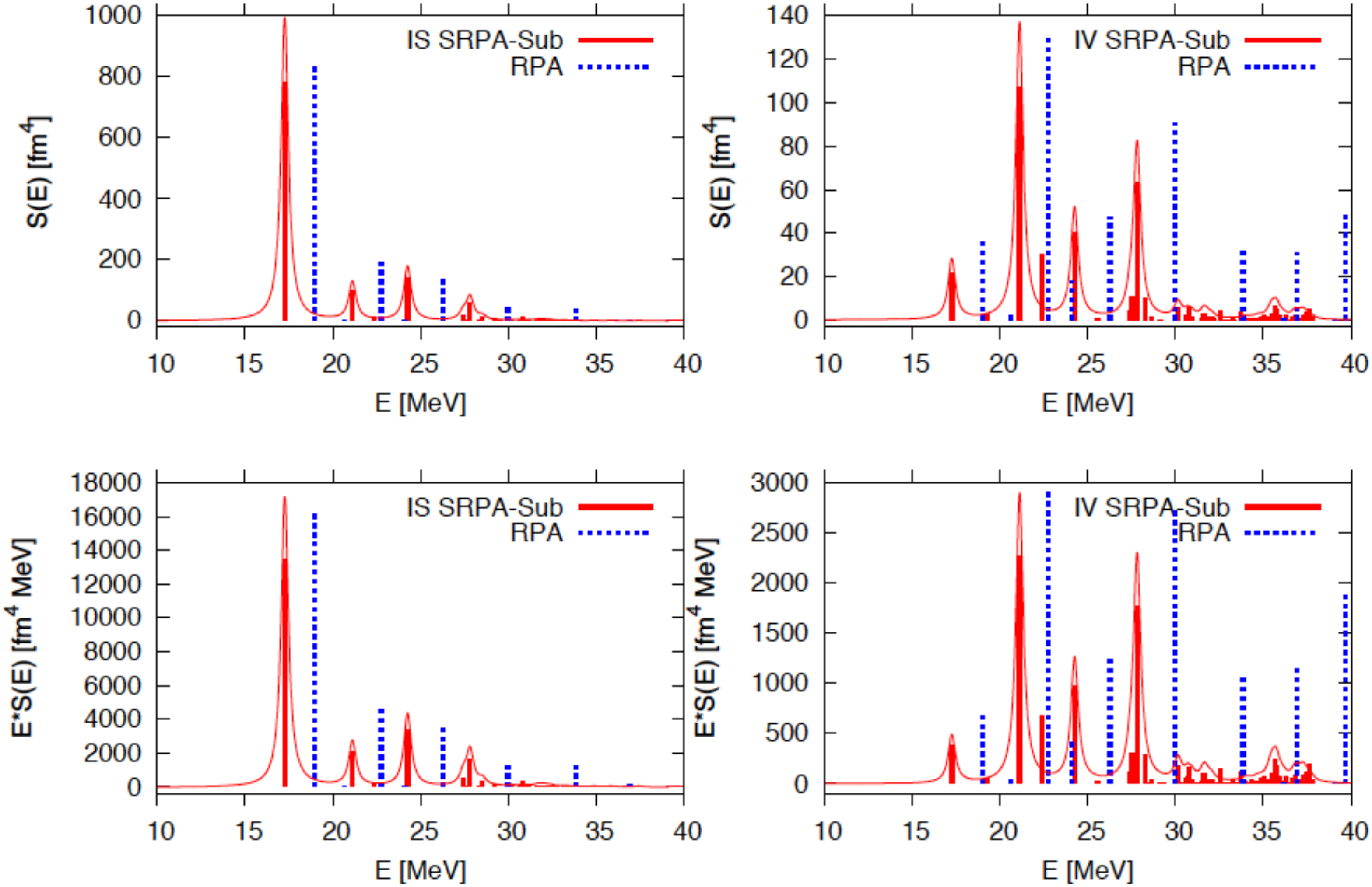
Fine structure of the isoscalar giant quadrupole resonance in ^{40}Ca due to Landau damping?

[Usman,...PP,...et al,PLB698(2011)]



Subtraction method

❖ Monopole response of ^{40}Ca



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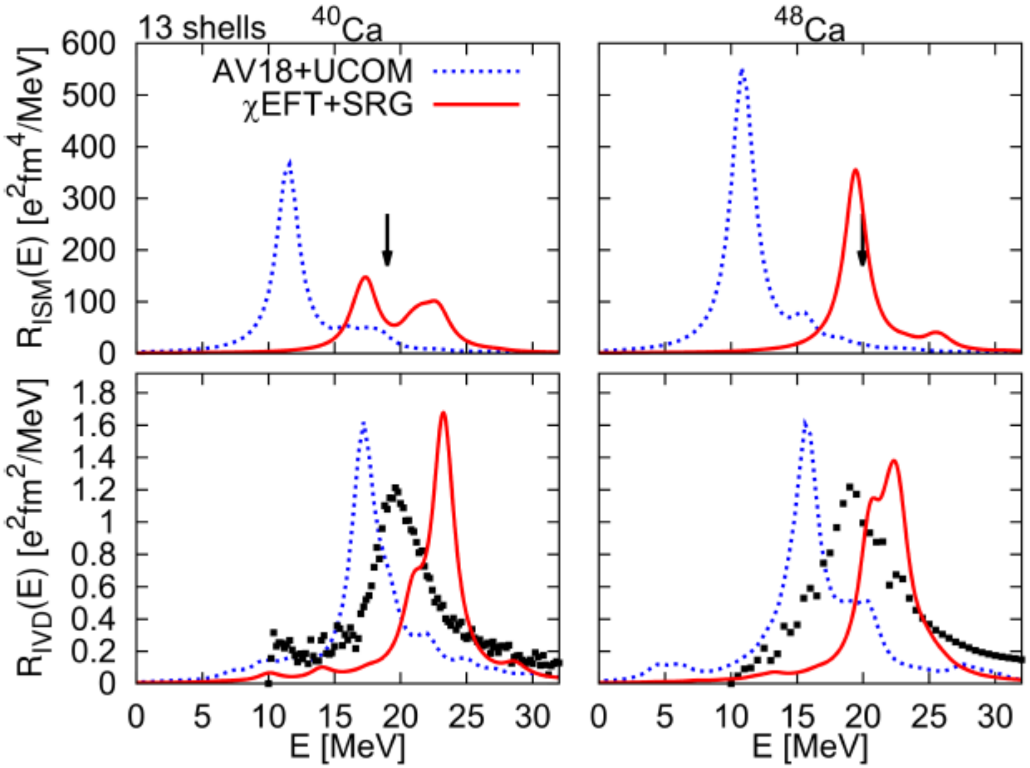
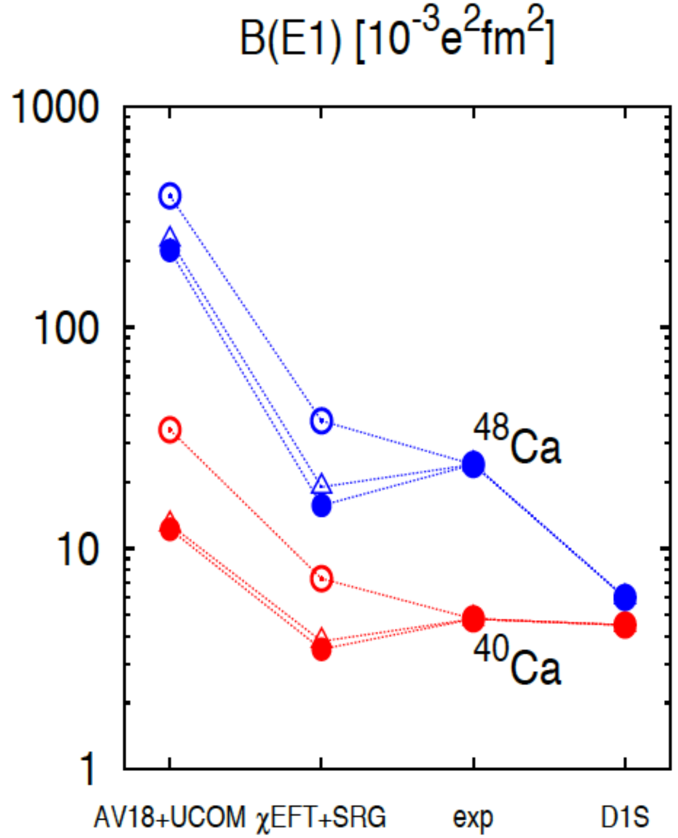
Chiral NN+NNN and extended RPA

PP,Trippel,Roth,APPB48(Zakopane2016)





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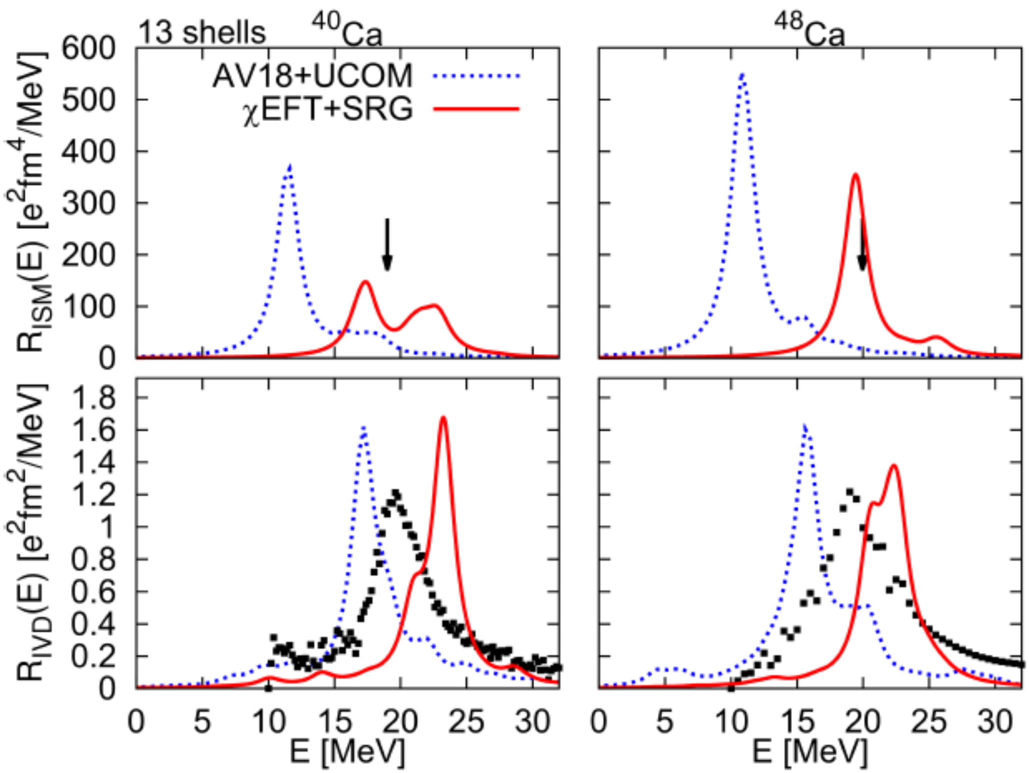

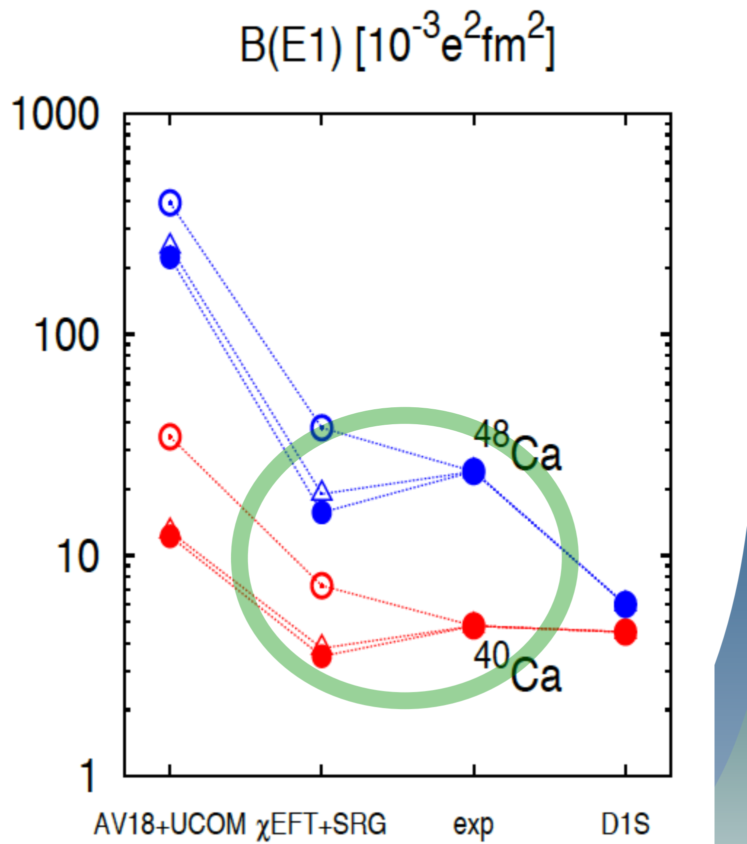

Strength of in-medium 3N force?

- Phenomenological functionals: a density-dependent term is required for saturation
- Without it, the ground state collapses to large density
- In the present approaches, most nuclei saturate already at the 2N level; the 3N only improves the saturation point.

- ***3N force an order of magnitude weaker than a typical density-dependent interaction, e.g.:***

$$t_3\ (\text{UCOM-SRG}) * \rho_0 \quad \div \quad t_3 * \rho_0^{\alpha}\ (\text{D1S}) \quad \sim \quad 0.08$$

  - A handle on the “acceptable” strength of a (in-medium) DDI vs. momentum dependence etc.?

H.Hergert, PP, R.Roth, PRC83,064317; A.Günther, PP, R.Roth, JPG41,115107(2014);

Effects of correlations

P. PAPAKONSTANTINOU AND R. ROTH PHYSICAL REVIEW C **81**, 024317 (2010)

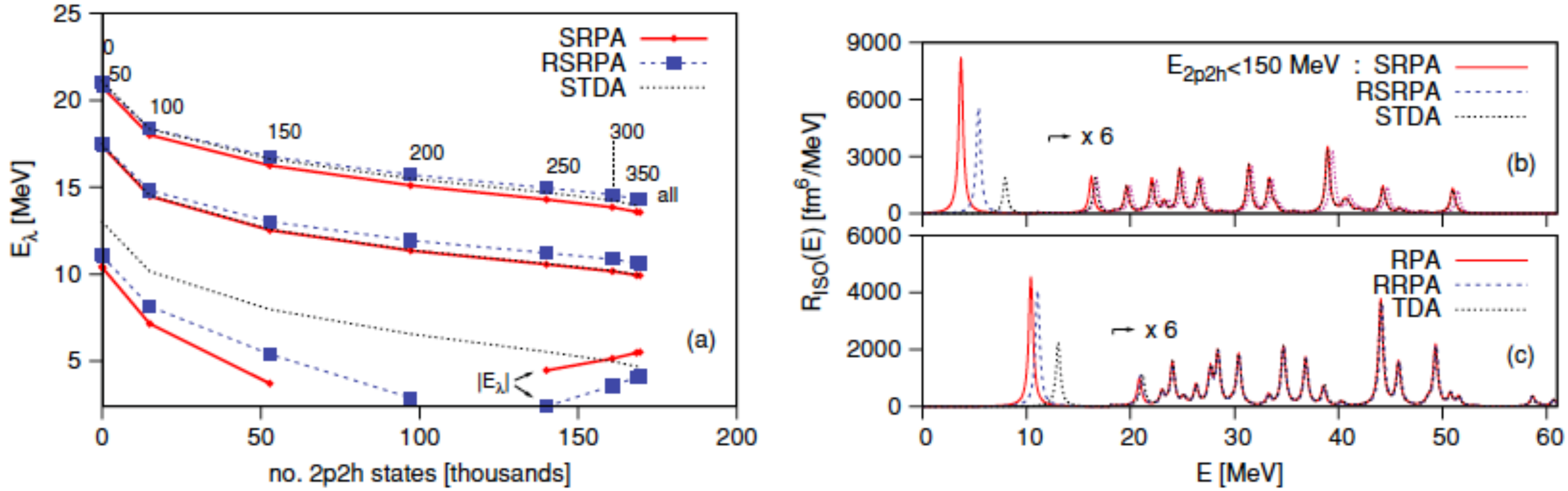

FIG. 9. (Color online) Exploring the influence of ground-state correlations: Isoscalar 3^- response of ^{16}O ($\epsilon_{max} = 14$, $\ell_{max} = 10$) within RPA or SRPA and "renormalized" RPA or SRPA, (RRPA, RSRPA; see text), as well as TDA and STDA (SRPA with $B = 0$). (a) Evolution of the three lowest (closest to zero) eigenvalues as we increase the number of 2p2h configurations considered, starting from zero and up to all 2p2h states available in the single-particle space, N_2. The numbers assigned to the various points indicate the corresponding 2p2h-energy cutoff imposed, $E_{2p2h,max}$, in MeV. The energy of the lowest eigenstate becomes imaginary for larger N_2 in (R)SRPA; then its amplitude is indicated. Note that the STDA and SRPA results for the second state almost coincide. (b) Strength function for $E_{2p2h,max} = 150$ MeV in SRPA, RSRPA, and STDA ($\Gamma = 2$ MeV). (c) Strength function in RPA, RRPA, and TDA.

See also: PP et al., PRC75,014310

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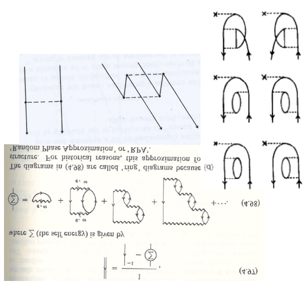
- Introducing three-nucleon interactions
  - Phenomenological
  - Realistic (chiral)
- Outlook

Outlook

- ❖ Model space and Hamiltonian meeting each other
- ❖ Collective phenomena (GRs) and their decays: SRPA represents a smart (tailored) choice of model space.
- ❖ Unitarily transformed interactions are perturbative and do not produce double counting
- ❖ Missing:
  - 3N effects: However, only a small correction is required
  - GSC: could affect substantially less collective states – M1, GT, etc.
  - How dressed should the particles be?
- ❖ A consistent framework is required



Thank you!

감사합니다~

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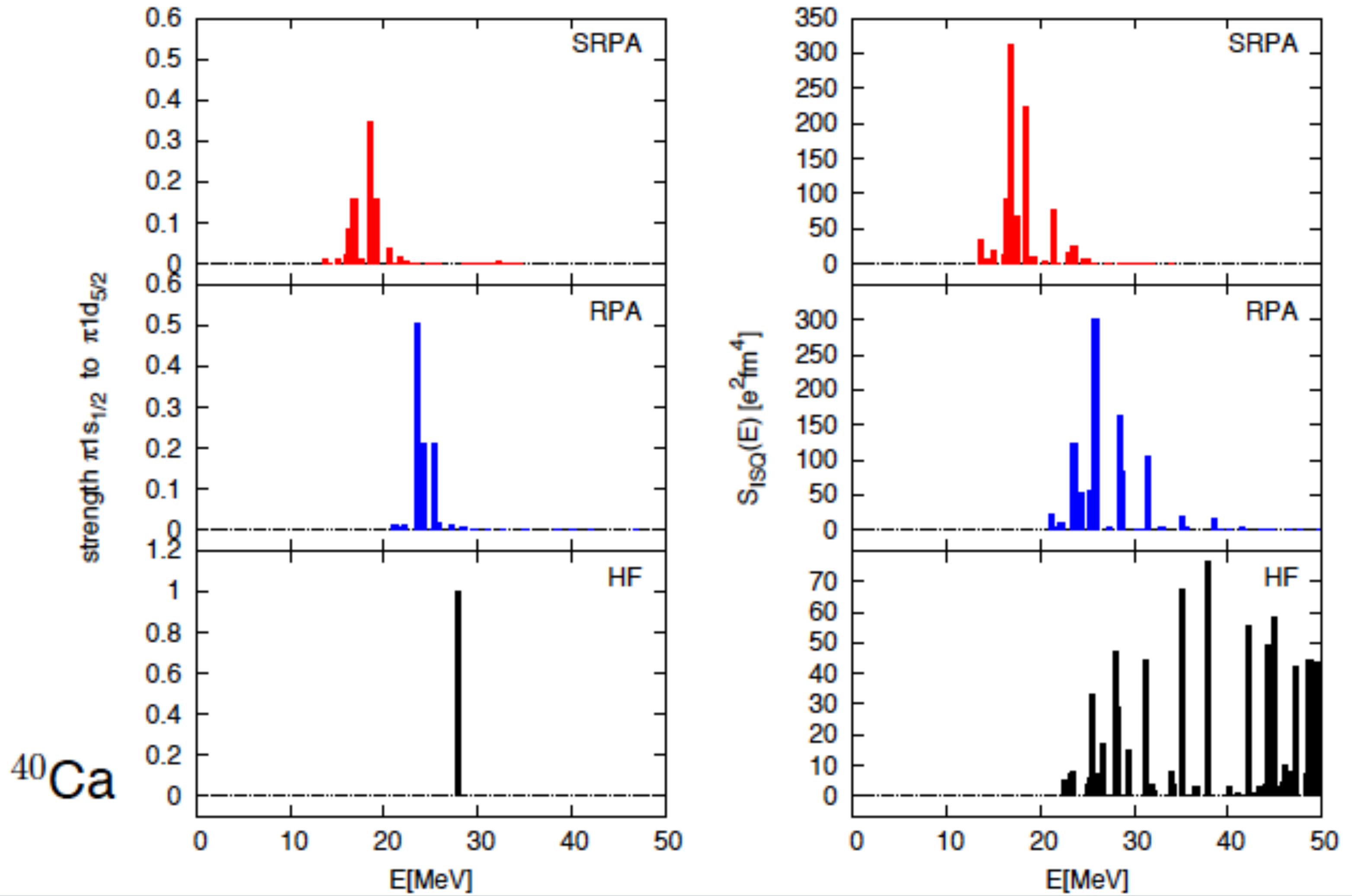
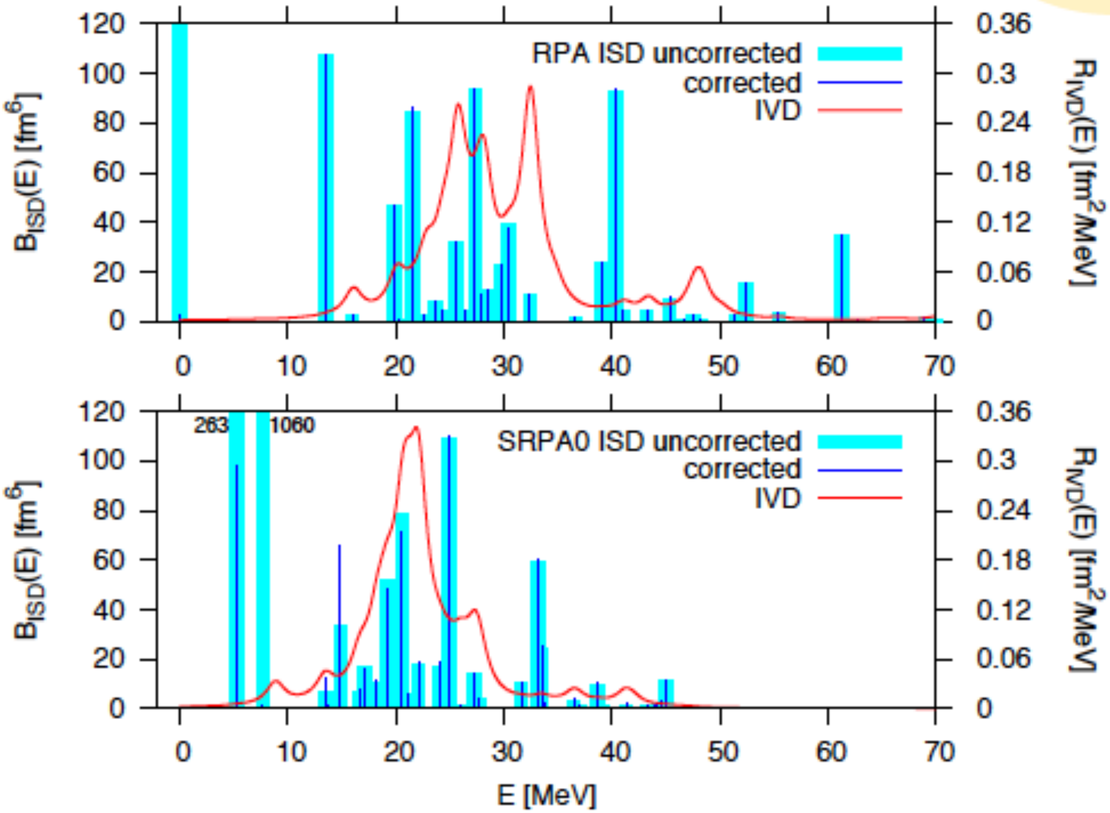
Fragmentation of ph strength



Spurious states!

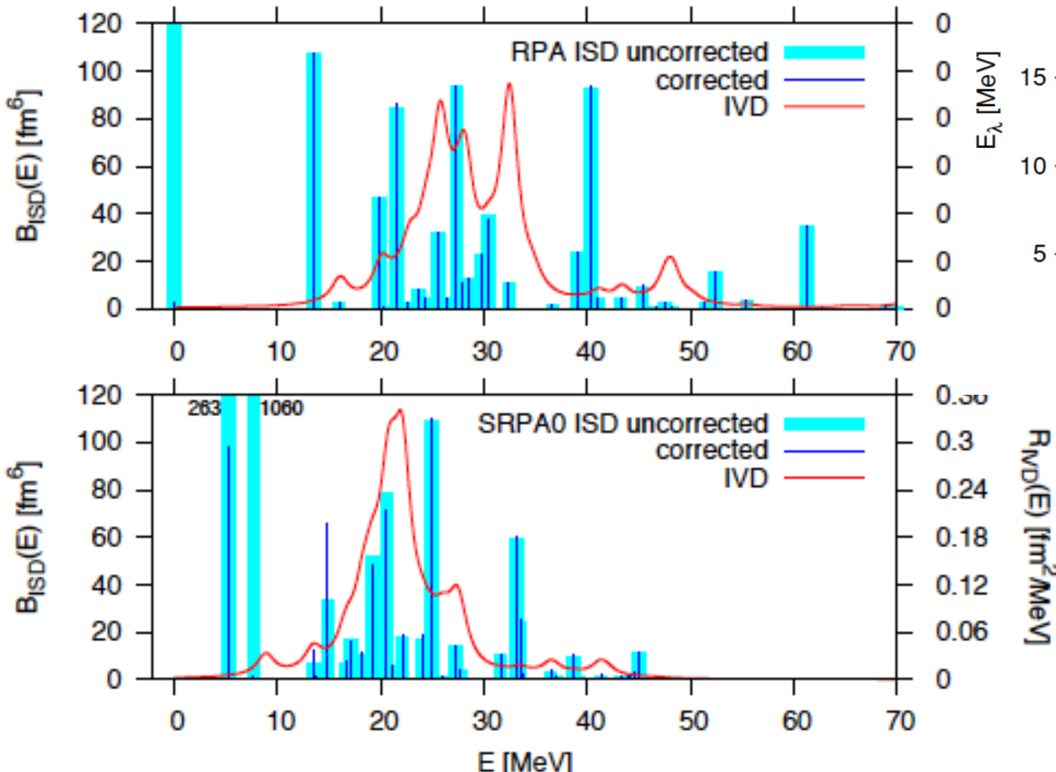
ISD corrected radial operator $r^3 - \frac{5}{3}\langle r^2 \rangle r$ vs r^3

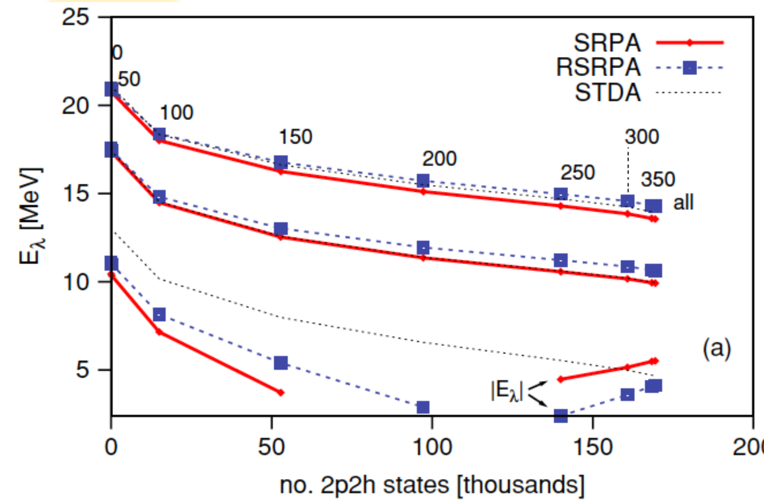
^{16}O
$N_{max} = 12$



Spurious states and instabilities in SRPA

ISD corrected radial operator $r^3 - \frac{5}{3}\langle r^2\rangle r$ vs r^3





Instability of low-energy states

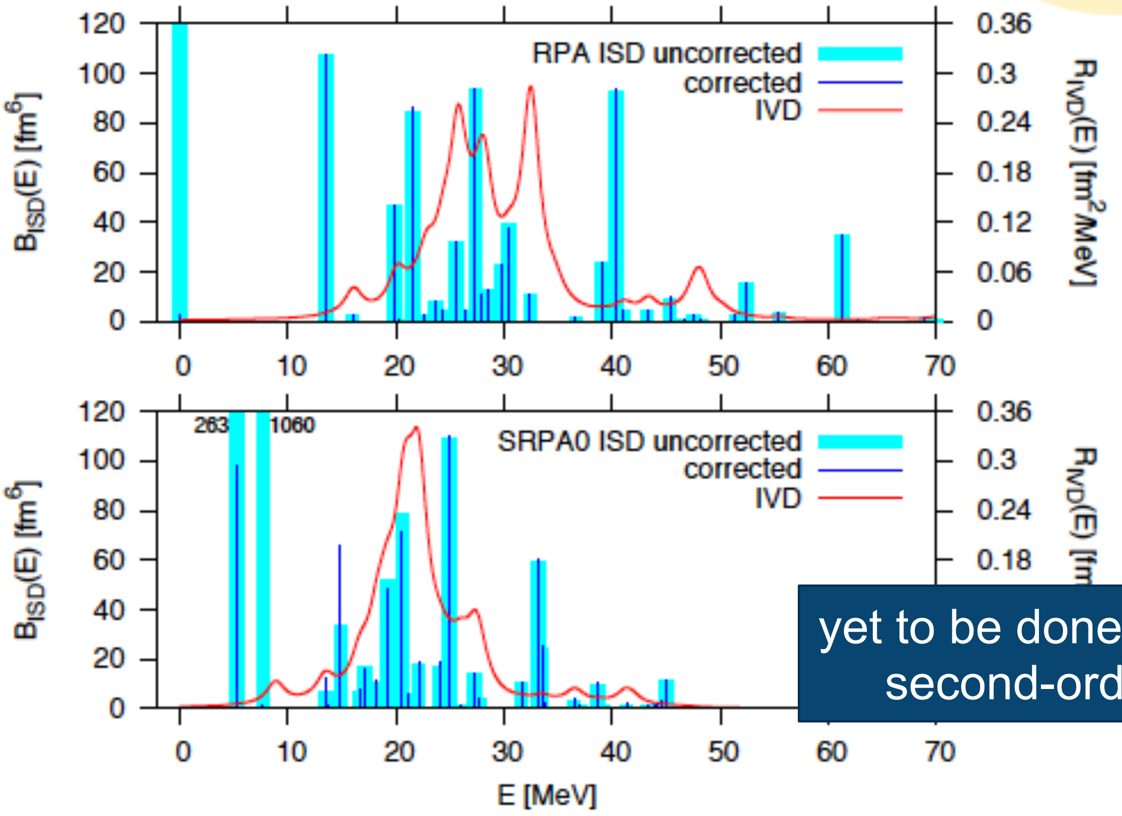
spurious admixtures

?

Spurious states!

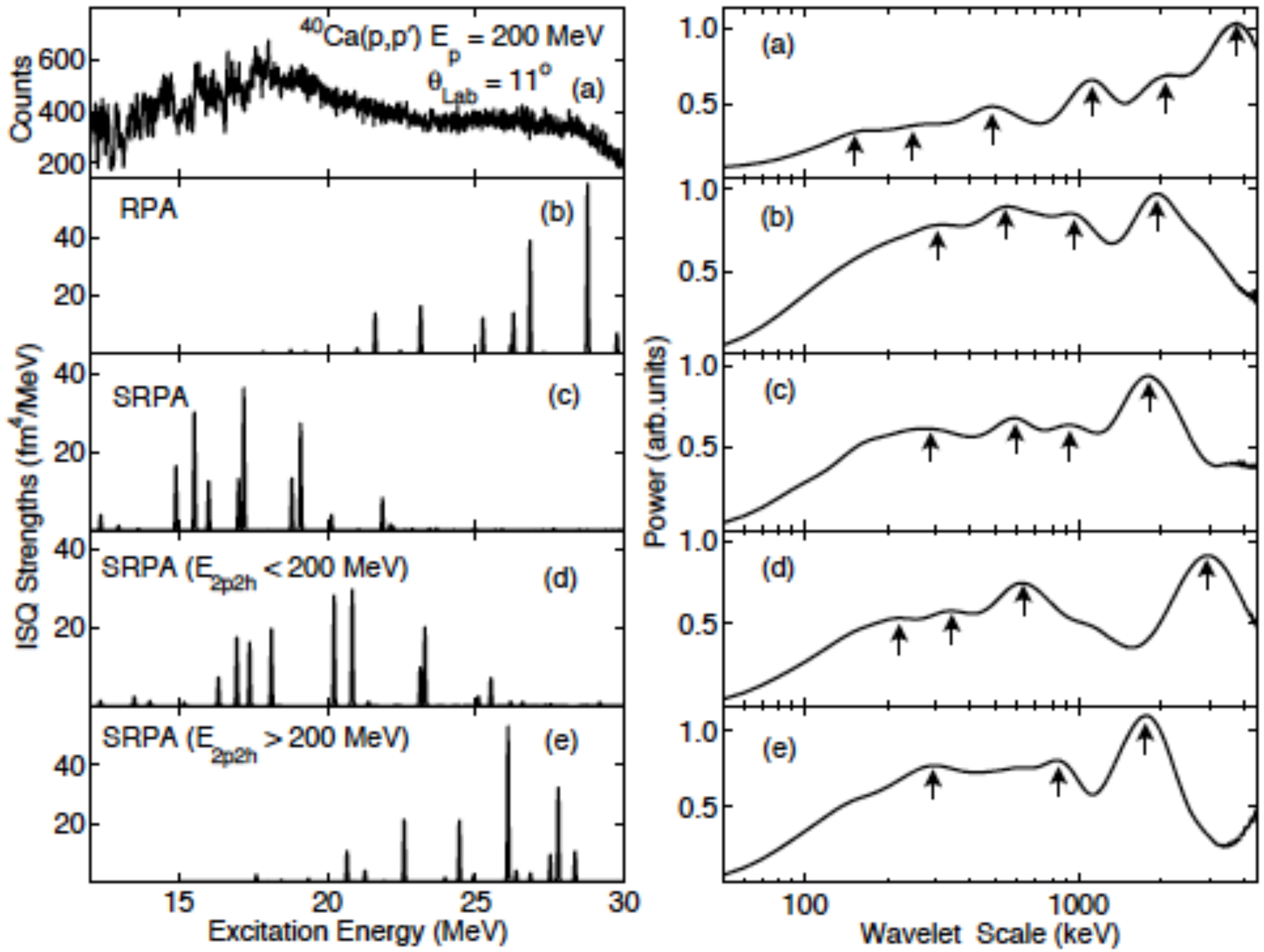
ISD corrected radial operator $r^3 - \frac{5}{3}\langle r^2 \rangle r$ vs r^3

^{16}O
$N_{\max} = 12$



yet to be done: self-consistent second-order formalism

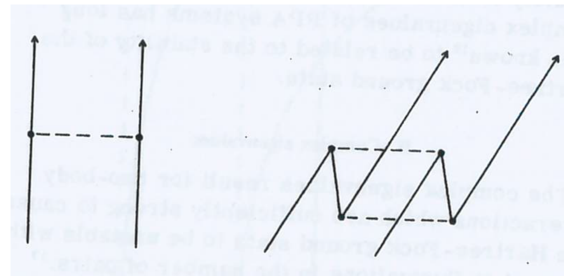
^{40}Ca (p,p’) and UCOM-SRPA

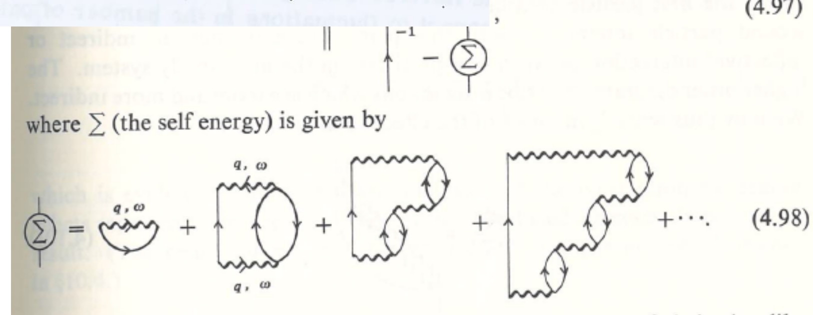


PLB698(2011)

Nomenclature: Which RPA?

- pp-RPA
  - From A to A±2 system
  - Pairing interaction
  - Ladder diagrams
- ph-RPA
  - Same number of particles
  - Phonon excitations
  - Ring diagrams
- ph-2p2h-RPA (Second RPA)
  - Extension of ph-RPA
  - Re

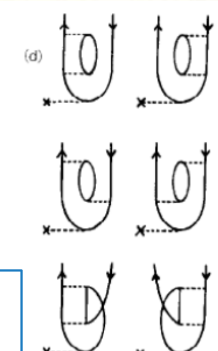



(4.97)

where $\sum$ (the self energy) is given by

(4.98)

The diagrams in (4.98) are called 'ring' diagrams because of their ring-like structure. For historical reasons, this approximation for G is called the 'Random Phase Approximation' or 'RPA'.



- HF reference state
- Spherical systems (J good quantum number)
- Everything antisymmetrized

And for which purpose?

- Ground-state correlations
  - Correction to the Hartree-Fock energy via the backward amplitudes
  - Correction real in the absence of phase transitions (e.g. superfluidity in pp-RPA)

- Excited states
  - pp-RPA: 2-particle transfer, spectroscopy
  - ph-RPA, ph-2p2h-RPA: Vibrational states, sound waves

And for which purpose?

RAON

- Ground-state correlations
  - Correction to the Hartree-Fock energy via the backward amplitudes
  - Correction real in the absence of phase transitions (e.g. superfluidity in pp-RPA)

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  - pp-RPA: 2-particle transfer, spectroscopy
  - ph-RPA, ph-2p2h-RPA: Vibrational states, sound waves

just “RPA” here

“Second RPA” (SRPA)

First SRPA results

- Softened realistic Hamiltonian (AV18, UCOM)
- No arbitrary truncations
- Explored: Energetic shift; EWSR; ph fragmentation; etc.