Mesoscopic physics society
2019 Summer school

Slides courtesy : 정제성, 김기호

Quantum control, decoherence, and quantum error correction

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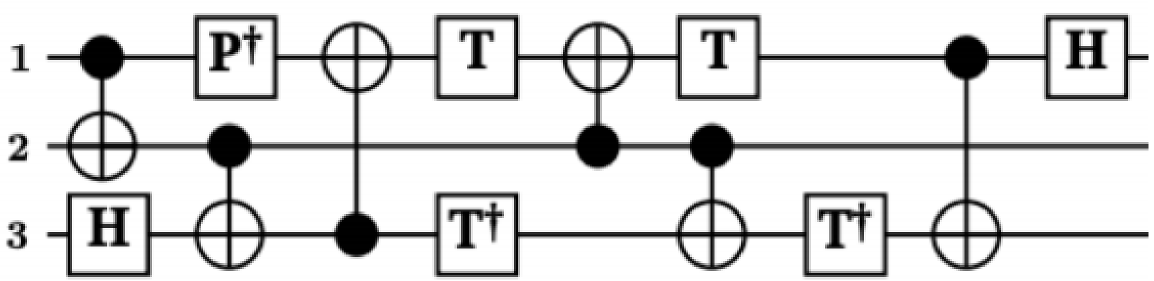
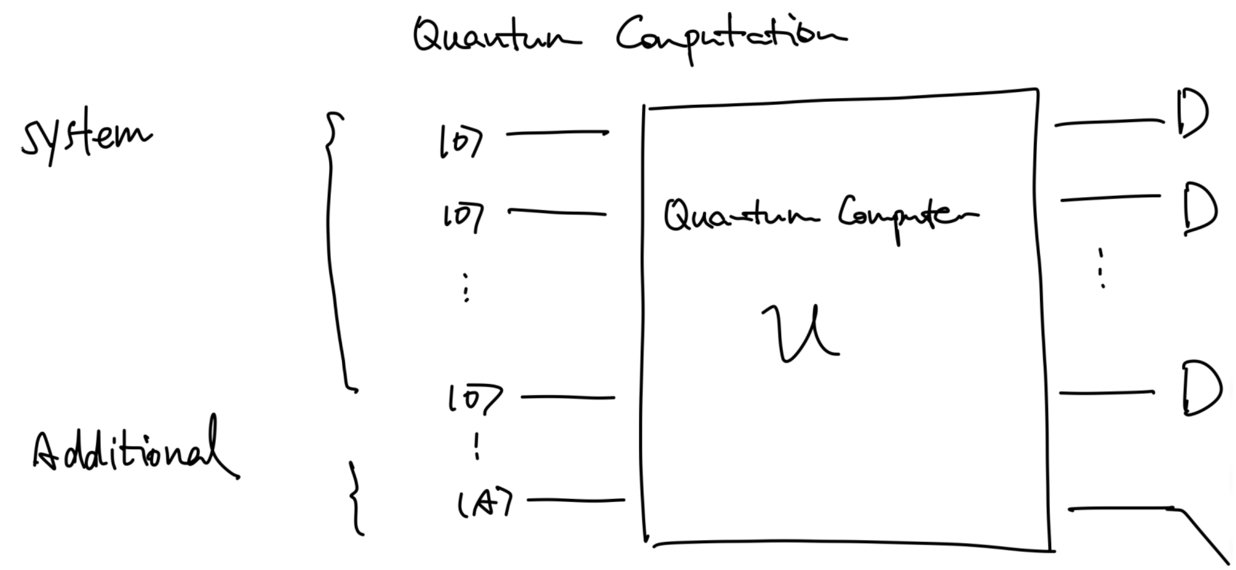
Aim of this lecture

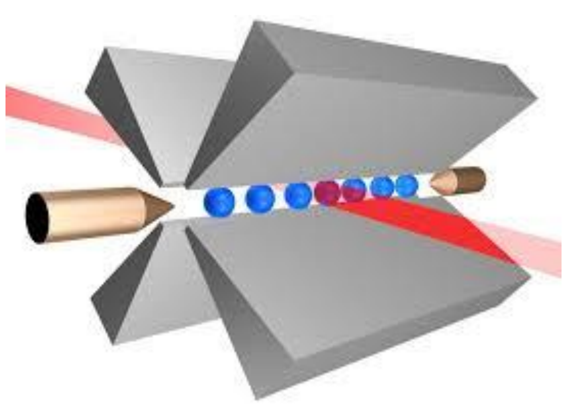
전반부

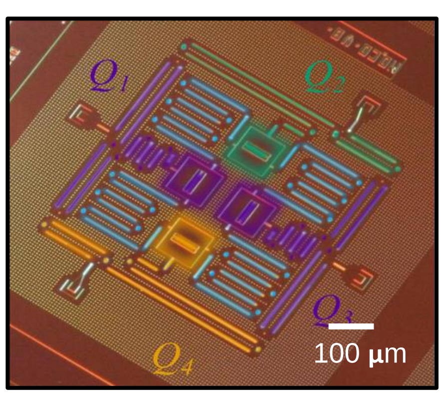


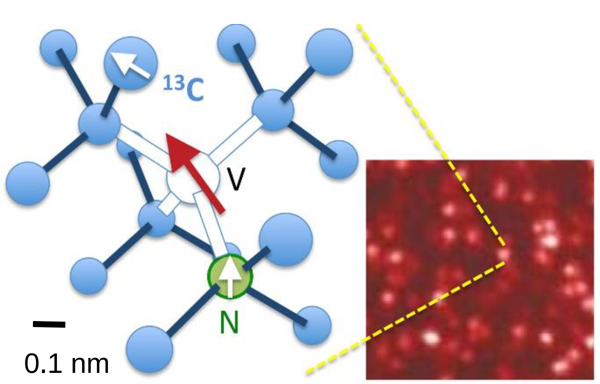


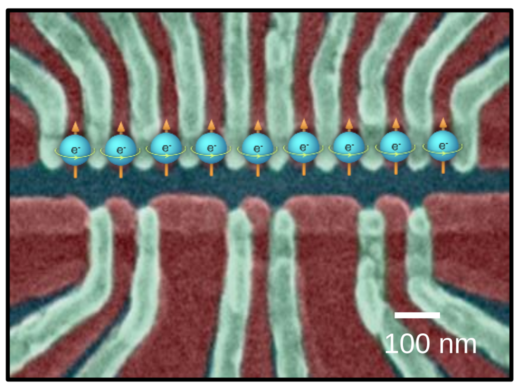
본 강의는, bridging the two part

후반부









Outline

Introduction

Quantum control and decoherence

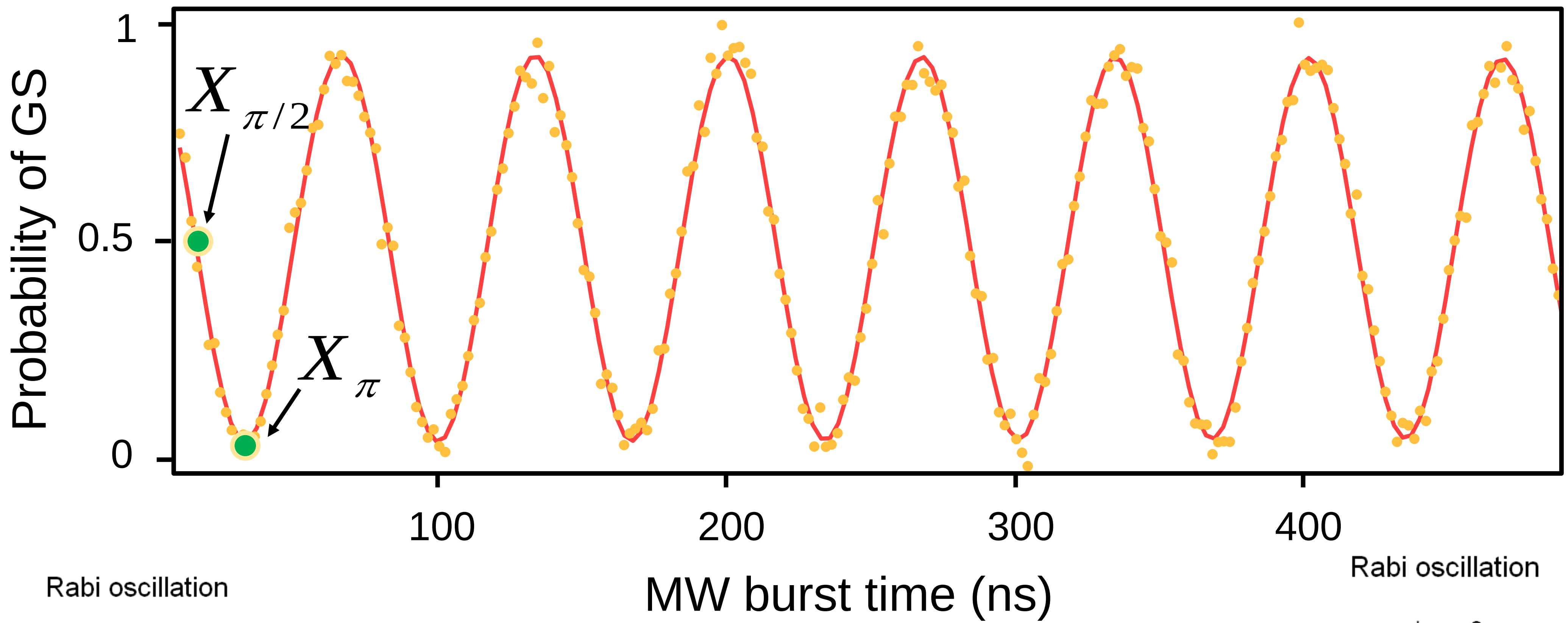
- 1Q, 2Q gate
- Master equations in the Markov approximation
- Quantum noise channels

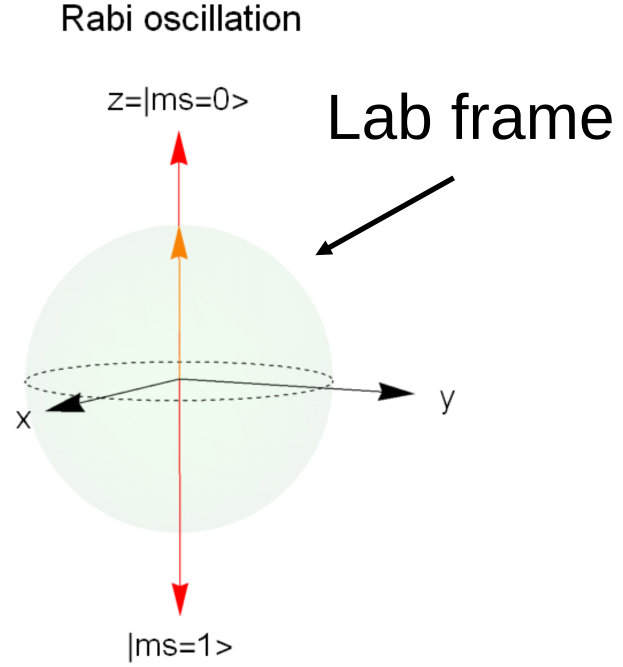
Quantum error correction

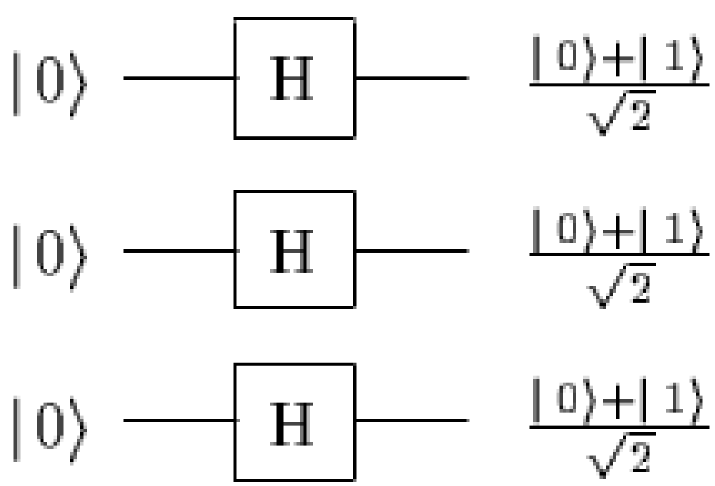
- Basic concepts
- Bit-flip and Phase-flip error correction
- Experimental examples

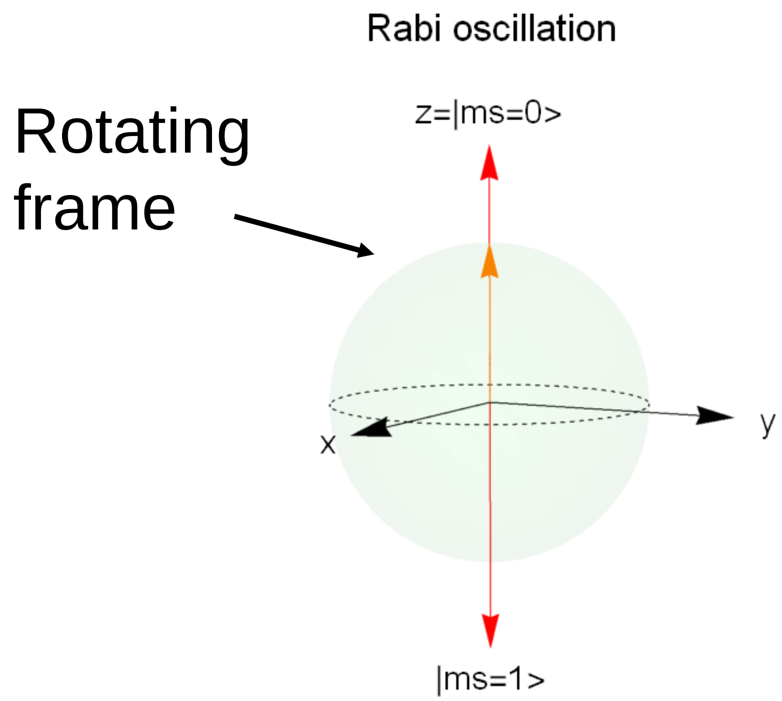
Single qubit gate : coherent rotation

Coherent Rabi pulse + Phase control = Single qubit rotation gates









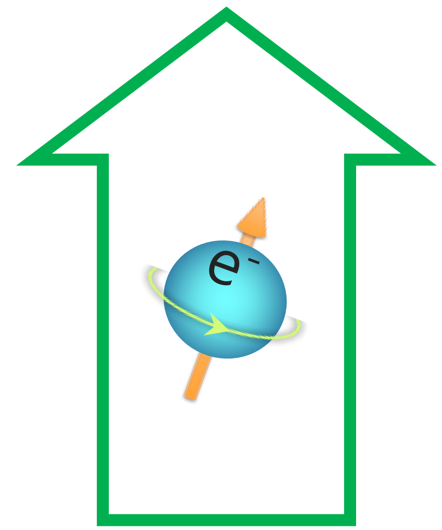
Control of quantum two level system

Rabi oscillation

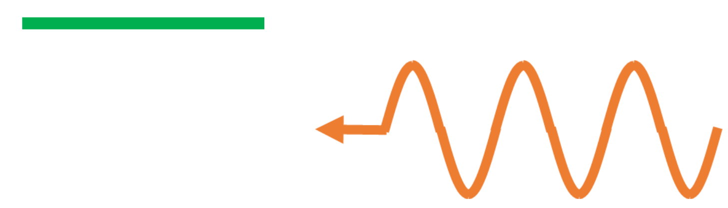
Two level system, with

$$\hat{H} = \boxed{\frac{\hbar\omega_0}{2}\hat{\sigma}_z} + \boxed{\hbar\eta(\hat{\sigma}_x \cos\omega_{MW} t)}$$

Apply harmonic radiation

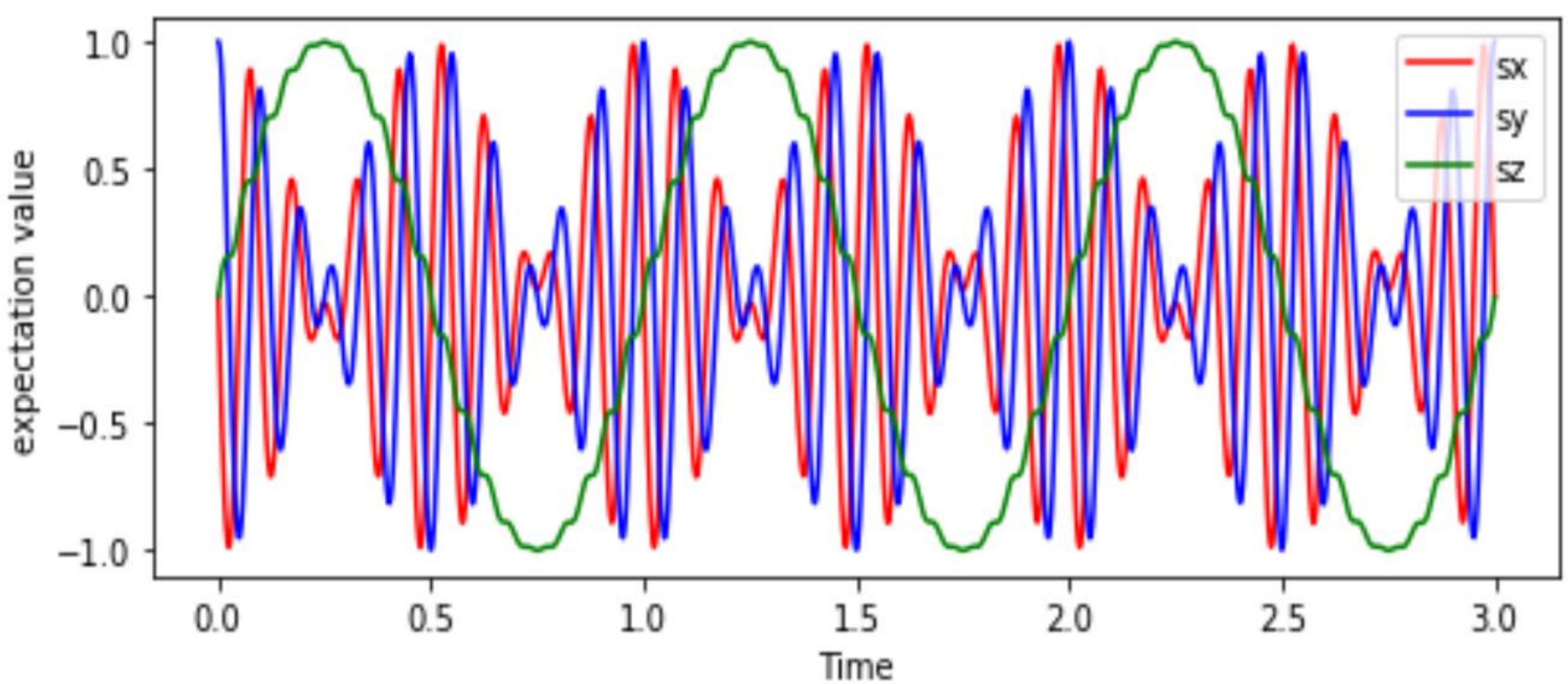


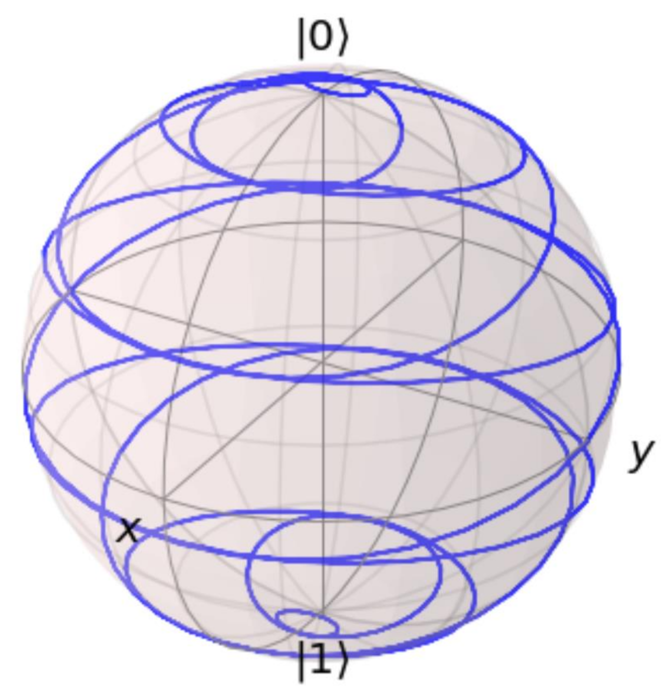
$\omega_0 = (E_1 - E_0)/\hbar$



ω_{MW}

On resonance, $\omega_0 = \omega_{MW}$





Control of quantum two level system

Rotating frame: RWA approximation

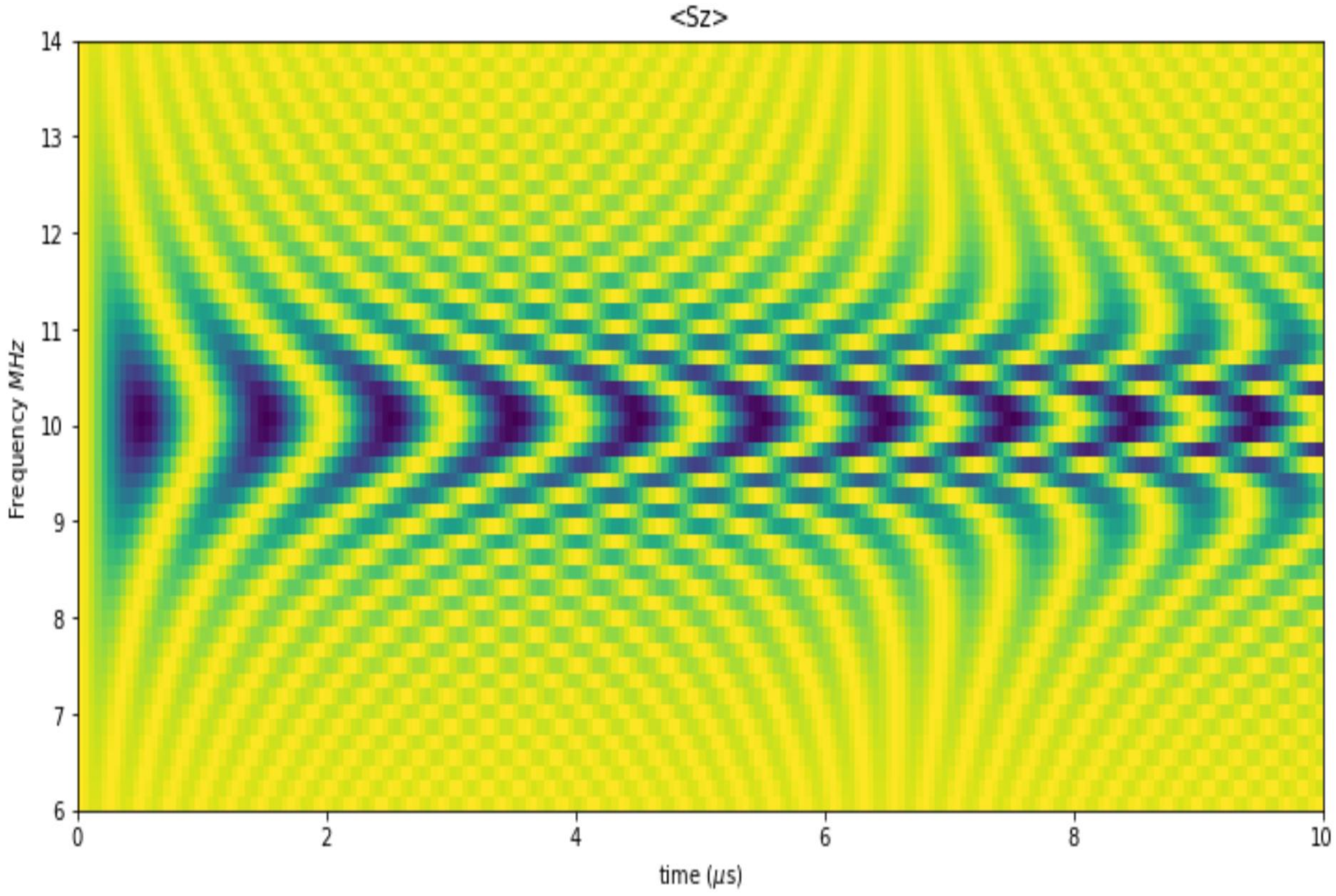
$$\hat{H}_{rot} = \frac{\hbar}{2}(\omega_0 - \omega_{MW})\hat{\sigma}_z + \frac{\hbar\eta}{2}\hat{\sigma}_x$$

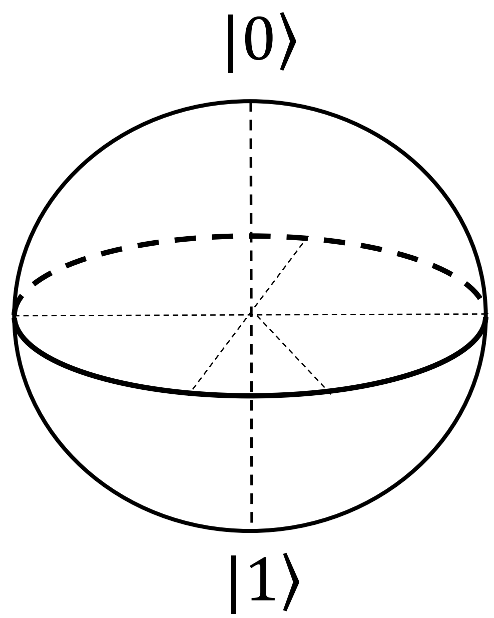
$$\delta = \omega_0 - \omega_{MW}$$

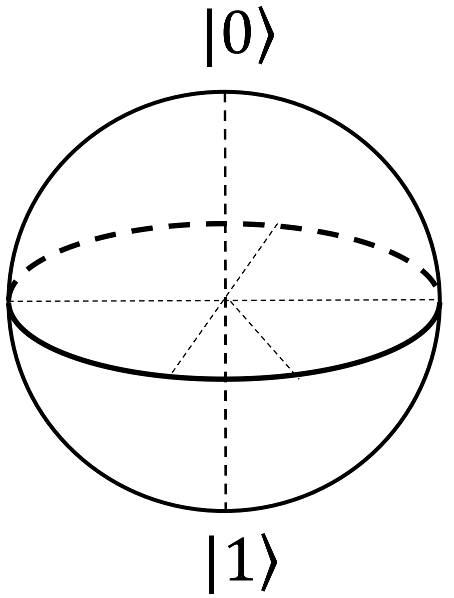
ω_0 의 intrinsic rot. 사라짐
$\hat{\sigma}_z, \hat{\sigma}_x$ 성분의 벡터합이 도는 축을 결정

Q : Hadamard Gate ?

낙서





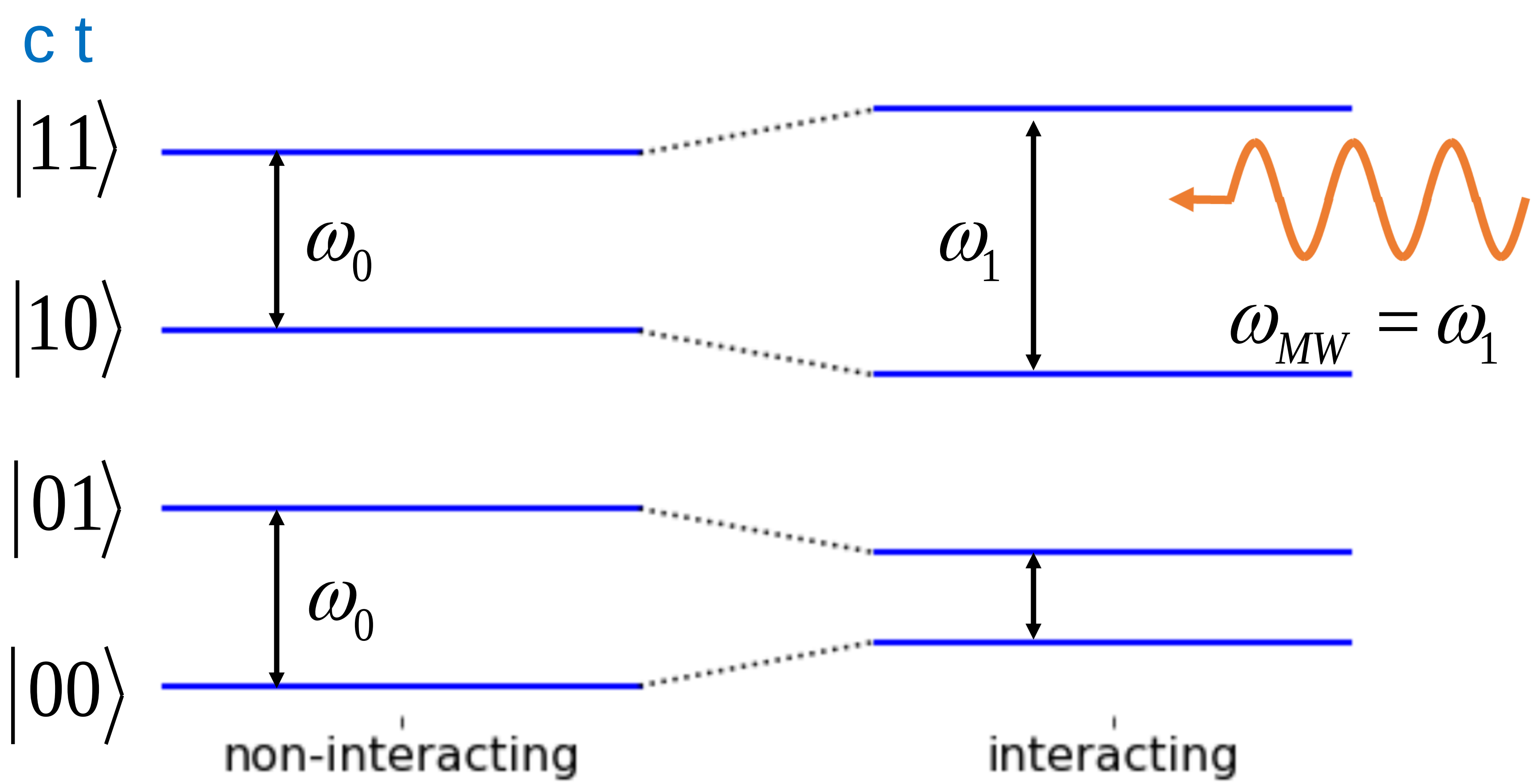


Control of quantum two level system

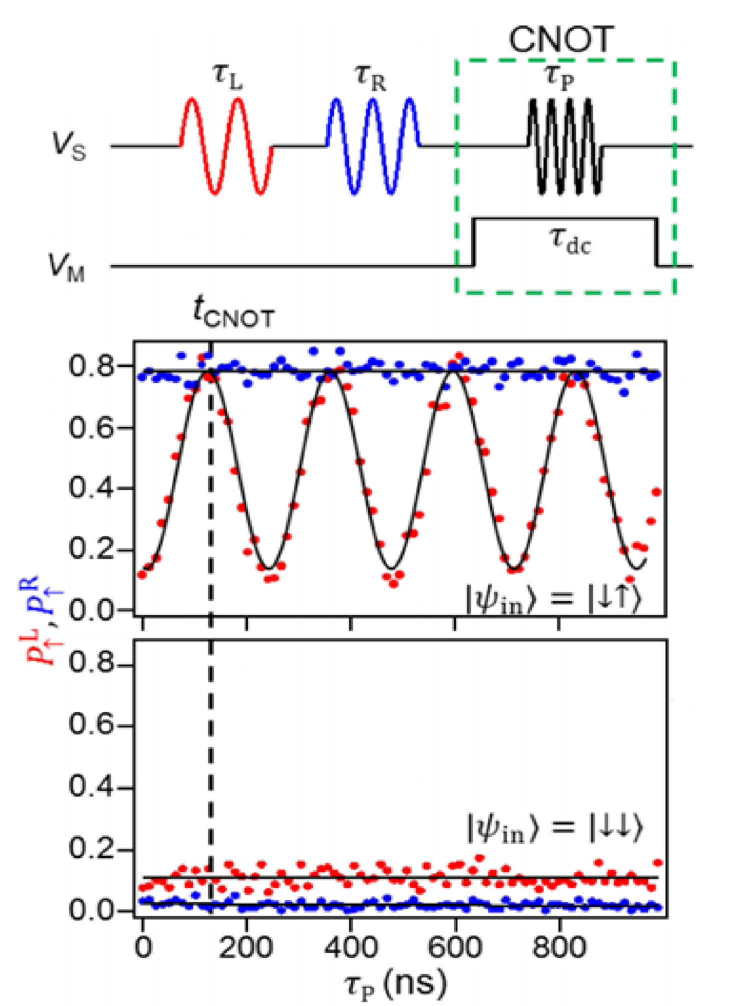
Two qubit gate

Ex. Calibrated Rabi π pulse under two body interaction = CNOT

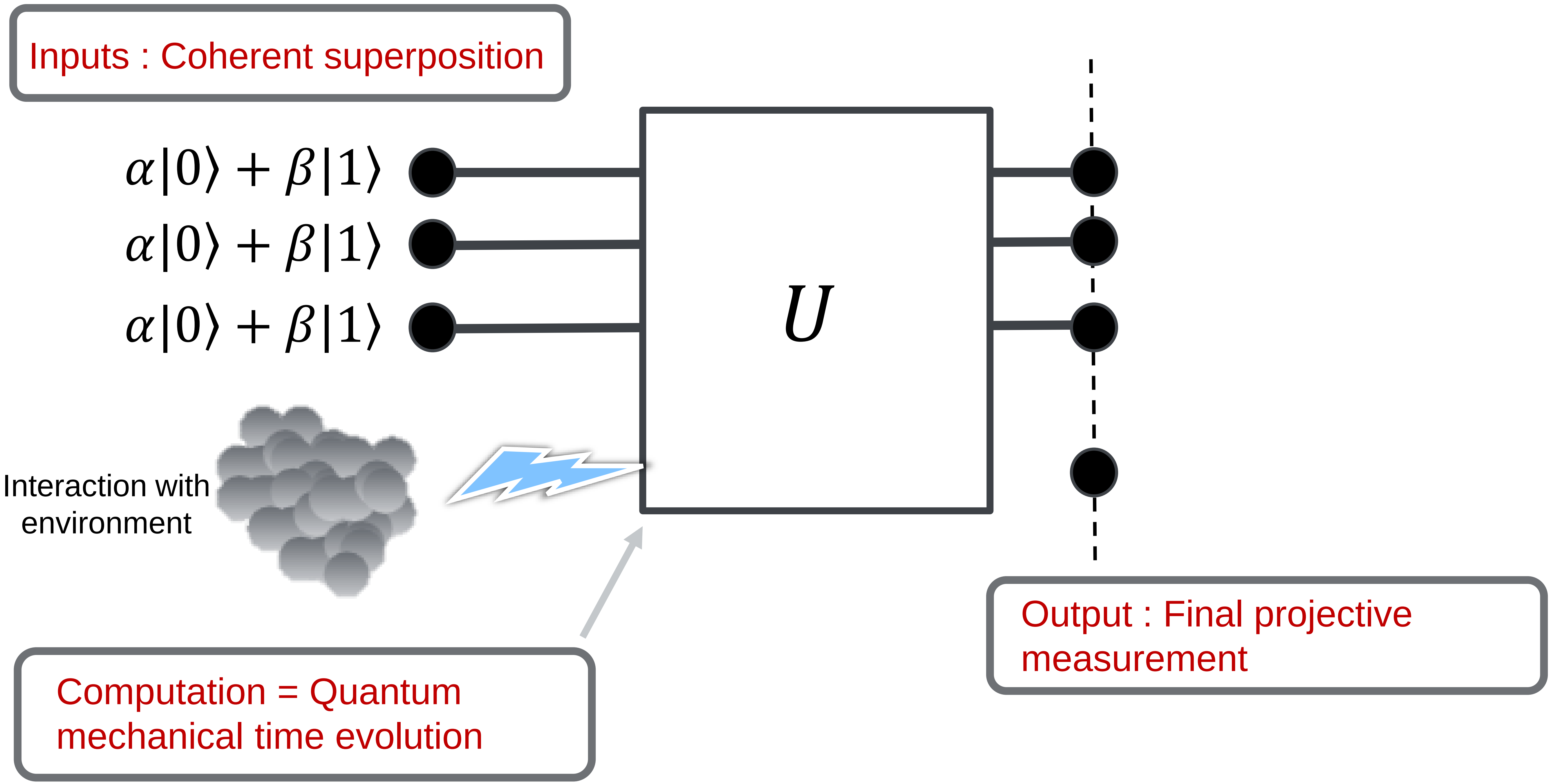
$$\hat{H} = \frac{\hbar\omega_0}{2}(2\hat{\sigma}_{z1}\otimes I + I\otimes\hat{\sigma}_{z2}) + \hbar g(\hat{\sigma}_{z1}\otimes\hat{\sigma}_{z2})$$



반도체 스핀 큐빗의 예

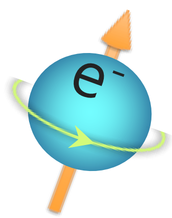


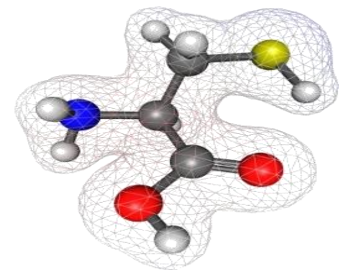
Coherent time evolution : computation



Understanding / controlling system – environment interaction is crucial

Quantum to classical transition



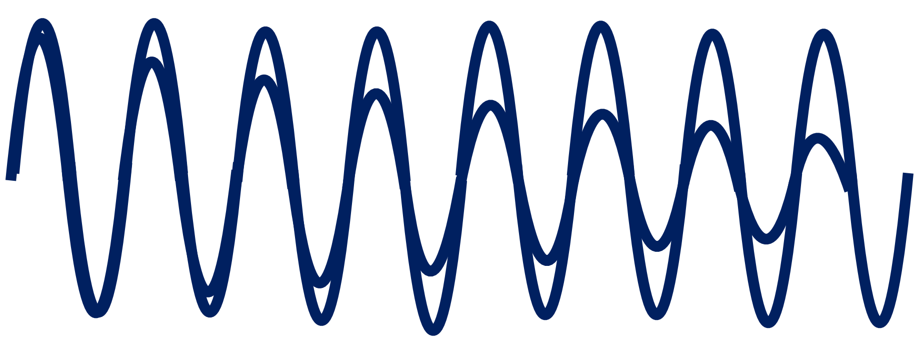


Decoherence

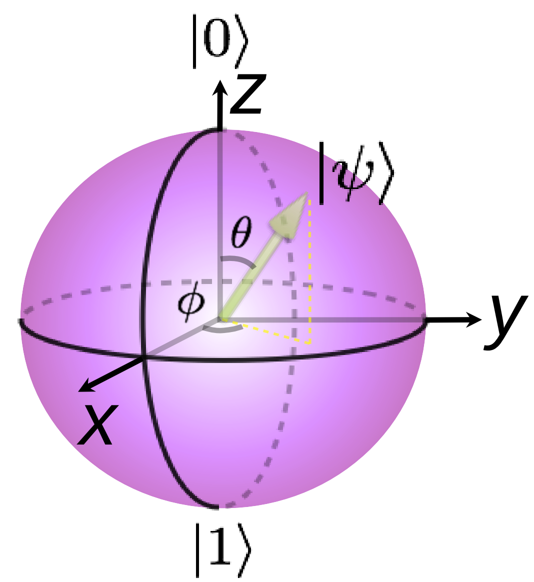
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\Downarrow$$

$$\rho = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$$



Quantum noise = decoherence, control error, etc.

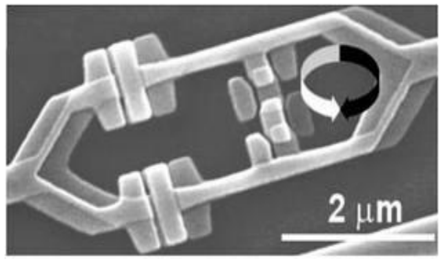


$$\alpha|0\rangle + \beta|1\rangle$$

$$\alpha = \cos(\frac{\theta}{2})$$

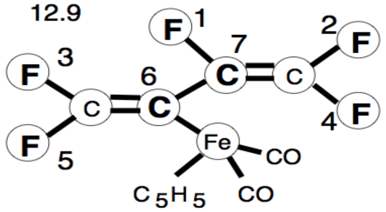
$$\beta = e^{i\phi}\sin(\frac{\theta}{2})$$

Superconductor



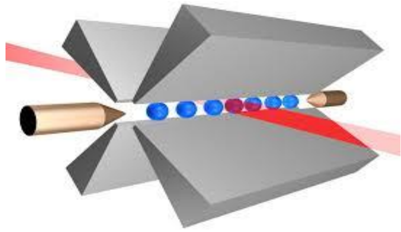
Y. Nakamura, et al., *Nature* **398**, 786 (1999)

NMR



L.M.K. Vandersypen, et al., *Nature* 414, 883 (2001)

Trapped Ions/Atoms



C. Monroe, et al., *Nature* **417**, 709 (2002)

Optics

E. Knill, et al., *Nature* **409**, 46 (2001)

Open quantum system

From closed to open quantum system

	Closed	Open
1. State	Ket vector $\|\Psi\rangle$	Density Matrix $\hat{\rho}$
2. Dynamics	Schrodinger	Master eq.
3. Measurement	Projective	Generalized (weak) Measurement

Density matrix

$$\hat{\rho} = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$$

p_i : probability to be in i th quantum state.

Properties

$$\hat{\rho}^\dagger = \hat{\rho}$$

$Tr(\rho^2)$ is called *purity*

$$Tr(\rho) = 1$$

$\rho^2 = \rho$ iff pure.

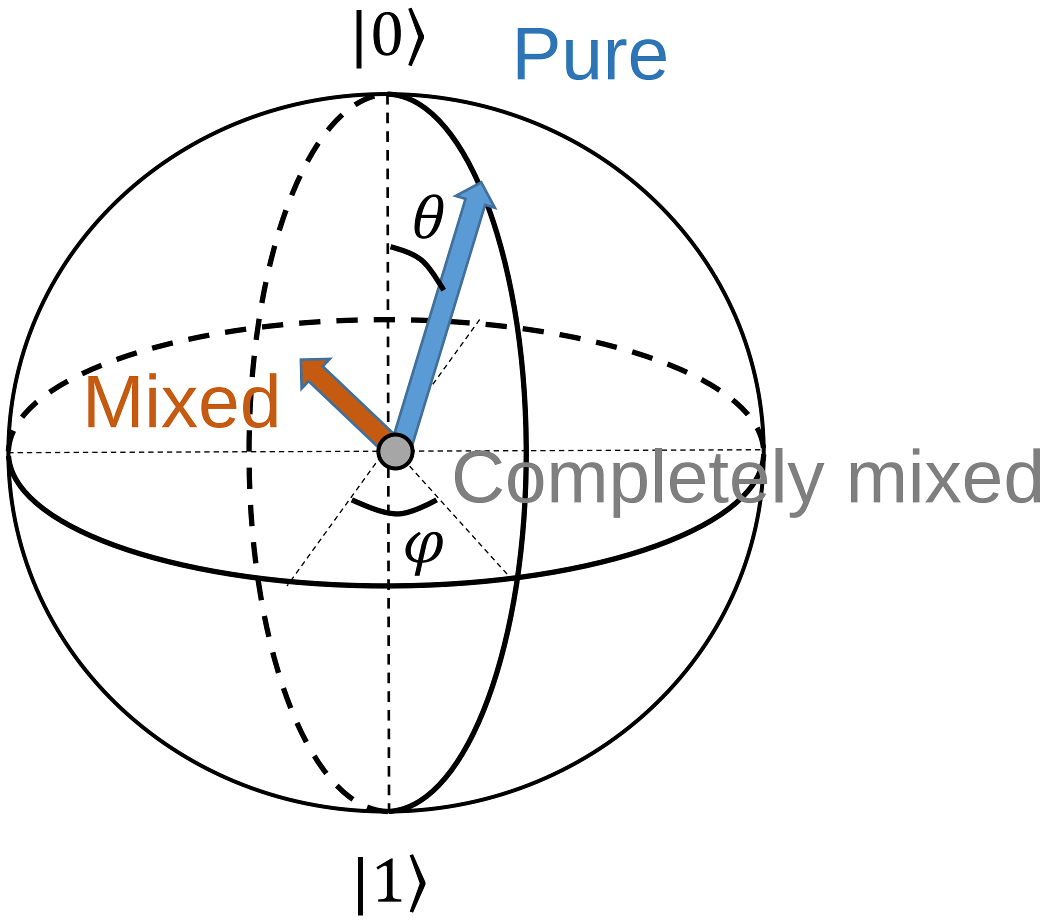
$Tr(\rho^2) = 1$ iff pure.

Open quantum system: Two level system

Pauli representation

$$\hat{\rho} = \frac{1}{2}\left(I + \sum_i m_i \sigma_i \right)$$

σ_i : Pauli matrix spans space of 2x2 matrices



Purity in Pauli rep.

$$Tr(\rho^2) = \frac{1}{4} Tr(I + 2\sum m_i \sigma_i + \sum m_i m_j \sigma_i \sigma_j)$$

$$= \frac{1}{2}(1 + \sum m_i^2)$$: By orthogonality

$$\frac{1}{2} \leq purity \leq 1 \Rightarrow |\vec{m}| \leq 1$$ Bloch Sphere rep.

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Closed : Unitary evolution

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{\mathrm{H}}, \hat{\rho}]$$

: Liouville von-Neumann equation

Open: System + Environment 의 전체 time evolution 중 system의 상태만 보면 어떻게 변화하는가?

$$\frac{d\hat{\rho}_S}{dt} = -\frac{i}{\hbar}[\hat{\mathrm{H}}, \hat{\rho}_S(t)] + \sum_{\mu}\left(-\frac{1}{2}L_{\mu}L_{\mu}^{\dagger}\hat{\rho}_S - \frac{1}{2}\hat{\rho}_S L_{\mu}L_{\mu}^{\dagger} + L_{\mu}\hat{\rho}_S L_{\mu}^{\dagger}\right)$$

: The master equation in the Lindblad form

Examples of Quantum channel

Application of L form to two-level system

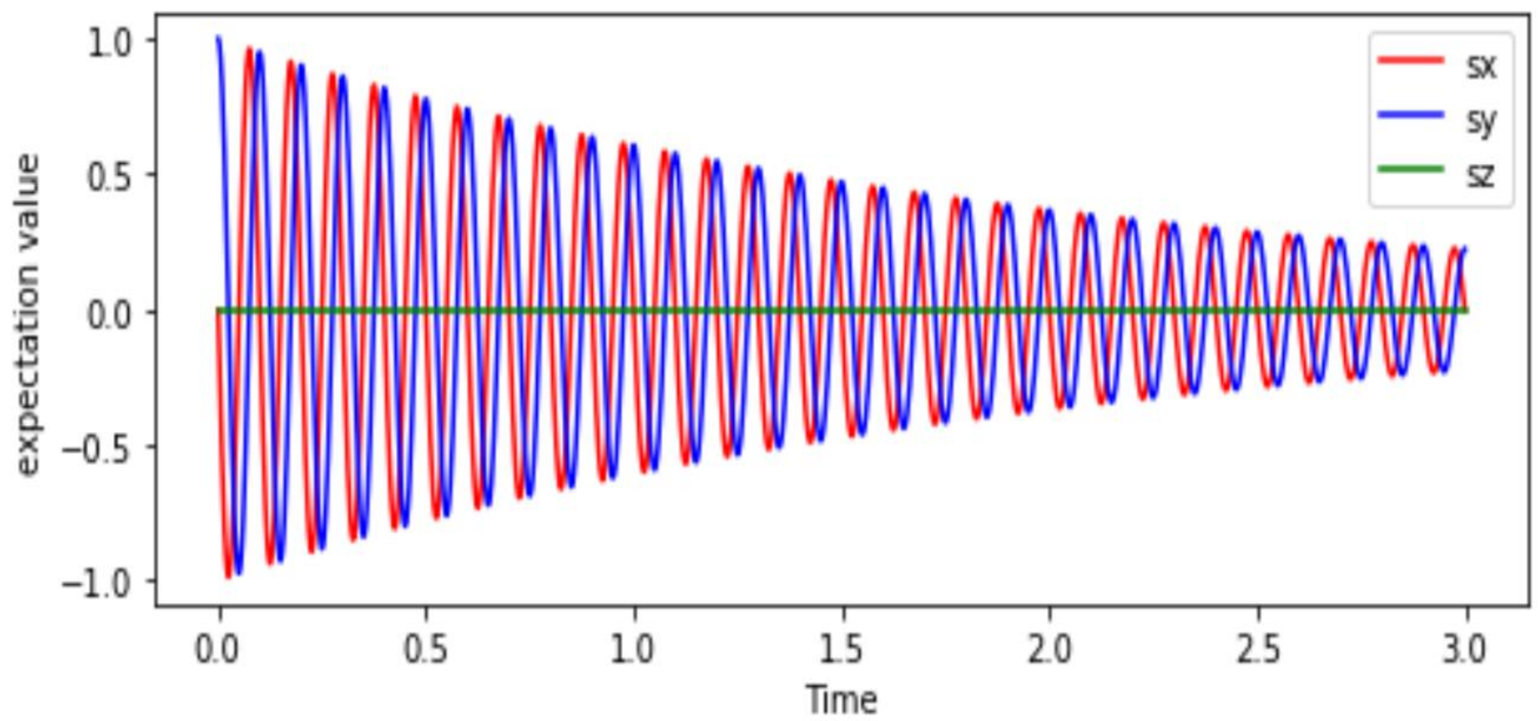
Pure dephasing channel

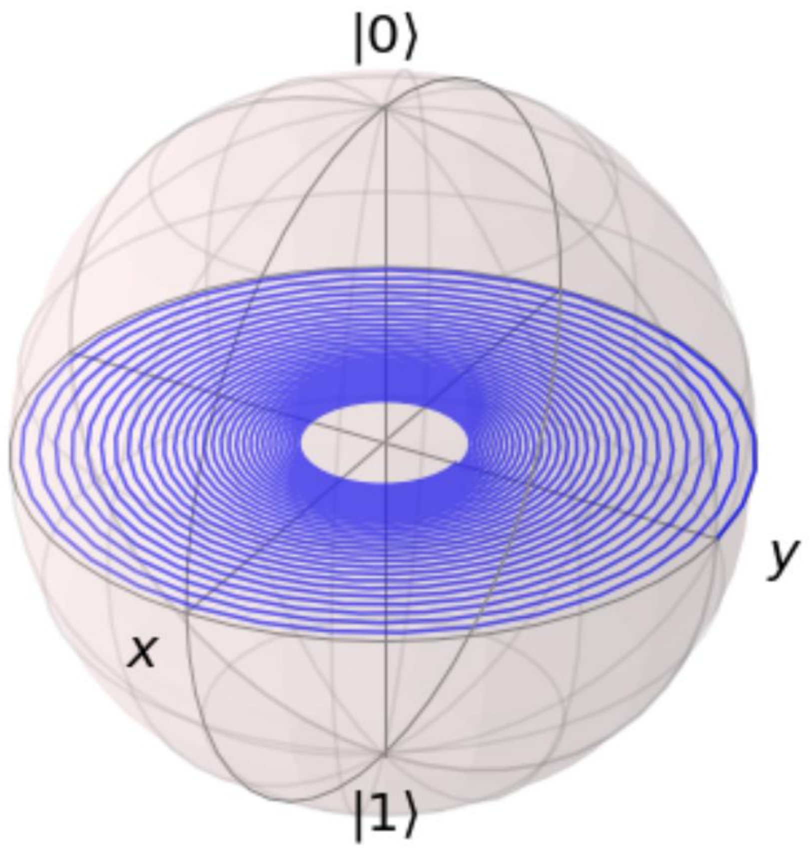
$$H_S = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix} \qquad L_1 = L_1^{\dagger} = \sqrt{\gamma}\sigma_+\sigma_- = \sqrt{\gamma}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Fluctuating energy levels

This time scale is called, T_ϕ pure dephasing time

In the Lab frame,





Examples of Quantum channel

Application of L form to two-level system

Amplitude damping (relaxation) channel

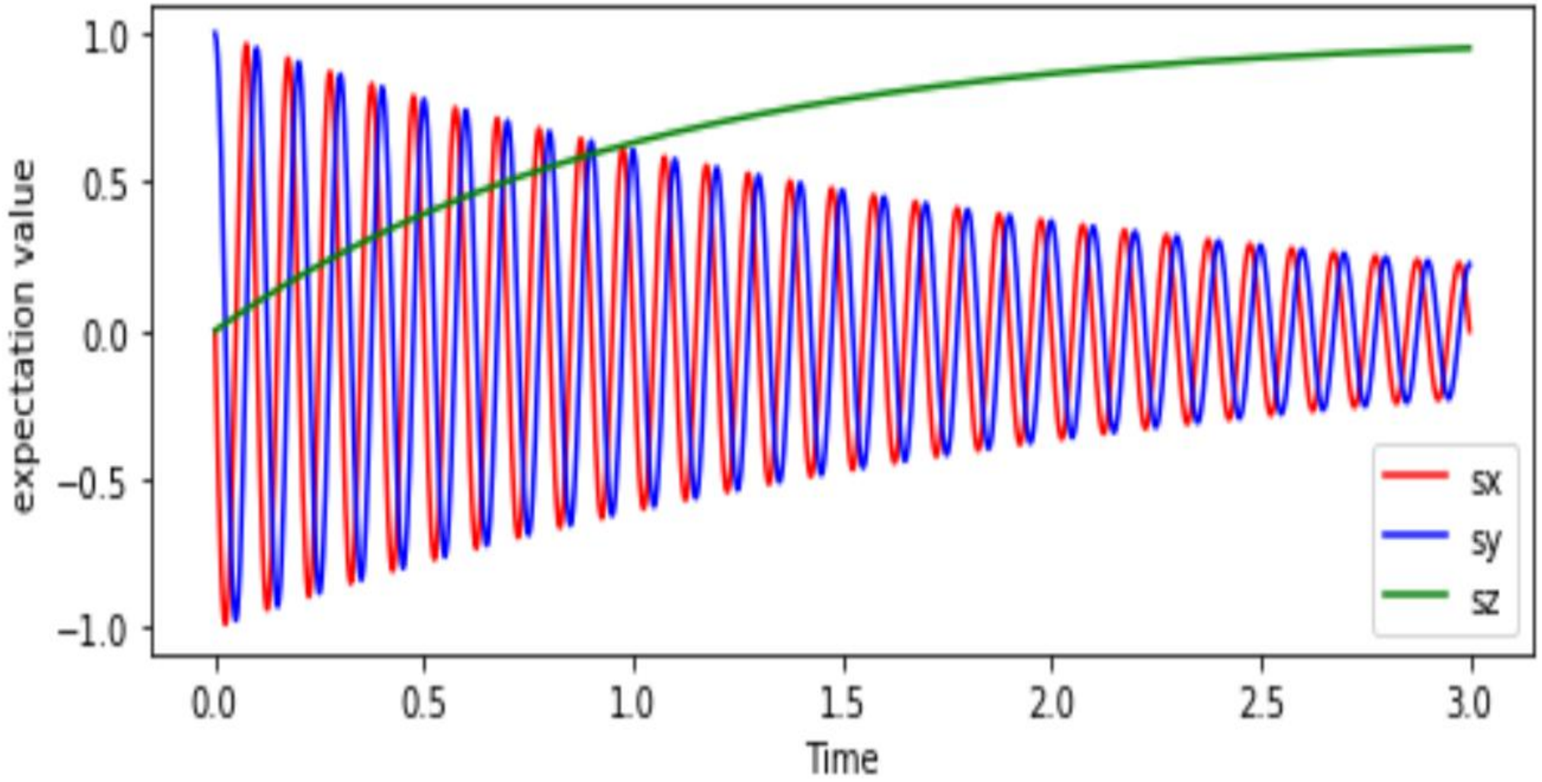
$$L_2 = \sqrt{\gamma}\sigma_- = \sqrt{\gamma}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

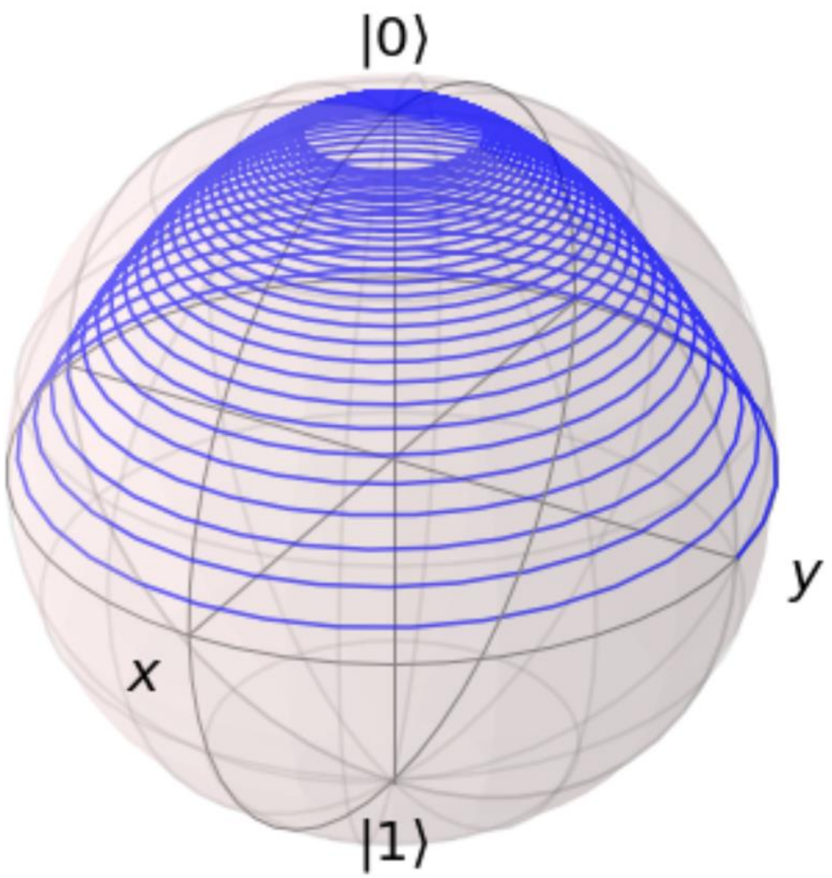
Energy relaxation

Total decoherence rate set by,

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}$$

This time scale is called, T_1 relaxation time





Also, depolarizing channel.. Etc.

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: The master equation in the Lindblad form

Dynamics of open quantum system

Time evolution: Kraus operators

$$H_{SE} = H_S \otimes H_E$$

Unitary evolution: U_{SE}

Environment orthonormal basis: $\left\{ \left| \mu \right\rangle_E \right\}$

Special proposition: $\hat{\rho}_{SE}(t) = \hat{\rho}_S(0) \otimes \hat{\rho}_E(0) = \hat{\rho}_S(0) \otimes \left| 0 \right\rangle_E \left\langle 0 \right|_E$

$$\hat{\rho}_{SE}(t) = U_{SE}(t) \hat{\rho}_{SE}(0) U^{\dagger}_{SE}(t)$$

$$\hat{\rho}_S(t) = Tr_E \left(U_{SE}(t) \hat{\rho}_{SE}(0) U^{\dagger}_{SE}(t) \right)$$

$$= \sum_{\mu} \left\langle \mu \right|_E U_{SE}(t) \left| 0 \right\rangle_E \hat{\rho}_S(0) \left\langle 0 \right|_E U^{\dagger}_{SE}(t) \left| \mu \right\rangle_E$$

$$= \sum_{\mu} \hat{M}_{\mu}(t) \hat{\rho}_S(0) \hat{M}^{\dagger}_{\mu}(t)$$

$\hat{M}_{\mu}(t)$: Kraus operators

Dynamics of open quantum system

Operator-sum representation　　　　　　　　　낙서

$$\rho_s(t) = \sum_\mu M_\mu(t)\hat{\rho}_S(0)M^\dagger_\mu(t)$$

$$= a[\hat{\rho}_S(0)]$$

Unitary evolution?
in general no.

Special case : pure state

$$\rho_s(t) = \sum_\mu M_\mu(t)\left|\psi_s(0)\right\rangle\left\langle\psi_s(0)\right| M^\dagger_\mu(t)$$

$$= \sum_\mu \left|\psi_s(t)\right\rangle\left\langle\psi_s(t)\right|$$

$$= \sum_\mu p_\mu(t)\left|\psi_s(t)\right\rangle\left\langle\psi_s(t)\right|$$

$$\left|\psi_s(t)\right\rangle \equiv M_\mu(t)\left|\psi_s(0)\right\rangle$$

$$\left|\psi_s(t)\right\rangle \equiv \frac{\left|\psi_s(t)\right\rangle}{\left\|\left|\psi_s(t)\right\rangle\right\|}$$

Not unitary operator
Not diagonal rep.
Generally mixed state

action of unitary operator on quantum system in general create entanglement

General properties of quantum map

1. Linearity

$$a[\lambda\hat{\rho}_1 + (1-\lambda)\hat{\rho}_2] = \lambda a[\hat{\rho}_1] + (1-\lambda)a[\hat{\rho}_2]$$

2. Completely Positive (CP condition)

$\hat{\rho}_{out} = a[\hat{\rho}_{in}]$: Physical state $\hat{\rho}_{out} = \hat{\rho}^{\dagger}_{out}$

$\Rightarrow \hat{\rho}_{out} \geq 0$ ($\hat{\rho}^{\dagger}_{out}\hat{\rho}_{out}$ non-negative eigenvalue)

3. Trace preserving (TP condition)

$$Tr_S[a[p_S]] = Tr_S(\sum_{\mu} M_{\mu}(t)\hat{\rho}_S(0)M^{\dagger}_{\mu}(t))$$

$$= Tr_S(\sum_{\mu}(M_{\mu}(t)M^{\dagger}_{\mu}(t))\hat{\rho}_S(0))$$

$$\sum_{\mu} M_{\mu}(t)M^{\dagger}_{\mu}(t) = \sum_{\mu} \langle 0|_E U^{\dagger}_{SE}(t)(|\mu\rangle_E \langle\mu|_E)U_{SE}(t)|0\rangle_E$$

$$= \langle 0|_E U^{\dagger}_{SE}(t)U_{SE}(t)|0\rangle_E = \hat{I}_S$$

Dynamics of open quantum system

Closed quantum system

$$\rho(t+dt) = U(t+dt,t)\rho(t)U^{\dagger}(t+dt,t)$$: generator of time-translation

$$U(t+dt,t) = I - \frac{i}{\hbar}H(t)dt$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho]$$: Liouville von-Neumann equation

Open quantum system

what is generator of time-translation?

$$\hat{\rho}_S(t_2) = a[\hat{\rho}_S(t_1)]$$

$$\hat{\rho}_S(t+dt) = a(t+dt,t)[\hat{\rho}_S(t)]$$: Markov approx.

$$= \sum_{\mu} M_{\mu}(t+dt,t)\hat{\rho}_S(t)M^{\dagger}_{\mu}(t+dt,t)$$

Dynamics of open quantum system

Lindblad operator

$$M_\mu(t+dt,t) \equiv L_\mu(t)\sqrt{dt}$$

$$M_0(t+dt,t) \equiv \hat{I} + G(t)dt$$

CPTP condition

$$\sum_\mu M_\mu M^\dagger_\mu = \hat{I}$$

$$\Rightarrow M_0 M^\dagger_0 + \sum_{\mu\neq 0} M_\mu M^\dagger_\mu = \hat{I} + (G+G^\dagger)dt + \sum_\mu L_\mu L_\mu^\dagger dt = \hat{I}$$

$$\Rightarrow G + G^\dagger = -\sum_\mu L_\mu L_\mu^\dagger$$

$$\Rightarrow G \equiv K - \frac{i}{\hbar}H = -\frac{i}{\hbar}H - \frac{1}{2}\sum_\mu L_\mu L_\mu^\dagger \equiv -\frac{i}{\hbar}H_{eff}$$

Dynamics of open quantum system

$$\rho_S(t+dt) = M_0 \hat{\rho}_S(t) M^\dagger{}_0 + \sum_{\mu\neq 0} M_\mu \hat{\rho}_S(t) M^\dagger{}_\mu$$

$$= \left(\hat{I} - \frac{i}{\hbar} H_{eff}\, dt\right) \hat{\rho}_S(t) \left(\hat{I} - \frac{i}{\hbar} H_{eff}\, dt\right) + \sum_\mu L_\mu \hat{\rho}_S(t) L^\dagger{}_\mu dt$$

$$\Rightarrow \frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H_{eff}, \hat{\rho}_S(t)] + \sum_\mu L_\mu \hat{\rho}_S(t) L^\dagger{}_\mu$$

expanding effective Hamiltonian

$$\frac{d\rho_S}{dt} = -\frac{i}{\hbar}[H, \hat{\rho}_S(t)] + \sum_\mu \left(-\frac{1}{2} L_\mu L_\mu^\dagger \hat{\rho}_S - \frac{1}{2} \hat{\rho}_S L_\mu L_\mu^\dagger + L_\mu \hat{\rho}_S L_\mu^\dagger \right)$$

: The master equation in the Lindblad form

The differential eqn. whose integral is CPTP map has to be this 'Lindblad form'

Composite systems

Hilbert Space of composite system

$$H = H_A \otimes H_B$$

Two observable algebra A, B

Product State of composite systems

$$|\phi_A\rangle \otimes |\phi_B\rangle,$$

$$where, |\phi_A\rangle \in H_A, |\phi_B\rangle \in H_B$$

Pure product states have density operators

$$\hat{\rho} = |\phi_A\rangle\langle\phi_A| \otimes |\phi_B\rangle\langle\phi_B| = \hat{\rho}_A \otimes \hat{\rho}_B$$

Q. All states are product state?

States which aren't products are correlated

$$\hat{\rho} = p|00\rangle\langle 00| + (1-p)|11\rangle\langle 11|$$

Q. How to check if the state is product state or not?

Schmidt decomposition for pure state

$$|\psi\rangle = \sum_{n=1}^{d} \sqrt{p_i}\, |e_i\rangle |f_j\rangle$$

$$then, \rho_A = \sum_{n=1}^{d} p_j |e_j\rangle\langle e_i|$$

Composite systems: Entanglement

Partial Trace

$$\rho_{AB} = \sum_i p_i \left|\Psi_i\right\rangle_{AB} \left\langle\Psi_i\right|_{AB}$$

'Trace out' environment -> information loss?

$$\rho_A = Tr_B \rho_{AB} = \sum_j \left\langle\psi_i\right|_B p_i \left|\Psi_i\right\rangle_{AB} \left\langle\Psi_i\right|_{AB} \left|\psi_j\right\rangle_B$$

Ex) For singlet state

$$\hat{\rho} = \frac{1}{2}(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)$$

$$\hat{\rho}_A = \frac{1}{2}\left\langle\uparrow\right|_B (\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)\left|\uparrow\right\rangle_B$$

$$+\frac{1}{2}\left\langle\downarrow\right|_B (\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle)(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|)\left|\downarrow\right\rangle_B$$

$$\Rightarrow \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

Completely Mixed state

Entropy: How much entangled?

$$S = -\sum_\mu p_\mu \log p_\mu$$

p_μ : Schmidt coefficient

More on entanglement

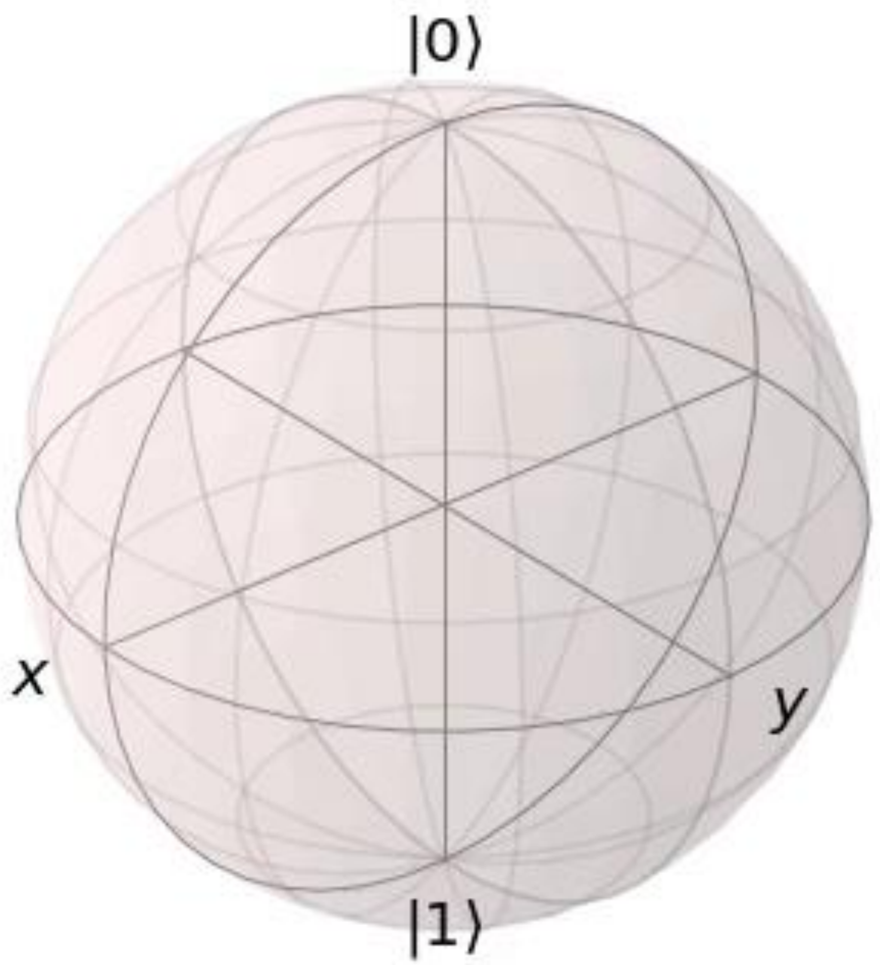
$$\hat{H} = \frac{J_1}{2}(\hat{\sigma}_{z1} \otimes I) + \frac{J_2}{2}(I \otimes \hat{\sigma}_{z2}) + \frac{J_{12}}{2}((\hat{\sigma}_{z1} - I) \otimes (\hat{\sigma}_{z2} - I))$$

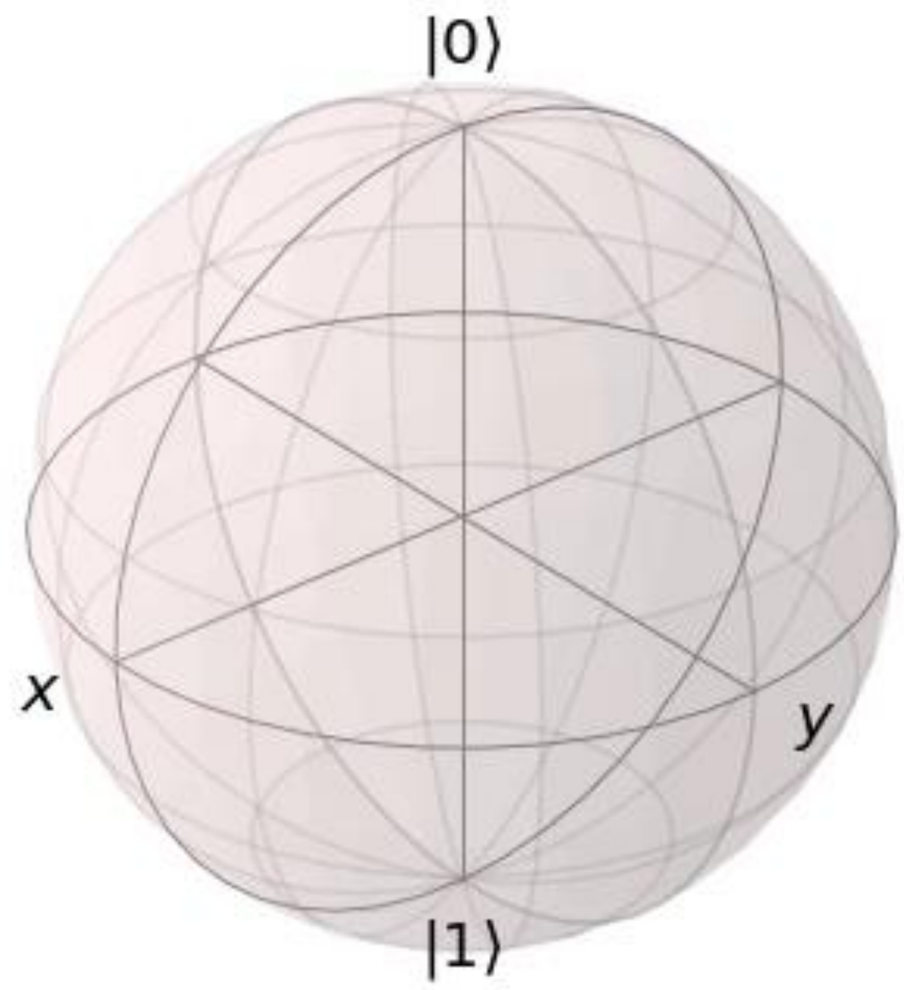
Initial state : $|++\rangle$

Note : no decoherence as a whole (no L)

Qubit 1 (system)

Qubit 2 (Env.)





Lab frame