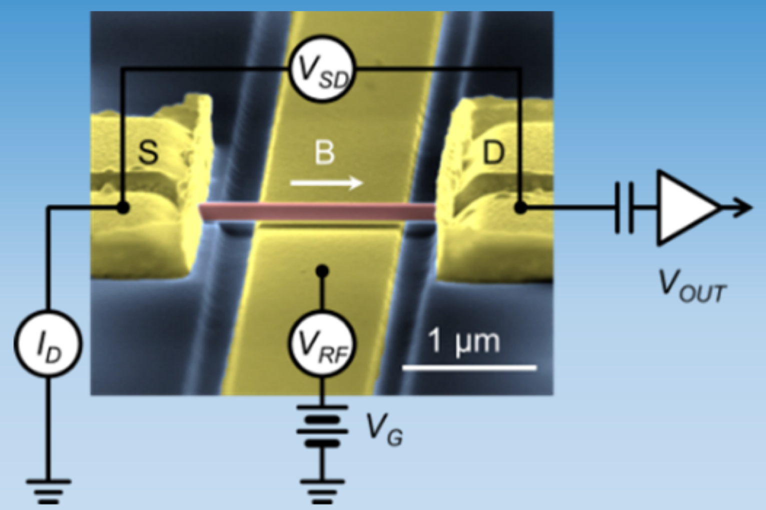

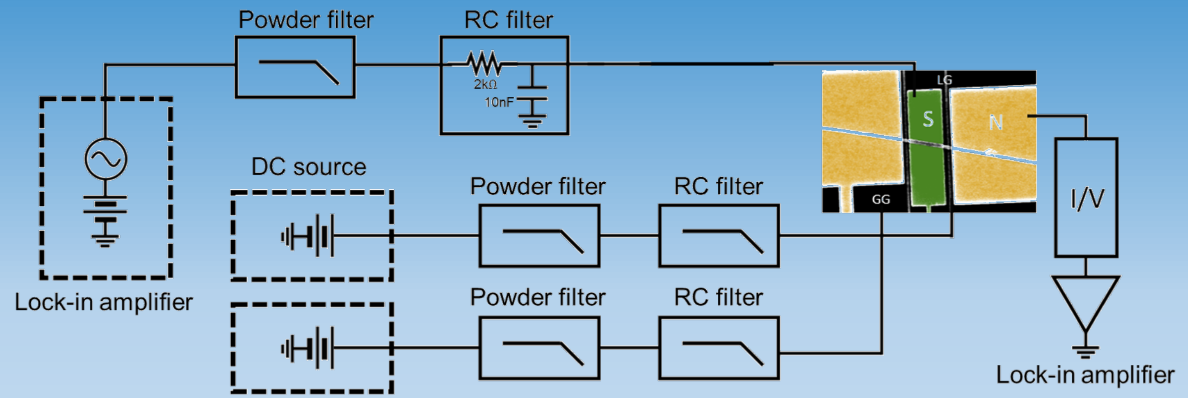

Electronics for mesoscopic physics

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Outline

- Basic concepts (1 hour)
  - Voltage, current
  - Resistance, inductance, capacitance
  - Impedance, admittance
  - Signal, noise, interference
  - High frequency circuit concepts
  - Q&A + break (10 min)

- Examples (1 hour)
  - Low-noise & low temperature conductance measurement
  - Inductive/capacitive/microwave detection of mechanical oscillator
  - Q&A + break (10 min)

References

- Art of Electronics (Horowitz and Hill)
- Noise reduction techniques in electronic systems (Henry W. Ott)
- Microwave engineering (Pozar)
- Low Level Measurements Handbook (Keithley)
- Spectrum Analysis Basics (Application Note 150, Agilent)
- Manuals and application notes for your equipments

Outline

- Basic concepts (13:30 ~ 14:30)
  - Voltage, current
  - Resistance, inductance, capacitance
  - Impedance, admittance
  - Signal, noise, interference
  - High frequency circuit concepts
  - Q&A + break (10 min)

- Examples (14:30~ 15:30)
  - Low-noise & low temperature conductance measurement
  - Inductive/capacitive/microwave detection of mechanical oscillator
  - Q&A + break (10 min)

Voltage & current

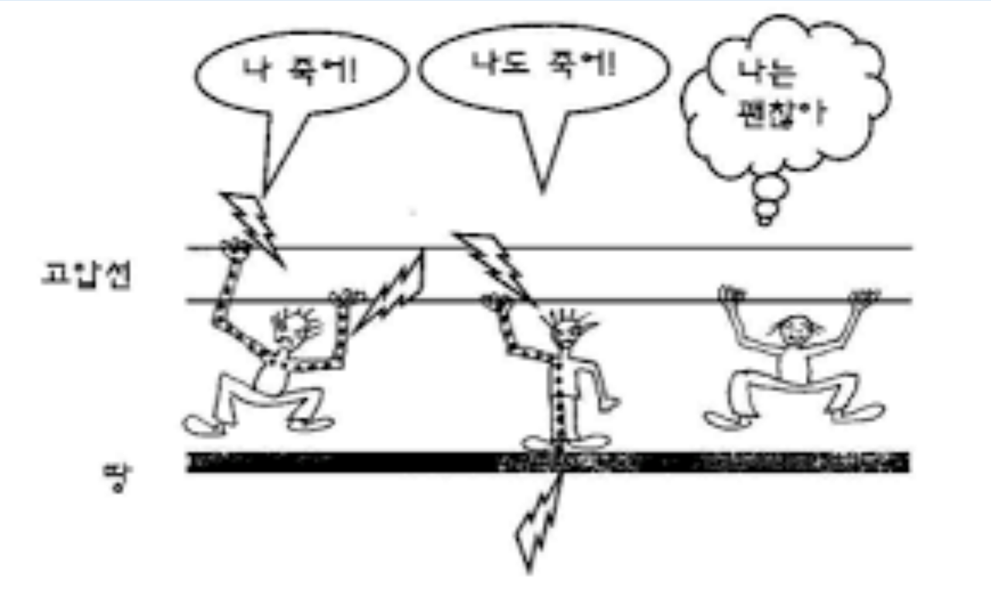
- Voltage (“V”)
  - Electric potential DIFFERENCE
  - Requires REFERENCE (i.e. 0 V or ground)

- Current (“I”)
  - Amount of charge flowing per unit time
  - ”Source” and “Sink” (or return) always exist

→ SIGNALS carrying information about mesoscopic physics

Notes

- Voltage reference (i.e. ground) can be at ANY electric potential
- Earth potential (i.e. earth, protective earth (PE), safety ground) is typically selected to protect people from electric shock



Notes

- When dealing with low level voltages (pV, nV, uV,…), remember small voltage develops in a current-carrying wire: could be ground connection.
- This small voltage can be seen as noise (or interference) obscuring the signal. (i.e. “*ground loop*”)

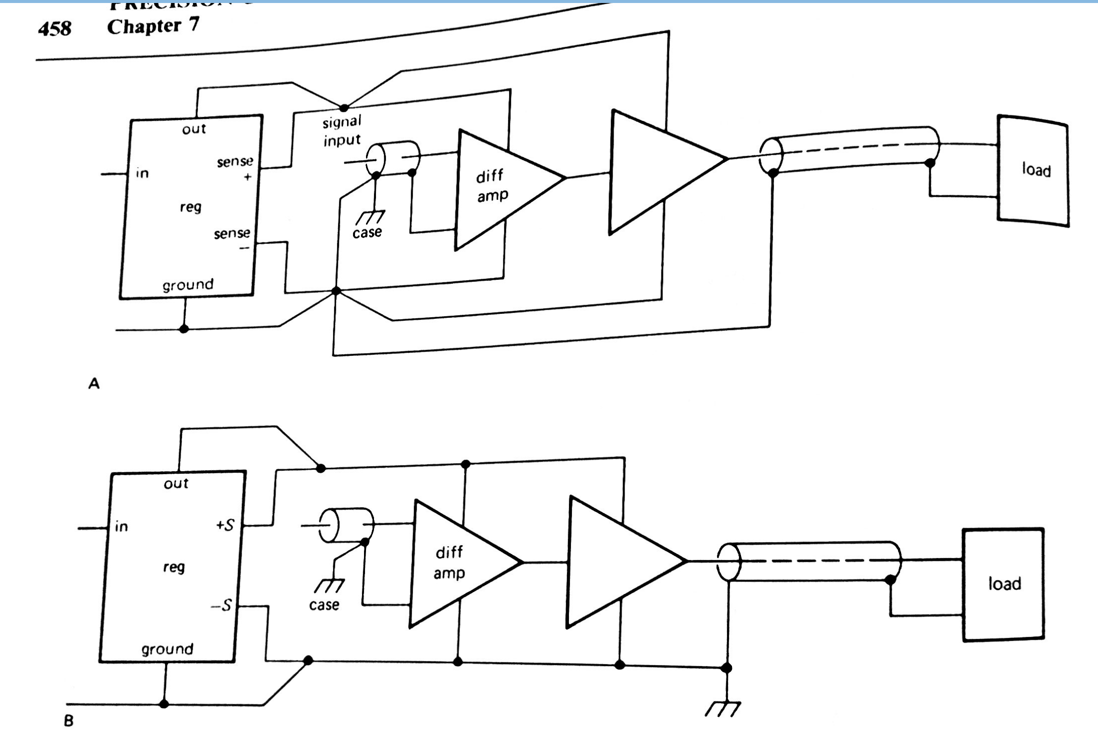


Figure 7.67. Ground paths for low-level signals.
A. Right
B. Wrong

Notes

- Voltage and current become "operators" in the quantum description of electromagnetic circuit: uncertainty, vacuum noise, etc.
- See "Introduction to quantum noise, measurement, and amplification" by A.A. Clerk (RMP 82, 1155 (2010)) for example:

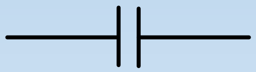
$$\hat{V}_q(t) = \int_0^\infty \frac{d\omega}{2\pi}(\hat{V}_q[\omega]e^{-i\omega t} + \text{H.c.}), \qquad \hat{V}_q[\omega] = \sqrt{\frac{\hbar\omega}{2}Z_q}(\hat{q}_{\text{in}}[\omega] + \hat{q}_{\text{out}}[\omega]),$$

$$\hat{I}_q(t) = \sigma_q \int_0^\infty \frac{d\omega}{2\pi}(\hat{I}_q[\omega]e^{-i\omega t} + \text{H.c.}), \qquad \hat{I}_q[\omega] = \sqrt{\frac{\hbar\omega}{2Z_q}}(\hat{q}_{\text{in}}[\omega] - \hat{q}_{\text{out}}[\omega]).$$

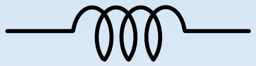
Resistance, capacitance, inductance (Passives)

- Resistance ("R")
  - R = V / I
  - Conductance G = 1/R = I / V

- Capacitance ("C")
  - C = charge / V = I / (dV/dt)

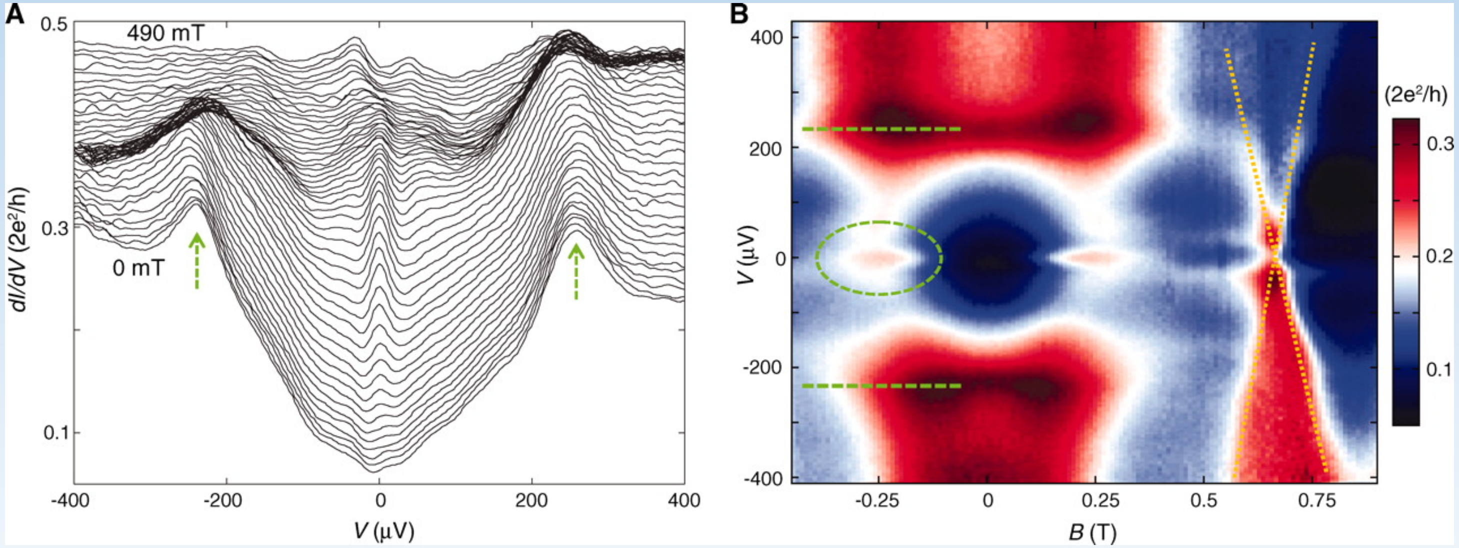


- Inductance ("L")
  - L = (flux) / I = V / (dI/dt)



Notes

- Typical R,L,C are independent of V or I
- Many interesting mesoscopic devices are not; e.g. differential conductance (= dI / dV) is useful



* V. Mourik *et.al.*, *Science* **336**, 6084 (2012).

Impedance, Admittance (AC circuits)

- For alternating voltage and current (“AC”), use complex notation as voltage V exp(jwt) and I exp(jwt) at a given angular frequency w; real parts carry the measured voltage and current
- Impedance (“Z”)
  - Z = V/I
  - R for resistance R, jwL for inductance L, 1/(jwC) for capacitance C
- Admittance (“Y”)
  - Y = I/V
- Ohm’s law obeyed with impedances and admittances

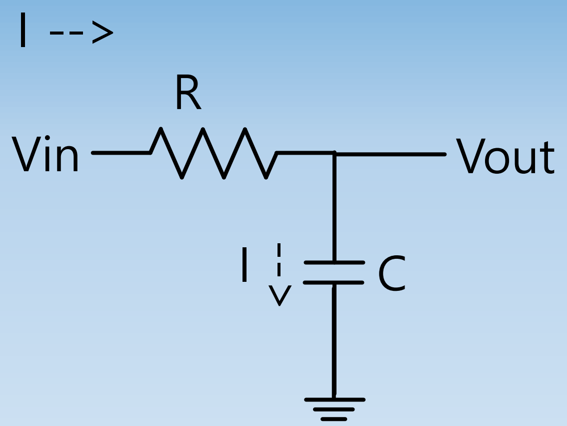
Notes

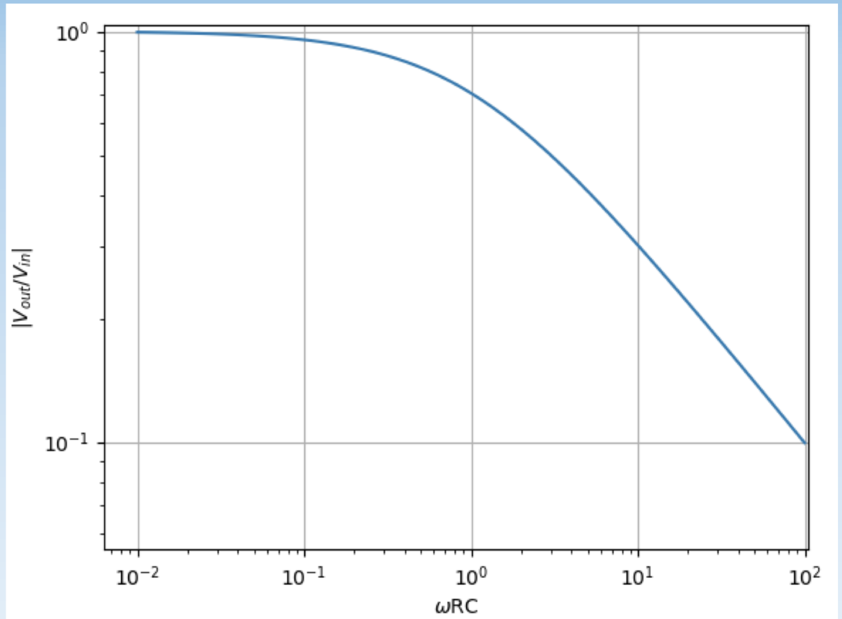
- Transmission lines (e.g. coaxial cables) have "characteristic impedance" (usually denoted as "Z_0")
- Z_0 is the ratio of voltage and current at a given point of transmission line
- Z_0 is real for lossless transmission lines and equal to $\sqrt{(L/C)}$ for the inductance and capacitance per unit length
- When a transmission line is terminated by a load with impedance equal to Z_0, it becomes reflectionless (i.e. perfect power transmission)
- Typical high frequency circuits require Z0 = 50 ohm, but there are cases with different values (e.g. 75 ohm)
- High frequency connectors (e.g. BNC,SMA,…) are designed to work with a specific Z_0; 50 ohms are most common

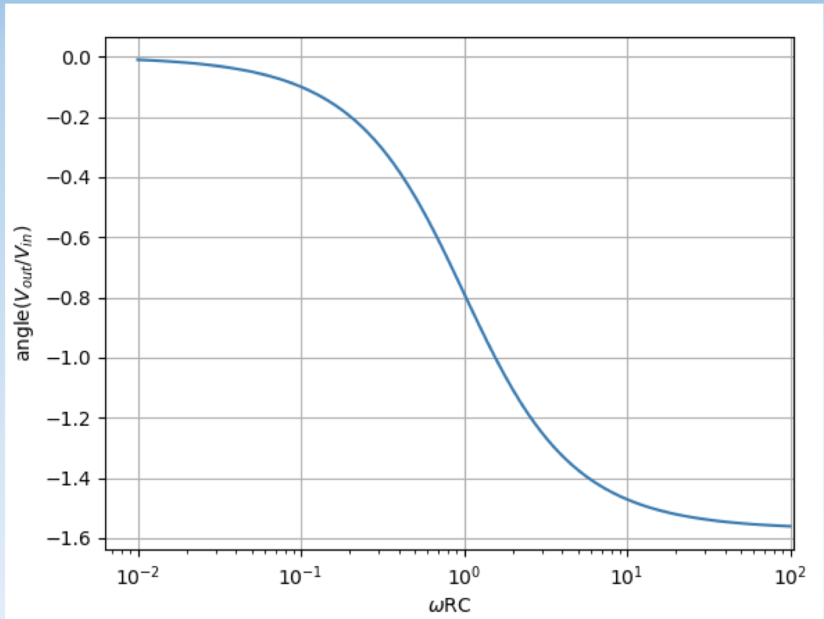
Kirchhoff's law

- Voltage
  - Sum of voltage drops around any closed circuit is zero
  - Energy conservation
- Current
  - Sum of currents into a point in a circuit equals the sum of the currents out
  - Charge conservation

Example: Low-pass filter







$$I = Vin / (R+1/jwC)$$
$$= Vout / (1/jwC)$$

$$Vout = [(1/jwC) / (R+1/jwC)]\ Vin$$
$$= Vin/(1+jwRC)$$

$$abs(Vout/Vin) = 1 / \sqrt{(1+(wRC)^2)}$$
$$angle(Vout/Vin) = \tan^{-1}(-wRC)$$

Voltage & current source

- Ideal voltage source provides a fixed voltage regardless of load
- In reality, any voltage source has finite maximum current and finite output resistance. i.e. voltage drop
- Usually high-frequency sources has 50 ohm output resistance for matching to 50 ohm transmission lines and loads

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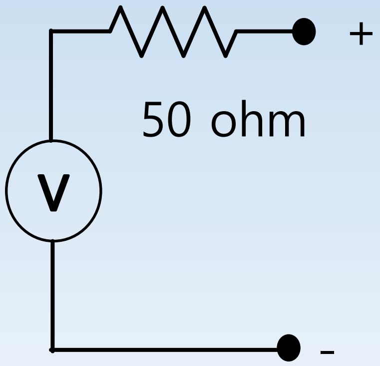

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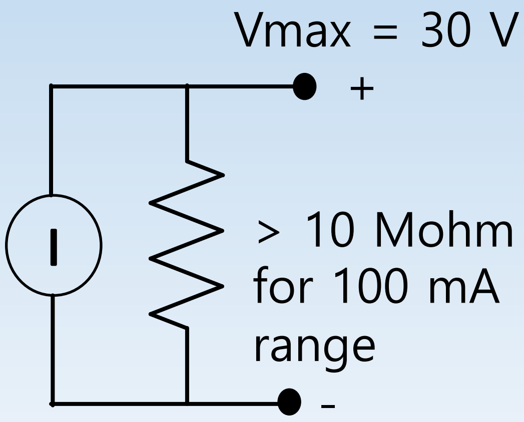

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Voltage & current measurement

- Voltage is amplified and converted to numbers by analog-to-digital converters (ADC); ADC compares the input voltage to a reference voltage to generate a digital number.

Voltage & current measurement

- Current is commonly converted to voltage either via simple resistor or current-to-voltage converter (i.e. current preamplifier like Ithaco 1211), and the resulting voltage is measured.

Voltage & current measurement

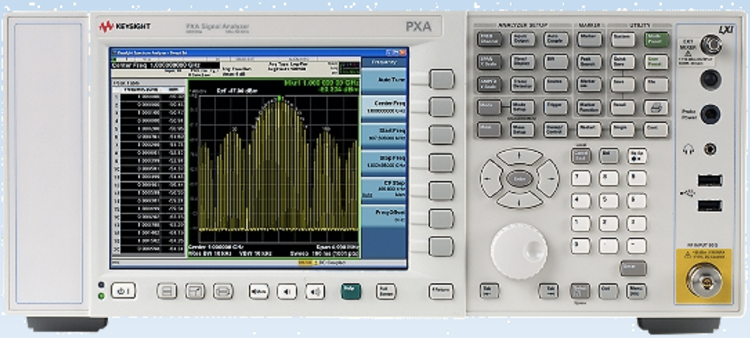
- All measurement electronics have input resistance (also called input impedance for high frequency circuits)
- Voltage meters have high input resistance (>10 Mohm typ.)
- Current meters have low input resistance (e.g. Ithaco 1211 has 0.5 ohm for 1V/mA sensitivity)
- High frequency measurement electronics have 50 ohm input impedance to reduce reflection (i.e. matching)

Voltage & current measurement

- All measurement electronics have input resistance (also called input impedance for high frequency circuits)
- Always check maximum voltage/current ratings (!)

10 Mohm to 10 Gohm

0.5 ohm @ 1V/mA
to 2 Mohm @ 0.1V/pA



50 ohm

Signal & Noise

- Voltage, current, or power (mostly voltage) carrying the information about what we want to know is signal
- In most cases, *noise* means something random(e.g. thermal noise, shot noise, 1/f noise) and *interference* refers other bad effect caused by something periodic or with relative spectral purity (e.g. 60 Hz interference from AC power, kHz interference from switching power supply, MHz interference from FM radio station…)

Decibels (dB)

- Relative strength of signal in log scale
- dB = 20 log10 (Signal/Reference) for amplitude signal (e.g. voltage)

or = 10 log10 (Signal/Reference) for power signal (e.g. electric power)

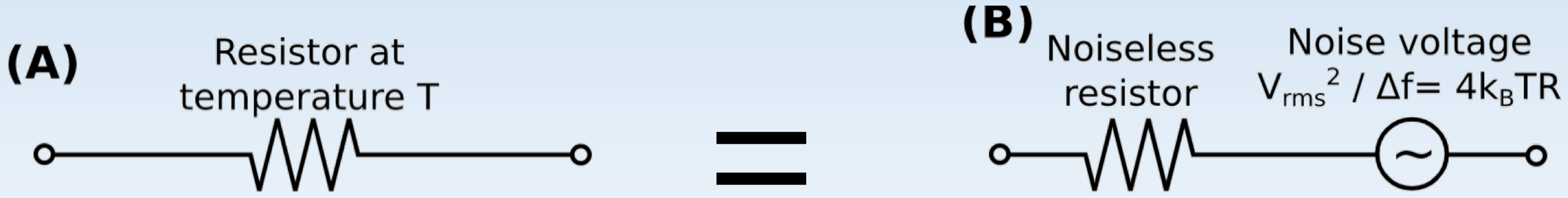
- Many variation's of dB's:
  - dBm = decibels relative to 1 mW power
  - dBV = decibels relative to 1 Vrms amplitude
  - dBc = decibels relative to power in the carrier frequency

Noise

- Noise voltage/current is random, but can have spectral properties; spectral density is convenient (e.g. voltage noise density e_n, current noise density i_n)
- RMS noise voltage or current is the integral of e_n or i_n over the measurement bandwidth (if measurement covers frequency range from f_1 to f_2, the bandwidth $B = f_2 – f_1$)
- Noise voltage/current from uncorrelated sources (most cases) should be combined by rms sum; $v_n = \sqrt{(v_1^2+v_2^2)}$

Noise

- Thermal fluctuation generates noise voltage across a resistor (Johnson noise)
  - Noise voltage (spectral) density $e_n = \sqrt{(4kTR)}$ (unit of $V/\sqrt{Hz}$)
  - White; no dependence in frequency
  - RMS noise voltage = $\sqrt{(4kTRB)}$



Noise

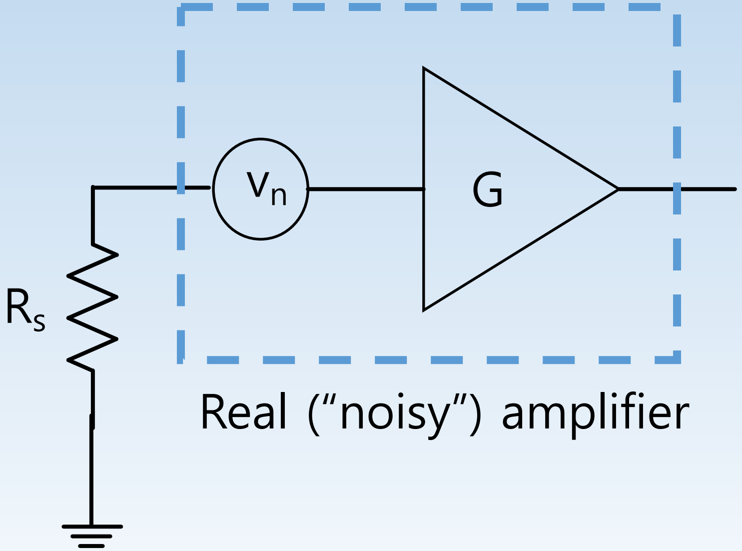
- Shot noise
  - Discreteness of charge generates fluctuation in current
  - Current noise density $i_n = \sqrt{(2eI)}$
- 1/f noise (flicker noise)
  - Real resistors have fluctuation in resistance; sources vary case by case
  - Noise power is close to 1/f (“pink”)

Signal to noise ratio (SNR)

- Usually expressed in dB (e.g. 10 log10(v_s^2/v_n^2))
- Measurement bandwidth and frequency dependent
- Any device with finite noise only *adds* noise; SNR worsens even for the best amplifier in the world!
- Noise is random, signal is not; SNR improves with averaging
- Usually, noise is wideband, signal is not; SNR improves with narrower bandwidth

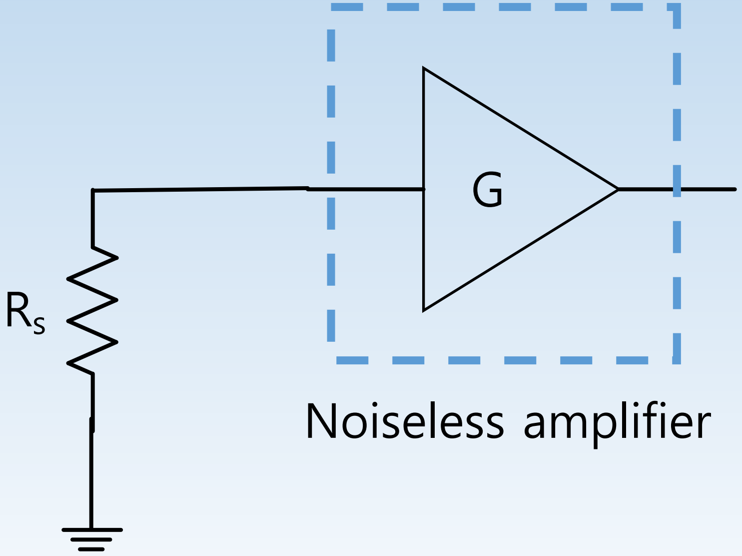
Noise figure (NF)

- Ratio in dB of the output of the real amplifier to the output of a noiseless amplifier with the same gain and source resistance.
- $NF = 10 \log10 [(4kTR_s + v_n{}^2)/(4kTR_s)]$



Real ("noisy") amplifier

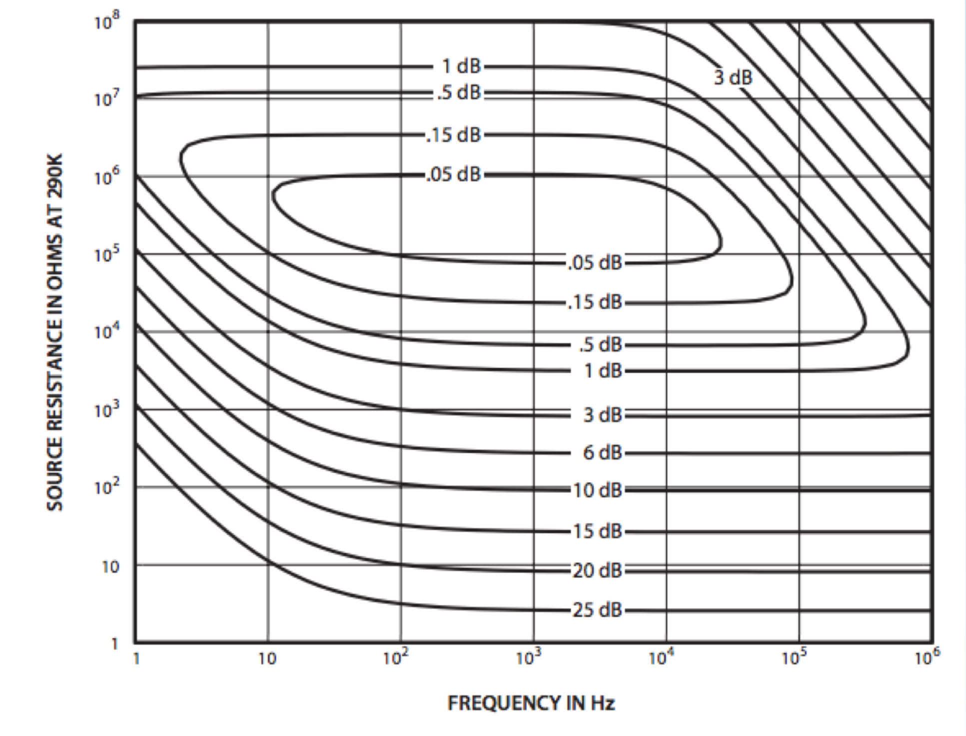
VS



Noiseless amplifier

Noise figure

- Ratio in dB of the output of the real amplifier to the output of a noiseless amplifier with the same gain and source resistance.
- NF = 10 log10 [$(4kTR_s + v_n^2)/(4kTR_s)$]
- Frequency and source resistance dependent



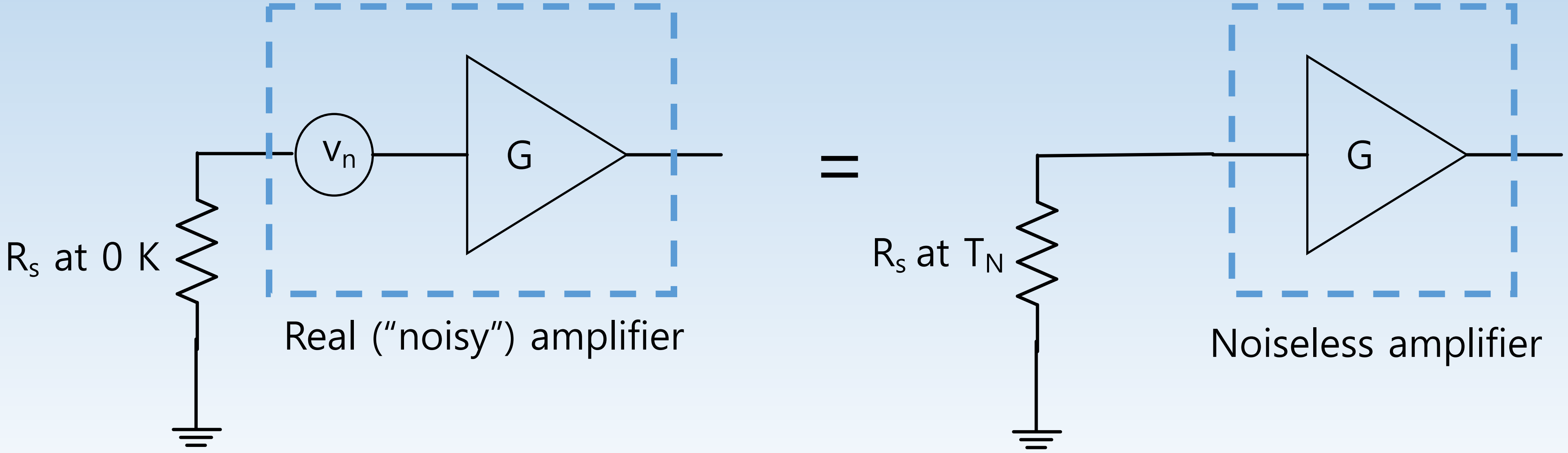
NF of SR560 amplifier

Noise figure

- Ratio in dB of the output of the real amplifier to the output of a noiseless amplifier with the same gain and source resistance.
- $NF = 10 \log10 [(4kTR_s + v_n^2)/(4kTR_s)]$
- Frequency and source resistance dependent
- $SNR = 10 \log10 (v_s^2/4kTR_s) - NF$

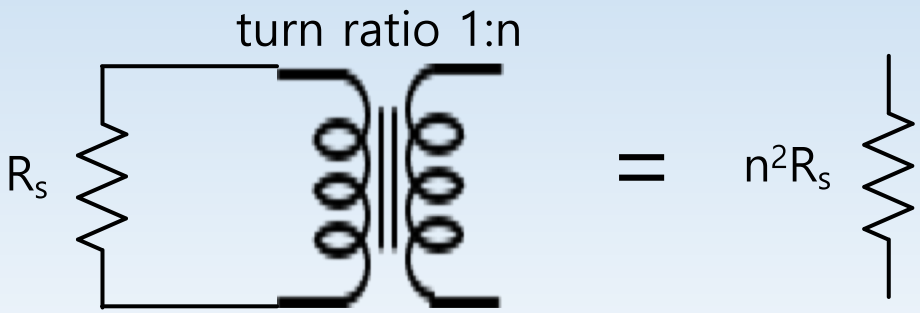
Noise temperature

- The temperature generating equal amount of amplifier noise at given source resistance (T_N)
- $NF = 10 \log10 [(T + T_N)/T]$



Note

- Adding resistors at the source to approach optimum source resistance *does not improve SNR* because of increased source Johnson noise
- Possible to reach optimum source resistance for the best NF (and best SNR) by converting the source impedance with reactive components. (e.g. transformer); “noise matching”



Interference

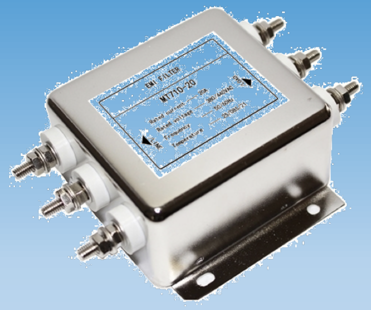
- Unwanted interfering signals other than random noises
- e.g. pickup from 60Hz power line pickup, switching mode power supply (20kHz ~ 2MHz)
- Can be reduced by filtering, shielding, grounding, etc.
- How they couple to circuit:
  - Capacitive(electrostatic) : high-impedance point of circuit
  - Magnetic : flux through closed loop
  - Electromagnetic : small section of wire acting as antenna
  - Current through ground lines or power lines (”common mode”)

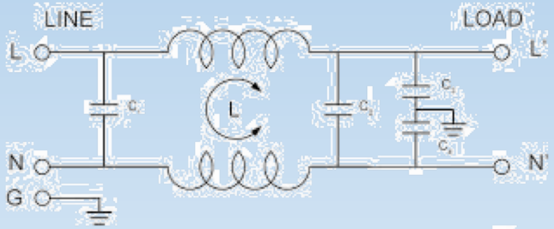
Interference

- How to avoid:
  - Raise signal(!)
  - Watch environmental factors:
    - Radio/TV station
    - Subway
    - High-voltage lines
    - Motors, elevators, heater, air conditioner etc.(consume high-current; generate power-line spikes)
    - Large transformer (magnetic coupling)
    - Welding equipment (high voltage & current)
  - Use components, shield, filters… (next)

Interference

- How to avoid:
  - Power line EMI ("electromagnetic interference") filters
  - Input/output filters
    - Signal at DC~kHz : low pass filter
    - Signal at MHz and above : band pass filter
  - Add shield (capacitive interference)
  - Twisted pair wire, mu-metal shield (magnetic interference)
  - Keep leads short, avoid loops, use ferrite beads (RF interference)





Note

- Shield should be at the circuit reference potential (i.e. ground)
- Avoid signal current flowing through shield; could interfere with others
- Be careful for shield-ground connection; avoid unnecessary ground loops
- Modern fast digital circuits (e.g. computer for data taking) could be the source of high-frequency interference; consider isolating via optical interface (e.g. GPIB optical isolator, RS-232 optical isolator…)

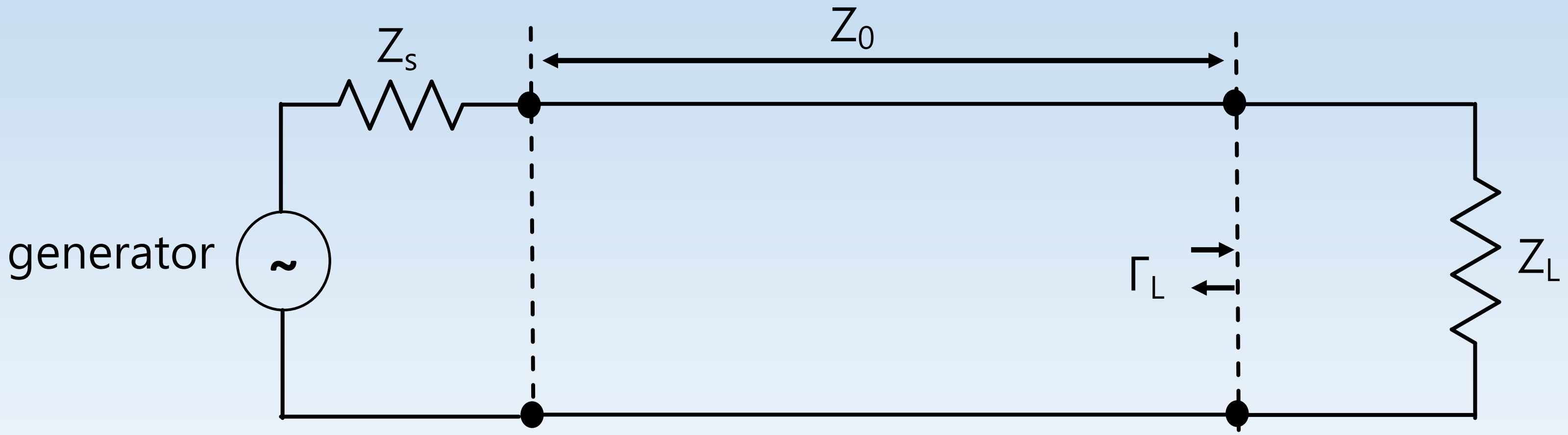
High-frequency circuit concepts

- Required when need to consider finite speed of light or wave-nature of electromagnetic propagation; i.e. length scale of circuit ~ wavelength
  - e.g. 2m BNC cable ~ 100 MHz wavelength (v_p ~ 0.8c); safely from 10 MHz and above in a typical lab set-up
- Common issues
  - Signals through cables can reflect back from load and make interference to generate frequency dependent transmission (“resonance”)
  - Delay through resonant part of amplifier feedback circuit generates self-oscillation (“instability”)
  - Pulse shape distortion (“overshoot” or “undershoot”)

Reflection coefficient (Γ)

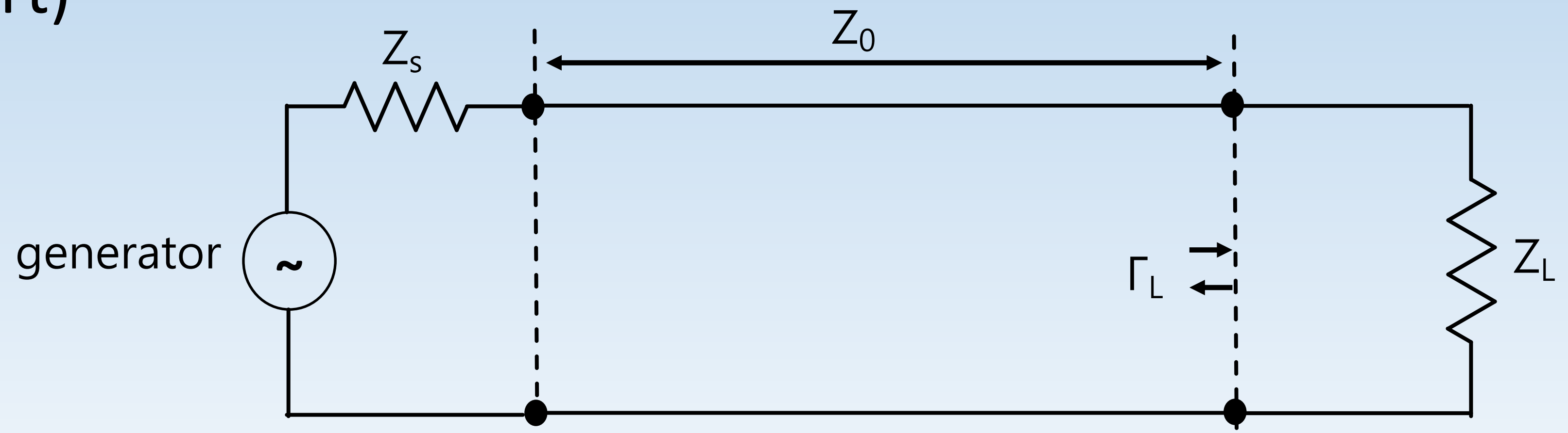
- Consider a load impedance Z_L at the end of a transmission line with characteristic impedance Z_0, reflection at the load:

$\Gamma_L = V_{reflected}/V_{incident} = (Z_L - Z_0)/(Z_L + Z_0)$



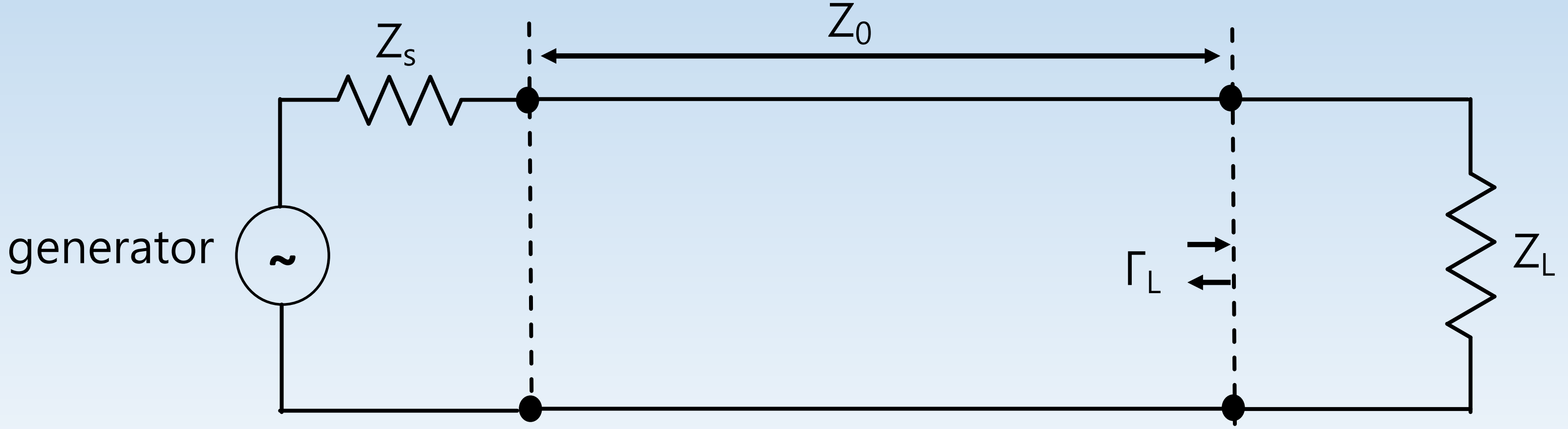
Reflection coefficient (Γ)

- $\Gamma_L = V_{reflected}/V_{incident} = (Z_L - Z_0)/(Z_L + Z_0)$
- Complex number affecting amplitude and phase of reflected wave
- Γ_L=0 when Z_L=Z_0(matched); Γ_L=1 when Z_L=inf (open); Γ_L=-1 when Z_L=0 (short)

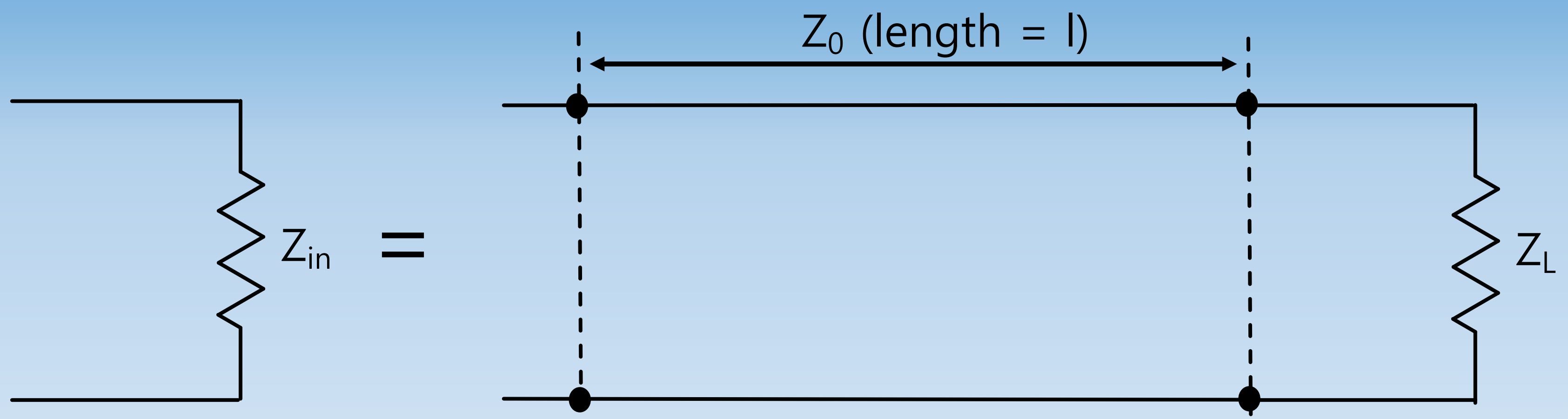


Reflection coefficient (Γ)

- $\Gamma_L = V_{reflected}/V_{incident} = (Z_L - Z_0)/(Z_L + Z_0)$
- If source and load are both matched, no reflection from either side of transmission line; perfect power transfer from generator(i.e. power dissipation at source = power dissipation at load; "power matching")

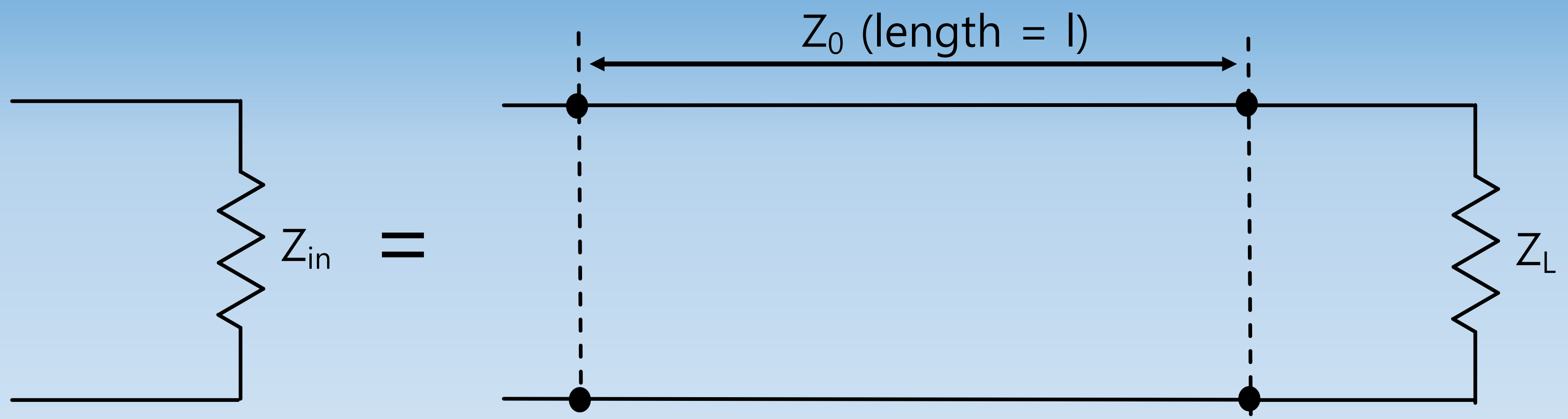


Input impedance of transmission line



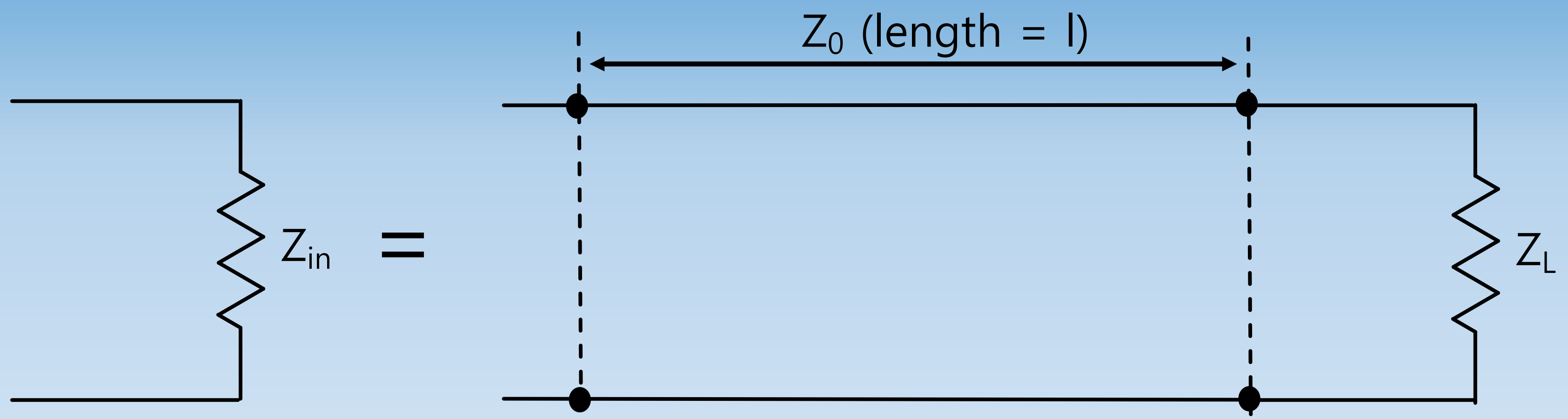
- $Z_{in} = Z_0 (Z_L + j Z_0 \tan kl)/(Z_0 + j Z_L \tan kl)$ ($k = 2\pi/\lambda$; λ=wavelength)
- Section of transmission line looks like a controllable impedance(!)

Input impedance of transmission line



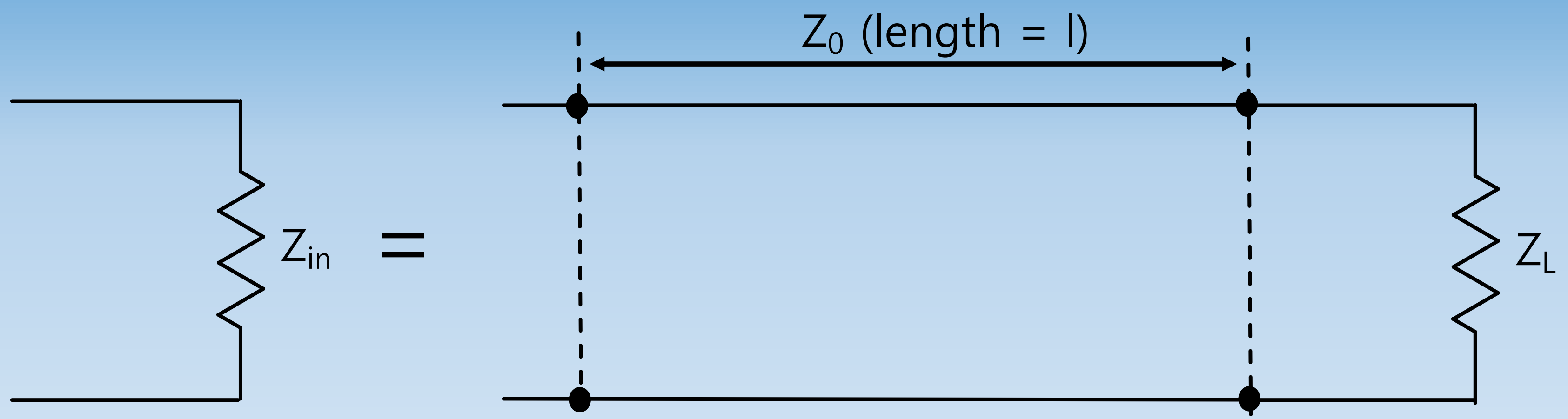
- $Z_{in} = Z_0 (Z_L + j Z_0 \tan kl)/(Z_0 + j Z_L \tan kl)$ ($k = 2\pi/\lambda$; λ=wavelength)
- $Z_{in} = Z_L$: i) $l=0$ (trivial) , ii) $k=0$ (DC), iii) $l= n\lambda/2$ (absent when multiple of half wavelength)   iv) $Z_L= Z_0$ (matched)

Input impedance of transmission line



- $Z_{in} = Z_0 (Z_L + j Z_0 \tan kl)/(Z_0 + j Z_L \tan kl)$ ($k = 2\pi/\lambda$; λ=wavelength)
- Z_L = 0 or inf (short or open): $Z_{in} = j Z_0 \tan kl$ or $-j Z_0 \cot kl$ (pure reactance)
- $l = \lambda/4$: Z_0^2/Z_L (quarter-wave transformer)

Input impedance of transmission line

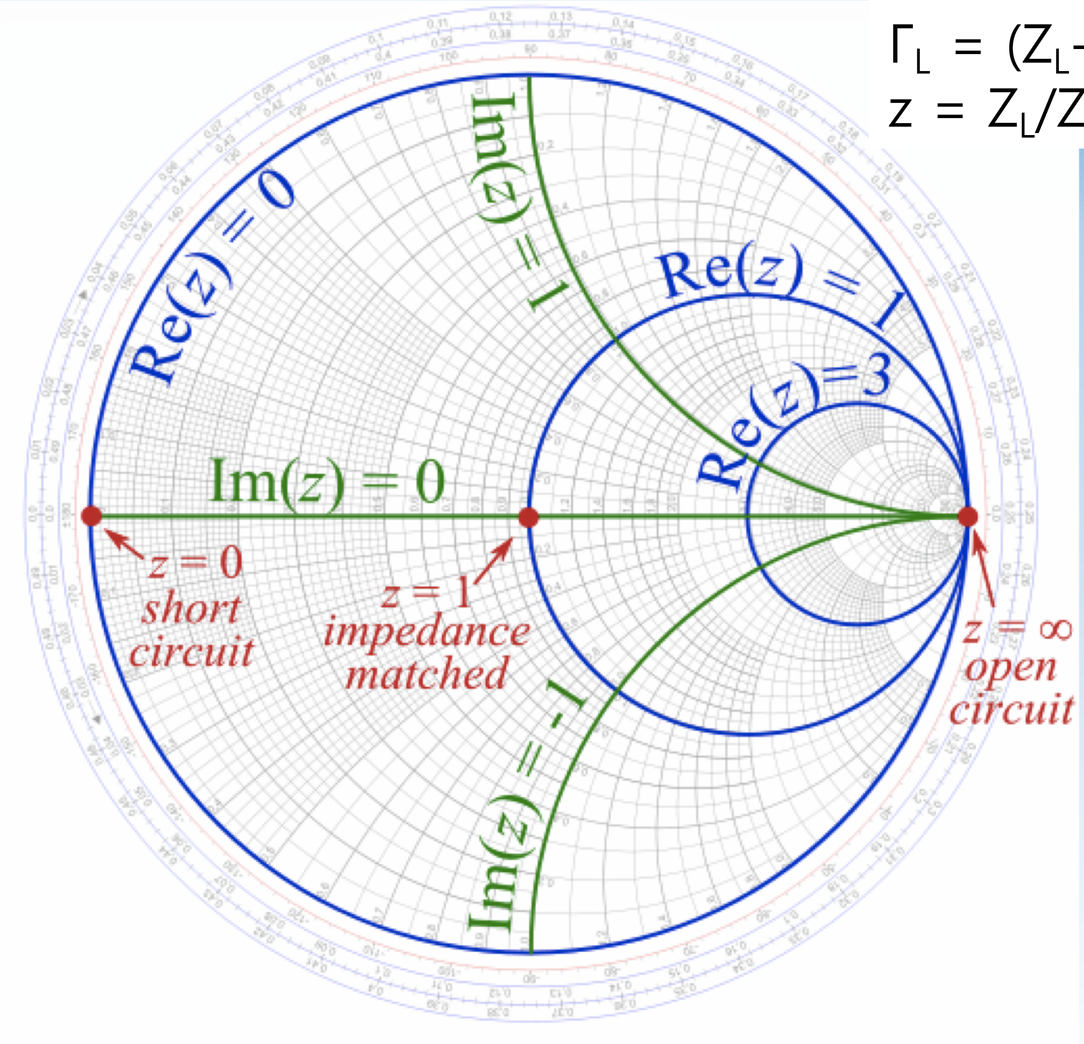


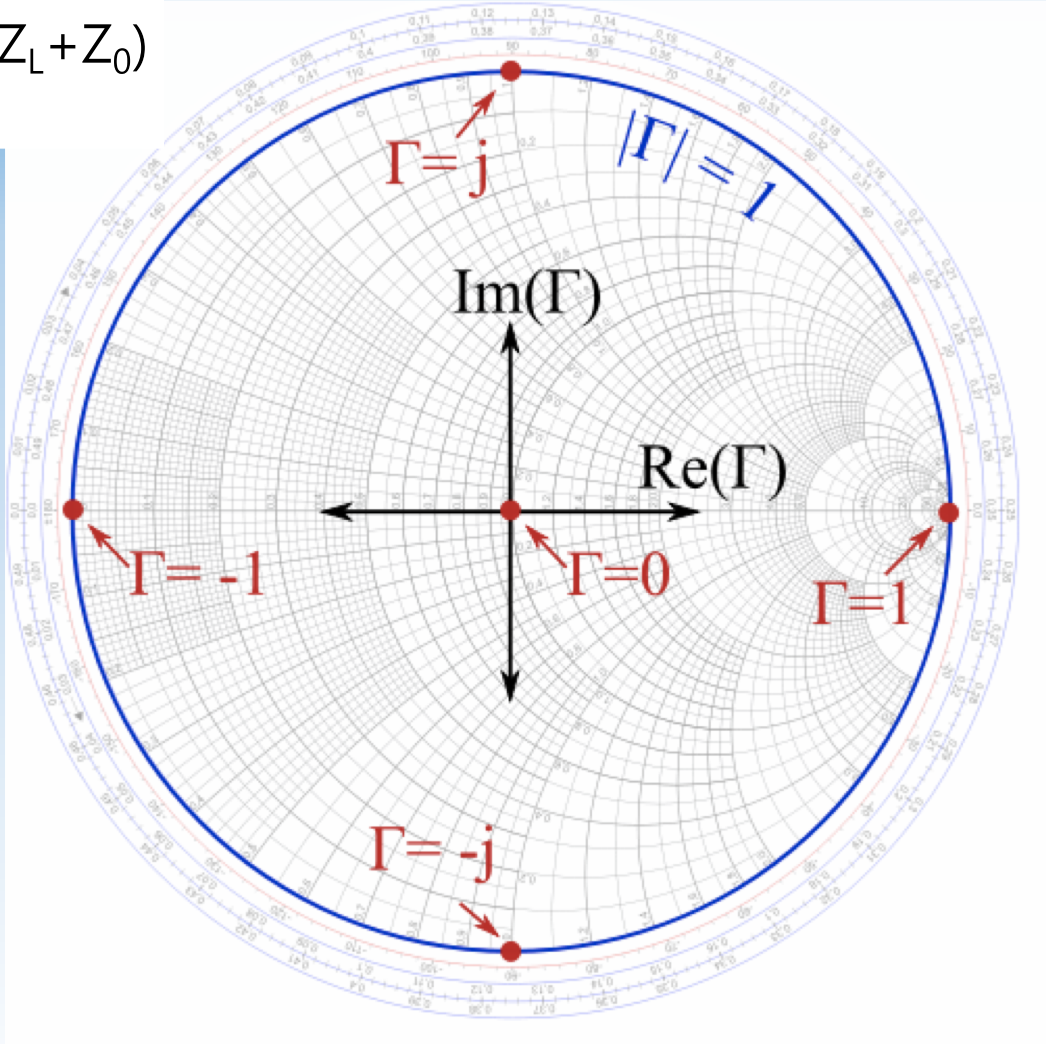
- $Z_{in} = Z_0\,(Z_L + j\,Z_0 \tan kl)/(Z_0 + j\,Z_L \tan kl)$ ($k = 2\pi/\lambda$; λ=wavelength)
- For any Z_L at certain frequency, always possible to choose length to turn $Re(Z_{in})=Z_0$, then add a section of transmission line to cancel $Im(Z_{in})$: "stub tuning for impedance matching" (next class)

Smith chart: graphical representation of reflection coefficient

$\Gamma_L = (Z_L - Z_0)/(Z_L + Z_0)$

$z = Z_L/Z_0$





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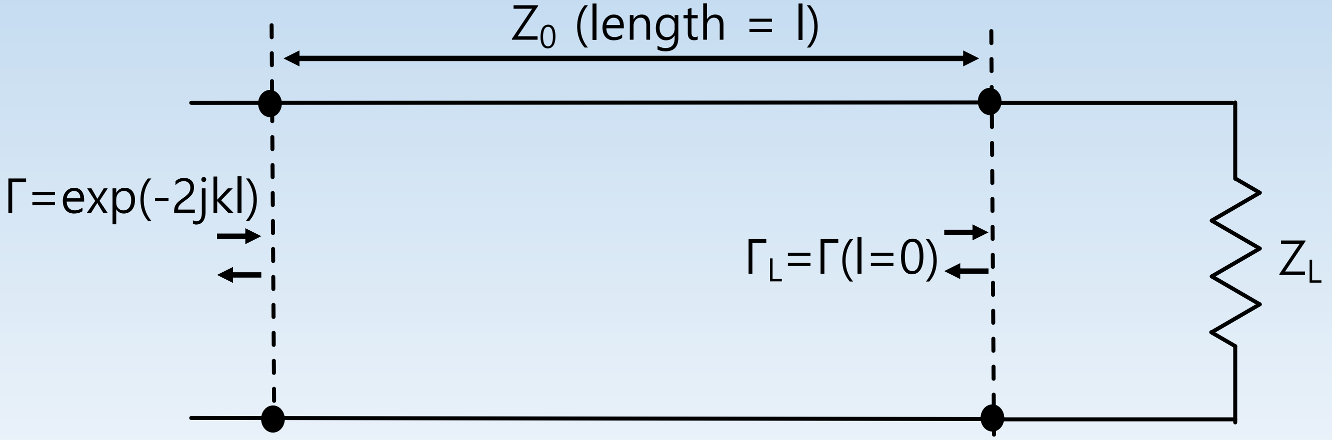
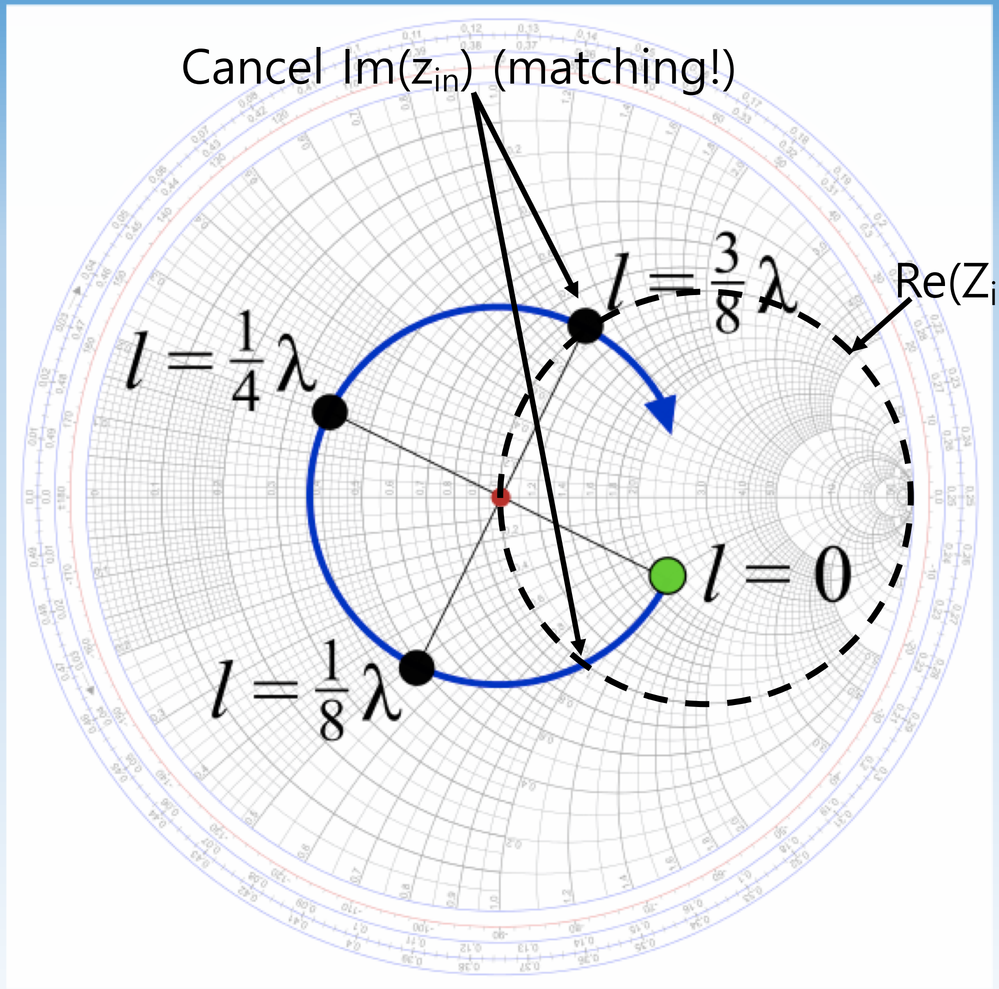
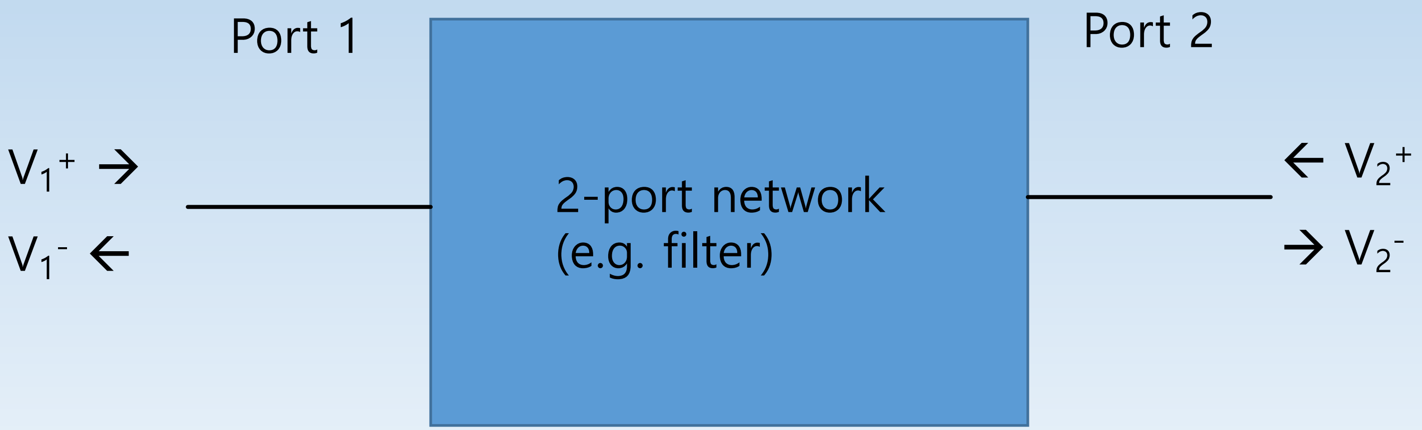




Scattering parameter (S-parameter)

- 2-port case (can be expanded for N-port S-parameters similarly)
- Assume all ports are terminated with Z_0 (i.e. no reflections)



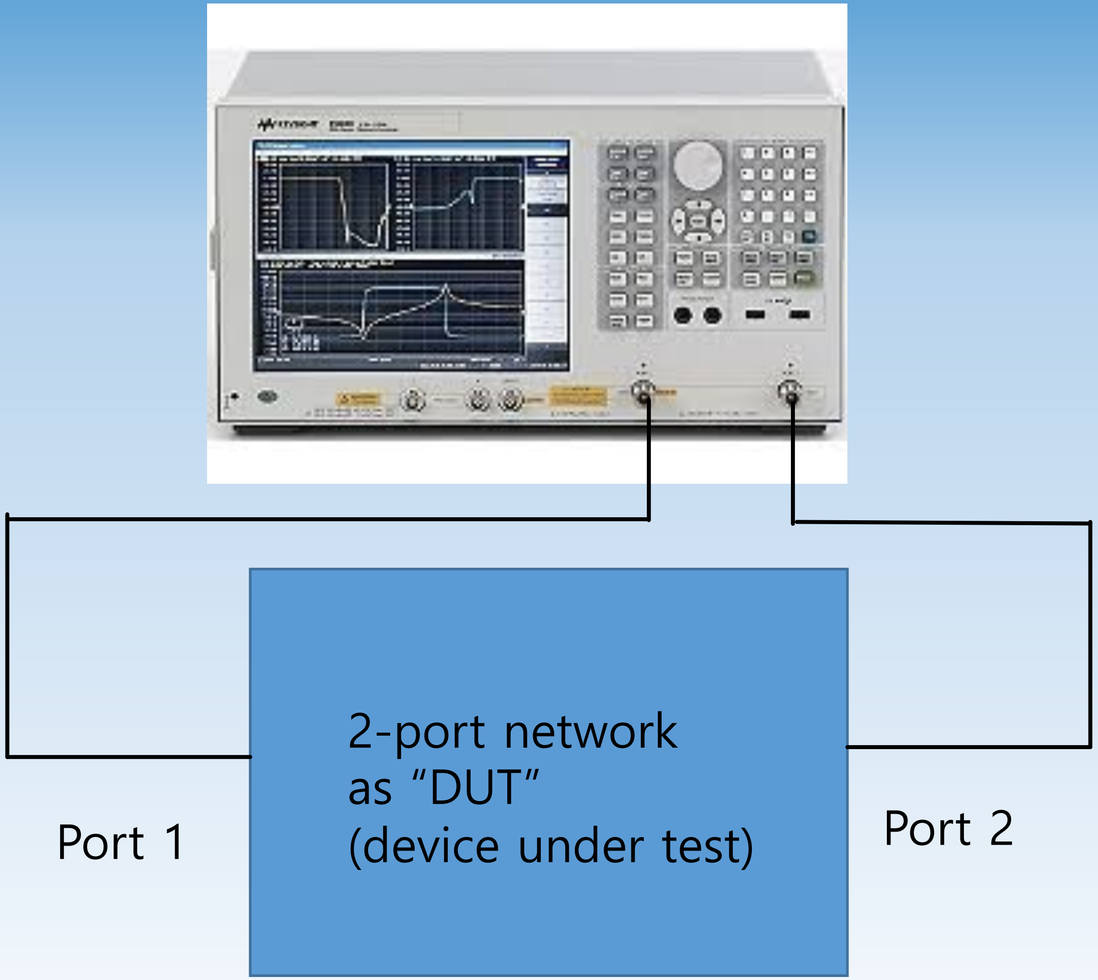
$$S = \begin{pmatrix} S11 & S12 \\ S21 & S22 \end{pmatrix}$$

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S11 & S12 \\ S21 & S22 \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}$$

- Outgoing voltage = S * incoming voltage
- S11 : port 1 reflection coefficient
- S21 : port 2 to 1 voltage (forward) gain
- S12 : port 1 to 2 voltage (reverse) gain
- S22 : port 2 reflection coefficient

Vector network analyzer (VNA)

- Instrument for acquiring scattering parameters
- Usually 2 port; n-port options provided
- Cable effect needs to be calibrated out with standards(short,open,thru, teminations); refer manual for details



Spectrum Analyzer (or signal analyzer)

- Instrument for analyzing spectral information of RF signal
- Measure RF power at each frequency within certain bandwidth (resolution bandwidth;"RBW")
- Modern analyzers provide FFT (fast digital fourier transform)

Note

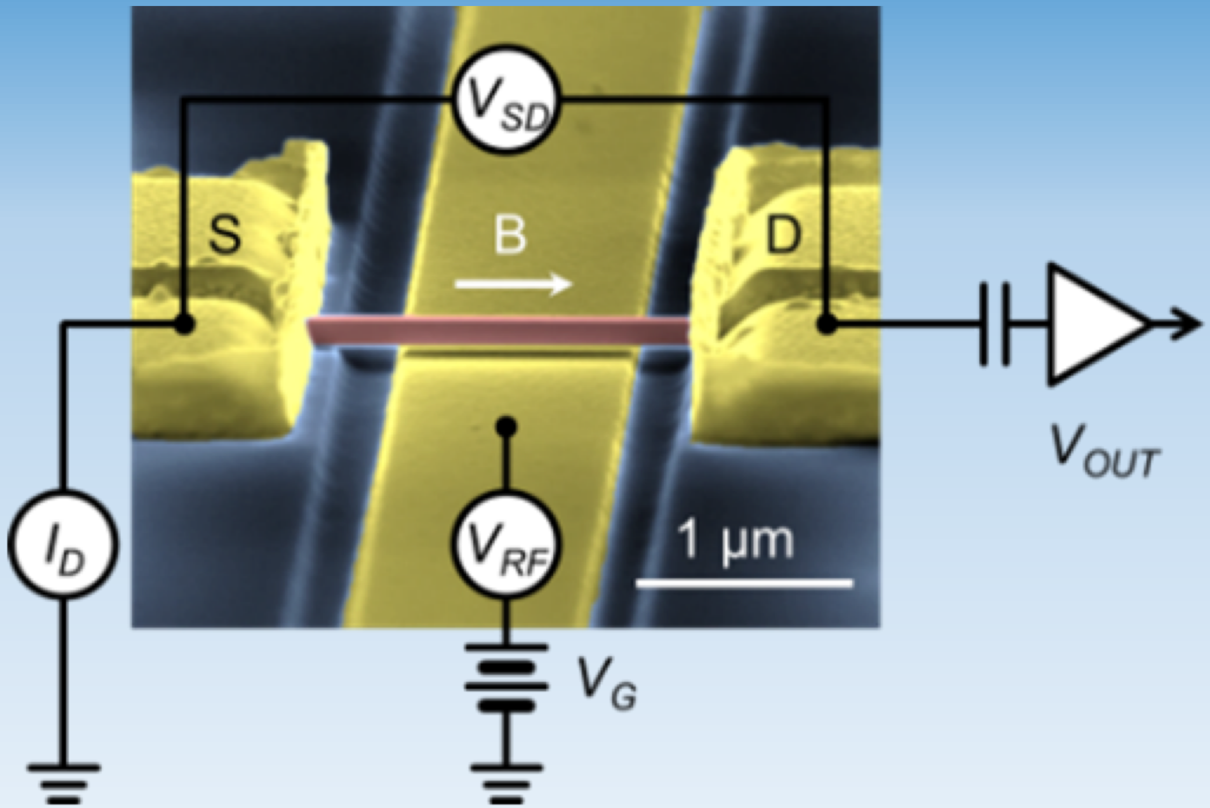
- Handle high-frequency cables and connectors carefully for optimal performance
- Use torque wrenches for connectors that require tightening nuts (e.g. SMA)
- RF instruments usually hate DC voltages and current; read manual when doubt
- RF instruments have maximum power that can be handled; check manual! it will either perform non-linear or even break with too much power

Outline

- Basic concepts (1 hour)
  - Voltage, current
  - Resistance, inductance, capacitance
  - Impedance, admittance
  - Signal, noise, interference
  - High frequency circuit concepts
  - Q&A + break (10 min)

- Examples (1 hour)
  - Low-noise & low temperature conductance measurement
  - Inductive/capacitive/microwave detection of mechanical oscillator
  - Q&A + break (10 min)

Example: DC and RF measurements of nanowire

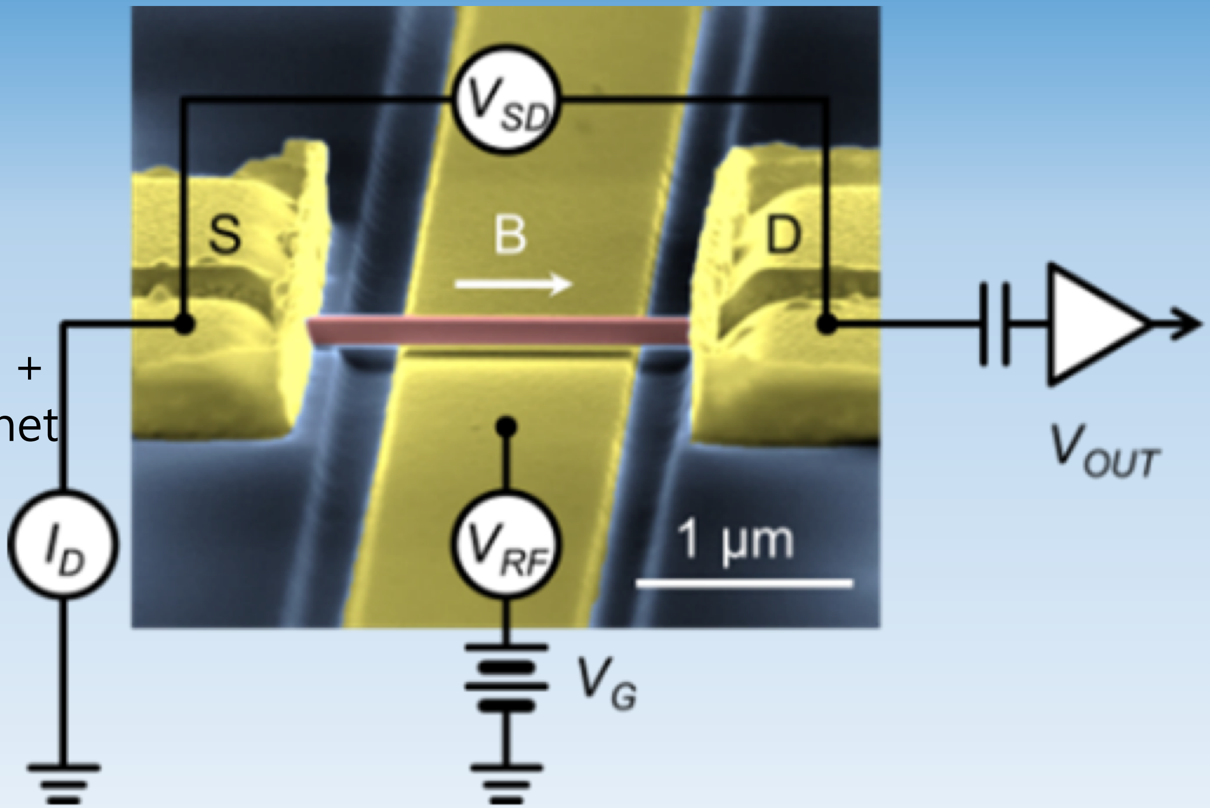


*Nanomechanical characterization of quantum interference in a topological insulator nanowire, *Nature Comm.* **10**, 4522 (2019).

Example: DC and RF measurements of nanowire

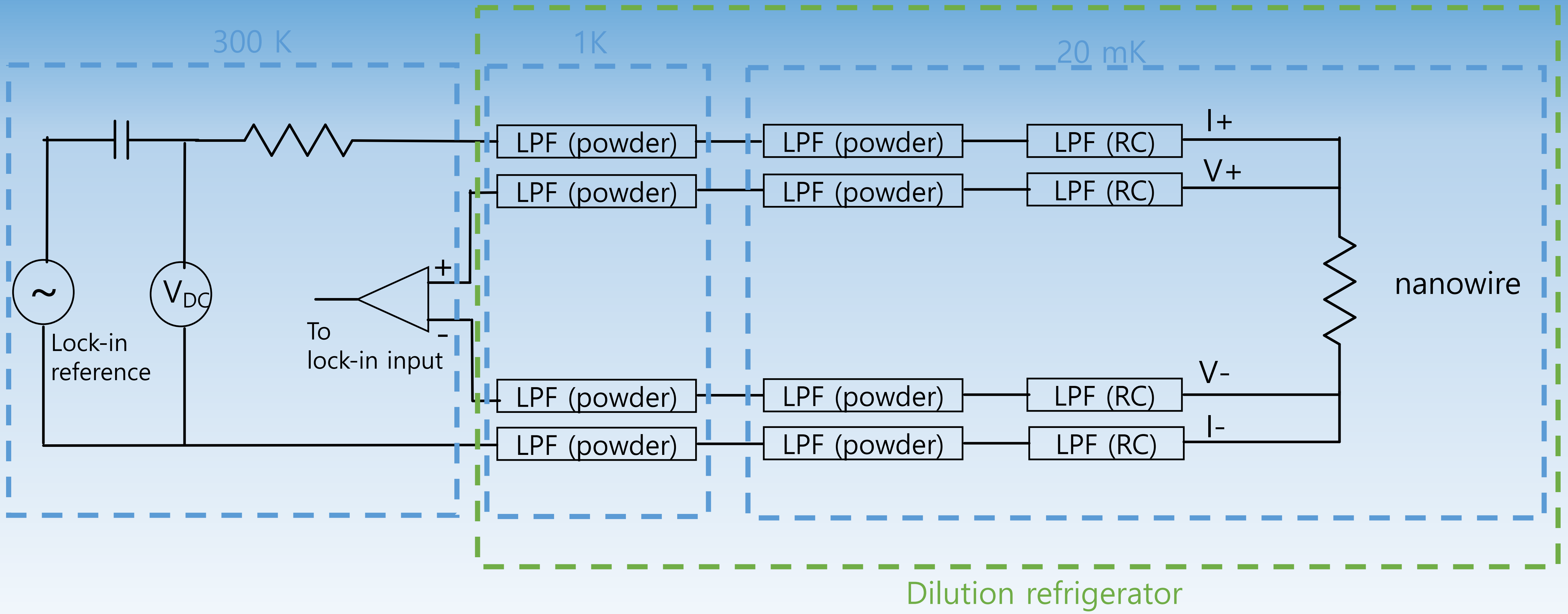
T = 20 mK
B = 0 to 9 T

= Dilution refrigerator + superconducting magnet



*Nanomechanical characterization of quantum interference in a topological insulator nanowire, *Nature Comm.* **10**, 4522 (2019).

DC conductance measurement



Lock-in amplifier

- Amplifies signal modulated at certain frequency and phase
- “synchronous” detection
- averages weak periodic signal under random noise (SNR ~ 1/√(# of averages))

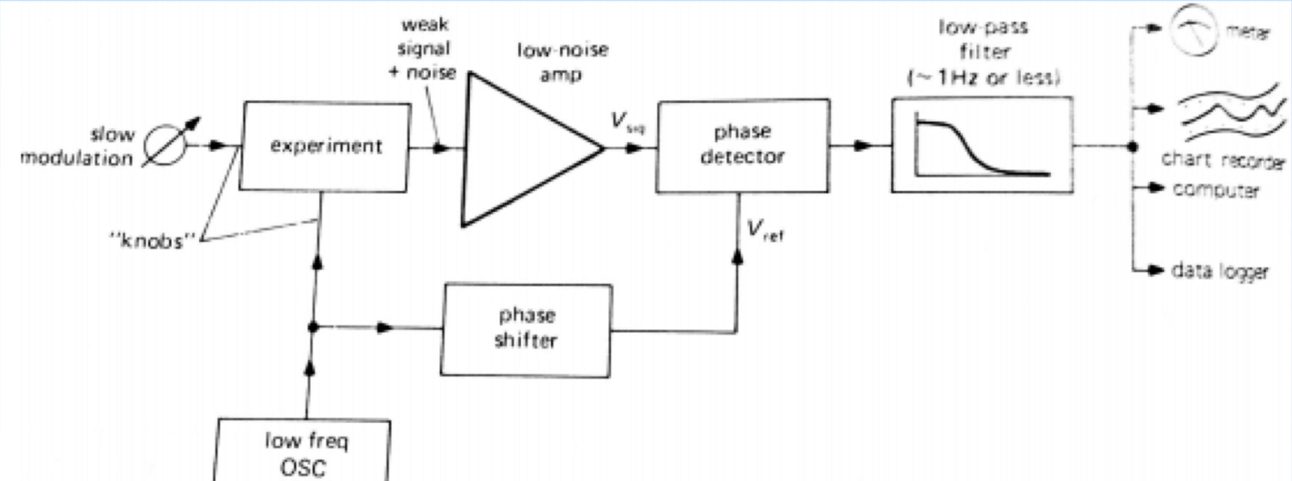


Figure 15.38. Lock-in detection.

Lock-in amplifier

- Enable differential conductance (dI/dV) measurement
- AC measurement; averages out slow drift in bias current

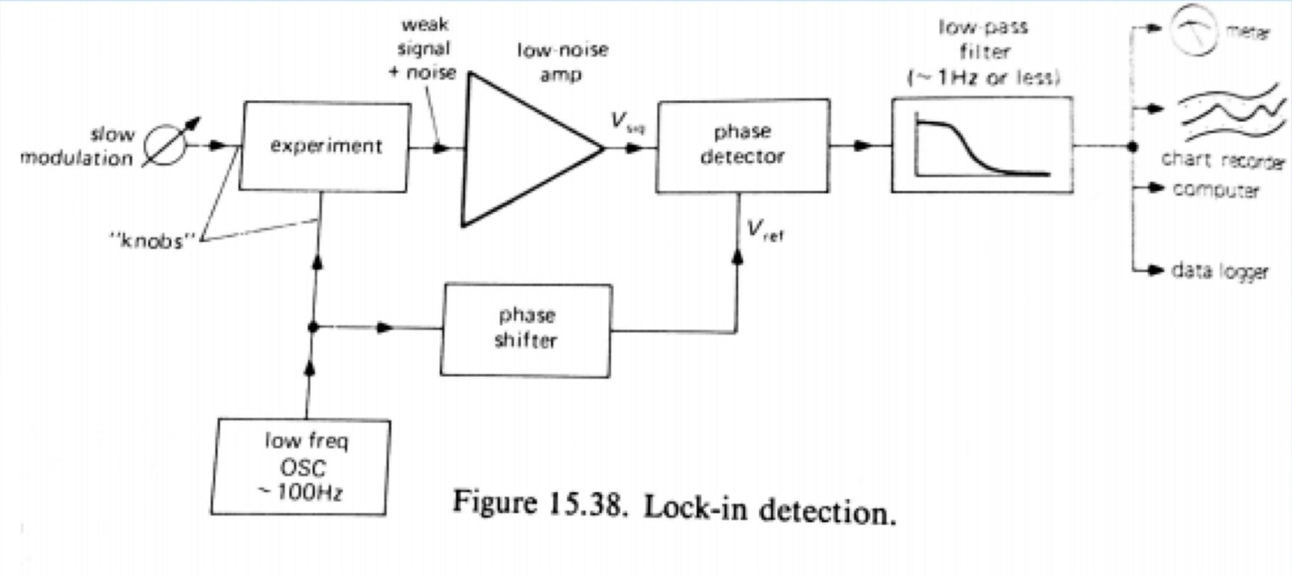
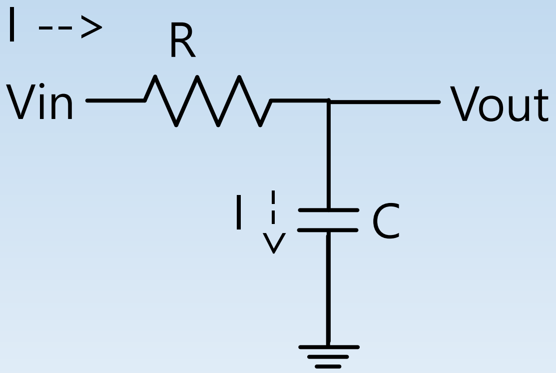


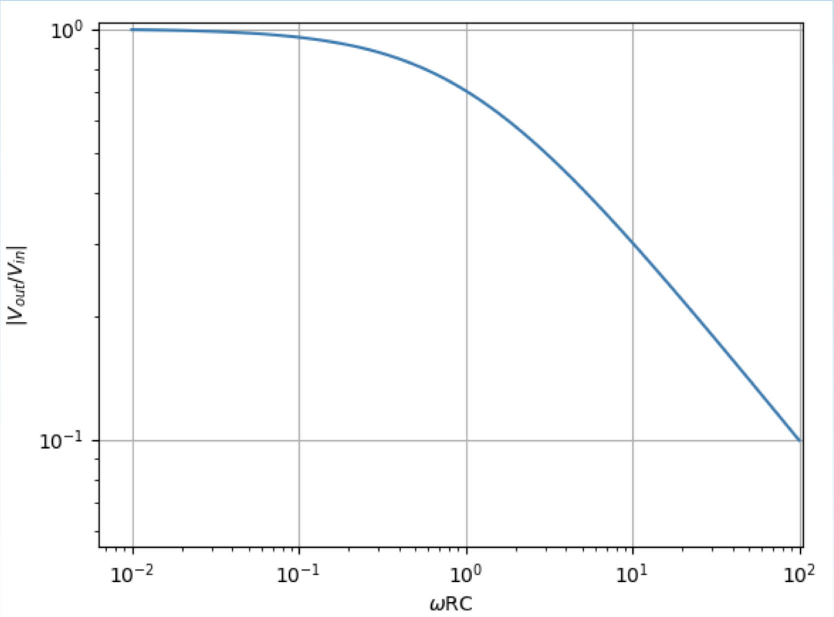
Figure 15.38. Lock-in detection.

Low-pass filter

- Attenuates thermal noise from room temperature
- Filters unwanted interference
- Cold bath for cooling electrons in the circuit



Simple RC filter; works up to ~MHz



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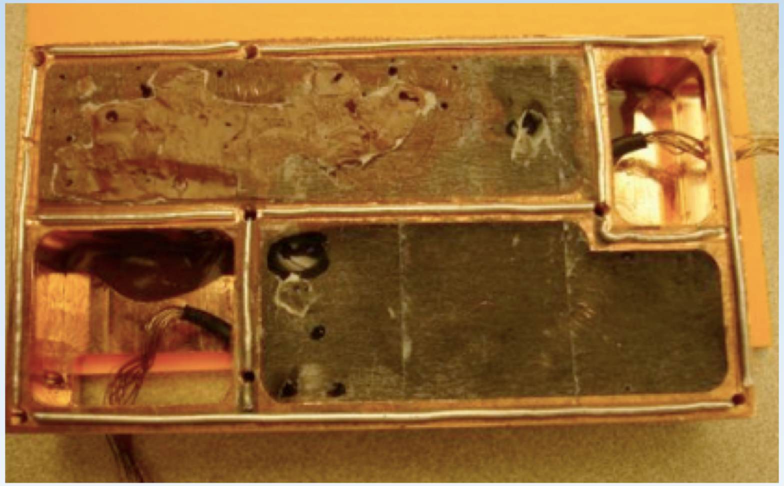
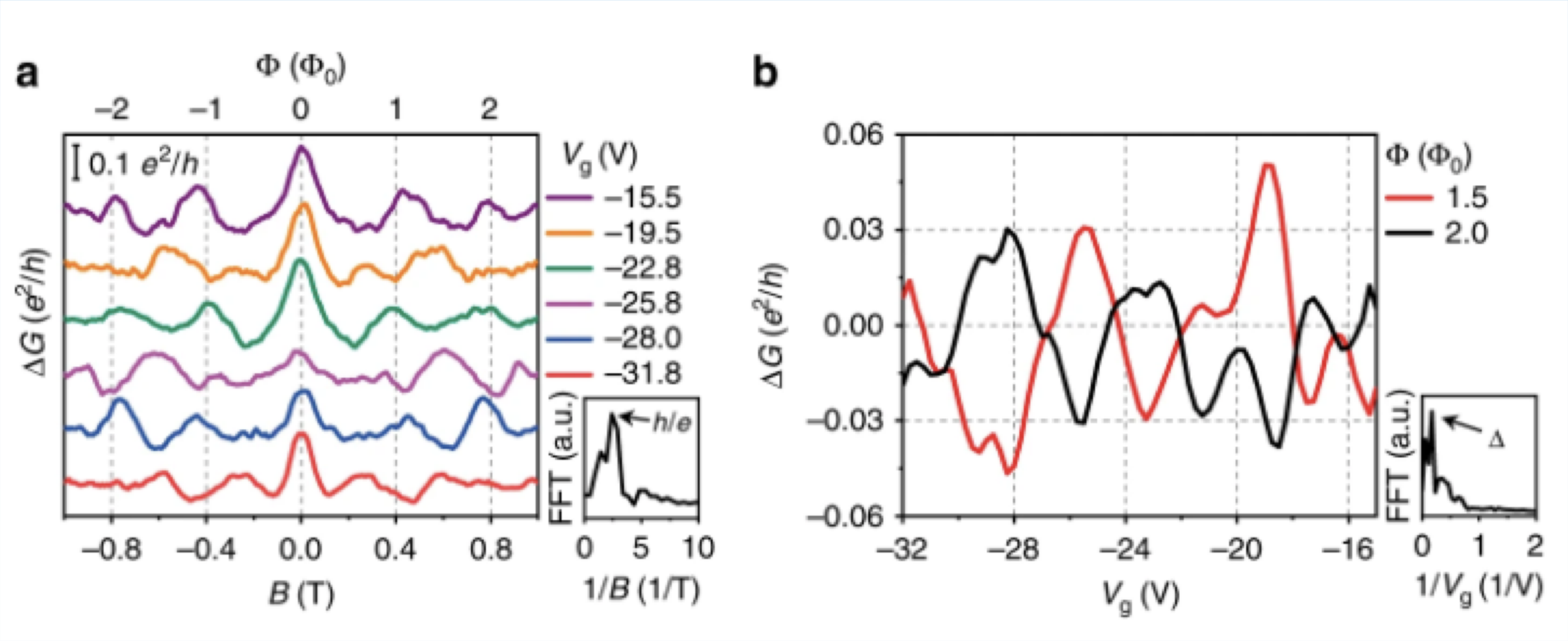


Powder filter; works up to >~20GHz

- Wires immersed in metal powder (diameter ~ 10um) and epoxy mixture
- Skin depth reduces as frequency goes up; higher attenuation
- Filtering GHz and above can be also done with: lossy coaxial cables (e.g.thermocoax), silver epoxy around wires; many literatures…

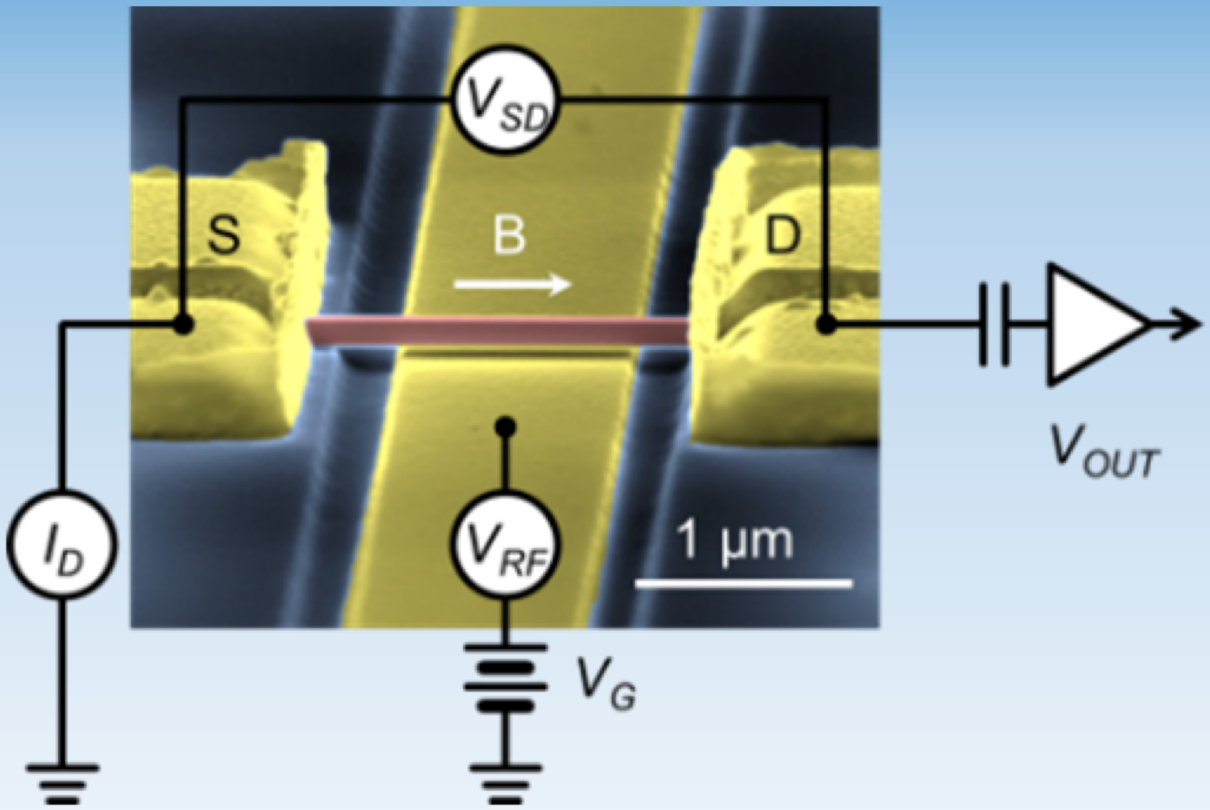
Aharonov-Bohm oscillation of conductance from TI surface states



*Nanomechanical characterization of quantum interference in a topological insulator nanowire, *Nature Comm.* **10**, 4522 (2019).

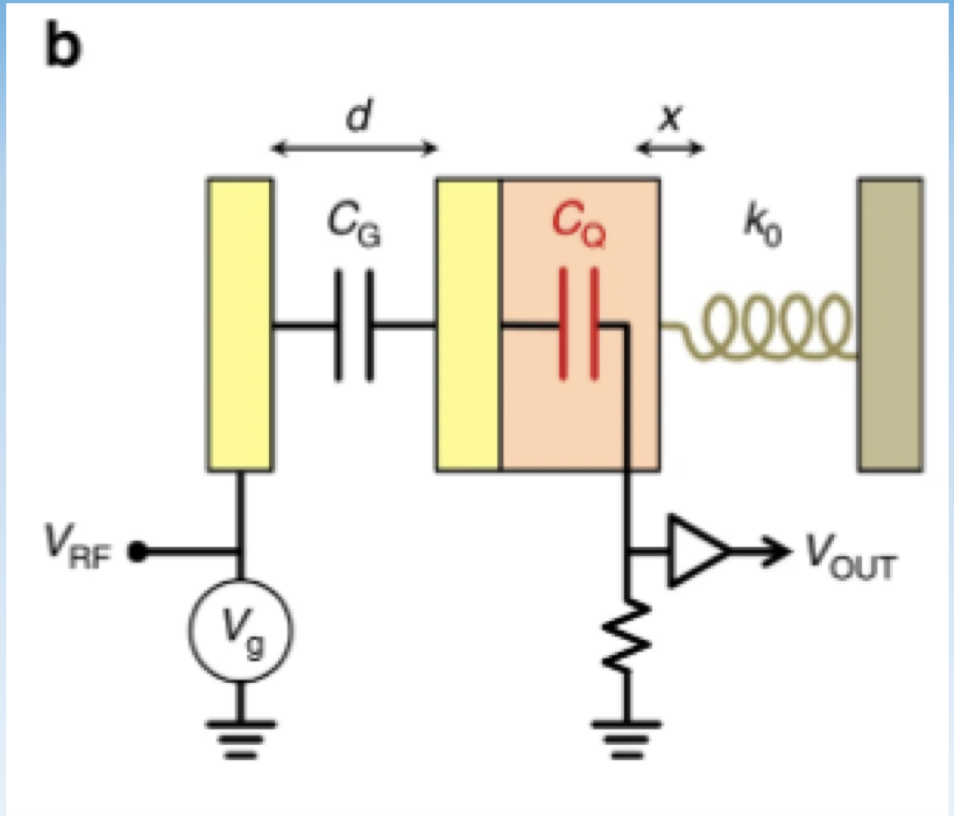
RF measurements of nanowire

- Goal: detect mechanical resonance of suspended nanowire
- Method: apply DC and RF voltage on the gate electrode to excite mechanical resonance; measure RF current generated by mechanical resonance via gate capacitance



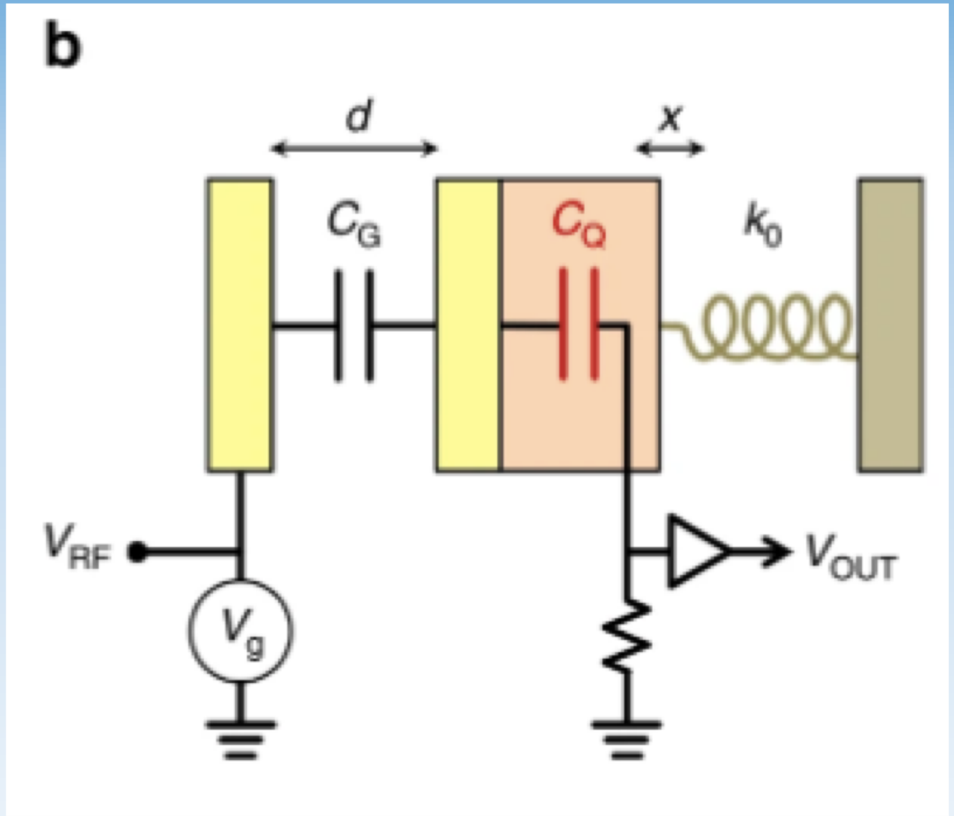
RF measurements of nanowire

- Assume metal wire (C_Q=inf)
- Mechanical motion "x" changes gate capacitance C_G
- Change in C_G induces current as, $I_{RF} = V_g (dC_G/dx)(dx/dt)$
- I_{RF} through input impedance of amplifier generates RF voltage that is amplified and recorded



RF measurements of nanowire

- C_G ~ 10 aF, V_g ~ 10 V
- Mechanical resonance ~ 100 MHz, gap between gate and nanowire ~ 100 nm, mechanical vibration amplitude ~ 100 pm
- I_{RF} ~ 100 pA, V_{RF}~5 nV
- Cryogenic amplifier T_N~10 K; e_n~ 70 pV/√ Hz
- Reach SNR = 0 dB at 5 kHz bandwidth



RF measurements of nanowire

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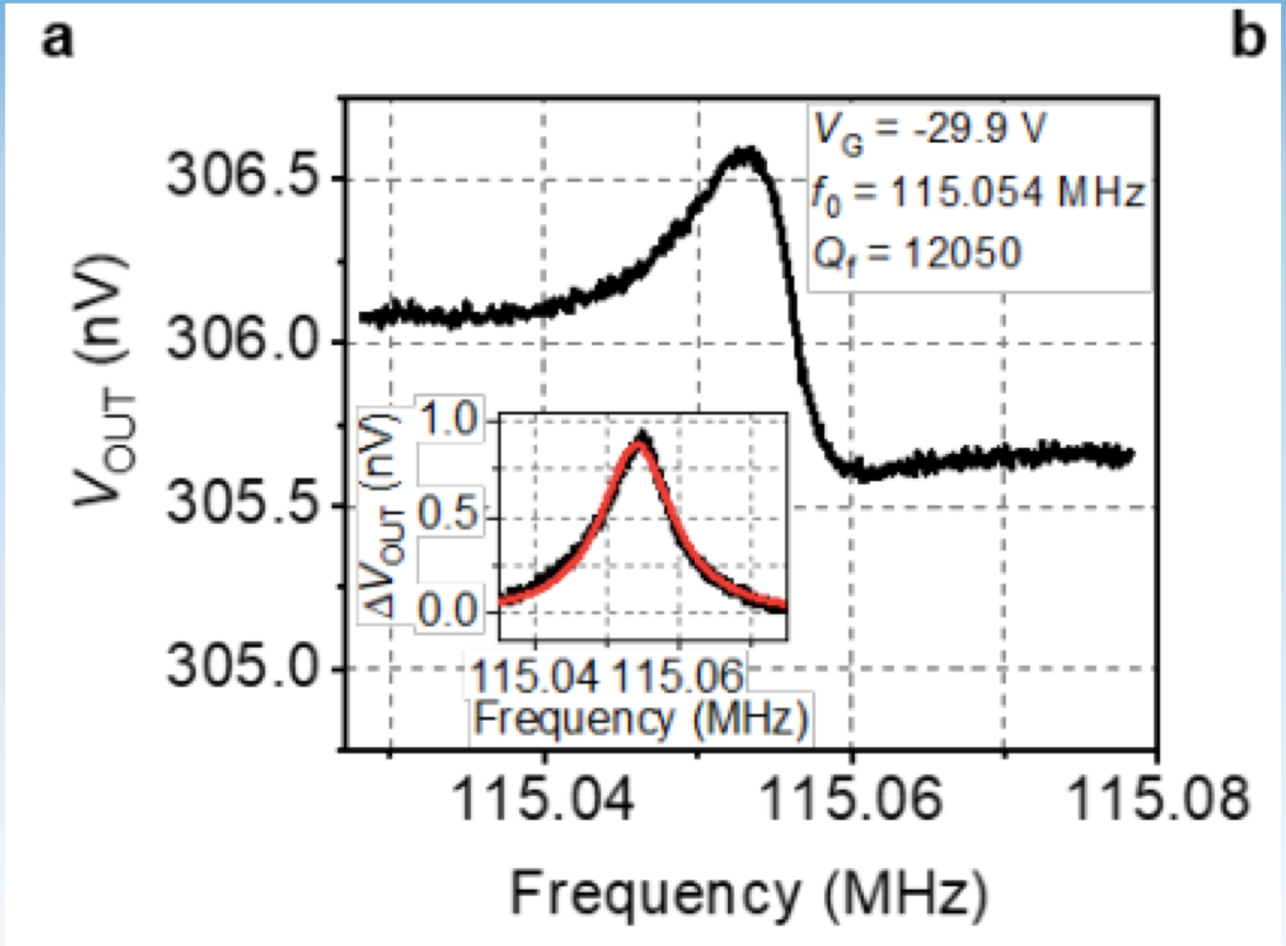
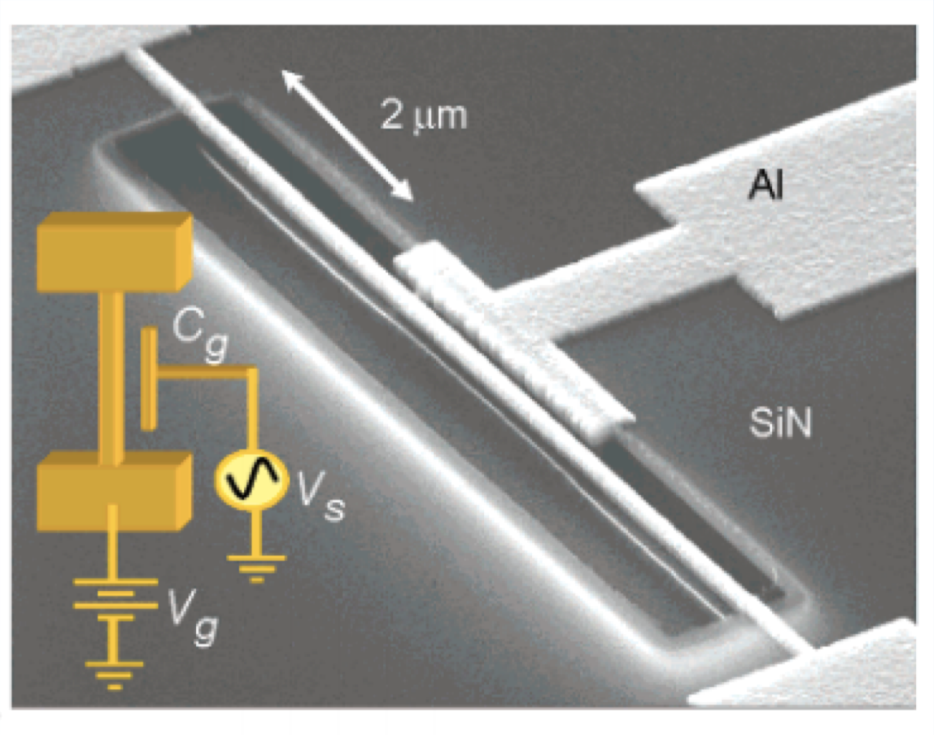
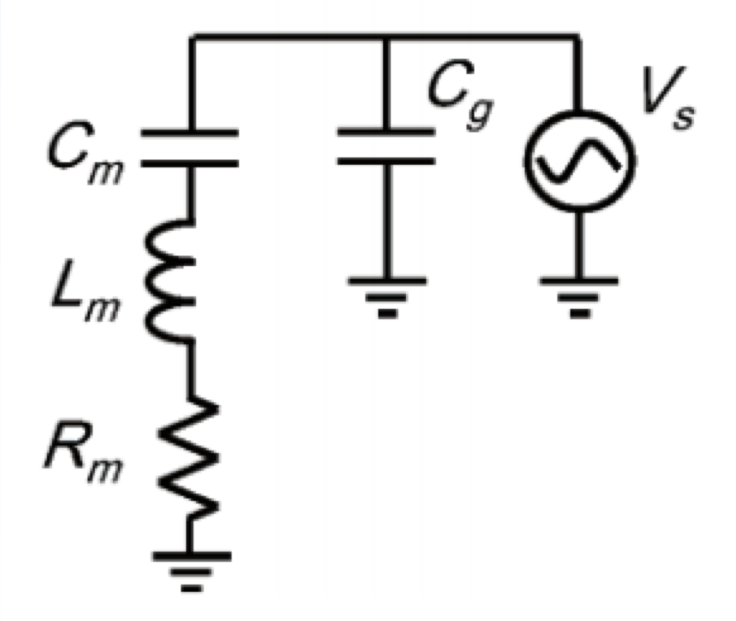


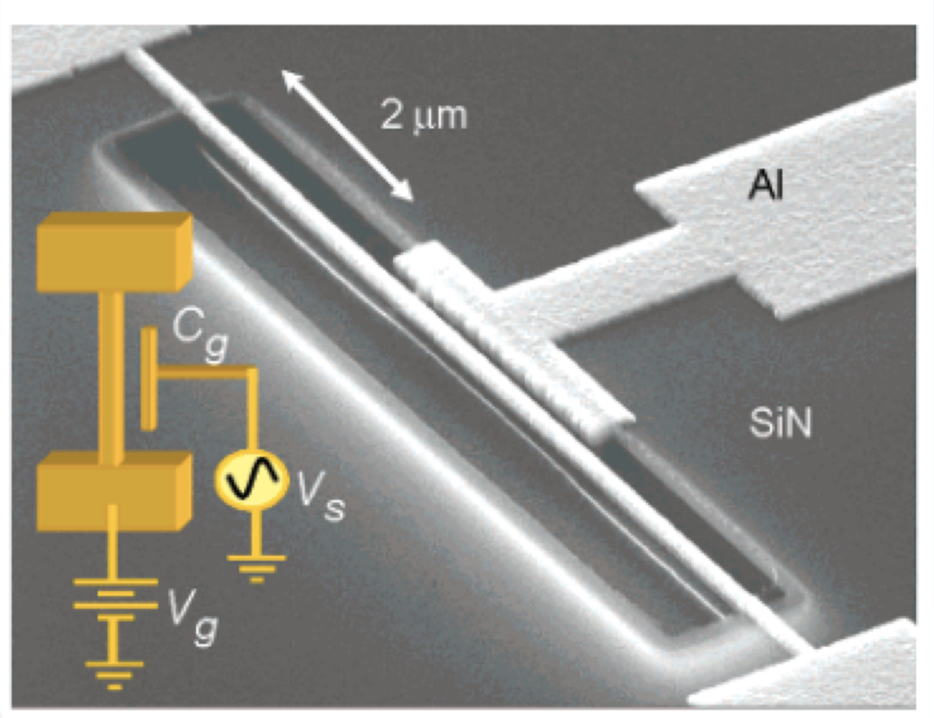
Capacitive detection of nanomechanical motion

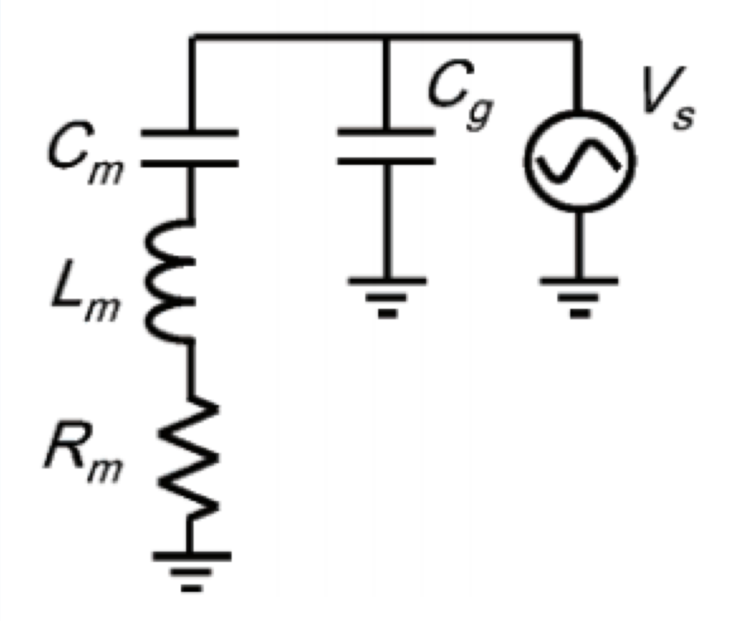




* Truitt *et.al.*, Efficient and Sensitive Capacitive Readout of Nanomechanical Resonator Arrays, *Nano Lett.* **7**, 120 (2007).

Capacitive detection of nanomechanical motion



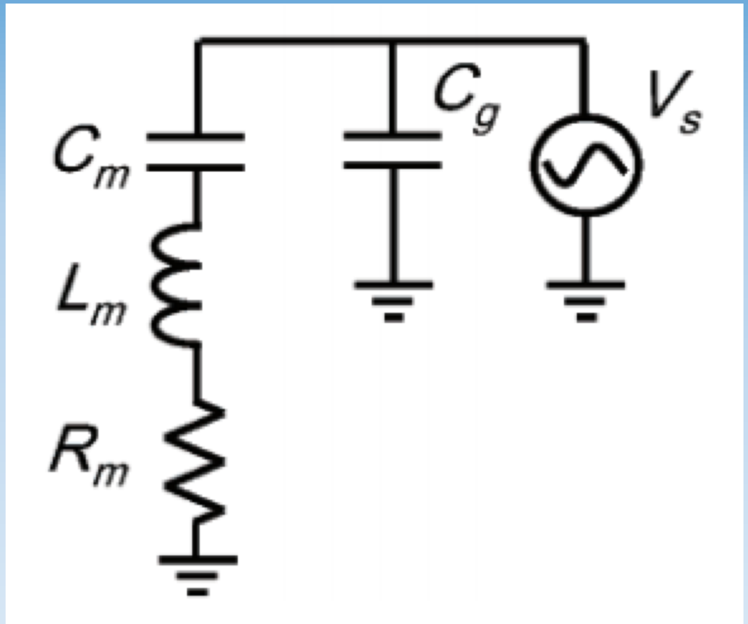


- Consider force due to V_s and V_g: $F = (dC_g/dx)V_sV_g$
- Force induces mechanical motion "x" and changes gate capacitance C_G; Change in C_G induces current as, $I_{RF} = V_g (dC_G/dx)(dx/dt)$
- Equivalent impedance = V_g/I_{RF} equal to the circuit diagram

Capacitive detection of nanomechanical motion

a series RLC circuit, where $L_m = d^2 m / V_g^2 C_g^2$, $C_m = V_g^2 C_g^2 / \omega_0^2 d^2 m$, and $R_m = d^2 m\, \omega_0 / (V_g^2 C_g^2 Q)$. Here, m is the effective (lumped) resonator mass, d is the equilibrium gap between the gate and the resonator center, and ω_0 and Q are the nanomechanical resonant angular frequency and the quality factor, respectively. We have made the approximation $\partial C_g / \partial x \cong -C_g / d$,

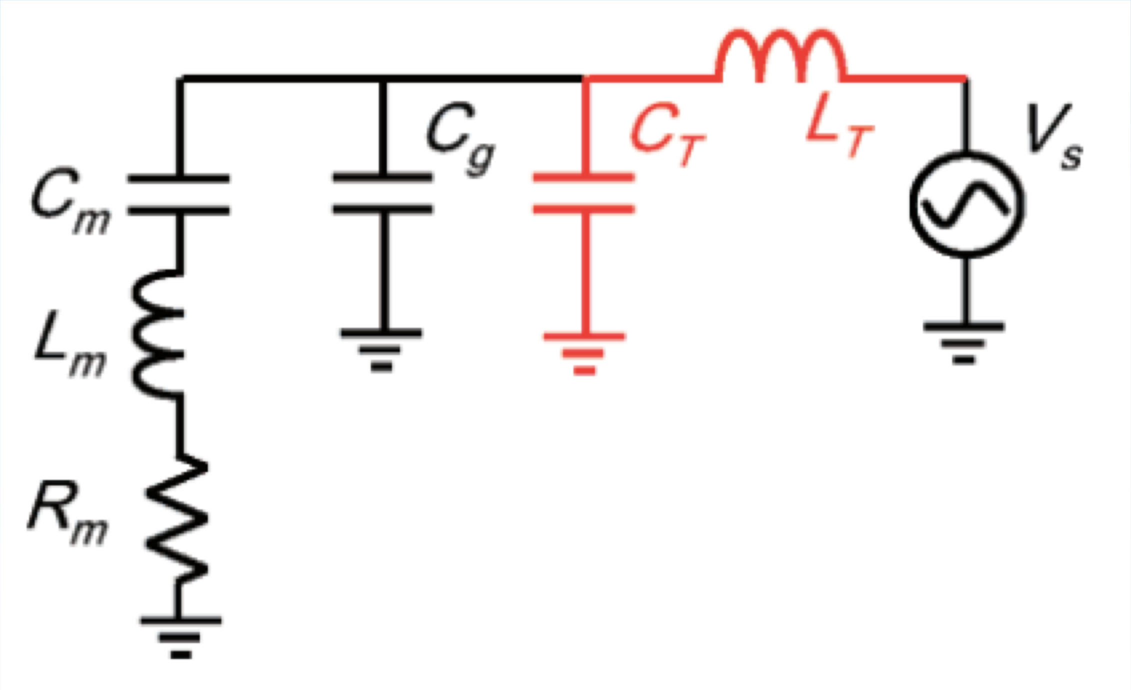
$m \approx 1.2 \times 10^{-15}$ kg, $Q \approx 26500$, $\omega_0 / 2\pi \approx 11$ MHz, and $C_g / d \approx 0.3$ aF/nm, with $C_g \approx 54$ aF, $d \approx 180$ nm), one finds that $L_m \approx 59$ H, $C_m \approx 3.6$ aF, and $R_m \approx 153$ kΩ for $V_g = 15$ V.



- High impedance mismatch; $R_m \gg Z_0$ (50 ohm)
- most RF power reflects back without generating motion; need matching circuit

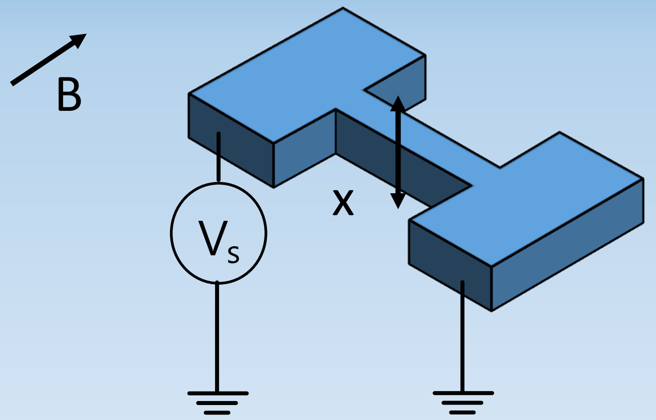
Capacitive detection of nanomechanical motion

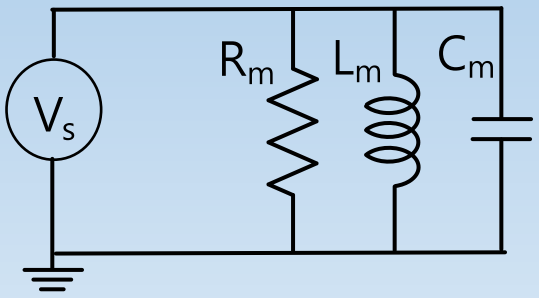
If the resonance of the LC network ω_{LC} is degenerate with the nanomechanical resonance, i.e., $\omega_{LC} = \omega_0$, the equivalent on-resonance impedance at the input of the LC network will be given by $Z_{Total}(\omega_0) \approx Z_{LC}^2/R_m + R_T$. Here, $Z_{LC} = \sqrt{L_T/C_T}$ is the characteristic impedance of the LC resonator, and R_T is any additional ohmic impedance that may come from the losses in the inductor (typically important) and the ohmic resistance of the nanomechanical resonator (typically unimportant).



- Lumped element resonant circuit works well up to ~100MHz (chip inductor and capacitor)
- Possible to find a combination of L_T and C_T so that Z_{total} ~ 50 ohm; most RF power is absorbed to actuate the nanomechanical motion

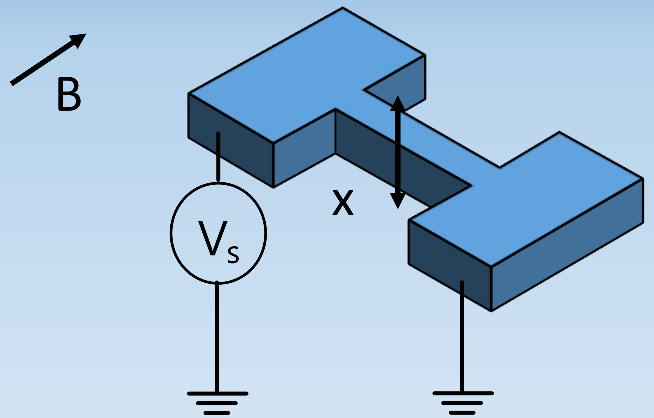
Magnetomotive (i.e. inductive) detection of nanomechanical motion

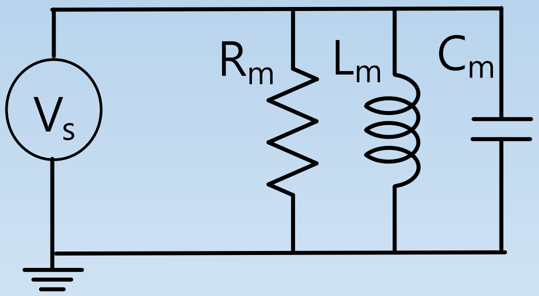




* A.N.Cleland & M.L.Roukes, External control of dissipation in a nanometer-scale radiofrequency mechanical resonator. *Sensors and Actuators a-Physical* **72**, 256-261 (1999).

Magnetomotive (i.e. inductive) detection of nanomechanical motion





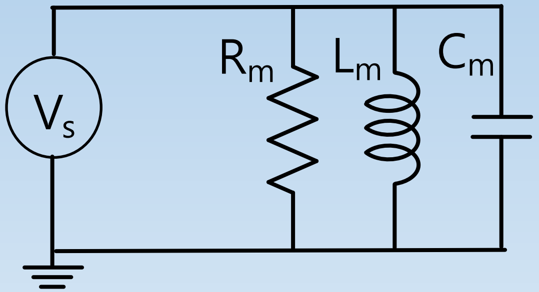
- Consider current through nanobeam I_s and its Lorentz force : $F = BLI_s$ (L: nanobeam length)
- Force induces mechanical motion "x" and generates electromotive force $V_s = BL(dx/dt)$
- Equivalent impedance = V_s/I_s equal to the circuit diagram

Magnetomotive (i.e. inductive) detection of nanomechanical motion

$$C_m = \frac{m}{l^2 B^2}$$

$$L_m = \frac{l^2 B^2}{\omega_0^2 m}$$

$$R_m = \frac{l^2 B^2}{\omega_0 m} Q_0 .$$



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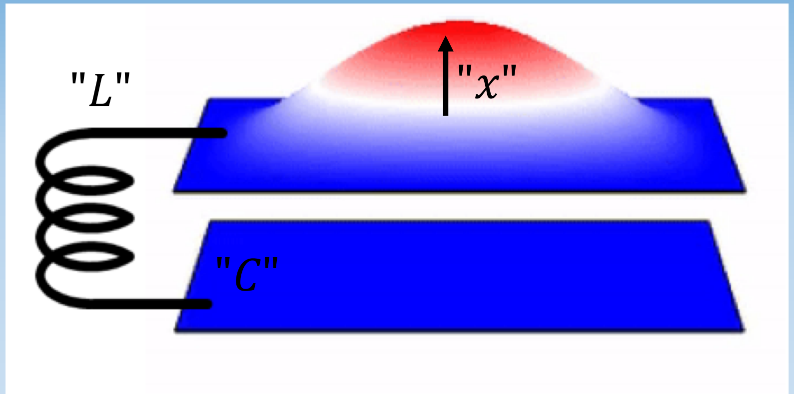
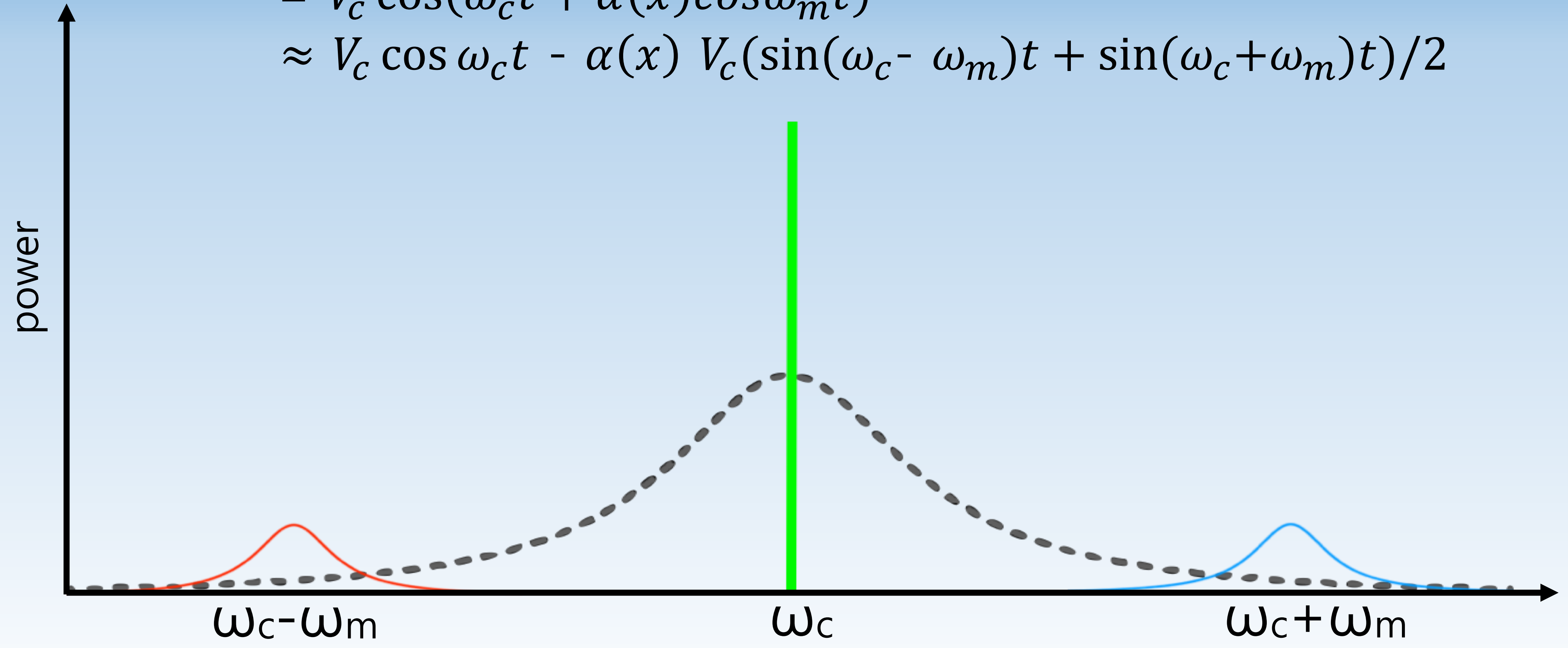
Microwave detection of mechanical motion



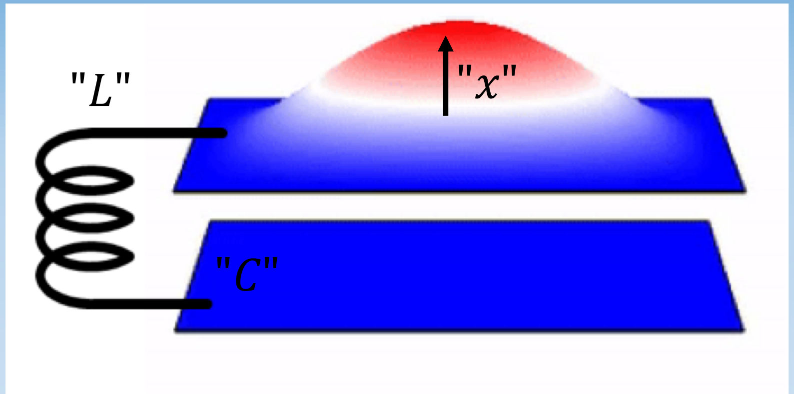
$$\omega_c = \sqrt{\frac{1}{LC(x)}} \approx \omega_c(x=0) + \left(\frac{\partial \omega_c}{\partial x}\right) x$$

Sidebands at mechanical resonance

$$V_{out} \propto V_c \cos\left(\omega_c t + \varphi_m(t)\right)$$
$$= V_c \cos(\omega_c t + \alpha(x) cos\omega_m t)$$
$$\approx V_c \cos \omega_c t \ - \ \alpha(x) \ V_c(\sin(\omega_c - \ \omega_m)t + \sin(\omega_c + \omega_m)t)/2$$



Microwave detection of mechanical motion



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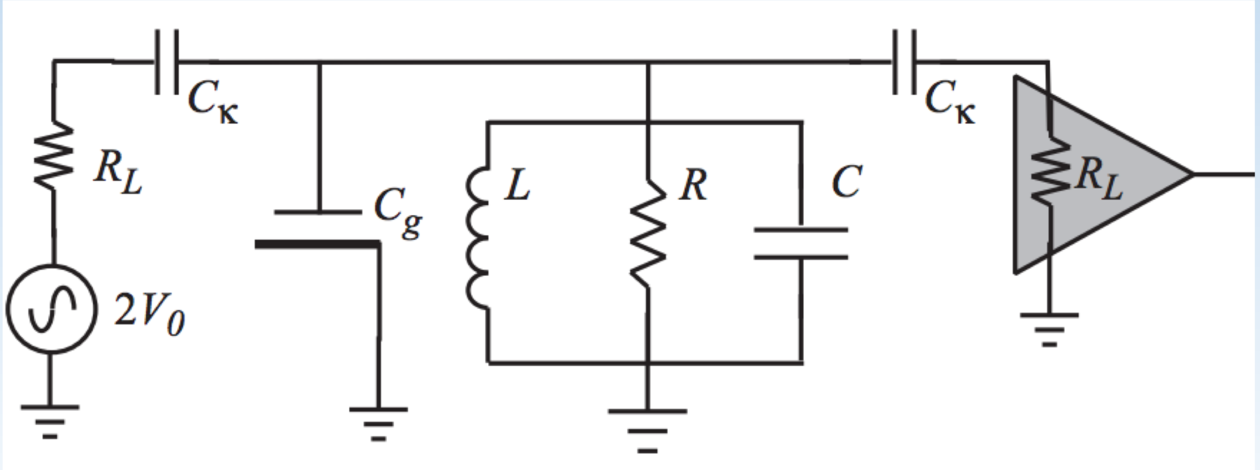
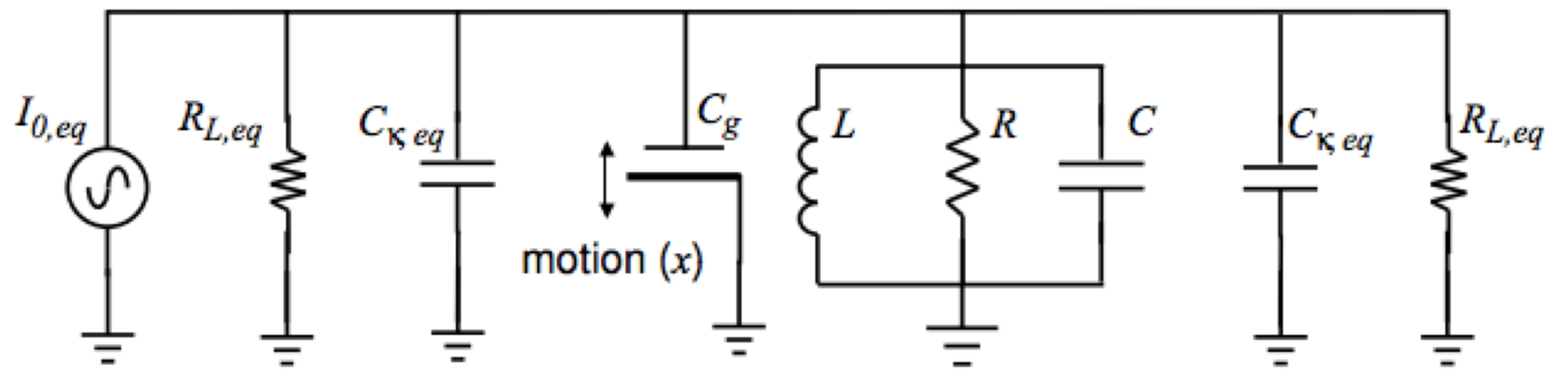


* J.Hertzberg, Back-Action Evading Measurements of Nanomechanical Motion Approaching Quantum Limits, Ph.D. Thesis, Univ. of Maryland (2009).

Microwave detection of mechanical motion



$$I_{0,eq}\cos(\omega_p t) = \frac{\partial}{\partial t}(C_{tot}V) + \frac{1}{R_{tot}}V + \frac{1}{L}\int V\,dt$$

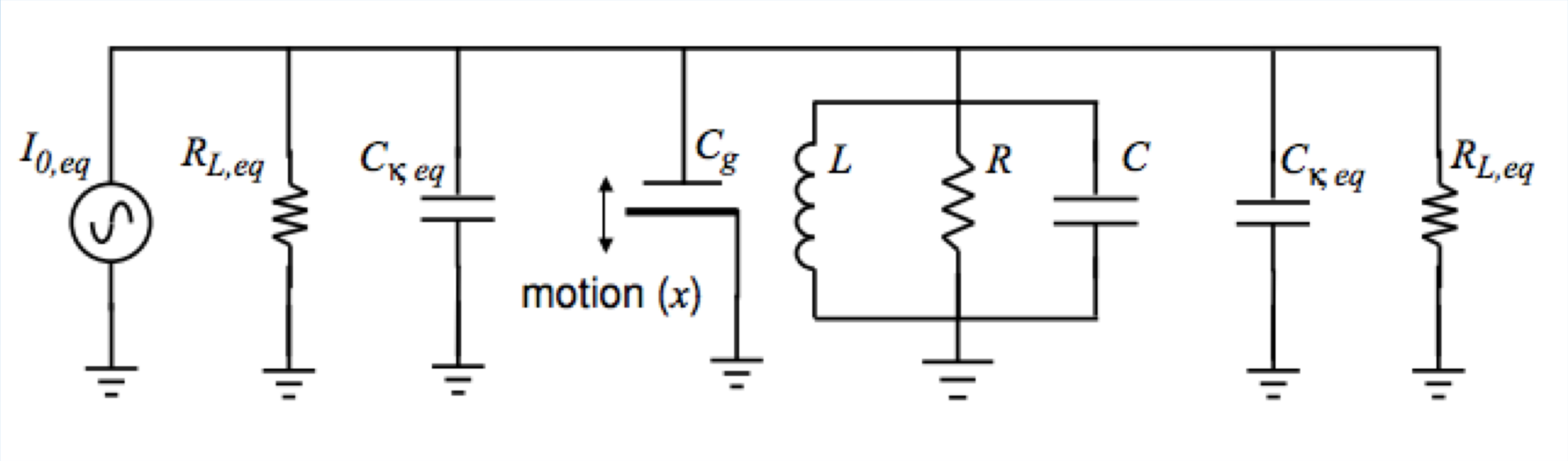
where $\frac{1}{R_{tot}} = \frac{1}{R} + \frac{2}{R_{L,eq}}$. We wish to solve for the voltage $V(t)$ within the SMR.

Differentiating once and plugging in our expression for C_g we have the differential equation

$$\begin{aligned}-I_{0,eq}\omega_p\sin(\omega_p t) = V&\left(\frac{1}{L} - \omega_{NR}^2\frac{\partial C_g}{\partial x}x_0\cos(\omega_{NR}t+\phi_{NR})\right)\\ +\dot{V}&\left(\frac{1}{R_{tot}} - 2\omega_{NR}\frac{\partial C_g}{\partial x}x_0\sin(\omega_{NR}t+\phi_{NR})\right)\\ +\ddot{V}&\left(C_{tot} + \frac{\partial C_g}{\partial x}x_0\cos(\omega_{NR}t+\phi_{NR})\right)\end{aligned}$$

* J.Hertzberg, Back-Action Evading Measurements of Nanomechanical Motion Approaching Quantum Limits, Ph.D. Thesis, Univ. of Maryland (2009).

Microwave detection of mechanical motion



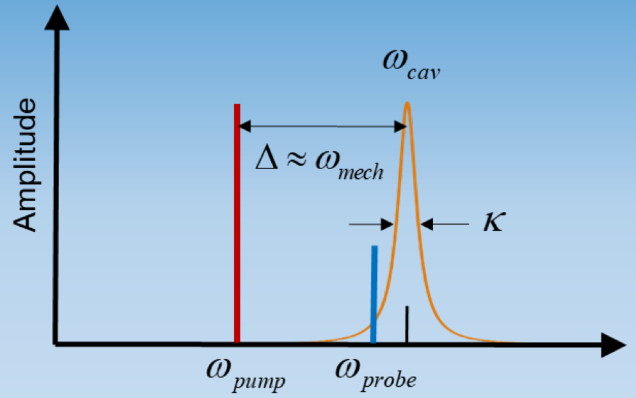
Voltage at sum and difference of mechanical and microwave resonance frequencies appear as,

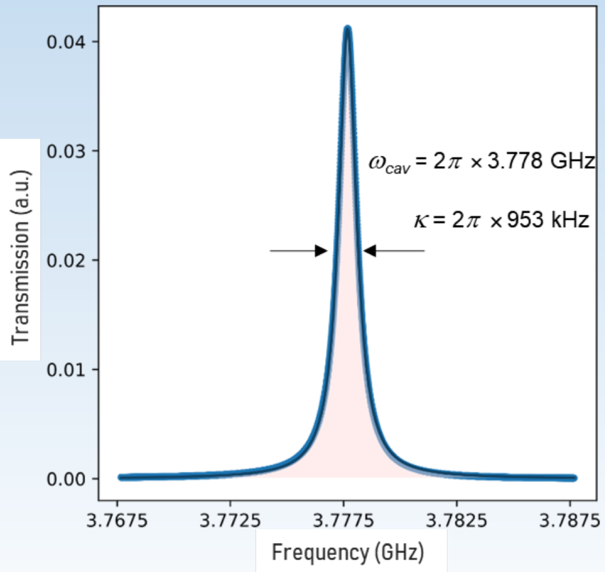
$$V_{s,amp} = \frac{-1}{\omega_{SMR}} \frac{\partial \omega_{SMR}}{\partial x} x_0 \cdot \frac{\omega_{SMR}}{\sqrt{\kappa^2 + 4\Delta\omega_s^2}} \cdot \frac{\kappa_{ext}}{\sqrt{\kappa^2 + 4\Delta\omega_p^2}} \cdot V_{p,0}$$

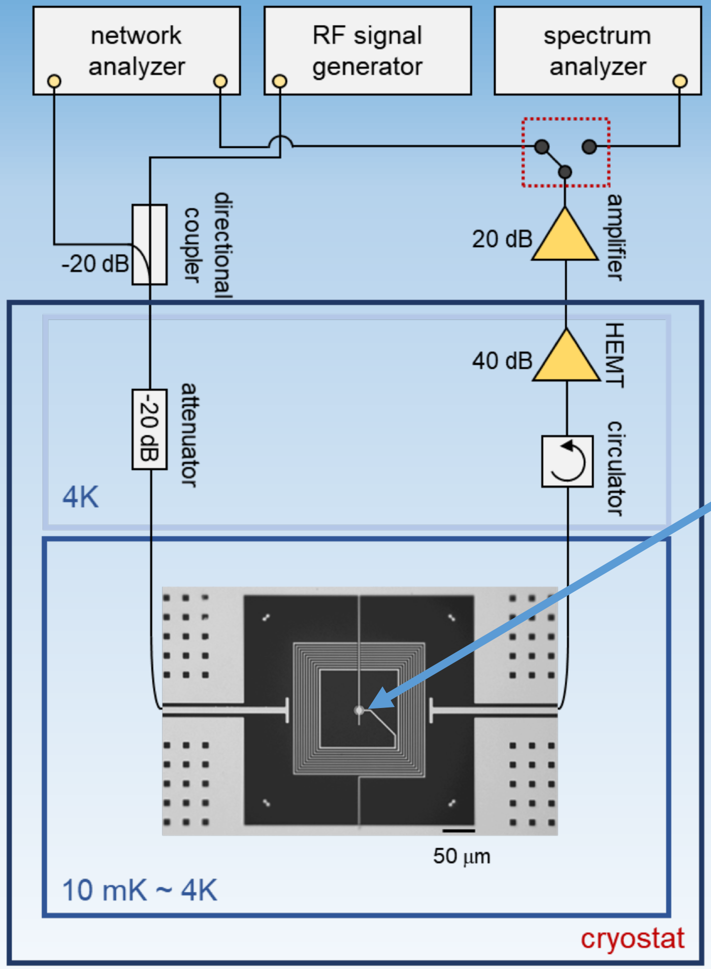
$$V_{d,amp} = \frac{-1}{\omega_{SMR}} \frac{\partial \omega_{SMR}}{\partial x} x_0 \cdot \frac{\omega_{SMR}}{\sqrt{\kappa^2 + 4\Delta\omega_d^2}} \cdot \frac{\kappa_{ext}}{\sqrt{\kappa^2 + 4\Delta\omega_p^2}} \cdot V_{p,0}$$

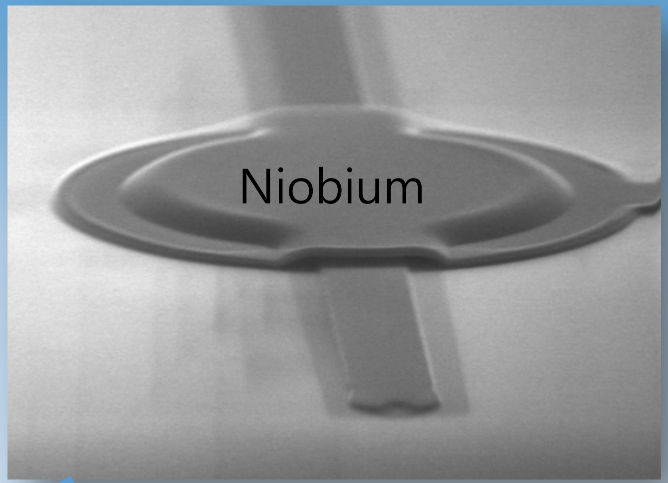
* J.Hertzberg, Back-Action Evading Measurements of Nanomechanical Motion Approaching Quantum Limits, Ph.D. Thesis, Univ. of Maryland (2009).

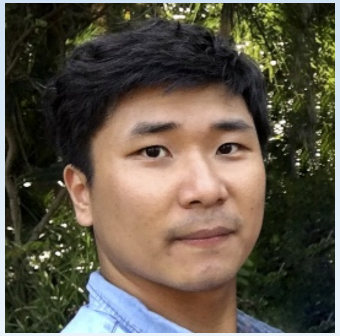
Microwave detection of mechanical motion





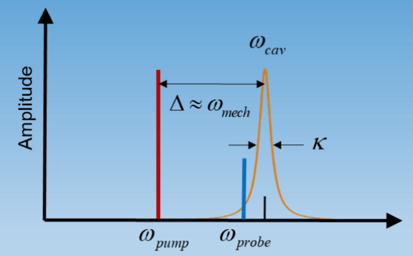


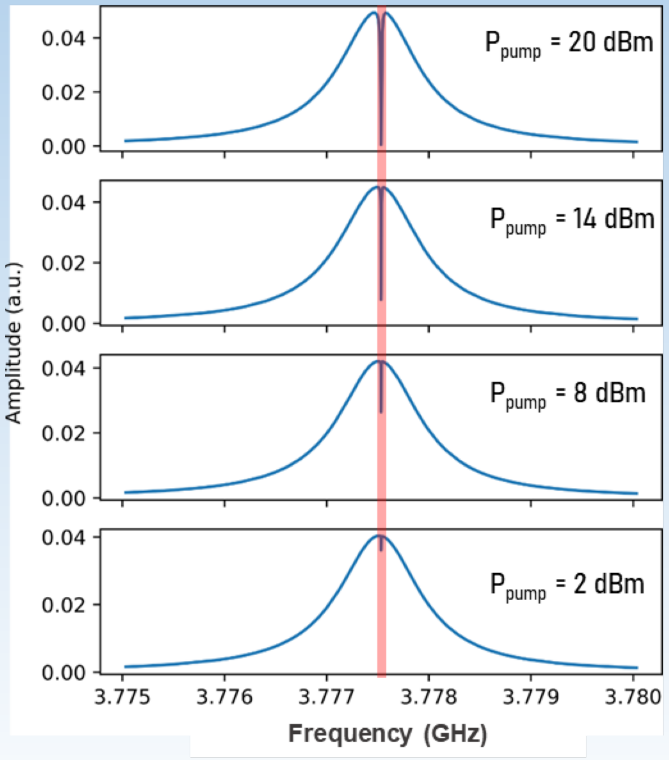


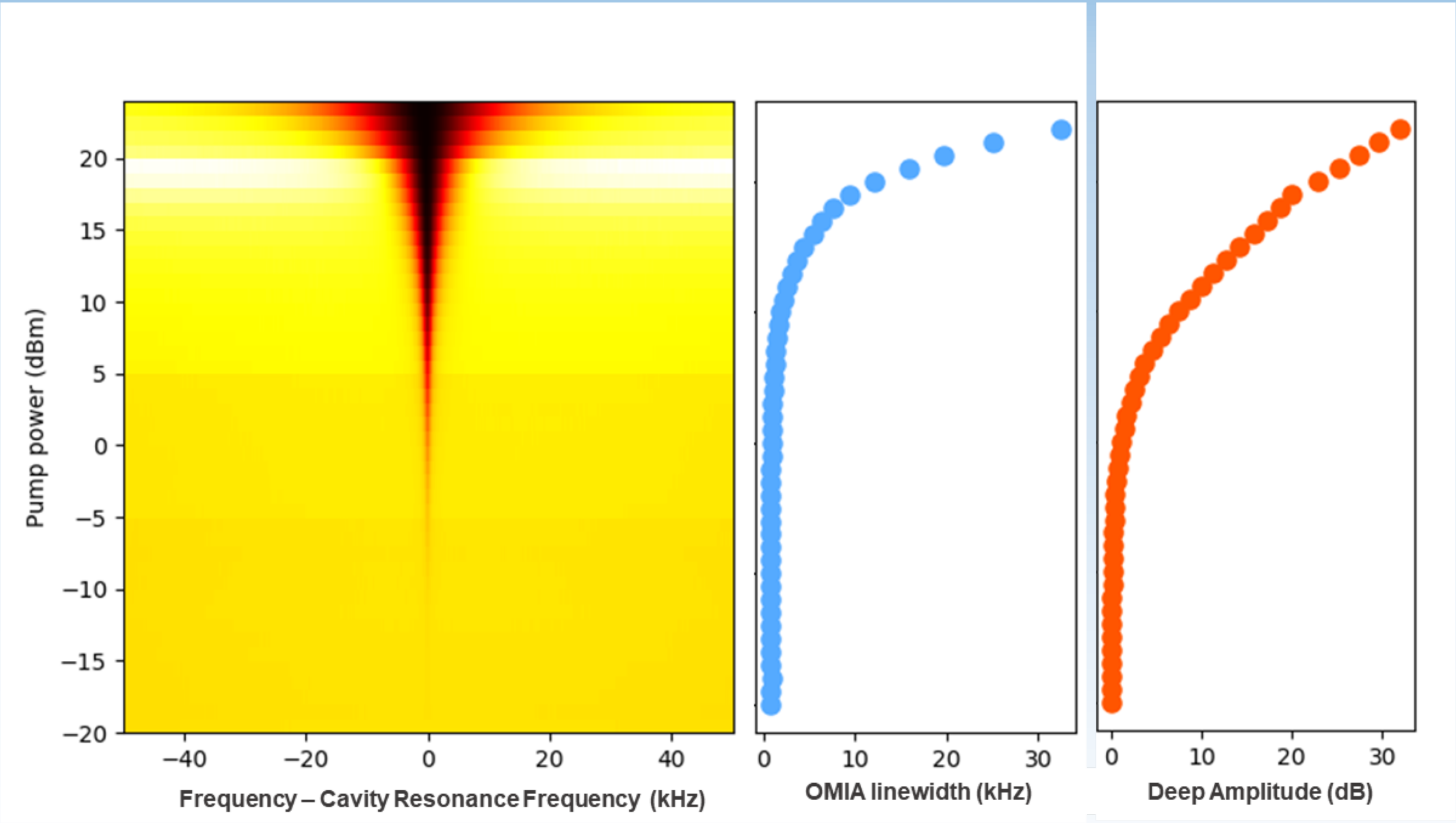


Dr. Jinwoong Cha (KRISS)

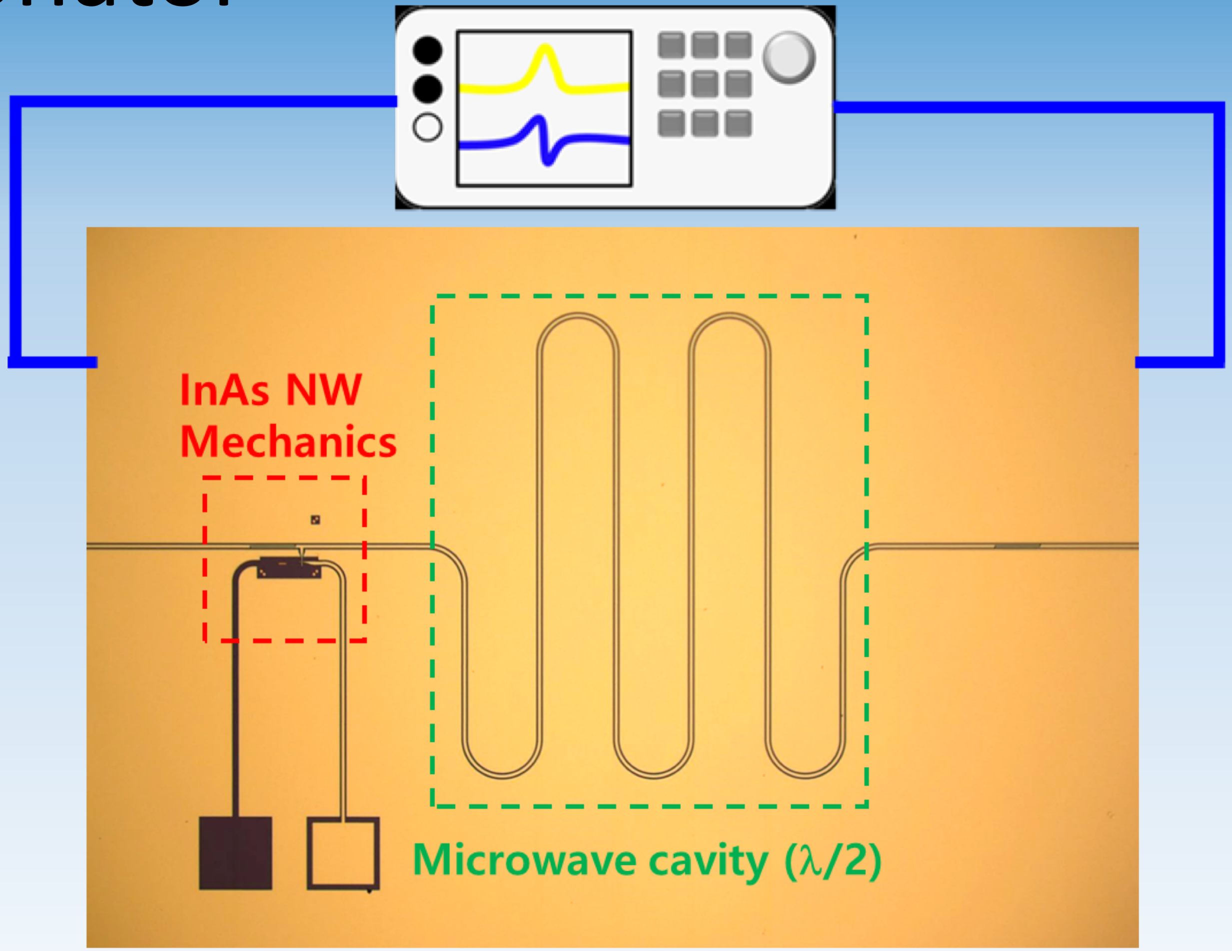
Mechanical sidebands modify microwave transmission





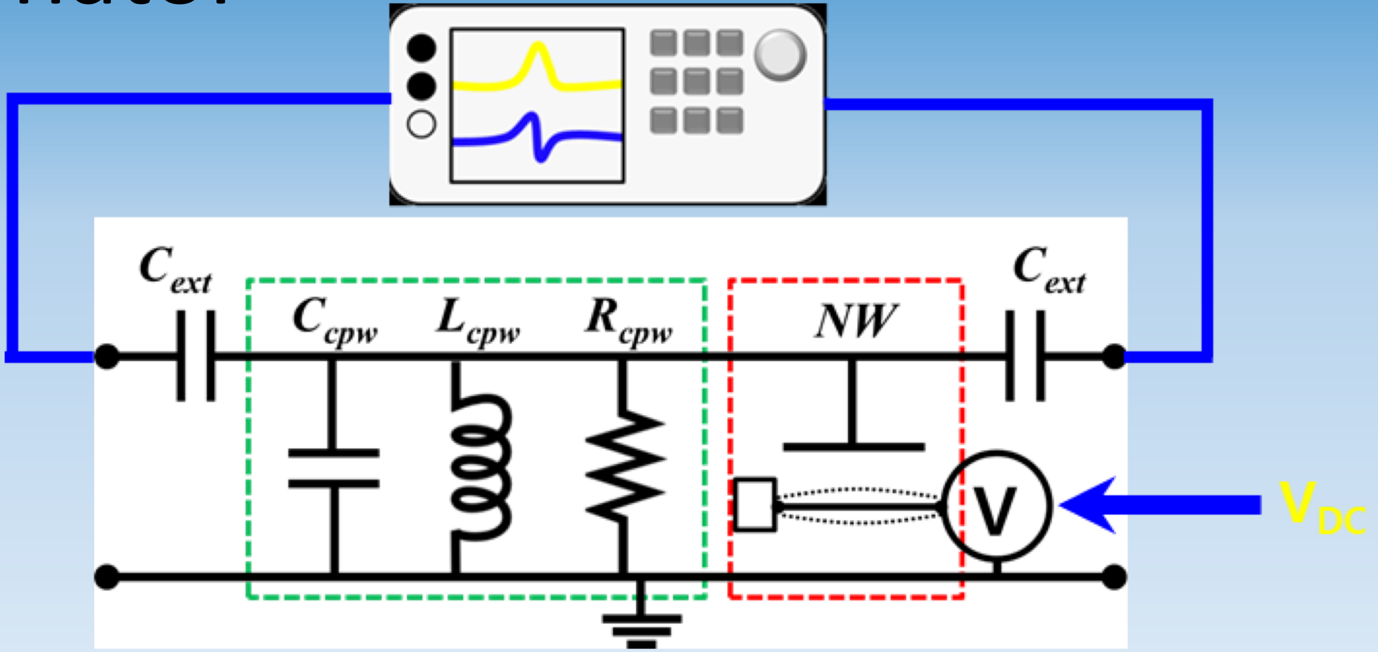


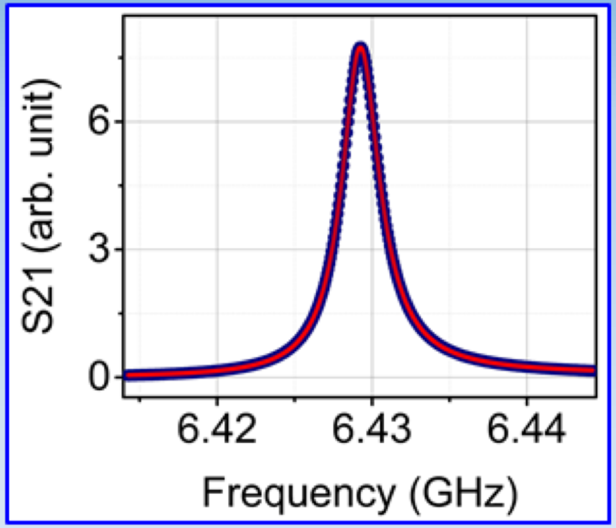
Microwave detection of nanowire mechanical resonator



Dr. Jihwan Kim (KAIST)

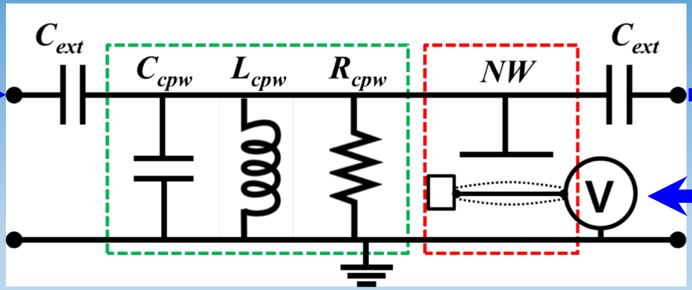
Microwave detection of nanowire mechanical resonator





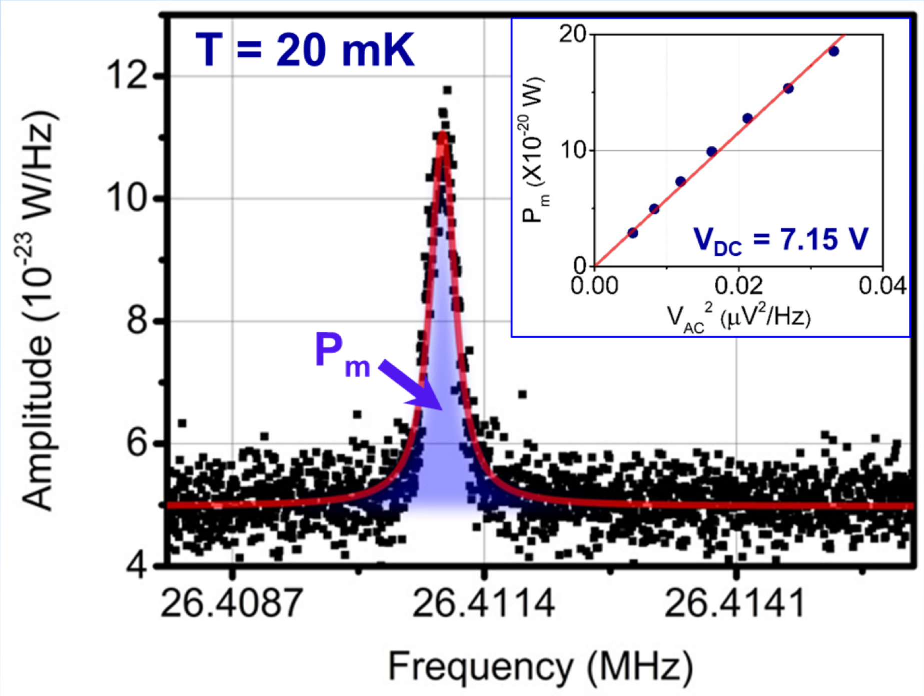
Mechanical sideband

Drive microwave (ω_c-ω_m)



Sideband signal at ω_c

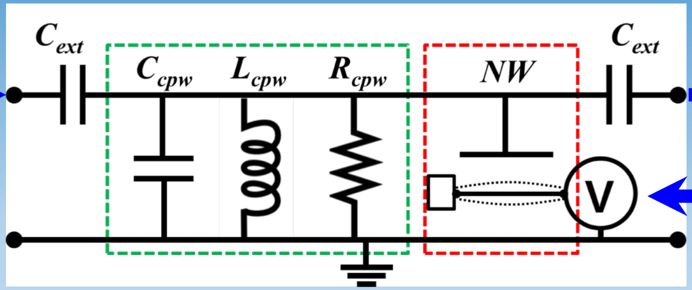
V_{DC} + V_{AC}



- P_m is proportional to actuation

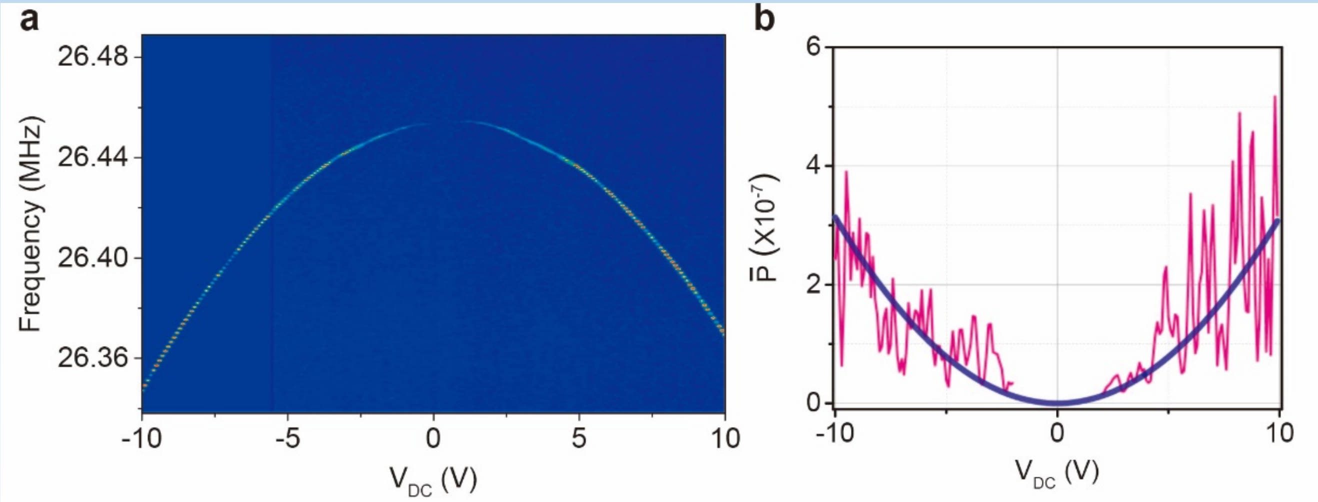
Mechanical sideband

Drive microwave (ω_c-ω_m)



Sideband signal at ω_c

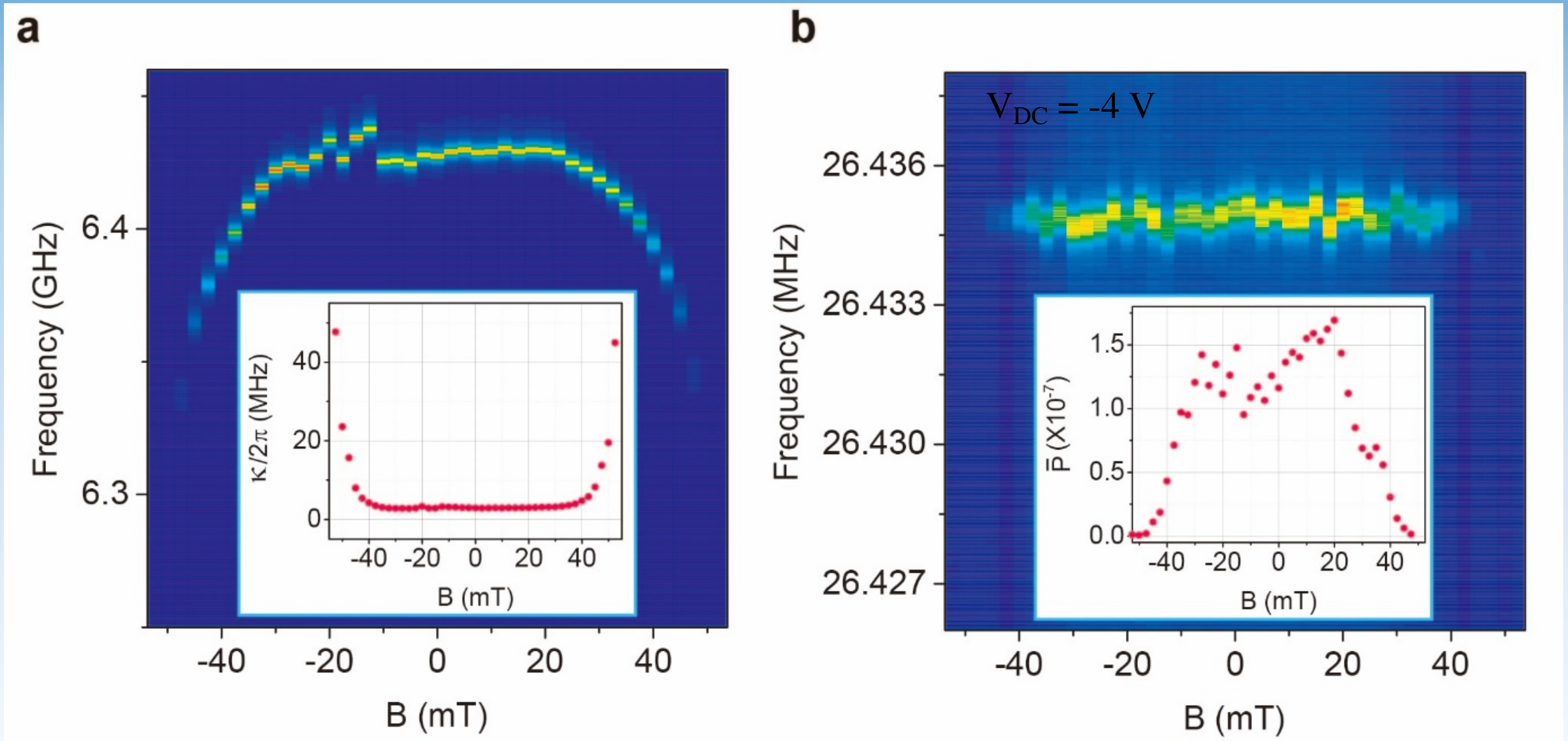
V_{DC} + V_{AC}



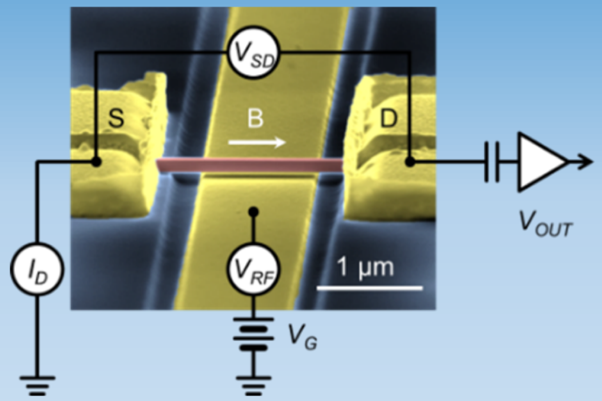
a. Capacitive frequency softening (well-known)

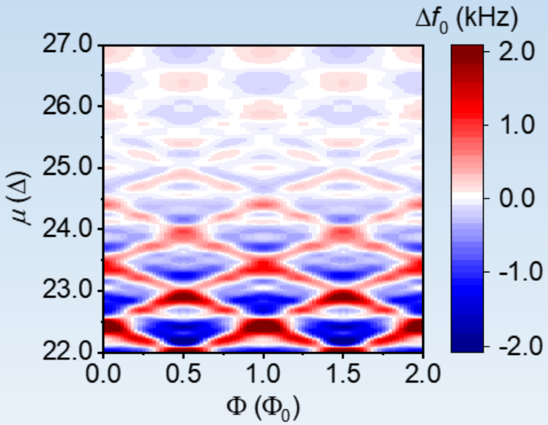
b. Sideband power oscillates; deviation from simple capacitance modulation model

Microwave resonance necessary for mechanical sideband

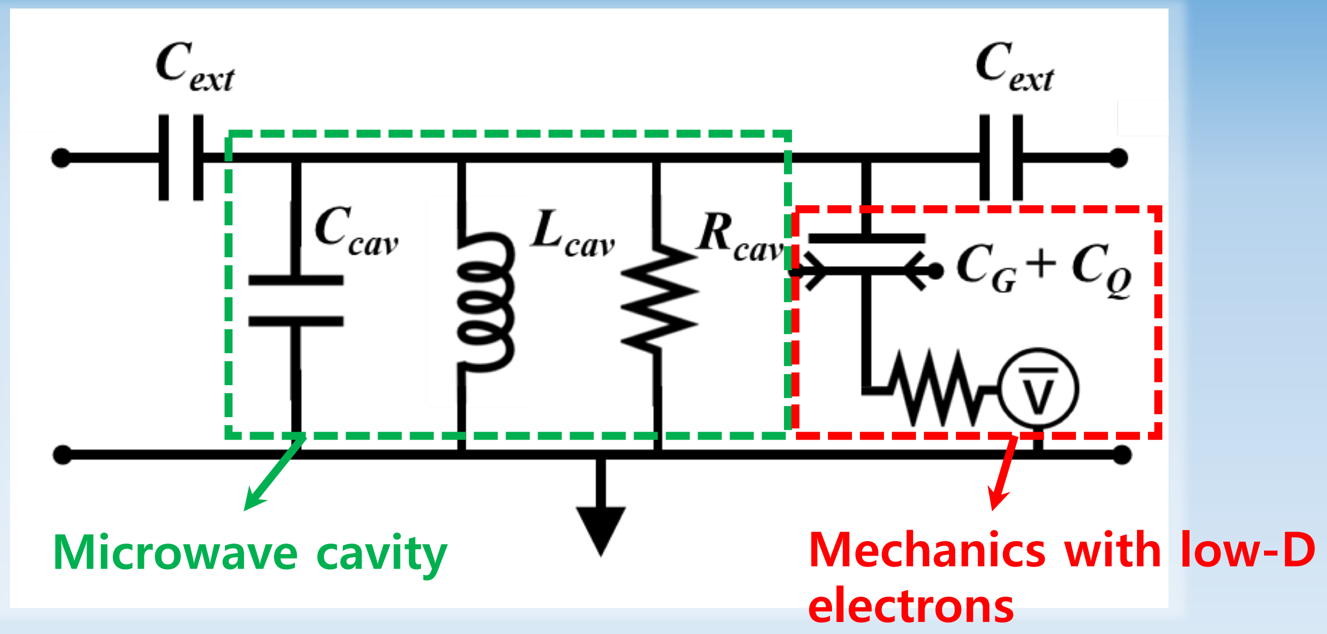


Outlook: nanomechanical study of mescopic electron states





*Nanomechanical characterization of quantum interference in a topological insulator nanowire, Nature Comm. **10**, 4522 (2019).



- ✓ High sensitivity mechanical probe via microwave detection
- ✓ 0D (quantum dot) or 1D in nanowire, 2D in vdW
- ✓ Quantum coherent control of mechanical motion with electrons

Outline

- Basic concepts (1 hour)
  - Voltage, current
  - Resistance, inductance, capacitance
  - Impedance, admittance
  - Signal, noise, interference
  - High frequency circuit concepts
  - Q&A + break (10 min)

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- Examples (1 hour)
  - Low-noise & low temperature conductance measurement
  - Inductive/capacitive/microwave detection of mechanical oscillator
  - Q&A + break (10 min)