Weyl Anomaly and Vacuum Response
in
BCFT and Holographic BCFT

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1701.04275, 1701.07202, proposal of Holo. BCFT (also WZ Guo),
1706.09652, Casimir effects from Weyl anomaly
1803.03068, magnetization current from Weyl anomaly
1804.01648, magnetization current from Holography
1812.10273, string current and Weyl anomaly in 6d BCFT
1903.02817, prediction of N^3 dof in M5 branes
2004.05780, Fermi condensate from Weyl anomaly
2005.12975, Fermi comdensate from Holography

Outline

Motivation 1: Novel Boundary Effects

- The presence of boundary allows novel phenomena, e.g. graphene, topological insulator, boundary critical behavior, self-dual string BCFT of M2 ending on M5-brane etc.
- Casimir effect arises from energetic response of the vacuum to the presence of boundary.

  M1. A motivation of our work is to discover new nontrivial boundary effects of QFT.

  New boundary phenomena we find :

  1. mechanical response of vacuum $\rightarrow$ (mechanical) Casimir effect
  2. magnetic response of vacuum: induced current $\rightarrow$ magnetic Casimir effect
  3. Higgs response of vacuum: Fermi condensate $\rightarrow$ potential Casimir effect
- These can be derived from exact analysis of BCFT (section 1) and AdS/BCFT (section 2).

Motivation 2: Holographic BCFT

- It would be nice to have handle on the nontrivial boundary phenomena beyond perturbation theory. e.g. $\longrightarrow$ Holography.
- Previously, an elegant proposal of AdS/BCFT has been proposed by Tadashi Takayanagi (2011). However there appears to have some restrictions in the proposal.
- M2. Resolving the restriction and improving our understanding of holography for BCFT was another motivation of our work.

Outline

1.1. Casimir effect in BCFT

General covariance and conservation law implies that for a d-dimensional BQFT, the vev of renormalized stress tensor has the asymptotic behaviour near boundary:

$$\langle T_{ij} \rangle = x^{-d} T_{ij}^{(d)} ... + x^{-1} T_{ij}^{(1)} + \cdots, \quad x \sim 0,$$

x is proper distance from boundary, and

$$T_{ij}^{(d)} = \alpha_0 h_{ij}, \qquad T_{ij}^{(d-1)} = 2\alpha_1 \bar{k}_{ij}, \quad \text{etc}$$

where h_{ij} and $\bar{k}_{ij}$ are the induced metric and the traceless part of extrinsic curvature of the boundary P.

- The boundary stress tensor has the structure: numerical coefficients (QFT data) $\times$ geometric tensor of bdy
- The Casimir coefficient α_0, α_1 etc fixes the shape dependence of the Casimir effects of BCFT.

  Q. How to calculate them in general?

General Casimir effect from Weyl Anomaly

Boundary Weyl anomaly

Weyl anomaly

$$\mathcal{A} := \partial_\sigma W[e^{2\sigma} g_{ij}]|_{\sigma=0} = \int_M \langle T_i^i \rangle,$$

where

$$\left\langle T_i^i \right\rangle = \left\langle T_i^i \right\rangle_M + \delta(x_\perp) \left\langle T_a^a \right\rangle_P .$$

- Weyl anomaly are well studied and classified in terms of curvature invariants.
  e.g. 3d: $\left\langle T_i^i \right\rangle = \delta(x)[b_1 \mathcal{R} + b_2 \mathrm{Tr}\bar{k}^2]$
- Bulk central charges c do not depend on BC. Boundary central charges b_i depend on BC in general.

- We find a surprising relation: Consider BQFT, the variation of the Weyl anomaly under an arbitrary variation of the metric can be measured by the 1-point function of the renormalized stress tensor (Chu, Miao 17, 18)

$$\boxed{(\delta\mathcal{A})_{\partial M_\epsilon} = \left(\frac{1}{2}\int_{x\geq\epsilon}\sqrt{g}\,T^{ij}\delta g_{ij}\right)_{\log(1/\epsilon)}}$$

where $(\delta\mathcal{A})_{\partial M}$ is the boundary terms in the variations of Weyl anomaly and T^{ij} is the renormalized bulk stress tensor.

- integrable relation: the right hand side must give an exact variation, this imposes strong constraints on the possible form of the stress tensor near the boundary.

E.g. 3d BCFT, Weyl anomaly has only boundary contributions

$$\mathcal{A} = \int_{\partial M} \sqrt{h}(b_1 \mathcal{R} + b_2 \mathrm{Tr}\bar{k}^2),$$

$$LHS = (\delta \mathcal{A})_{\partial M} = b_2 \int_{\partial M} \sqrt{h}\Big[(\frac{\mathrm{Tr}\bar{k}^2}{2} h^{ab} - 2\bar{k}^a_c k^{cb})\delta h_{ab} + 2\bar{k}^{ab}\delta k_{ab}\Big].$$

For RHS, sub. $T_{\mu\nu}$, int. over x and pick up the log divergent term

$$\begin{aligned} RHS = & - \alpha_1 \int_P \sqrt{h}[(\frac{\mathrm{Tr}\bar{k}^2}{2} h^{ab} - 2\bar{k}^a_c k^{cb})\delta h_{ab} + 2\bar{k}^{ab}\delta k_{ab}] \\ & + \int_P \sqrt{h}[(\frac{\beta_3}{2} - \alpha_1) k \bar{k}^{ab}\delta h_{ab} + \frac{\beta_4}{2}\lceil k^a_c k^{cb}\rceil \delta h_{ab}]. \end{aligned}$$

We obtain $\boxed{\alpha_1 = -b_2, \quad \beta_3 = -2b_2, \quad \beta_4 = 0.}$

4d BCFT. Similar analysis and results for 4d.

- It is remarkable that the Casimir coefficients (α's) are completely determined by the boundary central charges (b's). i.e. only needs boundary central charges, Casimir coefficients are dependent.
- The relations $\alpha_n = \alpha_n(b_i)$ are universal: the relations are independent of BC and the QFT.

1.2. Induced current from background B-field

Chiral anomaly give rises to (famously) induced transport of charges:

- CME: chiral magnetic effect
  (Vilenkin 80; Giovannini Shaposhnikov 98; Froehlich etal 98)

  $$\mathbf{J}_V = \sigma_{(\mathcal{B})V}\mathbf{B} \quad \mathbf{J}_A = \sigma_{(\mathcal{B})A}\mathbf{B},$$

  where the chiral magnetic conductivities are

  $$\sigma_{(\mathcal{B})V} = \frac{e\mu_A}{2\pi^2}, \quad \sigma_{(\mathcal{B})A} = \frac{e\mu_V}{2\pi^2}$$

- There is also a CVE: chiral vortical effect (Kharzeev, Zhitnitsky 07; Erfmemger etal 09; Son etal 09; Landsteiner etal 11)
- Note that these anomalous transport occurs only in a material system where the chemical potentials are non-vanishing.

Q. Is it possible to have anomalous transport occur in vacuum? if so, which anomaly is responsible?

Induced current from Weyl anomaly

- Just as energy momentum tensor, the renormalized current also admits an asymptotic expansion near the boundary:

$$\langle J_i \rangle = x^{-3} J_i^{(3)} + x^{-2} J_i^{(2)} + x^{-1} J_i^{(1)}, \quad x \sim 0,$$

  where x is the proper distance from the boundary and $J_i^{(n)}$ depend only on the background geometry and the background vector field strength.

- Imposing current conservation

$$\nabla_i \langle J^i \rangle = 0,$$

  we obtain the gauge invariant solutions

$$J_\mu^{(3)} = 0, \qquad J_\mu^{(2)} = 0,$$
$$J_\mu^{(1)} = \alpha_1 F_{\mu\nu} n^\nu + \alpha_2 \mathcal{D}_\mu k + \alpha_3 \mathcal{D}_\nu k_\mu^\nu + \alpha_4 \star F_{\mu\nu}\, n^\nu$$

- Under an arb. A_μ, one can establish the integrability condition

$$\boxed{(\delta \mathcal{A})_{\partial M} = \left(\int_M \sqrt{g} J^\mu \delta A_\mu \right)_{\log \frac{1}{\epsilon}}}$$

  Using it, the current coefficients α_n can be determined completely in terms of central charges.
- Consider background $U(1)$ gauge field in QED, we obtain the induced current near the boundary:

$$\boxed{\mathbf{J} = \frac{e^2 c}{\hbar} \frac{4 b_1 \mathbf{n} \times \mathbf{B}}{x}, \quad x \sim 0.}$$

1.3. Fermi condensate induced by Yukawa potential

- Fermi condensation is an interesting quantum phenomena e.g. Cooper pair in BCS theory of superconductivity, chiral condensate in QCD
- The condensation of fermion is usually attributed to the effects of strongly coupled dynamics and hence it can be used as an order parameter characterizing the phases of the theory.
- Consider Dirac fermion ψ coupled to a scalar field ϕ:

$$S = \int_M \sqrt{-g}\big(i\bar{\psi}\gamma^\mu \nabla_\mu \psi + \phi\bar{\psi}\psi\big).$$

One can establish the response relation between Weyl anomaly and Fermi condensate:

$$(\delta\mathcal{A})_{\partial M_\epsilon} = \left(\int_M \sqrt{-g}\langle\bar{\psi}\psi\rangle\delta\phi\right)_{\log\epsilon}$$

- The Weyl anomaly due to the scalar potential ϕ can be obtained using the heat kernel method. For example, for the bag BC $(1 \pm \gamma_5 \gamma_n)\psi|_{\partial M} = 0$, we obtain the Weyl anomaly $\mathcal{A}(g_{\mu\nu}, \phi)$ at one loop as

$$\mathcal{A} = \frac{1}{8\pi^2} \left(\int_M \sqrt{-g} \left[-(\nabla\phi)^2 + \frac{R\phi^2}{6} + \phi^4 \right] + \int_{\partial M} \sqrt{-h} \frac{k\phi^2}{3} \right).$$

- Hence we obtain the Fermi condensate near the boundary

$$\boxed{\langle \bar{\psi}\psi \rangle = \frac{1}{4\pi^2} \frac{\nabla_n \phi - \frac{1}{3} k\phi}{x} + O(\ln x), \quad x \sim 0.}$$

Remarks:

1. In general the Wess-Zumino consistency condition gives the general expression of Weyl anomaly due to a background scalar field as

$$\mathcal{A} = a_1\mathcal{A}_1 + a_2\mathcal{A}_2 + \sum_{n=1}^{4} b_n\mathcal{B}_n,$$

where

$$\mathcal{A}_1 = \int_M \sqrt{|g|}[-(\nabla\phi)^2 + \frac{1}{6}R\phi^2] + \int_{\partial M}\sqrt{|h|}\frac{1}{3}k\phi^2, \quad \mathcal{A}_2 = \int_M \sqrt{|g|}\phi^4,$$

$$\mathcal{B}_1 = \int_{\partial M}\sqrt{|h|}\phi^3, \quad \mathcal{B}_2 = \int_{\partial M}\sqrt{|h|}[k\phi^2 + 3\phi\nabla_n\phi],$$

$$\mathcal{B}_3 = \int_{\partial M}\sqrt{|h|}[R\phi + 6\Box\phi], \quad \mathcal{B}_4 = \int_{\partial M}\sqrt{|h|}[\mathrm{Tr}\bar{k}^2\phi]$$

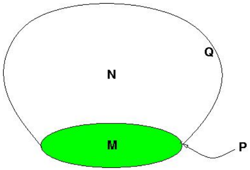
and a_n, b_m are the corresponding bulk and boundary central charges. One can use this to obtain the general expression of the FC.

2. The condensation has its physical origin in the nontrivial response of the fermion vacuum to changes in the background spacetime (either boundary location or the background metric), and can be felt when a background scalar field is turned on.
3. The scalar field can be, for example, the Higgs field in a fundamental theory or the phonon in condensed matter system.
4. Due to its universal nature, this anomaly induced Fermi condensate can be expected to have a wide range of implications in physics.

Outline

2.1. Statement of AdS/BCFT

- Consider d dimensional CFT defined on $R^{1,d-1}$. There is a $SO(2,d)$ conformal symmetry. This is realized in holography as isometries of the bulk of AdS$_{d+1}$ space
- When a boundary is introduced, the full conformal symmetry is reduced at the boundary. Takayanagi proposed to extend that the gravity dual is given by gravity in a $d+1$ dimensional manifold N whose boundary is given by M and Q. (Takayanagi 11)



The bulk gravity action is given by

$$I = \int_N \sqrt{G}(R - 2\Lambda) + 2\int_M \sqrt{g}K + 2\int_Q \sqrt{h}(K - T) + 2\int_P \sqrt{\sigma}\theta,$$

T measures the boundary degrees of freedom (g-function). differential equation for Q.

- The central issue is the determination of the location of Q in the bulk. Takayanagi proposed to impose Neumann boundary condition on Q to fix its position:

$$\text{EOM of } Q: \quad K_{\alpha\beta} - (K - T)h_{\alpha\beta} = 0, \qquad (*1)$$

This gives a second order DE.

- However since Q is of co-dimension one,the location of Q is determined by a single embedding function:

$$z = z(x^i), \quad \text{here } x^i = \text{coordinates of } M$$

The embedding equation

$$K_{\alpha\beta} - (K - T)h_{\alpha\beta} = 0, \qquad (*1)$$

generally imposes too many constraints and (*1) does not has solution for general shape P of BCFT.

Alternatively, we proposed to impose on Q a mixed BC and this leads to:

$$\boxed{(1-d)K + dT = 0} \qquad (*2)$$

(Chu, Guo, Miao 17; Chu, Miao 17)

- (*2) is natural as there is only one embedding function for Q and we expect one condition for it.
- This is a consistent proposal and describes the duals for general BCFT.

2.2 Applications: 1.Holographic boundary Weyl anomaly

One can derive the Weyl anomaly from the trace of the holographic stress tensor. (Balasubramanian, Krauss; Haro, Solodukhin, Skenderis).

- First we construct a finite action by adding covariant counterterms

$$\begin{aligned} I_{\rm reg} &= \int_N \sqrt{G}(R-2\Lambda) + 2\int_M \sqrt{g}(K - 2 - \frac{1}{2}R_M) \\ &+ 2\int_Q \sqrt{h}(K-T) + 2\int_P \sqrt{\sigma}(\theta - \theta_0 - K_M), \end{aligned}$$

  where we have included on M the usual counterterms in holographic renormalization, $\theta_0 = \theta(z=0)$ is a constant and K_M is the Gibbons-Hawking term for R_M on M.

- From this one obtain the Brown-York stress tensor on P

$$B_{ab} = 2(K_{Mab} - K_M\sigma_{ab}) + 2(\theta - \theta_0)\sigma_{ab}$$

- The boundary Weyl anomaly is given by

$$\langle T^a_a \rangle_P \;=\; \lim_{z\to 0} \frac{B^a_a}{z^2} = \lim_{z\to 0} \frac{4(\theta-\theta_0)-2K_M}{z^2}$$

- Solving for the shape of the bulk surface Q using our proposal, we get the correct boundary Weyl anomaly: $A^{\alpha}{}_{\beta} \neq \delta^{\alpha}{}_{\beta}$. This is a strong support to our proposal.

2.2 Applications: 2.Induced current near boundary

- To investigate the renormalized current in holographic models of BCFT, we add a Maxwell action:

$$I = \int_N \sqrt{G}[R - 2\Lambda - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}] + 2\int_Q \sqrt{\gamma}[K - T]$$

- Consider 4d and planar boundary, one can solve for the bulk Maxwell equation and BC, and obtain

$$\mathcal{A}_a = F_{xa}\sqrt{x^2 + z^2},$$

where F_{xa} is the field strength at the boundary.

- The holographic current agrees precisely with QFT

$$\langle J^a \rangle = \lim_{z\to 0} \frac{\delta I}{\delta A_a} = \lim_{z\to 0} \sqrt{G}\mathcal{F}^{za} = -\frac{F_{ax}}{x} + \cdots$$

- One can also derive the Fermi condensate holographically (skipped here)

Outline

3.1 Boundary string current from holography

- Charged particle moving on a worldline C: $x^\mu = x^\mu(\tau)$ and couples minimally to gauge field:

$$J^\mu(x) = \delta^{(d-1)}(x - x(\tau)) \frac{dx^\mu(\tau)}{d\tau}.$$

$$\int_M J_\mu A^\mu = \int_C A_\mu dx^\mu$$

- Similarly, movement of strings gives the higher 2-form current and a coupling to the 2-form potential $B_{\mu\nu}$:

$$J_{\mu\nu} = \delta^{(d-2)}(x - x(\sigma,\tau)) \epsilon^{\alpha\beta} \frac{\partial x^\mu}{\partial \sigma^\alpha} \frac{\partial x^\nu}{\partial \sigma^\beta}.$$

$$\int_M J_{\mu\nu} B^{\mu\nu} = \int_\Sigma B_{\mu\nu} dx^\mu dx^n.$$

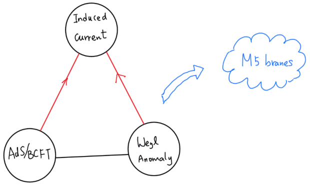
- Consider a 6d BCFT dual to the the bulk action with an H-field

$$I = \int d^7x\sqrt{G}(R - 2\Lambda - \frac{1}{12}H^2_{\mu\nu\lambda})$$

Using holography, we predict remarkably a string current parallel to the boundary when a H-field strength is turned on:

$$\boxed{J_{ab} = b_1 \frac{H_{abx}}{x}}.$$

- Recall that



Using this idea, we may learn something about the Weyl anomaly on M5-branes.

3.2. Prediction for the Weyl anomaly in 6d

Consider a BCFT in 6d. The gravitational part of the Weyl anomaly $\mathcal{A}$ is well understood: (Deser, Schwimmer)

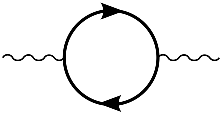
$$\langle T^{\mu}_{\mu} \rangle = \frac{1}{(4\pi)^{d/2}} \left(\sum_j c_{dj} I_j^{(d)} - (-1)^{\frac{d}{2}} a_d E_d \right).$$

Q. What about the contribution from background gauge field?

- The relation $(\delta\mathcal{A})_{\partial M} = \left(\int_{M_\epsilon} J_{\mu\nu}\delta B^{\mu\nu} \right)_{\log 1/\epsilon}$ predicts the Weyl anomaly in the 6d CFT:

$$\boxed{\mathcal{A} = \int_M \frac{b_1}{12} H^2}$$

- It is interesting to understand how matter fields would couple to the $B_{\mu\nu}$ field (covariant derivatives?) and give rises to the Weyl anomaly.



M5-branes system

- For a system of N M5-branes, the gravity dual is given by $AdS_5 \times S^5$.
- Restoring the units, b_1 is given by

$$b_1 = \frac{R^5}{16\pi G_7} = \frac{N^3}{3\pi^3},$$

where $G^{(7)} = G^{(11)}/R_S^4$, $R_S = l_P(\pi N)^{1/3}$ is the 4-sphere radius and $R = 2R_S$ is the AdS_7 radius.

- Therefore for a system of N M5-branes with boundary, one finds for the singlet current

$$\boxed{J_{ab} = \frac{N^3}{3\pi^3}\frac{H_{abx}}{x}.}$$

This results suggest that there is N^3 degrees of freedom in the interacting (2,0) theory.

Outline

Conclusions and Discussions

Summary:

1. Novel boundary CFT phenomena discovered: magnetization current, Fermi condensate
   1. they can be derived from the Weyl anomaly and hence are determined entirely in terms of the central charges
   2. they can be understood as some kinds of generalized Casimir effect of the vacuum in respond to external field
2. Holographic BCFT: Based on the original work of Takayanagai, we have constructed an alternative proposal of holographic BCFT which applies to general boundary.
3. Weyl anomaly for the M5-branes system: a prediction of N^3 dof from the scaling behavior of the Weyl anomaly

Conclusions:

- The interaction of QFT and holography is fruitful.
- The discovered effects should be observable
- Due to the universality of the Weyl anomaly, these effects should have applications in wide range of physical systems

Thank you!