Parity doublet model in dense matter and more

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Nuclear Many-Body Theories: Beyond the mean field approaches
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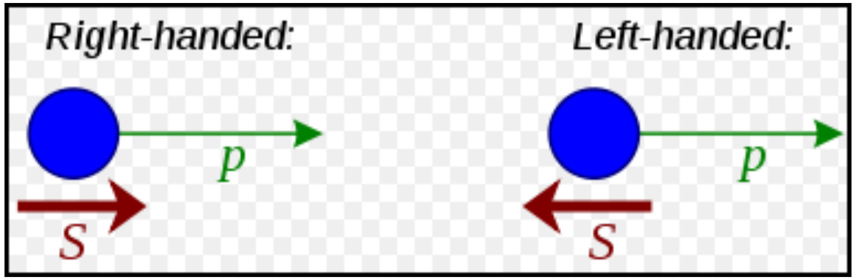
Parity doublet model in dense matter

- **Chiral symmetry**



wikipedia

$$\mathcal{L} = i\bar{\psi}_j \partial\!\!\!/ \psi_j$$

$$\Lambda_V : \ \psi \longrightarrow e^{-i\frac{\vec{\tau}}{2}\vec{\Theta}}\psi \simeq (1 - i\frac{\vec{\tau}}{2}\vec{\Theta})\psi$$

: vector transform

$$\begin{aligned} i\bar{\psi}\partial\!\!\!/\psi &\longrightarrow i\bar{\psi}\partial\!\!\!/\psi - i\vec{\Theta}\left(\bar{\psi}i\partial\!\!\!/\frac{\vec{\tau}}{2}\psi - \bar{\psi}\frac{\vec{\tau}}{2}i\partial\!\!\!/\psi\right) \\ &= i\bar{\psi}\partial\!\!\!/\psi \end{aligned}$$

$$V_\mu^a = \bar{\psi}\,\gamma_\mu \frac{\tau^a}{2}\,\psi$$

$$\Lambda_A : \qquad \psi \longrightarrow e^{-i\gamma_5\frac{\vec{\tau}}{2}\vec{\Theta}}\psi = (1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\Theta})\psi$$

: axial-vector transform

$$\begin{aligned} i\bar{\psi}\partial\!\!\!/\psi &\longrightarrow i\bar{\psi}\partial\!\!\!/\psi - i\vec{\Theta}\left(\bar{\psi}\,i\partial_\mu\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\psi + \bar{\psi}\,\gamma_5\frac{\vec{\tau}}{2}i\partial_\mu\gamma^\mu\,\psi\right) \\ &= i\bar{\psi}\partial\!\!\!/\psi \end{aligned}$$

$$A_\mu^a = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau}{2}\psi$$

- **Chiral symmetry breaking**

$$\delta \mathcal{L} = -m\,(\bar{\psi}\psi)$$

$$\Lambda_A :\; m\,(\bar{\psi}\psi) \longrightarrow m\bar{\psi}\psi - 2im\vec{\Theta}\left(\bar{\psi}\frac{\vec{\tau}}{2}\gamma_5\psi\right)$$

→ **Explicit chiral symmtery breaking**

$$\frac{m}{\Lambda_{\rm QCD}} \sim 0.05$$ → **chiral limit: m=0**

$$< \bar{q}q >^{1/3} / \Lambda_{\rm QCD} \sim 1$$ → **SSB of chiral symmetry**

$$m \sim (5-10)\ {\rm MeV},\ \ \Lambda_{\rm QCD} \sim 200\ {\rm MeV}, < \bar{q}q >^{1/3} \simeq -240\ {\rm MeV}$$

- **Mesons and chiral symmetry**

pion-like state: $\vec{\pi} \equiv i\bar{\psi}\vec{\tau}\gamma_5\psi$; sigma-like state: $\sigma \equiv \bar{\psi}\psi$

rho-like state: $\vec{\rho}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\psi$; a_1-like state: $\vec{a_1}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\gamma_5\psi$

$$\begin{aligned} \pi_i : \quad i\bar{\psi}\tau_i\gamma_5\psi \quad &\longrightarrow \quad i\bar{\psi}\tau_i\gamma_5\psi + \Theta_j\left(\bar{\psi}\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi + \bar{\psi}\gamma_5\frac{\tau_j}{2}\tau_i\gamma_5\psi\right) \\ &= \quad i\bar{\psi}\tau_i\gamma_5\psi + \Theta_i\bar{\psi}\psi \end{aligned}$$

→ $\vec{\pi} \longrightarrow \vec{\pi} + \vec{\Theta}\sigma$

$\sigma \longrightarrow \sigma - \vec{\Theta}\vec{\pi}$ $\qquad$ $\vec{\rho}_\mu \longrightarrow \vec{\rho}_\mu + \vec{\Theta} \times \vec{a_1}_\mu$

- **Linear sigma model**

$$\Lambda_V : \quad \pi^2 \longrightarrow \pi^2; \qquad \sigma^2 \longrightarrow \sigma^2$$

$$\Lambda_A : \quad \vec{\pi}^2 \longrightarrow \vec{\pi}^2 + 2\sigma\Theta_i\pi_i; \qquad \sigma^2 \longrightarrow \sigma^2 - 2\sigma\Theta_i\pi_i$$

$$(\vec{\pi}^2 + \sigma^2) \stackrel{\Lambda_V, \Lambda_A}{\longrightarrow} (\vec{\pi}^2 + \sigma^2)$$

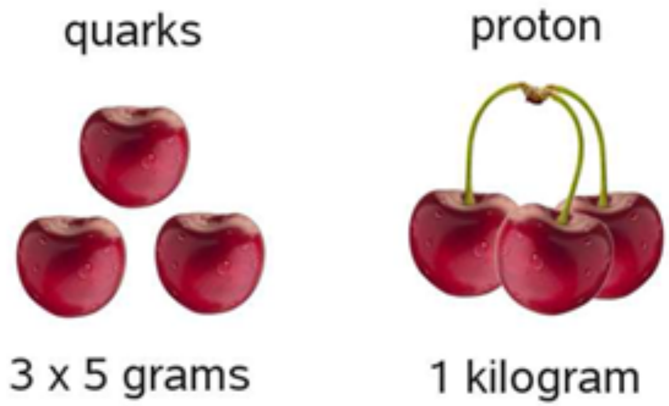
* SSB →

$$V = V(\pi^2 + \sigma^2) = \frac{\lambda}{4}\left((\pi^2 + \sigma^2) - f_\pi^2\right)^2$$

$$\mathcal{L}_{L.S.} = \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\lambda}{4}\left((\pi^2 + \sigma^2) - f_\pi^2\right)^2$$

$$\delta\mathcal{L} = -g_\pi \left[\left(i\bar{\psi}\gamma_5\vec{\tau}\psi \right) \vec{\pi} + \left(\bar{\psi}\psi \right) \sigma \right]$$

$$\begin{aligned} < \sigma > &= \sigma_0 = f_\pi \\ < \pi > &= 0 \\ M_N &= g_\pi \sigma_0 = g_\pi f_\pi \end{aligned}$$



Parity doublet model

Introduce two nucleon fields that transform in a mirror way under chiral transformations:

$$\mathrm{SU}_L(2) \times \mathrm{SU}(2)_R$$

$$\psi_{1R} \to R\psi_{1R}, \quad \psi_{1L} \to L\psi_{1L},$$
$$\psi_{2R} \to L\psi_{2R}, \quad \psi_{2L} \to R\psi_{2L}.$$

$$m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2)$$
$$= m_0(\bar{\psi}_{2L}\psi_{1R} - \bar{\psi}_{2R}\psi_{1L} - \bar{\psi}_{1L}\psi_{2R} + \bar{\psi}_{1R}\psi_{2L})$$

Or

,

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi = i\bar{\psi}_L\gamma_\mu\partial^\mu\psi_L + i\bar{\psi}_R\gamma_\mu\partial^\mu\psi_R,$$

$$\psi_R = \frac{1}{2}\left(1+\gamma_5\right)\psi, \quad \psi_L = \frac{1}{2}\left(1-\gamma_5\right)\psi.$$

$$\psi_R \rightarrow \exp\left(\imath\frac{\theta_R^a\tau^a}{2}\right)\psi_R; \quad \psi_L \rightarrow \exp\left(\imath\frac{\theta_L^a\tau^a}{2}\right)\psi_L$$

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix},$$

where the Dirac bispinors Ψ_+ and Ψ_- have positive and negative parity, respectively.

$$\Psi_R = \frac{1}{\sqrt{2}}(\Psi_+ + \Psi_-)\,; \quad \Psi_L = \frac{1}{\sqrt{2}}(\Psi_+ - \Psi_-)\,,$$

$$\begin{aligned}\mathcal{L} &= i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi \\ &= i\bar{\Psi}_+\gamma^\mu\partial_\mu\Psi_+ + i\bar{\Psi}_-\gamma^\mu\partial_\mu\Psi_- - m\bar{\Psi}_+\Psi_+ - m\bar{\Psi}_-\Psi_-.\end{aligned}$$

Baryon parity doublets and chiralspin symmetry

M. Catillo and L. Ya. Glozman

$$\mathcal{L} = \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2)$$
$$+ a\bar{\psi}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_1 + b\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2$$

$$m_{N\pm} = \frac{1}{2}\left(\sqrt{(a+b)^2\sigma^2 + 4m_0^2} \mp (a-b)\sigma\right)$$

The state N+ is the nucleon N(938). while N- is its parity partner conventionally identified with N(1500).

the decay width $\Gamma_{N\pi}$ for $N^*(1535) \rightarrow N + \pi$, $m_0 = 270$ MeV

“Linear sigma model with parity doubling,” C. E. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989)

Cold, dense nuclear matter in a SU(2) parity doublet model

$$\begin{aligned}\mathcal{L} \;=\;& \bar{\psi}_1 i\partial\!\!\!/\psi_1 + \bar{\psi}_2 i\partial\!\!\!/\psi_2 + m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\ & + a\bar{\psi}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_1 + b\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2 \\ & - g_\omega\bar{\psi}_1\gamma_\mu\omega^\mu\psi_1 - g_\omega\bar{\psi}_2\gamma_\mu\omega^\mu\psi_2 + \mathcal{L}_M,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_M \;=\;& \frac{1}{2}\partial_\mu\sigma^\mu\partial^\mu\sigma_\mu + \frac{1}{2}\partial_\mu\vec{\pi}^\mu\partial^\mu\vec{\pi}_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + g_4^4(\omega_\mu\omega^\mu)^2 \\ & + \frac{1}{2}\bar{\mu}^2(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + \epsilon\sigma,\end{aligned}$$

D. Zschiesche, L. Tolos, Jurgen Schaffner-Bielich, Robert D. Pisarski, Phys.Rev. C75 (2007) 055202

PHYSICAL REVIEW C **92**, 025201 (2015)

Asymmetric nuclear matter in a parity doublet model with hidden local symmetry

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We construct a model to describe dense hadronic matter at zero and finite temperatures, based on the parity doublet model of DeTar and Kunihiro [C. E. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989)], including the isosinglet scalar meson σ as well as ρ and ω mesons. We show that, by including a six-point interaction of the σ meson, the model reasonably reproduces the properties of normal nuclear matter with the chiral invariant nucleon mass m_0 in the range from 500 to 900 MeV. Furthermore, we study the phase diagram based on the model, which shows that the value of the chiral condensate drops at the liquid-gas phase transition point and at the chiral phase transition point. We also study asymmetric nuclear matter and find that the first-order phase transition for the liquid-gas phase transition disappears in asymmetric matter and that the critical density for the chiral phase transition at nonzero density becomes smaller for larger asymmetry.

Y. Motohiro, M. Harada, YK, Erratum: Phys.Rev. C95 (2017) 059903

$$
\begin{aligned}
\mathcal{L} = {} & \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) \\
& + g_1\bar{\psi}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_1 + g_2\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2 \\
& - g_{\omega NN}\bar{\psi}_1\gamma_\mu\omega^\mu\psi_1 - g_{\omega NN}\bar{\psi}_2\gamma_\mu\omega^\mu\psi_2 \\
& - g_{\rho NN}\bar{\psi}_1\gamma_\mu\vec{\rho}^\mu\cdot\vec{\tau}\psi_1 - g_{\rho NN}\bar{\psi}_2\gamma_\mu\vec{\rho}^\mu\cdot\vec{\tau}\psi_2 \\
& - e\bar{\psi}_1\gamma^\mu A_\mu\frac{1-\tau_3}{2}\psi_1 - e\bar{\psi}_2\gamma^\mu A_\mu\frac{1-\tau_3}{2}\psi_2 + \mathcal{L}_M ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_M = {} & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} - \frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + \frac{\bar{\mu}^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + \frac{\lambda_6}{6}(\sigma^2 + \vec{\pi}^2)^3 + \epsilon\sigma \\
& + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu
\end{aligned}
$$

To fix the parameters in our model with fixed m0

TABLE I. The inputs from free space (in MeV).

m_+	m_-	m_ω	m_ρ	f_π	m_π
939	1535	783	776	93	138

$$\frac{E}{A} - m_N = -16 \text{ MeV}\,, \quad n_0 = 0.16 \text{ fm}^{-3},$$
$$K = 240 \pm 40 \text{ MeV}\,, \quad E_{\text{sym}} = 31 \text{ MeV}\,.$$

Delta matter in a parity doublet model (within MFA)

Yusuke Takeda, YK, Masayasu Harada, Phys. Rev. C97 (2018) 065202

* In symmetric matter, Delta enters into matter at (1-4) times the saturation density.
The stable Δ-nucleon matter is realized around 4 times the saturation density, and the phase transition from nuclear matter to Δ-nucleon matter is of first order in the wide parameter region.

* In asymmetric matter, the phase transition from the nuclear matter to the stable Δ-nucleon matter can be of the second order for most parameter region. The onset density is smaller than that in symmetric matter.

* In symmetric dense matter, larger chiral invariant nucleon mass tends to lower the transition density to the stable N-Δ phase.

* Partial restoration of chiral symmetry is enhanced by Delta matter.

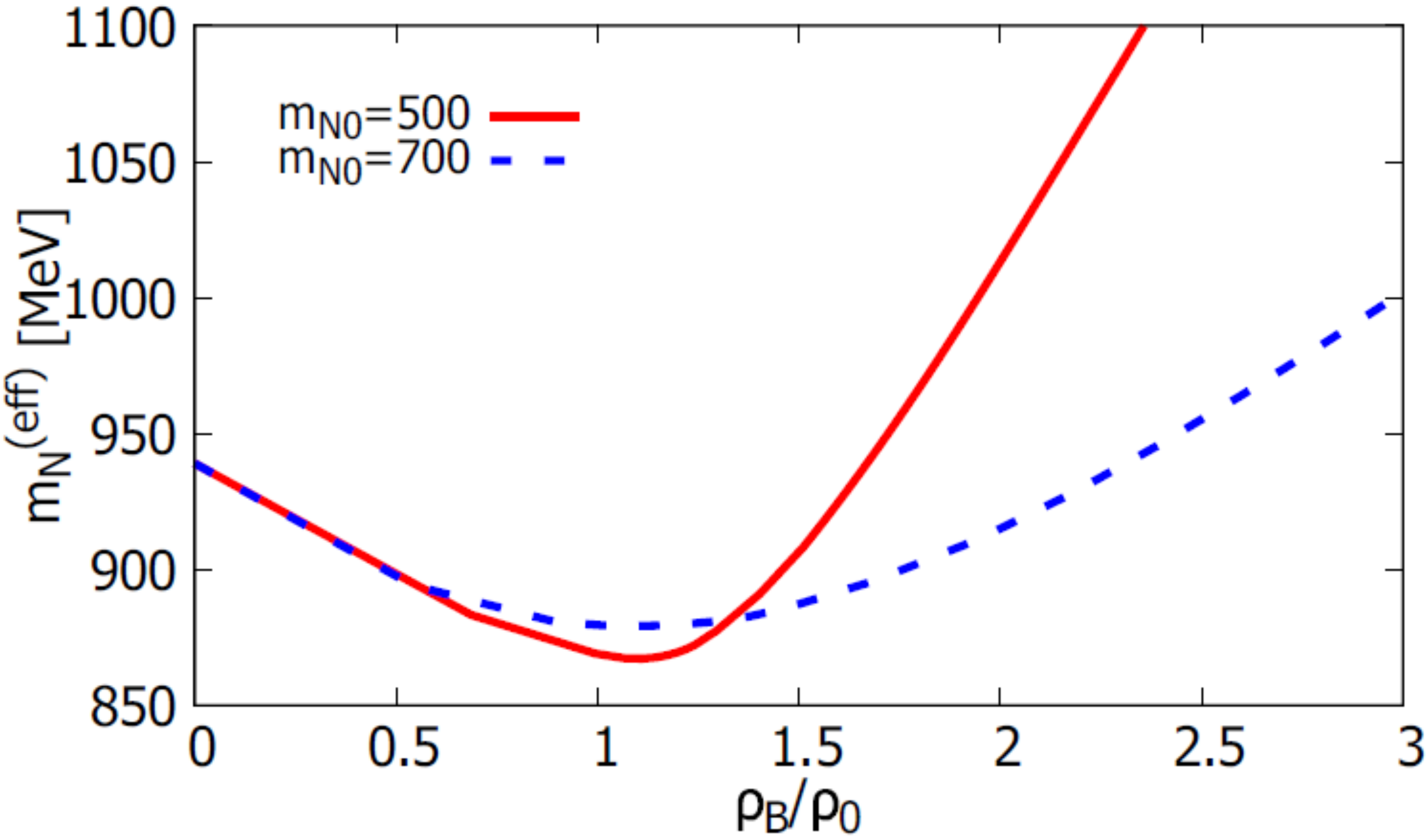


FIG. 2. Density dependence of effective mass of nucleon for $m_{N0} = 500\,\text{MeV}$ (red solid curve) and $700\,\text{MeV}$ (blue dashed curve) in symmetric nuclear matter.

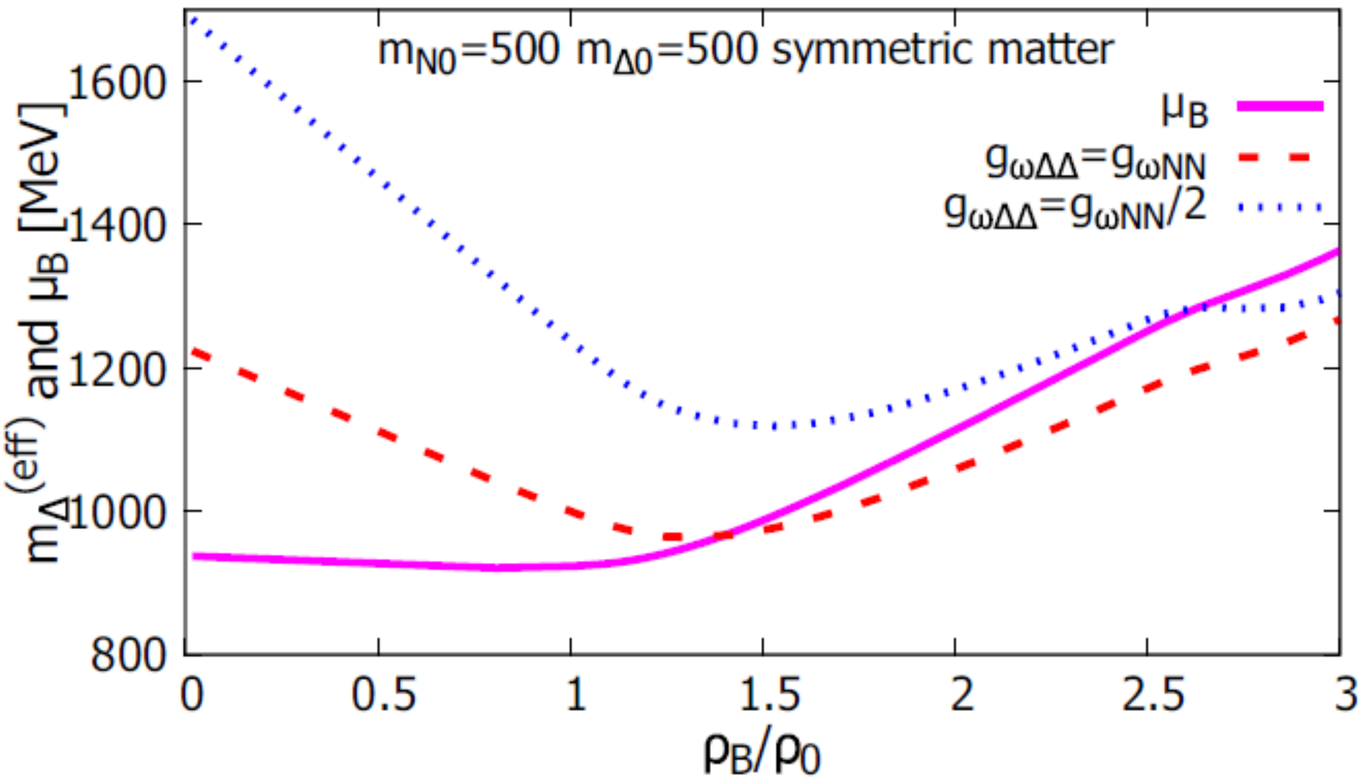


FIG. 3. Density dependence of the effective masses of Δ for $g_{\omega\Delta\Delta} = g_{\omega NN}$ (red dashed curve) and $g_{\omega\Delta\Delta} = g_{\omega NN}/2$ (blue dotted curve) with fixed values of $m_{N0} = m_{\Delta 0} = 500\,\text{MeV}$ in symmetric nuclear matter. The pink solid curve shows the density dependence of the baryon chemical potential μ_B.

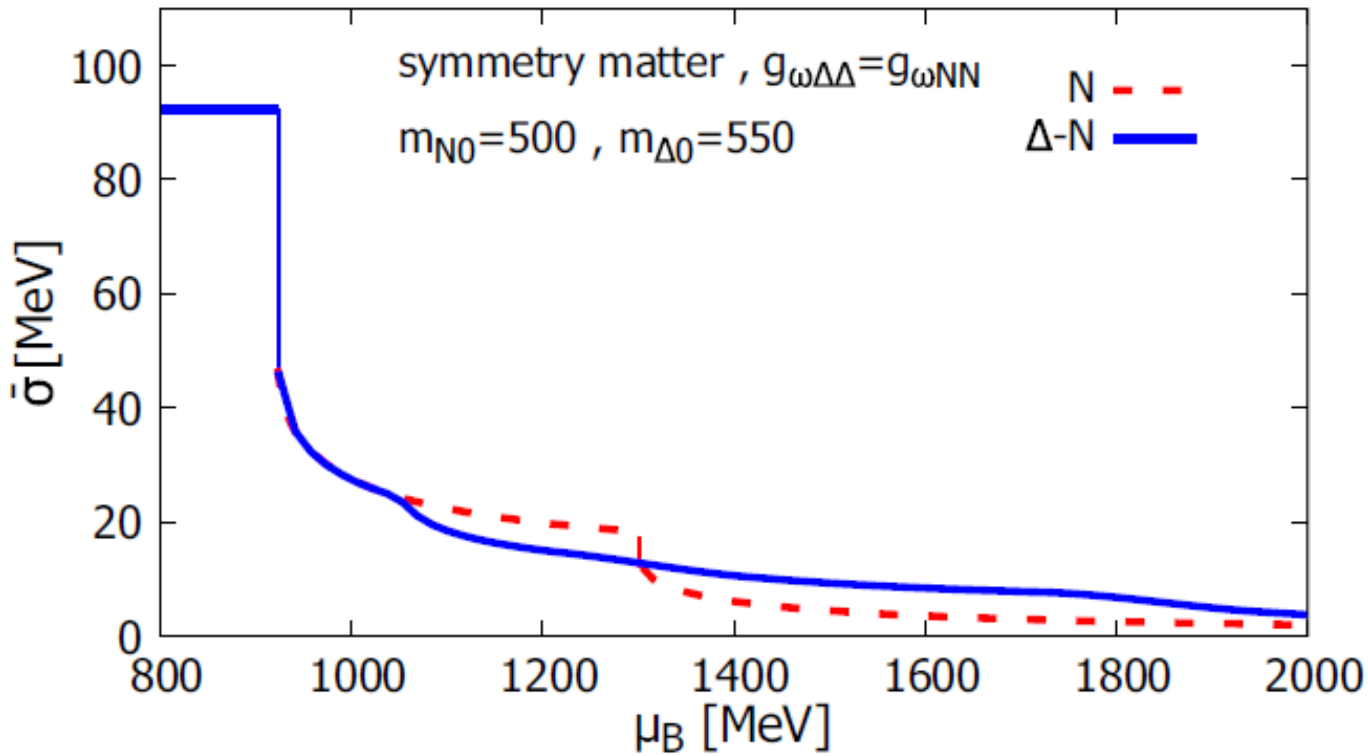


FIG. 13. Chemical potential dependence of the chiral condensate $\bar{\sigma}$ (blue solid curve). The red dashed curve shows the one with assuming no Δ in matter. Horizontal axis shows the baryon number chemical potential in unit of MeV, while vertical axis shows the value of the chiral condensate $\bar{\sigma}$ in unit of MeV. The parameters are chosen as $m_{N0} = 500\,\mathrm{MeV}$, $m_{\Delta 0} = 550\,\mathrm{MeV}$, and $g_{\omega\Delta\Delta} = g_{\omega NN}$.

Parity doublet model in relativistic continuum Hartree-Bogoliubov theory

RCHB theory properly takes into account the pairing correlation and the coupling to (discretized) continuum via Bogoliubov transformation in a microscopic and self-consistent way.

[J. Meng, et al, Prog. Part. Nucl. Phys. 57 (2006) 470]

Spherical RCHB code was provided by Jie Meng (Peking Univ.).

Main difference is the behavior of sigma mean field.

Revised the code to incorporate the difference.

Ik Jae Shin, Won-Gi Paeng, Masayasu Harada, YK, 1805.03402 in nucl-th

The equations of motion (EoM) for the stationary mean fields $\tilde{\sigma}$, ω_0, ρ_0^3 and A_0 read

$$
\begin{aligned}
\left(-\vec{\nabla}^2+m_\sigma^2\right)\langle\tilde{\sigma}(\vec{x})\rangle &= -\bar{N}(\vec{x})N(\vec{x})\,\frac{\partial\, m_N(\tilde{\sigma})}{\partial\tilde{\sigma}}\bigg|_{\tilde{\sigma}=\langle\tilde{\sigma}(\vec{x})\rangle} \\
&\quad+\left(-3f_\pi\lambda+10f_\pi^3\lambda_6\right)\langle\tilde{\sigma}(\vec{x})\rangle^2 \\
&\quad+\left(-\lambda+10f_\pi^2\lambda_6\right)\langle\tilde{\sigma}(\vec{x})\rangle^3 \\
&\quad+5f_\pi\lambda_6\langle\tilde{\sigma}(\vec{x})\rangle^4+\lambda_6\langle\tilde{\sigma}(\vec{x})\rangle^5 \\
\left(-\vec{\nabla}^2+m_\omega^2\right)\langle\omega_0(\vec{x})\rangle &= g_{\omega NN}N^\dagger(\vec{x})N(\vec{x})\,, \\
\left(-\vec{\nabla}^2+m_\rho^2\right)\langle\rho_0^3(\vec{x})\rangle &= g_{\rho NN}N^\dagger(\vec{x})\tau^3N(\vec{x})\,, \\
-\vec{\nabla}^2\langle A_0(\vec{x})\rangle &= eN^\dagger(\vec{x})\frac{1-\tau_3}{2}N(\vec{x})\,.
\end{aligned}
$$

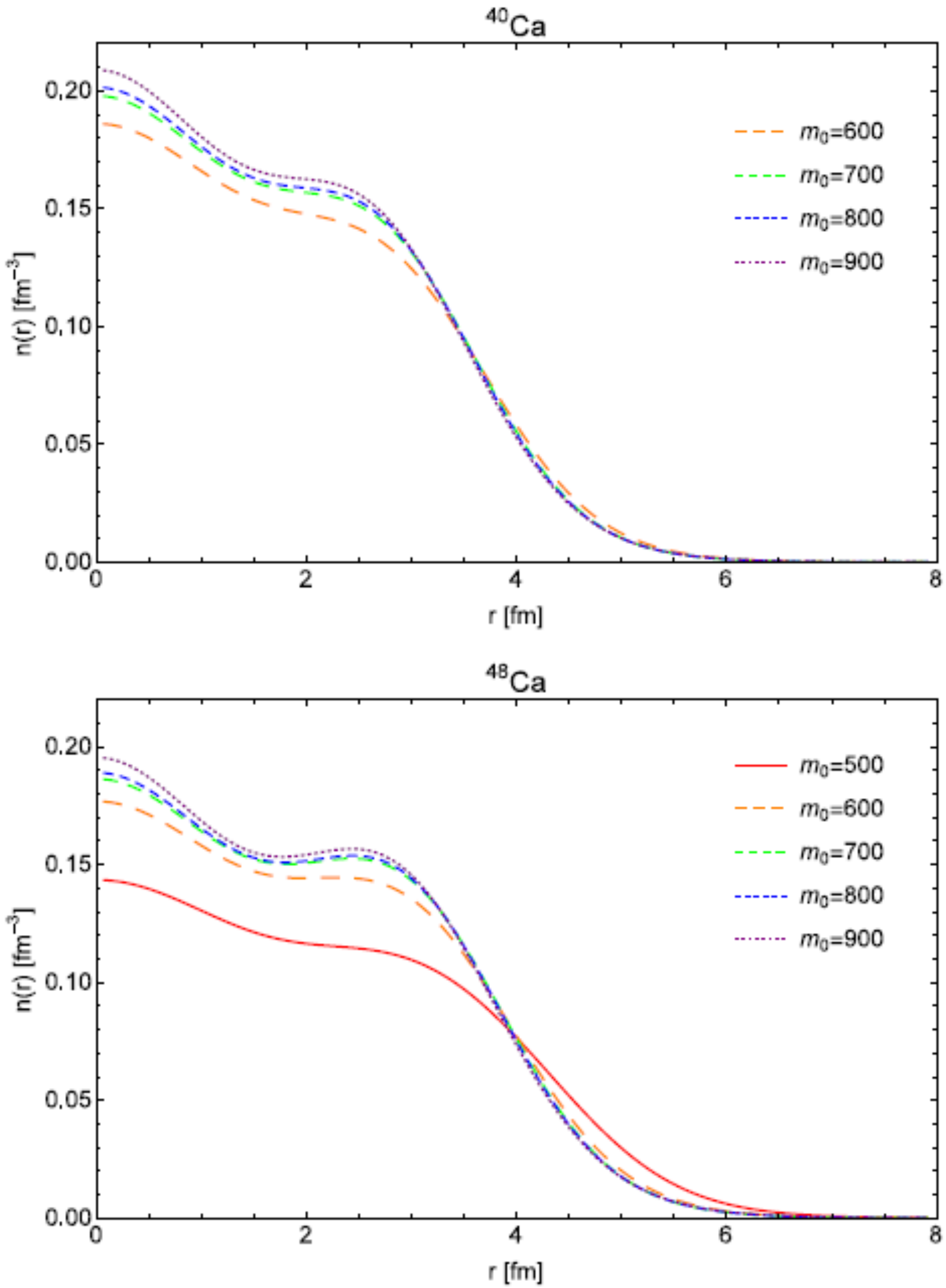


FIG. 1. (Color online) Nucleon density profile in ^{40}Ca and ^{48}Ca calculated with the parameter set 1.

	BE (MeV)			R_C (fm)	
	PDM	PC-PK1	Exp.	PDM	Exp.
^{16}O	8.04	7.96	7.98	2.76	2.70
^{24}O	7.06	7.12	7.04	2.82	–
^{32}Mg	7.83	7.91	7.80	3.14	–
^{38}Si	7.59	7.90	7.89	3.28	–
^{38}Ar	8.51	8.62	8.61	3.39	3.40
^{40}Ca	8.57	8.58	8.55	3.46	3.48
^{48}Ca	8.42	8.65	8.67	3.52	3.48
^{42}Ti	8.16	8.32	8.26	3.58	–
^{58}Ni	8.12	8.69	8.73	3.84	3.78
^{72}Kr	8.22	8.31	8.43	4.11	4.16
^{208}Pb	7.86	7.87	7.87	5.53	5.50

!

TABLE VIII. The neutron(ν) and proton(π) spin-orbit splittings of ^{40}Ca and ^{48}Ca

State	^{40}Ca		^{48}Ca	
	PDM	Exp.	PDM	Exp.
$\nu 1d$	1.52	6.75	1.28	5.30
$\nu 1f$			1.75	8.01
$\nu 2p$	0.48	2.00	0.44	1.67
$\pi 1d$	1.52	5.94	1.33	5.01
$\pi 2p$			0.45	2.14

A way go beyond the MFA: FRG

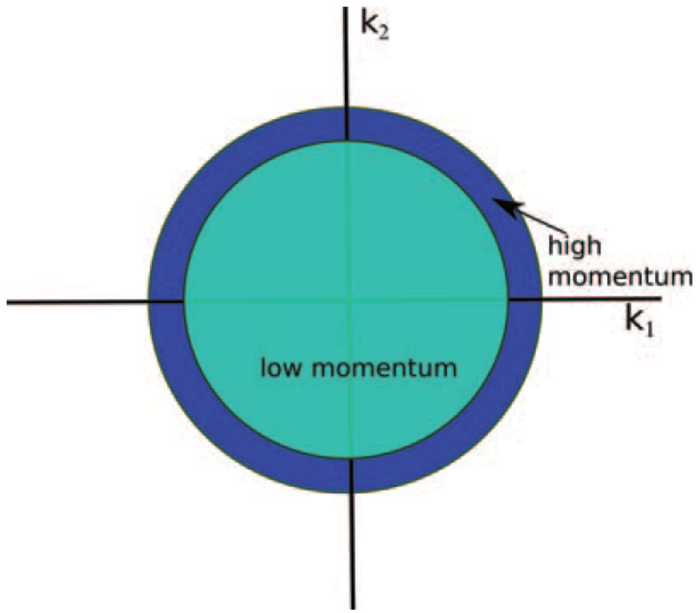
- Nucleons, though massive, can fluctuate near the Fermi surface as p-h excitations.
- The pion and sigma mesons can fluctuate.
- Vector mesons such as omega mesons may not fluctuate because they are massive. Only as mean field.
- FRG is a good way to handle those fluctuations.

Renormalization group method with different goals

- **To remove infinities (UV divergences)**
- **To describe the scale dependence of physical parameters**
- **To re-sum the perturbation expansion in QFT**
- **To solve strongly coupled theories**
- **…**

Effective action in QFT:

- The generating functional of the 1PI Green functions.
- The field equations derived from the effective action include all quantum effects. Knowledge of the effective action is in a sense equivalent to the “solution” of a theory.
- In thermal and chemical equilibrium the effective action includes in addition the thermal fluctuations and depends on the temperature and chemical potential.
- In statistical physics it corresponds to the free energy as a functional of some (space dependent) order parameter.



Analogy

Magnetic system	Q.F.T
$s(x)$	$\phi(x)$
H	$J(x)$
$\mathcal{H}(s)$	$\mathcal{L}(\phi)$
$Z(H)$	$Z[J]$
$F(H)$	$W[J]$
M	$\varphi\ (\langle\phi\rangle)$
$G(M)$	$-\Gamma[\langle\phi\rangle]$

- flow of Schwinger functional $W_k[j]$: Polchinski equation
- flow of effective action $\Gamma_k[\varphi]$: Wetterich equation
- flow from classical action $S[\varphi]$ to effective action $\Gamma[\varphi]$
- applied to variety of physical systems
  - strong interaction
  - electroweak phase transition
  - asymptotic safety scenario
  - condensed matter systen
    e.g. Hubbard model, liquid He^4, frustrated magnets, superconductivity ...
  - effective models in nuclear physics
  - ultra-cold atoms

The average action Γ_k is a simple generalization of the effective action, with the distinction that only fluctuations with momenta $q^2 \gtrsim k^2$ are included.

Γ_k interpolates between the classical action S and the effective action Γ as k is lowered from the ultraviolet cutoff Λ to zero: $\lim_{k\to\Lambda} \Gamma_k = S$, $\lim_{k\to 0} \Gamma_k = \Gamma$.

Wetterich Equation

$$Z[J] = \int D\phi \, e^{-S[\phi] + J\cdot\phi}$$

$$J\cdot\phi = \int d^4x \, J_a(x)\, \phi_a(x)$$

$$\langle \phi^n \rangle = \frac{1}{Z} \frac{\delta^n Z}{\delta^n J} = \frac{1}{Z} \int D\phi \, \phi^n e^{-S+\phi\cdot J}$$

$$W[J] = \ln Z[J]$$

↳ schwinger functional

$$G = \frac{\delta^2 W}{\delta^2 J} = \frac{\delta}{\delta J}\left(\frac{1}{Z}\frac{\delta Z}{\delta J}\right)$$

$$= \frac{1}{Z}\frac{\delta^2 Z}{\delta^2 J} - \frac{1}{Z^2}\frac{\delta Z}{\delta J}\frac{\delta Z}{\delta J}$$

$$= \langle \phi\phi \rangle - \langle \phi \rangle \langle \phi \rangle$$

$$\equiv \langle \phi\phi \rangle_c$$

Introduce a cutoff ΔS_k that vanishes in the IR.

$$W_k[J] = \ln Z_k[J]$$
$$= \ln \int D\phi\, e^{-S[\phi] + J\cdot\phi - \Delta S_k[\phi]}$$

k: renormalization scale, we are probing.

$$\Delta S_k[\phi] = \frac{1}{2}\phi \cdot R_k \cdot \phi$$
$$= \frac{1}{2}\int_{xy} \phi_a(x) R_{k,ab}(x,y) \phi_b(y)$$

↳ momentum dependent mass
UV and IR regulator!

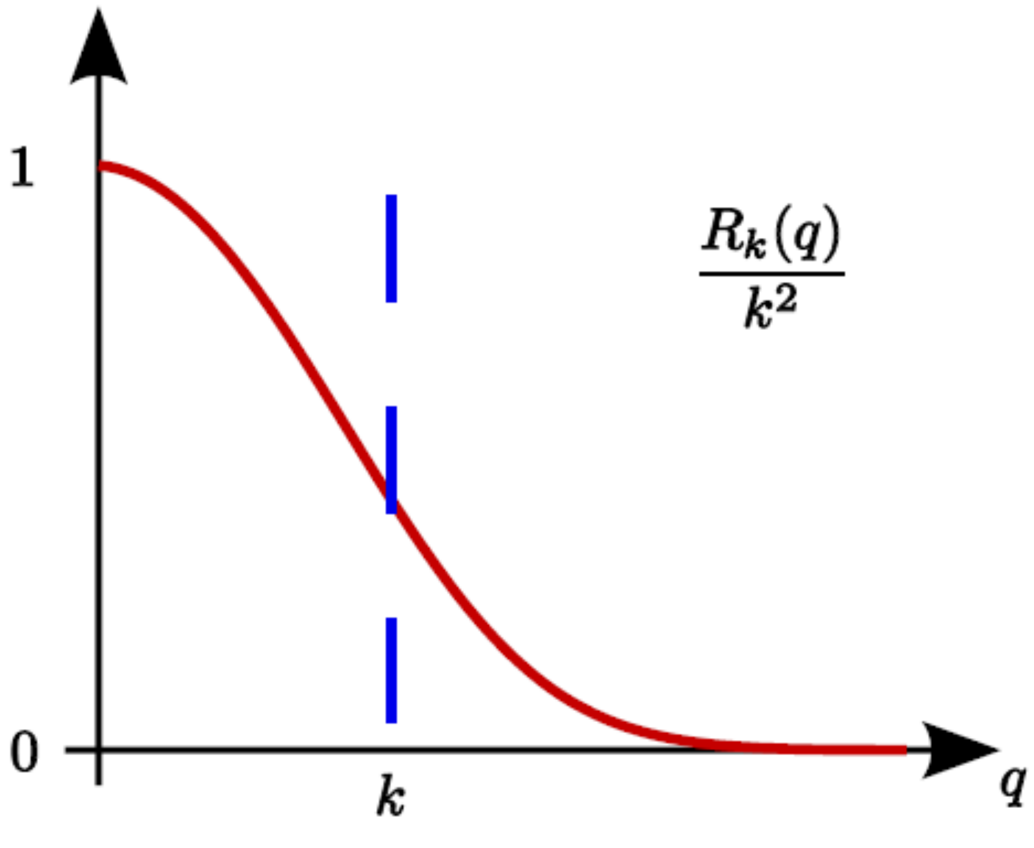
At fixed J,

$$\partial_k W_k[J] = -\frac{1}{Z_k}\int D[\phi]\left(\partial_k \Delta S_k[\phi]\right) e^{-S + J\cdot\phi - \Delta S_k}$$

$$= -\frac{1}{2}\langle \phi\, \partial_k R_k\, \phi \rangle$$

Using $\langle\phi\phi\rangle = \langle\phi\phi\rangle_c + \langle\phi\rangle\langle\phi\rangle$,
↳ φ

$$= -\frac{1}{2}\left(\langle\phi\phi\rangle_c + \varphi\varphi\right)\partial_k \cdot R_k$$



Typical form of the regulator function R_k

In terms of W_k the average action is defined via a modified Legendre transform

$$\Gamma_k[\phi] = -W_k[J] + \int d^d x J_a(x)\phi^a(x) - \Delta S_k[\phi]$$

where we have subtracted the term $\Delta S_k[\phi]$ on the r.h.s. This subtraction of the infrared cutoff term as a function of the macroscopic field ϕ is crucial for the definition of a reasonable coarse grained free energy with the property $\lim_{k\to\Lambda}\Gamma_k = S$.

$$\langle \phi\phi \rangle_c \equiv W_k^{(2)}$$

$$= \frac{\delta W_k}{\delta J} = \frac{\delta \varphi}{\delta J}$$

Now we arrive at Polchinski's equation.

$$\partial_k W_k[J] = -\frac{1}{2} \mathrm{Tr} \left[W_k^{(2)} \partial_k R_k \right]$$

$$-\frac{1}{2} \varphi (\partial_k R_k) \varphi$$

↳ Integration over x ($\sim p$) and summation over a, b.

$$\mathrm{Tr}\left[(\partial_k R_k) W_k^{(2)} \right]$$

$$= \int_{x,y} W_{k,ab}^{(2)}(x,y)\, \partial_k R_{k,ab}(x,y)$$

Effective action

($J \longleftrightarrow \varphi$) why $\phi_{cl} \neq \phi_{cl} + \delta\phi^{1\text{-loop}}$, etc

is a function in the full QFT

gives the exact value of $\langle \phi \rangle \Rightarrow \varphi$

$$\tilde{\Gamma}_k[\varphi] = J\cdot\varphi - W_k[J]$$

$$\frac{\partial \tilde{\Gamma}_k}{\partial \varphi} = J_k$$

~~$$\frac{\partial^2 \tilde{\Gamma}_k}{\partial^2 \varphi} = \frac{\partial J_k}{\partial \varphi} \left(= \tilde{\Gamma}_k^{(2)} \right)$$~~

$$\frac{\partial^2 \tilde{\Gamma}_k}{\partial^2 \varphi} = \frac{\partial J_k}{\partial \varphi} \left(= \tilde{\Gamma}_k^{(2)} \right)$$

↳ inverse propagator

? $\left(\tilde{\Gamma}_k^{(2)} \, W_k^{(2)}\right)_{ab}(x, y)$

$$= \int_z \frac{\delta J_c(z)}{\delta \varphi_a(x)} \frac{\delta \varphi_b(y)}{\delta J_c(z)}$$

$$= \frac{\delta \varphi_b(y)}{\delta \varphi_a(x)} \to \delta_{ab}\, \delta(x-y)$$

$$\therefore\ W_k^{(2)} = \left(\tilde{\Gamma}_k^{(2)}\right)^{-1}$$

$$= \left(\Gamma_k^{(2)} + R_k\right)^{-1}$$

for fixed φ

$$\partial_k \tilde{\Gamma}_k = \varphi\, \partial_k J - \partial_k W_k[J]$$

$$- \frac{\delta W}{\delta J} \frac{\partial J}{\partial k} \quad \left(\frac{\delta W}{\delta J} \to \varphi\right)$$

$$= - \partial_k W_k [J]$$

$$\Gamma_k[\varphi] = \hat{\Gamma}_k[\varphi] - \Delta S_k$$

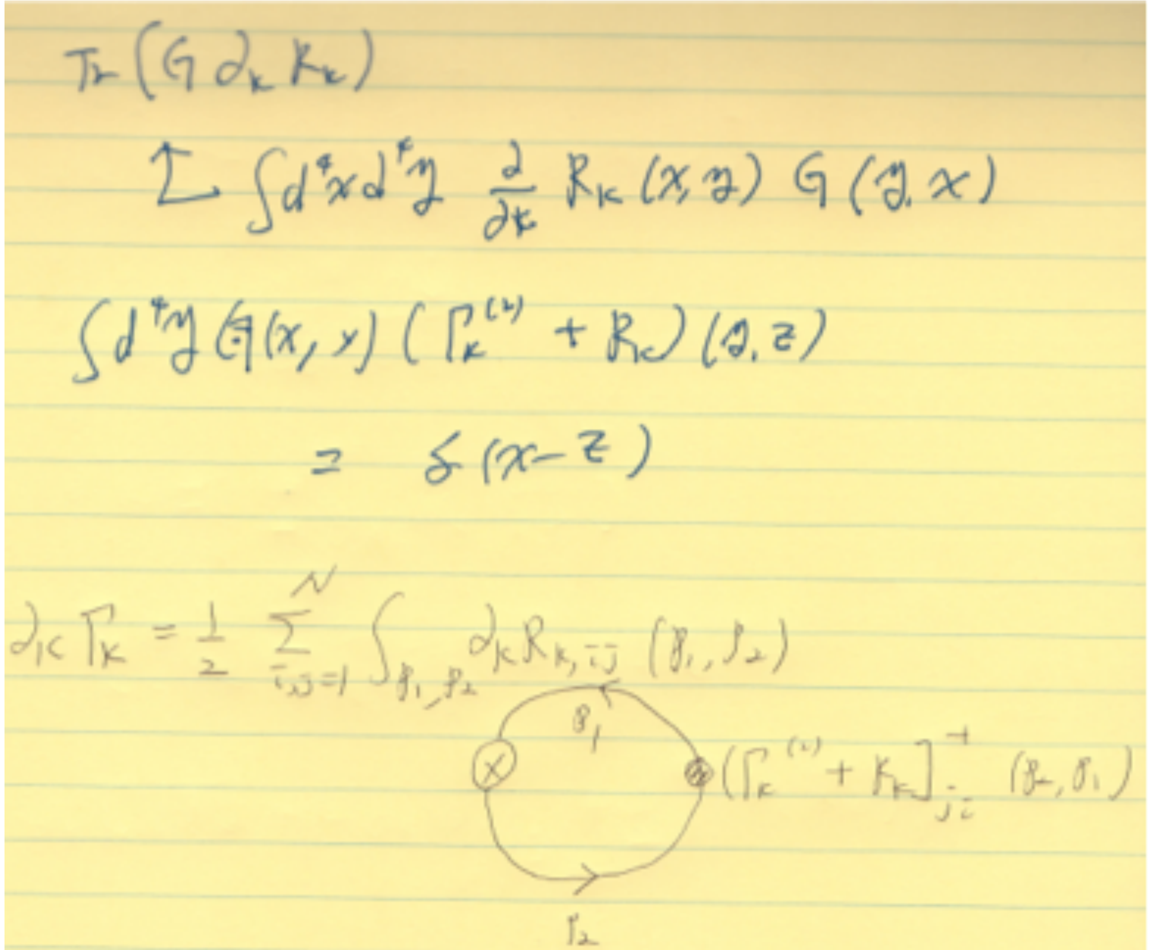
$$\partial_k \Gamma_k[\varphi] = - \partial_k W_k \big|_J - \frac{1}{2} \varphi (\partial_k R_k) \varphi.$$

$$\left(- \frac{1}{2} \mathrm{Tr}[W_k^{(2)} \partial_k R_k] - \frac{1}{2} \varphi (\partial_k R_k) \varphi \right)$$

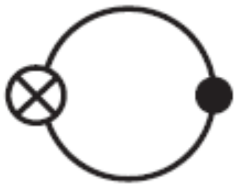
$$= + \frac{1}{2} \mathrm{Tr}\left[W_k^{(2)} \partial_k R_k \right]$$

$$= \frac{1}{2} \mathrm{Tr}\left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_k R_k \right]$$

< Wetterich eq. >



$$k\,\frac{\partial \Gamma_k}{\partial k} =$$



$$= \frac{1}{2}\,\mathrm{Tr}\,\frac{k\,\frac{\partial R_k}{\partial k}}{\Gamma_k^{(2)} + R_k},$$

Finite temperature and density?

At nonzero temperature T and chemical potential μ our ansatz for Γ_k reads

$$\Gamma_k = \int_0^{1/T} dx^0 \int d^3x \Big\{ i\overline{\psi}^a (\gamma^\mu \partial_\mu + \mu\gamma^0)\psi_a + \overline{h}_k \overline{\psi}^a \left[\frac{1+\gamma^5}{2} \Phi_a{}^b - \frac{1-\gamma^5}{2} (\Phi^\dagger)_a{}^b \right] \psi_b$$

$$+ Z_{\Phi,k} \partial_\mu \Phi^*_{ab} \partial^\mu \Phi^{ab} + U_k(\overline{\rho}; \mu, T) \Big\} .$$

$$\mathcal{Z} = \mathrm{Tr}\, e^{-\beta \hat{H}} = \sum_n \langle n | e^{-\beta \hat{H}} | n \rangle = \sum_n e^{-\beta E_n}, \qquad Z(\beta) = \int dx\, \langle x | e^{-\beta H} | x \rangle$$

$${}_H\langle x_f, t_f | x_i, t_i \rangle_H = \langle x_f | e^{-\frac{i}{\hbar} H (t_f - t_i)} | x_i \rangle = \int \mathcal{D}x\, e^{\frac{i}{\hbar} S[x]}$$

$$T = -i\beta$$

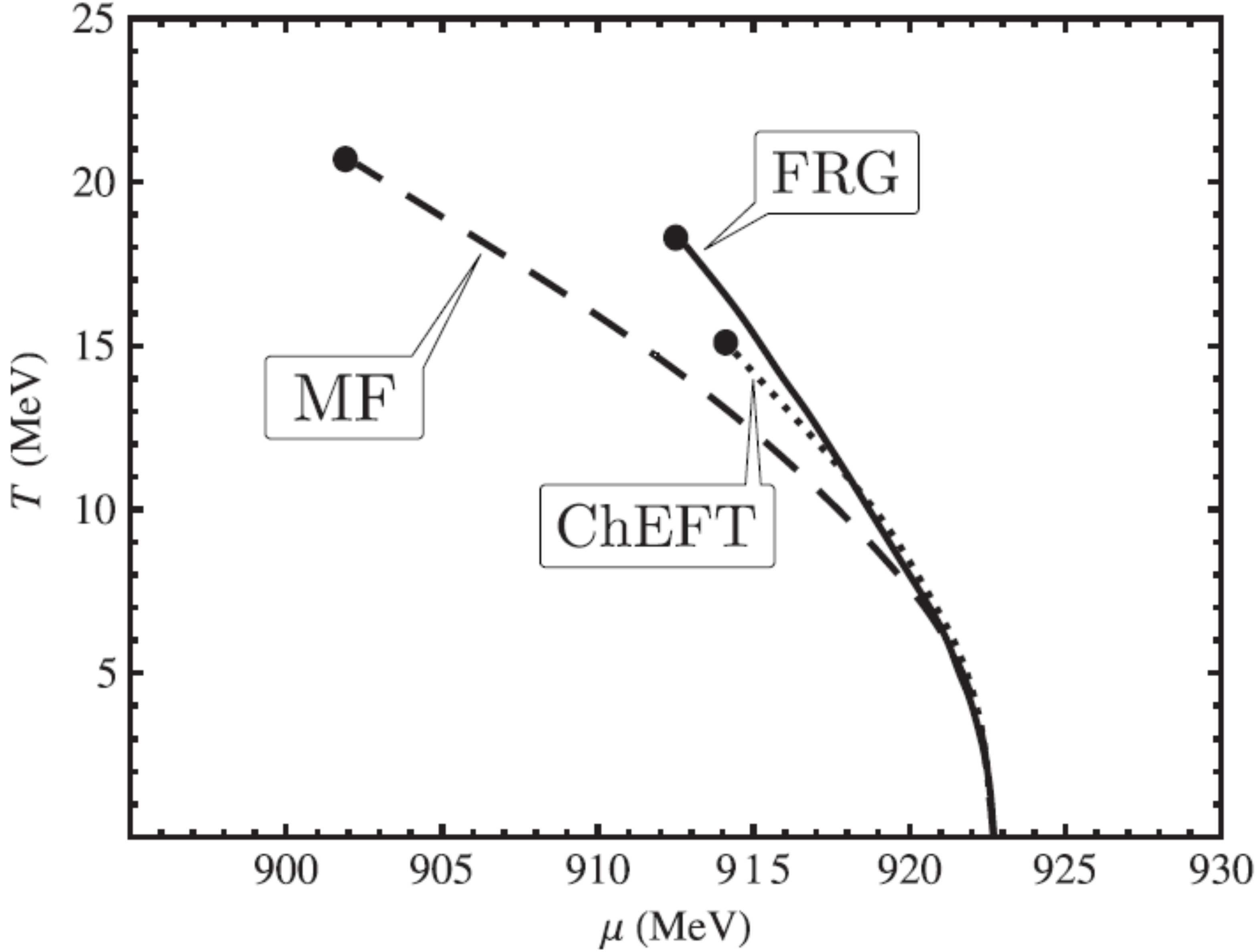
$$S_E[x] = \int_0^\beta dt\, L_E(x, \dot{x})$$

$$Z(\beta) = \int \mathcal{D}x\, e^{-S_E[x]}$$

$$x(\beta) = x(0)$$

$$\omega_n = \begin{cases} \frac{2n\pi}{\beta} & \text{for bosons} \\ \frac{(2n+1)\pi}{\beta} & \text{for fermions} \end{cases}$$

Liquid–gas phase transition in a $T - \mu$ diagram.



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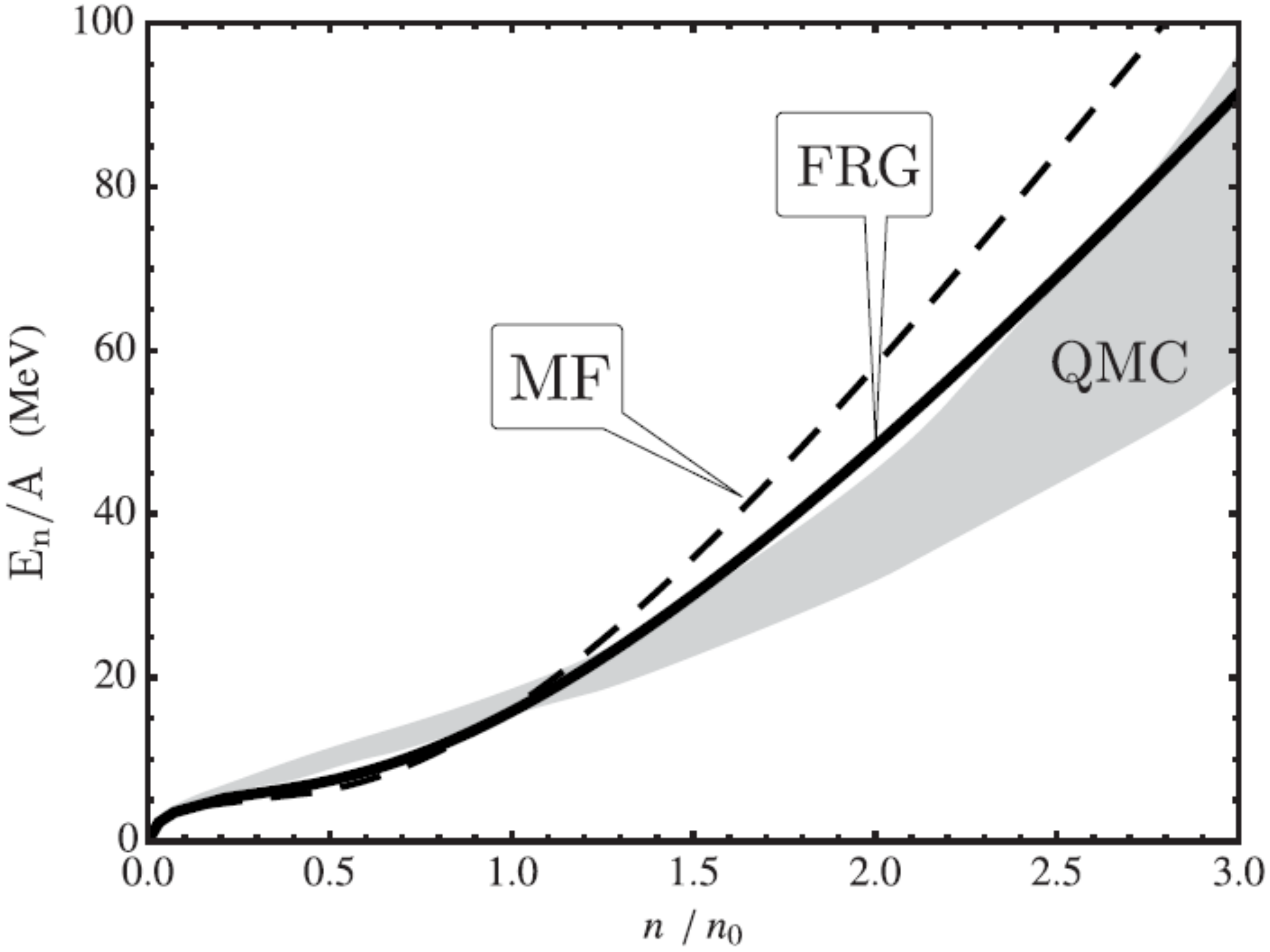
The equation of state for pure neutron matter at $T = 0$ with $E_{sym} = 32$ MeV



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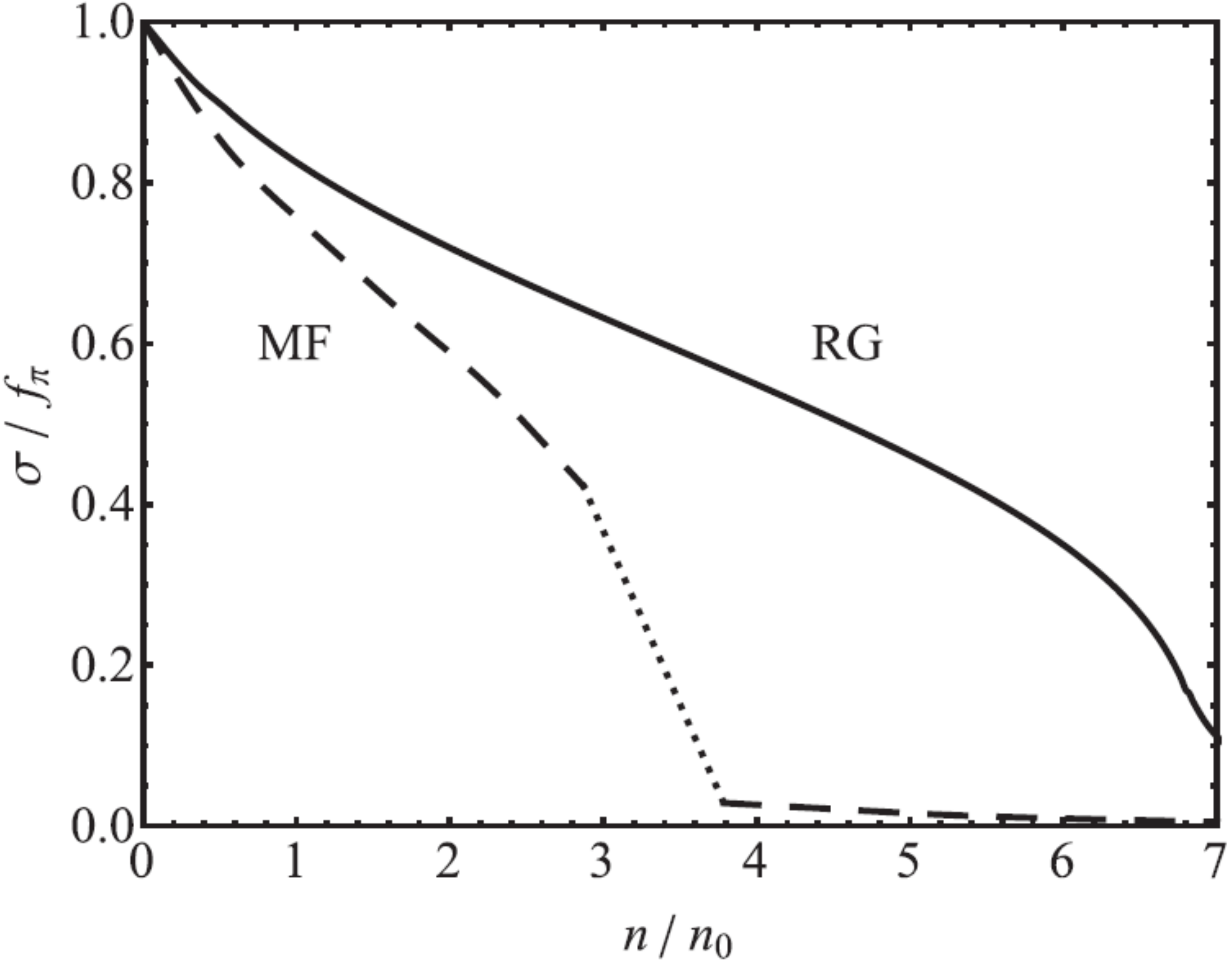

1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
σ / fπ
n / n0
MF
RG