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Instanton Effects on the Charmonium Spectra

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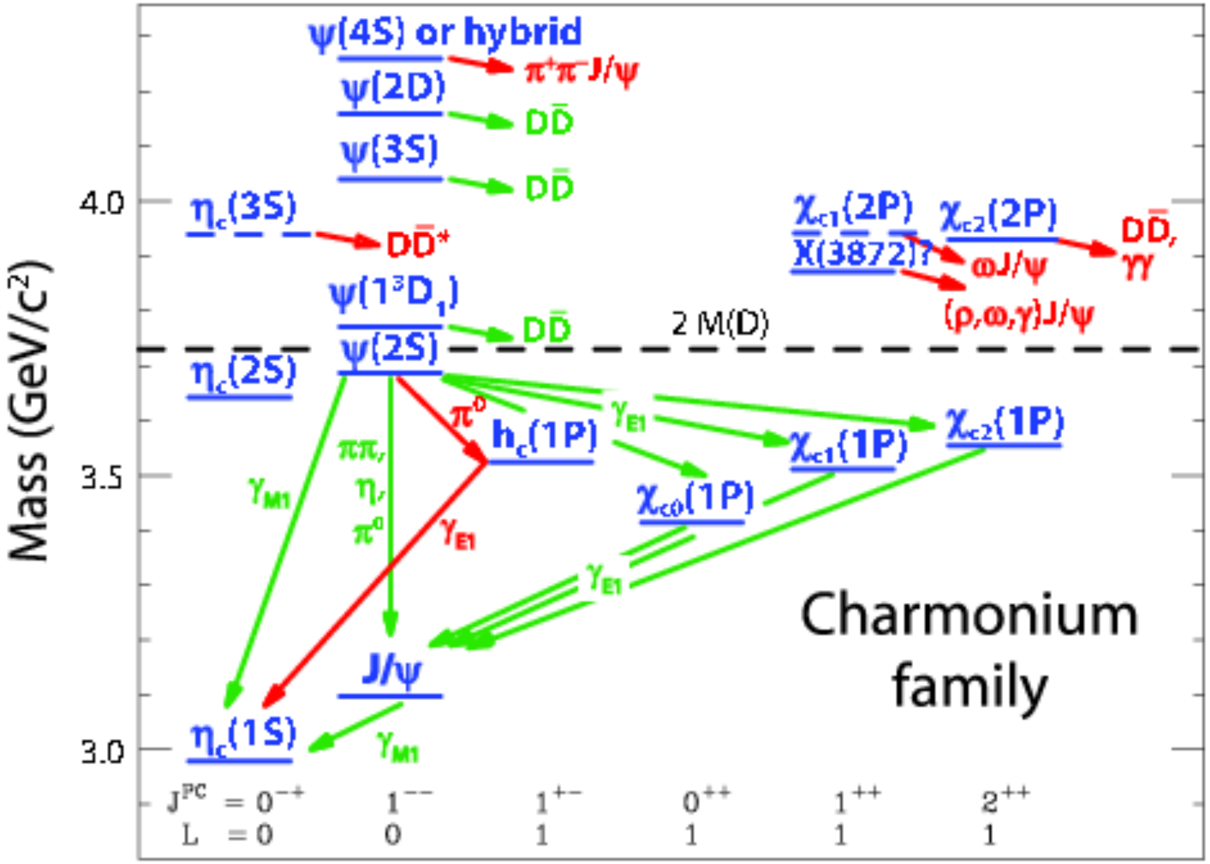
Talk @ Korea-Japan Joint Workshop
“The Present and the Future in Hadron Physics at J-PARC”
March 4-5, 2019, Busan, Korea

Content

Charmonium spectrum

- Below $D\bar{D}$ threshold (Region I - “low” energies)
- Above $D\bar{D}$ threshold (Region II - “high” energies)



$$m_{c\bar{c}} - 2m_c^{(\text{current})} \geq m_c^{(current)}$$

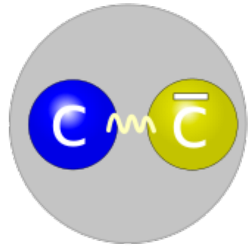
$$m_{c\bar{c}} - 2m_c^{(\text{current})} < m_c^{(current)}$$

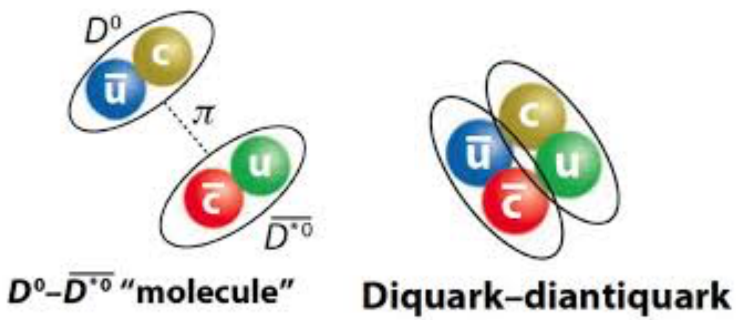
$$m_c^{(current)} = 1.275^{+0.025}_{-0.035}\ \text{MeV}$$

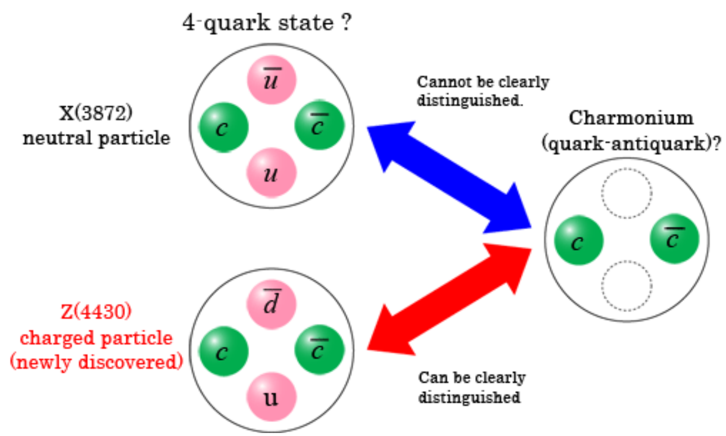
Charmonium

Interesting features

- Region I (potential models could be applied)
  - transitions are mainly EM type
  - two body non-relativistic system
  - one has readily the Schrodinger approach
- Region II (field-theoretical approaches seem to be more favourable)
  - the light degrees of freedom are more involved
  - meson molecules
  - diquark-antidiquark
  - tetra-quark picture
  - etc







Phenomenological Potential Models

- Light-light quark sector
  - computed fully in a non-perturbative way
  - only available general method is lattice QCD
  - relativistic models should be constructed

- Light-heavy quark sector
  - light quarks are fully relativistic
  - heavy quarks are moving slowly (v/c expansion is valid - Nonrelativistic QCD approach)

- Heavy-heavy quark sector
  - static $Q\bar{Q}$ potential approach is good approximation

Phenomenological Potential Models

- Static Coulomb-type potential based on perturbative one gluon exchange

$$V^{(\mathrm{OGE})}(r) = -\frac{4\alpha_s(p^2)}{3r}$$

$$\alpha_s(p^2) = \frac{4\pi}{\beta_0}\frac{1}{\ln(p^2/\Lambda^2_{\mathrm{QCD}})}, \qquad \beta_0 = \frac{11N_c - 2N_f}{3}$$

- Confining potential has fully phenomenological nature

$$V^{(\mathrm{conf})}(r) = \kappa r, \qquad \kappa \text{ is string constant}$$

Phenomenological Potential Models

- Leading order central potential is a sum of scalar (S) and vector (V) exchange terms

$$V_C(r) = V^{(\mathrm{conf})}(r) + V^{(\mathrm{OGE})}(r) \equiv V_S(r) + V_V(r)$$

- Spin-dependent parts appear as the next-to-leading order corrections

$$V_{\mathrm{sd}}(r) = V_{SS}(r)(\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}}) + V_{LS}(r)(\mathbf{L} \cdot \mathbf{S}) + V_T(r)\{3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - \mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}}\}$$

$$V_{SS} = \frac{1}{3m_Q^2}\nabla^2 V_V$$

$$V_{LS} = \frac{1}{2m_Q^2 r}\left(3\frac{\mathrm{d}V_V}{\mathrm{d}r} - \frac{\mathrm{d}V_S}{\mathrm{d}r}\right)$$

$$V_T = \frac{1}{6m_Q^2}\left(\frac{\mathrm{d}^2V_V}{\mathrm{d}r^2} - \frac{1}{r}\frac{\mathrm{d}V_V}{\mathrm{d}r}\right)$$

Phenomenological Potential Models

- Full potential containing a confining potential and a Coulomb-type potential based on perturbative one gluon exchange

$$V_C(r) = \kappa r - \frac{4\alpha_s}{3r}$$

$$V_{SS}(r) = \frac{32\pi\alpha_s}{9m_Q^2}\delta_\sigma(r)$$

$$V_{LS}(r) = \frac{1}{2m_Q^2}\left(\frac{4\alpha_s}{r^3} - \frac{\kappa}{r}\right)$$

$$V_T(r) = \frac{4\alpha_s}{3m_Q^2 r^3}$$

$$\delta_\sigma(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}$$

Fit to the data gives:

$$\kappa = 0.1425\,\mathrm{GeV}^2$$

$$\alpha_s = 0.5461,$$

$$\sigma = 1.0946\,\mathrm{GeV}$$

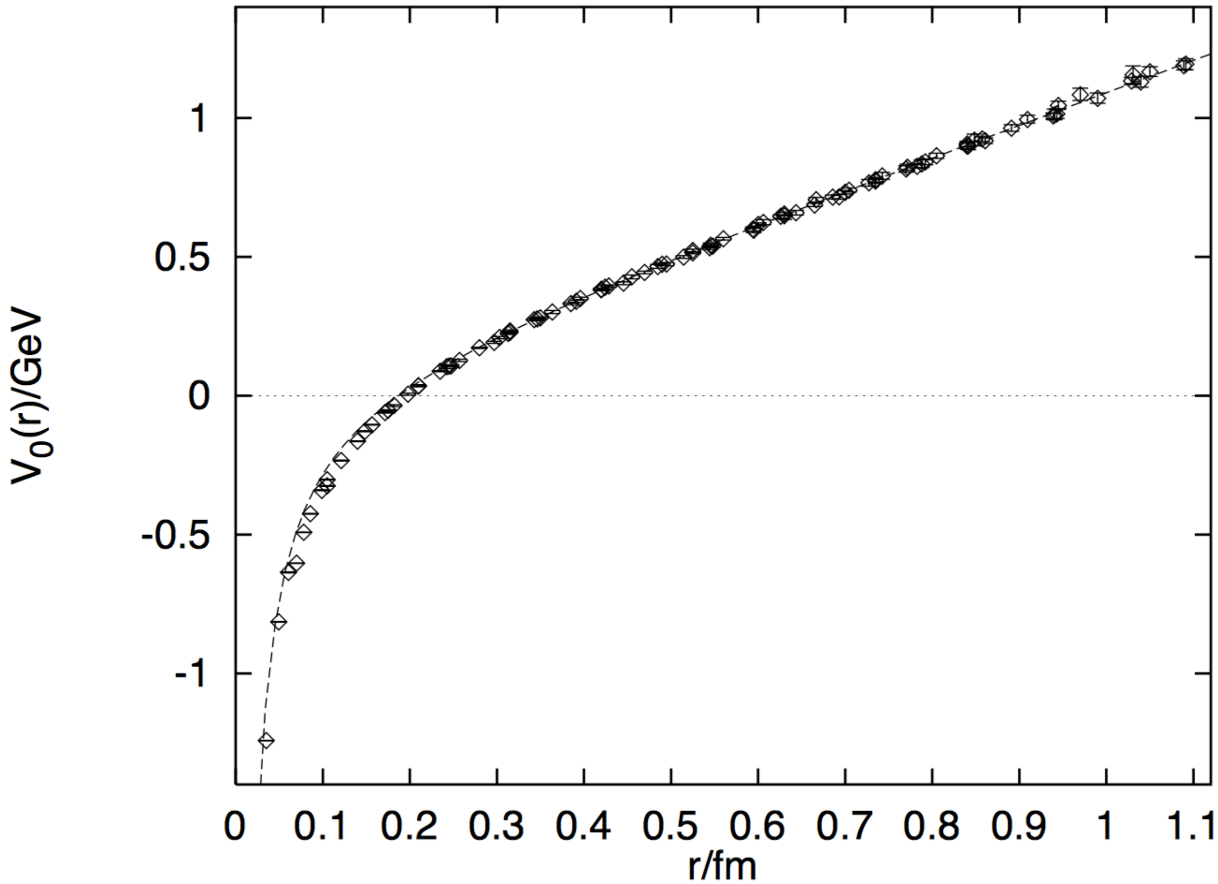
$$m_c = 1.4794\,\mathrm{GeV}$$

(Referred as NA)

[T. Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005)]

Phenomenological Potential Models

- Comparison with the Lattice QCD results (central potential)



$$V_C(r) = \kappa r - \frac{4\alpha_s}{3r}$$

From [G.S. Bali, K.Schilling, A. Wachter, arXiv:hep-lat/9506017]

Phenomenological Potential Models

TABLE I. Parameters of the nonrelativistic potential models: NR corresponds to the potential model in Ref. [10] where *eleven* charmonium states are used as an input, NR4 describes the present work with the potential parameters corresponding to the *four* charmonium states as an input, respectively.

The model	m_c [GeV]	α_s [GeV]	κ [GeV^2]	σ [GeV]
NR	1.4794	0.5461	0.1425	1.0946
NR4	1.4863	0.5525	0.1433	1.1286

[UY, to be published]

Phenomenological Potential Models

TABLE II. Experimental and theoretical spectrum of $c\bar{c}$ states. All energy states are given in MeV and the output results are rounded up to 1 MeV. Authors of NR model in Ref. [10] used 11 input states and their values are shown in the second column, respectively. To reproduce NR4 results in this work only 4 states are used as an input, and their values are shown in the fourth column.

State	Ref. [10]		This work		Exp. [13]
	Input	NR	Input	NR4	
$J/\psi(1^3S_1)$	3097	3090	3097	3098	3096.900 ± 0.006
$\eta_c(1^1S_0)$	2979	2982	2984	2983	2983.9 ± 0.5
$\psi(2^3S_1)$	3686	3672	3686	3684	3686.097 ± 0.025
$\eta_c(2^1S_0)$	3638	3630	3638	3640	3637.6 ± 1.2
$\psi(3^3S_1)$	4040	4072		4085	4039 ± 1
$\psi(4^3S_1)$	4415	4406		4421	4421 ± 4
$\chi_{c2}(1^3P_2)$	3556	3556		3561	3556.17 ± 0.07
$\chi_{c1}(1^3P_1)$	3511	3505		3511	3510.67 ± 0.05
$\chi_{c0}(1^3P_0)$	3415	3424		3414	3414.71 ± 0.30
$h_c(1^1P_1)$		3516		3527	3525.38 ± 0.11
$\chi_{c2}(2^3P_2)$		3972		3979	3927.2 ± 2.6
$\chi_{c0}(2^3P_0)$		3852		3881	3862^{+26+40}_{-32-13}
$\psi_2(1^3D_2)$		3800		3813	3822.2 ± 1.2
$\psi(1^3D_1)$	3770	3785		3794	3778.1 ± 1.2
$\psi(2^3D_1)$	4159	4142		4153	4191 ± 5

[UY, to be published]

Phenomenological Potential Models

- Origin of potential parameters

The model	m_c [GeV]	α_s [GeV]	κ [GeV^2]	σ [GeV]
NR	1.4794	0.5461	0.1425	1.0946
NR4	1.4863	0.5525	0.1433	1.1286

- m_c: Dynamical quark mass
- α_s: Defined by the energy range
- κ: Phenomenological parameter
- σ: “Technical” (smearing) parameter

Phenomenological Potential Models

- Dynamical quark mass (contribution due to dynamics)

$$\Delta m_c = m_c - m_c^{(current)}$$
$$\approx 200 \text{ MeV}$$

Due to nonperturbative dynamics?

- Strong coupling constant

$$\alpha_s(m_c^{(current)}) \approx \alpha_s(m_c) \approx 0.4$$

- Fitted values (for comparison)

The model	m_c [GeV]	α_s [GeV]	κ [GeV^2]	σ [GeV]
NR	1.4794	0.5461	0.1425	1.0946
NR4	1.4863	0.5525	0.1433	1.1286

Features of Strong Interactions at Low Energies

- QCD Lagrangian (defines the dynamics)

$$\begin{aligned} \mathcal{L} &= \bar{\psi}_\alpha (i \not{D}_{\alpha\beta} - m_{\alpha\beta})\psi_\beta - \tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} \\ D_\mu &= \partial_\mu + ig\, t^a A^a_\mu \\ G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu \end{aligned}$$

is gauge invariant

$$\begin{aligned} \psi &\rightarrow S\psi \\ gA_\mu &\rightarrow S(gA_\mu - i\partial_\mu)S^\dagger \end{aligned}$$

- Low energies (non-perturbative methods)
  - Confinement (understood only qualitatively)
  - Chiral symmetry and its spontaneous breakdown

- High energies (perturbative methods)
  - Asymptotic freedom (quark-gluon plasma)

Features of Strong Interactions at Low Energies

Non-perturbative methods must be related to QCD

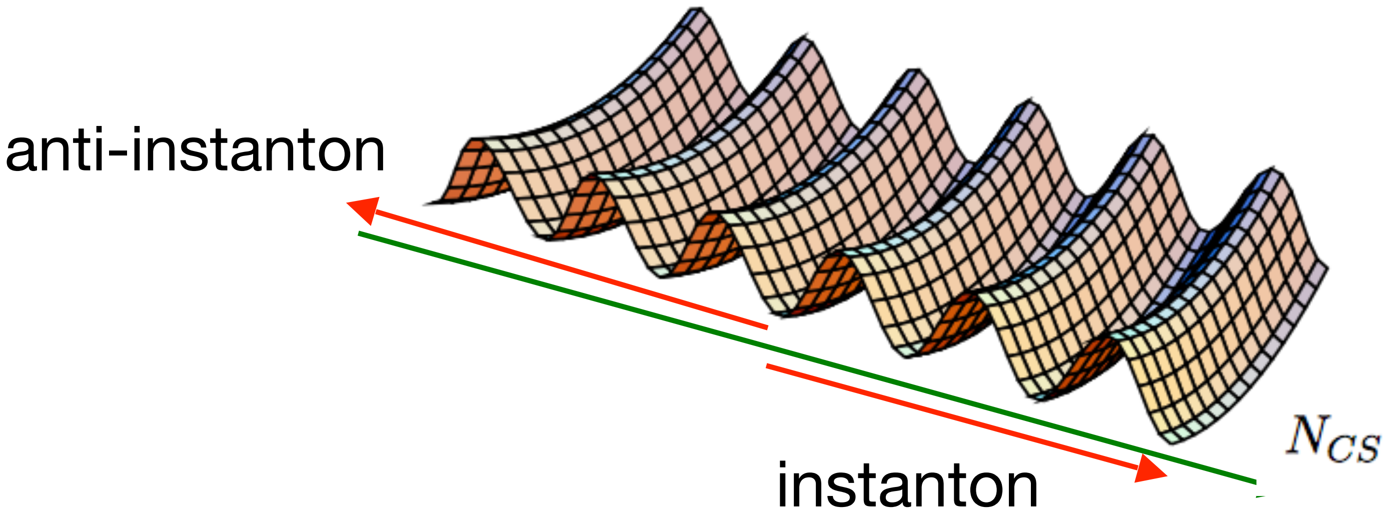
- Confinement
  - Analytically is not proven
  - Numerically can be demonstrated by Lattice QCD
- Chiral symmetry
  - Partially restored at high energies and strongly broken at low energies
  - Can be better understood from the analysis of vacuum structure of QCD and the possible fluctuations around the vacuum
- Instanton (example of pseudo-particle like fluctuation)

Features of Strong Interactions at Low Energies

Vacuum structure

There are topologically nontrivial structures

- stable and with finite energy
- in pure Yang-Mills theory only in 4D space (instanton)
- represent tunnelling transitions between vacuum states

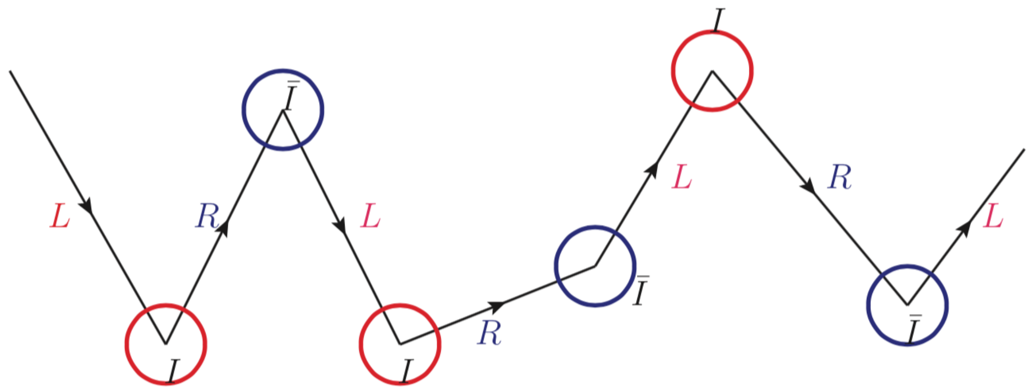


$$N_W = \frac{1}{24\pi^2} \int d^3\mathbf{x}\, \epsilon^{ijk} \left[(U^\dagger \partial_i U)(U^\dagger \partial_j U)(U^\dagger \partial_k U) \right]$$

Features of Strong Interactions at Low Energies

Istantons explain the spontaneous chiral symmetry breaking

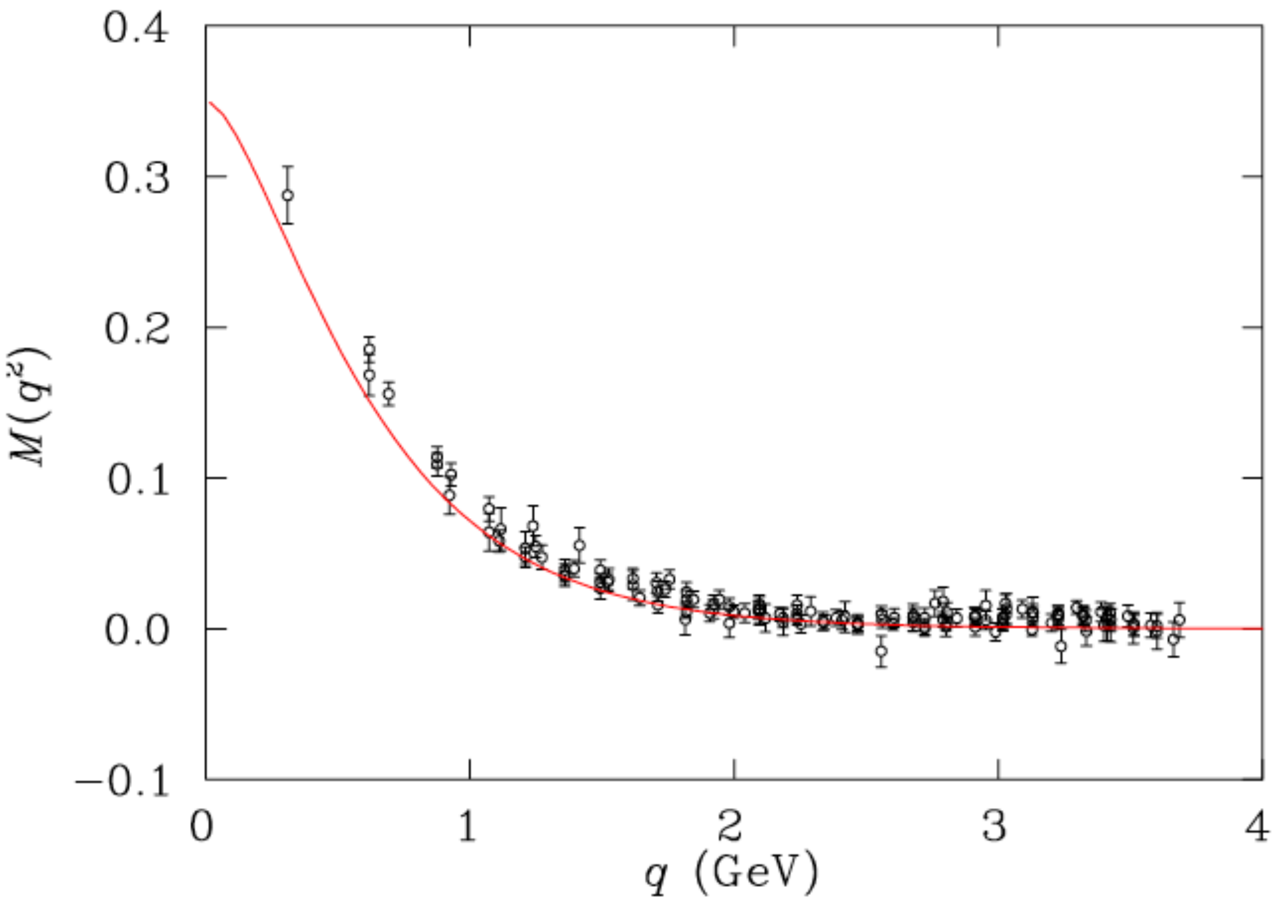
- Helicity of a light quark flips from instanton to anti-instanton and vice versa
- Quark acquires the dynamical mass *M*(*p*)



Features of Strong Interactions at Low Energies

Chiral symmetry breaking

The dynamical quark mass → $M(0) = 345\,\text{MeV}$

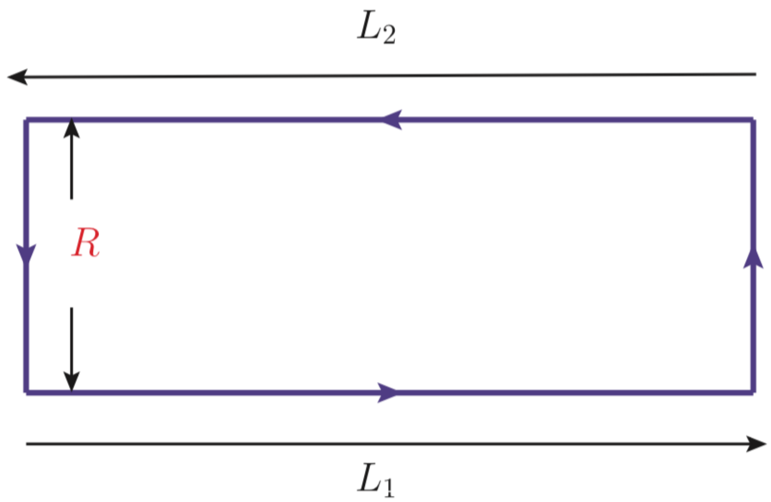


P. Bowman, U. Heller, D. Leinweber and A. Williams, hep-lat/0209129

Instanton Potential

- Evaluation of Wilson-loop for the two Heavy quarks separated at fixed distance in the instanton medium

$$W(L_1 L_2) \sim \exp(-V(R)T)$$



- Gives the central part of $Q\bar{Q}$ potential from instants

$$V(R) = \frac{N}{2VN_c}\int d^3 z_I \mathrm{Tr}_c \left[1 - P\exp\left(i\int_{L_1} dx_4 A_{I4}\right) P\exp\left(-i\int_{L_2} dx_4 A_{I4}\right)\right] + (I \to \bar{I})$$

D. Diakonov, V. Petrov, P. Pobylitsa, PLB **226**, 372 (1989)

Instanton Potential

- Spin-dependent parts appear from the next order in the heavy-quark mass expansion

$$V_{Q\bar{Q}}(r)=V(r)+V_{SS}(r)(\mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}})+V_{LS}(r)(\mathbf{L}\cdot\mathbf{S})$$
$$+V_T(r)\left[3(\mathbf{S}_Q\cdot\mathbf{n})(\mathbf{S}_{\bar{Q}}\cdot\mathbf{n})-\mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}}\right].$$

- Spin-spin part $$V_{SS}(r)=\frac{1}{3m_Q^2}\nabla^2V(r),$$
- Spin-orbit part $$V_{LS}(r)=\frac{1}{2m_Q^2 r}\frac{dV(r)}{dr},$$
- Tensor part $$V_T(r)=\frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV(r)}{dr}-\frac{d^2V(r)}{dr^2}\right).$$

Instanton Potential

- Central part of the $Q\bar{Q}$ Potential can be expressed in terms of the dimensionless function I

$$
\begin{aligned}
V(r) &= \frac{4\pi N\rho^3}{VN_c} I\left(\frac{r}{\rho}\right) = \frac{4\pi\rho^3}{N_c R^4} I\left(\frac{r}{\rho}\right), \\
I(x) &= \int_0^\infty y^2 dy \int_{-1}^1 dt \left[1 - \cos\left(\pi \frac{y}{\sqrt{y^2+1}}\right)\right. \\
&\times \cos\left(\pi\sqrt{\frac{y^2+x^2+2xyt}{y^2+x^2+2xyt+1}}\right) - \frac{y+xt}{\sqrt{y^2+x^2+2xyt}} \\
&\left.\times \sin\left(\pi\frac{y}{\sqrt{y^2+1}}\right) \sin\left(\pi\sqrt{\frac{y^2+x^2+2xyt}{y^2+x^2+2xyt+1}}\right)\right].
\end{aligned}
$$

Defines the strength of interaction

Defines the scale of interaction

- So, the spin dependent parts also expressed in terms of I

Instanton Potential

- The potential is sensitive to the instanton parameters
- There are different sets of parameters
- We discuss three of them, which we used in our calculations
  - Set I - phenomenological [E.V. Shuryak, Nucl.Phys. B203 (1982), D.Diakonov, V.Y.Petrov, Nucl.Phys. B245 (1984)]

$$\rho = \frac{1}{3}\,\mathrm{fm}, \qquad \bar{R} = 1\,\mathrm{fm}$$

  - Set IIa - 1/Nc meson-loop contributions in the light quark sector [H.C.Kim, M.Musakhanov, M.Siddikov, Phys.Lett. B633 (2006)]

$$\rho = 0.35\,\mathrm{fm}, \qquad \bar{R} = 0.856\,\mathrm{fm}$$

  - Set IIb - lattice simulations of instanton vacuum [e.g. M.C. Chu, J.M. Grandy, S.Huang, J.W. Negele, Phys.Rev. D49 (1994)]

$$\rho = 0.36\,\mathrm{fm}, \qquad \bar{R} = 0.89\,\mathrm{fm}$$

Instanton Potential

- All above mentioned sets of parameters and some other additional ones coincide with each-other within 10 – 15% uncertainty
- So, one has a comfortable possibility of independent averaging over instanton positions and orientations in the packing parameter value interval

$$\frac{N}{VN_c} \sim \pi^2 \left(\frac{\rho}{\bar{R}}\right) \in [0.1, 0.3]$$

- We note, that our results from Set IIa and Set IIb are close to each-other

Instanton Potential

- The contribution to central part of the $Q\bar{Q}$ Potential from the instantons (for practical purposes, 8 parametric description)

$$V_C^{(I)} = \frac{4\pi\rho^3}{N_c R^4} I\left(\frac{r}{\rho}\right)$$

$$I(x) = I_0 \left[1 + \sum_{i=1}^{3} a_i x^{2(i-1)} e^{-b_i x^2} + \frac{a_4}{x}\left(1 - e^{-b_4 x^3}\right)\right]$$

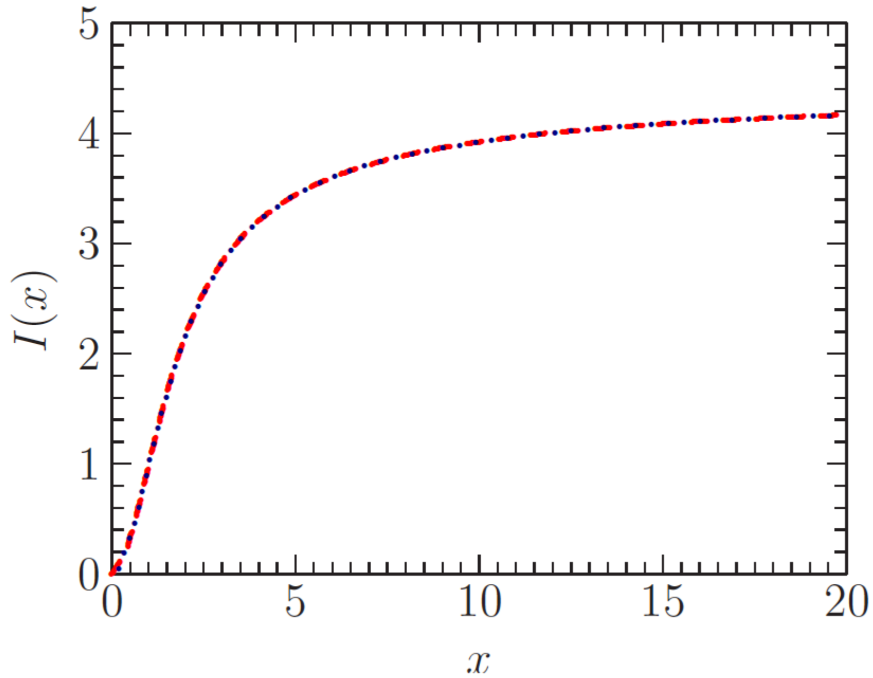
$$I_0 = -\frac{2\pi^3}{3}\left(J_0(\pi) + \frac{1}{\pi} J_1(\pi)\right)$$

$$a = \begin{pmatrix} -1 \\ 0.10184 \\ 0.00064 \\ -1.11267 \end{pmatrix}, \quad b = \begin{pmatrix} 0.25135 \\ 0.70255 \\ 0.18625 \\ 0.04644 \end{pmatrix}$$

UY, H.-Ch. Kim, E. Hiyama, PRD **98** (2018).

Instanton Potential

- Comparison of the exact solution with the parametrization



- red dashed (exact)
- blue dotted (parametrization)

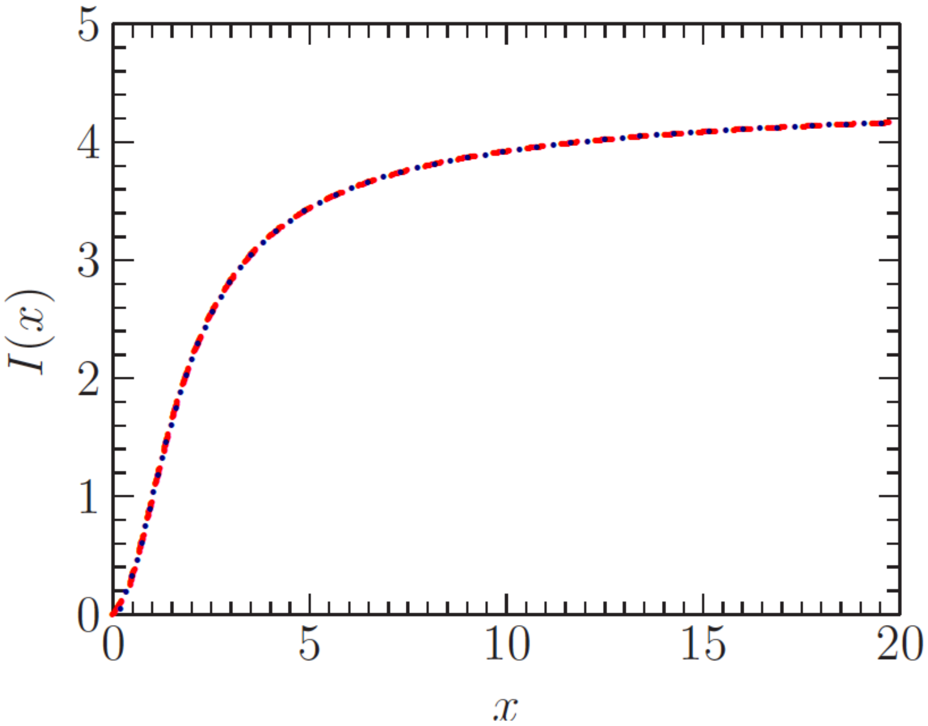
- Max value of relative error is at $x \to 0$

$$\lim_{x\to 0} \frac{I^{\text{exact}}(x) - I^{\text{param}}(x)}{I^{\text{exact}}(x)} = 1 - I_0(b_1 + a_2 + a_4 b_4)\left\{\frac{\pi^2}{3}\left[\frac{\pi}{16} - J_1(2\pi)\right]\right\}^{-1}$$

$$= 0.00972$$

Instanton Potential

- The contribution to central part of the $Q\bar{Q}$ Potential from the instantons



$$V(r) = \frac{4\pi N\rho^3}{VN_c} I\left(\frac{r}{\rho}\right)$$

$$V(r \to \infty) \simeq 2\Delta M_Q - \frac{g_{\mathrm{NP}}}{r}$$

$$g_{\mathrm{NP}} = \frac{2\pi^3 \bar{\rho}^4}{N_c \bar{R}^4}$$

Contribution to the confining potential from the instantons

Nonperturbative correction to the one gluon exchange perturbative calculations

- At large distances it is flattened and if we consider only instanton
  - no confinement
  - no highly excited state

UY, H.-Ch. Kim, E. Hiyama, PRD **98** (2018).

Potential Including Instanton Effects

- Reparametrization of the "restricted full potential"

$$V^{(total)} = V^{(phen)} + V^{(I)}$$

$$m_c - \text{ not free parameter}$$

$$\alpha_s - \text{ not free parameter}$$

$$\kappa - \text{ free parameter}$$

$$\sigma - \text{ free parameter}$$

TABLE I. Parameters corresponding to each model. MWOI represents the model without any instanton contributions, whereas M-I and M-IIb contain them as explained in Ref. [34].

The model	ρ [fm]	R [fm]	$\Delta m_{\rm I}$ [GeV]	α_s [GeV]	κ [GeV^2]	σ [GeV]
MWOI	-	-	-	0.2068	0.1746	5.0248
M-I	0.33	1.00	0.0676	0.3447	0.1520	0.9331
M-IIb	0.36	0.89	0.1357	0.4588	0.1279	0.5650

UY, H.-Ch. Kim, E. Hiyama, PRD **98** (2018).

Potential Including Instanton Effects

- Dynamical quark mass (contribution due to dynamics)

$$\Delta m_c = m_c - m_c^{(current)}$$

- Strong coupling constant

$$\alpha_s^{(\mathrm{MWOI})} = 0.4258$$

$$\alpha_s^{(\mathrm{M-I})} = 0.4137$$

$$\alpha_s^{(\mathrm{M-IIb})} = 0.4029$$

- Fitted values (for comparison)

The model	ρ [fm]	R [fm]	Δm_{I} [GeV]	α_s [GeV]	κ [GeV^2]	σ [GeV]
MWOI	-	-	-	0.2068	0.1746	5.0248
M-I	0.33	1.00	0.0676	0.3447	0.1520	0.9331
M-IIb	0.36	0.89	0.1357	0.4588	0.1279	0.5650

UY, H.-Ch. Kim, E. Hiyama, PRD **98** (2018).

Charmonium States Including Instanton Effects

State	Input	MWOI	M-I	M-IIb	Exp. [50]
$J/\psi(1^3S_1)$	3097	3084	3094	3096	3096.900 ± 0.006
$\eta_c(1^1S_0)$	2983	3027	2998	2983	2983.9 ± 0.5
$\psi(2^3S_1)$	3686	3635	3656	3675	3686.097 ± 0.025
$\eta_c(2^1S_0)$	3640	3590	3615	3638	3637.6 ± 1.2
$\psi(3^3S_1)$	4040	4067	4069	4071	4039 ± 1
$\psi(4^3S_1)$	4415	4443	4422	4398	4421 ± 4
$\chi_{c2}(1^3P_2)$		3428	3607	3740	3556.17 ± 0.07
$\chi_{c1}(1^3P_1)$		3437	3589	3715	3510.67 ± 0.05
$\chi_{c0}(1^3P_0)$		3415	3551	3673	3414.71 ± 0.30
$h_c(1^1P_1)$		3430	3599	3727	3525.38 ± 0.11
$\chi_{c2}(2^3P_2)$		3888	4039	4138	3927.2 ± 2.6
$\chi_{c0}(2^3P_0)$		3866	4006	4098	3862^{+26+40}_{-32-13}
$\psi_2(1^3D_2)$		3718	3836	3927	3822.2 ± 1.2
$\psi(1^3D_1)$		3730	3830	3914	3778.1 ± 1.2
$\psi(2^3D_1)$		4131	4241	4303	4191 ± 5

UY, H.-Ch. Kim, E. Hiyama, PRD **98** (2018).

Summary and Outlook

- Instantons contribute to the heavy quark sector too
- Compared to the charm sector, the bottom sector is less sensitive to instantons
- It will be interesting to take into account the gluon propagator renormalization in the instanton media (screened Coulomb type contribution to the potential)

Thank you very much for your attention!