KIDS energy density functional for nuclei and nuclear matter

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- **KIDS**

  Korea: IBS-Daegu-Sungkyunkwan

- **Collaboration**
- **First work**

  P. Papakonstantinou, Y. Lim (IBS), T.-S. Park (Sungkyunkwan Univ.)
- **Nuclear structures**

  H. Gil (Kyungpook) ,P. Papakonstantinou, (IBS), Y. Oh (Kyungpook)
- **Astrophysics and Heavy-ion collision**

  M. Kim (Pusan), Y. -M. Kim (UNIST), Y. Kim (IBS), C.-H. Lee (Pusan),

  S. Jeon (McGill)

Outline

EFT for two nucleons

Building a model

Application

Summary

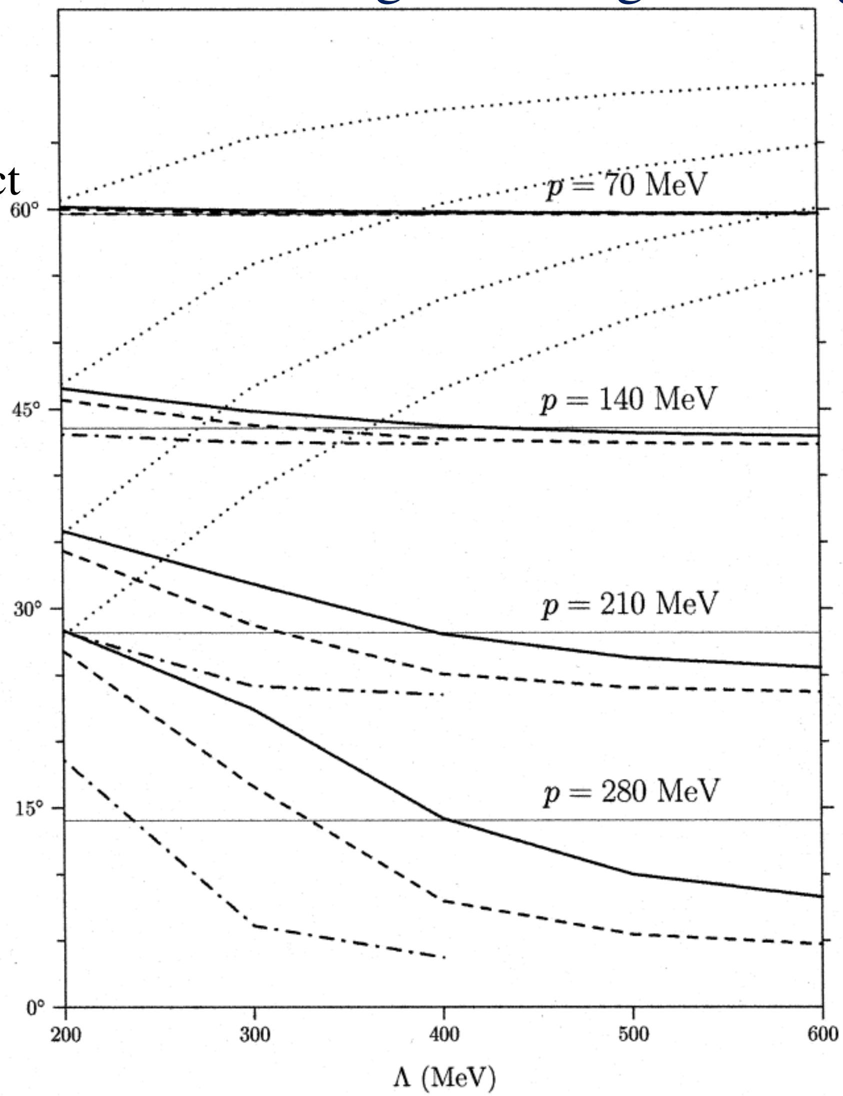
EFT for two nucleons

- Pionful EFT for the np scattering (CHH, T.-S. Park, D.-P. Min, PLB 473 (2000))

NN potential up to NLO following Weinberg counting rule

LO: OPE, 4N contact

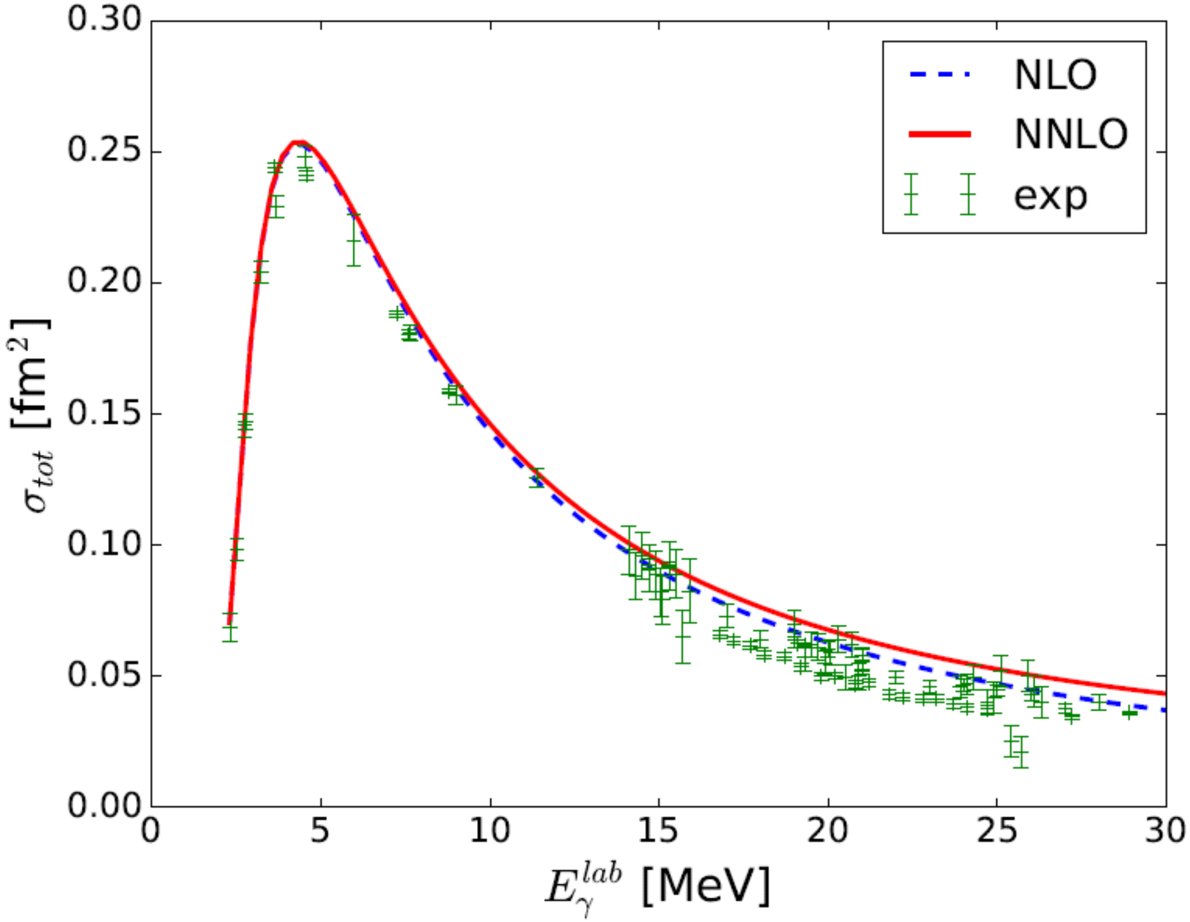
NLO: TPE, 4N contact



- np phase shifts
- LO: dotted
  - reproduce experiment only at small momentum
  - strongly cut-off dependent
- NLO: solid
  - less dependent on cut-off
  - good agreement at large momenta

- Pionless EFT for the np → dg (Y.-H. Song, S.-i. Ando, CHH, PRC 96 (2017))

Pionless EFT: Valid for *p* □ *m*p (*p* ~ *m*p, *E*g ~ 10 MeV)



- Can we have similar thing for the nuclear matter and nuclei?
  - Systematic and converging expansion
  - Uncertainty quantification
- Could be 'Yes'

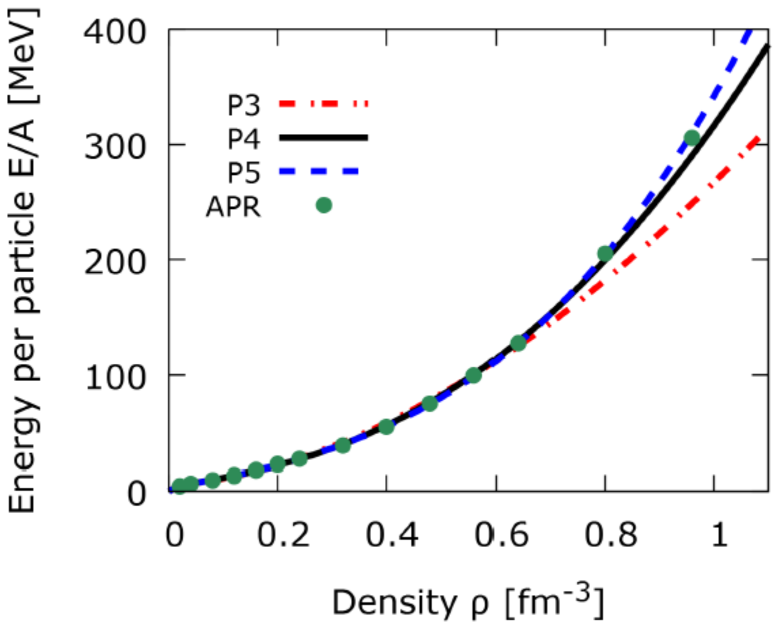


FIG. 5: Energy per particle of pure neutron matter with models P3, P4 and P5 presented in Table IV. Here, the symmetric EoS parameters α_i are fixed as model S3b in Table I.

(H. Gil, Y.-M. Kim, CHH, P. Papakonstantinou, Y. Oh, PRC 100 (2019))

Building a model

- Scale in nuclear matter
  - Momentum scale: kF (270 MeV for saturated symmetric matter)
  - kF/mr : less than 1 even at r = 8r0 (kF □ r1/3)
  - Expand the energy density in powers of kF/mr (mr: rho-meson mass)

- Rules
  - Rule1: Expand EDF for nuclear matter in powers of kF/mr
  - Rule2: Fit the parameters to the well-known nuclear matter properties
  - Rule3: Keeping the parameters unchanged, apply the model to nuclei

- KIDS Ansatz

$$\mathcal{E}(\rho,\delta) = \mathcal{T}(\rho,\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3}, \qquad \rho = \rho_n + \rho_p$$

$$c_i(\delta) = \alpha_i + \delta^2\beta_i, \qquad \delta \equiv (\rho_n - \rho_p)/\rho$$

- Fitting
  - *c*0(0), *c*1(0), *c*2(0): r0, E/A, K0 (assume *c*3(0) = 0)
  - *ci*(1) : APR PNM EoS (14 data in r = 0.02 – 0.96 fm-3)

- Fitting result

$$\chi^2(\delta) = \sum_j \exp\{-\beta\rho_j/\varrho_0\} \left(\frac{\mathcal{E}(\rho_j) - D_j}{\mathcal{T}(\rho_j)}\right)^2 \; ; \quad \beta \geq 0$$

- **Fitting to APR PNM EoS**

	c0	**c0 , c1**	**c0 - c2 (P3)**	**c0 - c3 (P4)**	**c0 - c5 (P5)**
c2	0.071632	0.001566	0.000529	0.000138	0.000115

- Fitting improves with more terms
- Improvement saturates at P5 (5 terms for PNM)
- **Double check: Fitting to QMC PNM EoS** (J. Carlson et al., Rev, Mod. Phys. 87 (2015))

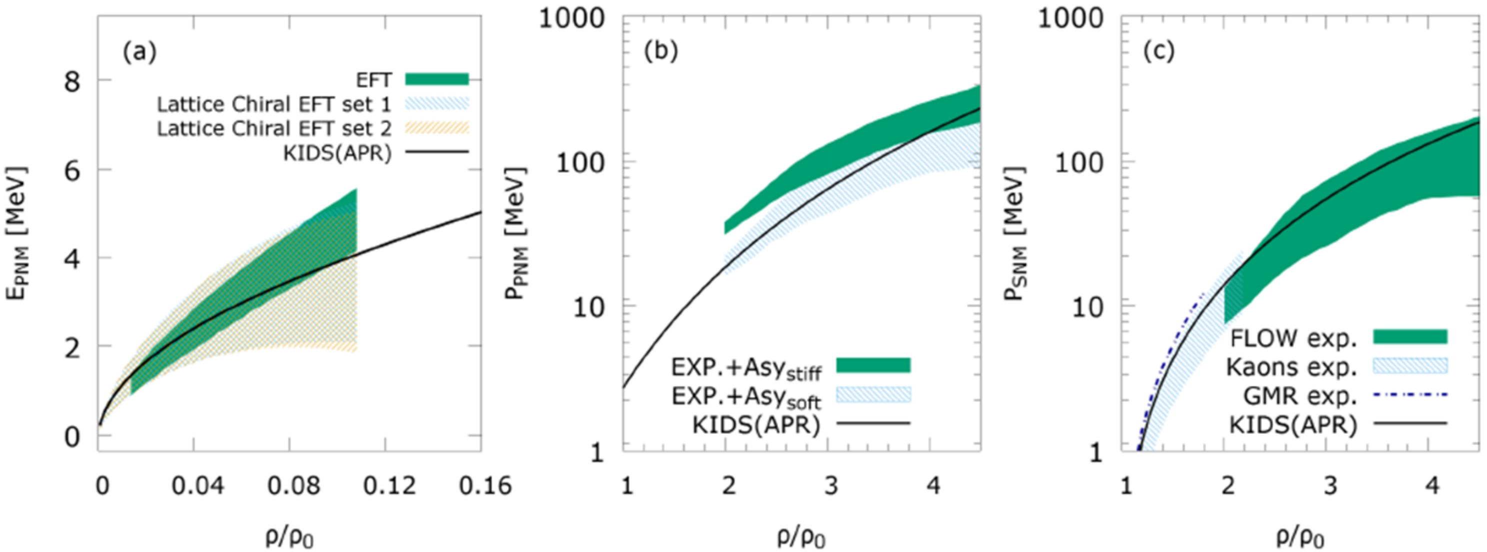
P4	**P5**	**P6**
[redacted]		

- Nuclear matter properties: P4 parameters

Dutra et al., PRC 85 (2012)

	K0 (MeV)	-Q0 (MeV)	J (MeV)	L (MeV)	Kt (MeV)	S(r0/2)/J	3PPNM/(L r0)
KIDS	240.00*	372.65	32.75	49.10	-377.06	0.667	1.03
Exp./Emp.	200-260	200-1200	30-35	40-76	-760,-372	0.57-0.86	0.90-1.10

*: Input data



Lattice chiral EFT: E .Epelbaum et al., EPJA40 (2009)

Application

Nuclei

Neutron star

- Skyrme type force

$$
\begin{aligned}
v_{ij} \;=\; & (t_0 + y_0 P_\sigma)\delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{6}\sum_{n=1}^{N-1}(t_{3n} + y_{3n}P_\sigma)\rho^{n/3}\delta(\mathbf{r}_i - \mathbf{r}_j) \\
& + \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k}^2 + \mathbf{k}'^2\delta(\mathbf{r}_i - \mathbf{r}_j)] + (t_2 + y_2 P_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k} \\
& + iW_0\,\mathbf{k}'\times\delta(\mathbf{r}_i - \mathbf{r}_j)\,\mathbf{k}\cdot(\sigma_i + \sigma_j),
\end{aligned}
$$

- Energy density

$$
\begin{aligned}
\mathscr{E} \;=\; & \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho - \frac{1}{8}(t_0 + 2y_0)\rho\delta^2 + \frac{1}{16}\sum_{n=1}^{N-1} t_{3n}\rho^{1+n/3} - \frac{1}{48}\sum_{n=1}^{N-1}(t_{3n} + 2y_{3n})\rho^{1+n/3}\delta^2 \\
& + \frac{1}{64}(9t_1 - 5t_2 - 4y_2)\frac{(\nabla\rho)^2}{\rho} - \frac{1}{64}(3t_1 + 6y_1 + t_2 + 2y_2)\frac{(\nabla\rho\delta)^2}{\rho} \\
& + \frac{1}{8}(2t_1 + y_1 + 2t_2 + y_2)\tau - \frac{1}{8}(t_1 + 2y_1 - t_2 - 2y_2)\sum_q \frac{\rho_q\tau_q}{\rho} \\
& + \frac{1}{2}W_0\left(\frac{\mathbf{J}\cdot\rho}{\rho} + \sum_q \frac{\mathbf{J}_q\cdot\nabla\rho_q}{\rho}\right),
\end{aligned}
$$

$$\tau = \frac{3}{5}\left(\frac{6\pi^2}{\nu}\right)^{2/3}\rho^{5/3}$$

KIDS EDF: $\mathscr{E}(\rho,\delta) = \mathscr{T}(\rho,\delta) + \sum_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3},$

- Transformation of coefficients

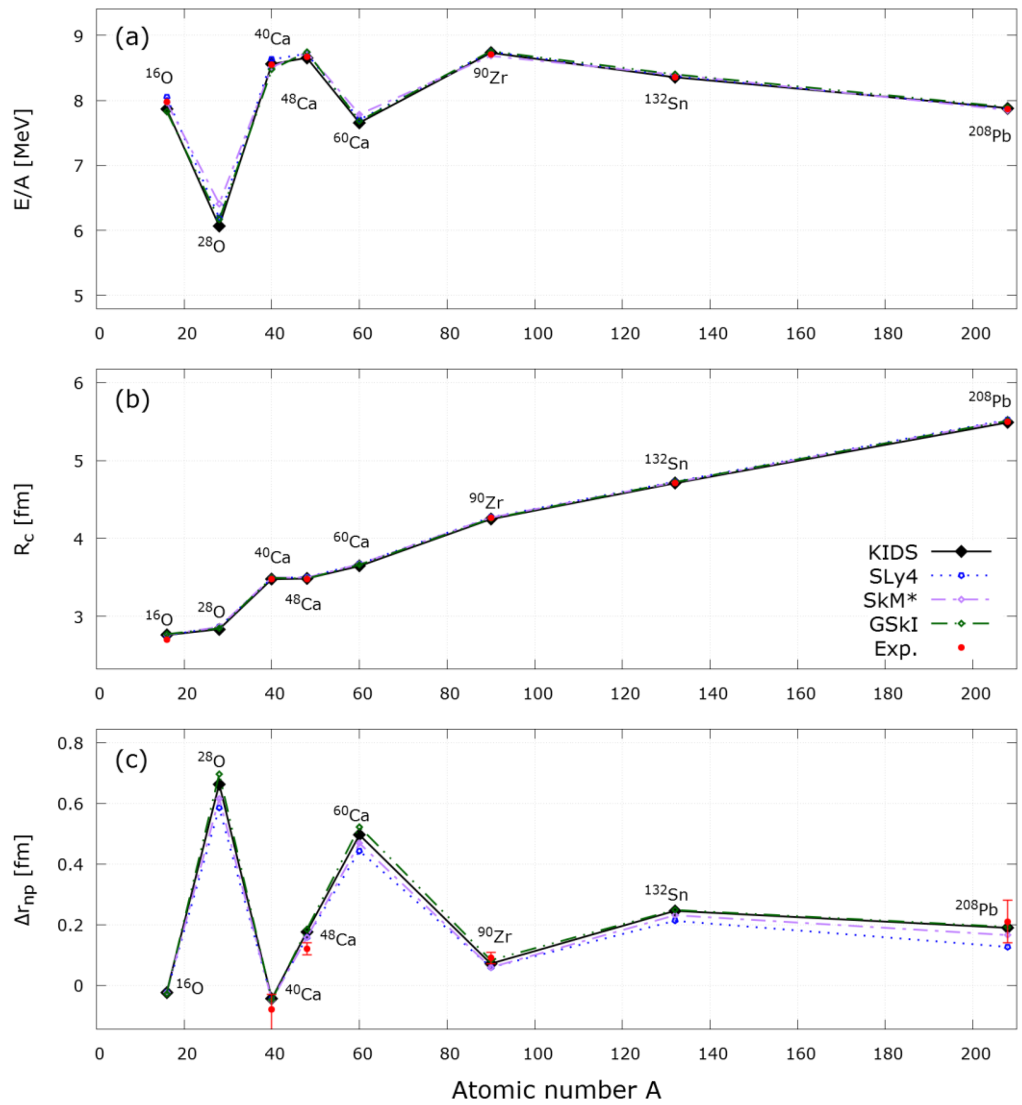
$$t_0 = \frac{8}{3}c_0(0)\,, \quad y_0 = \frac{8}{3}c_0(0) - 4c_0(1),$$
$$t_{3n} = 16c_n(0)\,, \quad y_{3n} = 16c_n(0) - 24c_n(1), \quad (n \neq 2)$$
$$t_{32} = 16c_2(0) - \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\theta_s \equiv 16c_2(0)(1-k)$$
$$y_{32} = 16c_2(0) - 24c_2(1) + \frac{3}{5}(3\pi^2)^{2/3}\left(3\theta_\mu - \frac{\theta_s}{2^{2/3}}\right) \equiv [16c_2(0) - 24c_2(1)](1-k')$$

with

$$\theta_s \equiv 3t_1 + 5t_2 + 4y_2 = \frac{5}{3}(3\pi^2)^{-2/3}16c_2(0)k$$
$$\theta_\mu \equiv t_1 + 3t_2 - y_1 + 3y_2 = -\frac{5}{9}(3\pi^2)^{-2/3}[16c_2(0) - 24c_2(1)]k' + \frac{\theta_s}{3 \cdot 2^{2/3}}.$$

k, k': fraction of gradient terms in the c2 term (r2/3)

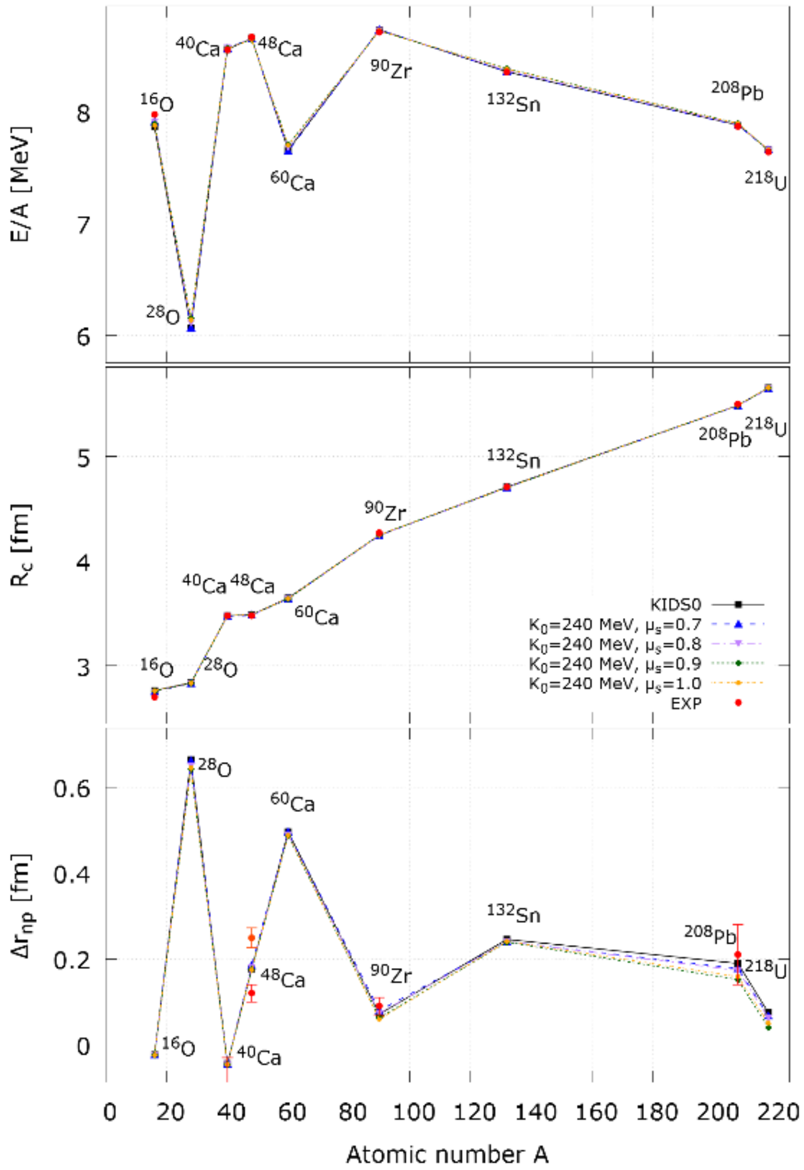
- Two parameter fitting
  - Assume *k*=*k*'
  - Assume *y*1=*y*2=0
  - Remaining parameters *k*, *W*0
  - Fit to E/A and Rc of 40Ca, 48Ca, 208Pb



- Dependence on the effective mass
  - Assume non-zero *y*1, *y*2
  - Fit to specific isoscalar
  - effective mass
  - Isoscalar effective mass

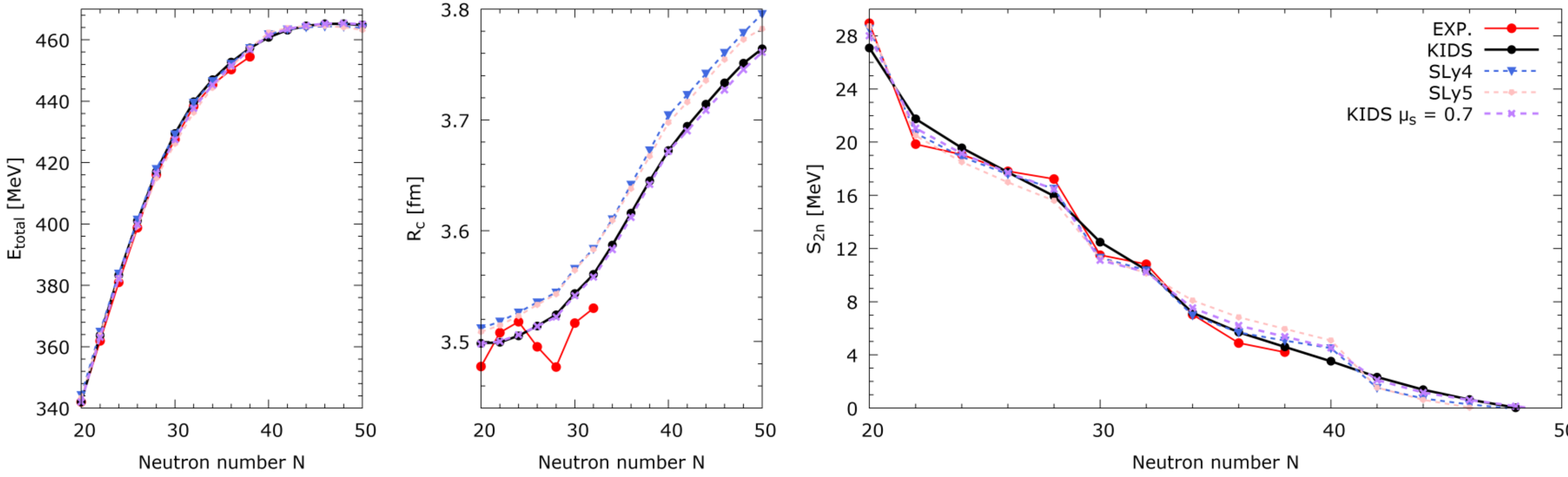
$$\mu_s^{-1} \equiv (m^*_{\rm IS}/m)^{-1} = 1 + \frac{m}{8\hbar^2}\rho\,\theta_s$$

$$\theta_s \equiv 3t_1 + 5t_2 + 4y_2$$

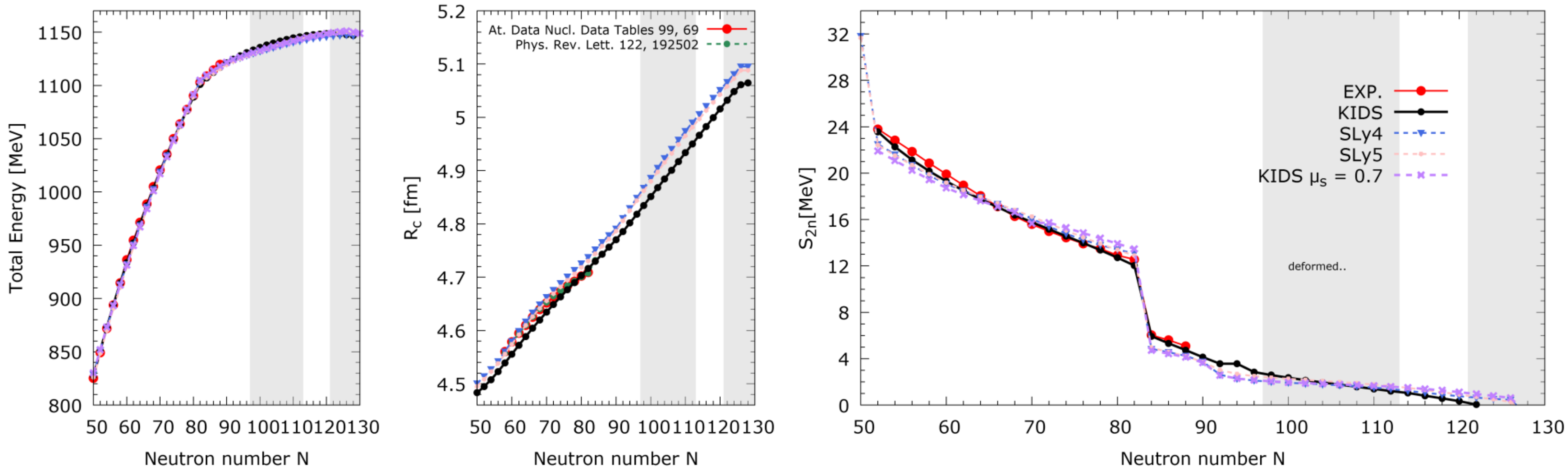


- Isotopes with pairing force

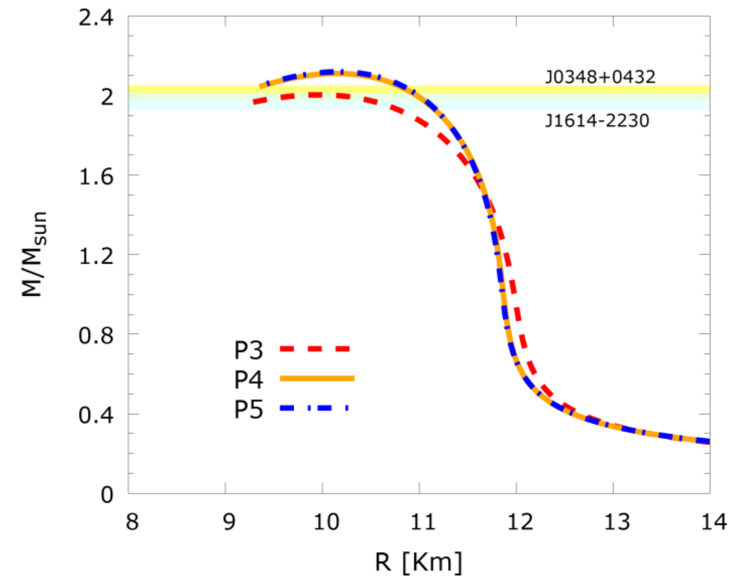
Ca isotope (Z = 20)



Sn isotope (Z = 50)



- Neutron star
  - Mass-radius



  - Tidal deformability

	GSkI	SLy4	SkI4	SGI	KIDS
$R_{1.4M_\odot}$ [km]	11.94	11.82	12.46	12.77	11.79
$k_2(1.4M_\odot)$	0.079	0.077	0.092	0.097	0.076
$\lambda(1.4M_\odot)$ $[10^{36}\text{g cm}^2\text{s}^2]$	1.906	1.770	2.772	3.292	1.737
$\Lambda(1.4M_\odot)$	337.2	312.9	490.9	583.0	307.5

Recent analysis (B. P. Abbott et al. arXiv:180511581v1 [g $\Lambda(1.4M_\odot) = 190^{+390}_{-120}$

Summary

- Link low energy EFT to nuclear matter and structures
- Novel EDF constructed with simple rules

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- Nuclear matter properties agree well with exp./emp. data
- Most updated data of neutron stars well reproduced
- Nuclear properties reproduced over wide range of mass number
- Effective mass controlled without altering bulk properties

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- Odd-odd, odd-even nuclei: tensor force, deformation
- Mass table whole nuclear chart
- Response to external probe