

Production of multi-charmed hadrons in heavy ion collisions

HaPhy-CENuM joint workshop:
The Future of Lattice-QCD studies in Korea

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Pukyong National University
Busan



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based on works in arXiv:1907.12786
in collaboration with Su Houg Lee at Yonsei

Outline

- Introduction
- Charmed hadrons in heavy ion collisions
- Hadron production by quark coalescence
- Transverse momentum distribution of charmed hadrons
- Conclusion

Physics Potential – some examples

- Heavy-flavor and quarkonia

- Multiply Heavy Flavoured hadrons. e.g.: Ξ_{cc} , Ω_{cc} , Ω_{ccc}
- $\chi_{c1,2}$ states
- Ultimate precision on B mesons at low p_T
- X, Y, Z charmonium-like states (e.g. X(3872))



Hadron formation from deconfined QGP

- Low-mass dielectrons

- Precision measurement of the thermal dilepton continuum, $0 < m < 3\text{GeV}$



Chiral symmetry restoration ρ -a1 sector

- Real soft photons

- down to 50MeV/c



QGP Radiation uncharted phase space region

- Real ultra-soft photons

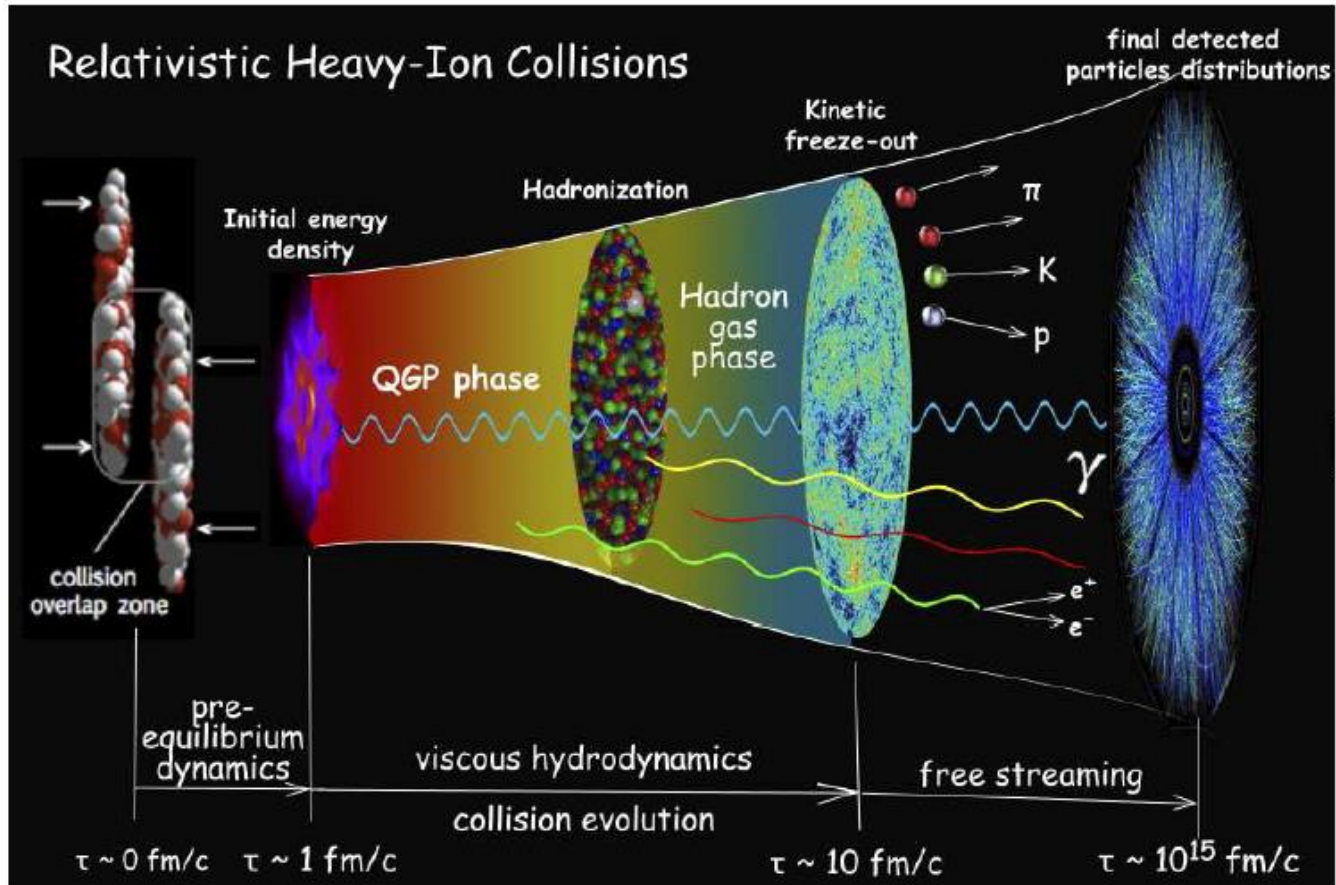
- Very low p_T photons: $1\text{MeV}/c < p_T^Y < 100\text{MeV}/c$
- dedicated small forward spectrometer at $3.5 < |\eta| < 5$



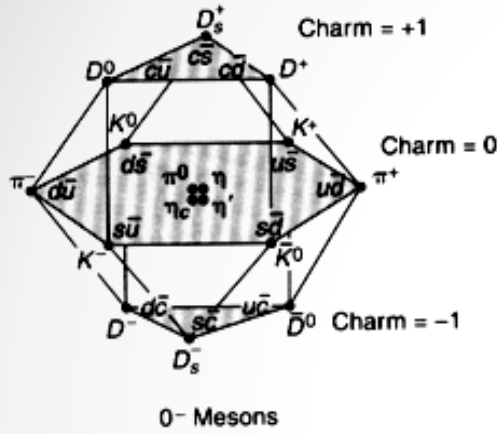
Test of soft theorems

Introduction

– Relativistic heavy ion collisions

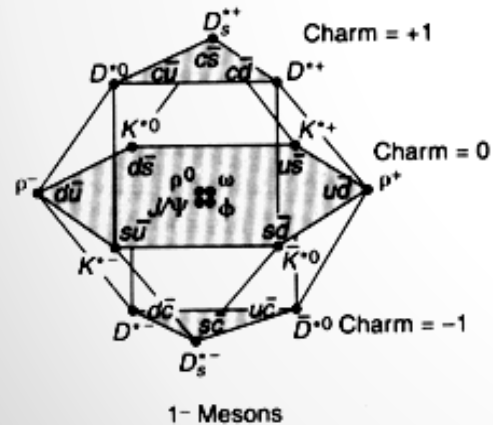


- Charmed hadrons, SU(4)

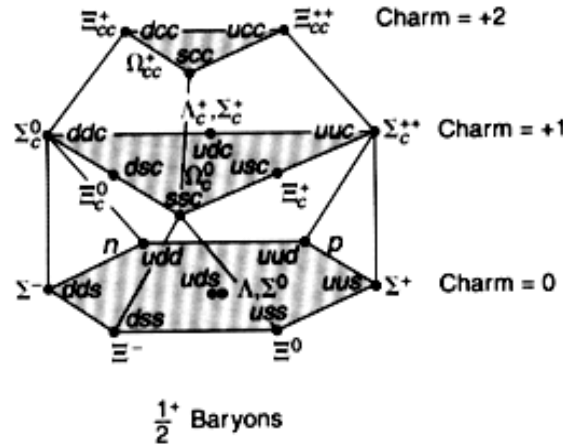


(a)

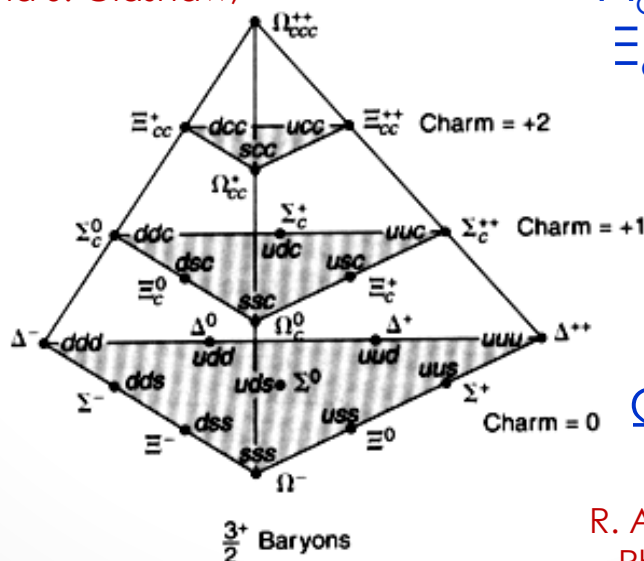
A. De Rújula, H. Georgi, And S. Glashaw, Phys. Rev. D **12**, 147 (1975)



(b)



(c)



(d)

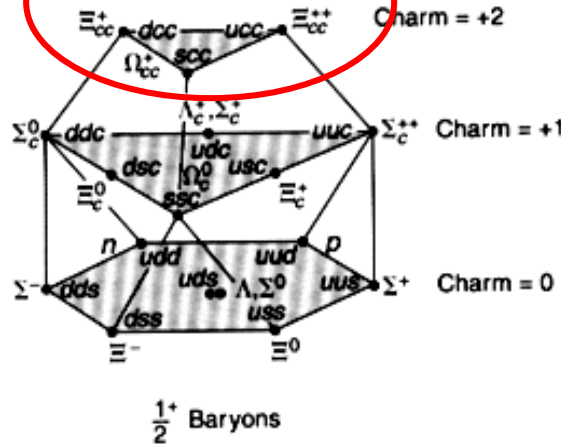
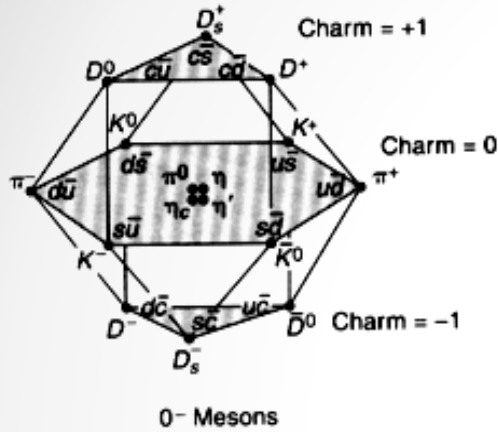
1) Charmonium states : $\eta_c, J/\psi, \chi_c, \psi'$

2) Charmed baryons and mesons : $D, D^*, D_s, D_s^*, \Lambda_c(2286), \Lambda_c(2595), \Lambda_c(2625), \Sigma_c(2455), \Sigma_c(2520), \Xi_c(2470), \Xi_c(2578), \Xi_c(2645), \Omega_c(2695), \Omega_c(2770)$

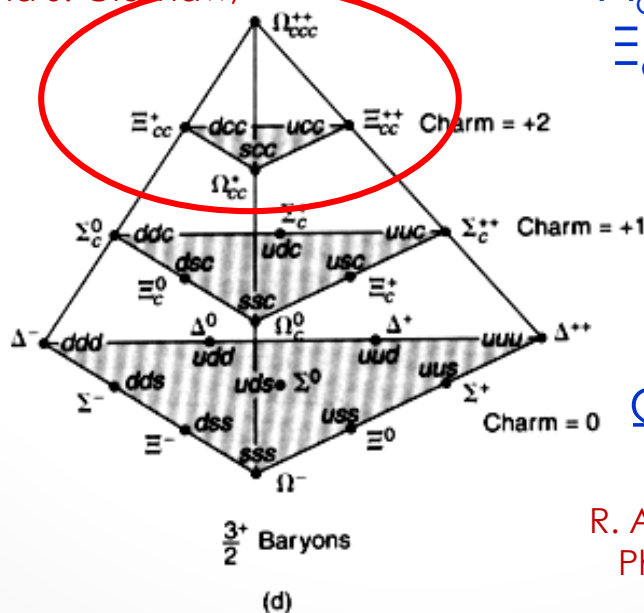
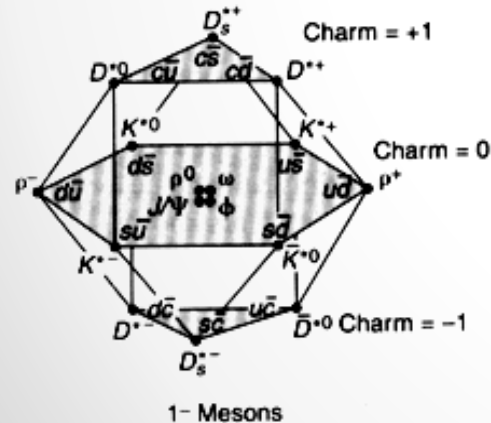
3) Doubly and triply charmed hadrons, exotic hadrons : $\Xi_{cc}^*, \Xi_{cc}^{*+}, \Omega_{cc}, \Omega_{cc}^*, \Omega_{ccc}, T_{cc}, X(3872)$

R. A. Briceño, H.-W. Lin, and D. R. Bolton, Phys. Rev. D **86** 094504 (2012)

- Charmed hadrons, SU(4)



(a) A. De Rújula, H. Georgi, And S. Glashow,
Phys. Rev. D **12**, 147 (1975)



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Phys. Rev. D **86** 094504 (2012)

HaPhy-CENuM joint workshop :
The future of Lattice-QCD in Korea

– Recent measurements of a doubly charmed baryon in 2017

PRL **119**, 112001 (2017)

PHYSICAL REVIEW LETTERS

WOOK CHUNG
15 SEPTEMBER 2017



Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.**

(LHCb Collaboration)

(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)

– T_{cc} ($ccqq$) mesons

Particle	m [MeV]	(I, J^P)
T_{cc}^1	3797	$(0, 1^+)$

S. Cho *et al.* (EXHIC Collaboration), Phys. Rev. C **84**, 064910 (2011)

S. Cho *et al.* (EXHIC Collaboration), Prog. Part. Nucl. Phys. **95**, 279 (2017)

J. Hong, S. Cho, T. Song, and S-H. Lee, Phys. Rev. C **98**, 014913 (2018)

– $X(3872)$ mesons

$X(3872)$

$$I^G(J^{PC}) = 0^+(1^{++})$$

Mass $m = 3871.68 \pm 0.17$ MeV

$m_{X(3872)} - m_{J/\psi} = 775 \pm 4$ MeV

$m_{X(3872)} - m_{\psi(2S)}$

Full width $\Gamma < 1.2$ MeV, CL = 90%

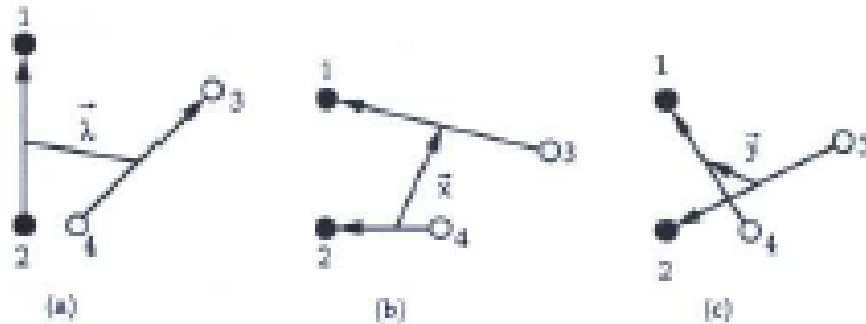
J. Beringer *et al.* (PDG), Phys. Rev. D **86**, 010001 (2012)

S.K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **90**, 242001 (2003)

- Internal structure of X(3872) mesons

1) Possible structures of X(3872) mesons, 3 independent relative coordinates

D. M. Brink and Fl. Stancu,
Phys. Rev. D 49, 4665 (1994)



2) The relative coordinates and momentum of X(3872) mesons

$$\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + m_4 \vec{r}_4}{m_1 + m_2 + m_3 + m_4}$$

$$\vec{r}'_1 = \vec{r}_1 - \vec{r}_2$$

$$\vec{r}'_2 = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} - \vec{r}_3$$

$$\vec{r}'_3 = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3} - \vec{r}_4$$

$$\vec{k} = \vec{p}'_{lT} + \vec{p}'_{\bar{l}T} + \vec{p}'_{cT} + \vec{p}'_{\bar{c}T},$$

$$\vec{k}_1 = \frac{m_{\bar{l}} \vec{p}'_{lT} - m_l \vec{p}'_{\bar{l}T}}{m_l + m_{\bar{l}}},$$

$$\vec{k}_2 = \frac{m_c (\vec{p}'_{lT} + \vec{p}'_{\bar{l}T}) - (m_l + m_{\bar{l}}) \vec{p}'_{cT}}{m_l + m_{\bar{l}} + m_c},$$

$$\vec{k}_3 = \frac{m_{\bar{c}} (\vec{p}'_{lT} + \vec{p}'_{\bar{l}T} + \vec{p}'_{cT}) - (m_l + m_{\bar{l}} + m_c) \vec{p}'_{\bar{c}T}}{m_l + m_{\bar{l}} + m_c + m_{\bar{c}}} \quad (A2)$$

$$\vec{k} = \vec{p}'_{lT} + \vec{p}'_{\bar{l}T} + \vec{p}'_{cT} + \vec{p}'_{\bar{c}T},$$

$$\vec{k}_1 = \frac{m_{\bar{l}} \vec{p}'_{lT} - m_l \vec{p}'_{\bar{l}T}}{m_l + m_{\bar{l}}},$$

$$\vec{k}_2 = \frac{m_c \vec{p}'_{cT} - m_{\bar{c}} \vec{p}'_{\bar{c}T}}{m_c + m_{\bar{c}}},$$

$$\vec{k}_3 = \frac{(m_c + m_{\bar{c}}) (\vec{p}'_{lT} + \vec{p}'_{\bar{l}T}) - (m_l + m_{\bar{l}}) (\vec{p}'_{cT} + \vec{p}'_{\bar{c}T})}{m_l + m_{\bar{l}} + m_c + m_{\bar{c}}}, \quad \bullet 8$$

Charmed hadrons in heavy ion collisions

– Charmonium states

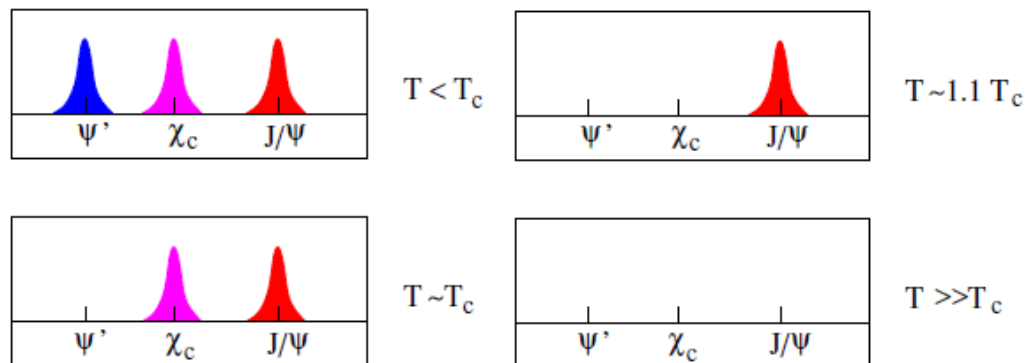
T. Matsui and H. Satz, Phys. Lett. B **178** 416 (1986)

1) J/ψ suppression and Debye screening

At $T > T_c$ color charges are Debye screened in QGP, and the Debye screening prevents the formation of the bound states

2) The different charmonium states melt sequentially as a function of their binding strength;

the most loosely bound state disappears first, the ground state last



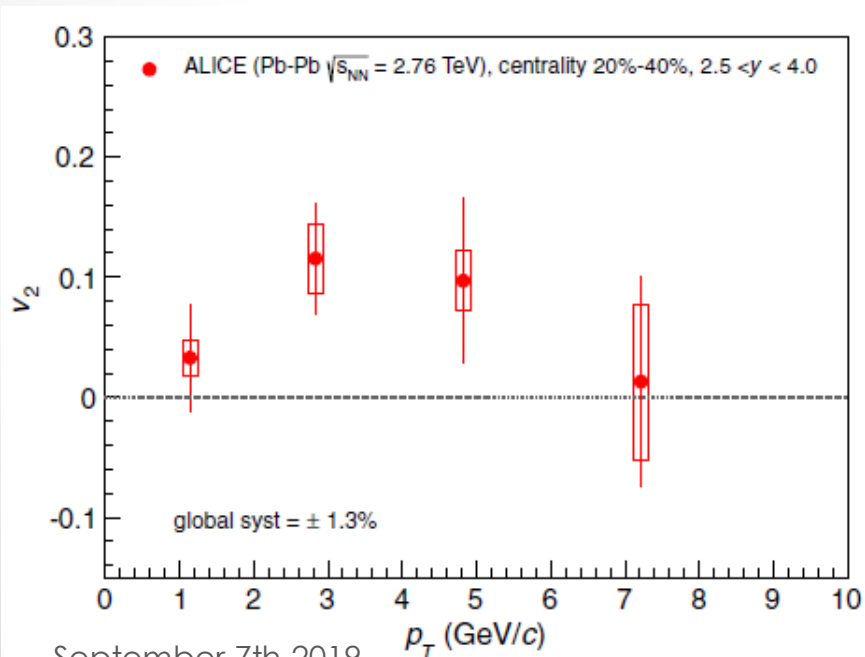
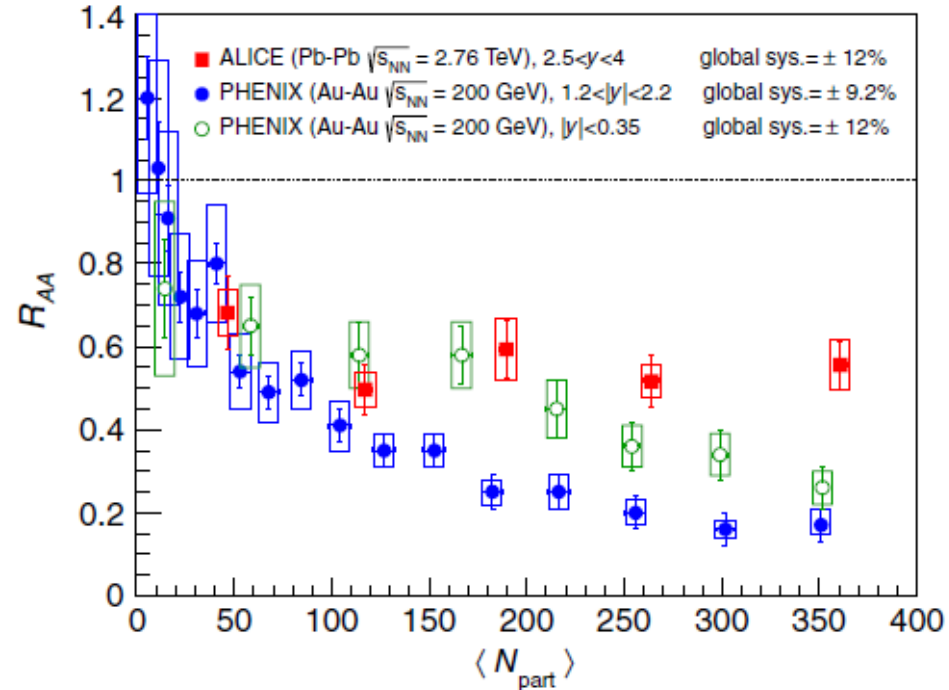
H. Satz, J. Phys. G.
32, R25 (2006)

$T \gg T_c$ ENuM joint workshop :
Lattice-QCD in Korea ● 9

– Regeneration of J/ψ mesons

1) The nuclear modification factor of J/ψ mesons

B. Abelev et al, (ALICE Collaboration),
Phys. Rev. Lett. **109**, 072301



2) Elliptic flow of the J/ψ

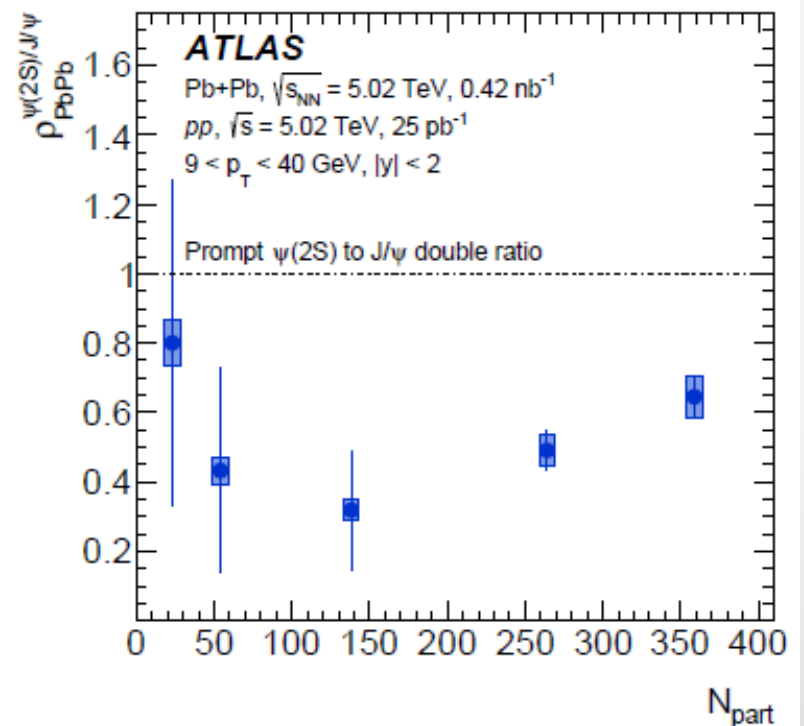
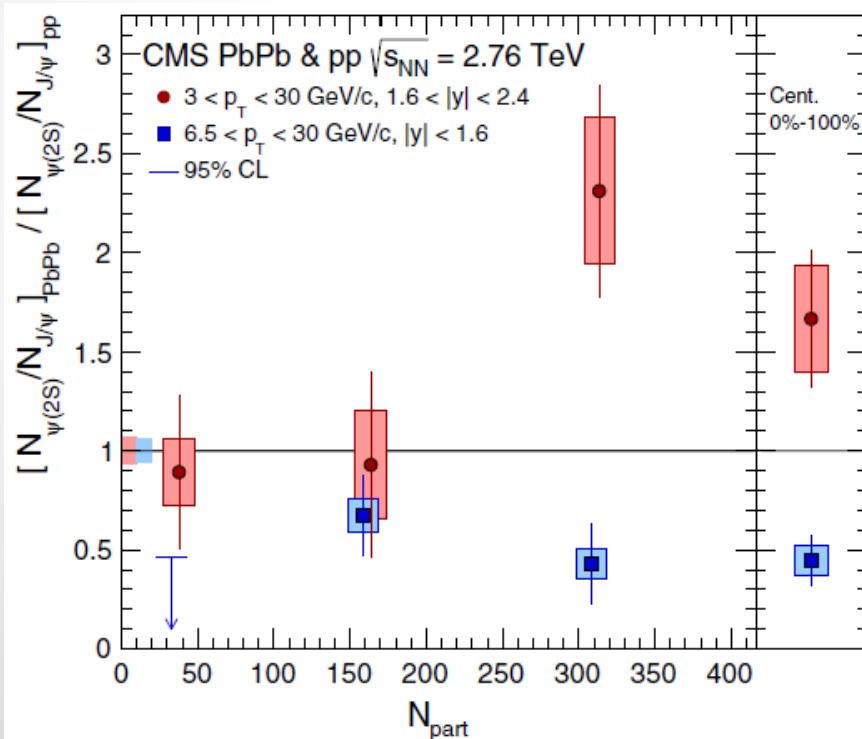
E. Abbas et al, Phys. Rev. Lett. **111**, 162301 (2013)

– Charmonium states in heavy ion collisions

1) The nuclear modification factor ratio between the J/ψ and the ψ'

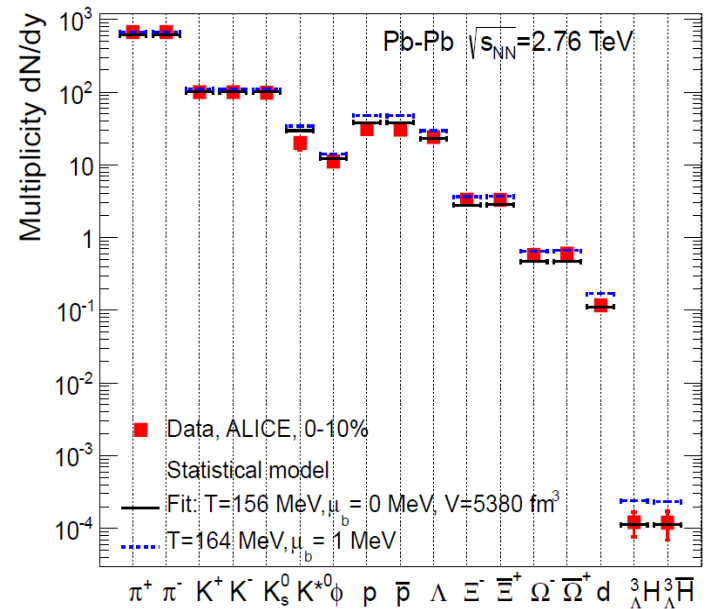
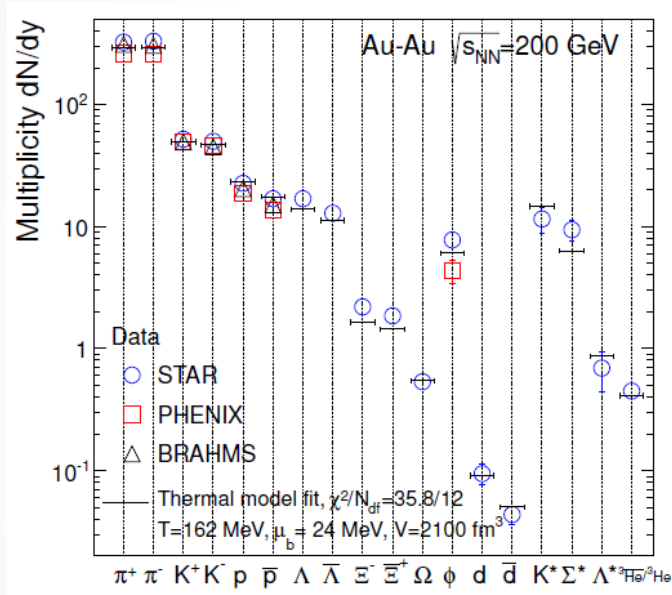
V. Khachatryan et al, Phys. Rev. Lett. **113**, 262301 (2014)

M. Aaboud et al, Eur. Phys. J. C **78**, 762 (2018)



– Multi-charm hadron production

1) Yields in statistical hadronization models



A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nucl. Phys. A **904-905**, 535c (2013)

J. Stachel, A. Andronic, P. Braun-Munzinger, and K. Redlich, J. Phys. Conf. Ser. **509**, 012019 (2014)

S. Cho *et al.* [ExHIC Collaboration], Prog. Part. Nucl. Phys. **95**, 279 (2017)

	RHIC		LHC	
	Stat.	Coal.	Stat.	Coal.
Ξ_{cc}	1.0×10^{-2}	1.3×10^{-3}	2.8×10^{-2}	4.9×10^{-3}
Ξ_{cc}^*	6.4×10^{-3}	9.0×10^{-4}	1.8×10^{-2}	3.3×10^{-3}
Ω_{scc}	2.8×10^{-3}	2.5×10^{-4}	8.0×10^{-3}	9.0×10^{-4}
Ω_{scc}^*	1.5×10^{-3}	1.6×10^{-4}	4.3×10^{-3}	6.0×10^{-4}

	RHIC		LHC	
	Stat.	Coal.	Stat.	Coal.
Ω_{ccc}	1.1×10^{-4}	1.1×10^{-6}	4.0×10^{-4}	5.3×10^{-6}
T_{cc}	8.9×10^{-4}	5.3×10^{-5}	2.7×10^{-3}	1.3×10^{-4}
X_2	5.7×10^{-4}	5.6×10^{-4}	1.7×10^{-3}	1.7×10^{-3}
X_4	5.7×10^{-4}	5.3×10^{-5}	1.7×10^{-3}	1.3×10^{-4}

Hadron production by quark coalescence



– Yields of hadrons in the coalescence model

V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003)

R. J. Freis, B. Muller, C. Nonaka, and S. Bass, Phys. Rev. C **68**, 044902 (2003)

$$N^{Coal} = g \int \left[\prod_{i=1}^n \frac{1}{g_i} \frac{p_i \cdot d\sigma_i}{(2\pi)^3} \frac{d^3 p_i}{E_i} f(x_i, p_i) \right] f^W(x_1, \dots, x_n : p_1, \dots, p_n)$$

1) The Wigner function, the coalescence probability function

$$\begin{aligned} f^W(x_1, \dots, x_n : p_1, \dots, p_n) \\ = \int \prod_{i=1}^n dy_i e^{p_i y_i} \psi^* \left(x_1 + \frac{y_1}{2}, \dots, x_n + \frac{y_n}{2} \right) \psi \left(x_1 - \frac{y_1}{2}, \dots, x_n - \frac{y_n}{2} \right) \end{aligned}$$

2) A Lorentz-invariant phase space integration of a space-like hyper-surface constraints the number of particles in the system

$$\int p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3 E_i} f(x_i, p_i) = N_i$$

– Hadron production by recombination

: Transverse momentum distributions of hadron yields

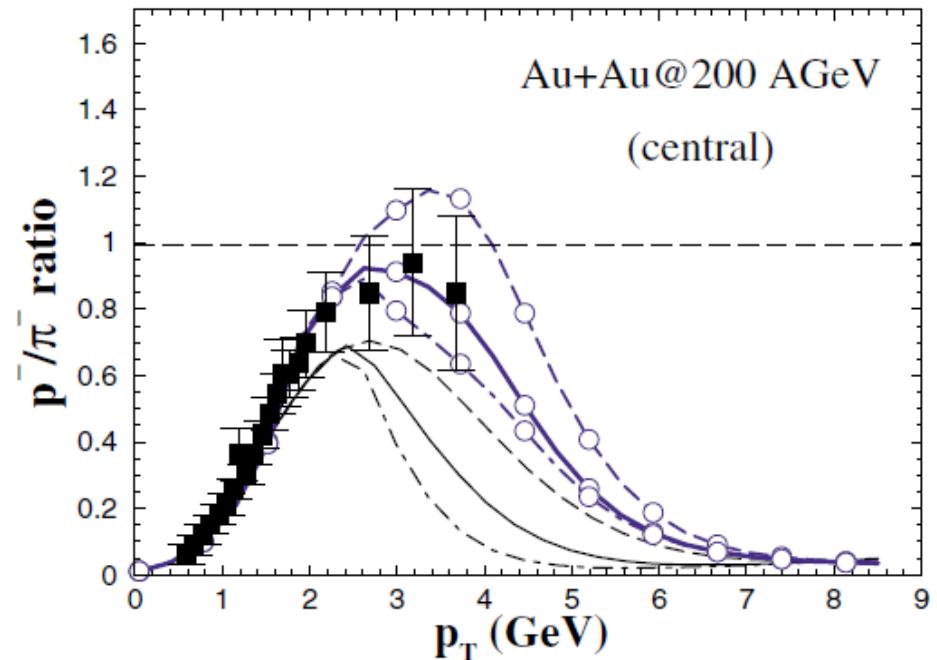
1) The puzzle in antiproton/pion ratio

V. Greco, C. M. Ko, and P. Levai, *Phys. Rev. Lett.* **90**, 202302 (2003)

R. J. Freis, B. Muller, C. Nonaka, and S. Bass, *Phys. Rev. Lett.* **90**, 202303 (2003)

originated from a competition between two particle production mechanisms

: A fragmentation dominates at large transverse momenta and a coalescence prevails at lower transverse momenta



2) The transverse momentum spectra

$$\frac{dN_M}{d^2\mathbf{p}_T} = g_M \frac{6\pi}{\tau\Delta y R_\perp^2 \Delta_p^3} \int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \left. \frac{dN_q}{d^2\mathbf{p}_{1T}} \right|_{|y_1| \leq \Delta y/2} \left. \frac{dN_{\bar{q}}}{d^2\mathbf{p}_{2T}} \right|_{|y_2| \leq \Delta y/2} \\ \times \delta^{(2)}(\mathbf{p}_T - \mathbf{p}_{1T} - \mathbf{p}_{2T}) \Theta\left(\Delta_p^2 - \frac{1}{4}(\mathbf{p}_{1T} - \mathbf{p}_{2T})^2 - \frac{1}{4}[(m_{1T} - m_{2T})^2 - (m_1 - m_2)^2]\right).$$

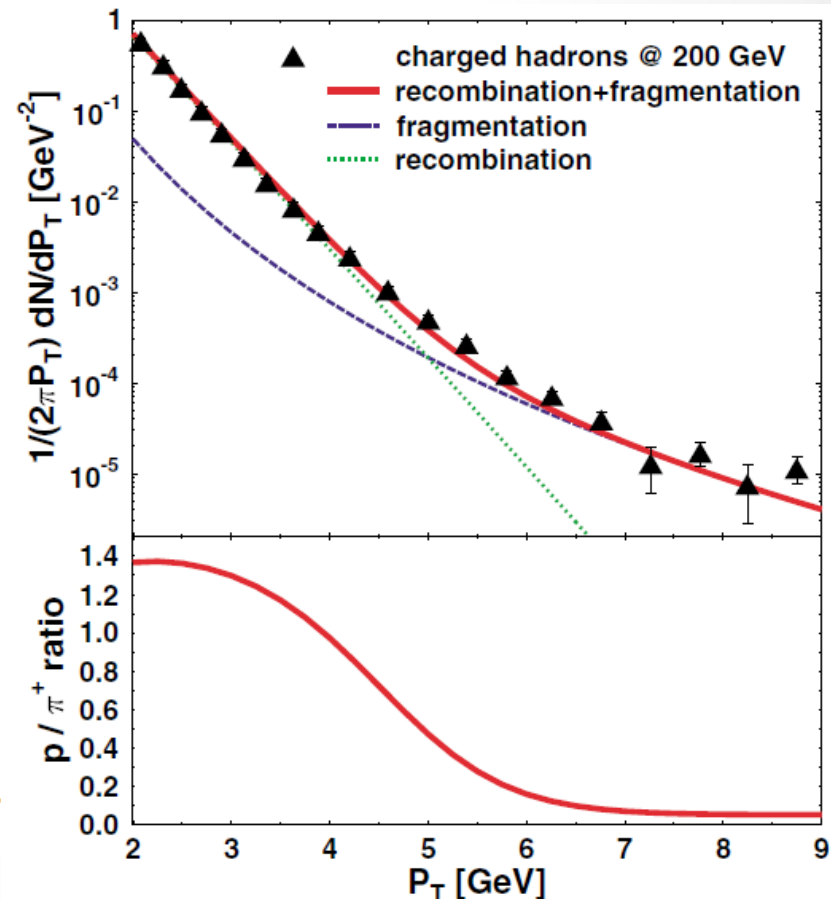
$$f_M(x_1, x_2; p_1, p_2) = \frac{9\pi}{2(\Delta_x \Delta_p)^3} \Theta(\Delta_x^2 - (x_1 - x_2)^2) \\ \times \Theta\left(\Delta_p^2 - \frac{1}{4}(p_1 - p_2)^2 + \frac{1}{4}(m_1 - m_2)^2\right).$$

and

$$E \frac{dN_M}{d^3P} = C_M \int_\Sigma \frac{d^3RP \cdot u(R)}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \\ \times w_a\left(R; \frac{\mathbf{P}}{2} - \mathbf{q}\right) \Phi_M^W(\mathbf{q}) w_b\left(R; \frac{\mathbf{P}}{2} + \mathbf{q}\right)$$

$$\Phi_M^W(\mathbf{q}) = \int d^3r \Phi_M^W(\mathbf{r}, \mathbf{q})$$

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \varphi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$



Transverse momentum distributions of charmed hadrons

– Charmonia production by recombination

S. Cho, Phys. Rev. C **91**, 054914 (2015)

1) Coalescence production of charmonium states

$$N_\psi = g_\psi \int p_c \cdot d\sigma_c p_{\bar{c}} \cdot d\sigma_{\bar{c}} \frac{d^3\vec{p}_c}{(2\pi)^3 E_c} \frac{d^3\vec{p}_{\bar{c}}}{(2\pi)^3 E_{\bar{c}}} f_c(r_c, p_c) f_{\bar{c}}(r_{\bar{c}}, p_{\bar{c}}) W_\psi(r_c, r_{\bar{c}}; p_c, p_{\bar{c}}),$$

The transverse momentum distribution of the charmonium yield

$$\frac{dN_\psi}{d^2\vec{p}_T} = \frac{g_\psi}{V} \int d^3\vec{r} d^2\vec{p}_{cT} d^2\vec{p}_{\bar{c}T} \delta^{(2)}(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d^2\vec{p}_{cT}} \frac{dN_{\bar{c}}}{d^2\vec{p}_{\bar{c}T}} W_\psi(\vec{r}, \vec{k})$$

$$W_s(\vec{r}, \vec{k}) = 8e^{-\frac{r^2}{\sigma^2} - k^2\sigma^2}$$

$$W_p(\vec{r}, \vec{k}) = \left(\frac{16}{3} \frac{r^2}{\sigma^2} - 8 + \frac{16}{3} \sigma^2 k^2 \right) e^{-\frac{r^2}{\sigma^2} - k^2\sigma^2}$$

$$W_{\psi_{10}}(\vec{r}, \vec{k}) = \frac{16}{3} \left(\frac{r^4}{\sigma^4} - 2\frac{r^2}{\sigma^2} + \frac{3}{2} - 2\sigma^2 k^2 + \sigma^4 k^4 - 2r^2 k^2 + 4(\vec{r} \cdot \vec{k})^2 \right) e^{-\frac{r^2}{\sigma^2} - k^2\sigma^2}.$$

2) Integration of the Wigner function over the spatial coordinates

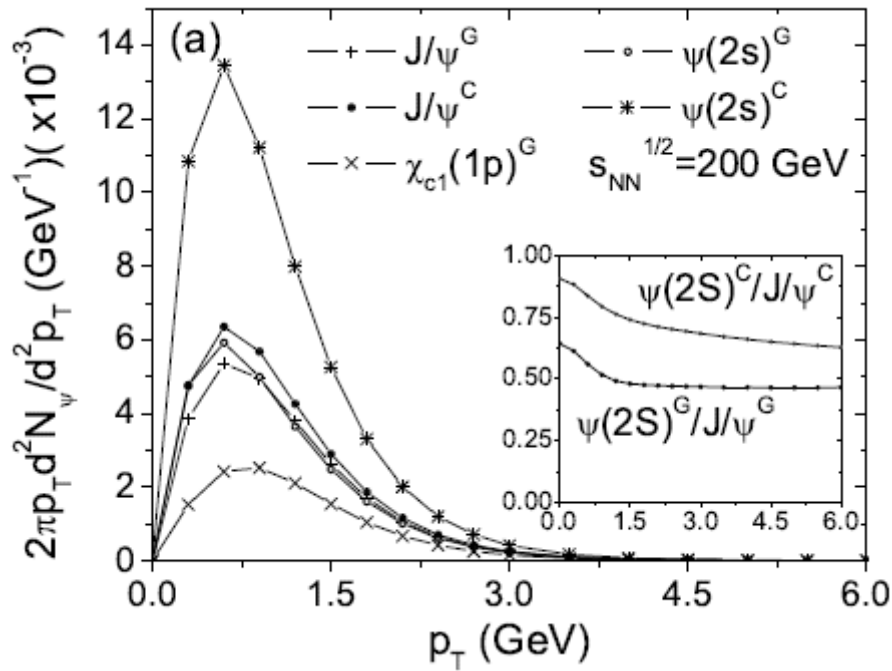
$$\int d^3\vec{r} W_\psi(\vec{r}, \vec{k}) = \begin{cases} (2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} & \psi_s^G; J/\psi \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} \sigma^2 k^2 & \psi_p^G; \chi_c \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} \left(\sigma^2 k^2 - \frac{3}{2}\right)^2 & \psi_{10}^G; \psi(2S) \\ 64\pi \frac{a_0^3}{(a_0^2 k^2 + 1)^4} & \psi_{1S}^C; J/\psi \\ 8\pi a_0^3 \frac{(a_0^2 k^2 - 1/4)^2}{(a_0^2 k^2 + 1/4)^6} & \psi_{2S}^C; \psi(2S) \end{cases}$$

$$\int d^3\vec{r} W(\vec{r}, \vec{k}) = |\tilde{\psi}(\vec{k})|^2$$

M. Hillery, R. F. O'Connell, M. O. Scully and E. P. Wigner, Phys. Rept. **106**, 121 (1984)

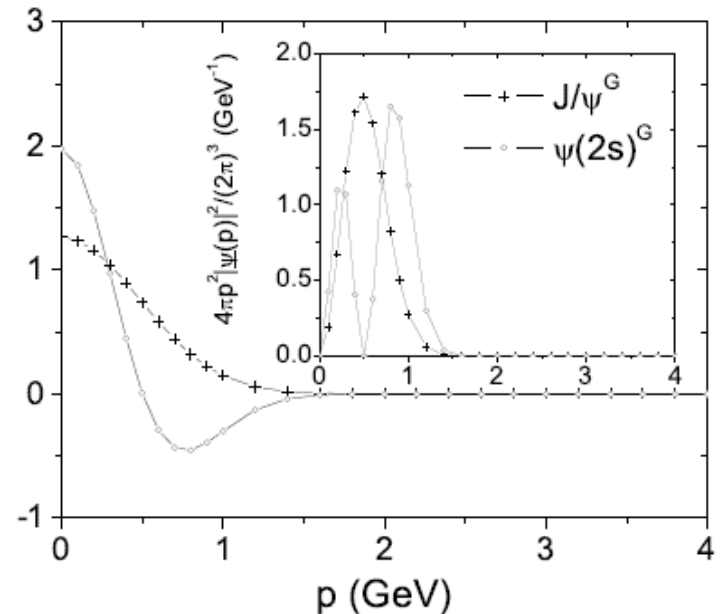
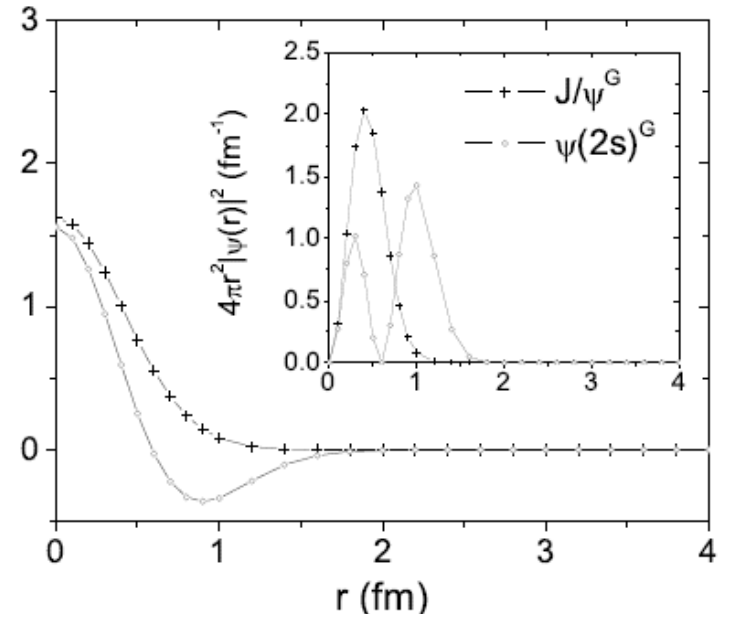
$$\frac{dN_\psi}{d\vec{p}_T} = \frac{g_\psi}{V} \int d\vec{p}_{cT} d\vec{p}_{\bar{c}T} \delta(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d\vec{p}_{cT}} \frac{dN_{\bar{c}}}{d\vec{p}_{\bar{c}T}} |\tilde{\psi}(\vec{k})|^2$$

3) Transverse momentum distributions of charmonium states



$\psi(r)$ ($\text{fm}^{-3/2}$)

$\underline{\psi}(p)/(2\pi)^{3/2}$ ($\text{GeV}^{-3/2}$)



– Production of multi-charm hadrons by recombination

1) Coalescence production of multi-charm hadrons

$$N_{\Xi_{cc}} = g_{\Xi_{cc}} \int p_l \cdot d\sigma_l p_{c_1} \cdot d\sigma_{c_1} p_{c_2} \cdot d\sigma_{c_2} \frac{d^3 \vec{p}_l}{(2\pi)^3 E_l} \frac{d^3 \vec{p}_{c_1}}{(2\pi)^3 E_{c_1}} \frac{d^3 \vec{p}_{c_2}}{(2\pi)^3 E_{c_2}} f_l(r_l, p_l) f_{c_1}(r_{c_1}, p_{c_1}) \\ \times f_{c_2}(r_{c_2}, p_{c_2}) W_{\Xi_{cc}}(r_l, r_{c_1}, r_{c_2}; p_l, p_{c_1}, p_{c_2})$$

$$N_X = g_X \int p_l \cdot d\sigma_l p_{\bar{l}} \cdot d\sigma_{\bar{l}} p_c \cdot d\sigma_c p_{\bar{c}} \cdot d\sigma_{\bar{c}} \frac{d^3 \vec{p}_l}{(2\pi)^3 E_l} \frac{d^3 \vec{p}_{\bar{l}}}{(2\pi)^3 E_{\bar{l}}} \frac{d^3 \vec{p}_c}{(2\pi)^3 E_c} \frac{d^3 \vec{p}_{\bar{c}}}{(2\pi)^3 E_{\bar{c}}} \\ \times f_l(r_l, p_l) f_{\bar{l}}(r_{\bar{l}}, p_{\bar{l}}) f_c(r_c, p_c) f_{\bar{c}}(r_{\bar{c}}, p_{\bar{c}}) W_X(r_l, r_{\bar{l}}, r_c, r_{\bar{c}}; p_l, p_{\bar{l}}, p_c, p_{\bar{c}})$$

2) The transverse momentum distributions

$$\frac{d^2 N_{\Xi_{cc}}}{d^2 \vec{p}_T} = \frac{g_{\Xi_{cc}}}{V^2} \int d^3 \vec{r}_1 d^3 \vec{r}_2 d^2 \vec{p}_{lT} d^2 \vec{p}_{c_1T} d^2 \vec{p}_{c_2T} \delta^{(2)}(\vec{p}_T - \vec{p}_{lT} - \vec{p}_{c_1T} - \vec{p}_{c_2T}) \frac{d^2 N_l}{d^2 \vec{p}_{lT}} \\ \times \frac{d^2 N_{c_1}}{d^2 \vec{p}_{c_1T}} \frac{d^2 N_{c_2}}{d^2 \vec{p}_{c_2T}} W_{\Xi_{cc}}(\vec{r}'_1, \vec{r}'_2, \vec{r}'_3, \vec{k}_1, \vec{k}_2, \vec{k}_3),$$

$$\frac{d^2 N_X}{d^2 \vec{p}_T} = \frac{g_X}{V^3} \int d^3 \vec{r}_1 d^3 \vec{r}_2 d^3 \vec{r}_3 d^2 \vec{p}_{lT} d^2 \vec{p}_{\bar{l}T} d^2 \vec{p}_{cT} d^2 \vec{p}_{\bar{c}T} \delta^{(2)}(\vec{p}_T - \vec{p}_{lT} - \vec{p}_{\bar{l}T} - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{d^2 N_l}{d^2 \vec{p}_{lT}} \frac{d^2 N_{\bar{l}}}{d^2 \vec{p}_{\bar{l}T}} \\ \times \frac{d^2 N_c}{d^2 \vec{p}_{cT}} \frac{d^2 N_{\bar{c}}}{d^2 \vec{p}_{\bar{c}T}} W_X(\vec{r}'_1, \vec{r}'_2, \vec{r}'_3, \vec{k}_1, \vec{k}_2, \vec{k}_3)$$

3) Transverse momentum distributions of charm and light quarks

$$\frac{dN_c}{d^2p_T} = \begin{cases} a_0 \exp[-a_1 p_T^{a_2}] & p_T \leq p_0 \\ a_0 \exp[-a_1 p_T^{a_2}] + a_3(1 + p_T^{a_4})^{-a_5} & p_T \geq p_0 \end{cases}$$

$$\frac{d^2N_l}{d^2p_T} = g_l \frac{V}{(2\pi)^3} m_T e^{-m_T/T_{eff}},$$

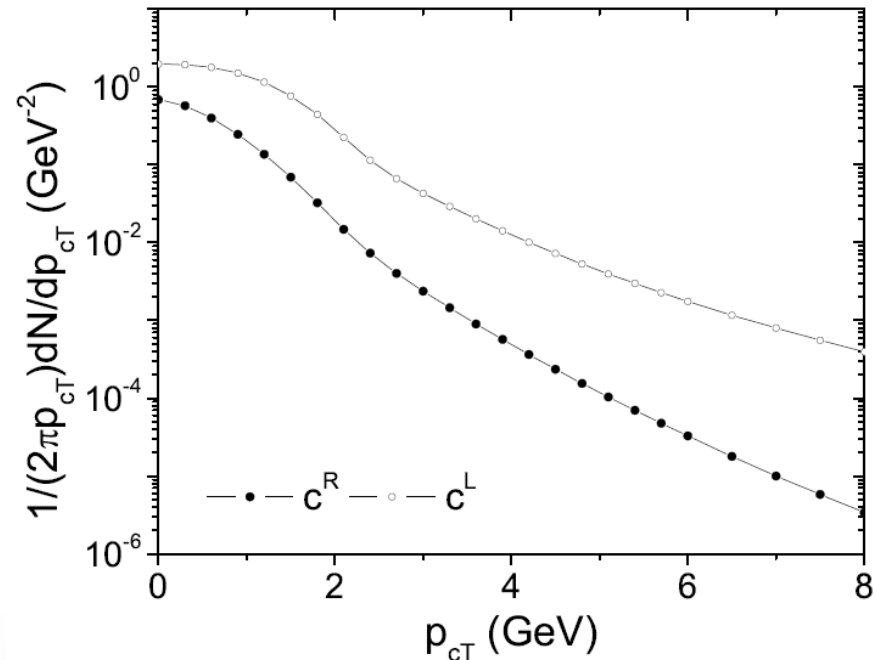
	a_0	a_1	a_2	a_3	a_4	a_5
RHIC						
$p_T \leq p_0$	0.69	1.22	1.57			
$p_T \geq p_0$	1.08	3.04	0.71	3.79	2.02	3.48
LHC						
$p_T \leq p_0$	1.97	0.35	2.47			
$p_T \geq p_0$	7.95	3.49	3.59	87335	0.5	14.31

S. Plumari, V. Minissale, S. K. Das, G. Coci and V. Greco, Eur. Phys. J. C **78**:348 (2017)

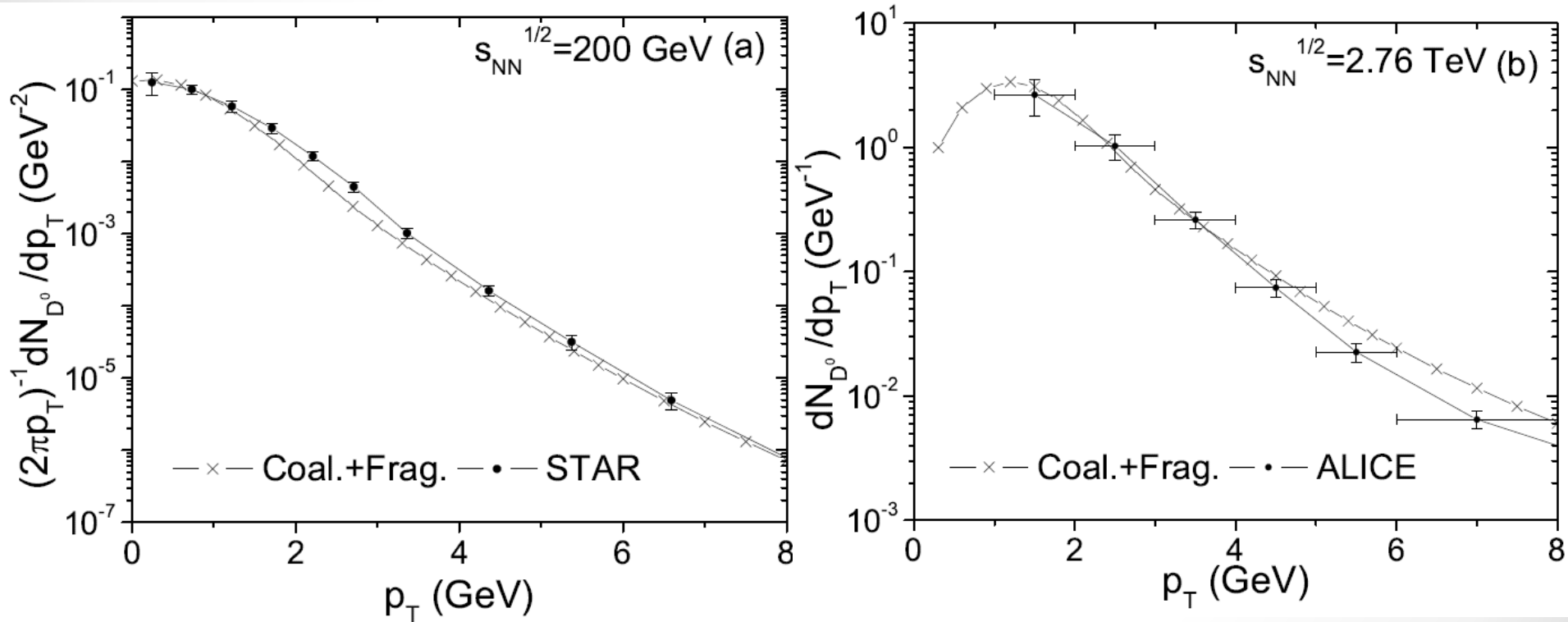
Y. Oh, C. M. Ko, S.-H. Lee, and S. Yasui, Phys. Rev. C **79** 044905 (2009)

S. Cho *et al.* (EXHIC Collaboration), Prog. Part. Nucl. Phys. **95**, 279 (2017)

	RHIC		LHC (2.76 TeV)	
	Sc. 1	Sc. 2	Sc. 1	Sc. 2
T_H (MeV)		162		156
V_H (fm ³)		2100		5380
μ_B (MeV)		24		0
μ_s (MeV)		10		0
γ_c				39
γ_b		4.0×10^7		8.6×10^8
T_C (MeV)	162	166	156	166
V_C (fm ³)	2100	1791	5380	3533
$N_u = N_d$	320	302	700	593
$N_s = N_{\bar{s}}$	183	176	386	347
$N_c = N_{\bar{c}}$		4.1		11
$N_b = N_{\bar{b}}$		0.03		0.44



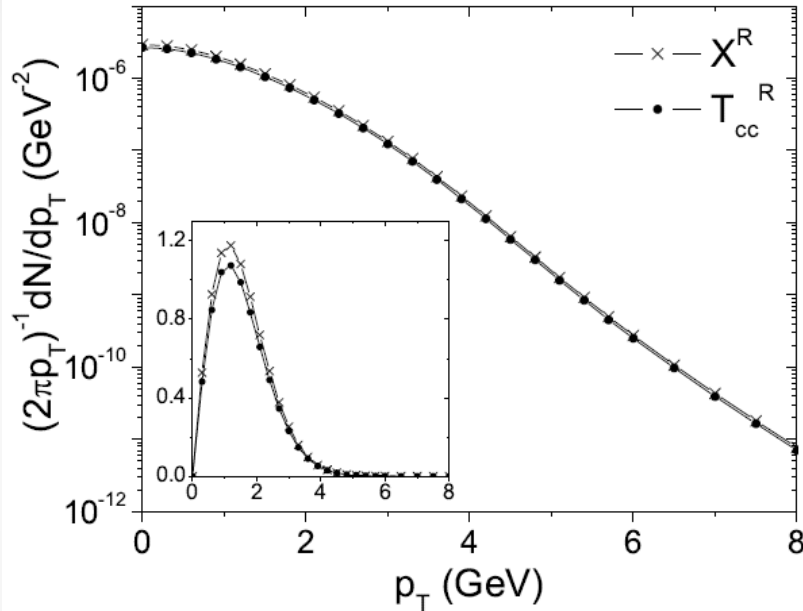
4) Transverse momentum distributions of D^0 mesons



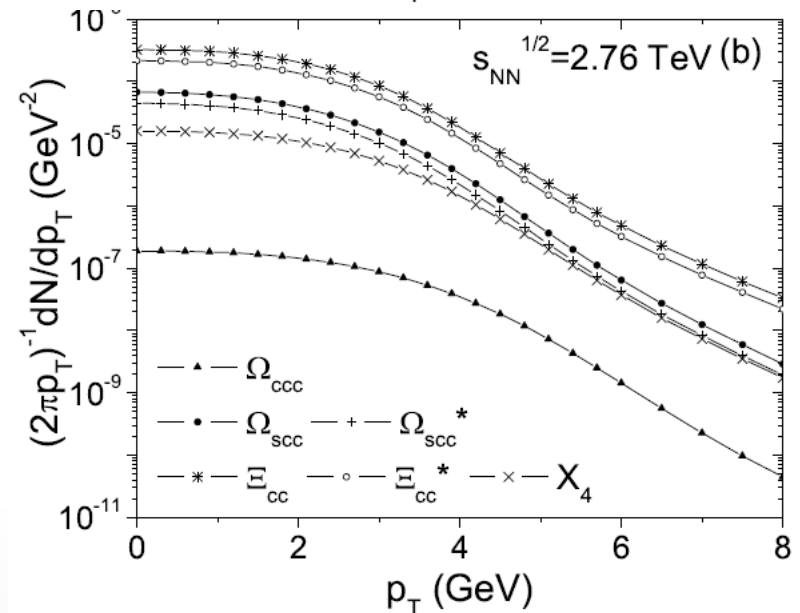
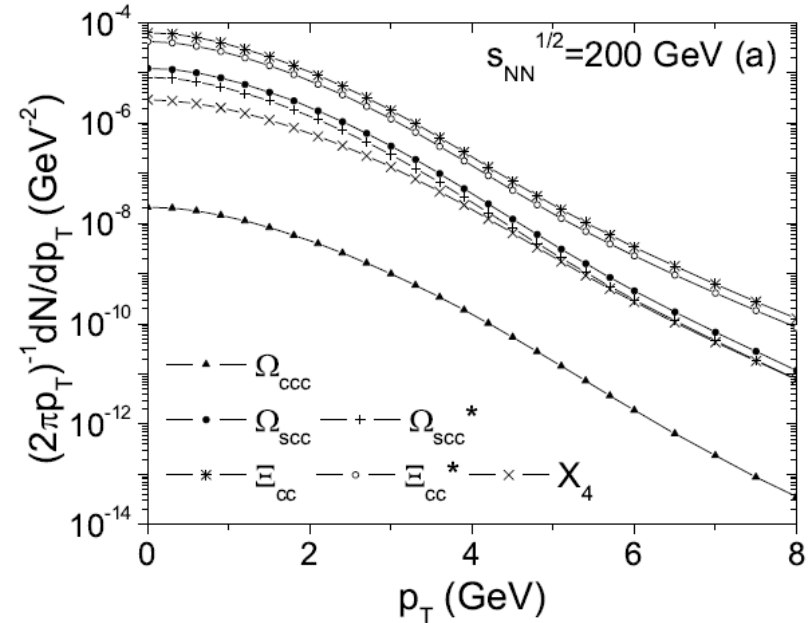
J. Adam et al. [STAR Collaboration], Phys. Rev. C **99**, no. 3, 034908 (2019).

J. Adam et al. [ALICE Collaboration], JHEP **1603**, 081 (2016).

5) Transverse momentum distributions of multi-charm hadrons

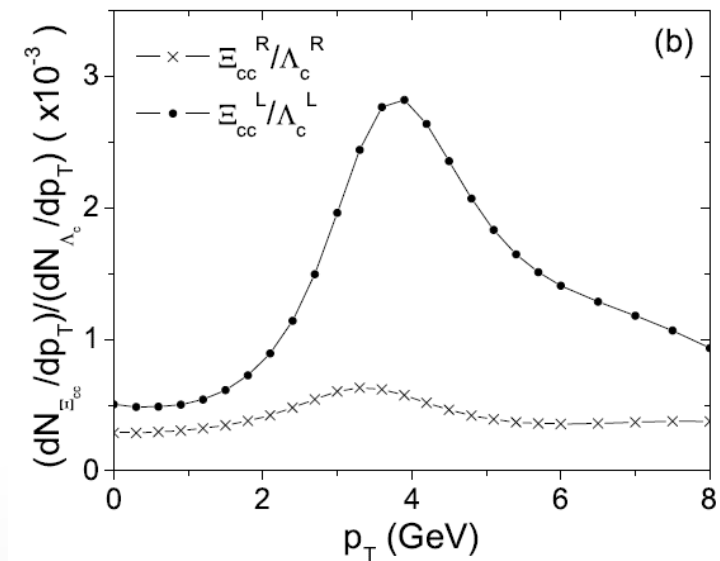
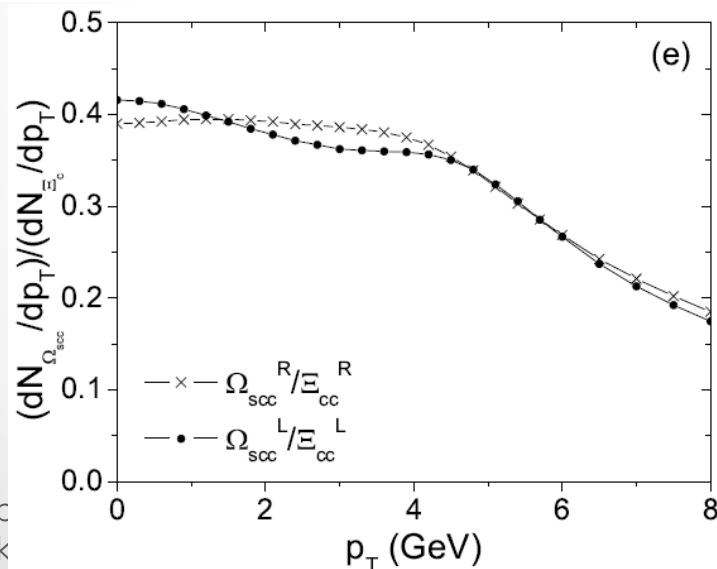
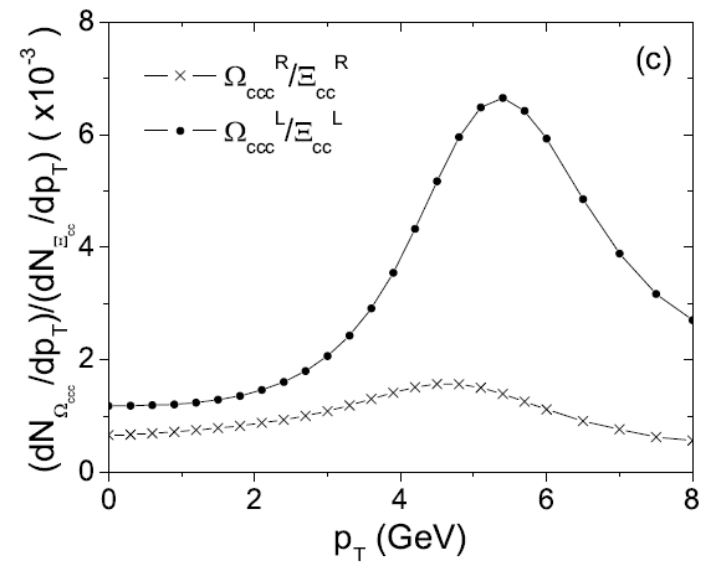
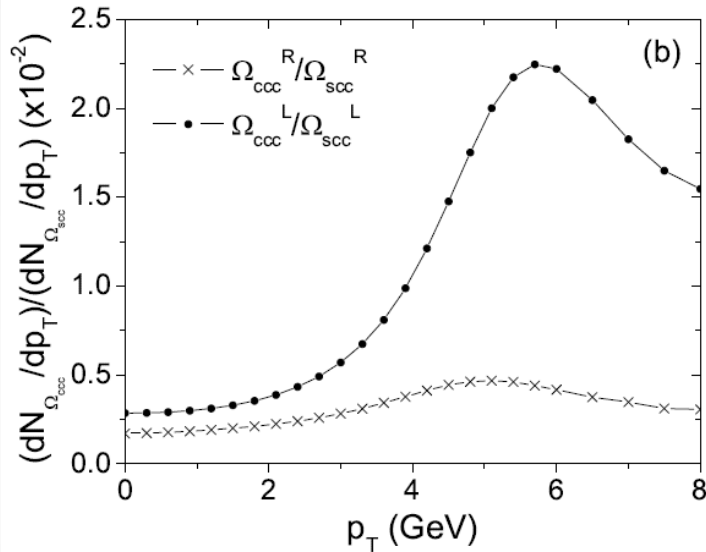


	RHIC	LHC
Ξ_{cc}	4.4×10^{-4}	6.7×10^{-3}
Ξ_{cc}^*	2.9×10^{-4}	4.5×10^{-3}
Ω_{scc}	8.6×10^{-5}	1.3×10^{-3}
Ω_{scc}^*	5.7×10^{-5}	8.5×10^{-4}
Ω_{ccc}	1.7×10^{-7}	5.9×10^{-6}
T_{cc}	2.2×10^{-5}	3.8×10^{-4}
X_4	2.4×10^{-5}	3.8×10^{-4}
X_2	2.6×10^{-4}	4.5×10^{-3}

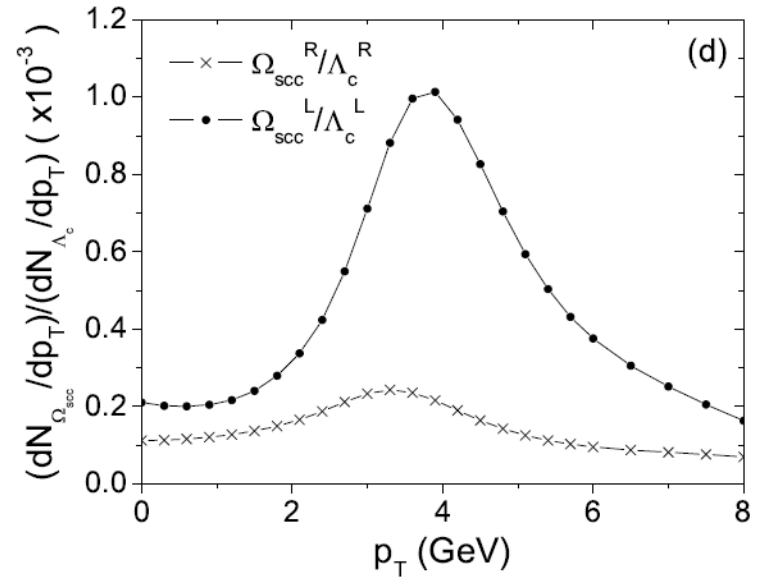
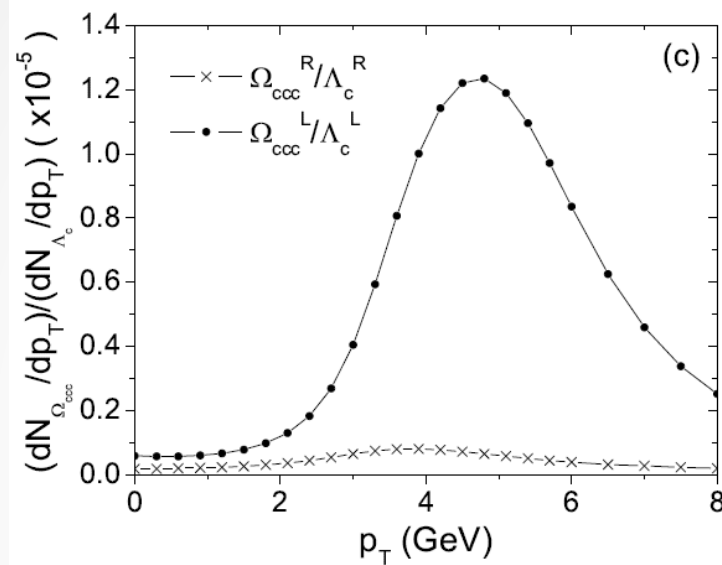


6) Transverse momentum distribution ratios

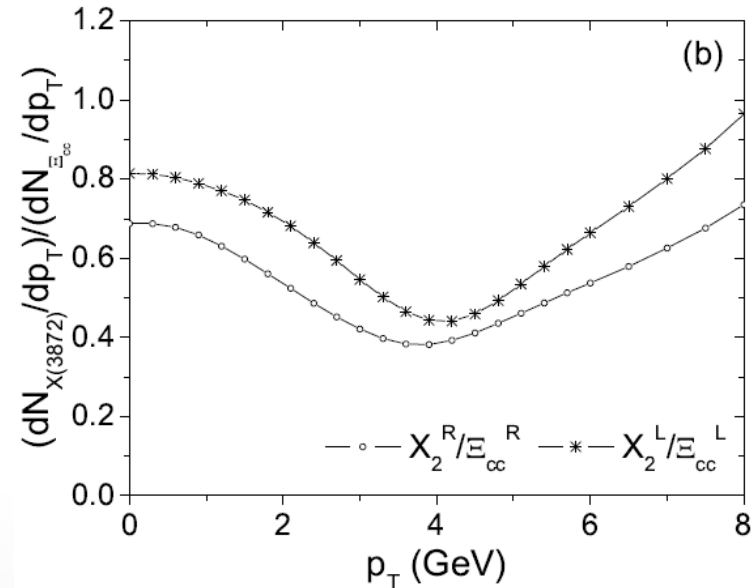
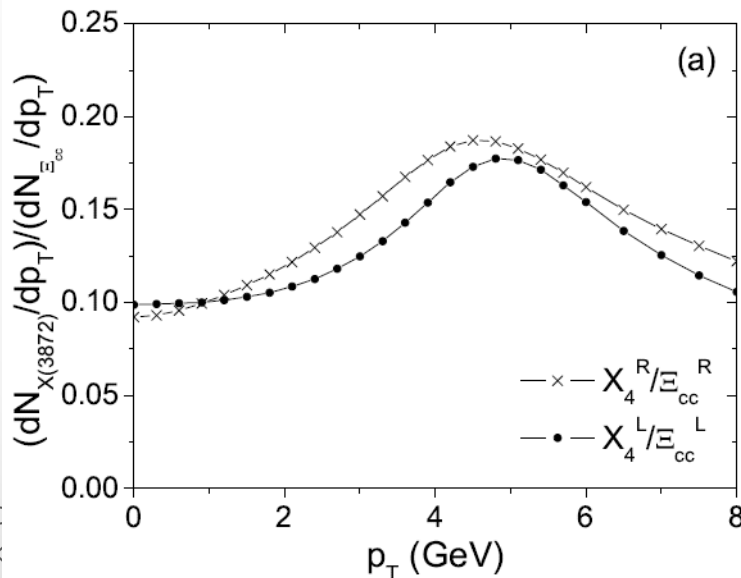
a) Baryon/baryon (ccc/ccq, ccc/ccs, ccq/ccs, and ccq/ccq)

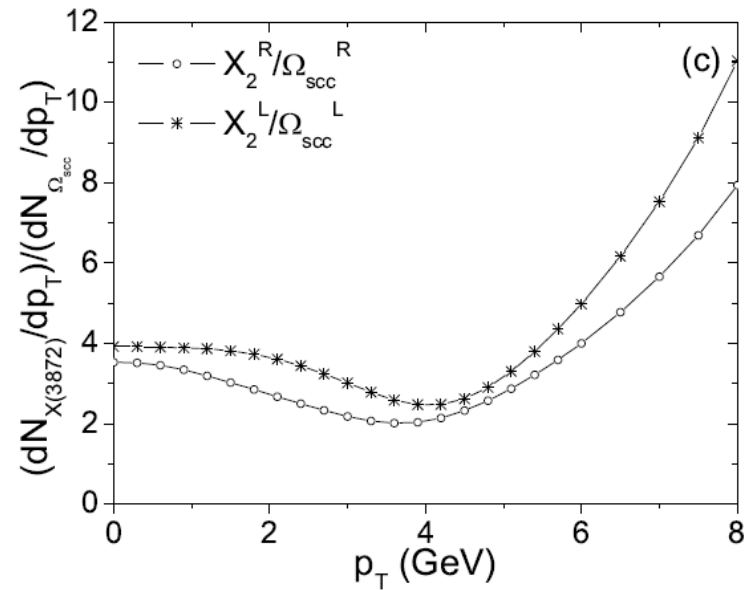
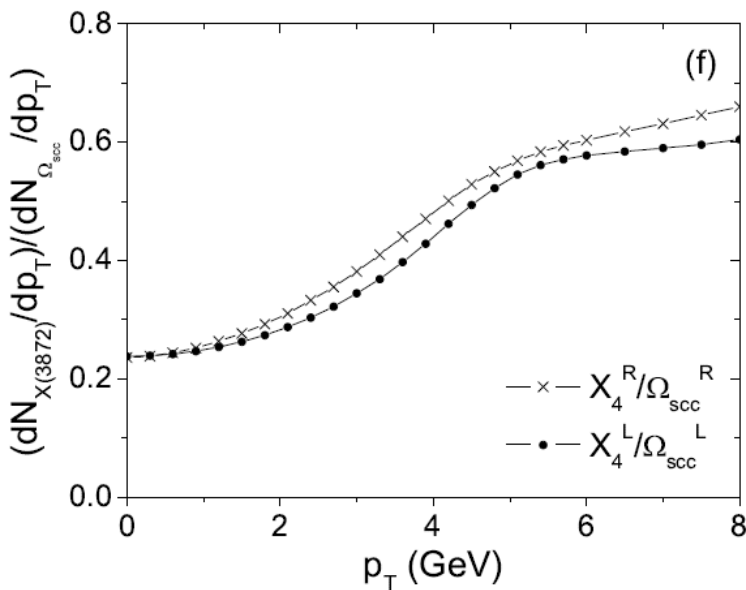
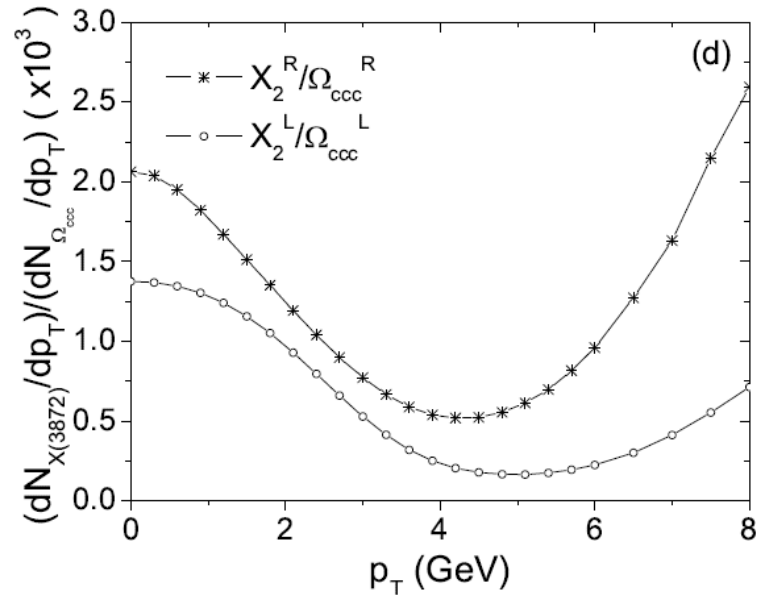
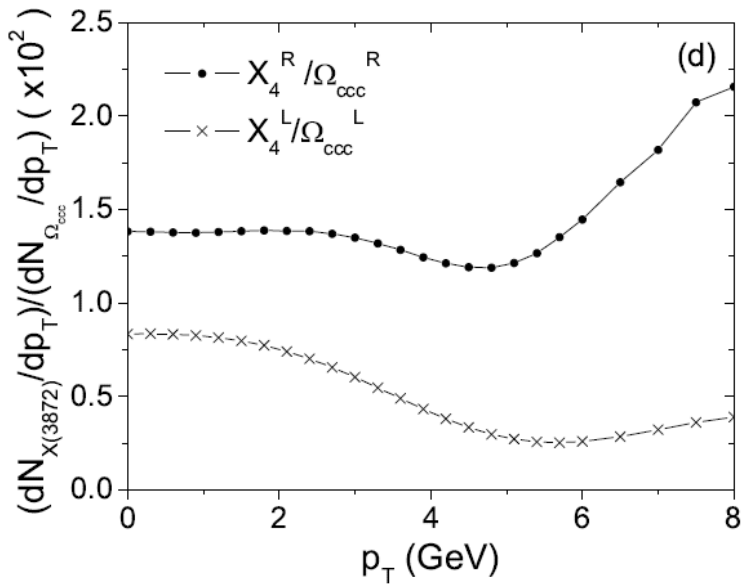


b) Baryon/baryon (ccc/cqq, or ccs/cqq)



c) Meson/baryon (ccqq/ccq, or cc/ccq)



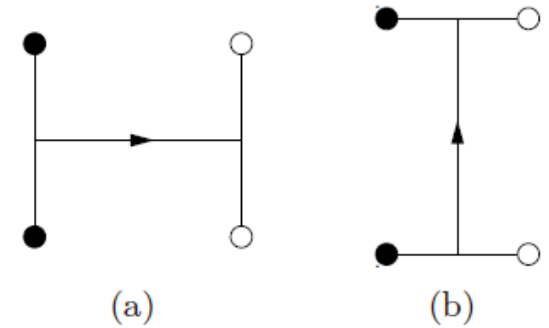


- Dependence of the transverse momentum distribution on internal relative coordinates

1) The Wigner function of the (3872) meson

$$W_X(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{k}_1, \vec{k}_2, \vec{k}_3)$$

$$= 8^3 \exp\left(-\frac{r_1^2}{\sigma_1^2} - \sigma_1^2 k_1^2\right) \exp\left(-\frac{r_2^2}{\sigma_2^2} - \sigma_2^2 k_2^2\right) \exp\left(-\frac{r_3^2}{\sigma_3^2} - \sigma_3^2 k_3^2\right)$$



$$\vec{k} = \vec{p}_{lT} + \vec{p}_{\bar{l}T} + \vec{p}_{cT} + \vec{p}_{\bar{c}T},$$

$$\vec{k}_1 = \frac{m_{\bar{l}}\vec{p}_{lT} - m_l\vec{p}_{\bar{l}T}}{m_l + m_{\bar{l}}},$$

$$\vec{k}_2 = \frac{m_c(\vec{p}_{lT} + \vec{p}_{\bar{l}T}) - (m_l + m_{\bar{l}})\vec{p}_{cT}}{m_l + m_{\bar{l}} + m_c},$$

$$\vec{k}_3 = \frac{m_{\bar{c}}(\vec{p}_{lT} + \vec{p}_{\bar{l}T} + \vec{p}_{cT}) - (m_l + m_{\bar{l}} + m_c)\vec{p}_{\bar{c}T}}{m_l + m_{\bar{l}} + m_c + m_{\bar{c}}}$$

$$\mu_1 = \frac{m_l m_{\bar{l}}}{m_l + m_{\bar{l}}}, \quad \mu_2 = \frac{(m_l + m_{\bar{l}})m_c}{m_l + m_{\bar{l}} + m_c},$$

$$\mu_3 = \frac{(m_l + m_{\bar{l}} + m_c)}{m_l + m_{\bar{l}} + m_c + m_{\bar{c}}},$$

$$\vec{k} = \vec{p}_{lT} + \vec{p}_{\bar{l}T} + \vec{p}_{cT} + \vec{p}_{\bar{c}T},$$

$$\vec{k}_1 = \frac{m_{\bar{l}}\vec{p}_{lT} - m_l\vec{p}_{\bar{l}T}}{m_l + m_{\bar{l}}},$$

$$\vec{k}_2 = \frac{m_{\bar{c}}\vec{p}_{cT} - m_c\vec{p}_{\bar{c}T}}{m_c + m_{\bar{c}}},$$

$$\vec{k}_3 = \frac{(m_c + m_{\bar{c}})(\vec{p}_{lT} + \vec{p}_{\bar{l}T}) - (m_l + m_{\bar{l}})(\vec{p}_{cT} + \vec{p}_{\bar{c}T})}{m_l + m_{\bar{l}} + m_c + m_{\bar{c}}},$$

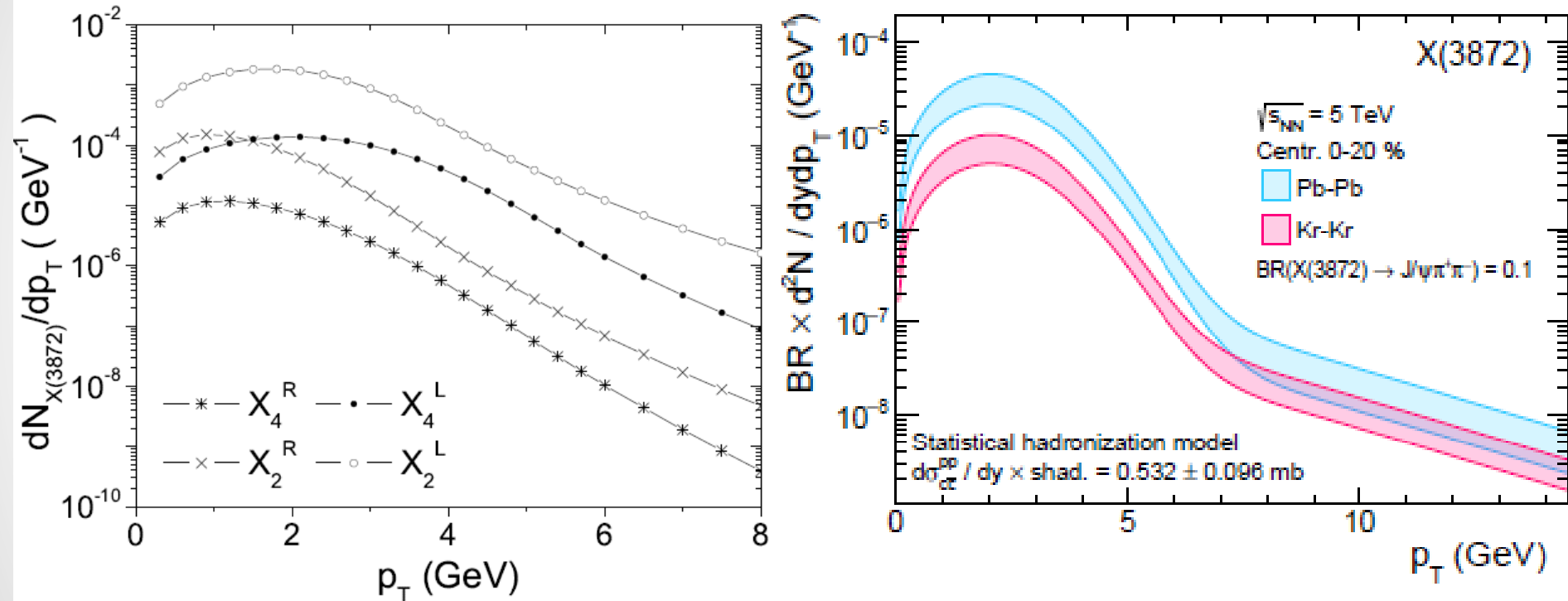
$$\mu_1 = \frac{m_l m_{\bar{l}}}{m_l + m_{\bar{l}}}, \quad \mu_2 = \frac{m_c m_{\bar{c}}}{m_c + m_{\bar{c}}},$$

$$\mu_3 = \frac{(m_l + m_{\bar{l}})(m_c + m_{\bar{c}})}{m_l + m_{\bar{l}} + m_c + m_{\bar{c}}}.$$

$$-\sigma_1^2 k_1^2 - \sigma_2^2 k_2^2 - \sigma_3^2 k_3^2 = -\frac{1}{\omega} \frac{1}{m_l + m_{\bar{l}} + m_c + m_{\bar{c}}} \left(\frac{m_{\bar{l}} + m_c + m_{\bar{c}}}{m_l} p_l'^2 + \frac{m_c + m_{\bar{c}} + m_l}{m_{\bar{l}}} p_{\bar{l}}'^2 + \frac{m_{\bar{c}} + m_l + m_{\bar{l}}}{m_c} p_c'^2 + \frac{m_l + m_{\bar{l}} + m_c}{m_{\bar{c}}} p_{\bar{c}}'^2 \right.$$

$$\left. -2(\vec{p}_l \cdot \vec{p}_{\bar{l}} + \vec{p}_{\bar{l}} \cdot \vec{p}_c + \vec{p}_c \cdot \vec{p}_{\bar{c}} + \vec{p}_{\bar{c}} \cdot \vec{p}_l + \vec{p}_l \cdot \vec{p}_c + \vec{p}_c \cdot \vec{p}_{\bar{l}} + \vec{p}_{\bar{l}} \cdot \vec{p}_{\bar{c}}) \right)$$

– Comparison with results from statistical hadronization model



A. Andronic, P. Braun-Munzinger, M. K. Kohler, K. Redlich and J. Stachel, arXiv:1901.09200

Conclusion

– Production of multi-charmed hadrons in heavy ion collisions

- 1) Heavy ion collision experiments provide better chances to study production of multi-charm hadrons as well as exotic hadrons
- 2) Transverse momentum distributions of charmonium states are affected by their intrinsic wave function distributions
- 3) Transverse momentum distribution ratios between multi-charm hadrons and $X(3872)$ mesons, or other combinations between heavy quark hadrons reflect the distribution of momentum among constituent quarks : the momentum of the heavy quark hadron is mostly carried by the heavy quark
- 4) The transverse momentum distribution is also dependent on the internal structure of the hadron



Thank you for your attention!