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Baryon properties in a strong magnetic field

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Nuclear Many-Body Theories: Beyond the Mean Field Approach
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Topological soliton models

Why topological models?

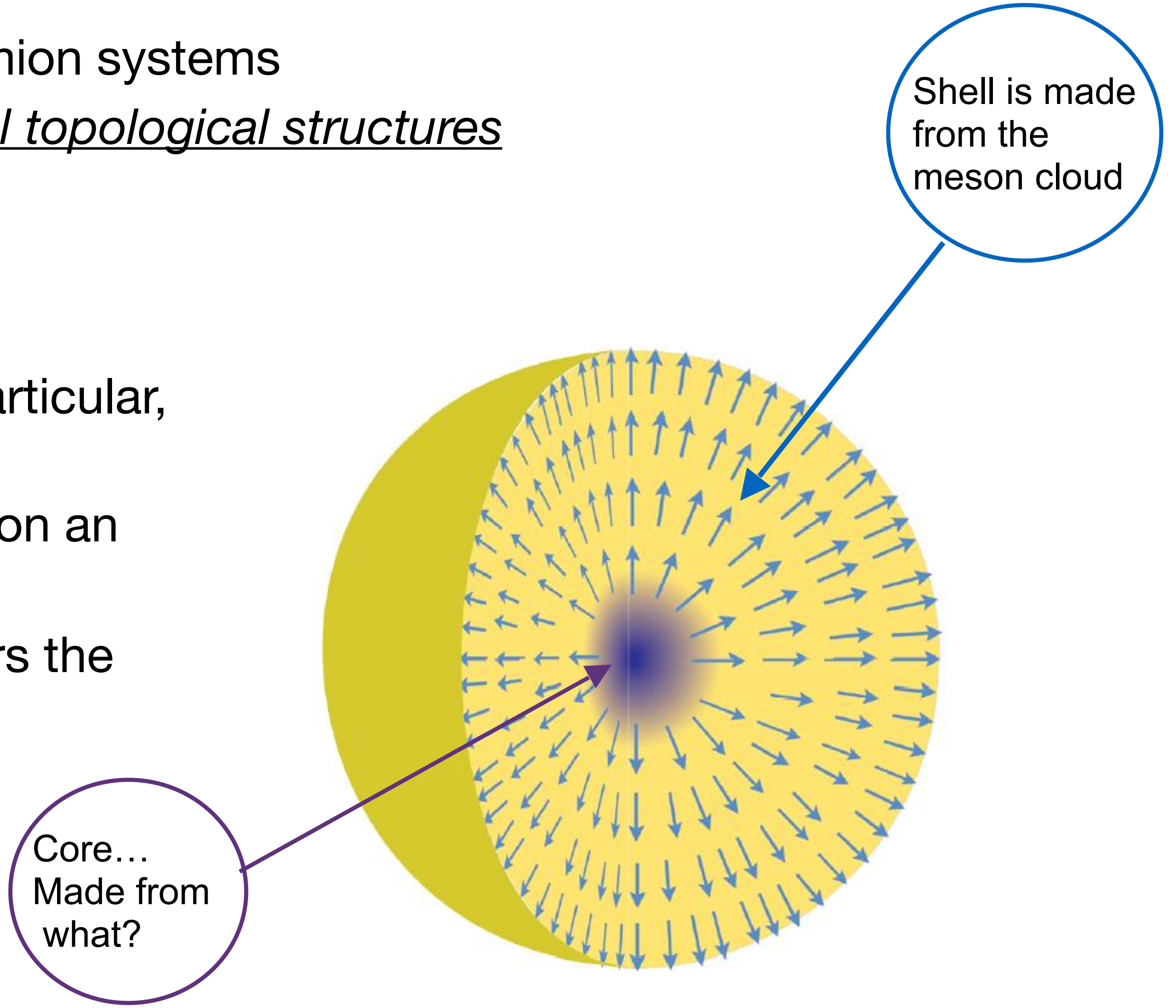
At fundamental level we may have

- fermions -> bosons are trivial fermion systems
- bosons -> fermions are *nontrivial topological structures*

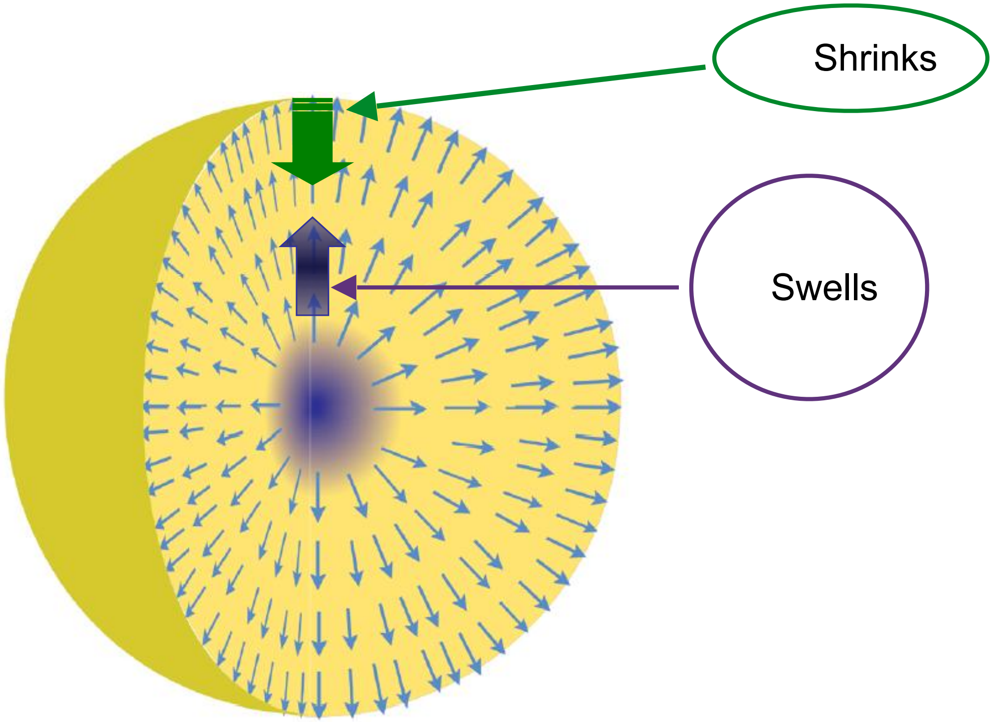
Structure

From what made a nucleon and, in particular, its core?

- The structure treatment depends on an energy scale
- At the limit of large number colours the core still has the mesonic content



Stabilisation mechanism



- Soliton has the finite size and the finite energy
- One needs at least two counter terms in the effective (mesonic) Lagrangian

Prototype: **Skyrme model**

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

- Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_\pi^2}{16} \mathrm{Tr}\,(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial^\nu U]^2$$

Shrinking term — Swelling term

Skyrme Model

Simple scaling analysis $\vec{r} \to \lambda \vec{r}$

- Lambda is a characteristics scale length, e.g. parameter defining the soliton size

- The energy of the static configuration (classical soliton mass) changes as

$$E = \int \mathrm{d}^3 r \left\{ \frac{F_\pi^2}{16} \mathrm{Tr}\,(\partial_i U^\dagger \partial^i U) + \frac{1}{32e^2} \mathrm{Tr}[U^\dagger \partial_i U, U^\dagger \partial^j U]^2 \right\}$$
$$\Rightarrow \int \mathrm{d}^3 r \left\{ \lambda \frac{F_\pi^2}{16} \mathrm{Tr}\,(\partial_i U^\dagger \partial^i U) + \lambda^{-1} \frac{1}{32e^2} \mathrm{Tr}[U^\dagger \partial_i U, U^\dagger \partial^j U]^2 \right\}$$

- Energy-momentum tensor form factors' analysis gives the more detailed information

Skyrme Model

Hedgehog solution (nontrivial mapping)

- Directions in isotopic space are related to the directions in ordinary 3D configuration space

$$U = \exp\left\{\frac{i\bar{\tau}\,\bar{\pi}}{2F_\pi}\right\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

- Rotations in isotopic and configuration spaces may compensate each other

$$SU(2) \Leftrightarrow O(3) \quad \text{mapping}$$

$$U \Rightarrow U' = AUA^\dagger = \exp\{iA\vec{\tau}A^\dagger\vec{n}F(r)\} = \exp\{i\tau_i D_{ij}(A)n_j F(r)\}$$

- As a result the energy of the system does not change (rotational zero mode fluctuations)

$$E = E'$$

- Nucleons appear after the zero mode quantuzations

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|\, S = T, s, t >= (-1)^{t+T}\sqrt{2T+1}D^{S=T}_{-t,s}(A)$$

Skyrme Model (Summary)

The free space Lagrangian (was widely in use)
[G.S.Adkins et al. Nucl.Phys. B228 (1983)]

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\,\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\,\mathrm{Tr}\left(U + U^\dagger - 2\right)$$

- Nontrivial structure: topologically stable solitons with the corresponding conserved topological number (baryon number) ***A***

$$U = \exp\{i\bar{\tau}\,\bar{\pi}/2F_\pi\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+\partial_\alpha U$$

$$A = \int d^3 r B^0$$

- Nucleon is quantized state of the classical soliton-skyrmion

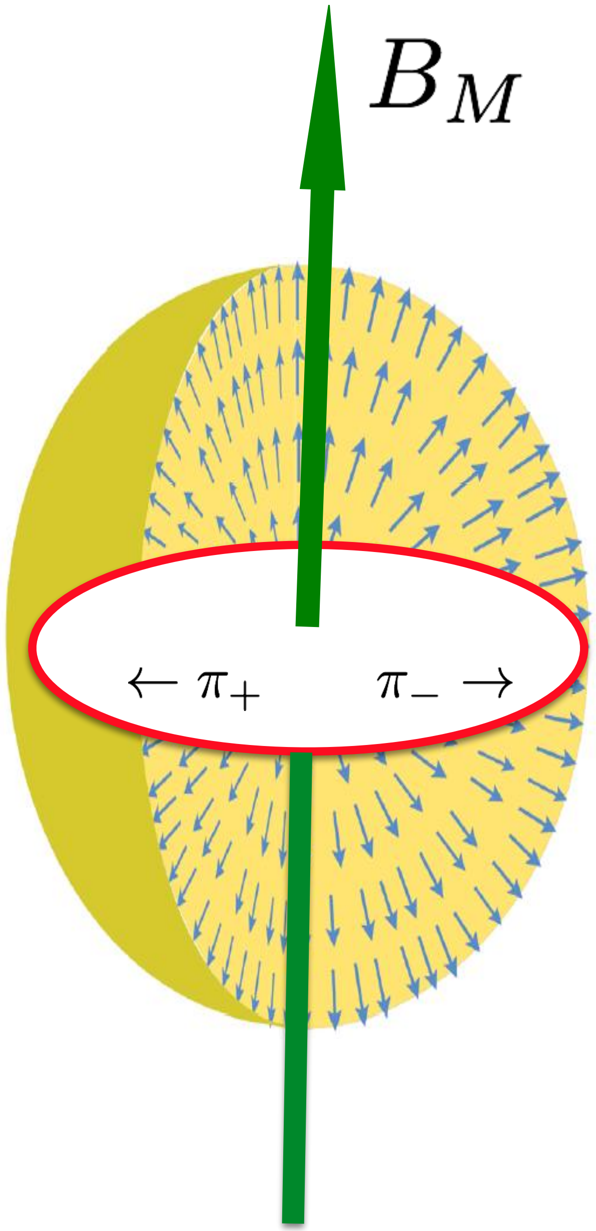
$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|\, S = T, s, t >= (-1)^{t+T}\sqrt{2T+1}D^{S=T}_{-t,s}(A)$$

Soliton in a magnetic field

Deformation of the soliton

- Soliton in a static magnetic field in z-direction has a deformed shape
- Spheroidal solitons due to the charged (nonlinearly interacting) pions in a magnetic field
- One should take into account the deformation effects



Soliton in a magnetic field

"Fully" deformed ansatz

- The most general form $$U(\vec{r}) = \exp\left\{i\vec{\tau}\cdot\vec{N}(\vec{r})P(\vec{r})\right\}$$
- Three profile functions
  - Spatial extent $$P = P(r,\theta,\varphi)$$
  - Non-spherical symmetry in isotopic space in terms of the two functions $$\vec{N} = \begin{pmatrix} \sin\Theta(r,\theta,\varphi)\cos\Phi(r,\theta,\varphi) \\ \sin\Theta(r,\theta,\varphi)\sin\Phi(r,\theta,\varphi) \\ \cos\Theta(r,\theta,\varphi) \end{pmatrix}$$

Axial symmetry for $\vec{B}_M || \vec{z}$

- Only two profile functions $$P = P(r,\theta) \quad \text{and} \quad \Theta = \Theta(r,\theta)$$
- Third one is in trivial form $$\Phi = \varphi$$

EM interactions in a Soliton Picture

Gauging the theory

- Introducing the standard covariant derivative with U(1) field

$$D_\mu U = \partial_\mu U + i q_e A_\mu [Q, U]$$

- The charge operator in SU(2) framework has the form

$$Q = \frac{1}{6}\,\mathbb{I} + \frac{1}{2}\,\tau_3$$

Axial symmetry for $\vec{B}_M || \vec{z}$ **gives**

- Gauge field A_μ is an external field

- In a symmetrically fixed gauge

$$A^\mu = \left(0, -\frac{1}{2} y B_M, \frac{1}{2} x B_M, 0\right)$$

Variational approach to the problem

Classical soliton energy (mass)

- The functional of two functions $E[P,\Theta] = \int \mathrm{d}^3 r \mathcal{M}(P(r,\theta),\Theta(r,\theta))$

- Integrand has the form

$$\mathcal{M}(P,\Theta) = \mathcal{M}(P(r,\theta),\Theta(r,\theta)) + \Delta\mathcal{M}(P(r,\theta),\Theta(r,\theta),B_M)$$

Second order polynomial form on the magnetic field

- If the magnetic field is zero

$$\mathcal{M}(P,\Theta) = \mathcal{M}(P(r),\theta) + \Delta\mathcal{M}(=0)$$

Gives spherically symmetric hedgehog's functional

Variational approach to the problem

Classical equations of motion

- The variation of functional gives the coupled partial differential equations (technically a difficult task)

$$\begin{cases} g(P_{rr}, P_{\theta\theta}, P_r, P_\theta, \Theta_r, \Theta_\theta, P, \Theta) = 0 \\ h(\Theta_{rr}, \Theta_{\theta\theta}, \Theta_r, \Theta_\theta, P_r, P_\theta, \Theta, P) = 0 \end{cases}$$

- Should satisfy “the baryon number equals to one” condition

$$B = -\frac{1}{\pi} \int_0^\infty dr \int_0^\pi d\theta \, (P_r \Theta_\theta - P_\theta \Theta_r) \sin^2 P = 1$$

Variational approach to the problem

Simplification of the problem

- Considering the functional

$$\mathcal{M}_{\text{spherical}}(P) = \mathcal{M}(P(r)) + \Delta\mathcal{M}(P(r), B_M)$$

- One gets an ordinary differential equation (nonlinear form)

$$f(P'', P', P, r, B_M) = 0$$

- At the linear approximation

"Confining nature" - big values of magnetic field

$$P''(r) + \frac{2}{r}P'(r) - \frac{2}{r^2}P(r) - \left(m_\pi^2 + \frac{2}{3}q_e B_M\right)P(r) - \frac{2}{15}(q_e B_M r)^2 P(r) = 0$$

"The pion mass modification" - small values of magnetic field

Variational approach to the problem

Solutions of linear equation

- At the small values of magnetic field (Yukawa type)

$$P(r) \sim \frac{1 + Ar}{r^2}\, e^{-Ar}, \qquad A = \left(m_\pi^2 + \frac{2}{3} q_e B_M\right)^{1/2}$$

- At the large values of magnetic field (gaussian type)

$$P(r) \sim \frac{1}{2^{1/4} r^2} \exp\left\{ - \frac{q_e B_M r^2}{\sqrt{30}} \right\} U\left(\frac{-3 + \sqrt{30}}{12} + \frac{\sqrt{30} m_\pi^2}{8 q_e B_M}, -\frac{1}{2}; \sqrt{\frac{2}{15} q_e B_M r^2} \right)$$

The confluent hypergeometric function of the second type

Variational approach to the problem

Parametrisation of solutions

$$P(r,\theta) = 2\arctan\left\{\frac{r_0^2}{r^2}(1+Ar)[1+u(\theta)]\right\}\exp\left\{-\beta_0 Ar - \beta_1 q_e B_M r^2)\right\}$$

$$u(\theta) = q_e B_M \sum_{n=1}^{\infty} \gamma_n \cos^n\theta$$

$$\Theta(r,\theta) = \theta + \zeta(r,\theta)$$

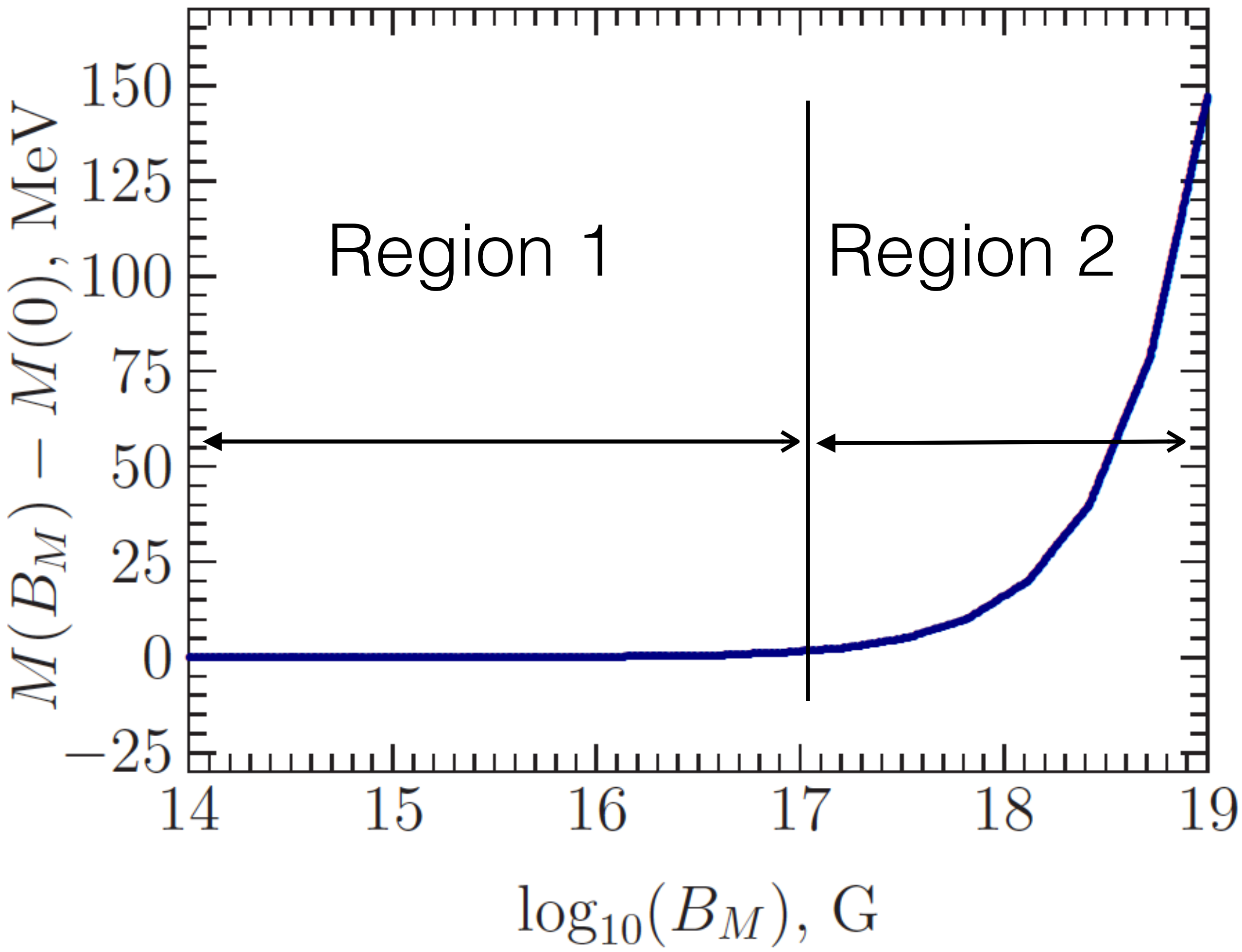
$$\zeta(r,\theta) = q_e B_M r e^{-\delta_0^2 r^2} \sum_{n=1}^{\infty} \delta_n \sin(2n\theta)$$

$$r_0,\ \beta_i,\ \gamma_i,\ \delta_i \quad \text{are variational parameters}$$

- Properly reproduces the asymptotic solutions
- Properly reproduces the solution at origin
- Nicely interpolates in between of these solutions
- Accuracy is very good (the deviations from the exact solutions at the spherical case within the 1%)

Soliton mass & baryon charge redistribution in a magnetic field

Dependence of the classical soliton mass on the magnetic field



UY, H.-Ch.Kim & M.Oka, PRD 99 (2019)

Soliton mass & baryon charge redistribution in a magnetic field

Analysing the regions (table from the Wikipedia)

- **10^{-9}–10^{-8} gauss** – the magnetic field of the human brain
- **10^{-6}–10^{-3} gauss** – the magnetic field of Galactic molecular clouds
- **0.25–0.60 gauss** – the Earth's magnetic field at its surface
- **25 gauss** – the Earth's magnetic field in its core[6]
- **50 gauss** – a typical refrigerator magnet
- **100 gauss** – an iron magnet
- **1500 gauss** - within a sun spot [7]
- **10000 to 13000 gauss** – remanence of a neodymium-iron-boron (NIB) magnet[8]
- **16000 to 22000 gauss** - saturation of high permeability iron alloys used in transformers[9]
- **3000–70,000 gauss** – a medical magnetic resonance imaging machine
- **10^{12}–10^{13} gauss** – the surface of a neutron star[10]
- **4×10^{13} gauss** – the quantum electrodynamic threshold
- **10^{15} gauss** – the magnetic field of some newly created magnetars[11]
- **10^{17} gauss** – the upper limit to neutron star magnetism[11]

and **10^19 gauss** during the heavy ion collisions (very short time interval)

Soliton mass & baryon charge redistribution in a magnetic field

Analysing the regions

- Linearised equation

$$P''(r) + \frac{2}{r}P'(r) - \frac{2}{r^2}P(r) - \left(m_\pi^2 + \frac{2}{3}q_e B_M\right)P(r) - \frac{2}{15}(q_e B_M r)^2 P(r) = 0$$

The three terms for comparing

- Region 1 (Yukawa type asymptotic)

$$r^2(q_e \times 10^{17}\,\mathrm{G})^2 \sim 10\,\mathrm{MeV}^2 \ll (m_\pi^2 + 2q_e B_M/3) \approx m_\pi^2 \sim 0.18\,\mathrm{GeV}^2$$

- Region 2 (quadratic term is more important - gaussian asymptotic)

$$m_\pi^2 \leq 2q_e B_M/3$$

Soliton mass & baryon charge redistribution in a magnetic field

TABLE I. Variational parameters for the profile functions P and Θ at some selected values of the external magnetic field B_M.

B_M	0	10^{15} G	10^{17} G	10^{19} G
r_0, fm^2	0.95646	0.95641	0.95200	0.97324
β_0	1.31568	1.31554	1.30447	0.93320
β_1	0	0	0	0.21958
γ_2, fm^2	0	−0.64430	0.12305	0.33700
γ_4, fm^2	0	0.30370	0.21985	0.08227
γ_6, fm^2	0	−0.10019	−0.14775	0.21615
δ_0, fm^{-2}	4.23604	3.90049	2.84256	3.21149
δ_1, fm	0	0.13997	0.09016	0.9366
δ_2, fm	0	0.24411	0.00207	0.00174

$$P(r,\theta) = 2\arctan\left\{\frac{r_0^2}{r^2}(1+Ar)[1+u(\theta)]\right\}\exp\{-\beta_0 Ar - \beta_1 q_e B_M r^2)\}$$

UY, H.-Ch.Kim & M.Oka, PRD 99 (2019)

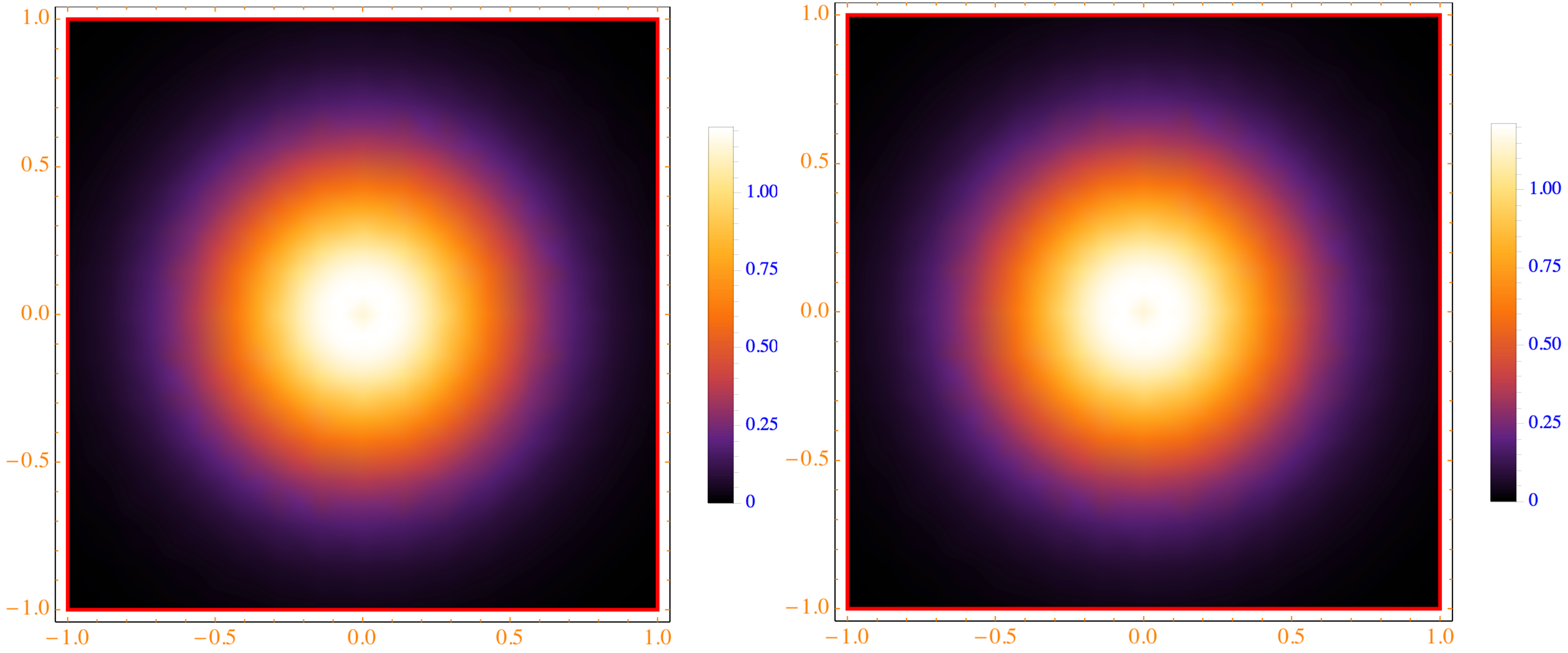
Soliton mass & baryon charge redistribution in a magnetic field

Baryon charge density

$$B_0(r,\theta) = -\frac{P_r\Theta_\theta - P_\theta\Theta_r}{2\pi^2 r^2}\left(\frac{\sin\Theta}{\sin\theta}\right)\sin^2 P$$

Soliton mass & baryon charge redistribution in a magnetic field

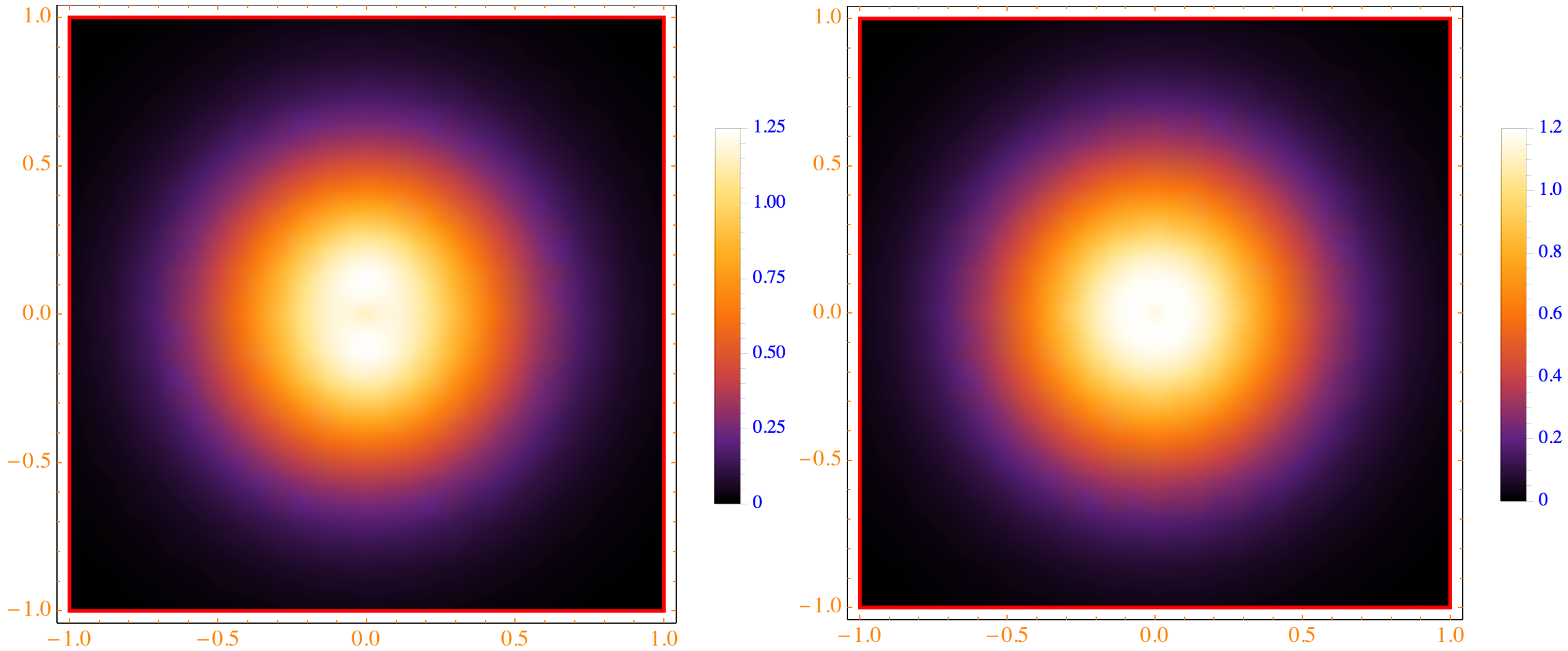
Baryon charge density distribution of soliton at $B_M = 0$



Baryon charge density $B_0(y,z)$ (left panel) and $B_0(x,y)$ (right panel) planes

Soliton mass & baryon charge redistribution in a magnetic field

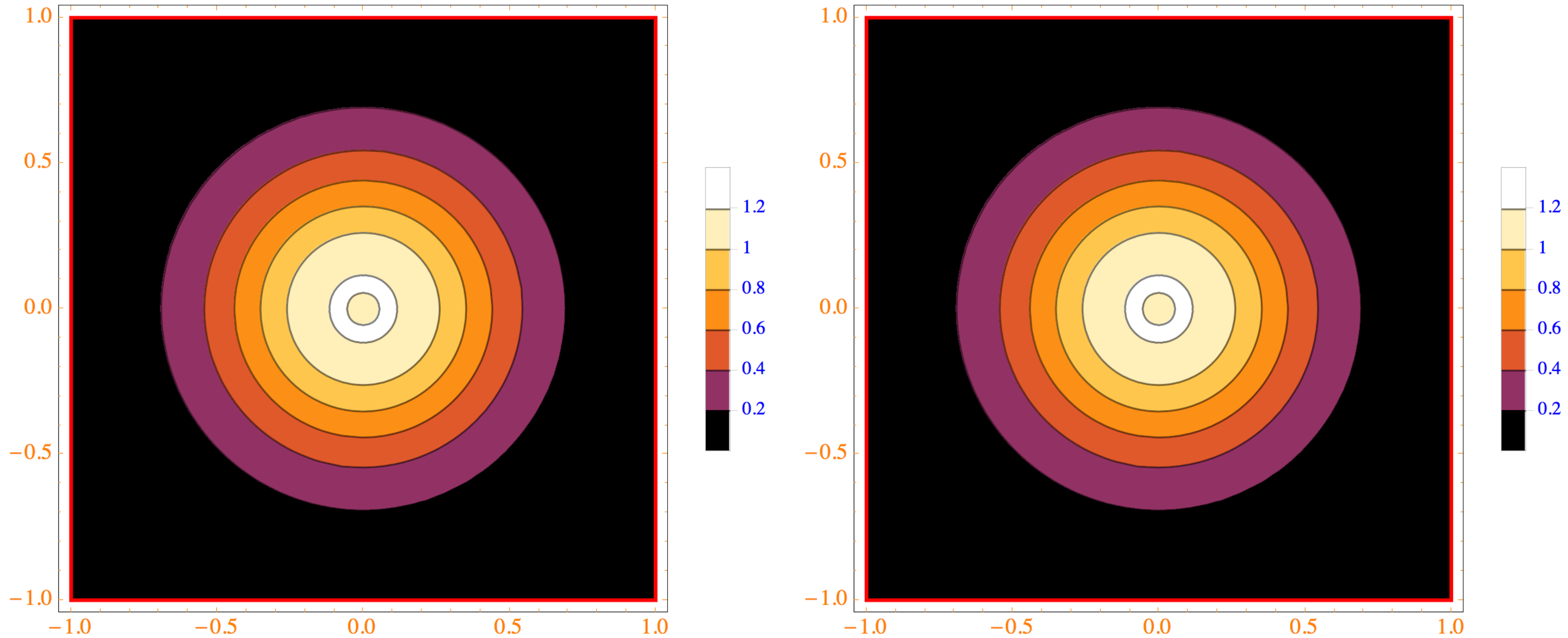
Baryon charge density distribution of soliton at $B_M = 10^{17}\,\mathrm{G}$



Baryon charge density $B_0(y,z)$ (left panel) and $B_0(x,y)$ (right panel) planes

Soliton mass & baryon charge redistribution in a magnetic field

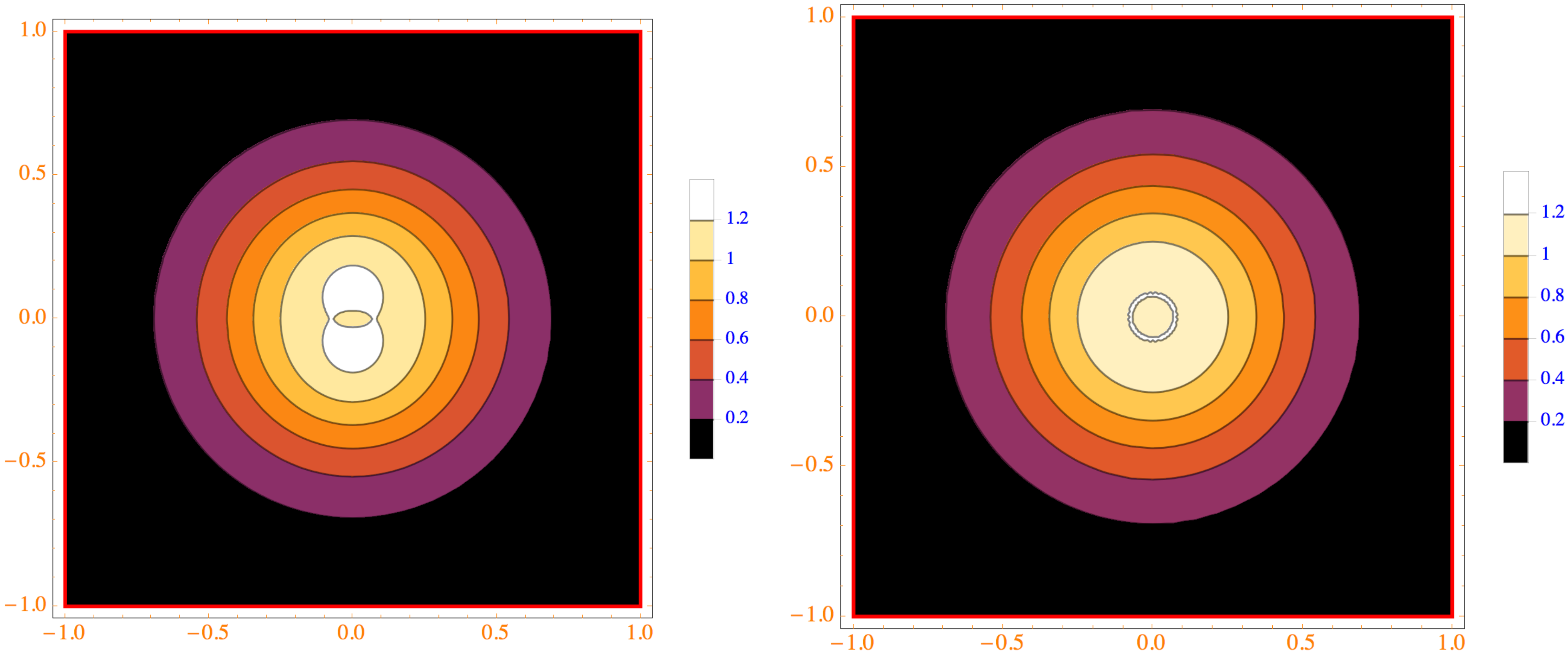
Equivibaryon charge contour lines at $B_M = 0$



Baryon charge density $B_0(y,z)$ (left panel) and $B_0(x,y)$ (right panel) planes

Soliton mass & baryon charge redistribution in a magnetic field

Equivibaryon charge contour lines at $B_M = 10^{17}\,\mathrm{G}$



Baryon charge density $B_0(y,z)$ (left panel) and $B_0(x,y)$ (right panel) planes

Soliton mass & baryon charge redistribution in a magnetic field

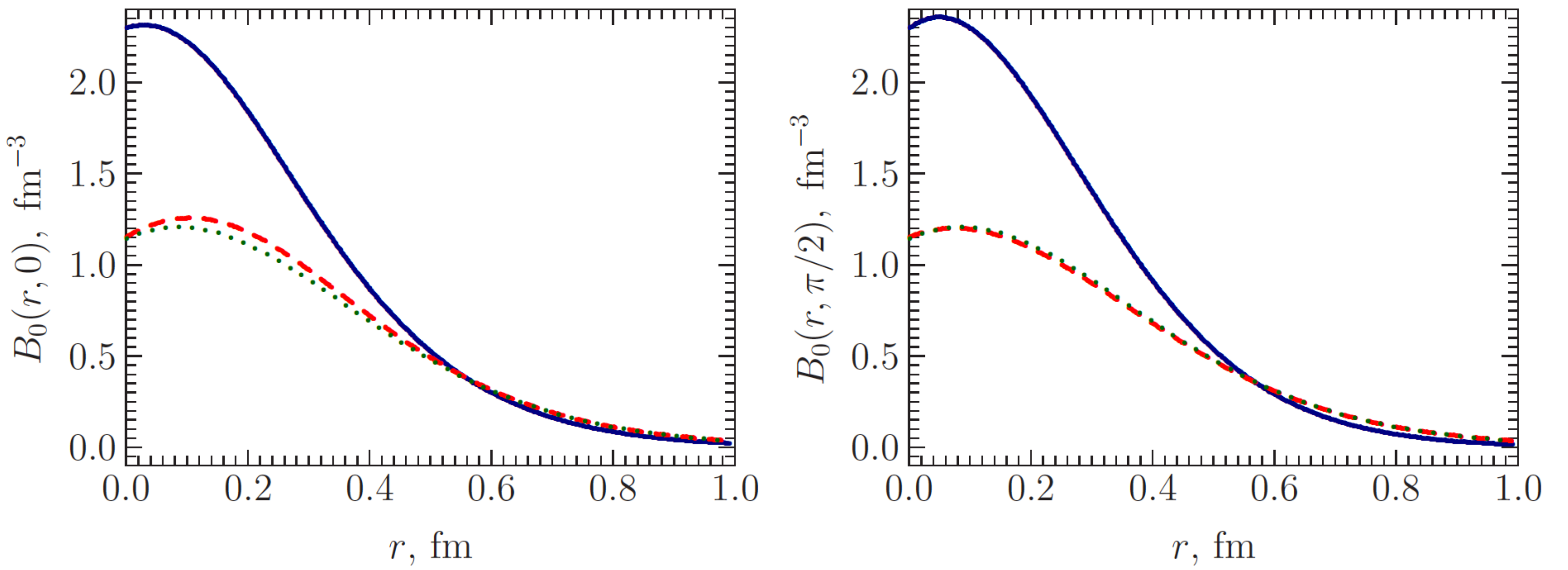


FIG. 2. (Color online) Results of the baryon charge distributions along the z direction (left panel) and in the perpendicular plane to the z axis (right panel), respectively. The solid curves depict the results with $B_M = 10^{19}$ G, the dashed ones draw those with $B_M = 10^{17}$ G, and the dotted ones correspond to the case of $B_M = 0$, respectively.

UY, H.-Ch.Kim & M.Oka, PRD 99 (2019)

Soliton mass & baryon charge redistribution in a magnetic field

$$\Delta B_0(r) \equiv B_0(r, \pi/2) - B_0(r, 0)$$

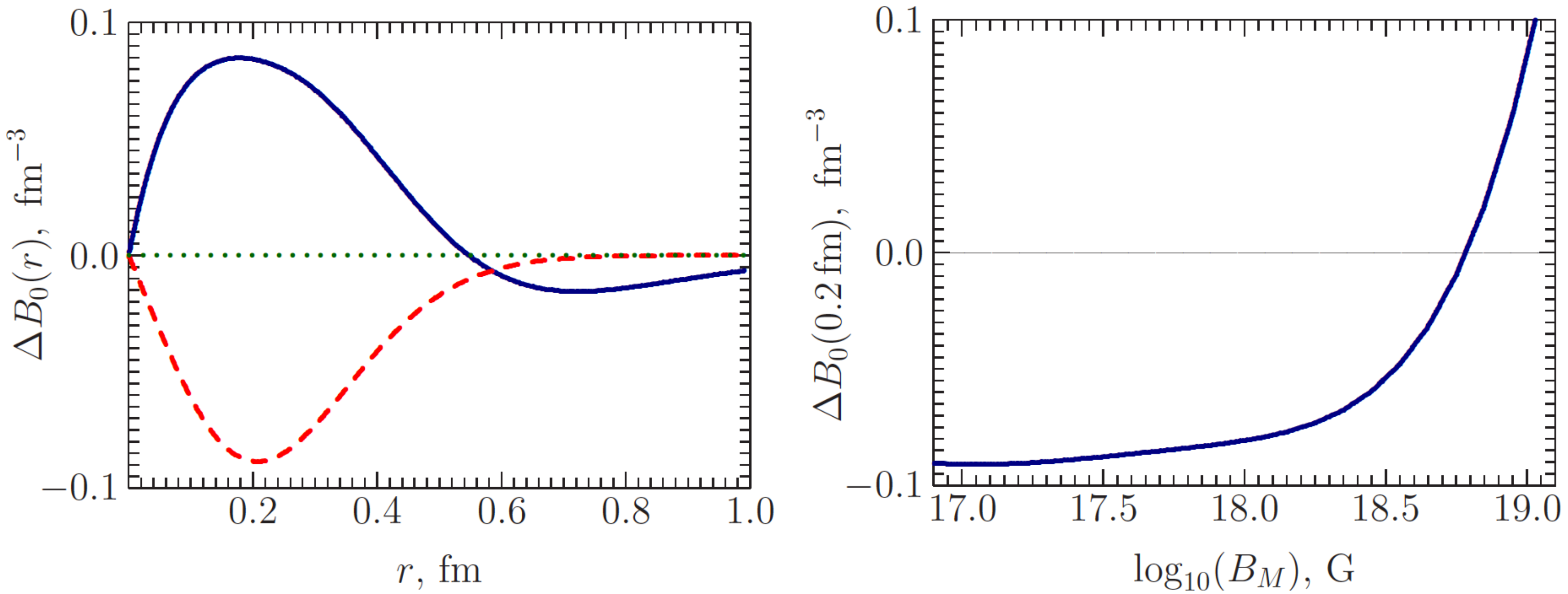


FIG. 3. (Color online) The left panel draws the results of the anisotropy $\Delta B_0(r)$ defined in Eq. (17) as functions of r, whereas the right panel shows the result of $\Delta B_0(0.2\,\text{fm})$ fixed at $r = 0.2\,\text{fm}$ as a function of the magnetic field. Notations in the left panel are the same as in Fig. 2.

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Soliton mass & baryon charge redistribution in a magnetic field

Baryon charge localization

$$B_{(1\,\mathrm{fm})} = \int_0^{1\,\mathrm{fm}} r^2 dr \int d\Omega \, B_0$$

$$B_{(1\,\mathrm{fm})} = 0.9014 \text{ for } B_M = 0,$$

$$B_{(1\,\mathrm{fm})} = 0.9024 \text{ for } B_M = 10^{17}\, G$$

$$\text{and } B_{(1\,\mathrm{fm})} = 0.9665 \text{ for } B_M = 10^{19}\, G$$

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Quantisation of spheroidal solitons

Global time independent rotations

- Performing separately SO(3) rotations in isotopic and configuration spaces

$$U = \exp\{i\tau_i D_{ij}(A) N_j(\vec{r}) P(\mathbf{R}^{-1}\vec{r})\}$$

- And defining the new vectors in a body-fixed (primed) frame

$$U = \exp\{i\vec{\tau}\vec{N}' P(\vec{r}\,')\}$$

- One has the same energy of the classical static configuration

$$E[P(\vec{r}), \Theta(\vec{r})] = E'[P(\vec{r}\,'), \Theta(\vec{r}\,')]$$

Quantisation of spheroidal solitons

Time dependent slow rotations

- Performing separately SO(3) rotations in isotopic and configuration spaces and

$$U = \exp\{i\tau_i D_{ij}(A(t))N_j(\vec{r})P(\mathbf{R}^{-1}(t)\vec{r})\}$$

- using the following relations

$$\partial_0 U = f_{1,i}(\vec{N}\,', \vec{r}\,')\partial_0 N_i'(t) + f_{2,i}(\vec{N}\,', \vec{r}\,')\partial_0 r_i'$$

$$\partial_0 N_i'(t) = \dot{D}_{ij}(t)N_j = \dot{D}_{ij}D_{jk}^{-1}D_{kl}N_l = \dot{D}_{ij}D_{jk}^{-1}N_k'$$

$$\partial_0 r_i'(t) = \dot{R}_{ij}^{-1}(t)r_j = \dot{R}_{ij}^{-1}R_{jk}R_{kl}^{-1}r_l = \dot{R}_{ij}^{-1}R_{jk}r_k'$$

- one generates the angular velocities in isotopic and configuration spaces

$$\dot{D}_{ij}D_{jk}^{-1} = i\epsilon_{ikl}\omega_l \quad \text{and} \quad \dot{R}_{ij}^{-1}R_{jk} = -i\epsilon_{ikl}\Omega_l$$

Quantisation of spheroidal solitons

Time dependent Lagrangian and canonical conjugate variables

- Lagrangian has form

$$L = -M + \frac{\omega_1^2+\omega_2^2}{2}\Lambda_{\omega\omega,12} - (\omega_1\Omega_1+\omega_2\Omega_2)\Lambda_{\omega\Omega,12} + \frac{\Omega_1^2+\Omega_2^2}{2}\Lambda_{\Omega\Omega,12} + \frac{(\omega_3-\Omega_3)^2}{2}\Lambda_{\omega\Omega,33}$$

- Canonical conjugate variables

$$T_i = \frac{\partial L}{\partial \omega_i} \quad \text{and} \quad J_i = \frac{\partial L}{\partial \Omega_i}$$

- Hamiltonian of the system

$$\hat{H} = M + \frac{\hat{T}_3^2}{2\Lambda_{\omega\Omega,33}} + \frac{(\hat{T}_1\hat{J}_1 + \hat{T}_2\hat{J}_2)\Lambda_{\omega\Omega,12}}{\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12} - \Lambda_{\omega\Omega,12}^2} + \frac{(\hat{T}_1^2 + \hat{T}_2^2)\Lambda_{\Omega\Omega,12} + (\hat{J}_1^2 + \hat{J}_2^2)\Lambda_{\omega\omega,12}}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12} - \Lambda_{\omega\Omega,12}^2)}$$

Quantisation of spheroidal solitons

Eigenstates and eigenenergies

- Eigenstates in a body-fixed reference frame

$$|T, T_3; J, J_3\rangle$$

- The grand spin zero low energy configuration

$$\vec{K} = \vec{T} + \vec{J} = 0$$

- Eigenenergies

$$E = M + \frac{\Lambda_{\omega\omega,12} + \Lambda_{\Omega\Omega,12} - 2\Lambda_{\omega\Omega,12}}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12} - \Lambda^2_{\omega\Omega,12})} T(T+1) - \frac{\Lambda_{\Omega\Omega,12} T_3^2 + \Lambda_{\omega\omega,12} J_3^2}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12} - \Lambda^2_{\omega\Omega,12})}$$
$$- \left(\frac{1}{2\Lambda_{\omega\Omega,33}} + \frac{\Lambda_{\omega\Omega,12}}{\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12} - \Lambda^2_{\omega\Omega,12}} \right) T_3 J_3$$

Quantisation of spheroidal solitons

Eigenstates and eigenenergies

- Limiting considerations

$$\lim_{B_M\to 0,\ \Theta(r,\theta)\to\theta,\ P(r,\theta)\to P(r)} \frac{\Lambda_{\omega\omega,12}+\Lambda_{\Omega\Omega,12}-2\Lambda_{\omega\Omega,12}}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12}-\Lambda^2_{\omega\Omega,12})}$$

$$= \lim_{\Lambda_{\omega\Omega,12}\to\Lambda}\left(\lim_{\Lambda_{\Omega\Omega,12}\to\Lambda_{\omega\Omega,12}}\left(\lim_{B_M\to 0,\ \Lambda_{\omega\omega,12}\to\Lambda_{\Omega\Omega,12}} \frac{\Lambda_{\omega\omega,12}+\Lambda_{\Omega\Omega,12}-2\Lambda_{\omega\Omega,12}}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12}-\Lambda^2_{\omega\Omega,12})}\right)\right)=\frac{1}{2\Lambda}$$

- Only $T_3=-J_3$ states are allowed

$$-\frac{\Lambda_{\Omega\Omega,12}T_3^2+\Lambda_{\omega\omega,12}J_3^2}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12}-\Lambda^2_{\omega\Omega,12})}-\left(\frac{1}{2\Lambda_{\omega\Omega,33}}+\frac{\Lambda_{\omega\Omega,12}}{\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12}-\Lambda^2_{\omega\Omega,12}}\right)T_3J_3$$

$$=-\frac{\Lambda_{\Omega\Omega,12}T_3^2+\Lambda_{\omega\omega,12}J_3^2+2\Lambda_{\omega\Omega,12}T_3J_3}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12}-\Lambda^2_{\omega\Omega,12})}-\frac{1}{2\Lambda_{\omega\Omega,33}}T_3J_3$$

Quantisation of spheroidal solitons

Eigenstates and eigenenergies

- Finally, we have

$$E = M + \frac{T_3^2}{2\Lambda_{\omega\Omega,33}} + \frac{\Lambda_{\Omega\Omega,12} + \Lambda_{\omega\omega,12} - 2\Lambda_{\omega\Omega,12}}{2(\Lambda_{\omega\omega,12}\Lambda_{\Omega\Omega,12} - \Lambda^2_{\omega\Omega,12})}\left(T(T+1) - T_3^2\right)$$

- Degeneracy of Delta energy states are partially lifted

$$m_{\mathrm{p}} = m_{\mathrm{n}}$$
$$m_{\Delta^{++}} = m_{\Delta^{-}} \neq m_{\Delta^{+}} = m_{\Delta^{0}}$$

Baryons in a magnetic field

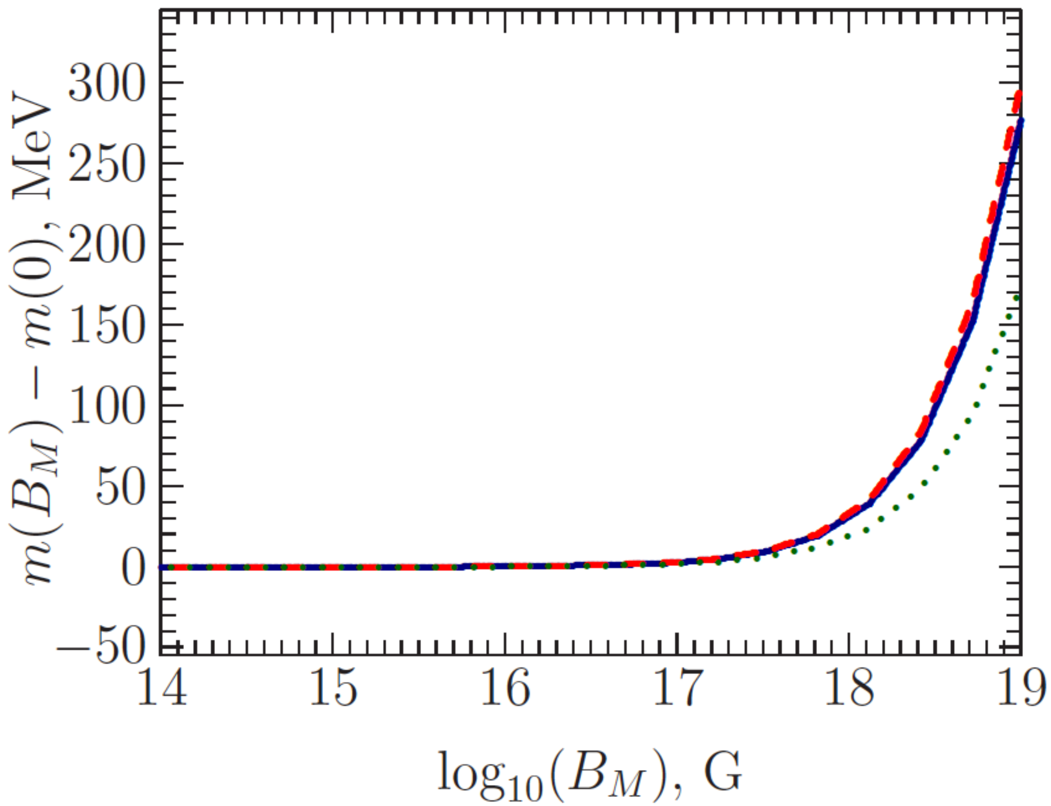

FIG. 4. (Color online) The changes of the baryon masses as a function of the magnetic field. The solid curve depicts m_{Δ^0}, whereas the dashed one draws m_{Δ^-}. The dotted one represents $m_{\rm n}$, respectively.

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Baryons in a magnetic field

The mass splittings

$$\begin{aligned}
\Delta m_{(0,-)}(B_M) &= [m_{\Delta^0}(B_M) - m_{\Delta^-}(B_M)] \\
&- [m_{\Delta^0}(0) - m_{\Delta^-}(0)], \\
\Delta m_{(0,\mathrm{n})}(B_M) &= [m_{\Delta^0}(B_M) - m_{\mathrm{n}}(B_M)] \\
&- [m_{\Delta^0}(0) - m_{\mathrm{n}}(0)], \\
\Delta m_{(-,\mathrm{n})}(B_M) &= [m_{\Delta^-}(B_M) - m_{\mathrm{n}}(B_M)] \\
&- [m_{\Delta^-}(0) - m_{\mathrm{n}}(0)].
\end{aligned}$$

Baryons in a magnetic field

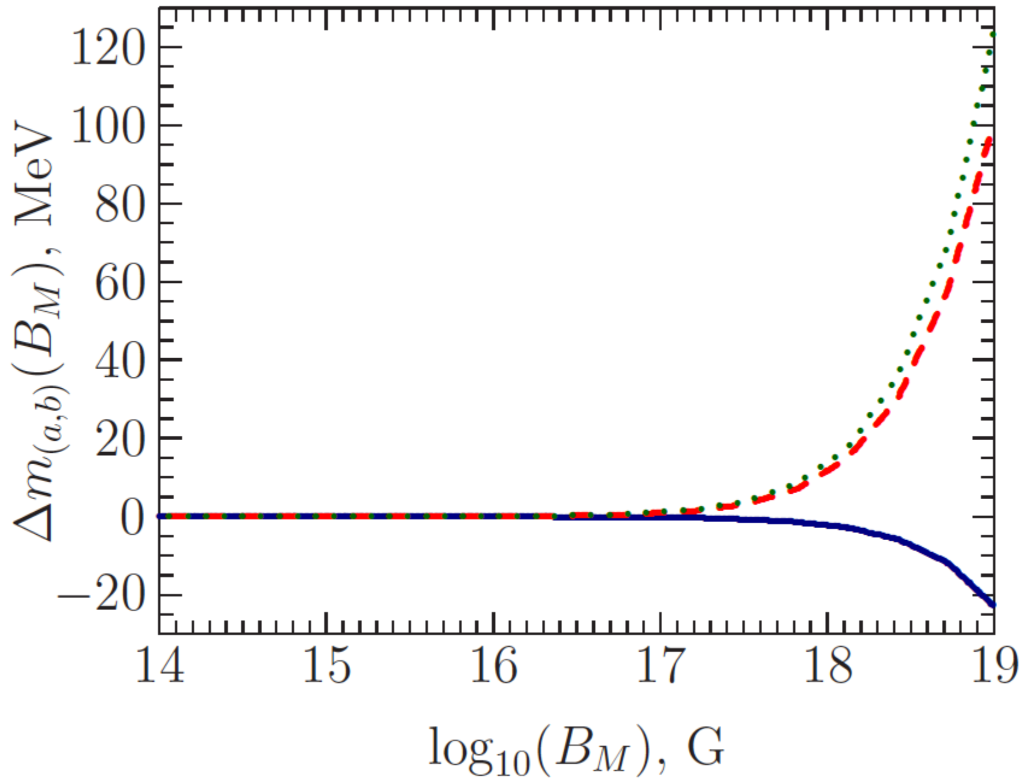

FIG. 5. (Color online) The change of the baryon mass splittings in the presence of the magnetic field. The solid curve draws the result of $\Delta m_{(0,-)}(B_M)$, whereas the dashed one depicts $\Delta m_{(0,\mathrm{n})(\mathrm{B_M})}$. The dotted one shows $\Delta m_{(-,\mathrm{n})}(B_M)$. For the definitions of $\Delta m_{(a,b)}$, see Eqs. (25)-(26).

UY, H.-Ch.Kim & M.Oka, PRD 99 (2019)

Summary

- Within the present approach
  - Baryons are deformed in a magnetic field
  - The changes are marginal up to the values of magnetic field existing in neutron stars
  - The changes are large at the large values of magnetic field corresponding to heavy-ion collision experiments

Thank you very much for your attention!