Structure of hadrons in a nuclear medium

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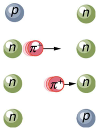
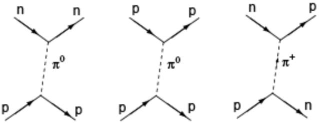
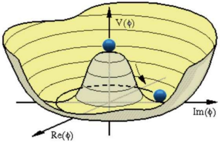
Introduction

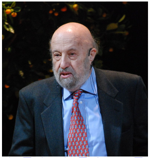
- Pion was introduced by Yukawa as a mediator (carrier) of the nuclear strong force (Proc. Phys. Math. Soc. Jpn. **17** (1935)). The light mesons have a important role in nucleon and nuclear structure





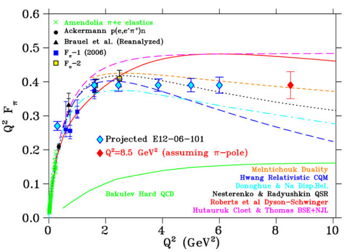
- Pion, the Goldstone bosons emerged as consequence of spontaneously breaking of global chiral symmetry in the favor SU(2), has a special place in QCD $\Longleftrightarrow$ Nambu-Jona-Lasinio introduced chiral symmetry and its breaking for generating mass and appearing pion (Phys. Rev. **124** & **122** (1961))

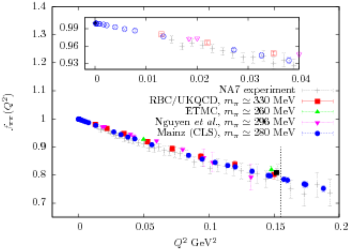




Introduction

- There have been many studies devoted to understand the internal structure of the pion in free space based on phenomenological approaches such as NJL, Instanton vacuum , Light-front, and Dyson-Schwinger models as well as lattice QCD simulations (Courtesy: Garth Huber Slide, EIC Meeting 2018 & Bastian. B. Brant, Int. J. Mod. Phys. E**22** (2013))





- However, in nuclear medium, only few theoretical studies have been reported so far on the pion[1] & kaon structures and no experimental data available (mostly nucleon in medium $\Longleftrightarrow$ EMC effect)

[1] Suzuki, Phys. Lett. B**368** (1996)

Introduction

- Recent work of Ref.[2] used a light-front constituent quark model to describe the pion in vacuum as well as in medium. There are a few aspects which require further investigations:
  - In the LF constituent quark model, the dressed mass value in vacuum is an input and treated as a parameter
  - There is no quark condensate which cannot explain the connection with spontaneous breaking of chiral symmetry of the vacuum
- We address this point in our work by using the NJL model which describes the spontaneous breaking of chiral symmetry and offers the dynamically generated quark mass through quark condensates
- Some observations such as EMC effect indicates the internal structure of hadrons may change in nuclear medium. The phenomena of medium modifications is therefore one the most interesting subject in nuclear and hadron physics[3]

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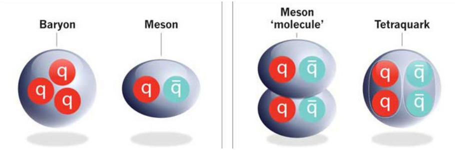
[2] J. P. B. C. de Melo, *et al.*, Phys. Rev. C**90** (2014)

[3] G. E. Brown, PRL **66** (1991), K.Saito, Prog. Part. Nucl. Phys. **58** (2007), Hayano, Rev.Mod. Phys. **82** (2010), Leupold, Int. J. Mod. Phys. E**19** (2010), and Metag, EPJ Web Conf. **34** (2017)

Introduction

- Since chiral symmetry has a big impact on the low-lying hadron mass spectrum, the partial restoration of chiral symmetry in a strongly interaction medium is important to understand the change of hadrons properties in nuclear medium
- As pions are the lightest bound states composed of dressed and quark-antiquark pair



- We focus on electroweak properties in nuclear medium in this study by calculating the weak decay constant of the in-medium pion, the pion-quark coupling constant in symmetric nuclear matter, and quark condensate in medium as well as the medium modifications of the pion & kaon form factors

Pion and Kaon in the BSE-NJL Model

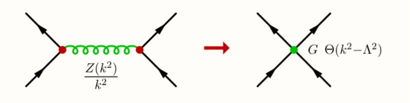
The three flavor NJL Lagrangian – containing only four fermion interactions

$$\mathscr{L}_{NJL} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}_q]\psi + G_\pi \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] - G_\rho \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] \quad (1)$$

- $\psi = (u, d, s)^T$ denotes the quark field with the flavor components
- G_π and G_ρ are four-fermion coupling constants
- $\lambda_1, \cdots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}\mathbb{1}$
- $\hat{m}_q = \mathrm{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

Pion and Kaon in the BSE-NJL Model

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction by integrating out the gluon field and absorbing into the coupling constant $\Longleftrightarrow$ *quark effective theory*



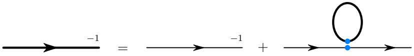
- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole). Therefore we regularize using the proper time regularization to simulate confinement (IC.Cloet, PRC**90** (2014), PTPH, PRC**94** (2016))

$$\frac{1}{\mathscr{X}^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\tau^{(n-1)}e^{-\tau\mathscr{X}} \rightarrow \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau\tau^{(n-1)}e^{-\tau\mathscr{X}} \quad (2)$$

where $\Lambda_{IR} \sim \Lambda_{QCD} \sim 0.24$ GeV and Λ_{UV} is determined.

Pion and Kaon in the BSE-NJL Model

- NJL Gap Equation is determined using quark propagator in momentum space $S_q^{-1}(p) = \not{p} - M_q + i\epsilon$



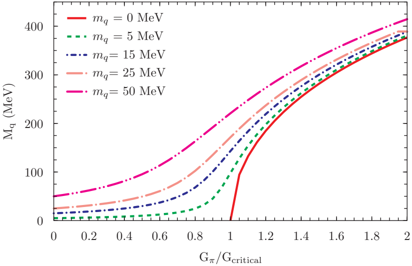
$$
\begin{aligned}
M_q &= m_q + M_q \frac{3 G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} \\
&= m_q - 2 G_\pi \langle \bar{\psi}\psi \rangle
\end{aligned} \quad (3)
$$

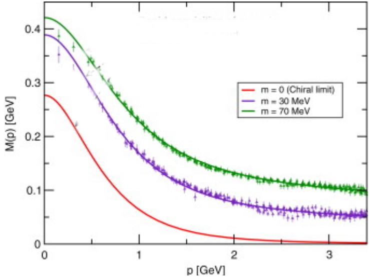
- Chiral quark condensates is defined by $\langle \bar{\psi}\psi \rangle = -\frac{3M_q}{2\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2}$
- Mass is generated through interaction vacuum $\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$

NJL Gap Equation

NJL and DSE gap equations PTPH *et al.*, PRC**94** (2016), C.D.Roberts, PPNP **61** (2008)

- The NJL constituent quark mass is a constant up to certain $p \sim 0.6$ GeV and it drops in higher p region

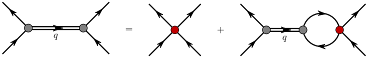




- The NJL model can be used for low momentum p and low energy E

Bethe Salpeter Equation for the pion and kaon

Mesons in the NJL model are quark-antiquark bound states whose properties are determined by solving the BSE



- In the NJL model, $\mathscr{T}$-matrix is given by

$$\mathscr{T}(q) = \mathscr{K} + \int \frac{d^4k}{(2\pi)^4} \mathscr{K} S(q+k) \mathscr{T}(q) S(k)$$

- The solution to the BSE in the pion and kaon

$$\mathscr{T}_\alpha(q)_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab}\, t_\alpha(q) \left[\gamma_t \lambda_\alpha^\dagger\right] \quad (4)$$

- The reduced t-matrix in this channel take a form

$$t_\alpha(q) = \frac{-2iG_\pi}{1+2G_\pi \Pi_\pi(q^2)}$$
$$t_\beta^{\mu\nu}(q) = \frac{-2iG_\rho}{1+2G_\rho \Pi_\beta(q^2)} \left(g^{\mu\nu} + 2G_\rho \Pi_\beta(q^2) \frac{q^\mu q^\nu}{q^2}\right) \quad (5)$$

Bethe Salpeter Equation of the pion and kaon

- The bubble diagrams appearing read

$$\begin{aligned}
\Pi_\pi(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D \left[\gamma_5 S_l(k) \gamma_5 S_l(k+q)\right], \\
\Pi_K(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D \left[\gamma_5 S_l(k) \gamma_5 S_s(k+q)\right], \\
\Pi_\nu^{aa}(q^2) &= 6i \int \frac{d^4k}{(2\pi)^4} Tr_D \left[\gamma^\mu S_a(k) \gamma^\nu S_a(k+q)\right]
\end{aligned} \tag{6}$$

- The kaon and pion masses is given by the pole of the t-matrix

$$\begin{aligned}
1 + 2G_\pi \Pi_\pi(k^2 = m_\pi^2) &= 0 \\
1 + 2G_\pi \Pi_K(k^2 = m_K^2) &= 0
\end{aligned} \tag{7}$$

Pion and Kaon Masses

The meson masses are defined by the pole in the corresponding t-matrix and therefore the kaon and pion masses are given by

$$
\begin{aligned}
m_\pi^2 &= \left[\frac{m}{M_l}\right] \frac{2}{G_\pi \mathcal{I}_{ll}(m_\pi^2} \\
m_K^2 &= \left[\frac{m_s}{M_s} + \frac{m}{M_l}\right] \frac{1}{G_\pi \mathcal{I}_{ls}(m_K^2} + (M_s - M_l)^2 \qquad (8)
\end{aligned}
$$

where $\mathcal{I}_{ll}$ and $\mathcal{I}_{ls}$ in the proper time regularization scheme are defined by

$$
\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \qquad (9)
$$

The meson-quark-quark coupling constants and Pion and Kaon Decay Constants

The residue at a pole in the $\bar{q}q$ $t-$matrix defines the effective meson-quark -quark coupling constants:

$$\begin{aligned} Z_\pi(q^2) &= -\frac{\partial \Pi_\pi(q^2)}{\partial q^2}\Big|_{q^2=m_\pi^2} \\ Z_K(q^2) &= -\frac{\partial \Pi_K(q^2)}{\partial q^2}\Big|_{q^2=m_K^2} \\ Z_\rho(q^2) &= -\frac{\partial \Pi_\rho(q^2)}{\partial q^2}\Big|_{q^2=m_\rho^2} \end{aligned} \quad (10)$$

Pion and kaon decay constant in the proper time regularization is given by

$$\begin{aligned} f_\pi &= \frac{N_C\sqrt{Z_\pi}M}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)} \\ f_K &= \frac{N_C\sqrt{Z_K}}{4\pi^2}[(1-x)M_2 + xM_1]\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)} \end{aligned} \quad (11)$$

QUARK-MESON COUPLING (QMC) MODEL

Lagrangian QMC model

The effective Lagrangian for a symmetric nuclear matter in the QMC model:

$$\mathscr{L}_{\rm QMC} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M_N^*(\sigma) - g_\omega\gamma_\mu\omega^\mu\right]\psi + \mathscr{L}_m, \qquad (12)$$

The free meson Lagrangian density:

$$\mathscr{L}_m = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{2}\partial_\mu\omega_\nu\left(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu\right) + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu$$

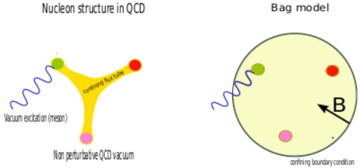


FIGURE: The QCD picture of the nucleon and the bag model [4]

[4] J.Stone *et al.*, Prog.Part.Nucl.Phys. **100** (2018)

Lagrangian QMC model

- In the QMC model, the nuclear matter is treated as a collection of the nucleons that are assumed to be non-overlapping MIT bags
- The Dirac equation for the light quarks inside the bags are given by

$$\left[i\gamma\cdot\partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q + \frac{1}{2}V_\rho^q\right)\right] \left(\begin{array}{c} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{array}\right) = 0$$

$$\left[i\gamma\cdot\partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q - \frac{1}{2}V_\rho^q\right)\right] \left(\begin{array}{c} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{array}\right) = 0$$

$$\left[i\gamma\cdot\partial_x - m_s\right] \left(\begin{array}{c} \psi_s(x) \\ \psi_{\bar{s}}(x) \end{array}\right) = 0, \quad (13)$$

where the effective current quark mass m_q^* is defined as

$$m_q^* \equiv m_q - V_\sigma^q, \quad (14)$$

where m_q is the current quark mass, where $q = (u, d, s)$ and V_σ^q is the scalar potential.

Lagrangian QMC model

The effective nucleon mass:

$$M_N^*(\sigma) \equiv M_N - g_\sigma(\sigma)\,\sigma, \tag{15}$$

Total energy per nucleon:

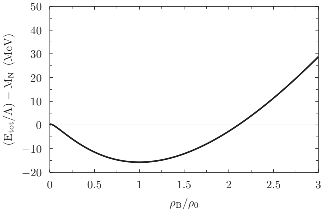
$$E^{\rm tot}/A = \frac{4}{(2\pi)^3\rho_B}\int d\mathbf{k}\,\theta(k_F - |\mathbf{k}|)\sqrt{M_N^{*2}(\sigma)+\mathbf{k}^2} + \frac{m_\sigma^2\sigma^2}{2\rho_B} + \frac{g_\omega^2\rho_B}{2m_\omega^2}. \tag{16}$$

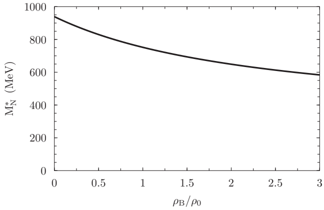
Table: Parameters of the QMC model and the obtained nucleon properties at saturation density $\rho_0 = 0.15$ fm^{-3} for two quark mass values in free space, $m_q = 5.0$, and 16.4 MeV. The m_q, M_N^*, and K are given in units of MeV. The parameters are fitted to the free space nucleon mass $M_N = 939$ MeV with $R_N = 0.8$ fm (input), and the nuclear matter saturation properties.

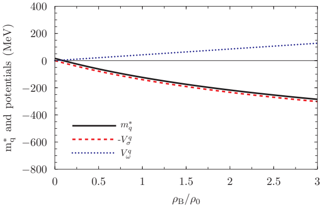
m_q	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	$B^{1/4}$	z_N	M_N^*	K
5	5.393	5.304	170.0	3.295	754.6	279.3
16.4	5.438	5.412	169.2	3.334	752.0	281.5

Lagrangian QMC model

Energy per nucleon ($E^{\text{tot}}/A - M_N$), effective nucleon mass M_N^* and effective quark mass (m_q^*) and the quark potentials (V_σ^q and V_ω^q) for symmetric nuclear matter in the QMC model for the current quark mass $m_q = 16.4$ MeV







PION AND KAON PROPERTIES IN A NUCLEAR MEDIUM

In-medium pion properties (NJL+QMC)

- Using the in-medium properties corresponding to $m_q = 16.4$ MeV calculated in the QMC model,
- we calculate the effective quark mass M_u^*, in-medium pion decay constant, in-medium quark condensate, and in-medium πqq coupling constant using the NJL model.
- The in-medium dressed quark propagator:

$$S_q^*(k^*) = \frac{\not{k} + V^0 + M_q^*}{(k + V^0)^2 - M_q^{*2} + i\epsilon}, \qquad (17)$$

where the medium modification enter as the shift of the quark momentum through $(k^*)\mu \rightarrow k^\mu + V^\mu$ where vector potential, $V^\mu = (\delta_0^\mu V^0, \vec{0})$[5]. The asterisk denotes the in-medium quantity

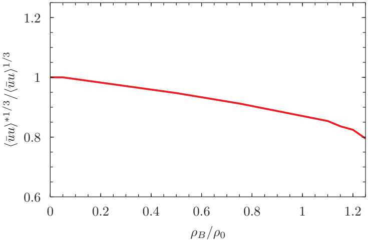
- The in-medium NJL gap mass in the proper-time regularization scheme:

$$M_q^* = m_q^* + \frac{3G_\pi M_q^*}{\pi^2} \int_{\frac{1}{\Lambda_{UV}^2}}^{\infty} \frac{d\tau}{\tau^2} e^{\left(-\tau (M_q^*)^2\right)} \qquad (18)$$

[5] Miller, Phys. Rev. Lett. **103** (2009)

In-medium pion properties (NJL+QMC)

The ratio of the in-medium to vacuum quark condensates as a function of ρ_B/ρ_0 with $\rho_0 = 0.15\ \text{fm}^{-3}$

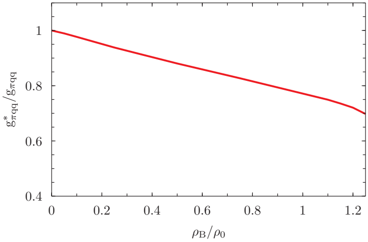


- The ratio of the in-medium to vacuum quark condensates decreases with increasing nuclear matter density
- The ratio at saturation nuclear matter density is estimated about 0.87
- This is somehow higher than obtained in Ref[6] which gives 0.63-0.57 via the relation $< \bar{q}q >^* / < \bar{q}q > \sim 1 - (0.37 \sim 0.43)\rho_B/\rho_0$

[6] Jido *et al.*, PLB **670** (2008)

IN-MEDIUM PION PROPERTIES (NJL+QMC)

The ratio of the in-medium to vacuum pion-quark coupling constant as a function of ρ_B/ρ_0 with $\rho_0 = 0.15$ fm^{-3}



- We observe $g^*_{\pi qq}/g_{\pi qq}$ decreases with increasing density, which is consistent with the results of Refs.[7]
- At normal density, we obtain $g^*_{\pi qq}/g_{\pi qq} = 0.77$ which is smaller than the value obtained in Ref.[8]

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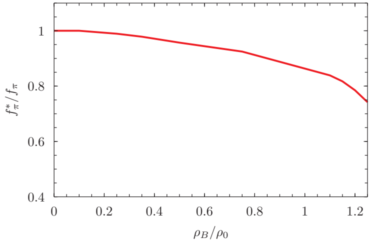
[7] V. Bernard *et al.*, PRD **38** (1988)

[8] Ramalho *et al.*, J. Phys. **G40** (2013)

In-medium pion properties (NJL+QMC)

The ratio of the in-medium to vacuum pion decay constant as a function of ρ_B/ρ_0 with $\rho_0 = 0.15\ \mathrm{fm}^{-3}$



- The ratio is found to decrease as density increases
- At normal density, we obtained $f_\pi^*/f_\pi = 0.87$, which is in a good agreement with results of Ref[9] $f_\pi^*/f_\pi = 0.80$
- Ratio is larger than the values obtained in Refs.[10] [11] by about 10-20 %

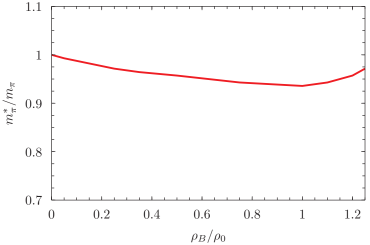
[9] P. Kienle *et al.*, Prog. Part. Nucl. Phys. **52** (2004)

[10] M. Kirchbach *et al.*, Nucl. Phys. A**616** (1997)

[11] U. G. Meissner *et al.*, Ann. Phys. (N.Y)**297** (2002)

In-medium pion properties (NJL+QMC)

The ratio of the in-medium to vacuum pion mass as a function of ρ_B/ρ_0 with $\rho_0 = 0.15$ fm^{-3}



- we confirm that the pion is almost unchanged up to $1.25\rho_0$ which is consistent with the results obtained by Bernard in the low density region
- The difference between the in-medium and free pion masses is within 6% up to nuclear density of $1.25\rho_0$
- This justifies the assumption (deeply bound pionic atom) that $m^*_\pi \sim m_\pi$ up to normal nuclear density or $1.25\rho_0$

In-medium pion properties (NJL+QMC)

The ratio of the in-medium to vacuum isovector nucleon axial-vector coupling constant as a function of ρ_B/ρ_0 with $\rho_0 = 0.15$ fm^{-3}.

- The Goldberger-Treiman relation at nucleon level is given by $g_A \equiv g_{\pi NN} f_\pi / M_N$. at the quark level, the ratio of the in-medium to vacuum isovector nucleon axial-vector coupling constant:

$$\left(\frac{g_A^*}{g_A}\right) = \left(\frac{g_{\pi qq}^*}{g_{\pi qq}}\right)\left(\frac{f_\pi^*}{f_\pi}\right)\left(\frac{M_q}{M_q^*}\right). \tag{19}$$

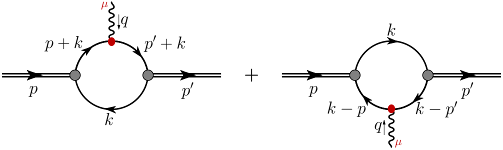
- we estimate $g_A^*/g_A = 0.99$, which is consistent with the quenching of g_A^* but *less amount of quenching* compared with the result of Ref.[12] that gives $g_A^*/g_A = 0.9$

[12] D. H. Lu *et al.*, arXiv:nucl-th/0112001

IN-MEDIUM MODIFICATION FORM FACTOR OF THE PION

In-Medium Modifications Form Factor in NJL model

Diagrammatic representation of the electromagnetic current for the pion and kaon

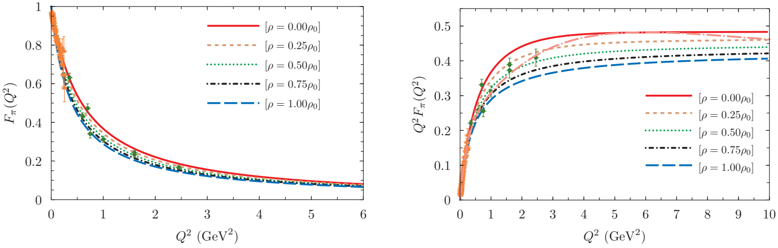


Feynman diagram for quark [left] and for the anti quark [right] The complete results for the pseudoscalar meson form factor in a nuclear medium – with a dressed quark-photon vertex – read

$$\begin{aligned} F^*_{\pi^+}(Q^2) &= \left[F^*_{1U}(Q^2) - F^*_{1D}(Q^2)\right] f^{*ll}_{\pi}(Q^2) \\ F^*_{K^+}(Q^2) &= F^*_{1U}(Q^2) f^{*ls}_{K}(Q^2) - F^*_{1S}(Q^2) f^{*sl}_{K}(Q^2) \\ F^*_{K^0}(Q^2) &= F^*_{1D}(Q^2) f^{*ls}_{K}(Q^2) - F^*_{1S}(Q^2) f^{*sl}_{K}(Q^2) \end{aligned} \quad (20)$$

In-Medium Modifications Pion Form Factor

Results for the in-medium space-like electromagnetic form factors of the pion (PTPH, Yongseok Oh, K. Tsushima, PRC**99**(2019))



- Our pion form factor results show that the in-medium pion electromagnetic form factor decreases with increasing density
- The medium effects on the suppression of the pion form factor are clearly seen and it is reduced by 20% at normal density

In-Medium Modifications Pion Form Factor

Results for the charge radii of the charged pion in the nuclear medium and its quark sector charge radii

Table: Results for the charge radii of the charged pion in the nuclear medium and its quark sector charge radii. This is calculated using the inputs from the QMC model. All charge radii in the medium along with the vacuum are in units of fm. The empirical result in the vacuum.

ρ_B/ρ_0	r_π	r_u	r^{expt}
0.00	0.629	0.629	0.672 ± 0.008
0.25	0.667	0.667	
0.50	0.705	0.704	
0.75	0.740	0.739	
1.00	0.771	0.771	

- The obtained charge radius of the pion in vacuum is in good agreement with the empirical data[13]

[13] C. Patrignani *et al.*, Chin. Phys. **C40** (2016)

In-Medium Modifications Kaon Form Factor

Results for the in-medium space-like electromagnetic form factors of the kaon (PTPH, Yongseok Oh, K. Tsushima, in preparation (2019))

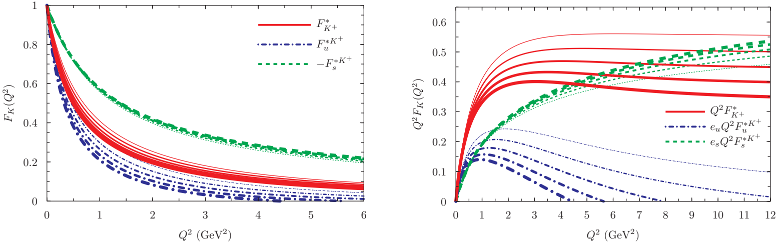


Figure: Results for the in-medium $Q^2 F^*_{K^+}(Q^2)$ (solid line) together with the charge-weighted quark-sector contributions. The in-medium form factors are calculated using the inputs from the calculated NJL model combined with the QMC model for $\rho_B/\rho_0 = [0.00, 0.25\ , 0.50, 0.75, 1.00]$. The difference densities are represented by thinner to thicker lines.

- Our kaon form factor results decrease with increasing density as the pion case

In-Medium Modifications Charge Radius

Results for the charge radii of the charged kaon in the nuclear medium and its quark sector charge radii

Table: Results for the in-medium charge radii of the charged kaon and its in-medium quark sector charge radii. This is calculated in the NJL model using the inputs from the standard QMC model. All in-medium charge radii along with the vacuum are in units of fm. The empirical result in the vacuum.

ρ_B/ρ_0	r_K	r_u	r_s	r^{expt}
0.00	0.59	0.65	0.44	0.56 ± 0.03
0.25	0.62	0.69	0.44	
0.50	0.65	0.73	0.44	
0.75	0.68	0.77	0.44	
1.00	0.71	0.81	0.44	

PION AND KAON PDFs IN A NUCLEAR MEDIUM

In collaboration with:

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  Catedra CONACyt, Departmento de Fisica, Mexico
- Kazuo Tsushima
  Laboratorio de Fisica Teorica e Computacional-LFTC, Universidade Cruzeiro do Sul
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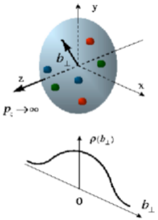
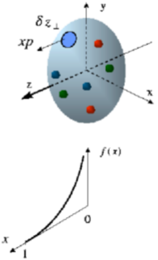
PARTON DISTRIBUTION FUNCTIONS IN THE BSE-NJL MODEL



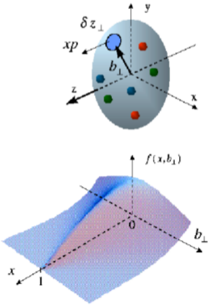
Elastic Scattering

transverse quark distribution in coordinate space



DIS

longitudinal quark distribution in momentum space

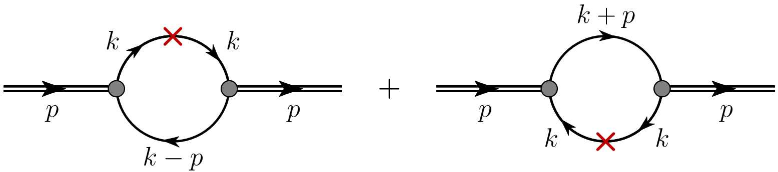


DES (GPDs)

fully-correlated quark distribution in both coordinate and momentum space

Parton distribution functions

The valence quark distribution functions of the pion or kaon are given by the two Feynman diagrams(PTPH, Ian Cloet, Anthony W. Thomas, PRC**94**(2016))



The operator insertion $\gamma^+ \delta\left(k^+ - xp^+\right) \hat{P}_q$, where $\hat{P}_q$ is the projection operator for quarks of flavor q:

$$\begin{aligned} \hat{P}_{u/d} &= \frac{1}{2}\left(\frac{2}{3}\mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right) \\ \hat{P}_s &= \frac{1}{3}\mathbb{1} - \frac{1}{\sqrt{3}}\lambda_8 \end{aligned} \tag{21}$$

Parton distribution functions

The valence quark and anti-quark distributions in the pion or kaon are given by

$$
\begin{aligned}
q_\alpha(x) &= iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ - xp^+\right) \\
&\times Tr\left[\gamma_5 \lambda_\alpha^\dagger S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha S(k-p)\right] \\
\bar{q}_\alpha(x) &= -iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ + xp^+\right) \\
&\times Tr\left[\gamma_5 \lambda_\alpha S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha^\dagger S(k+p)\right] \qquad (22)
\end{aligned}
$$

To evaluate these expression we first take the moments

$$
\mathcal{A}_n = \int_0^1 dx x^{n-1} q(x) \qquad (23)
$$

where $n = 1, 2, 3, \cdots$ is an integer.

Parton distribution functions

Using the Ward-like identity $S(k)\gamma^{+}S(k)=\frac{-\partial S(k)}{\partial k_{+}}$ and introducing the Feynman parameterization, the quark and anti-quark distributions can then be straightforwardly determined. For the valence quark and anti-quark distributions of the K^{+} we find:

$$\begin{aligned}
q_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2+xM_s^2+(1-x)M_l^2\right]} \\
&\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2-(m_l-M_s)^2\right]\right] \\
\bar{q}_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2+xM_l^2+(1-x)M_s^2\right]} \\
&\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2-(m_l-M_s)^2\right]\right] \qquad (24)
\end{aligned}$$

$\Rightarrow$ Results for the π^{+} are obtained by $M_s\to M_l$ and $Z_K\to Z_\pi$, giving the result $u_\pi(x)=\bar{d}_\pi(x)$

Parton distribution functions

The quark distributions satisfy the baryon number and momentum sum rules, which for the K^+ read:

$$\int_0^1 dx\,[u_{K^+}(x) - \bar{u}_{K^+}(x)] \;=\; \int_0^1 [\bar{s}_{K^+}(x) - s_{k^+}(x)] = 1 \qquad (25)$$

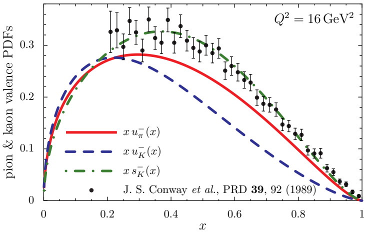
for the number sum rules and at the model scale the momentum sum rules is given by

$$\int_0^1 dxx\,[u_{K^+}(x) + \bar{u}_{K^+}(x) + \bar{s}_{K^+}(x) + s_{k^+}(x)] \;=\; 1 \qquad (26)$$

Analogous results holds for the remaining kaons and the pions.

Kaon PDFs Results

Results for the valence quark distributions of the π^+ and K^+ evolved from model scale to $Q^2 = 16\ GeV^2$ using NLO DGLAP equations[14] and compared empirical data for the pion valence PDF.



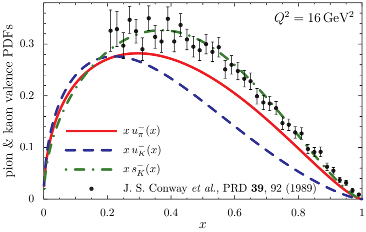
⇒ At model scale, the momentum fraction by the u and s quarks in the K^+, $< xu >= 0.42$ and $< xs >= 0.58$

⇒ The flavor breaking effects of $[< xs > - < xu >]/[< xs > + < xu >] \sim$ 16% which is similar to that seen in masses $[M_s - M_u]/[M_s + M_u] \sim 21\%$.

[14] M. Miyama and S. Kumano, Comput.Pys.Commun. **94**, 185

Kaon PDFs Results

Results for the valence quark distributions of the π^+ and K^+, evolved from the model scale using NLO DGLAP equations.

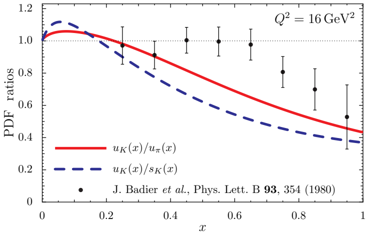


$\rightarrow$ $SU(3)$ flavor breaking at the model scale $u_K(x)$ peaks at $x_u = 0.237$ and $\bar{s}_K$ peaks at the $x_s = 1 - x_u = 0.763$

$\rightarrow$ This implies flavor breaking effects of around $[x_s - x_u]/[x_s + x_u] \sim 53\%$. For the pion the peak at $x = 0.5$ when $m_u = m_d$.

Kaon PDFs Results

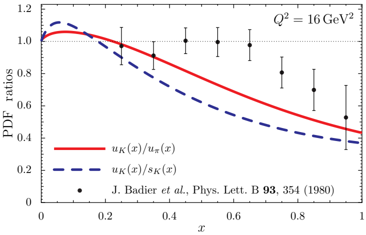
The ratio of the u quark distribution in the K^+ to the u quark distribution in the π^+, after NLO evolution to $Q^2 = 16$ GeV^2



→ The ratio of $u_K/u_\pi \to 0.434 \sim M_u/M_s$ as $x \to 1$, which is in a good agreement with existing data.

→ The x-dependence differs from much of data in the valence region. This may lie with the absence of the momentum dependence in the NJL Bethe Salpeter vertices, or with data itself.

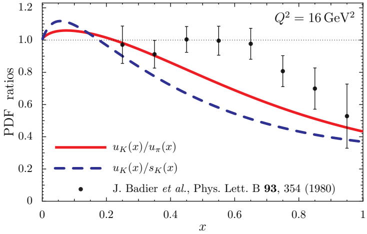
Kaon PDFs Results



$\rightarrow$ The ratio $u_K(x)/s_K(x)$ approaches 0.37 as $x \rightarrow 1$. It is evident that the flavor breaking effects have a sizable x dependence, being maximal at large x and becoming negligible at small x where the perturbative effects from DGLAP evolution dominate.

$\rightarrow$ The Drell-Yan-West (DYW) relation, $F(Q^2) \sim \frac{1}{Q^{2n}} \leftrightarrow q(x) \sim (1-x)^{2n-1}$. For the pion, $F_\pi \sim 1/Q^2$ and the DYW relation implies $q_\pi(x) \sim (1-x)$. Kaon PDF do behave as the pion.

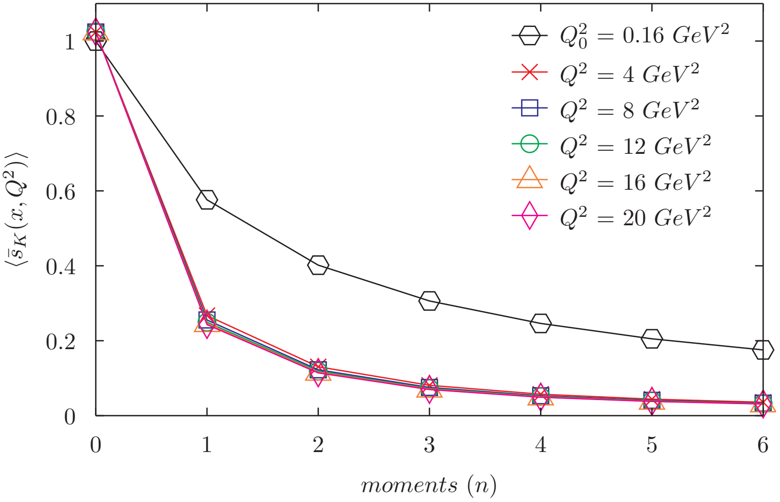
Kaon PDFs Results



$\rightarrow$ As reflection of the expectations of may be expected by DYW like relations, $u_K/s_K < 1$ as $x \rightarrow 1$ and $| F_K^u/F_K^s |< 1$ for $Q^2 >> \Lambda_{QCD}$.

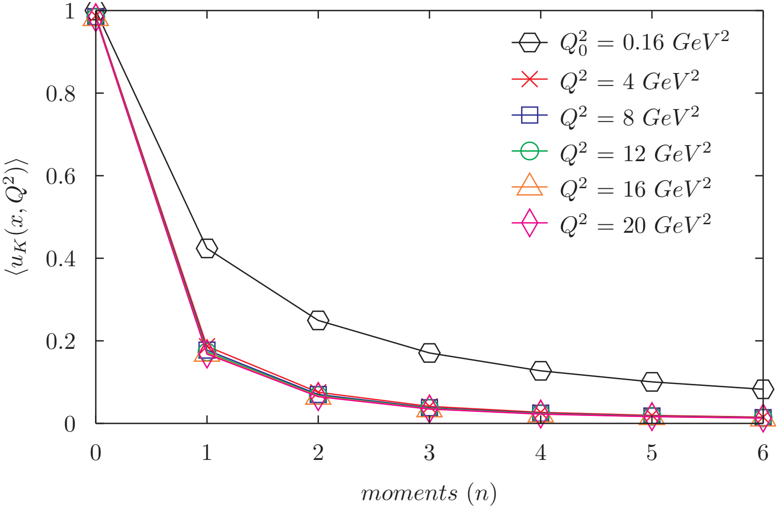
Kaon PDFs Results

Moments PDFs of the s quark in the kaon



Kaon PDFs Results

Moment PDFs of the u quark in the kaon



In-medium pion properties (NJL+QMC)

- Using the in-medium properties corresponding to $m_q = 16.4$ MeV calculated in the QMC model,
- we calculate the effective quark mass M_u^*, in-medium pion decay constant, in-medium quark condensate, and in-medium πqq coupling constant using the NJL model.
- The in-medium dressed quark propagator:

$$S_q^*(k^*) = \frac{\not{k} + V^0 + M_q^*}{(k + V^0)^2 - M_q^* + i\epsilon}, \tag{27}$$

where the medium modification enter as the shift of the quark momentum through $(k^*)\mu \to k^\mu + V^\mu$ where vector potential, $V^\mu = (\delta_0^\mu V^0, \vec{0})$[15]. The asterisk denotes the in-medium quantity

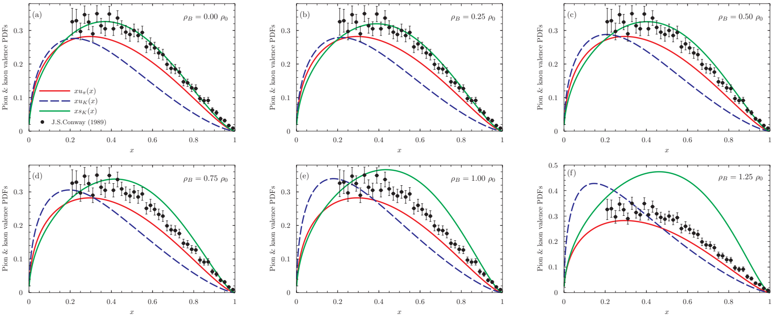
- The in-medium NJL gap mass in the proper-time regularization scheme:

$$M_q^* = m_q^* + \frac{3G_\pi M_q^*}{\pi^2} \int_{\frac{1}{\Lambda_{UV}^2}}^{\infty} \frac{d\tau}{\tau^2} e^{\left(-\tau (M_q^*)^2\right)} \tag{28}$$

[15] Miller, Phys. Rev. Lett. **103** (2009)

Numerical results of PDFs of the pion and kaon in a nuclear medium

PDFs of the pion and kaon in nuclear medium (*PRELIMINARY RESULT*)



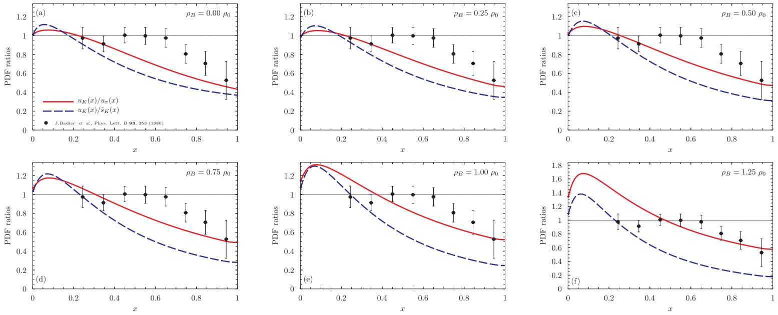
The effect of the vector field is then incorporated through scaling the quark distribution and shifting the Bjorken variable

$$q_{K^+}(x_a) = \frac{\epsilon_F}{E_F} q_{K^+}\left(\tilde{x_a} = \frac{\epsilon_F}{E_F} x_a - \frac{V_0}{E_F}\right) \tag{29}$$

where $\epsilon_F = \sqrt{k_F^q + M_q^2} \pm V_0 \equiv E_F \pm V_0$.

Numerical results of PDFs of the pion and kaon in a nuclear medium

Ratio of $u_{K^+}(x)/u_{\pi^+}(x)$ in nuclear medium (*PRELIMINARY RESULT*)



Summary and outlook

- We have studied a kaon and pion properties in vacuum as well as in medium.
- Our prediction on pion and kaon properties are in good agreement with other prediction
- We have extend our study on the in-medium modifications form factors of pion and kaon in order to understand the feature of form factors of the kaon and pion in the medium. The result looks very interesting and promising
- It would be interesting to extend calculation to the generalized parton distributions (GPDs) of the pion, kaon, and ρ, D, B meson in medium
- It could also be extended to octet and heavy baryons in medium

THANK YOU VERY MUCH FOR ATTENTION !!