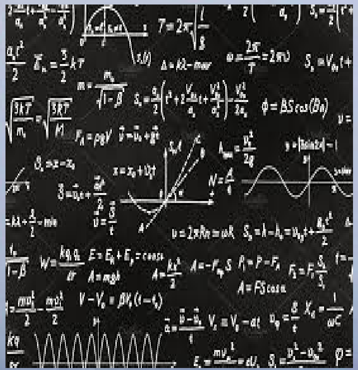
Matteo Baggioli

우도에서 썸타

Phases of Matter & Collective Excitations (in the eyes of a high energy theorist)

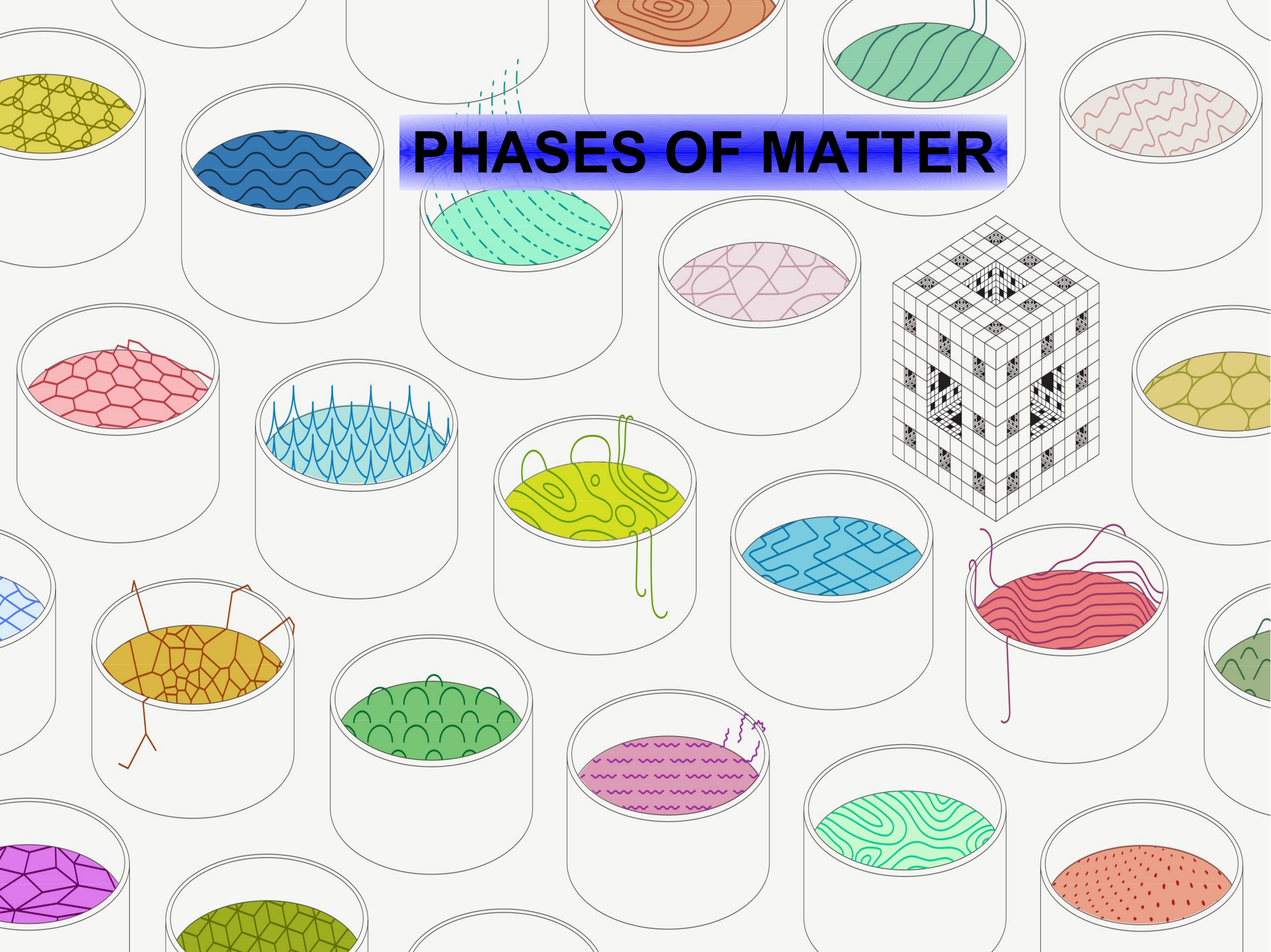


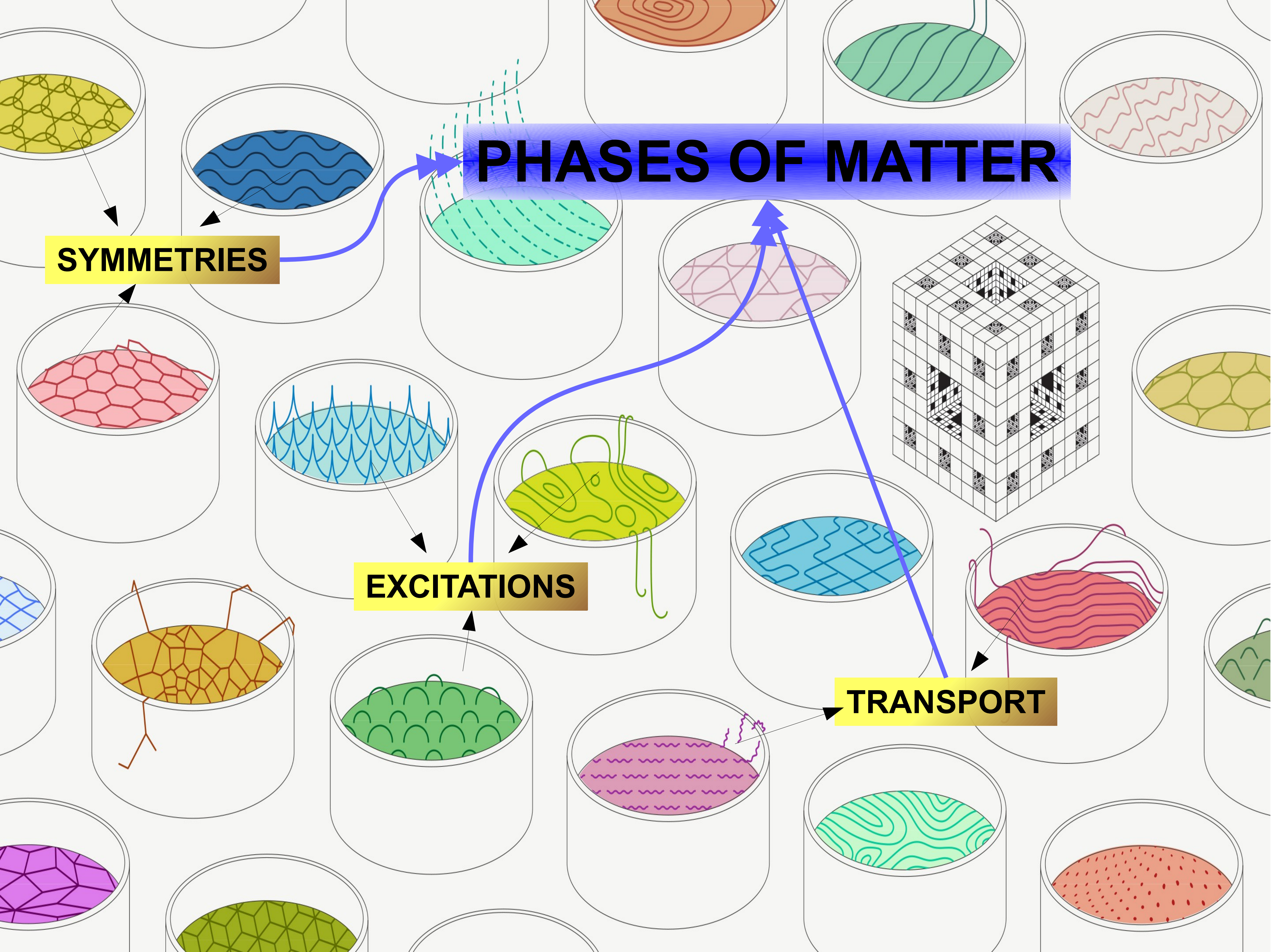
Quantum Matter and Quantum Information with Holography
August 23 (Sun), 2020 ~ August 31 (Mon), 2020

THANKS TO THE ORGANIZERS

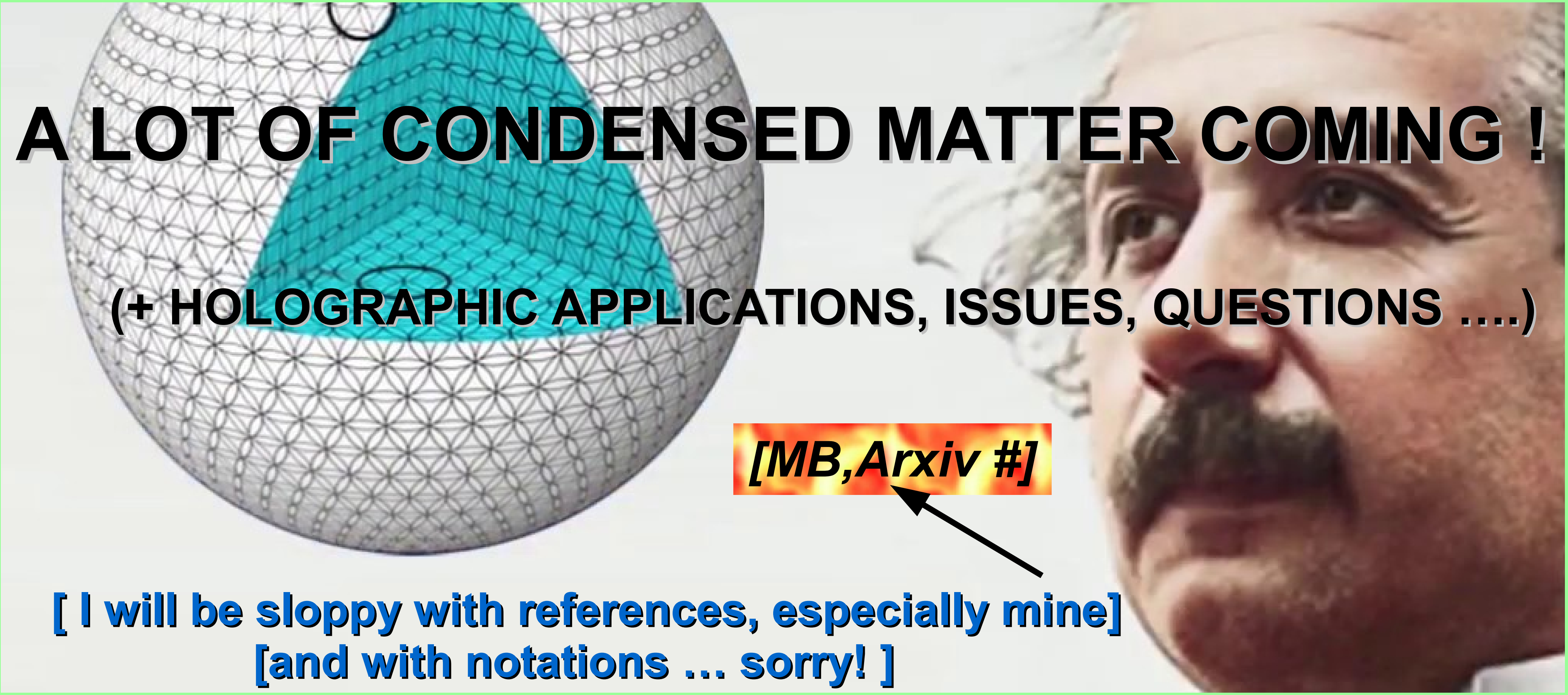
AND TO

APCTP
Asia Pacific Center for Theoretical Physics


PHASES OF MATTER


PHASES OF MATTER
SYMMETRIES
EXCITATIONS
TRANSPORT


DISCLAIMER


A LOT OF CONDENSED MATTER COMING !
(+ HOLOGRAPHIC APPLICATIONS, ISSUES, QUESTIONS ….)
[MB,Arxiv #]
[I will be sloppy with references, especially mine]
[and with notations … sorry!]

LECTURES

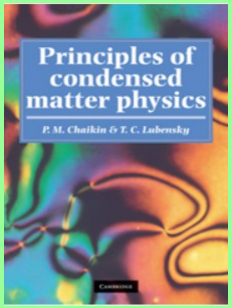
1 SOLIDS, LATTICES & SUPERLATTICES

2 LIQUIDS & VISCOELASTICITY

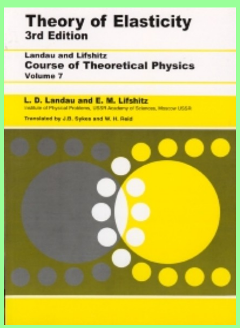
3 QUASICRYSTALS & INCOMMENSURATE STRUCTURES


Part
One

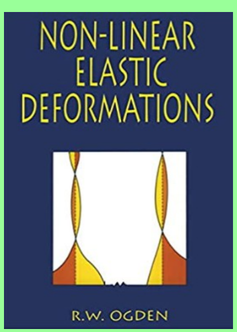
SOLIDS, LATTICES & SUPERLATTICES



[Principles of Condensed Matter Physics, Chaikin,Lubensky, Cambridge University Press]



[Theory of Elasticity, Volume 7, Landau- Lifshitz]



[Non-Linear Elastic Deformations, Ogden, Dover Civil and Mechanical Engineering]

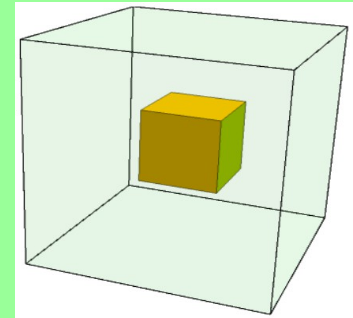
Zoology of condensed matter: Framids, ordinary stuff, extra-ordinary stuff

Alberto Nicolis, Riccardo Penco, Federico Piazza, Riccardo Rattazzi

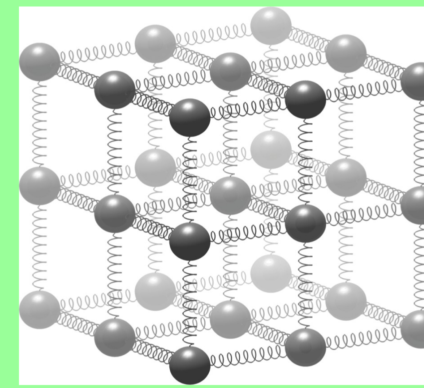
arXiv:1501.03845

What is a solid ?

- Shape & Volume fixed



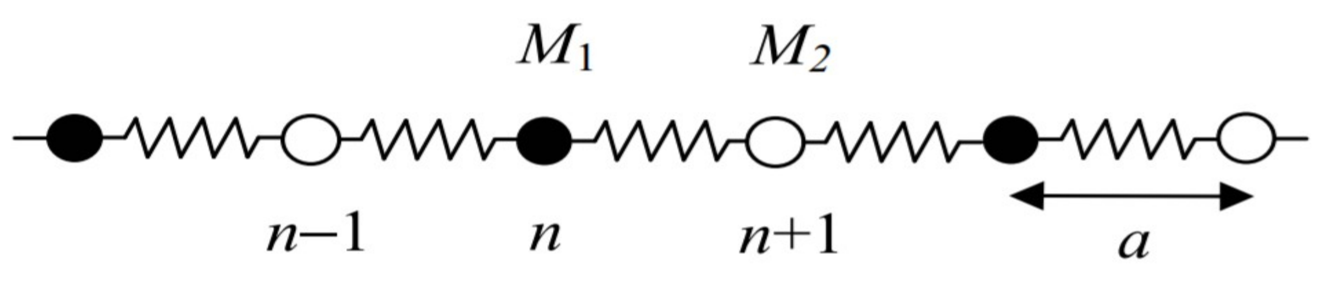
- Rigid ----> "elastic"



- Transverse sound (phonons)
- $g(\omega) \sim \omega^2 \quad : \quad \text{Debye}$
- $C(T) \sim T^3 \quad : \quad \text{Debye}$

Phonons from springs

1d chain



$$F_n = C(u_{n+1} - u_n) + C(u_{n-1} - u_n)$$

Hooke's law

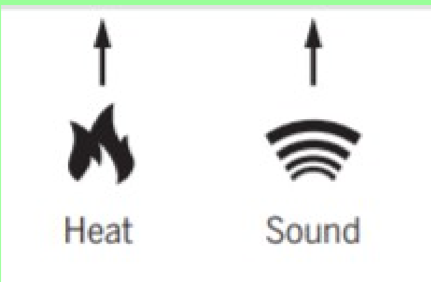
$$M_1 \frac{d^2 u_n}{dt^2} = -C(2u_n - u_{n+1} - u_{n-1})$$

$$M_2 \frac{d^2 u_{n+1}}{dt^2} = -C(2u_{n+1} - u_{n+2} - u_n)$$

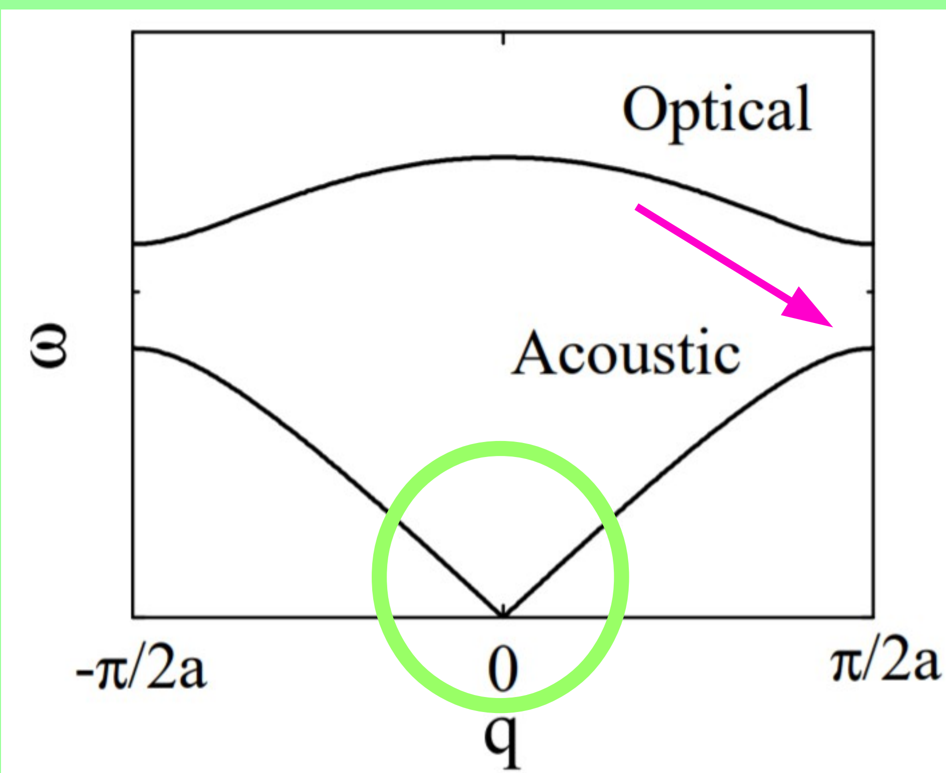
Phonons from springs

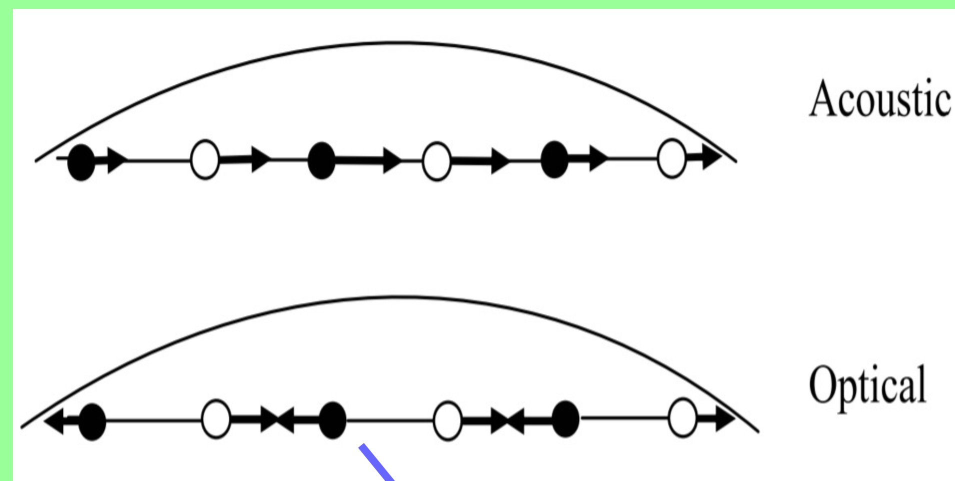
PHONONS :

$$\begin{bmatrix} 2C - M_1\omega^2 & -2C\cos qa \\ -2C\cos qa & 2C - M_2\omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0 \, .$$



$$\omega^2 = C\left(\frac{1}{M_1} + \frac{1}{M_2}\right) \pm C\sqrt{\left(\frac{1}{M_1} + \frac{1}{M_2}\right)^2 - \frac{4\sin^2 qa}{M_1 M_2}} \, .$$





$$\omega^2 = \frac{2\,C\,a^2}{M_1 + M_2}\, q^2 + \ldots$$

Vibrational density of states

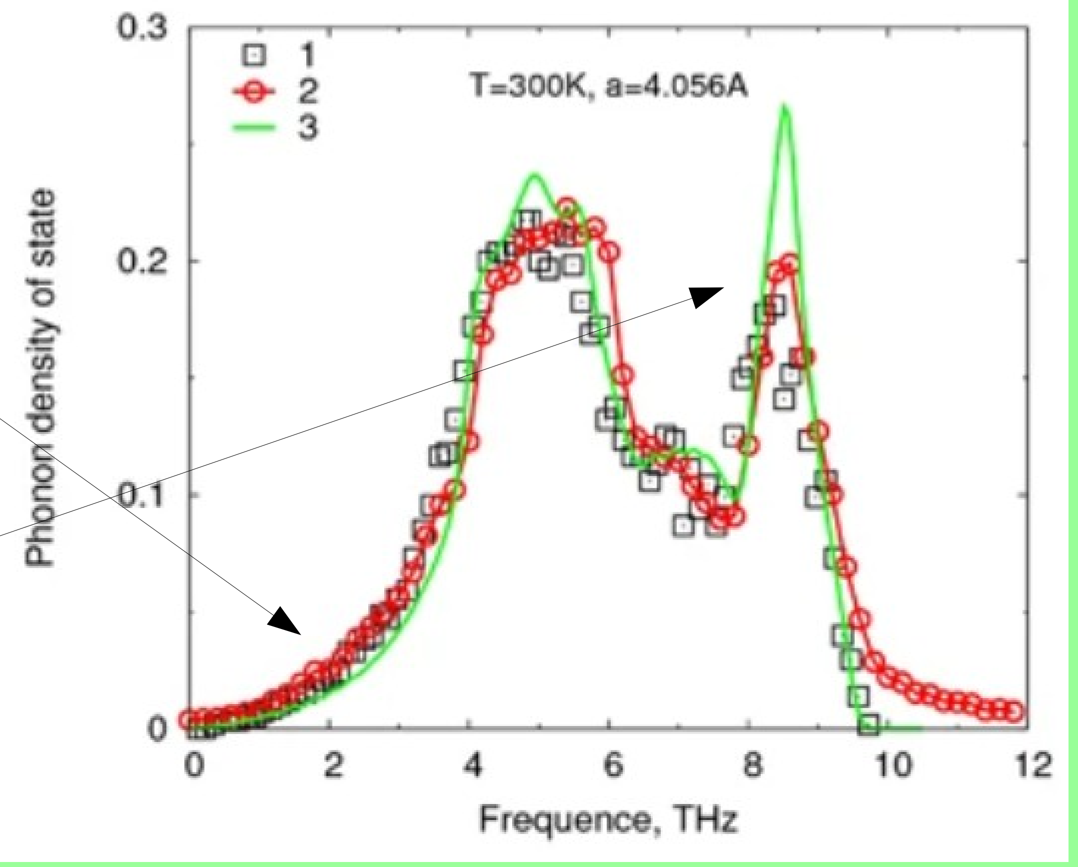
$$N_{q<k} = \left(\frac{L}{2\pi}\right)^3 \frac{4}{3}\pi k^3 = \frac{V k^3}{6\pi^2}$$

Debye model

$$g(\omega)\,d\omega = f(k)\,dk = \frac{k^2}{2\pi^2}\,dk$$

$$\omega = v\,k$$

$$g(\omega) = \frac{1}{2\pi^2 v^3}\,\omega^2$$



Van- Hove singularities

$$\frac{d\omega}{dk} = 0$$

Specific heat

$$U \;=\; 3 \int d\omega\, g(\omega)\, \hbar\, \omega\, n(\omega)$$

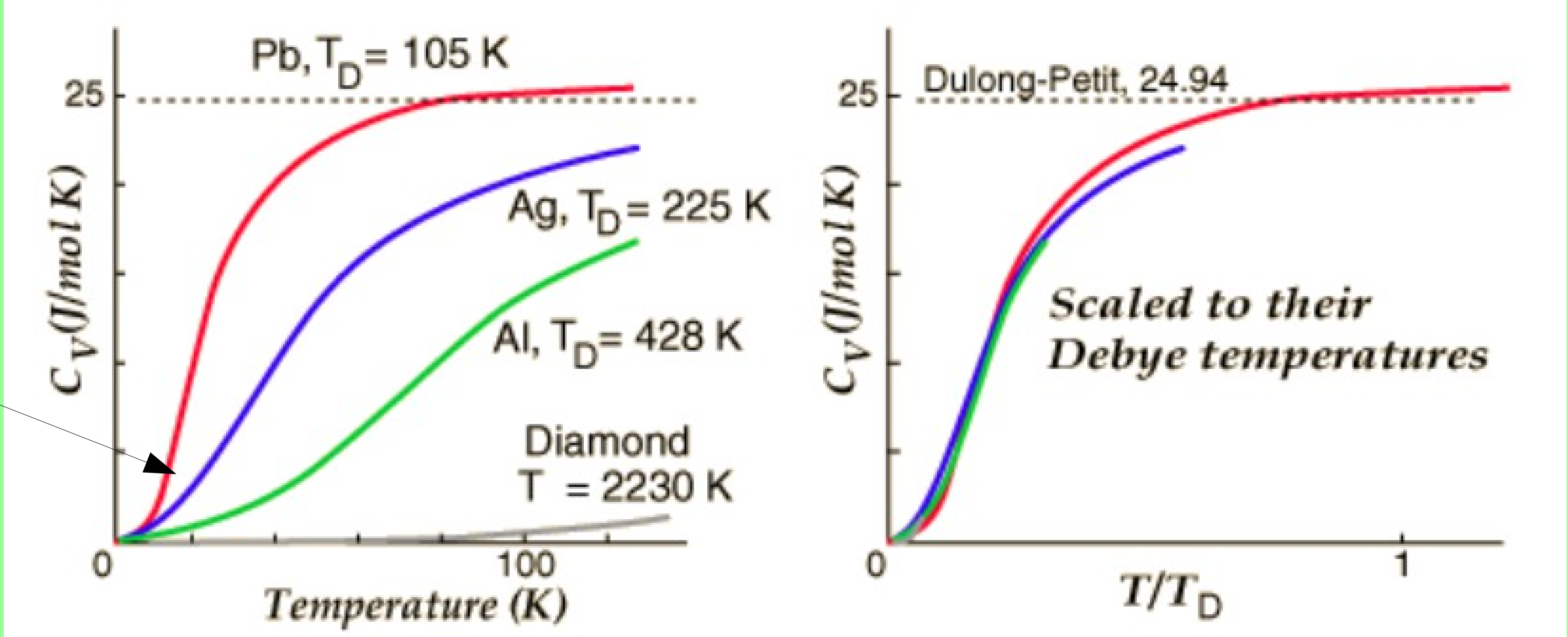
$$C \;=\; \frac{\partial U}{\partial T}$$

$$C(T) \;=\; k_B \int_0^\infty \left(\frac{\hbar\omega}{2\,k_B\,T}\right)^2 \sinh\left(\frac{\hbar\omega}{2\,k_B\,T}\right)^{-2} g(\omega)\, d\omega\,,$$

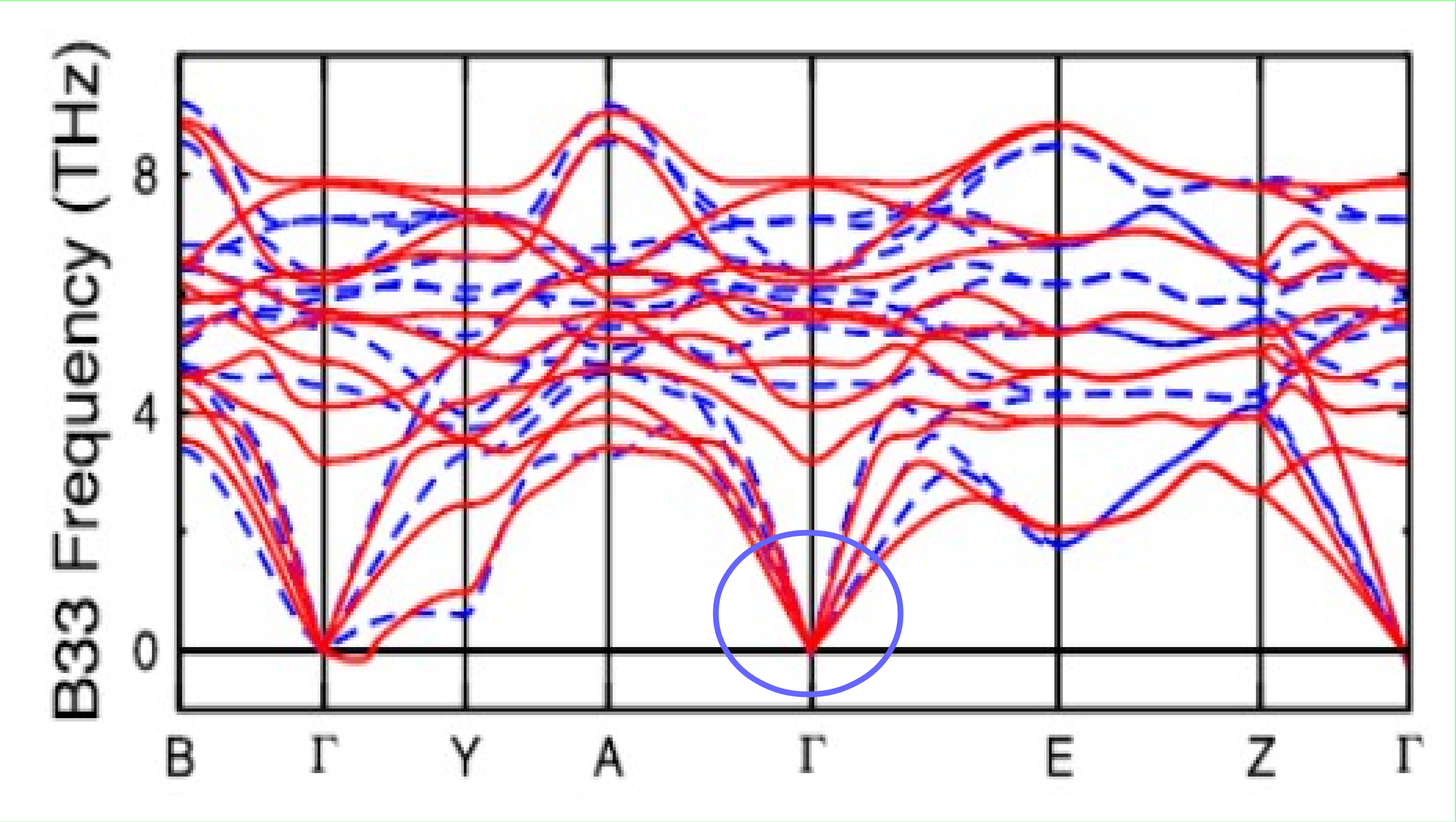
$$C(T) \;\sim\; T^d \quad [\leftarrow\; g(\omega) \sim \omega^{d-1}]$$

At low temperature !

Debye Law



Reality check

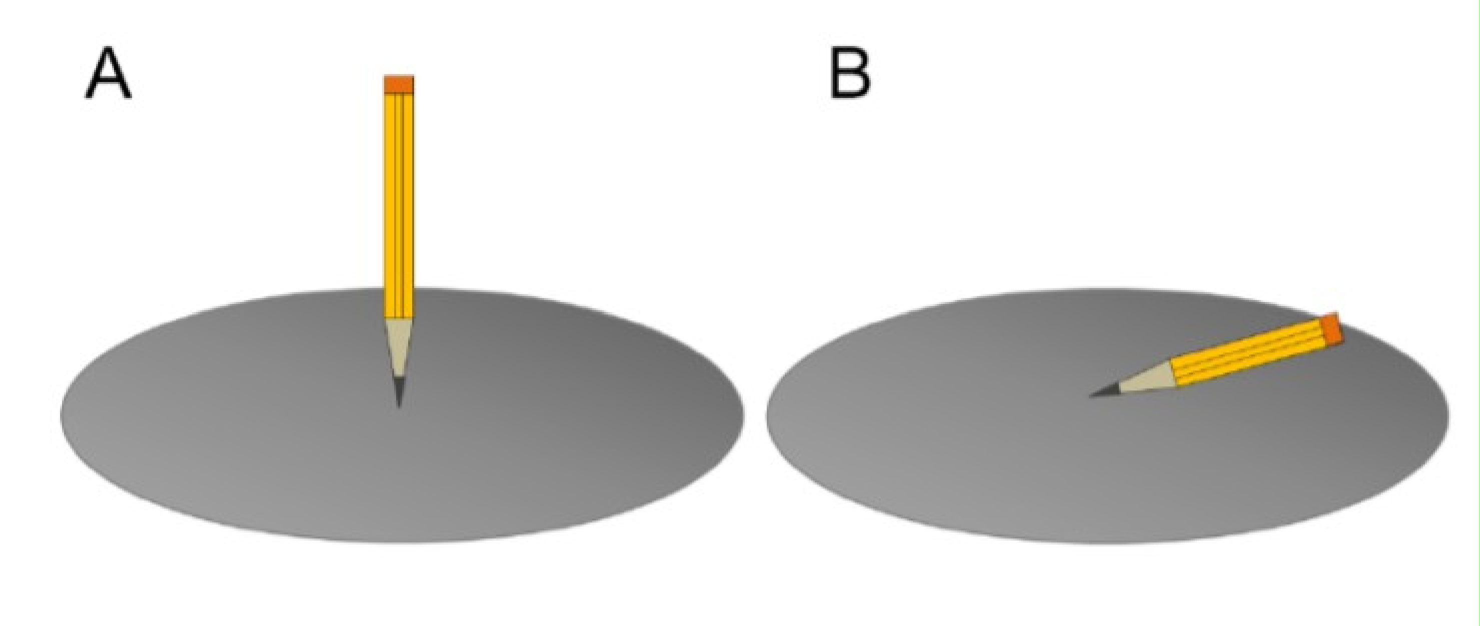


e.g. **Nickel titanium (Nitinol)**

https://www.youtube.com/watch?v=NcwBTn0zWHw

Towards a modern approach

Spontaneous symmetry breaking



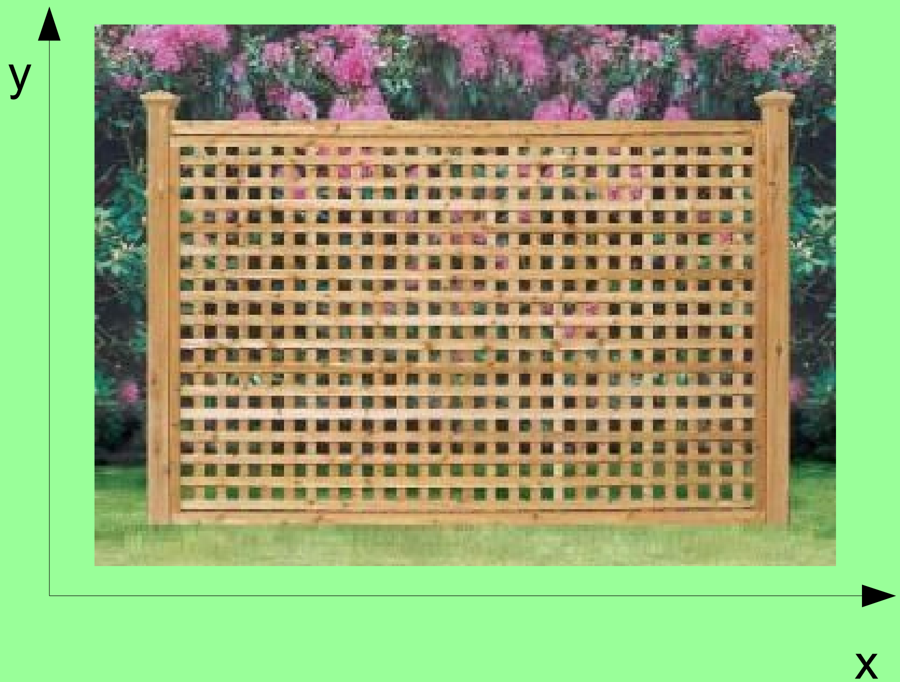
An Introduction to Spontaneous Symmetry Breaking
by Aron J. Beekman, Louk Rademaker, Jasper van Wezel
– Published as SciPost Phys. Lect. Notes 11 (2019)

SOLID = SYSTEM THAT BREAKS SPONTANEOUSLY TRANSLATIONS

PHONONS = CORRESPONDING GOLDSTONE BOSONS

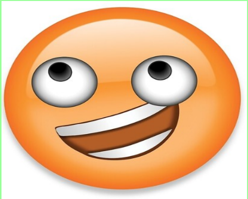
Caveats (1 example)

SPACETIME SYMMETRIES → **Be careful with the Goldstone Theorem**



How many broken generators ?

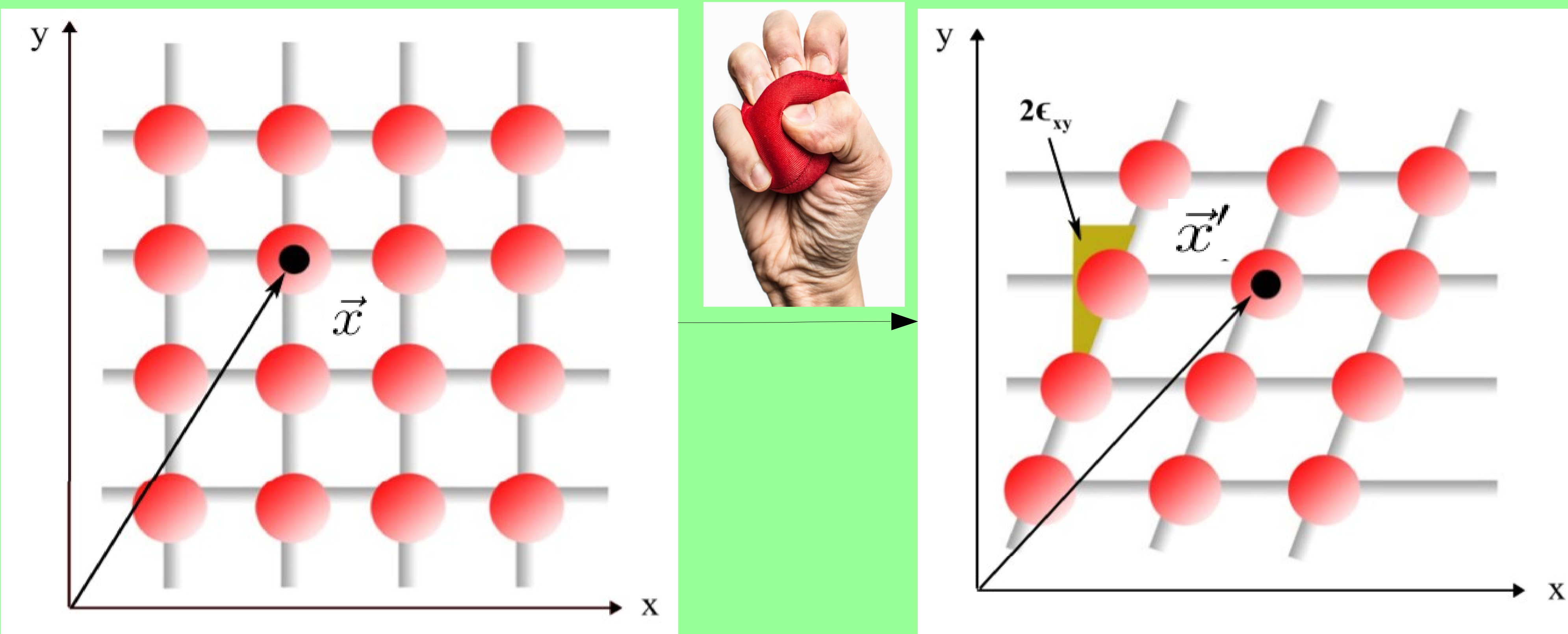
$\mathcal{P}_x \quad \mathcal{P}_y \quad \mathcal{J}$



And how many phonons (Goldstones) ?
(other ex. 2D membrane in 3D)

[Inverse Higgs constraint]

Theory of elasticity



$$\text{displacement } \vec{u} \equiv \vec{x}' - \vec{x}$$

$$\text{strain tensor } \epsilon_{ij} \equiv \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right)$$

$$u_i = \epsilon_{ij}\, dx^j$$

Theory of elasticity

dynamics

$$\rho\, \ddot{u}_i = \nabla_j\, \sigma_{ji}$$

+

constitutive equation

$$\sigma_{ij} = C_{ijkl}\, \epsilon_{kl}$$

ISOTROPIC SYSTEM

$$\sigma_{ij} = K\, \delta_{ij}\, \epsilon_{kk} + 2\, G \left(\epsilon_{ij} - \frac{1}{d} \delta_{ij}\, \epsilon_{kk} \right)$$

trace

transverse & traceless

bulk modulus

shear modulus

"shear"
It does not change the volume

Theory of elasticity

And finally we get:

$$\omega_{T,L} = \pm v_{T,L}\, k$$

$$v_T^2 = \frac{G}{\rho}\,, \qquad v_L^2 = \frac{G + K}{\rho}$$

OUR PHONONS

Few comments

$$K = -V\,\frac{dP}{dV}$$

- *Bulk modulus from thermodynamics*
- *No attenuation/dissipation*

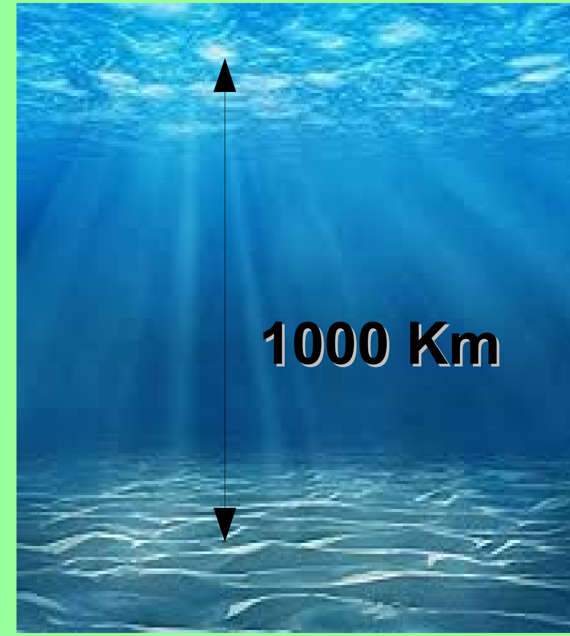
- *For relativistic theories*

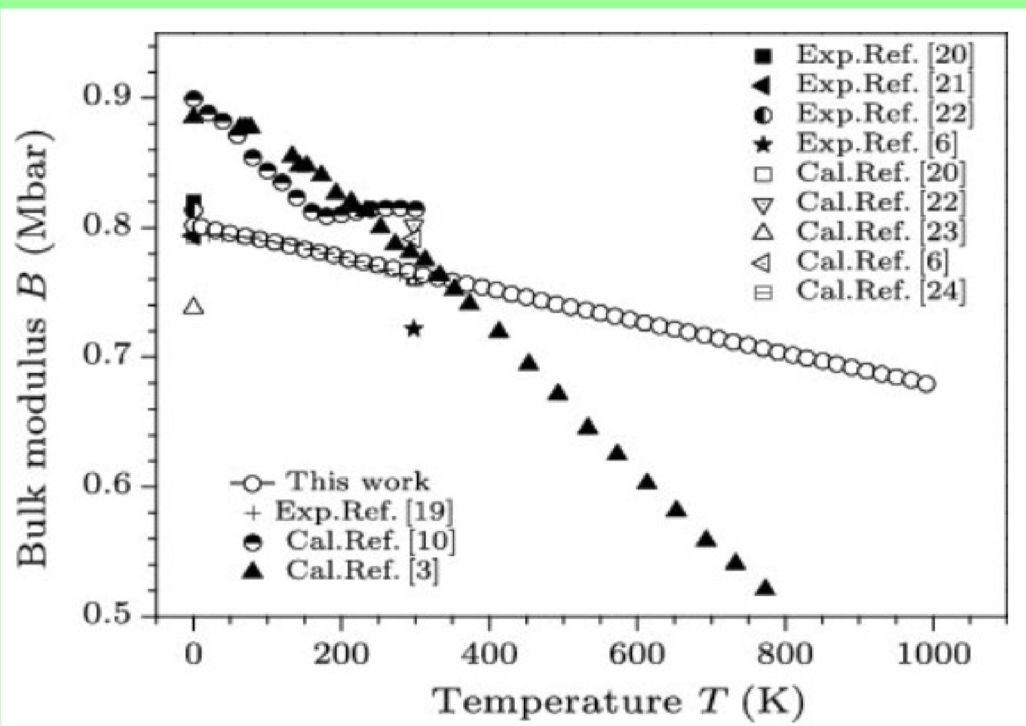
$$\rho \;\rightarrow\; \epsilon + p \quad (\chi_{\pi\pi})$$

Momentum susceptibility

Some experimental data

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Silicone rubber	0.001×10^{10}	0.2×10^{10}	0.0002×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}
Tendon (typical)	0.12×10^{10}	—	—





Rigidity diminishes
Increasing temperature

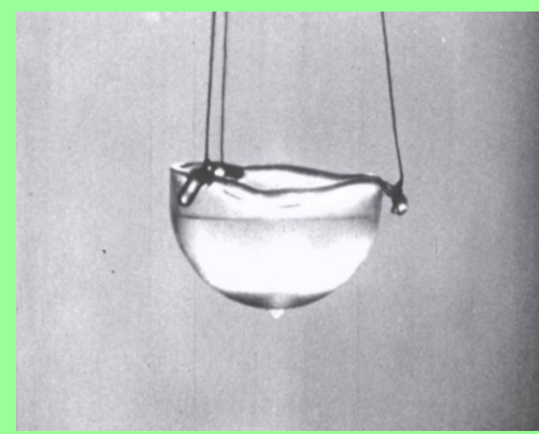
A systematic classification

E
N
E
R
G
Y

Relativistic lorentz invariance

Different spontaneous breaking patterns

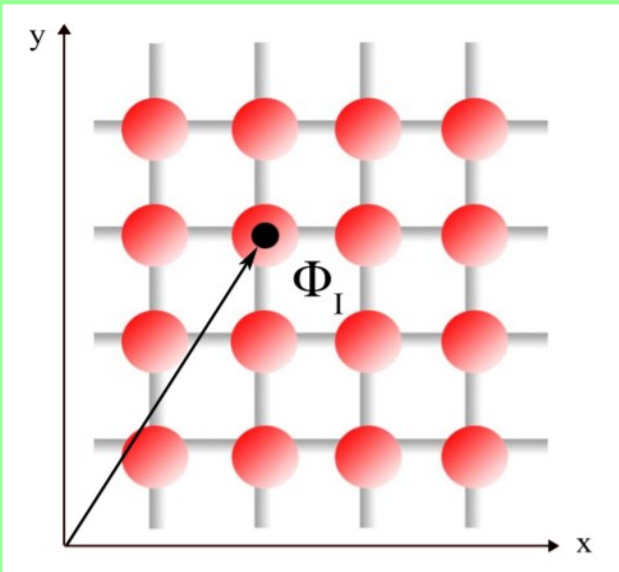
[Nicolis, Penco, Piazza, Rattazzi,2015]



~~**Relativistic lorentz invariance**~~

First step to be modern

Build a field theory language



Set of "comoving coordinates"

$$\Phi^i = x^i + \phi^i(t,x)$$

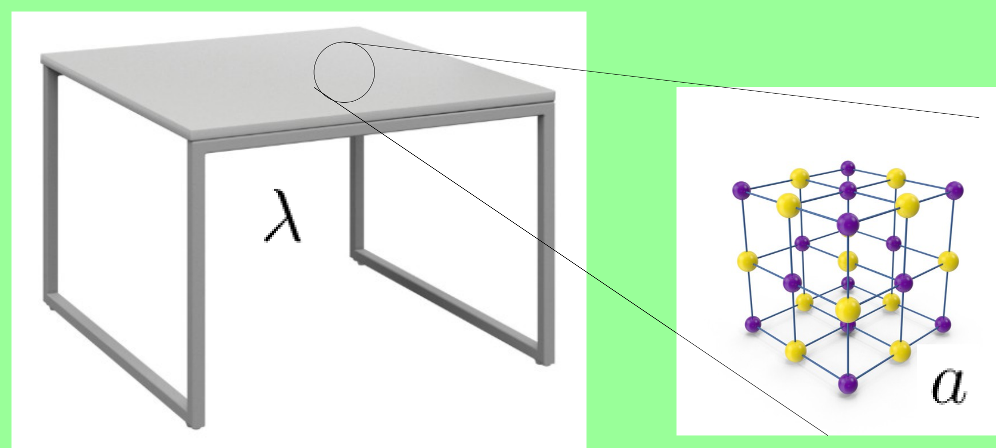
Equilibrium **PHONONS**

$$\varepsilon_{ij} = \frac{1}{2}\left(\partial_i \phi_j + \partial_j \phi_i\right)$$

Assumptions :

$$\lambda \gg a$$

Isotropy and homogeneity at large scales



The effective action

Lowest order : $$\mathcal{I}^{IJ} = g^{\mu\nu}\partial_\mu\Phi^I\partial_\nu\Phi^J$$

$$Z = \det\left(\mathcal{I}^{IJ}\right)$$

$$d-2 \text{ traces } X_n = \mathrm{tr}\left\{(I^{IJ})^n\right\}$$

Most general action

$$S = -\int d^dx\,\sqrt{-g}\,V\left(Z,\{X_n\}\right)$$

For the moment flat space

The formalism

SSB pattern

Spacetime translations — *Internal shifts*

$$x^I \to x^I + b^I \quad \times \quad \phi^I \to \phi^I + a^I$$

homogeneity

Diagonal subgroup

$$a = -b$$

Homogeneous stress tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}\Big|_{g=\eta} = -\eta_{\mu\nu} V + 2\,\partial_\mu\phi^I\partial_\nu\phi_I\, V_X + 2\left(\partial_\mu\phi^I\partial_\nu\phi_I\, X - \partial_\mu\phi^I\partial_\nu\phi^J\,\mathcal{I}_{IJ}\right) V_Z\,. \qquad (5)$$

Some phenomenology

[MB,1708.08477]

$$T^{tt} \equiv \rho = V\,,$$
$$T^x_x \equiv p = -V + X\,V_X + 2\,Z\,V_Z\,,$$
$$T^x_y = 2\,\partial_x\phi^I\partial_y\phi^I\,V_X\,,$$

Shear modulus

$$G = 2\,V_X$$

Bulk modulus

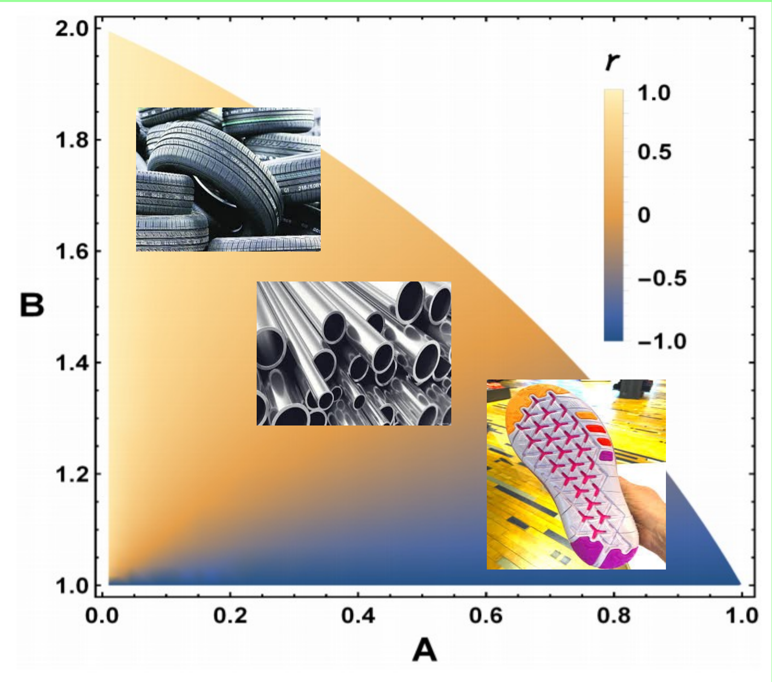
$$K = 2ZV_Z + 4Z^2V_{ZZ} + 4XZV_{XZ} + X^2V_{XX}$$

$$V(X,Z) = \rho_{\rm eq}\,X^A\,Z^{(B-A)/2}$$

$$G = \rho_{\rm eq}\,2^A\,A\,, \quad K = \rho_{\rm eq}\,2^A\,B\,(B-1)$$

Poisson's ratio

$$\mathfrak{r} \equiv \frac{K-G}{K+G}$$



The collective excitations (phonons)

$$\phi^I = \phi^I_{\rm str} + \pi^I$$

and expand the action at 2[nd] order

[MB,1708.08477]

$$\delta S_2 = \int d^3x \Big[N_T \dot{\pi}_T^2 + N_L \dot{\pi}_L^2 + 2 N_{TL} \dot{\pi}_T \dot{\pi}_L - c_T^2 (\partial_x \pi_T)^2 - c_L^2 (\partial_x \pi_L)^2 - 2 c_{TL}^2 \partial_x \pi_T \partial_x \pi_L \Big], \quad \text{(A3)}$$

$$\pi_{L/T} = a_{L/T}\, e^{i\omega t - ikx} \longrightarrow \begin{pmatrix} \#(\omega,k) & \#(\omega,k) \\ \#(\omega,k) & \#(\omega,k) \end{pmatrix} \begin{pmatrix} a_T \\ a_L \end{pmatrix}$$

$$\omega_{T,L} = \pm\, v_{T,L}\, k$$

$$v_T^2 = \frac{G}{\epsilon + p}, \qquad v_L^2 = \frac{G + K}{\epsilon + p}$$

Solid VS fluids part I

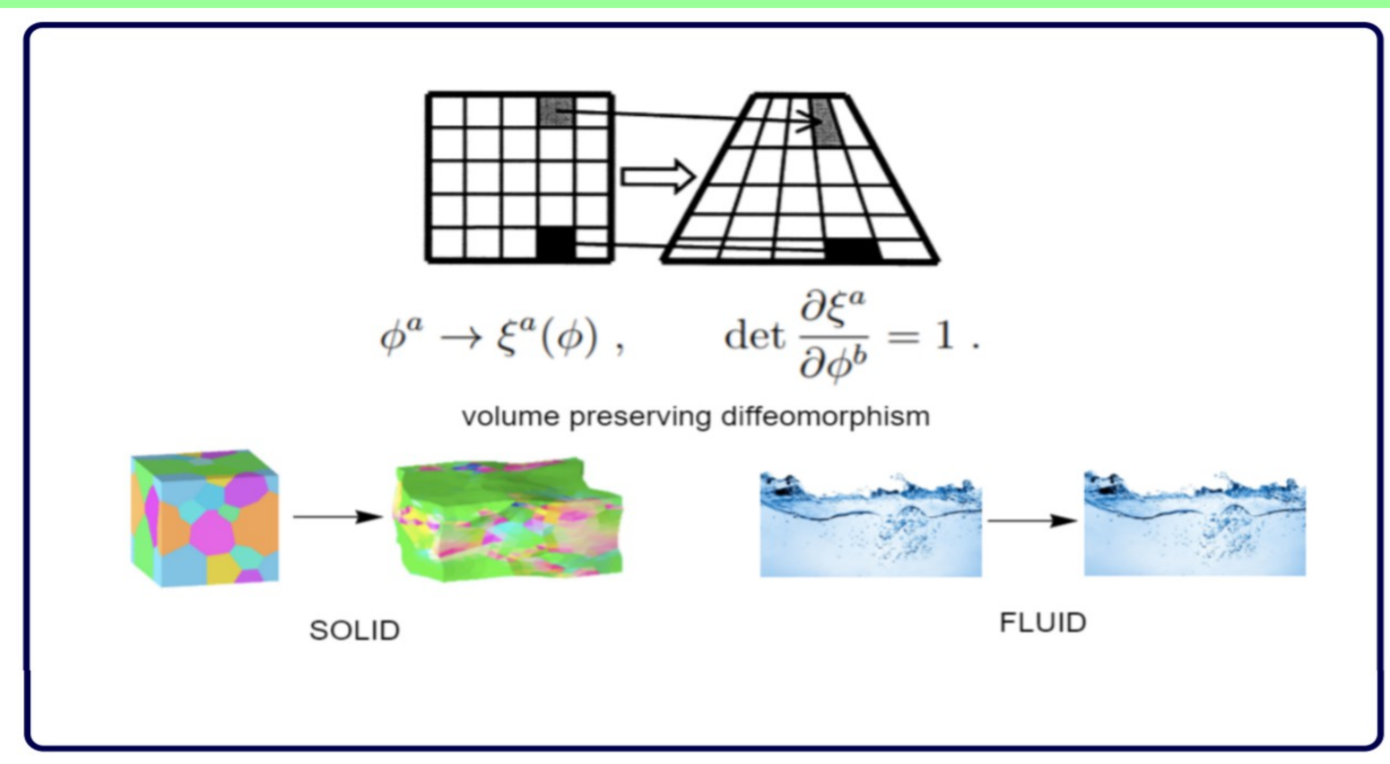
$$G = 2V_X$$

$$S = -\int d^d x \, V(Z)$$

No shear modulus

No propagating shear waves

IT'S A FLUID



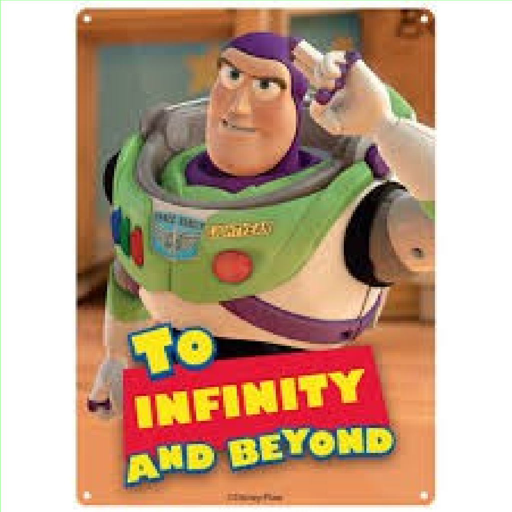
"A fluid takes the shape of the container …"

SYMMETRIES

EFT is nice but ...

1) **Very hard to include dissipation (problem of the field theory formalism)**

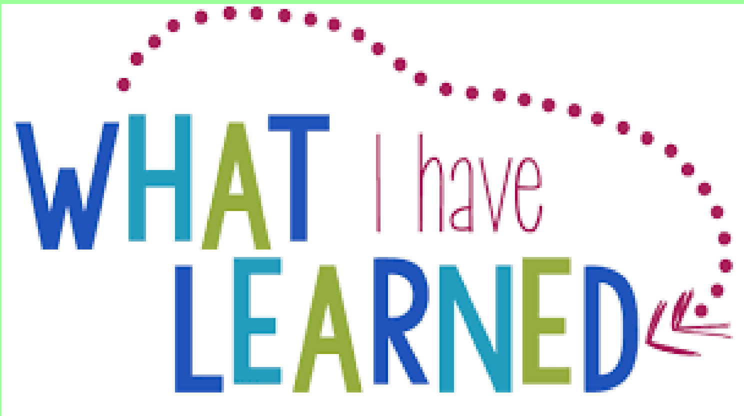
Fundamental for attenuation, viscous effects, Viscoelastic materials, finite T effects, ...

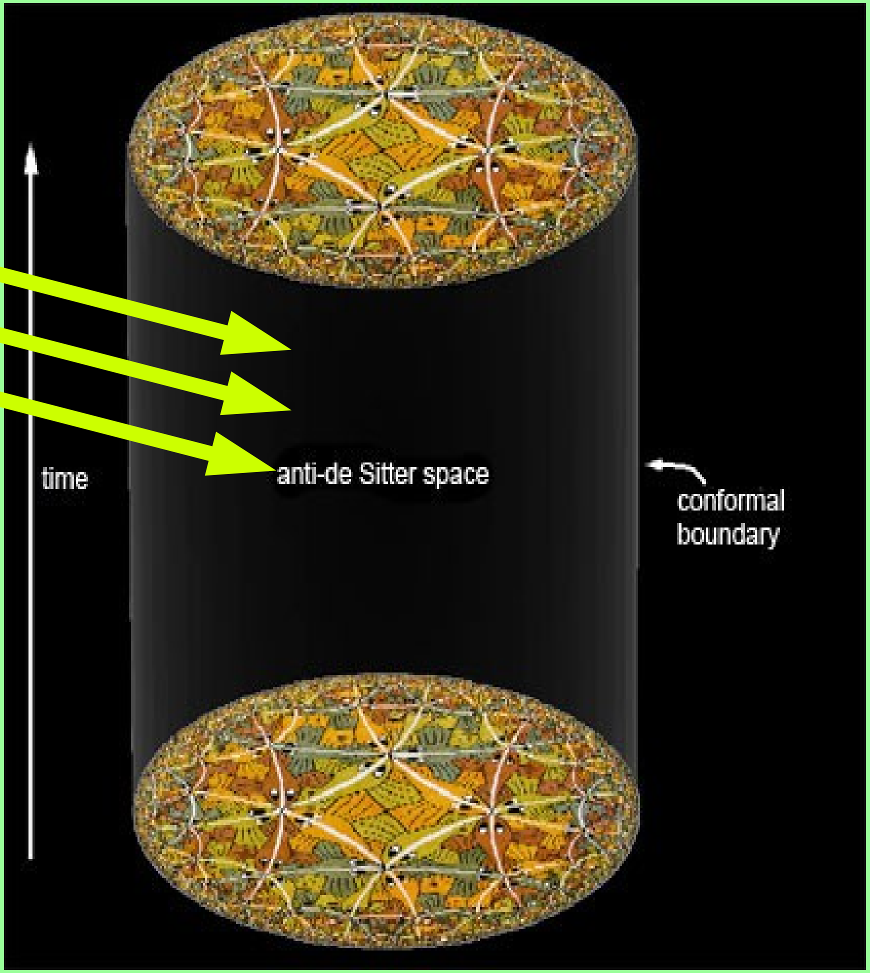


2) **It does not predict any transport coefficient (microscopics are needed)**

?

Holographic counterpart



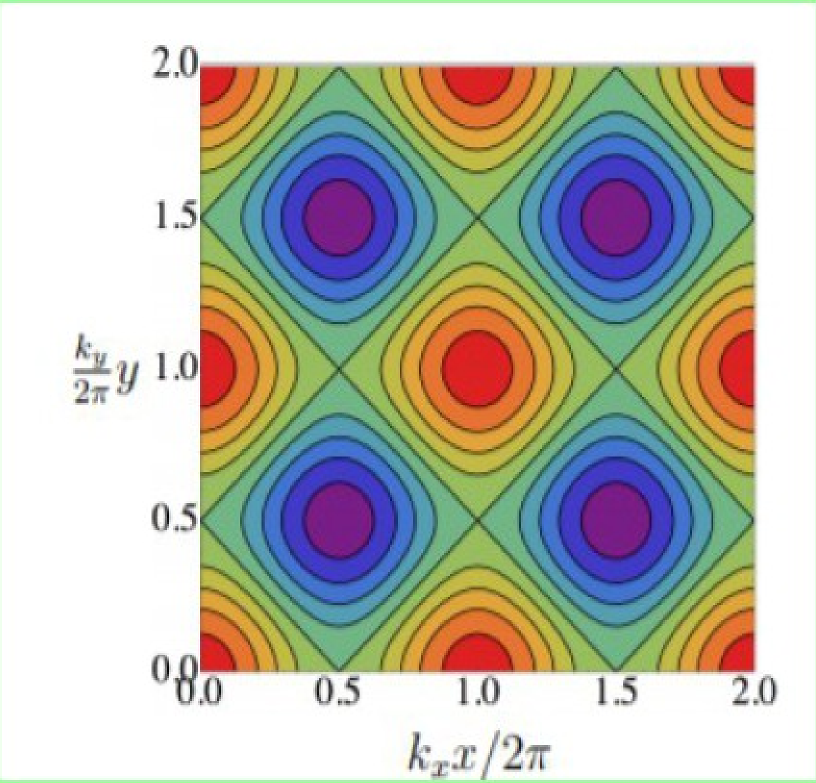


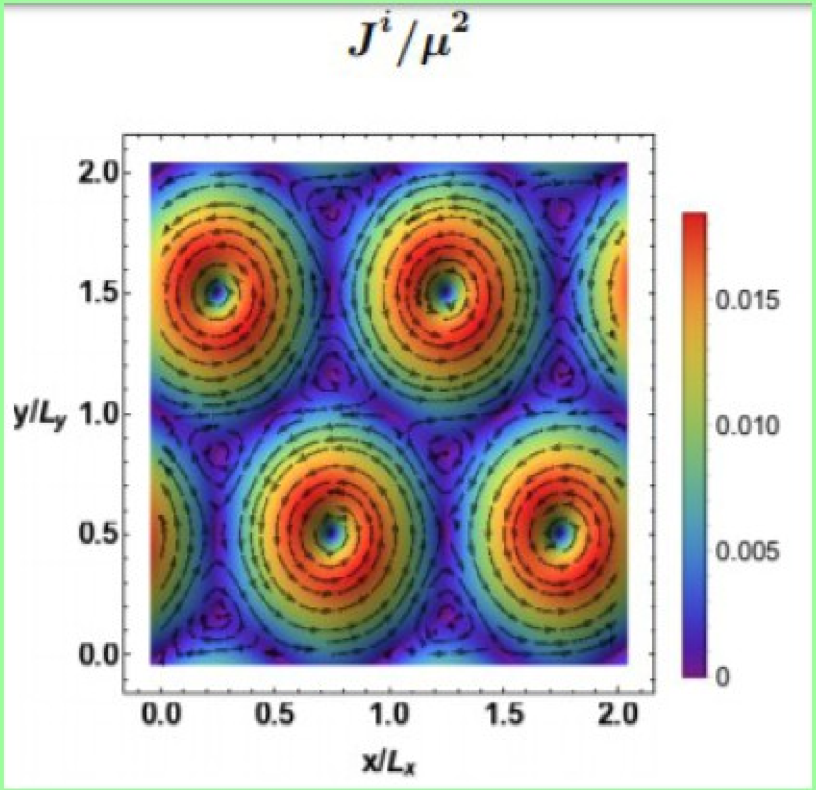
The key concept

Breaking translations Spontaneously !

OUR GOAL !

If you are strong & patient (not like me)





Black hole instabilities at finite momentum!

The solution wants to "fall" into a inhomogeneous background, "a lattice"

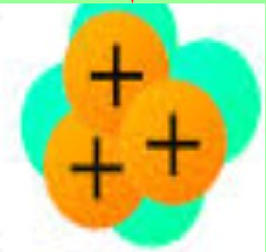
[Donos,Krikun, Andrade, Pantelidou, Li, Zaanen, Cremonini, Schalm, ...]

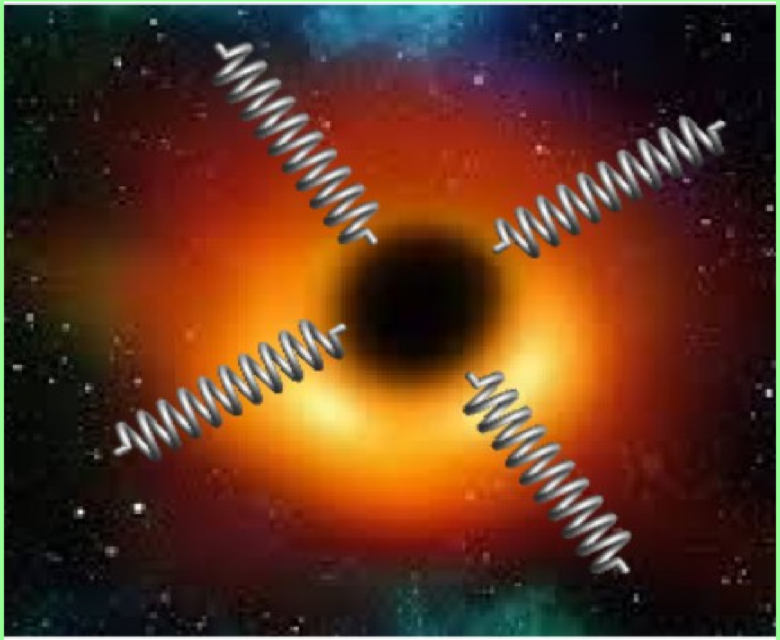
How to embed this into Holography

$$S = M_P^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + \frac{3}{\ell^2} - m^2 V(X) - \frac{1}{4} F^2 \right] ,$$

gravity

$$ds^2 = \frac{1}{u^2} \left[-f(u)\, dt^2 - 2\, dt\, du + dx^2 + dy^2 \right]$$





Asymptotically AdS ELASTIC black hole

(analytic background = easy)

[MB,1411.1003 ,1510.09089]

Lorentz-Violating Massive gravity

From the dictionary : ***[MB,1510.09089]***

$$\partial_\mu T^{\mu\nu}_{CFT} = 0$$ ⟷ **Diffeomorphisms invariance**

$$\partial_i T^{ij} \neq 0$$ ⟶ **Momentum dissipation**

(cf. Drude model)

Caveats: **1) Breaking lorentz invariance (energy conserved)**

2) We want to break it spontaneously !!!

Explicit VS Spontaneous

How do we do it: $\phi^I = x^I$

$$V(X) = X^n$$

$$\phi^I(u,x) = \phi^I_0(x) + \phi^I_1(x)\, u^{5-2n} + \dots$$

$n < 5/2$

$$x^I = \phi^I_0(x) \equiv source$$

Explicit breaking

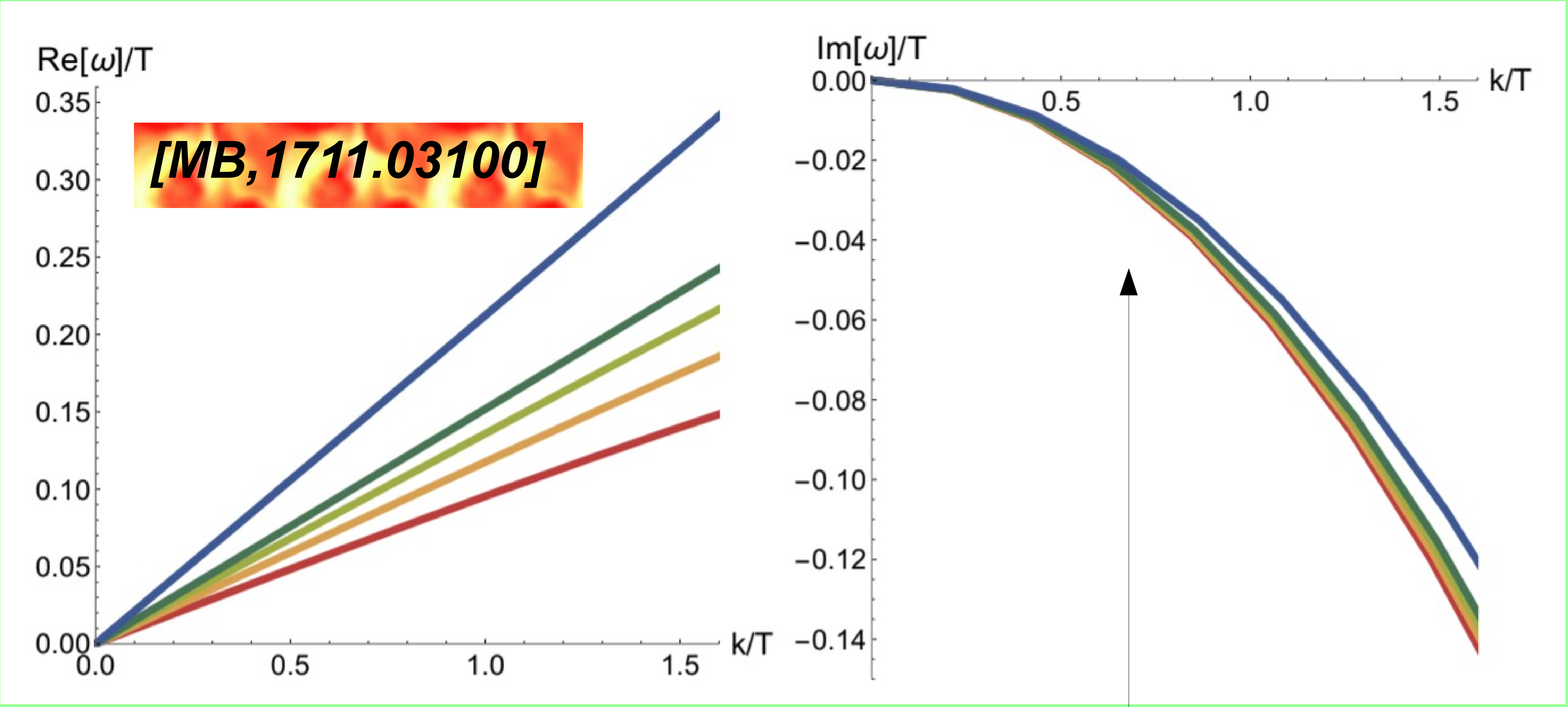
$n > 5/2$

$$x^I = \phi^I_0(x) \equiv \langle \mathcal{O}^I \rangle$$

Spontaneous breaking

[MB,1711.03100]

Where are the phonons



Transverse Phonons (it is a solid !)

$$\omega = v\,k - i\,\frac{\Gamma}{2}k^2 + \ldots$$

The elastic modulus

$$stress(\omega, k) = G\, strain - i\,\omega\,\eta\, strain + \dots$$

$\langle T^{ij} \rangle$ (stress) $\quad h_{ij}^{(0)}$ (strain)

[MB,1711.03100]

KUBO FORMULA

$$G = \mathrm{Re}\,\langle T^{ij} T^{ij} \rangle^R (\omega = k = 0)$$

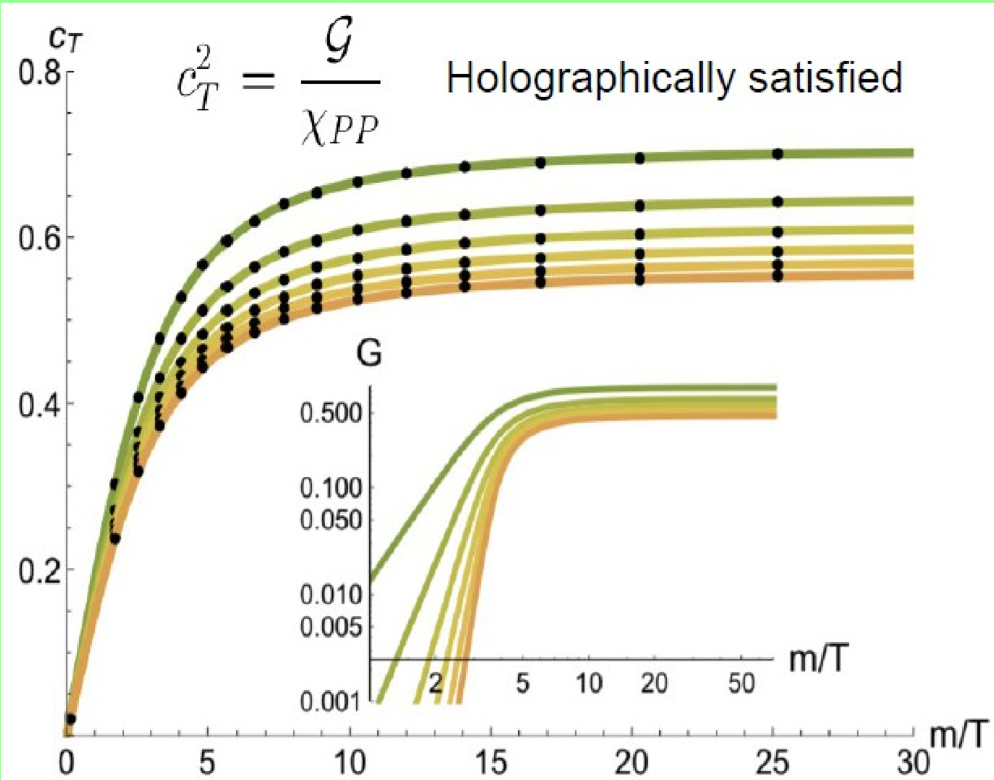
Remarks

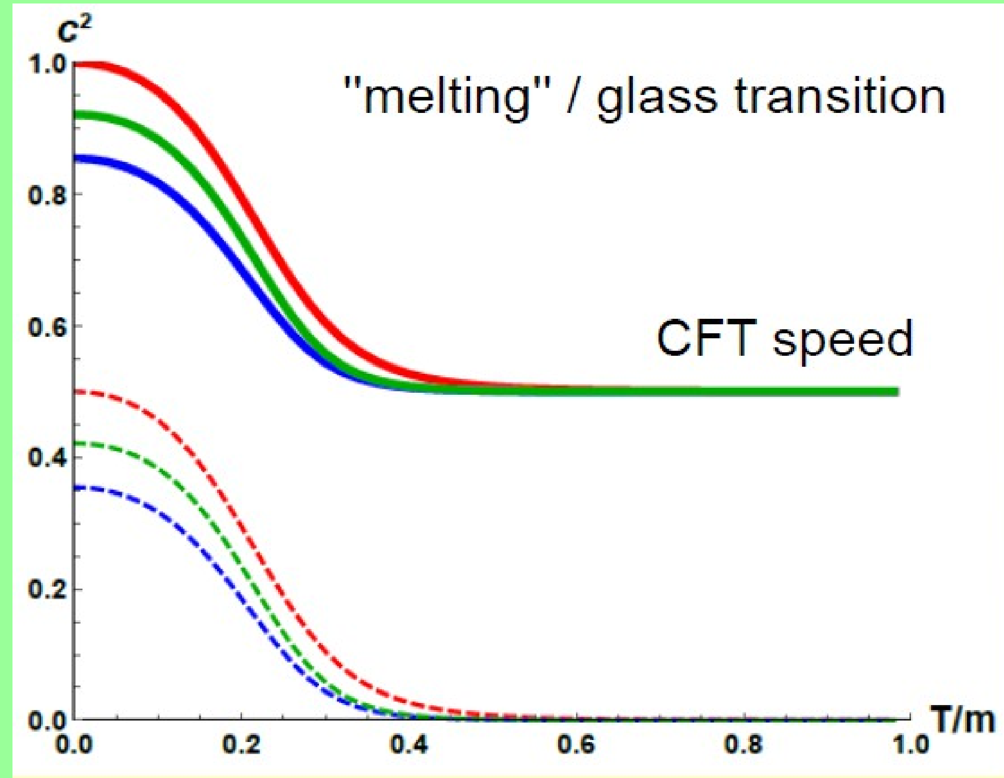
1) the source can also be implemented with the scalars (gauge choice) *[useful later]*

2) perturbative formula

$$G = m^2 \int_0^{u_h} \frac{V'\left(v^2\right)}{v^2}\, dv\,, \qquad m/T \ll 1\,.$$

How does a holographic solid sound





The formulas for the speeds works perfectly !

+

$$v_L^2 = \frac{1}{2} + v_T^2$$

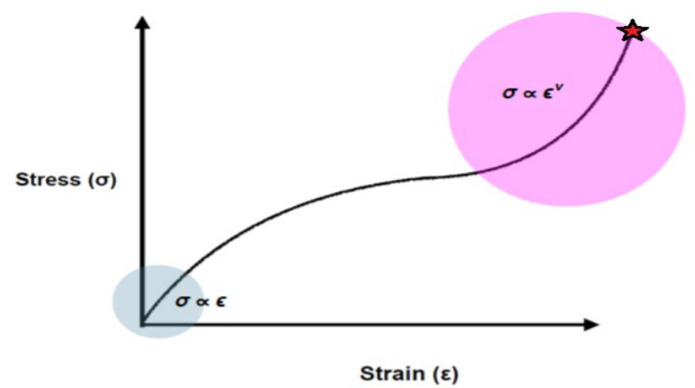
Conformal symmetry

[MB,1711.03100,1910.05281]

Beyond linear elasticity

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} + \dots \longrightarrow \sigma(\epsilon)$$

[MB,1807.07474,2006.10774]

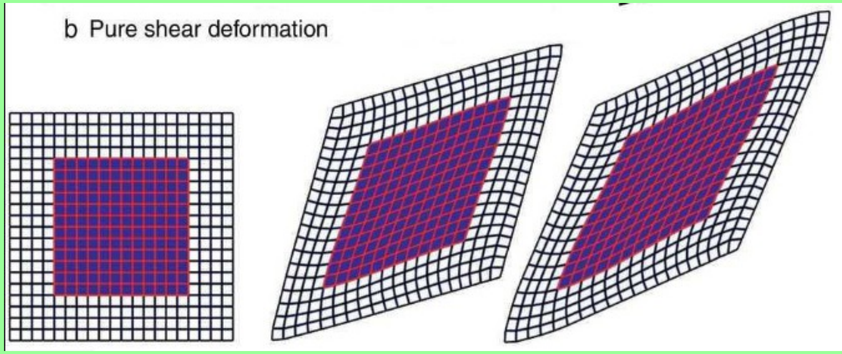


(NONLINEAR) STRESS-STRAIN CURVE

$$\phi^I_{\rm str} = O^I_J x^J\,, \quad O^I_J = \alpha \begin{pmatrix} \sqrt{1+\varepsilon^2/4} & \varepsilon/2 \\ \varepsilon/2 & \sqrt{1+\varepsilon^2/4} \end{pmatrix}$$

BULK STRAIN $\varepsilon_{ii} = 2(\alpha - 1)$

SHEAR STRAIN ε

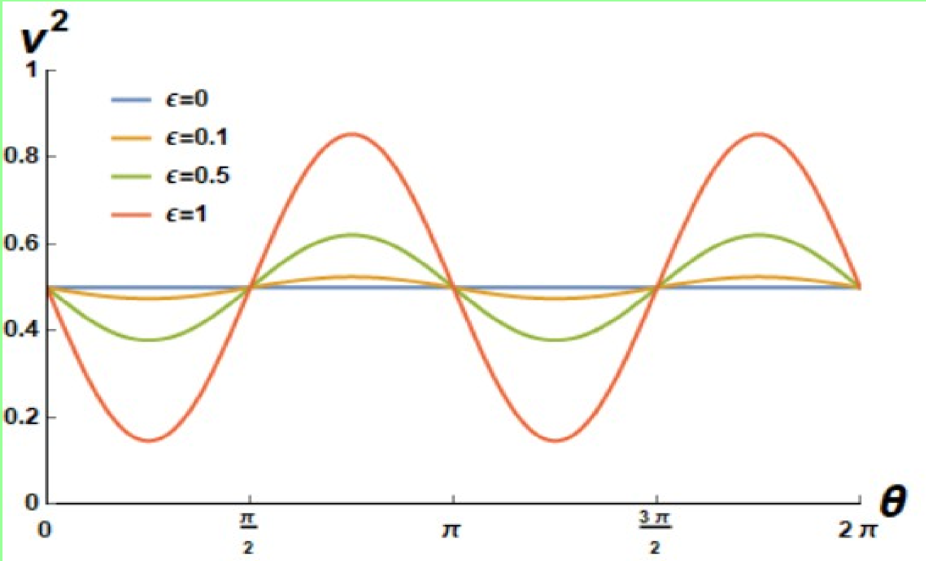


Beyond linear elasticity

Phonon speeds become anisotropic

$$\omega_{\pm} = c_{\pm}(\alpha, \varepsilon)\, k$$

[MB,1807.07474,2006.10774]

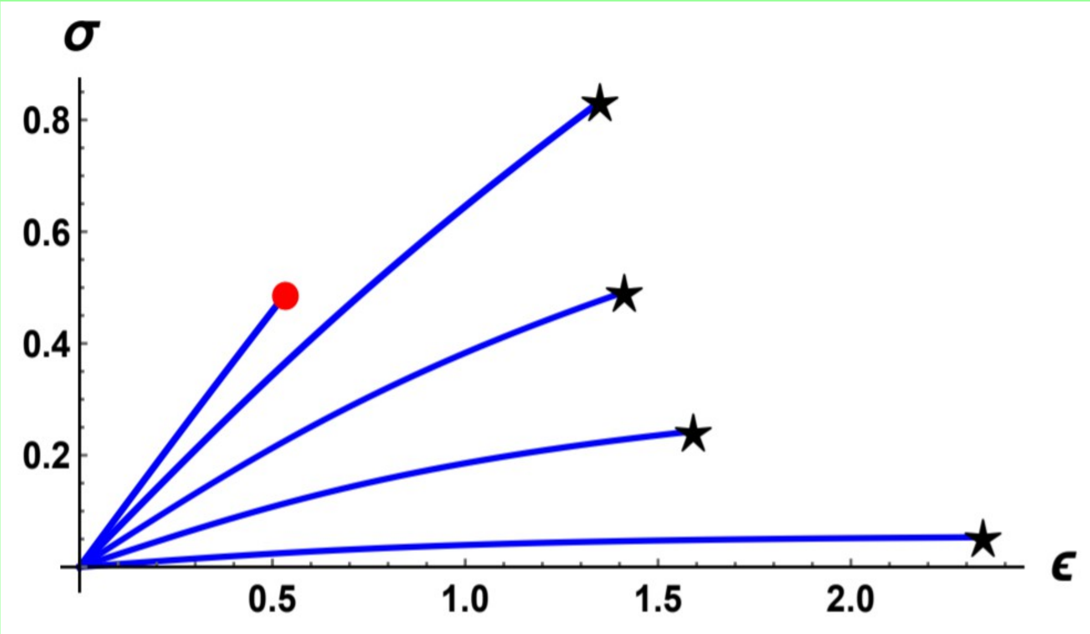


Non-linear large Deformations curves + breaking

$$V(X,Z) = \rho_{\rm eq}\, X^A\, Z^{(B-A)/2}$$

$$\sigma(\varepsilon) \sim A\, \varepsilon^{2A}$$

$$\Delta T_{ii}(\kappa) \sim (B-1)\, \kappa^{2B}$$



Beyond holographic linear elasticity

[MB,1807.07474,2006.10774]

Same as before :

$$\Phi^I(u,x) = O^I_j\, x^j$$

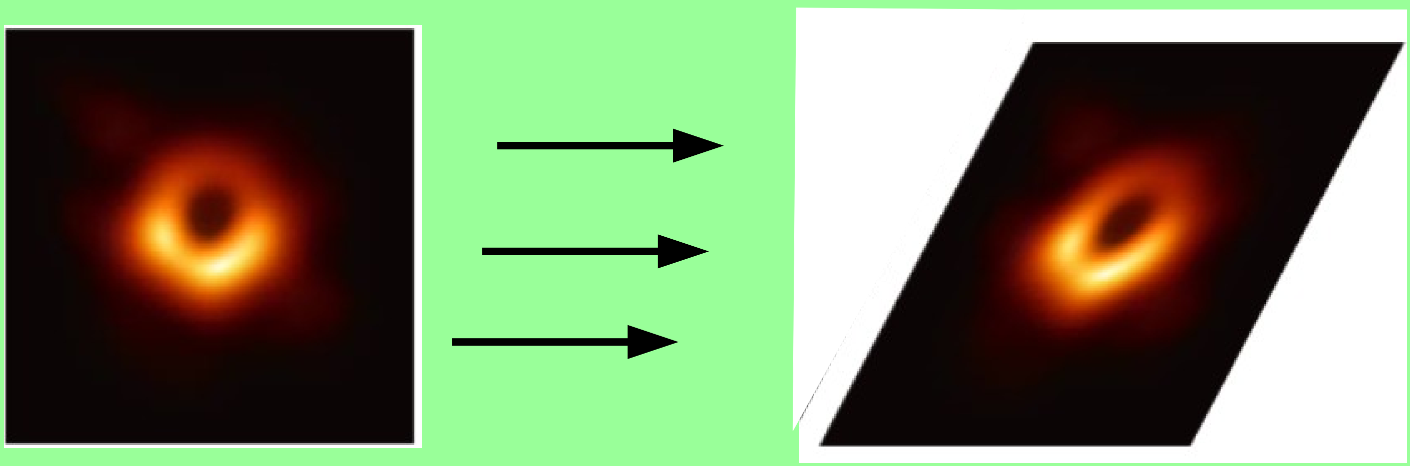
+ *:*

$$ds^2 = \frac{1}{u^2}\left(-f(u)\,e^{-\chi(u)}\,dt^2 + \frac{du^2}{f(u)} + \gamma_{ij}(u)\,dx^i dx^j\right)$$

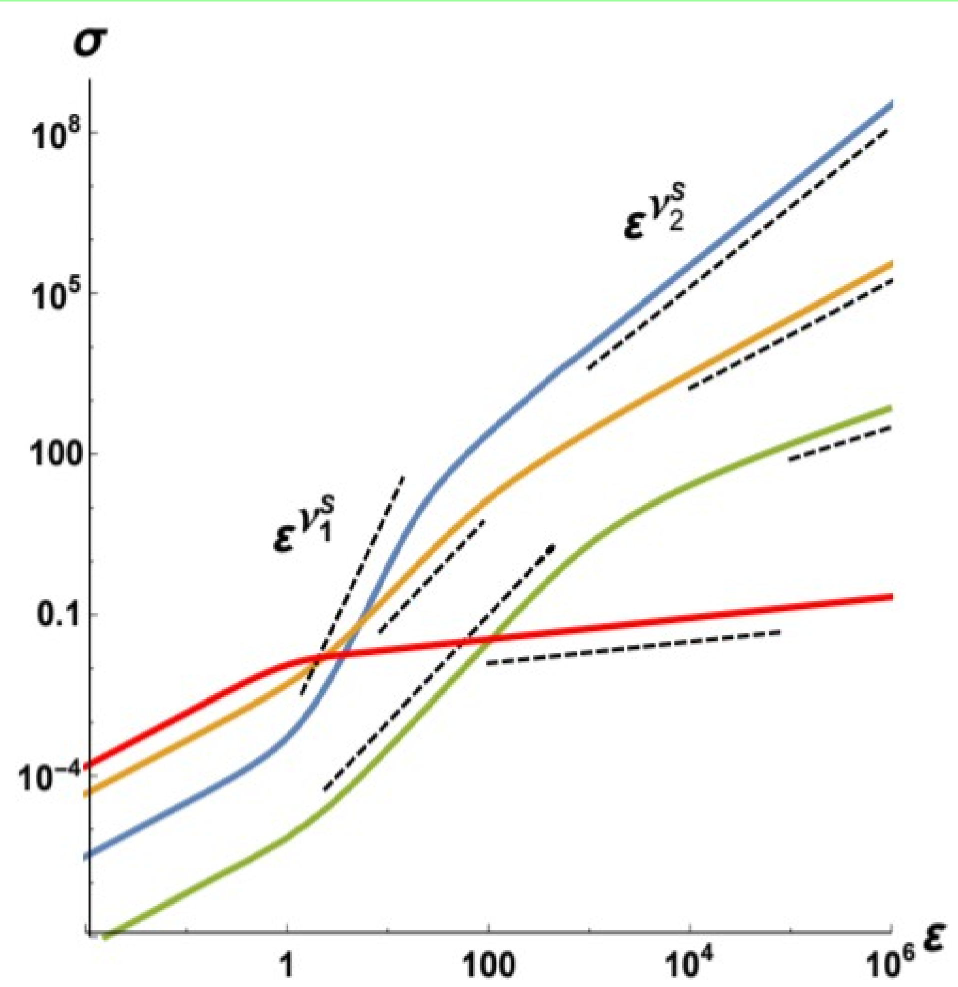
Anistropic black hole with finite background strain (spin 2 hair)

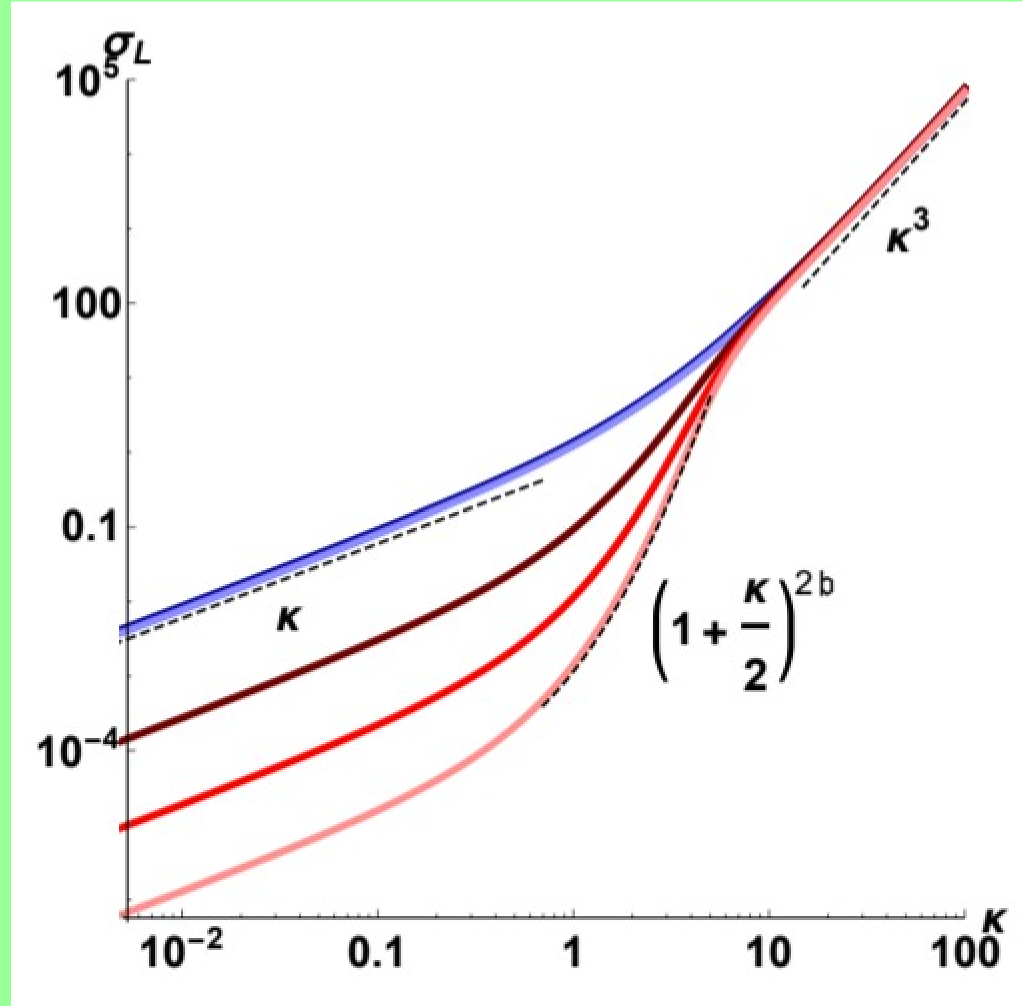
$$\widehat{\gamma} = \exp\left[h_+(u)\,\widehat{\sigma}_+ + h_\times(u)\,\widehat{\sigma}_\times\right]$$

$$h_\times = h\cos\theta \qquad h_+ = h\sin\theta$$



Beyond holographic linear elasticity





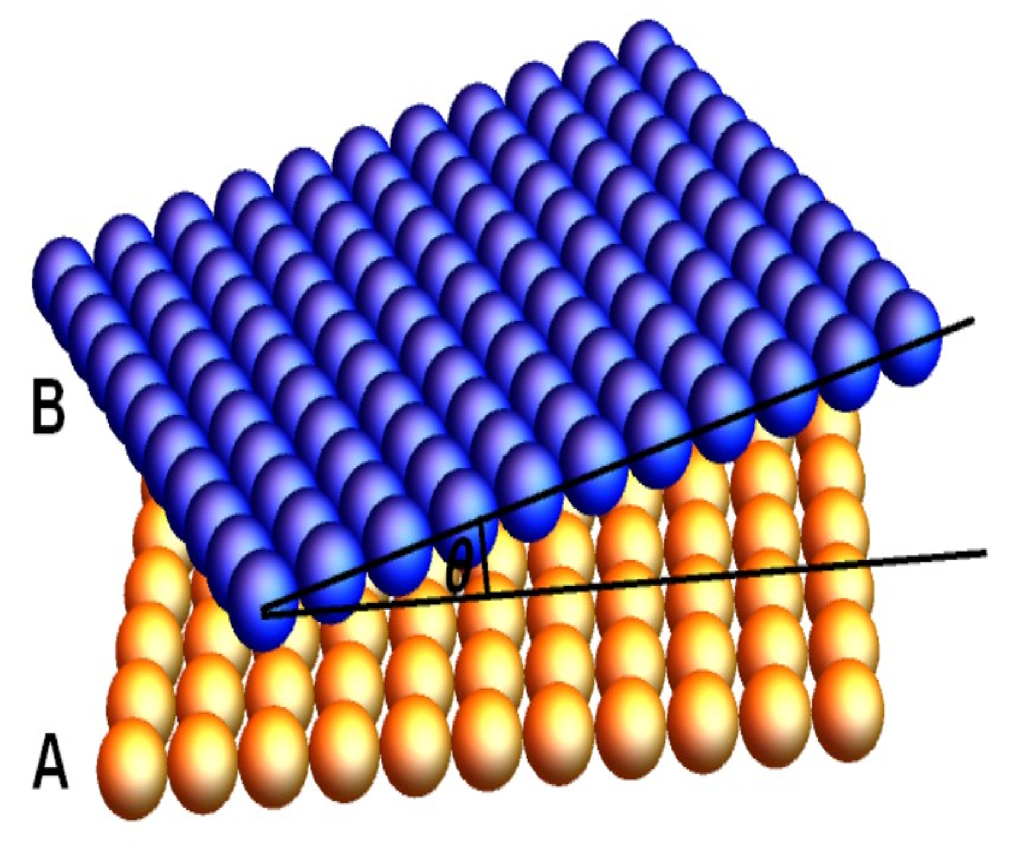
A lot of phenomeology to explore
+ a nice example of nonlinear response
In holography (cf.nonlinear conductivity) !

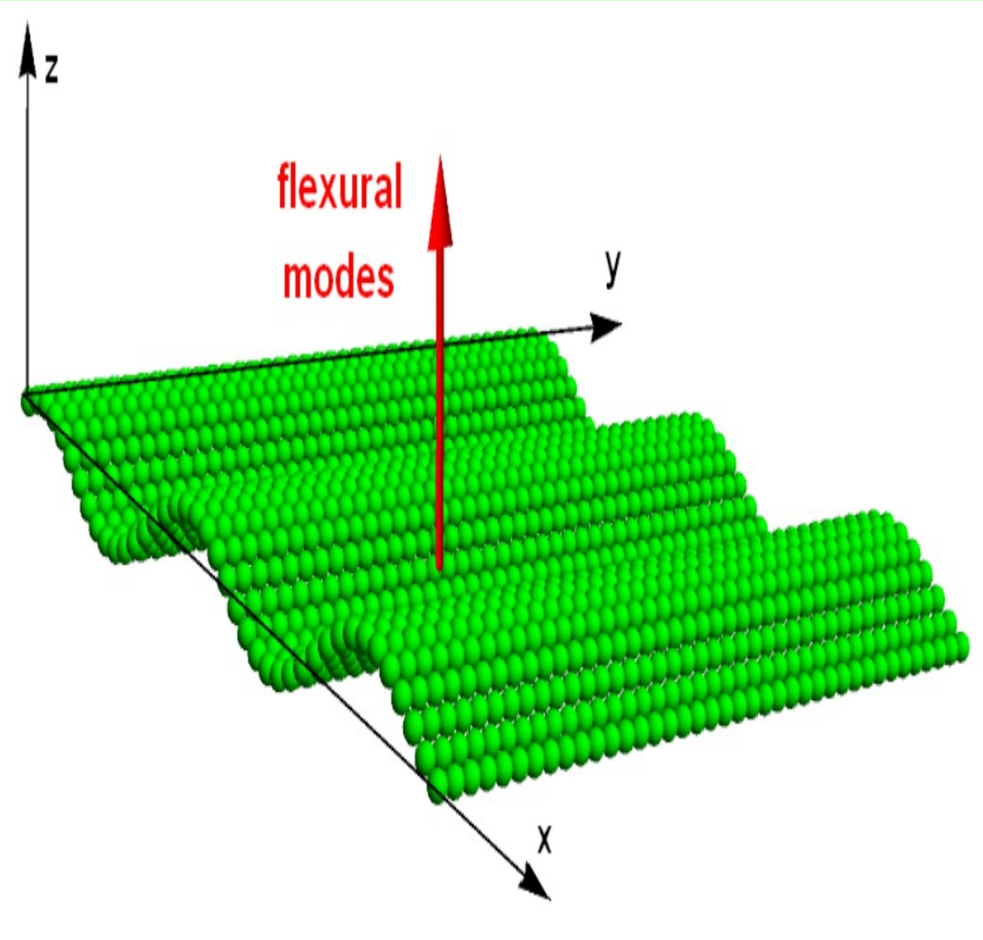
[MB,2006.10774]

More advanced topics

Most famous example:
twisted bilayer graphene

[MB,"coming soon"]





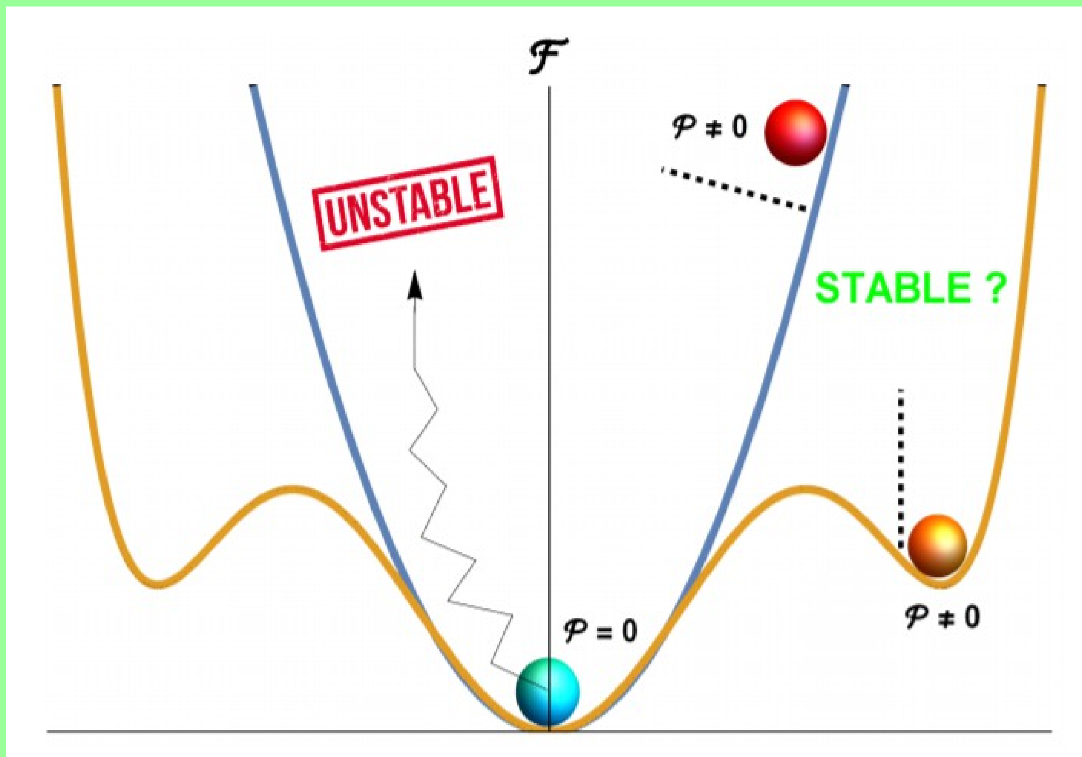
$$\phi^I_A = \alpha\, x^I$$

$$\phi^I_B = R^I_J(\theta)\, x^J$$

$$\omega = A\, k^2$$

More advanced topics

[MB,2001.05737]



Pattern formation outside of equilibrium

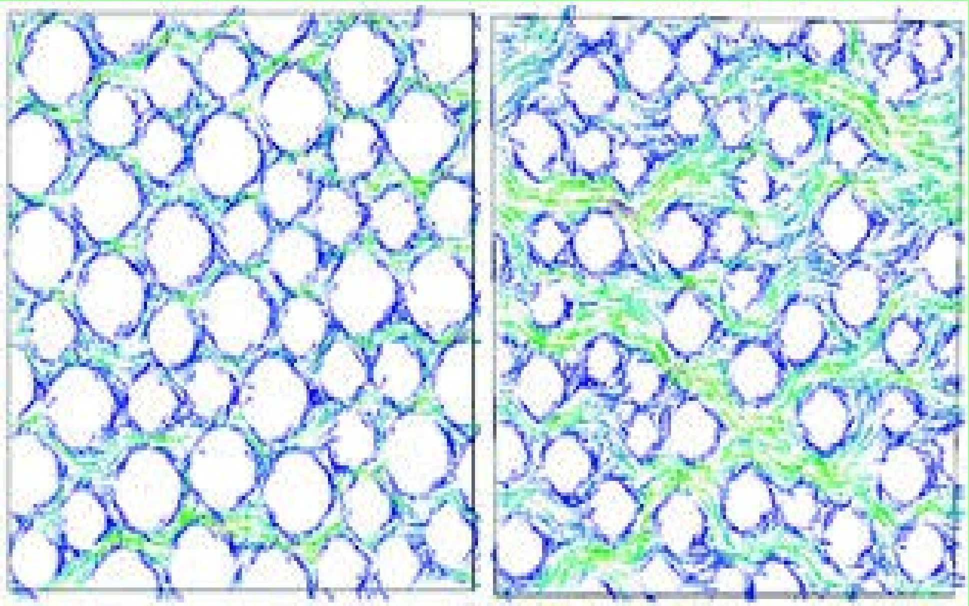
M. C. Cross and P. C. Hohenberg
Rev. Mod. Phys. **65**, 851 – Published 1 July 1993

More advanced topics

Hydrodynamics & Strain pressure

[MB,2001.05737]

[Armas, Jain]



Unified Hydrodynamic Theory for Crystals, Liquid Crystals, and Normal Fluids

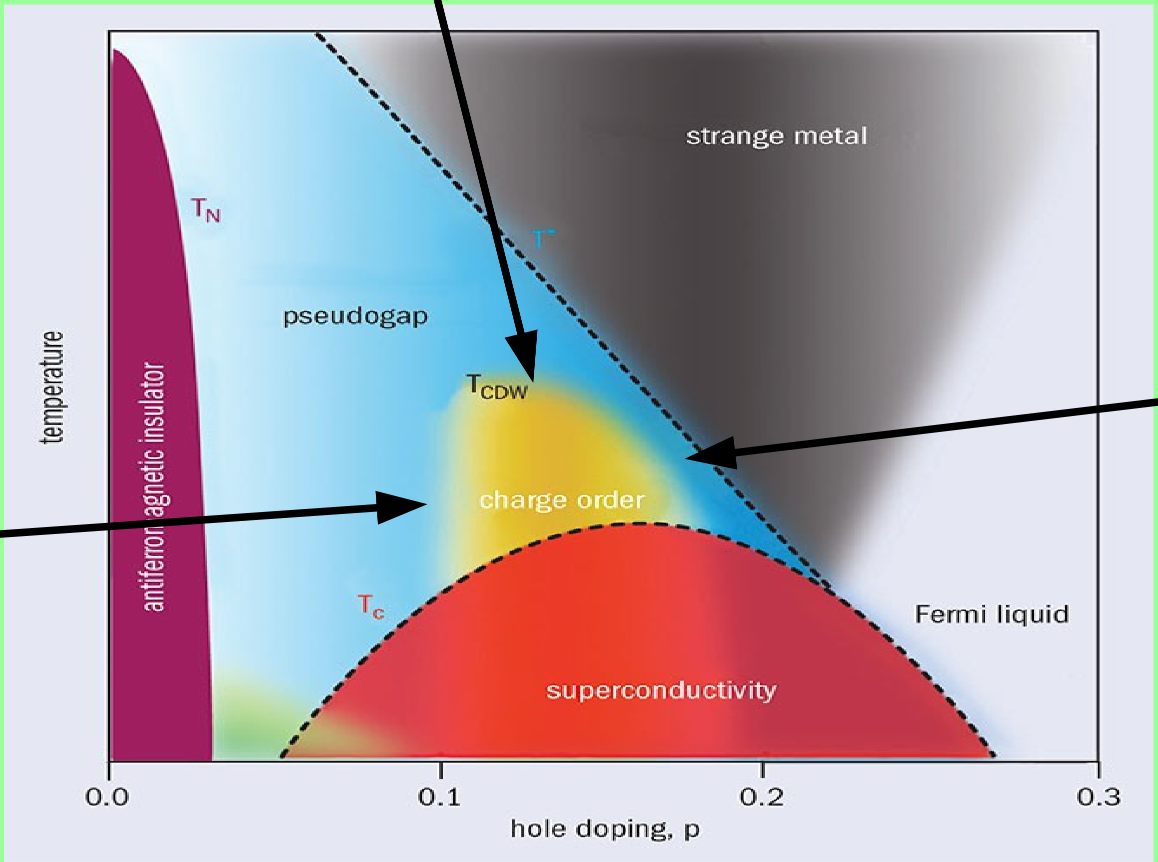
P. C. Martin, O. Parodi, and P. S. Pershan
Phys. Rev. A **6**, 2401 – Published 1 December 1972

More advanced topics

Phenomenology I

[Hartnoll, Gouteraux et Al.]

[Fradkin, Emery, Kivelson]



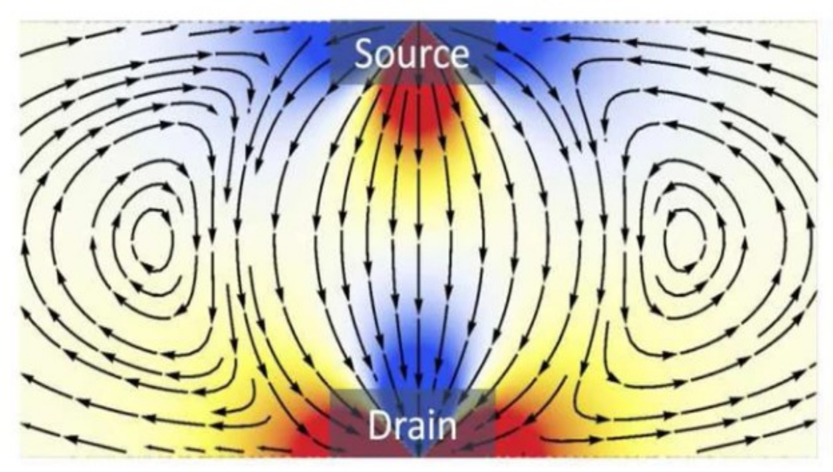
Colloquium: Theory of intertwined orders in high temperature superconductors

Eduardo Fradkin, Steven A. Kivelson, and John M. Tranquada
Rev. Mod. Phys. **87**, 457 – Published 26 May 2015

More advanced topics

Phenomenology II

Phonons in the critical region
(glassy features ?)



Phonons hydrodynamics

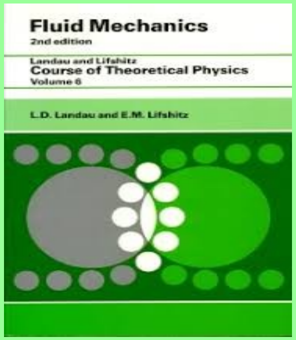

Discussion


SPEAKUP

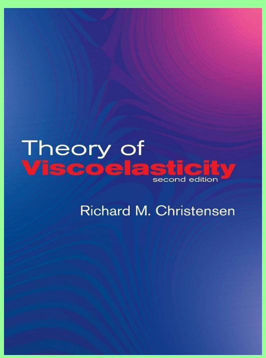


part 2

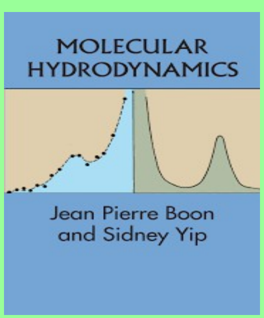
LIQUIDS & VISCOELASTICITY



[Fluid Mechanics, Landau & Lifshitz]



[Theory of Viscoelasticity, Christensen]



[Molecular Hydrodynamics, Boon & Yip]

Lectures on hydrodynamic fluctuations in relativistic theories

Pavel Kovtun

arXiv:1205.5040

What is a liquid ?

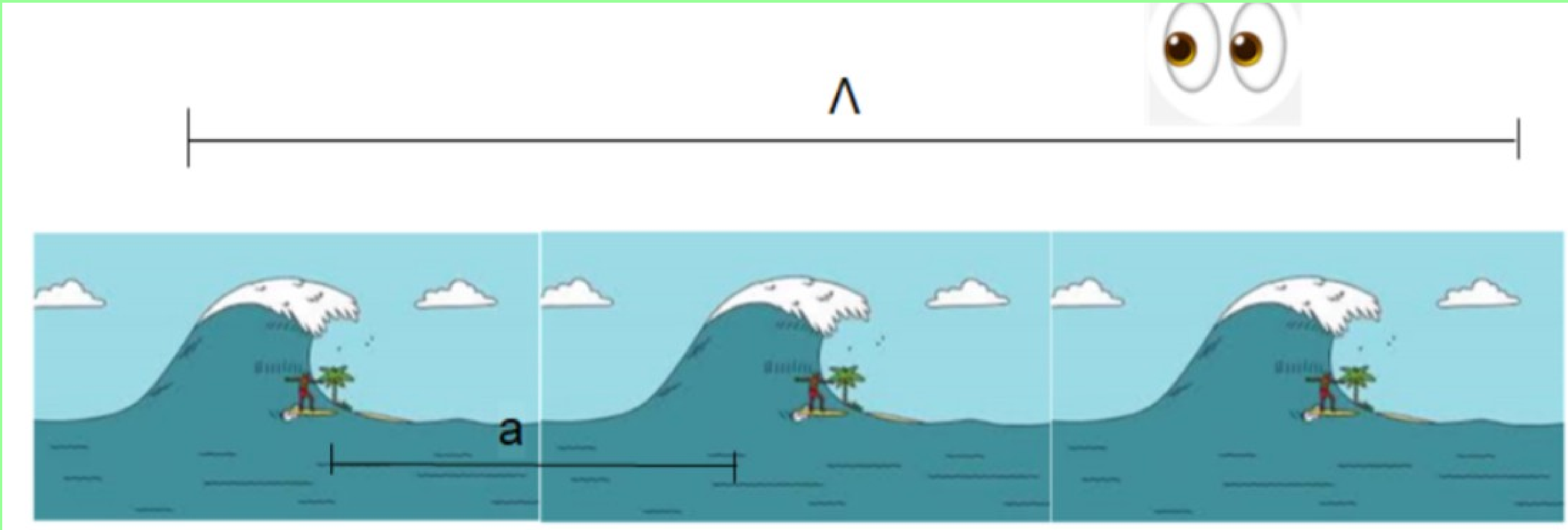
- Shape not fixed
  but volume fixed

- Not Rigid -----> "viscous"
  (response only to shear strain rate)

$$\sigma = \eta \dot{\epsilon}$$

- Shear diffusion

Hydrodynamics



$$\Lambda \gg a \quad \longrightarrow \quad \omega/T,\ k/T \ll 1$$

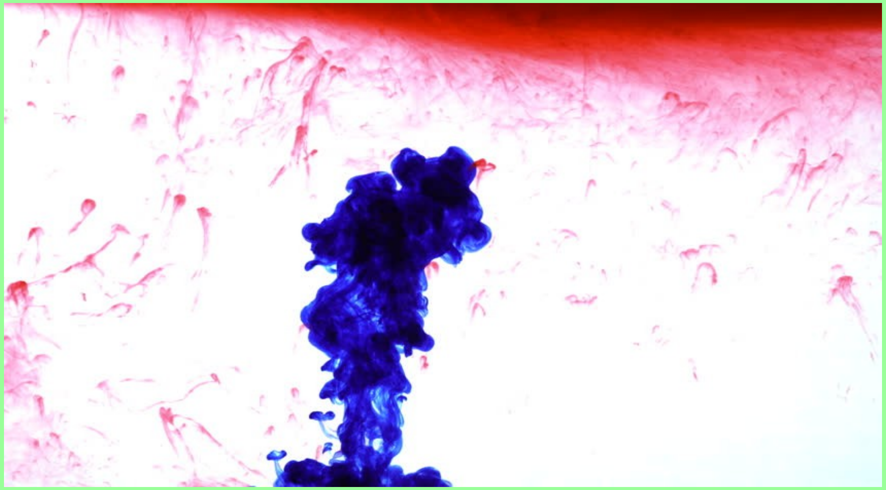
$$T^{\mu\nu} = (slow) + (fast) + (faster) + (much\ faster) + \ldots$$

$$\partial \qquad \partial^2 \qquad \partial^3 \qquad \partial^4$$

Gradient expansion

Perturbative expansion

Example: diffusion



Transverse sector :
Shear diffusion

Dissipation
Viscosity

Continuity equation :

$$\frac{\partial \phi}{\partial t} + \nabla \cdot \mathbf{j} = 0,$$

Fick's law :

$$\mathbf{j} = -D(\phi, \mathbf{r}) \, \nabla \phi(\mathbf{r}, t).$$

$$\omega = -i \, D \, k^2 + \ldots$$

Example: relativistic hydrodynamics

$$D = \frac{\eta}{s \, T} = \frac{1}{4 \, \pi \, T}$$

Difficulties with liquids

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu J^{\mu} = 0$$

+

$$T^{\mu\nu} = \cdots + \dots$$
$$J^{\mu} = \cdots + \dots$$

+ symmetries

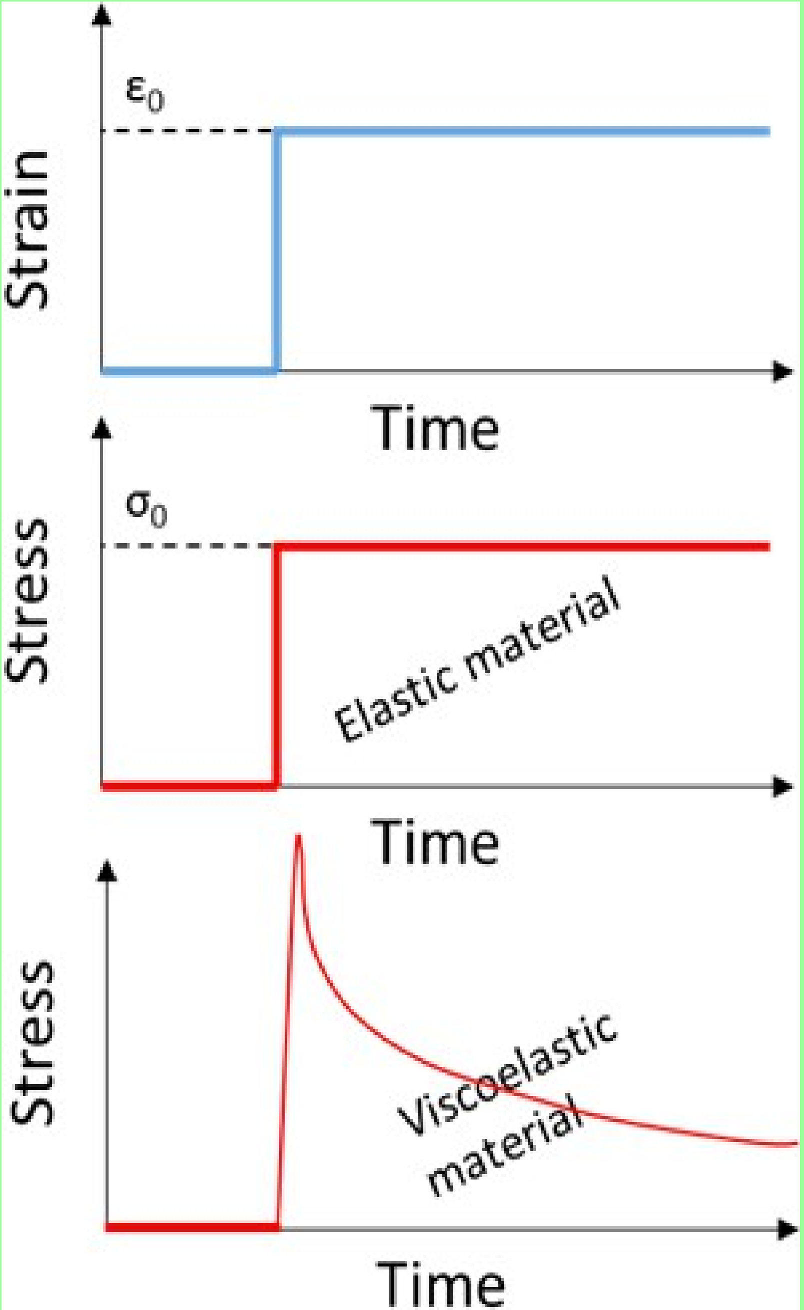
Very different from what we discussed for solids !

Where is the action ?
(e.g. action for a diffusive mode)

DISSIPATION

HERMITIAN ACTION

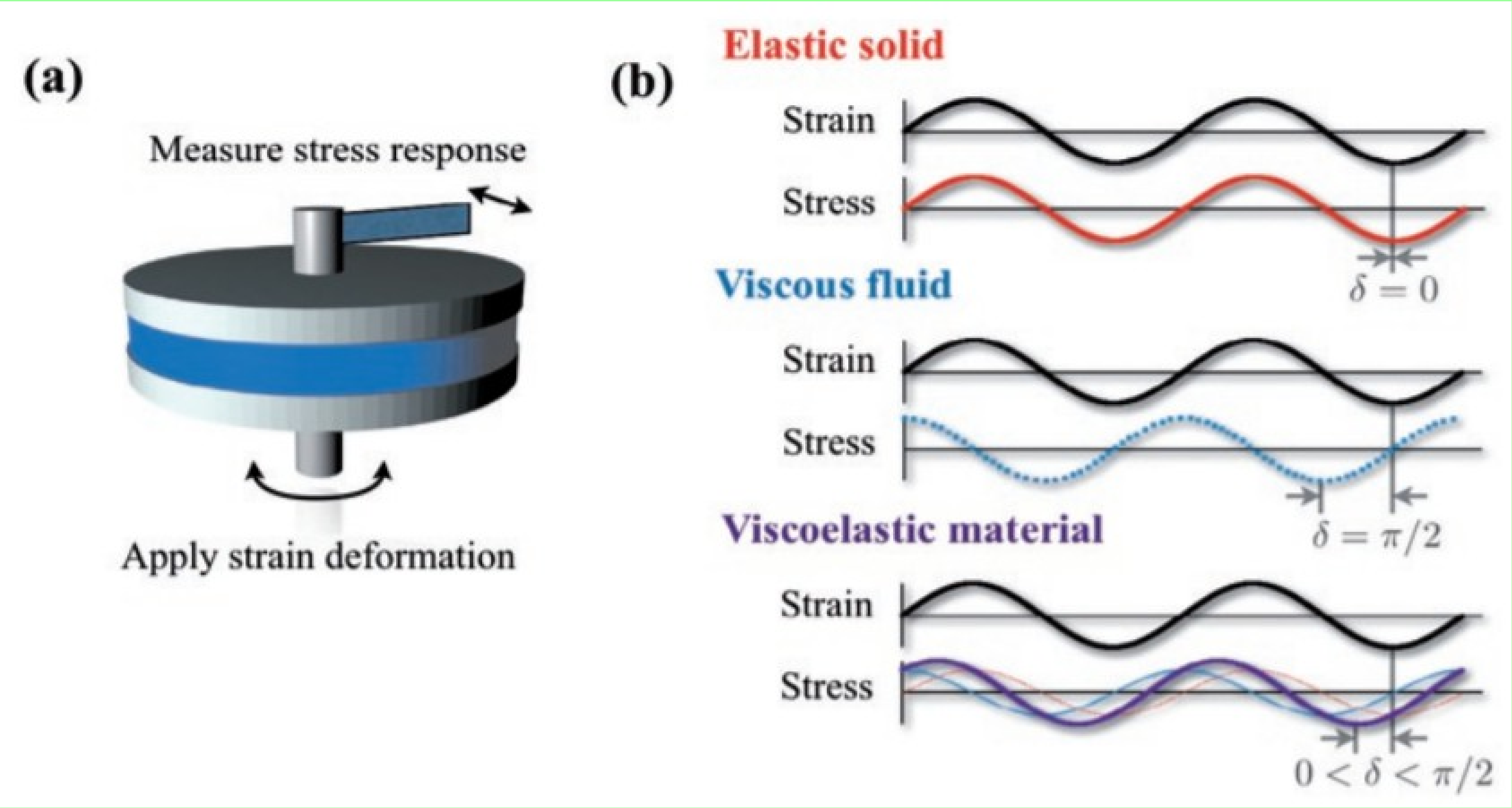
Viscoelastic materials



They combine elastic and viscous response

Complex dynamics

Experimental tests



stress $\sigma(t) = \sigma_0 \cos(\omega t)$ **strain** $\epsilon(t) = \epsilon_0 \cos(\omega t - \delta)$

Complex modulus
<Tij Tij>

$$\sigma(\omega) = G(\omega)\,\epsilon(\omega), \quad G = G' + i\,G''$$

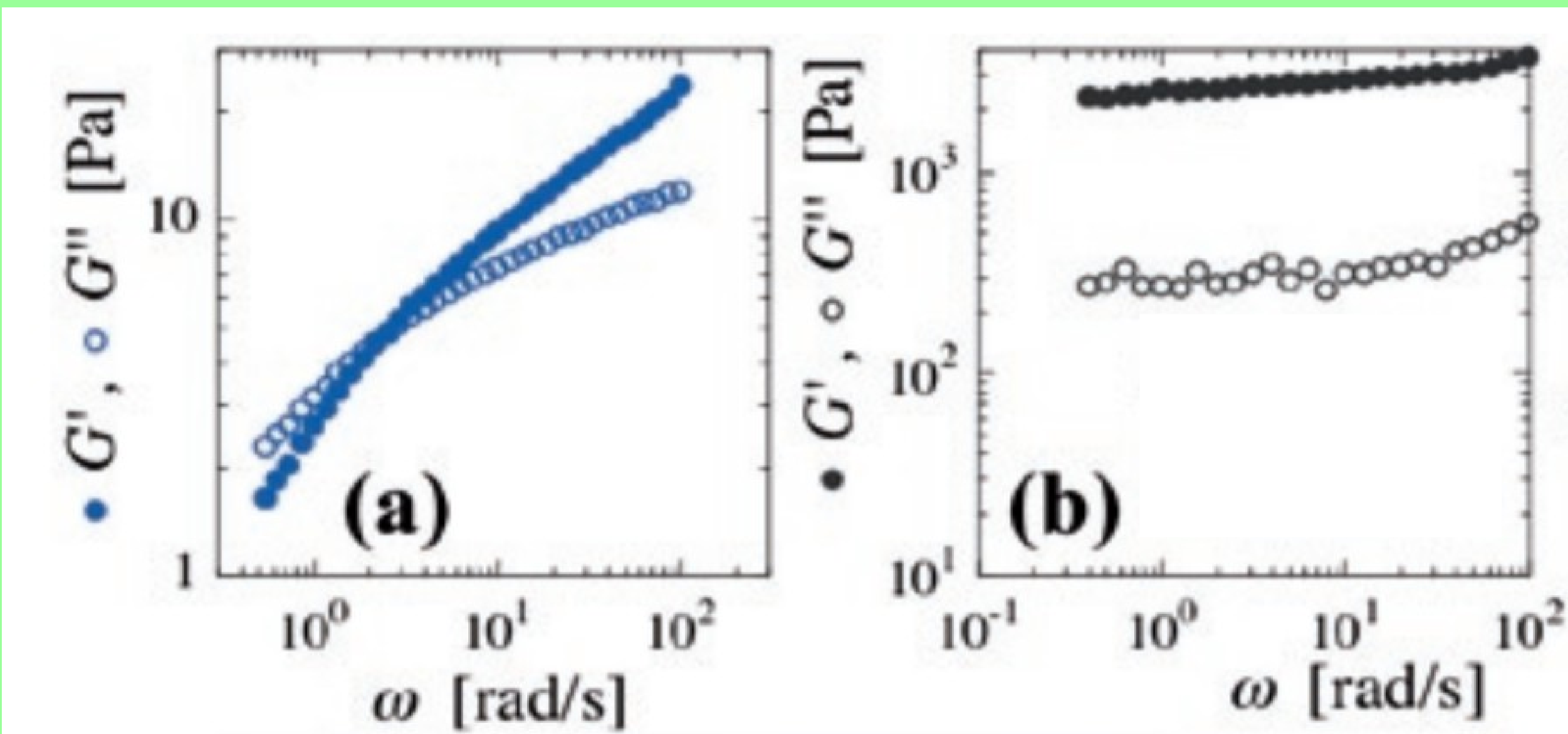
Solids VS fluids (again)

$$G = G' + i\,G''$$

Complex dynamical modulus

Elastic response
static and in phase

Dissipative/viscous response
response to a strain rate, out of phase

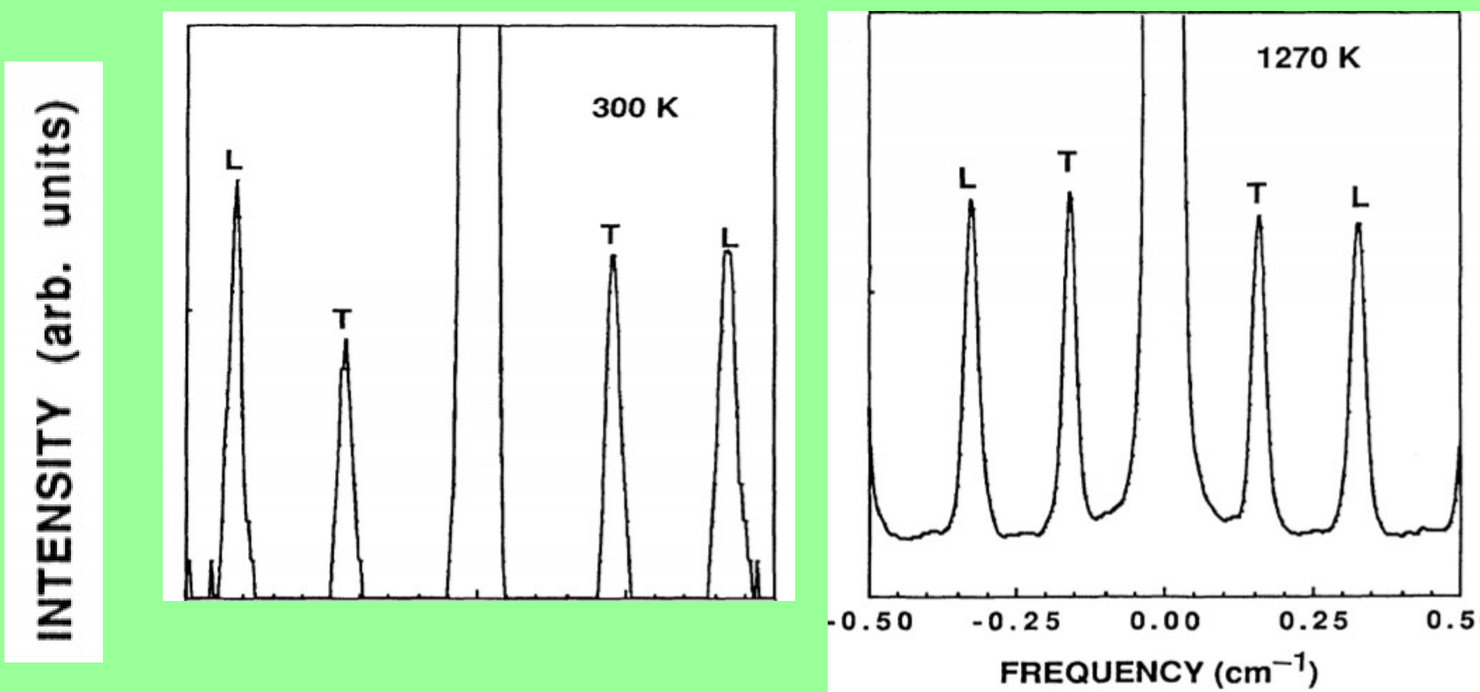


Are liquids and solids really different ?

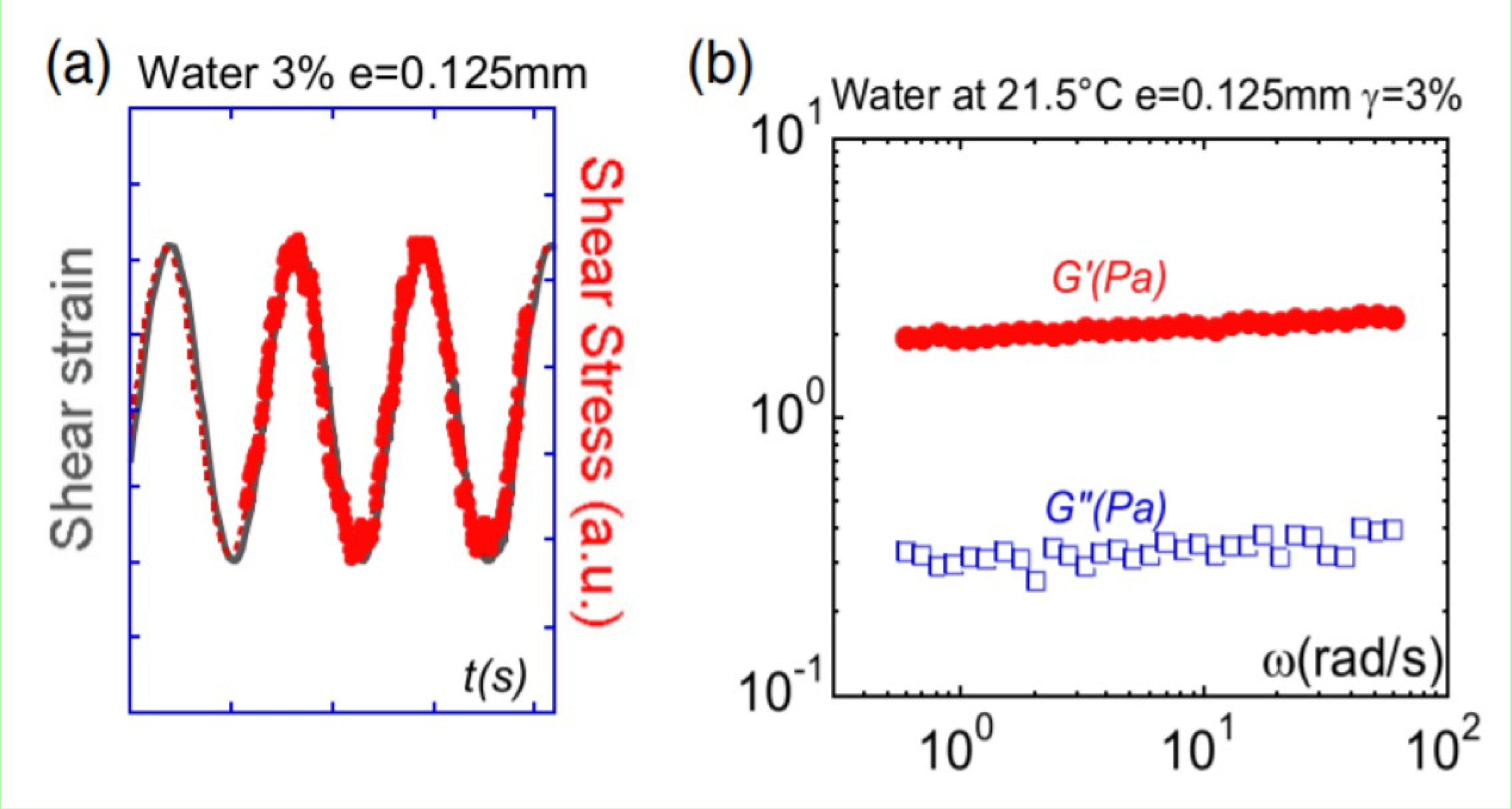
NOT IF $$\omega > \omega_F \equiv \frac{1}{\tau}$$

Proved in several experiments !!

High-frequency longitudinal and transverse dynamics in water



This is ….

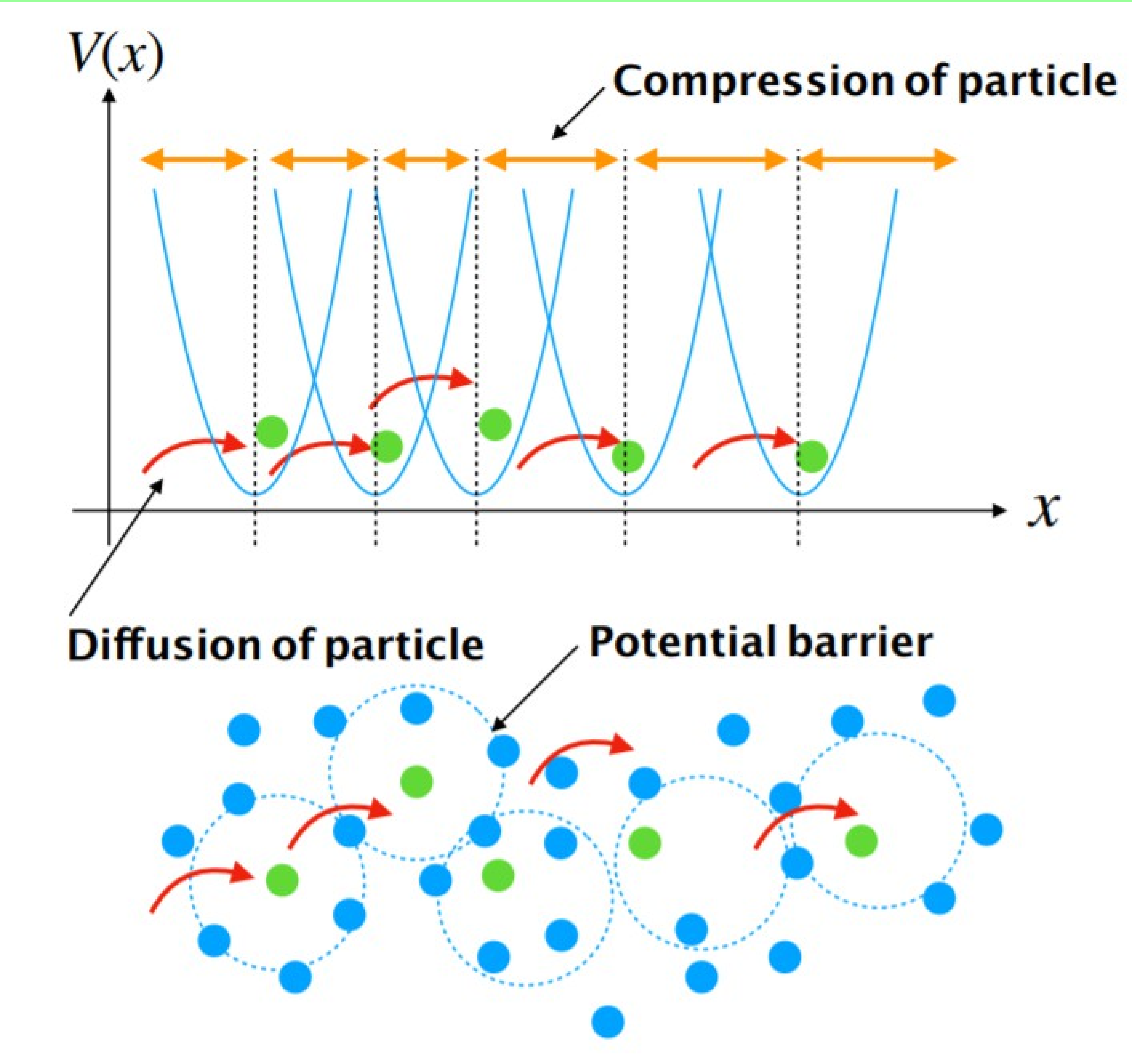


LIQUIDS SUPPORT PROPAGATING SHEAR WAVES ALSO AT LOW FREQUENCIES !!!

LIQUIDS BEHAVE LIKE ELASTIC SOLIDS ALSO AT LOW FREQUENCY !!!

[Noirez et al, J. Phys.: Condens. Matt. 2012]

Let us think more about liquids

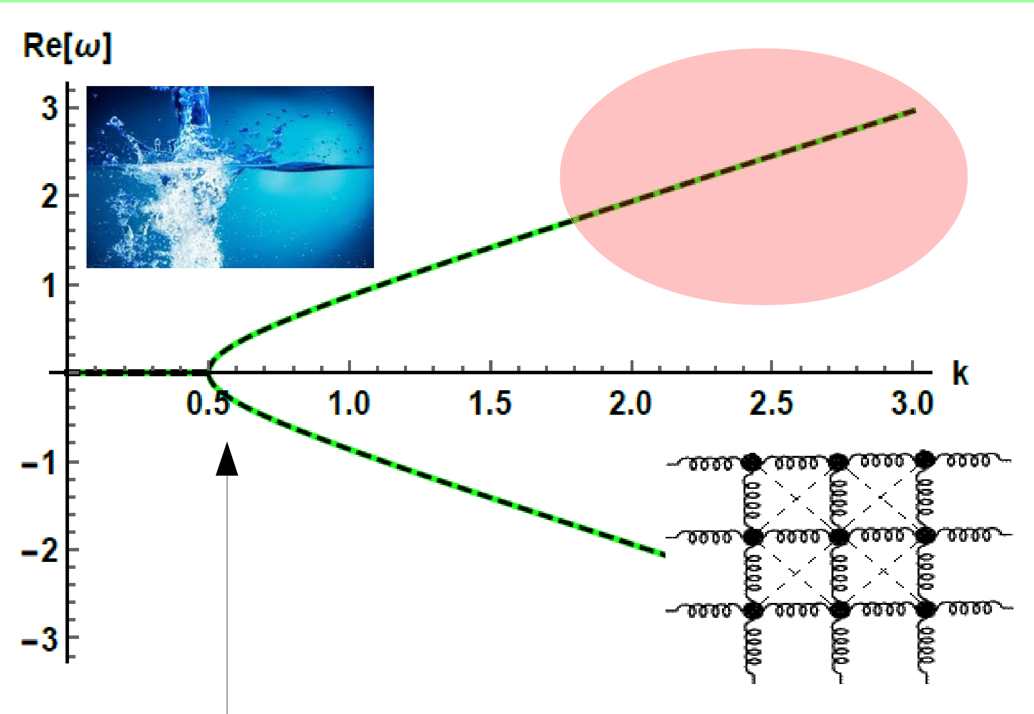


τ

Average time of rearrengements of molecules

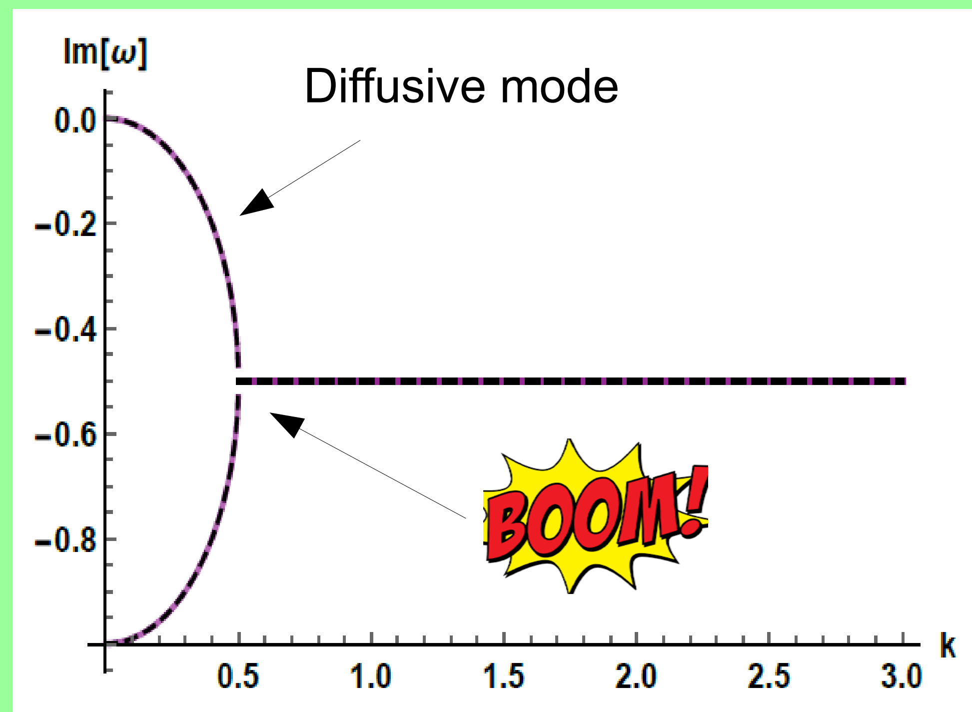
Average time of particles jumps through the potential barrier

The Maxwell-Frenkel approach



$$\omega^2 + \omega \frac{i}{\tau} - \mathcal{V}^2 k^2 = 0$$

$$\omega = -\frac{i}{2\tau} \pm \sqrt{\mathcal{V}^2 k^2 - \frac{1}{4\tau^2}}$$



K-gap

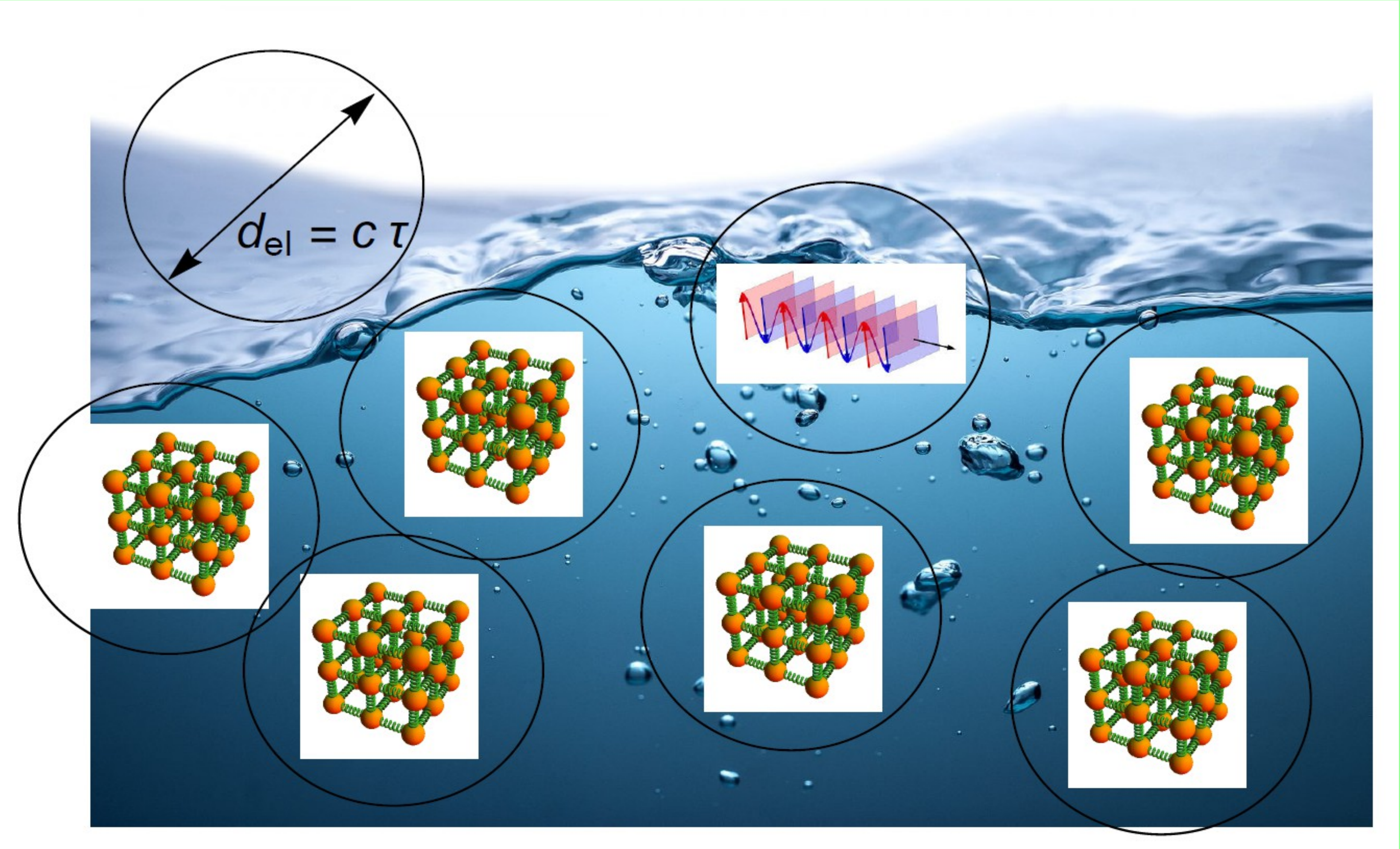
[MB,1904.01419]

Propagating shear mode at low frequency

From the collision of the diffusive mode and a lower non-hydro mode

Physical meaning

[MB,1904.01419]



$$k > k_g = \frac{1}{2c\tau}$$

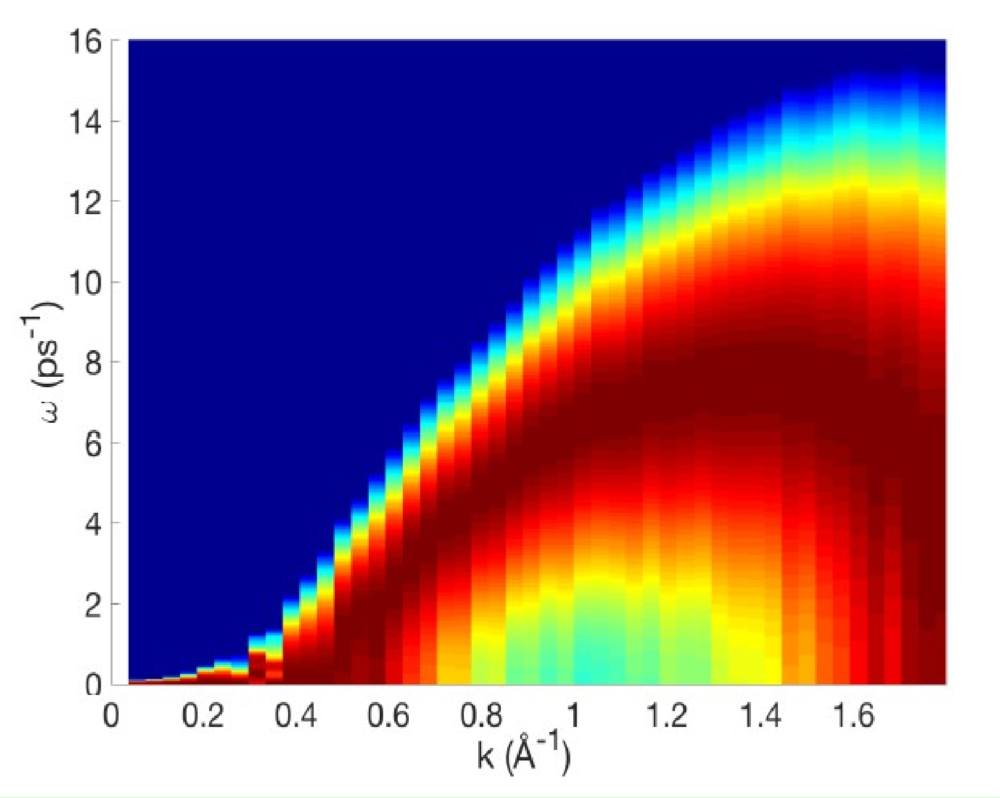
$$d_{\rm el} = c\tau$$

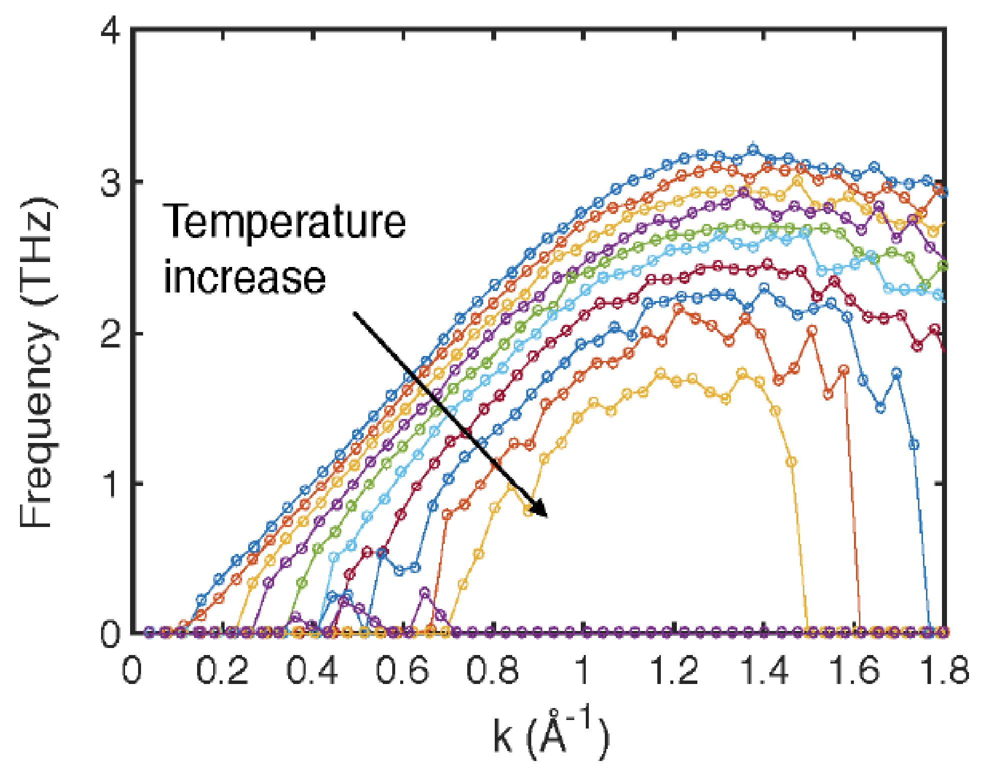
Molecular dynamic simulations

[C. Yang, M. T. Dove, V. V. Brazhkin, K. Trachenko, PRL 2017]

K-gap observed in molecular dynamical simulations

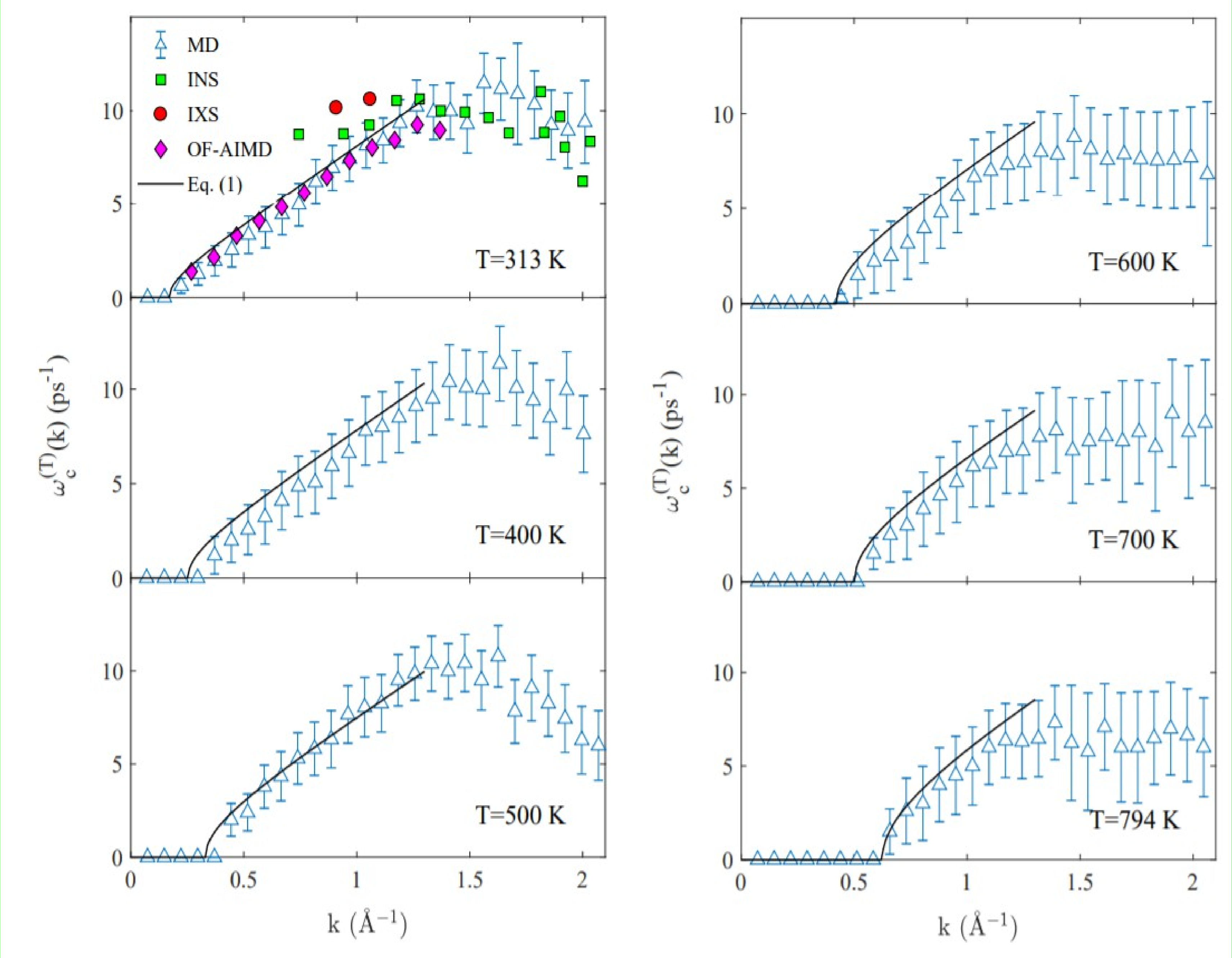
The K-gap increases with temperature in agreement with the idea that tau decreases.





Experiments (Ga)

[Trachenko et Al, 2020]



A field theory toy model

work W done to move the liquid. $W \propto F s$.

F is the viscous force $F \propto \eta \frac{ds}{dt}$

[MB,2004.13613]

New dissipative term

$$L_d \propto \phi_1 \frac{\partial \phi_2}{\partial t} - \phi_2 \frac{\partial \phi_1}{\partial t}$$

[It needs two scalar fields ...]

$$L_\psi = \frac{1}{2}\left[\left(\frac{\partial \psi_1}{\partial t}\right)^2 - c^2\left(\frac{\partial \psi_1}{\partial x}\right)^2 - \left(\frac{\partial \psi_2}{\partial t}\right)^2 + c^2\left(\frac{\partial \psi_2}{\partial x}\right)^2\right] + \tag{23}$$

$$+ \frac{1}{2\tau}\left(\psi_2 \frac{\partial \psi_1}{\partial t} - \psi_1 \frac{\partial \psi_2}{\partial t}\right)$$

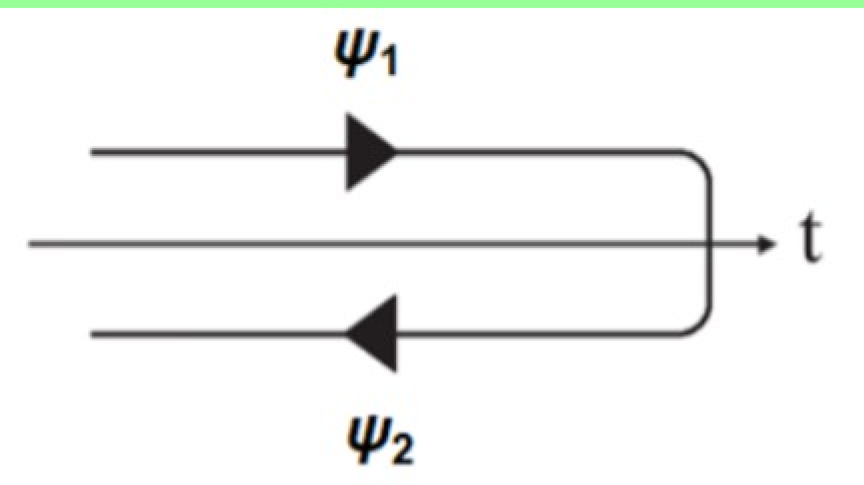
Non-hermitian
PT symmetric

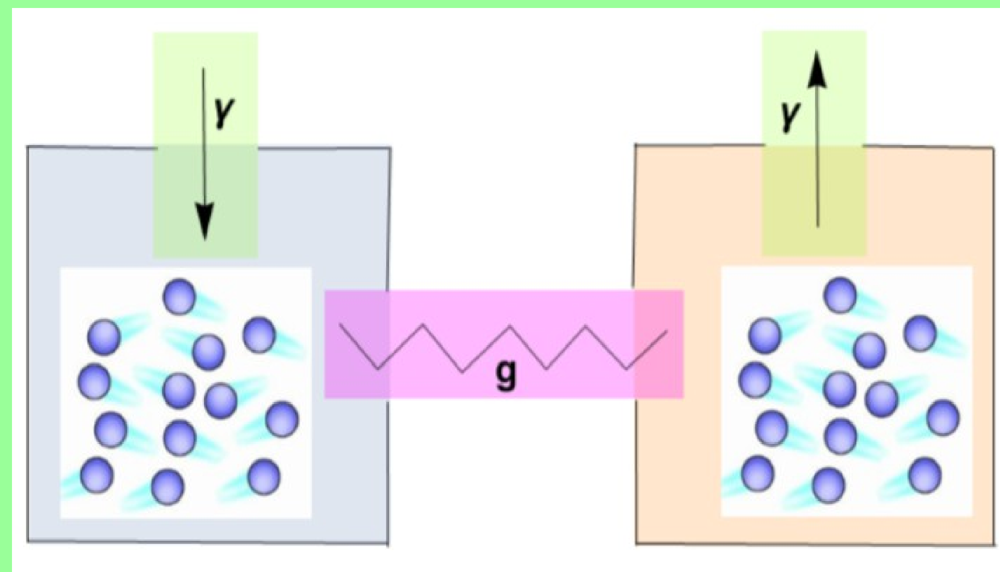
A field theory toy model

[MB,2004.13613]

Keldysh-Schwinger interpretation

$$e^{W(\zeta_1,\zeta_2)} = \int D\psi_1 D\psi_2\, e^{i\int_{\infty}^{t} dt(\mathcal{L}(\psi_1;\xi_1) - \mathcal{L}(\psi_2;\xi_2)) + i\,\mathcal{L}_{1,2}(\infty)}$$



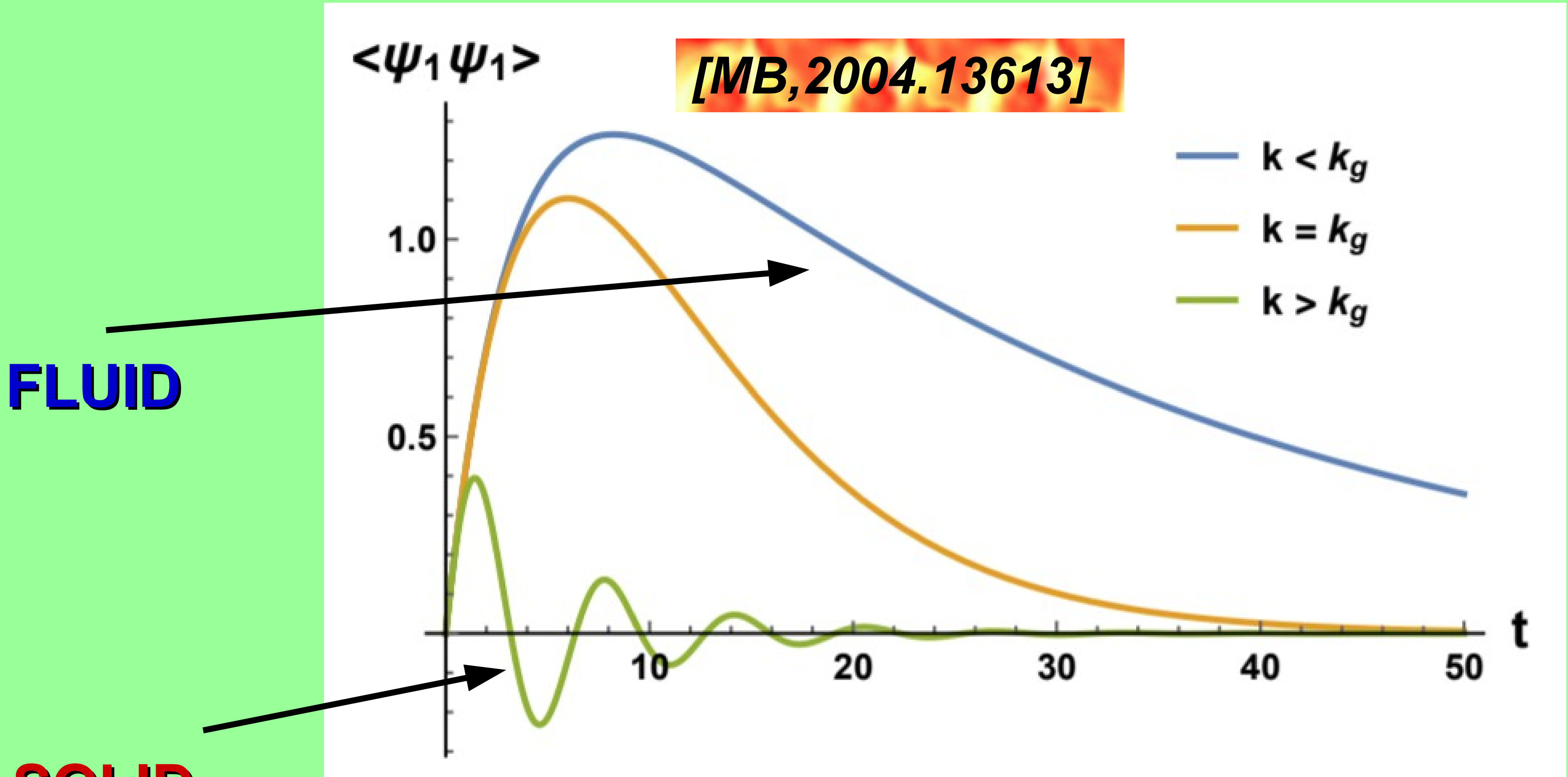


Non-hermitian (= dissipation)

PT symmetry (=unitary)

$$\frac{dO(t)}{dt} = i\, e^{i H^\dagger t} \left(H^\dagger O - O H\right) e^{-i H t}$$

A field theory toy model



$$\langle\psi_1\psi_1\rangle = -\frac{\omega^2 - c^2k^2}{(\omega^2 - c^2k^2)^2 + \tau^{-2}\omega^2} \quad \langle\psi_1\psi_1\rangle_t = \frac{\sin(\omega_R|t|)}{2\,\omega_R}e^{-\frac{|t|}{2\tau}}$$

$$\langle\psi_1\psi_2\rangle = \frac{i\,\omega\,\tau^{-1}}{(\omega^2 - c^2k^2)^2 + \tau^{-2}\omega^2} \quad \langle\psi_1\psi_2\rangle_t = \frac{\sin(\omega_R t)}{2\omega_R}e^{-\frac{|t|}{2\tau}}$$

A field theory toy model

Interaction potential

$$\mathfrak{D} = \frac{1}{(2\pi)^3} \int_0^\infty \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega^2 - c^2\mathbf{k}^2 + \frac{i\omega}{\tau}}\, d\mathbf{k}$$

$$\mathfrak{D} = -\frac{1}{4\pi c^2 r}\, e^{\frac{ir}{c}\sqrt{\omega^2 + \frac{i\omega}{\tau}}}$$

solid-regime $\omega\tau \gg 1$

$$\mathfrak{D} = -\frac{1}{4\pi c^2 r}\, e^{\frac{i\omega r}{c}}$$

hydro-regime $\omega\tau \ll 1$

$$\mathfrak{D} = -\frac{1}{4\pi c^2 r}\, e^{\frac{ir}{c}\sqrt{\frac{\omega}{2\tau}}}\, e^{-\frac{r}{c}\sqrt{\frac{\omega}{2\tau}}}$$

$d^2 \propto D\,T$

[MB,2004.13613]

Relativistic hydrodynamics

$$\omega = -i\,\frac{\eta}{\epsilon + p}\,k^2$$

SHEAR DIFFUSION
(linear order)

$$|v| = \left|\frac{\partial \omega}{\partial k}\right| \sim k > c$$

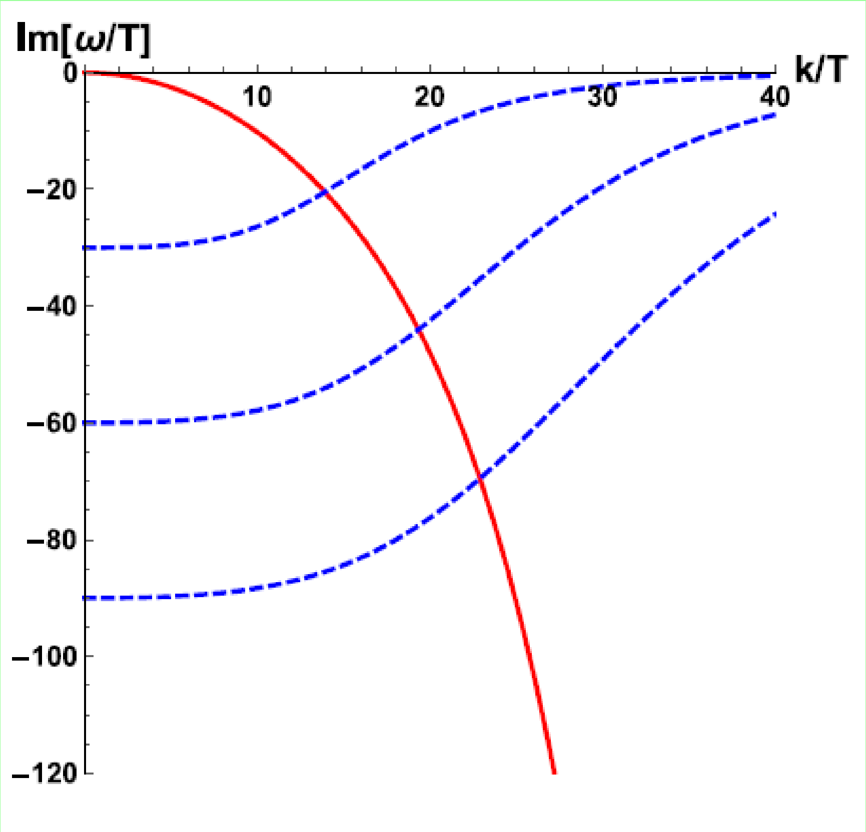
Not a real problem (in my opinion), only issue for simulations

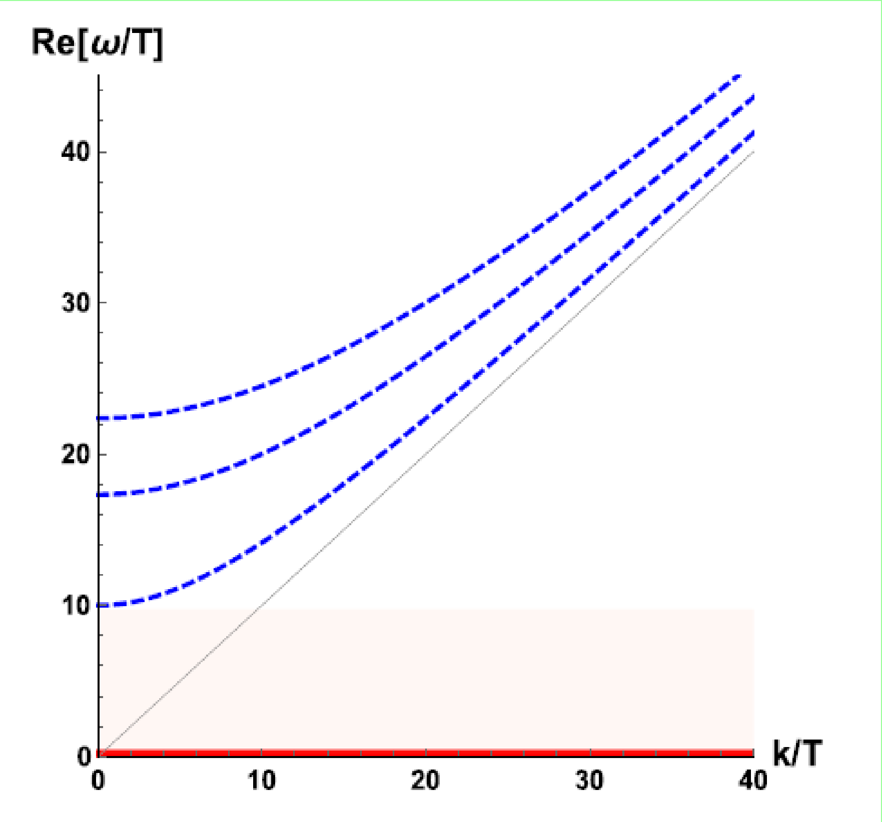
$$\omega^2 + \underline{i\,\omega\,\tau_\pi^{-1}} = v^2\,k^2\,, \qquad v^2 = \frac{\eta}{(\epsilon + p)\,\tau_\pi}\,,$$

Israel-Stewart formalism ⟶ *Same (telegraph) equation*

Holographic inputs (step 0)

Schwarzschild Black hole → *Relativistic hydrodynamics*



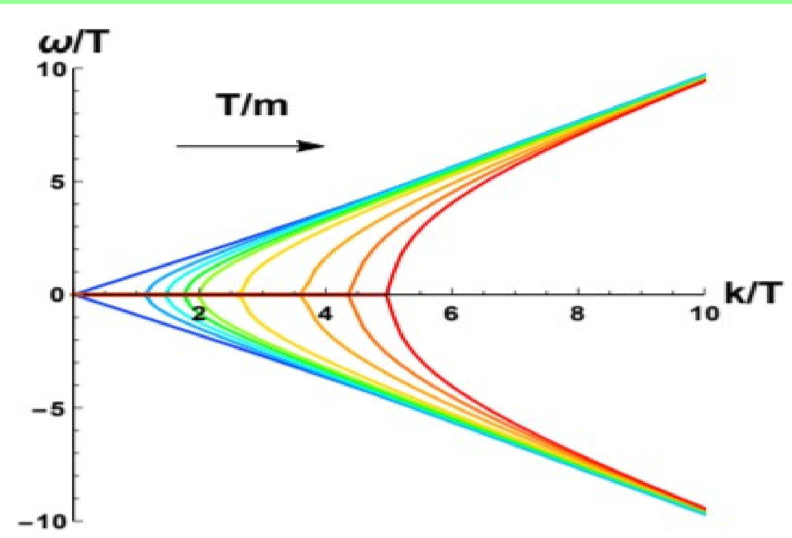


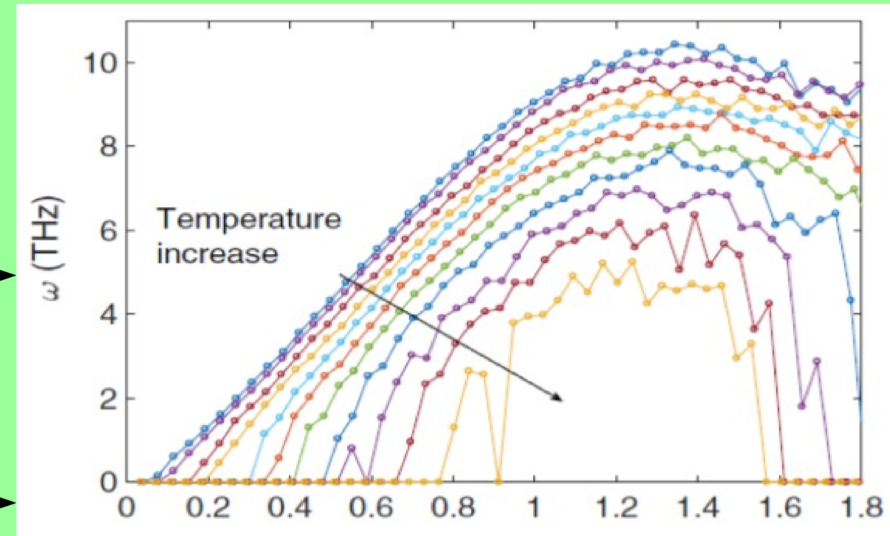
No k-gap, only crossing of modes ….

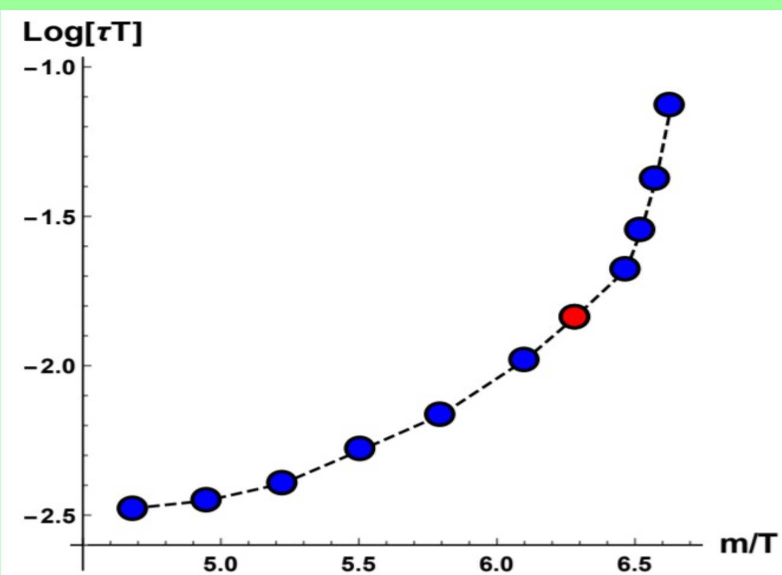
Holographic inputs

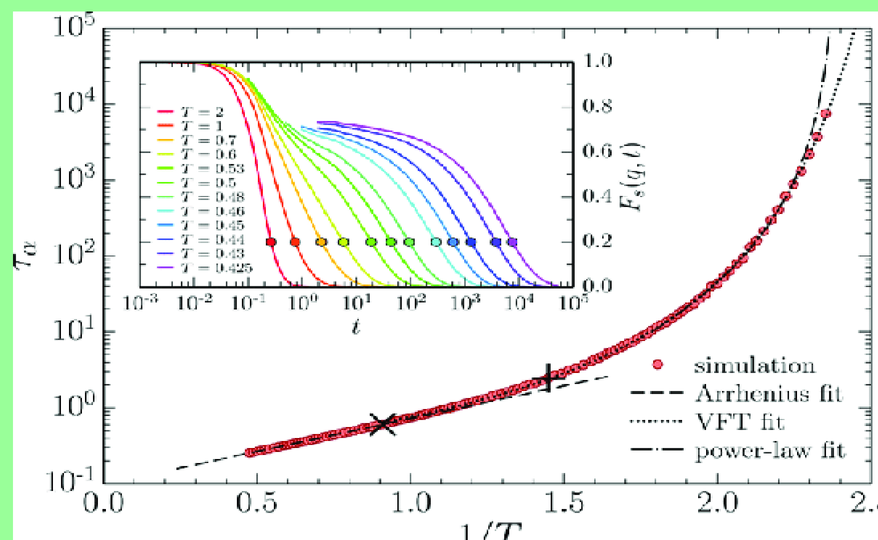
[MB,1807.10530, 1808.05391]

Let us add a scale
Example 1: linear axions







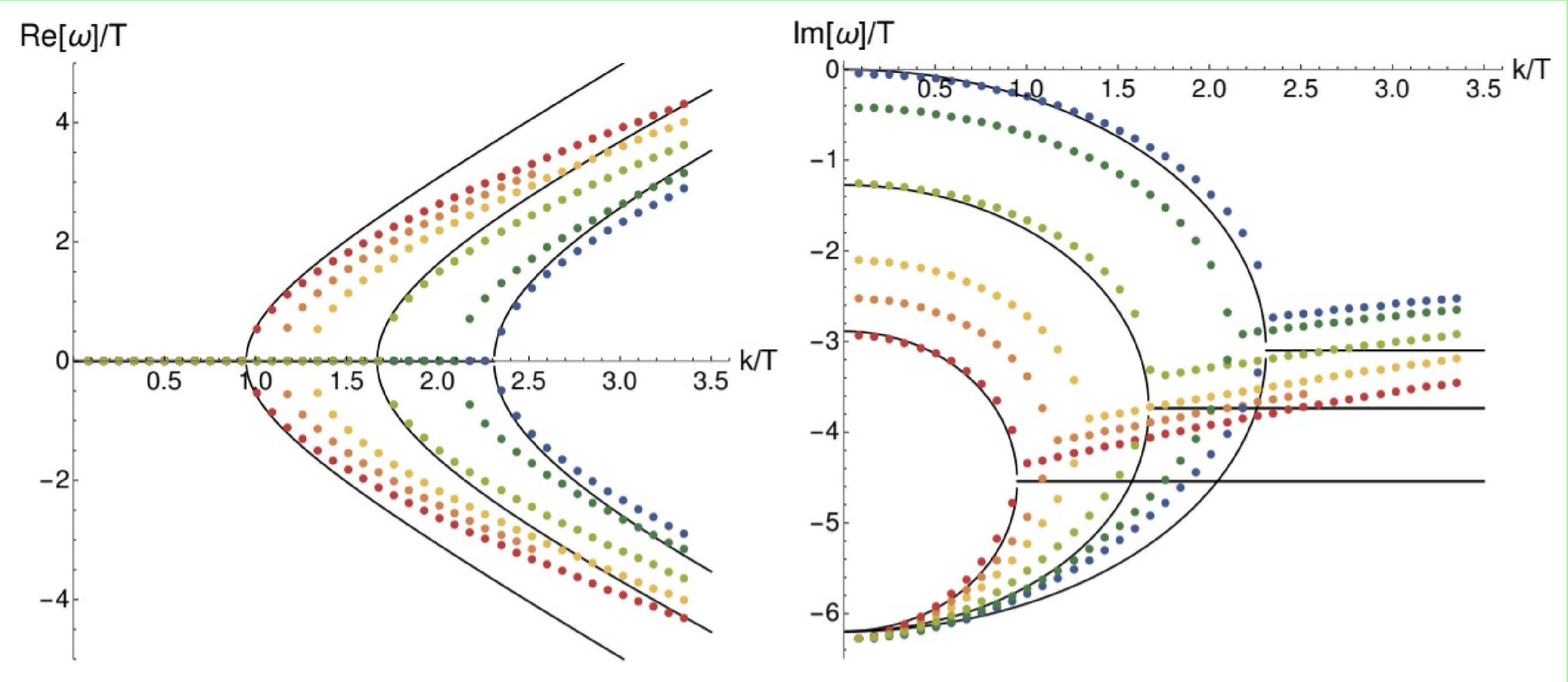


Holographic inputs

[MB,1905.00804, 1912.07321]

Coulomb interactions

$$\left(\omega^2\,\delta A_x^{(0)} + \lambda\,\delta A_x^{(1)}\right) = 0\,,$$



Hydro theory:

Relativistic magnetohydrodynamics

Juan Hernandez, Pavel Kovtun

arXiv:1703.08757

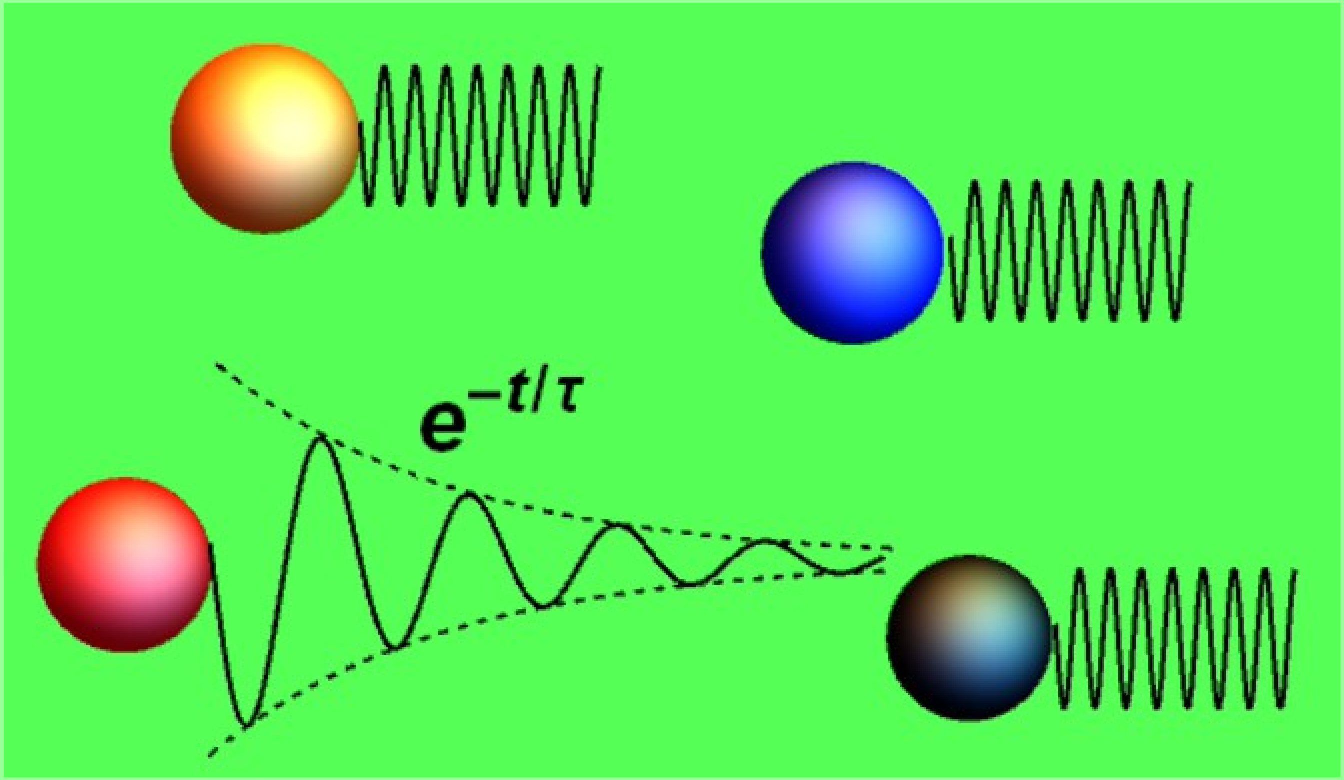
"Holographic" inputs

Holography and hydrodynamics with weakly broken symmetries

Sašo Grozdanov, Andrew Lucas, Napat Poovuttikul

arXiv:1810.10016

QUASI-HYDRODYNAMICS



$$\partial_t S + \partial_x \mathcal{J} = 0,$$
$$\partial_t \mathcal{J} + \frac{\mathfrak{D}}{\tau}\partial_x S = -\frac{1}{\tau}\mathcal{J}.$$

$$\omega^2 + \frac{i}{\tau}\omega - \frac{\mathfrak{D}}{\tau}k^2 = 0,$$

1 CHARGE NOT CONSERVED BUT SLOWLY RELAXING

Another example

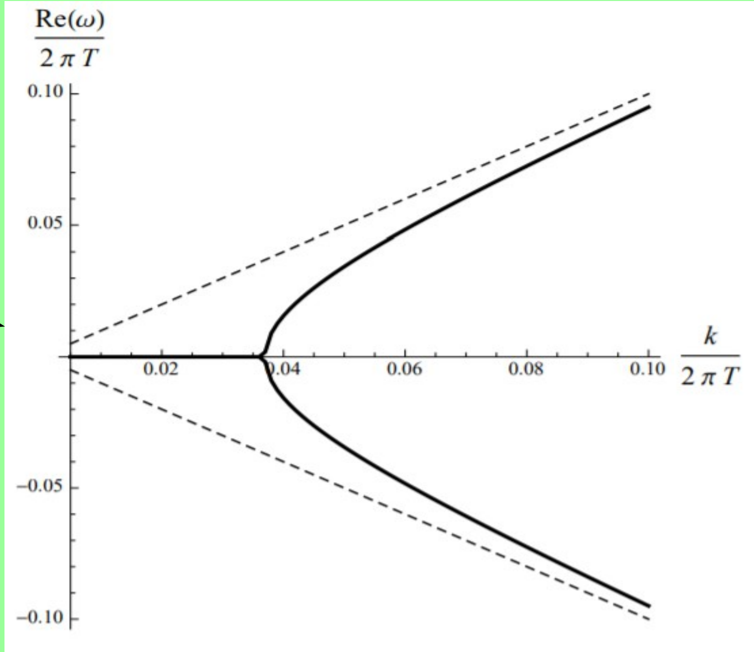
GENERALIZED GLOBAL SYMMETRIES (e.g. two-form dynamics)

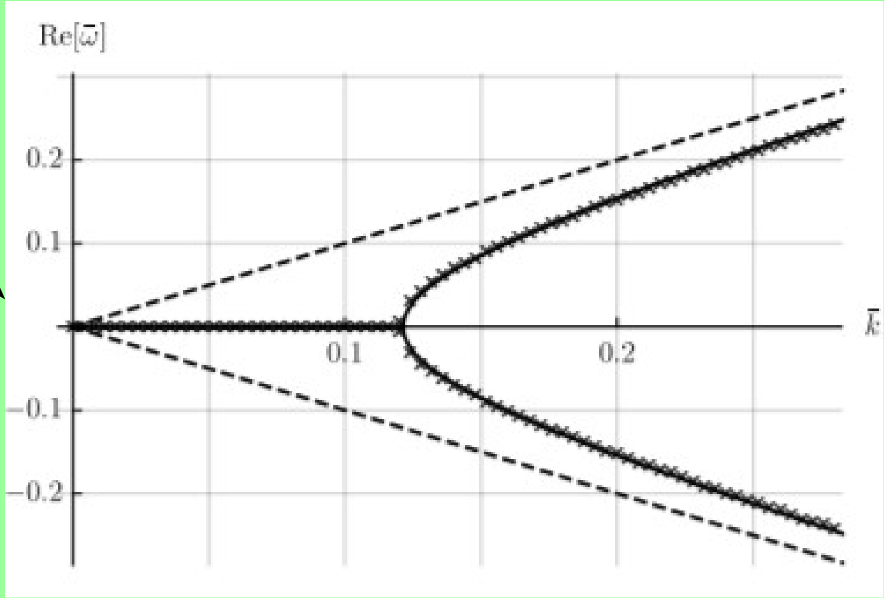
Generalized global symmetries and holography

Diego M. Hofman, Nabil Iqbal arXiv:1707.08577

Generalised global symmetries in states with dynamical defects: the case of the transverse sound in field theory and holography

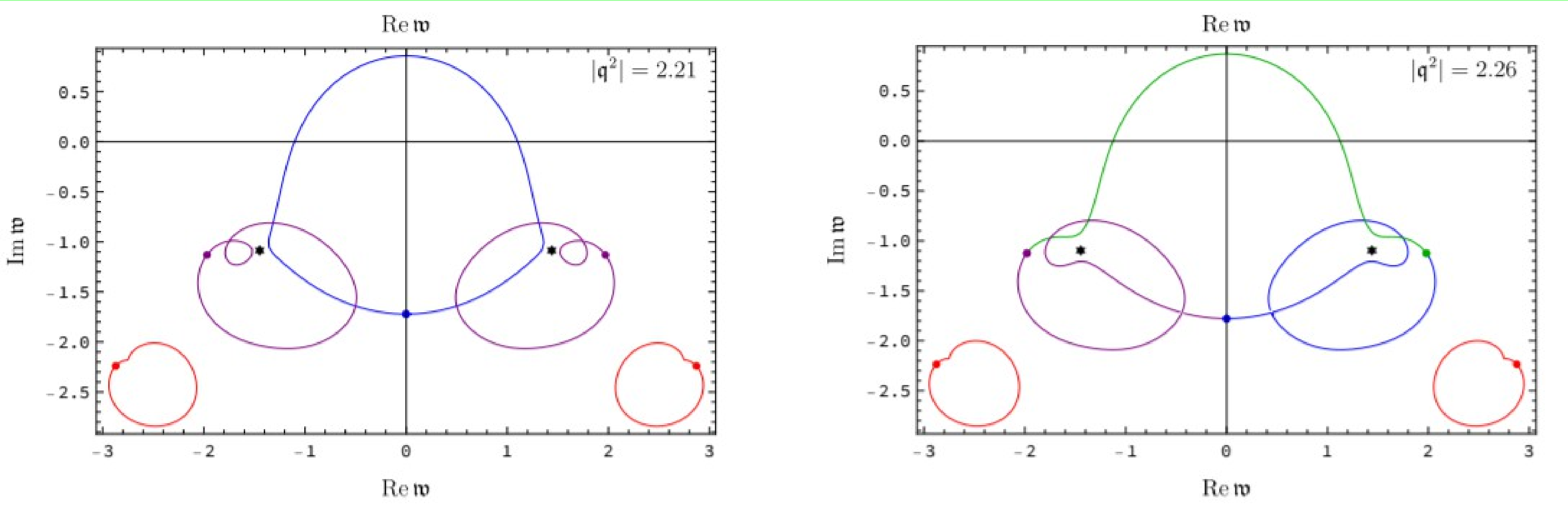
Sašo Grozdanov, Napat Poovuttikul arXiv:1801.03199

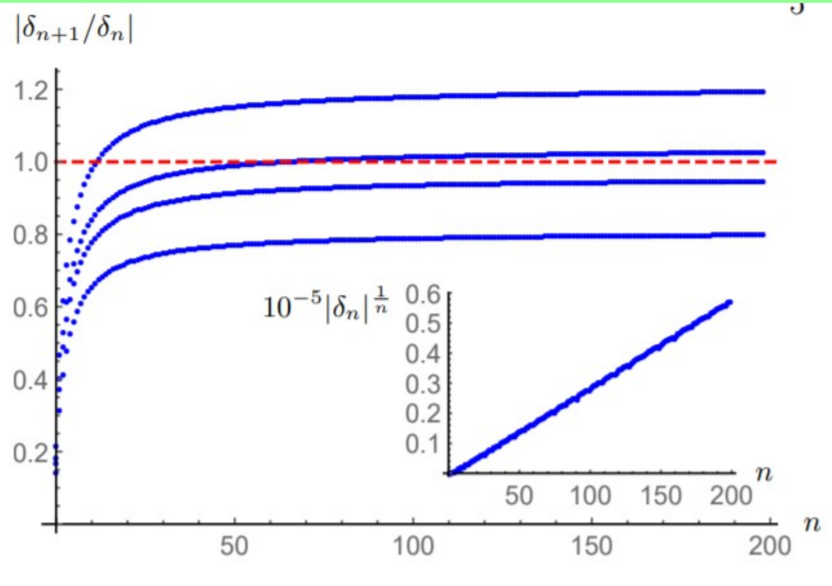




Convergenge of Hydrodynamics

Radius of convergence → ***Collisions in complex space***

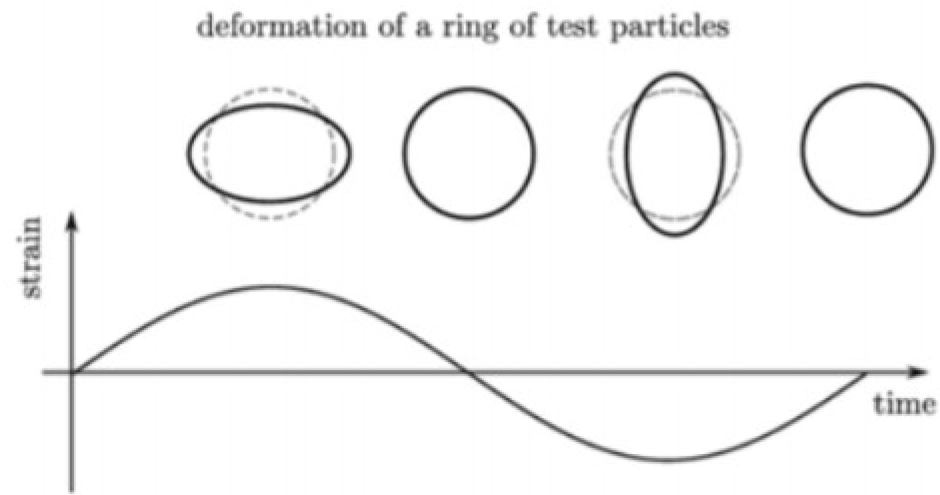


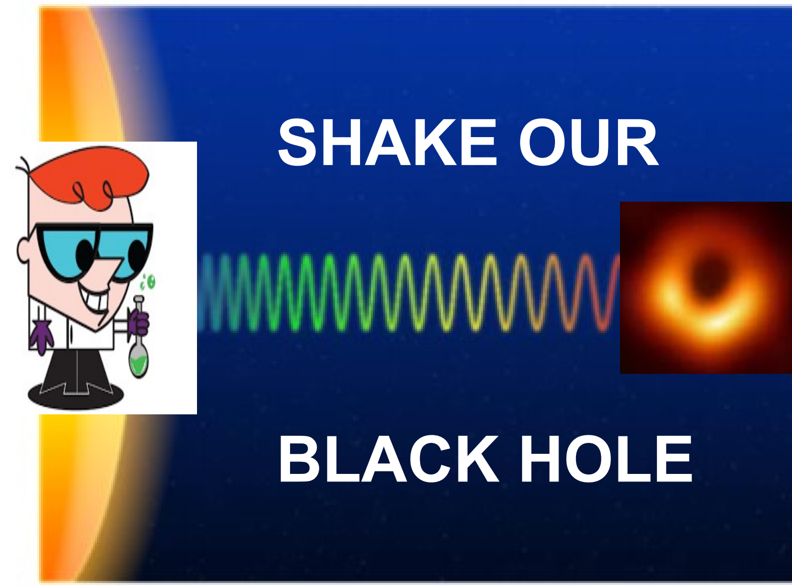


K-gap sets the radius of convergence of hydrodynamics

[Grozdanov et Al.]
[Heller et Al.]

Holographic viscoelasticity





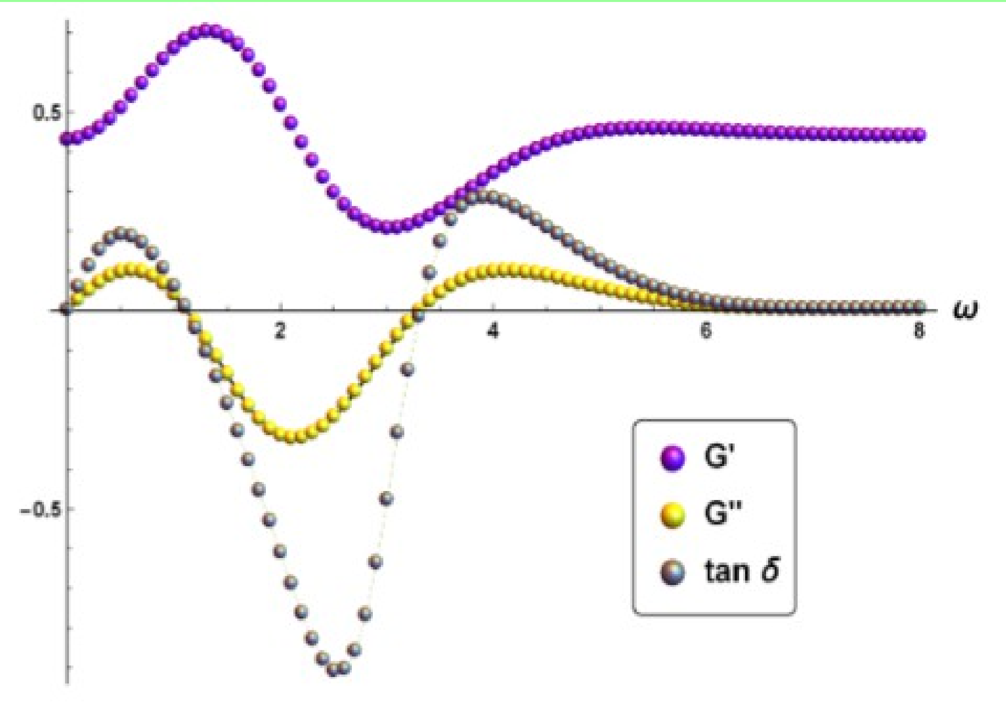
$$h_{xy}(t) = \gamma(t)$$

$$\gamma(t) = \gamma_0 \sin(2\pi\omega t)$$

$$\rightarrow T_{xy}(t)$$

[MB,1903.02859, 1910.06331]

Holographic viscoelasticity



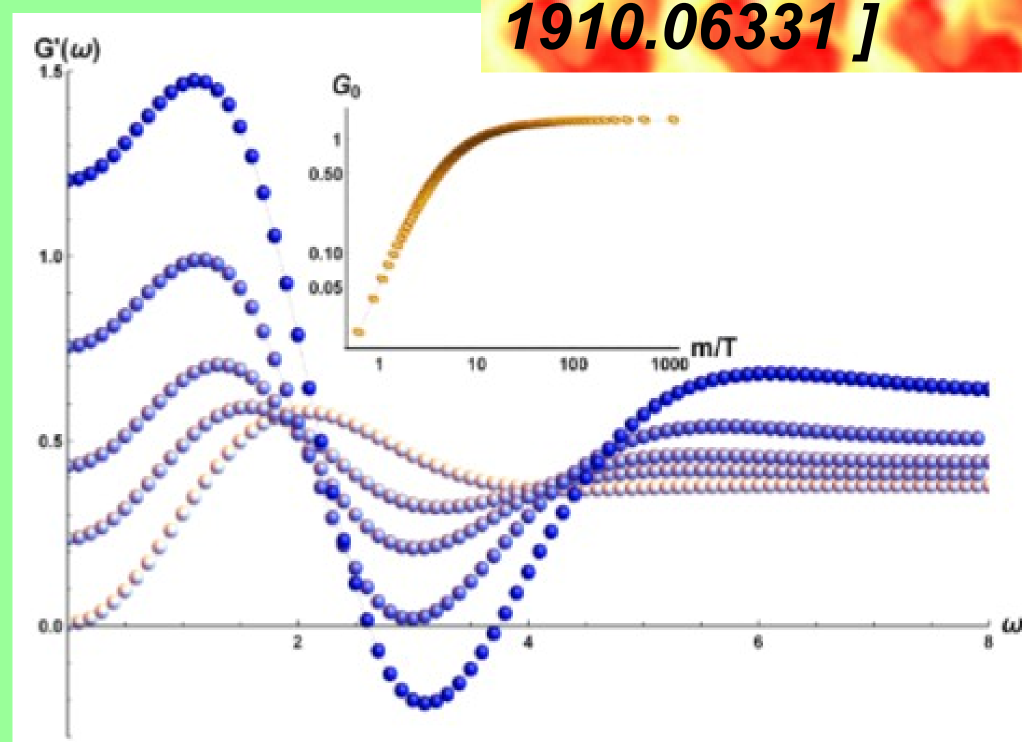
At large frequencies the viscosity goes to zero = the system is a solid

Well-known fact, Cf. Frenkel time

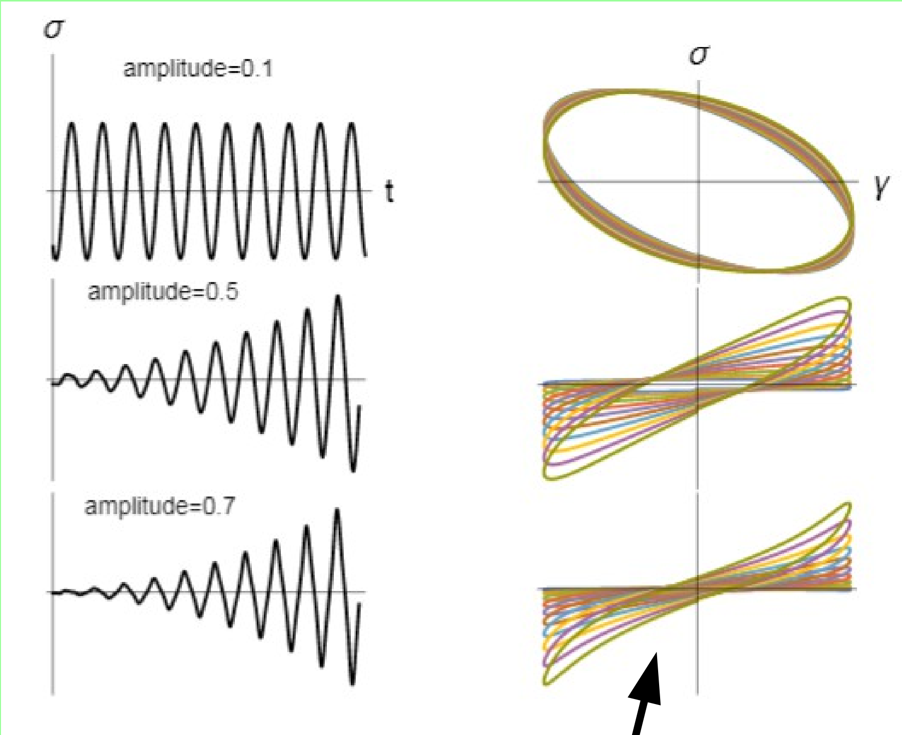
[MB,1903.02859, 1910.06331]

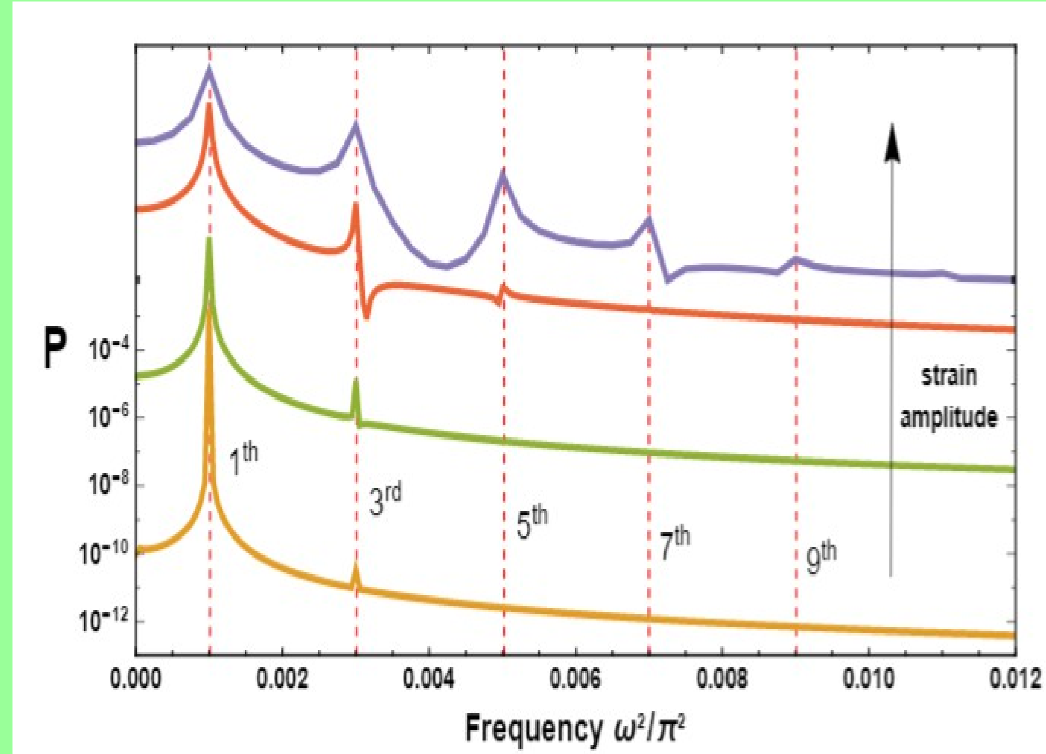
The larger the graviton mass, the more solid our system

FLUID ⟶ SOLID



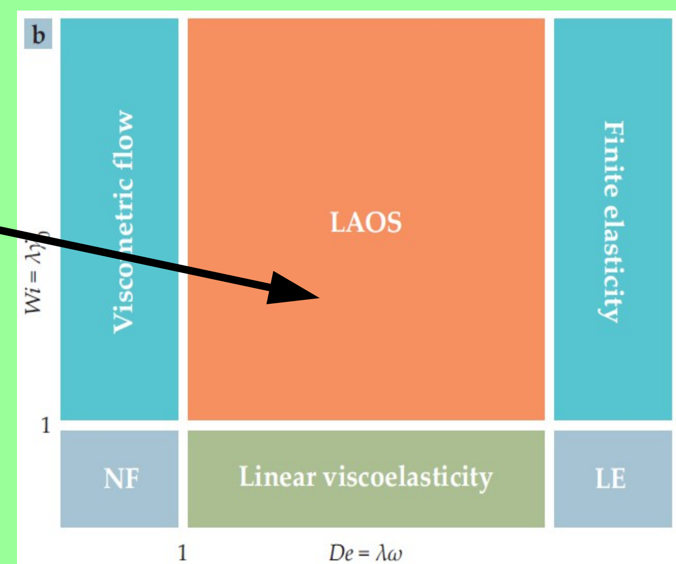
Holographic viscoelasticity



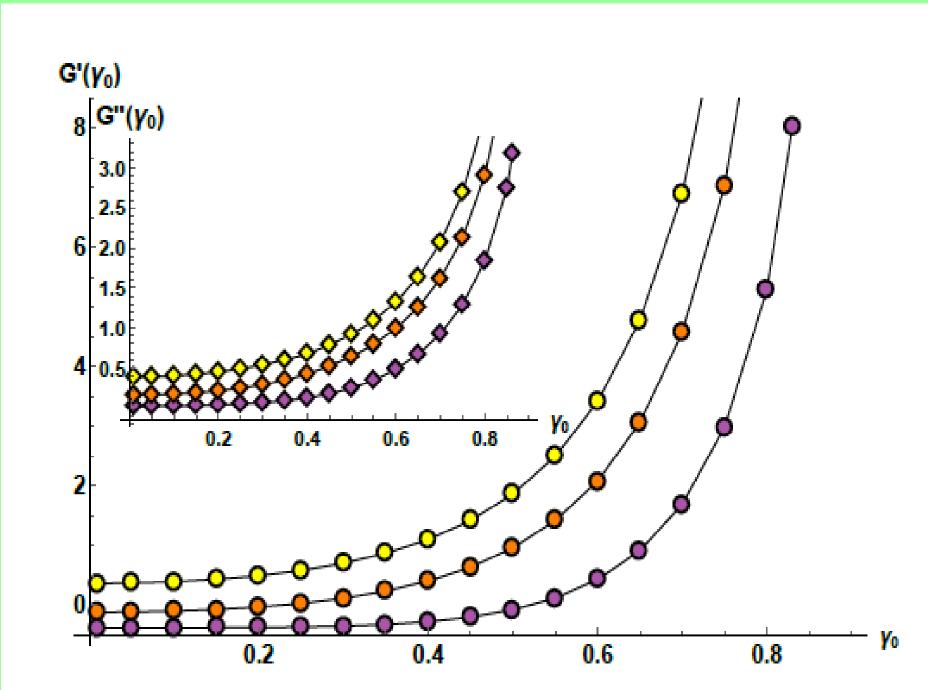


[MB,1910.06331]

Nonlinear regime ! LAOS !



Holographic viscoelasticity

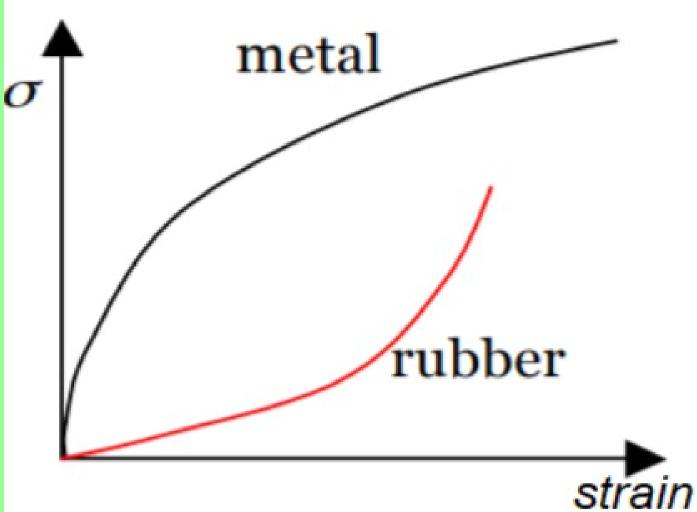


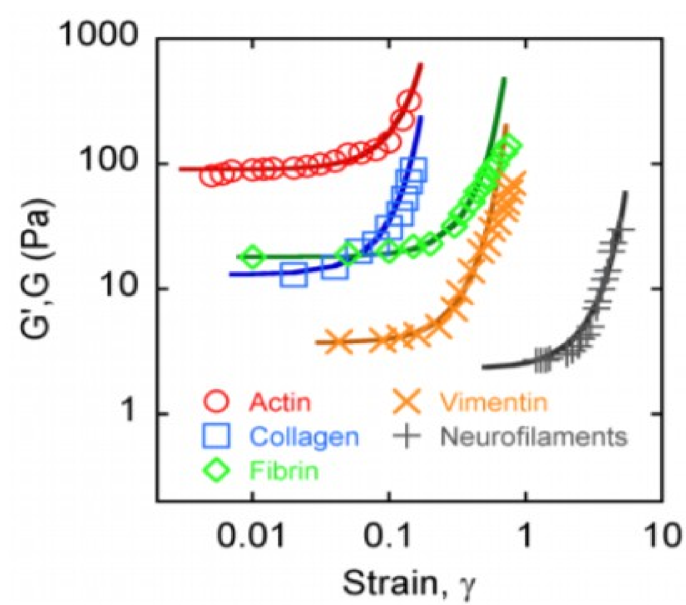
STRAIN STIFFENING

Nonlinear response similar to polymers and rubbers (not metals)

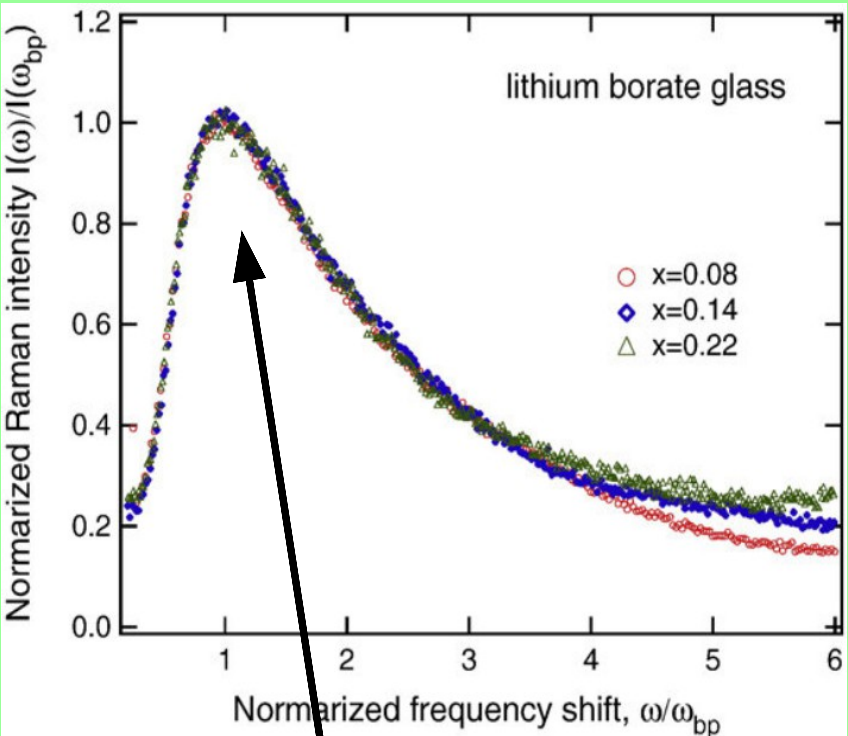
In agreement with negative thermal expansion coefficient

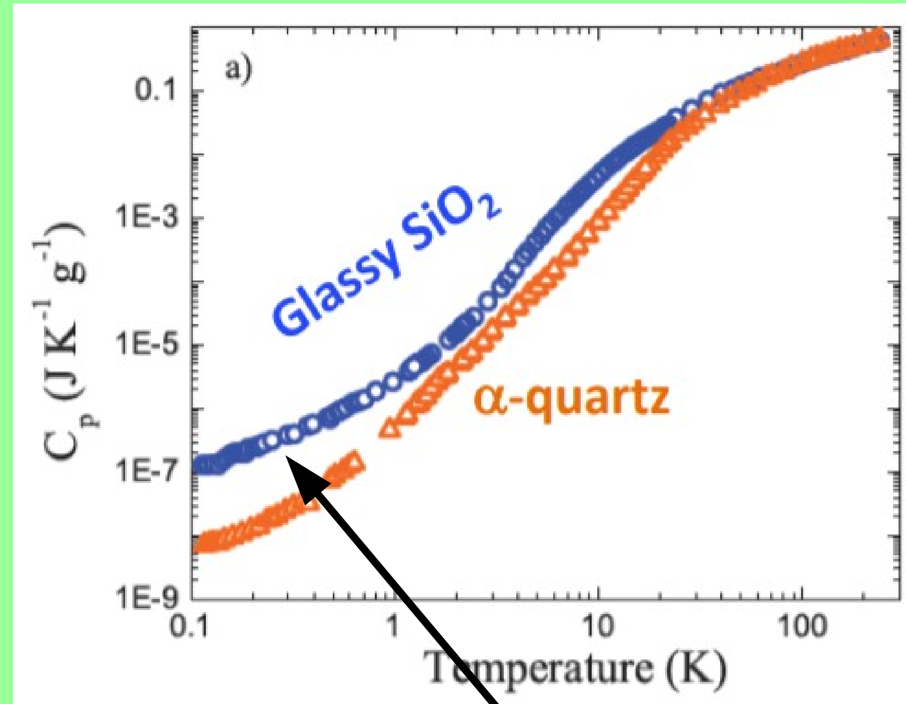
[MB,1910.06331]





Food for curiosity : glasses





"BOSON" PEAK (~1 Thz)
EXCESS OF STATES

$C_p \propto aT^3 + bT$

Peak at ~ 10K

Low-T anomalous properties

Explanations

Theory 1

Theory 2

Theory ...

Theory 246

BUT ...

Same properties observed now in ordered crystals and in incommensurate structures !!

That cannot be the end of the story nor the universal explanation

Methods (EFT-Hydro again)

DEBYE PHONONS

$$\omega = v\,k$$

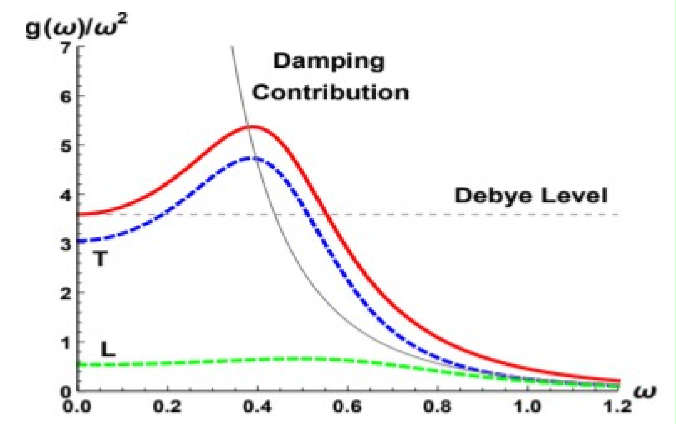
NATURE OF COLLECTIVE LOW-ENERGY DEGREES OF FREEDOM

A new paradigm

[MB,1810.09516,1911.03351]

1) Damping and anharmonicity universally induce a boson peak

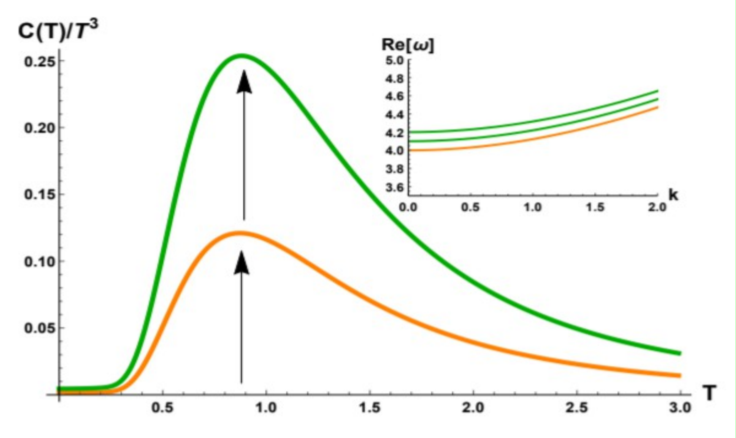
(theory predicts experimental density dependence for the 1st time and confirms correlation Boson peak an Ioffe-Regel frequency)



2) Piling up of soft optical-like gapped mode induces a boson peak

[MB,1812.07245,2008.01407]

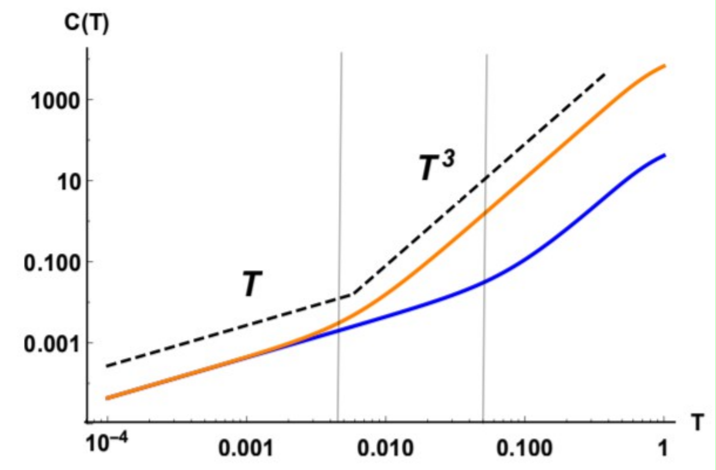
Confirmed by experiments [Moratalla et Al, Cano et Al.]



3) Low-energy diffusive modes (diffusons, phasons) give a linear in T specific heat

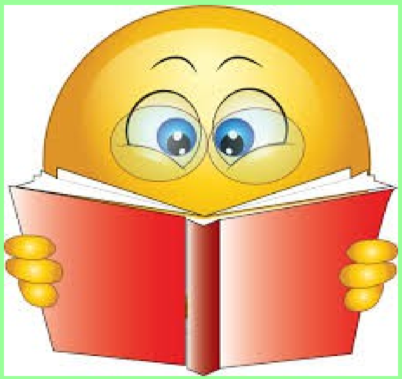
[MB,1905.03286,2008.01407]

Confirmed by experiments in quasicrystals [Cano et Al.]



More advanced topics/questions

HOLOGRAPHY – HYDRO
WHAT CAN WE LEARN MORE?



- VISCOELASTICITY
- BROKEN SYMMETRIES
- ACTION FOR HYDRO
- DISSIPATIVE-OPEN SYSTEMS

K-GAP IS EVERYWHERE !
LOTS OF OPEN QUESTIONS ...


ELSEVIER
Physics Reports
Volume 865, 15 June 2020, Pages 1-44

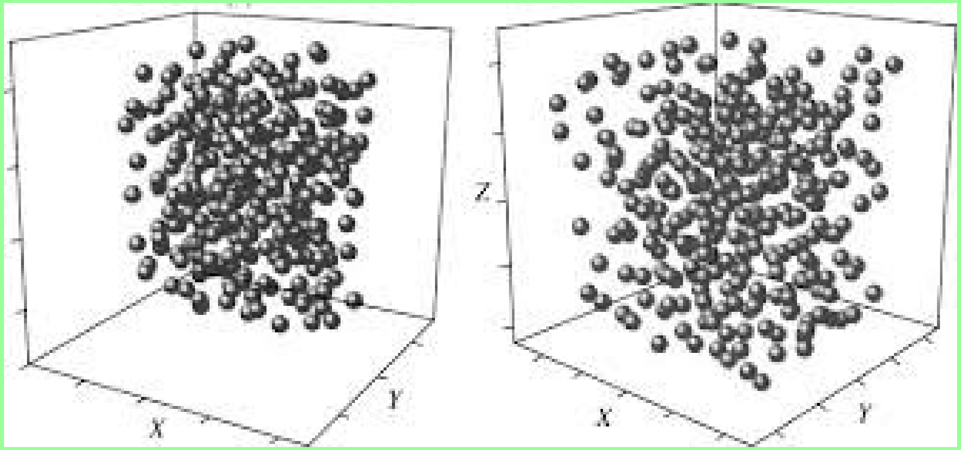
Gapped momentum states

Matteo Baggioli [a], Mikhail Vasin [b], Vadim Brazhkin [b], Kostya Trachenko [c]

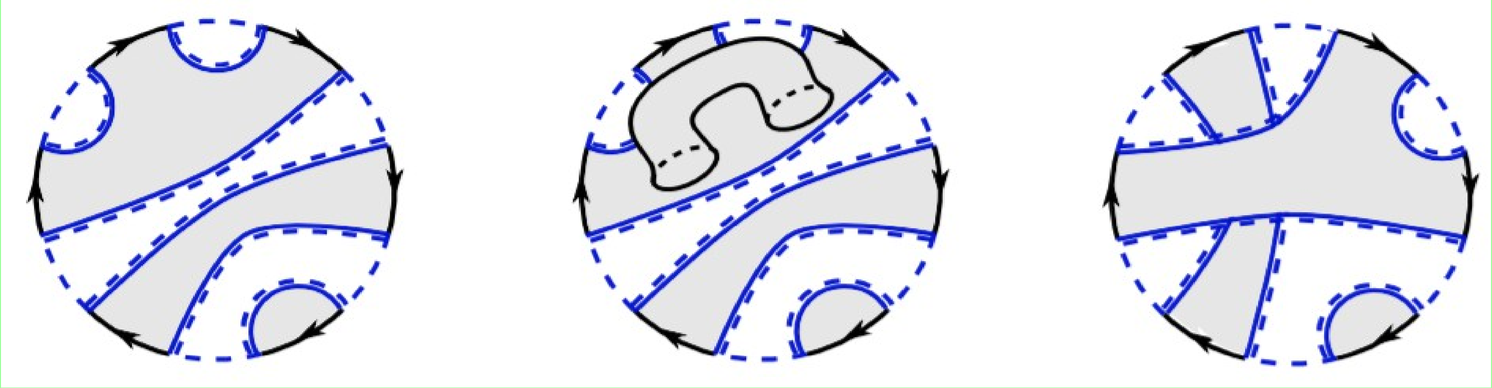
Show more

More advanced topics/questions

WHAT IS A GLASS AND ARE THOSE PROPERTIES REALLY ANOMALOUS?
WHAT ABOUT THERMAL CONDUCTIVITY ?



HOLOGRAPHIC GLASSES ?
GRAVITY AND GLASSES (see gravity ensemble)



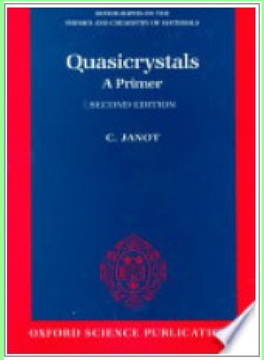

Discussion


SPEAKUP



PART 3

QUASICRYSTALS & INCOMMENSURATE



Quasicrystals: A Primer, Christian Janot

www.rsc.org/csr **TUTORIAL REVIEW**

Phonons, phasons and atomic dynamics in quasicrystals†

Marc de Boissieu*

Hydrodynamics of icosahedral quasicrystals

T. C. Lubensky, Sriram Ramaswamy, and John Toner
Phys. Rev. B **32**, 7444 – Published 1 December 1985

REVIEW ARTICLE

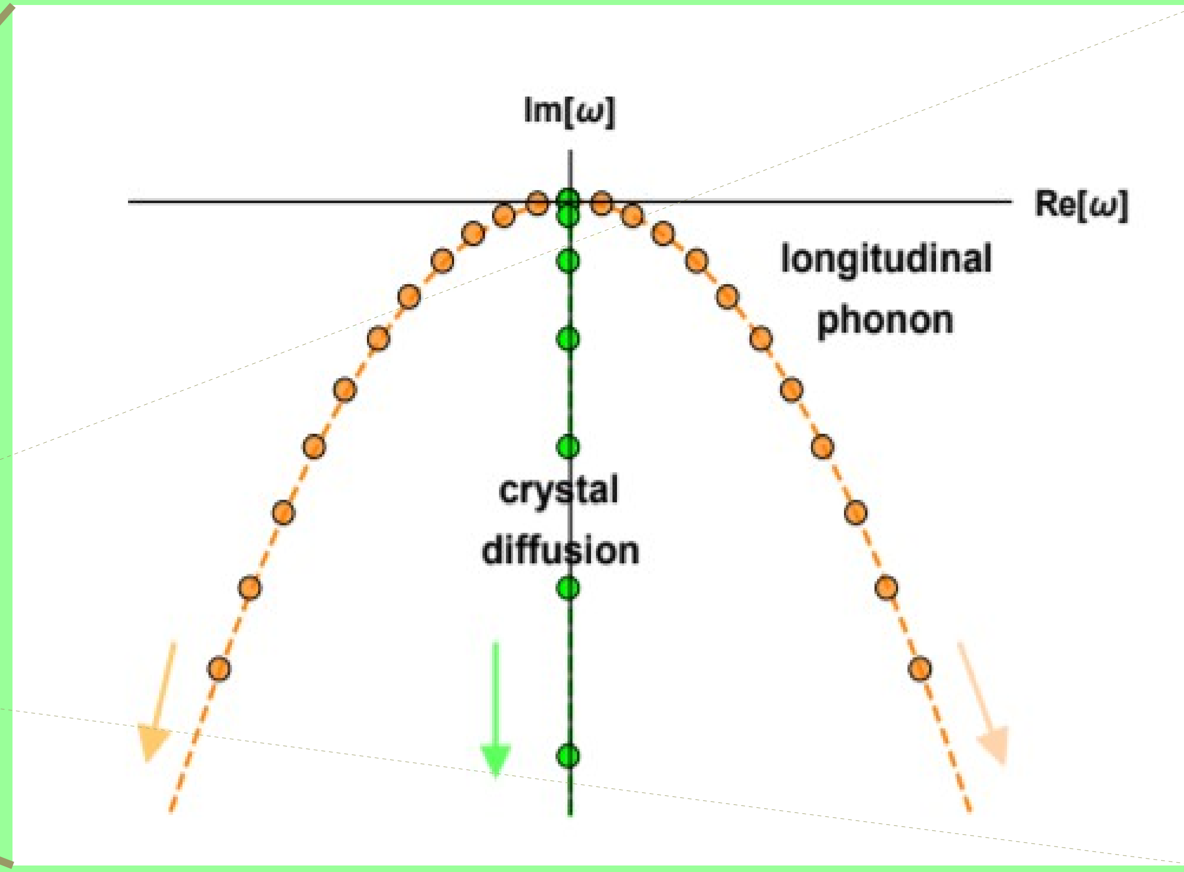
Commensurate phases, incommensurate phases and the devil's staircase

P Bak

Reports on Progress in Physics, Volume 45, Number 6

What brought me here

[MB,1905.09164 , 1905.09488]

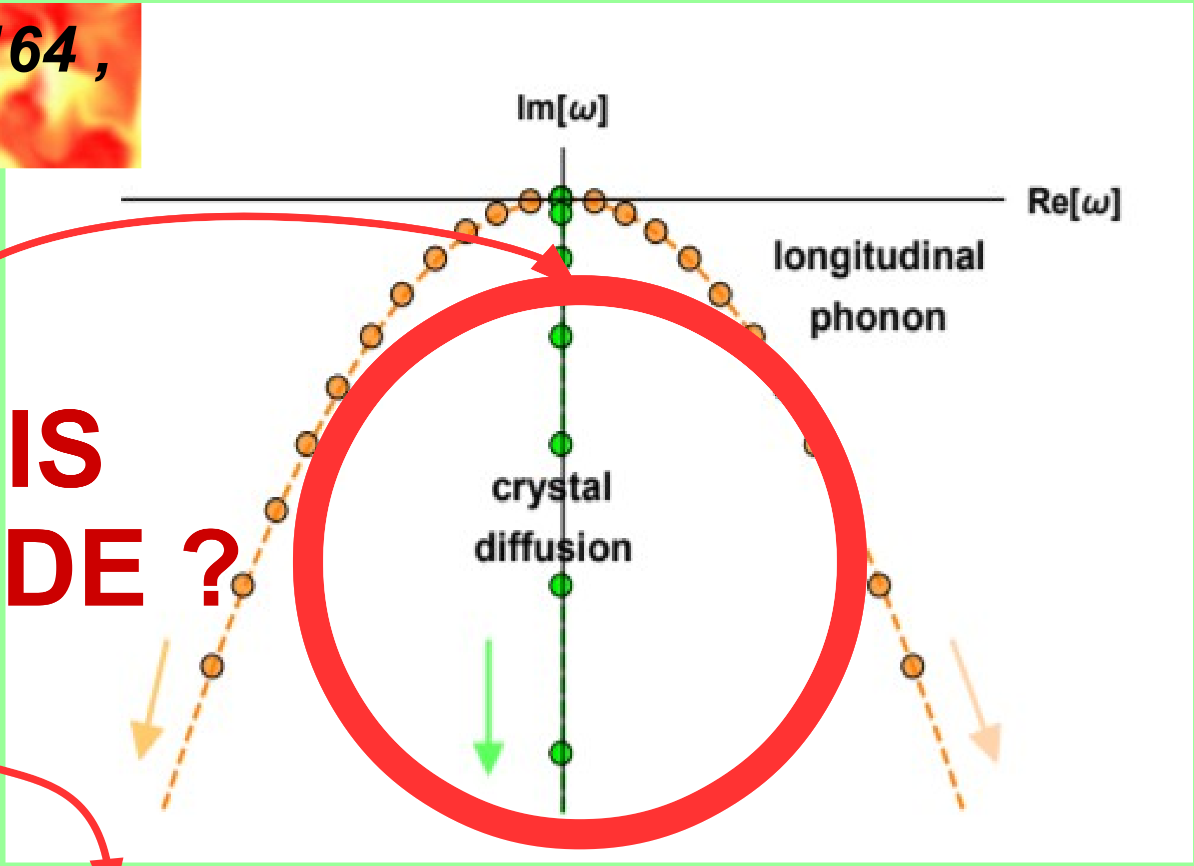


$$\omega = \pm c_L\, k - i\, D_p\, k^2 + \dots ,$$ → Longitudinal sound

$$\omega = -i\, D_\Phi\, k^2 + \dots ,$$ →

What brought me here

[MB,1905.09164 , 1905.09488]



WHAT IS THIS MODE ?

[Some people simply say it is mass diffusion or vacancy diffusion]

I will explain why that cannot be.

$$\omega = \pm c_L\, k - i\, D_p\, k^2 + \ldots ,$$

→ Longitudinal sound

$$\omega = -i\, D_\Phi\, k^2 + \ldots ,$$

→

Important points

Longitudinal diffusive mode

$$\omega = -i\, D_{||}\, k^2$$

1 - It does <u>not</u> come from the breaking of translations

2 - It comes from the SSB of the global internal symmetry

$$\phi \to \phi + a$$

[Donos,Martin,Pantelidou,Ziogas,2019]
[Amoretti, Arean, Gouteraux, Musso,2018]

3 - It is a **diffusive Goldstone boson**

Other considerations

Translations are not broken to a discrete subgroup.
There is no unit cell. The systems are not periodic.
(e.g. Incommensurate CDW)

The mode does not come from the conservation of
any local U1 current (e.g. mass diffusion, charge diffusion)

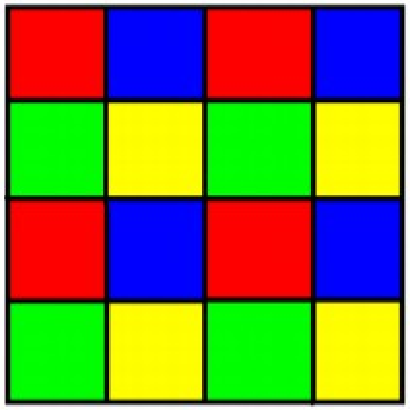
The are no commensurability effects.

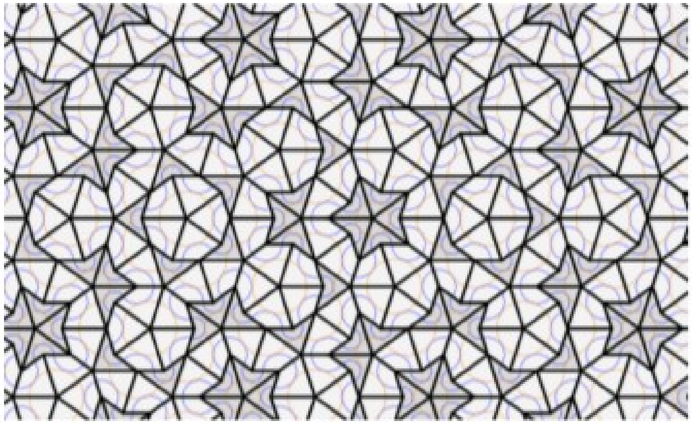
[Andrade,Krikun, 2015]

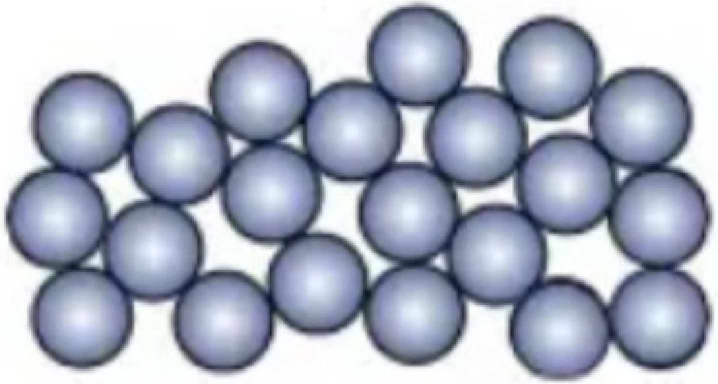
Most of the systems are metastable.
Actually there is NO known Q-lattice or axion model (with SSB)
which is stable in all senses (thermodynamics and dynamics)

Aperiodic crystals

Real ! (cf. Nobel Prize)







Periodic crystal

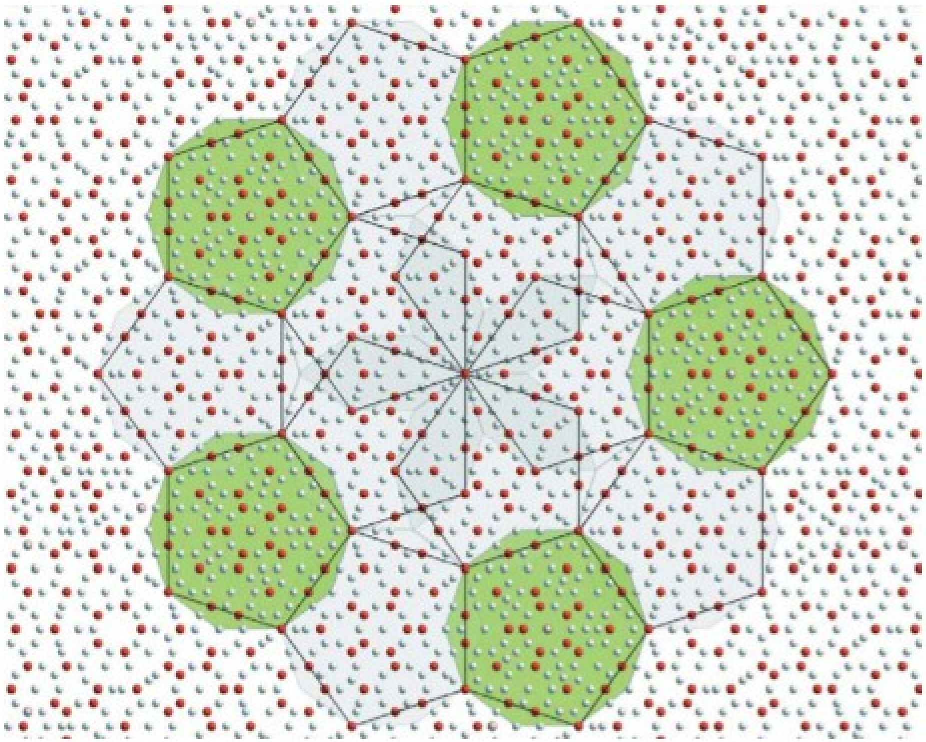
QUASICRYSTAL

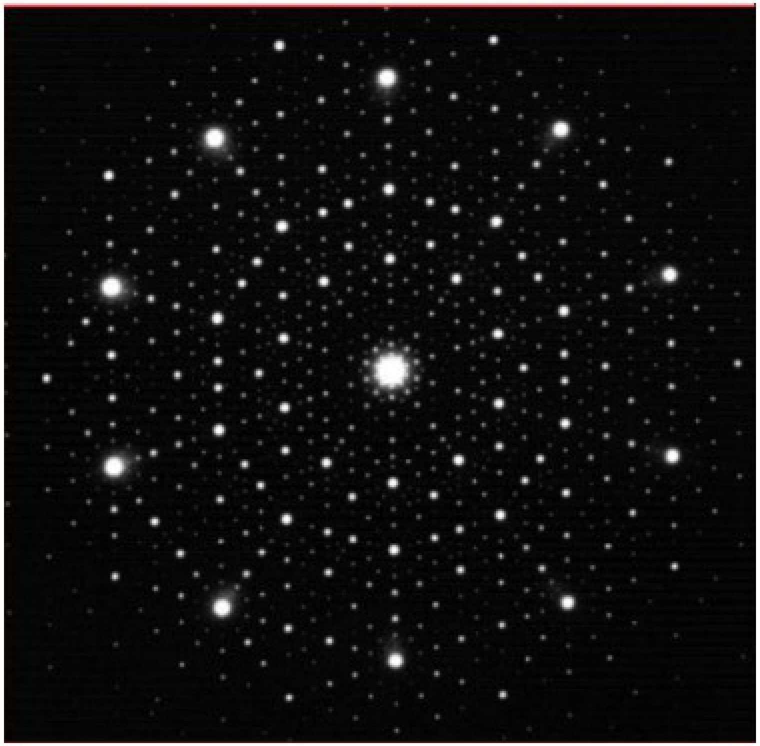
Amorphous system
(e.g. glasses)

LONG RANGE ORDER BUT NO PERIODICITY

(e.g. Penrose tilings , incommensurate CDWs)

Bragg peaks



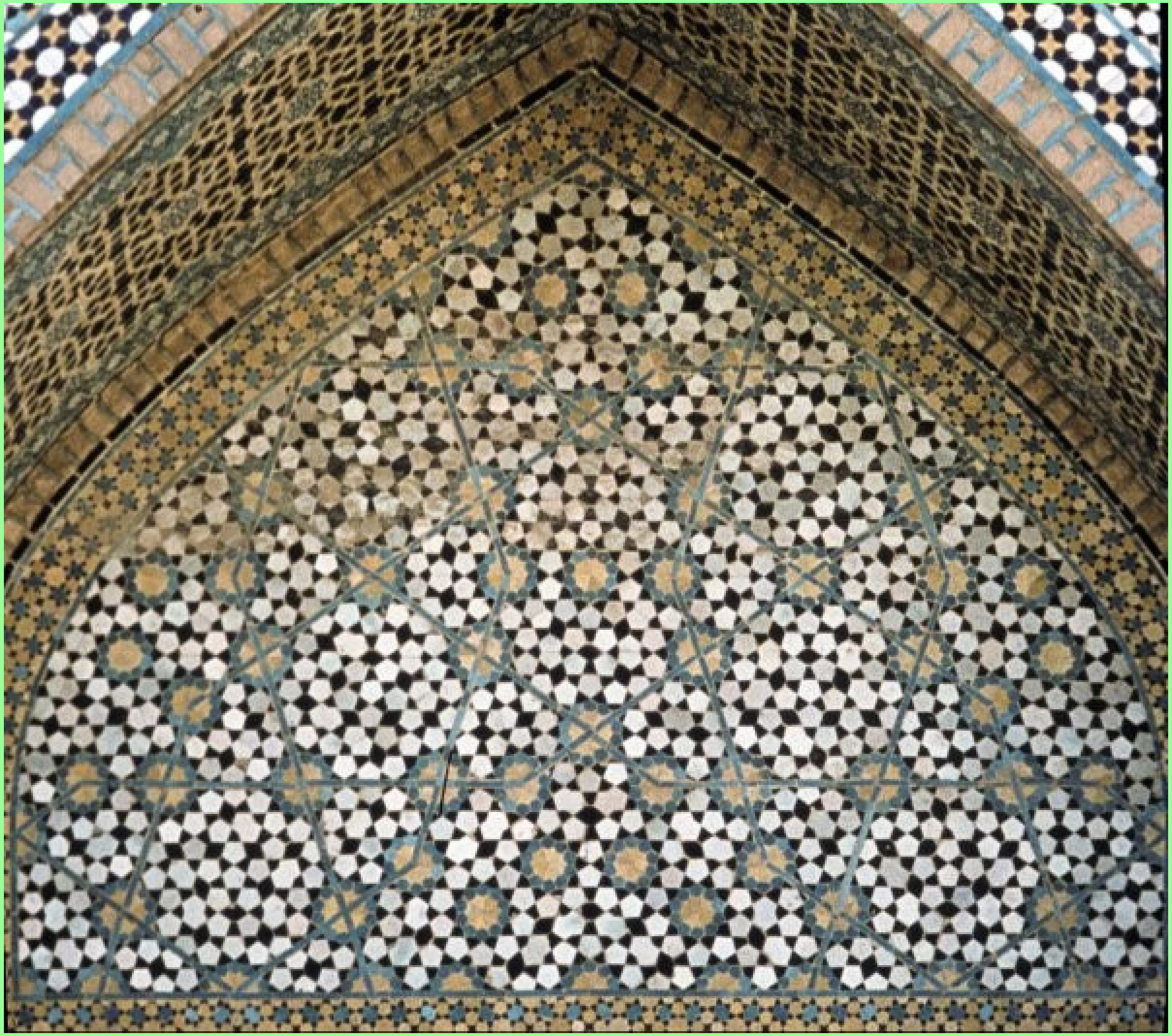


$$\rho(\vec{x}) = \frac{1}{V} \sum_{\vec{G}} \rho(\vec{G})\, e^{i\,\vec{G}\cdot\vec{r}}$$

$$\vec{G} = \underbrace{n_1\,\hat{x}_1 + n_2\,\hat{x}_2 + n_3\,\hat{x}_3}_{d\,\text{physical dimensions}} + \underbrace{\sum_{i=d}^{D-d} n_i\,\hat{x}_i}_{D-d}$$

Sharp bragg peaks but not periodic (+ modulated intensity)

Penrose tilings & art

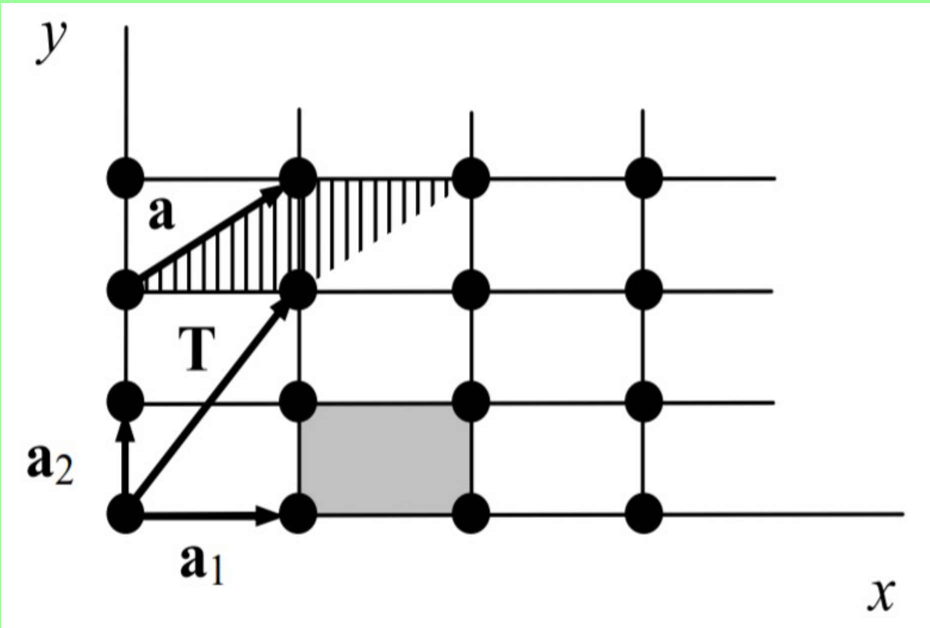


We need to change the tiles of the kitchen floor....

But, I do not want them periodic! They are boring.

Siri, call Roger Penrose.

Lattice structures

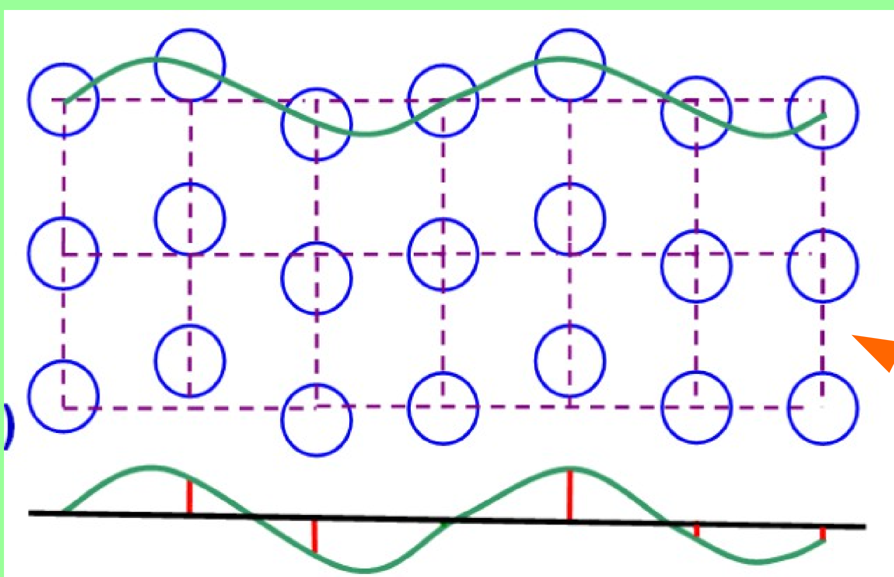


$$\mathbf{T} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2$$

Point in the crystal

<u>simple cubic</u>

$$\begin{cases} \mathbf{a}_1 = a\mathbf{x} \\ \mathbf{a}_2 = a\mathbf{y} \end{cases}$$



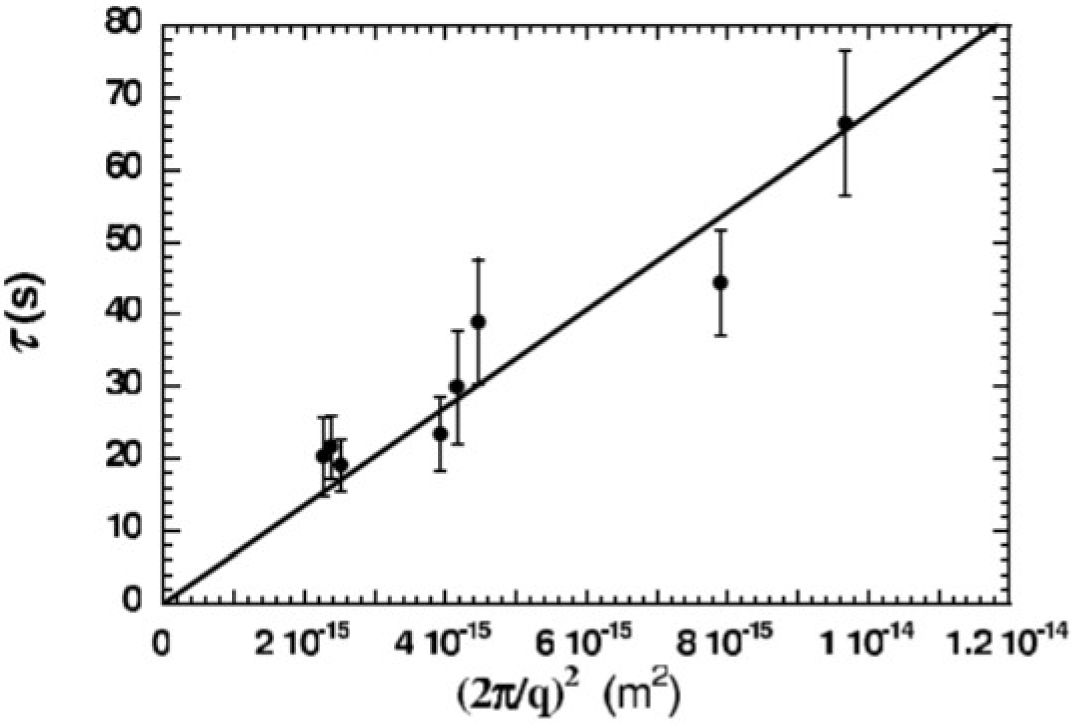
$$\vec{G} = \underbrace{n_1\,\hat{x}_1 + n_2\,\hat{x}_2 + n_3\,\hat{x}_3}_{d\,\text{physical dimensions}} + \underbrace{\sum_{i=d}^{D-d} n_i\,\hat{x}_i}_{D-d}$$

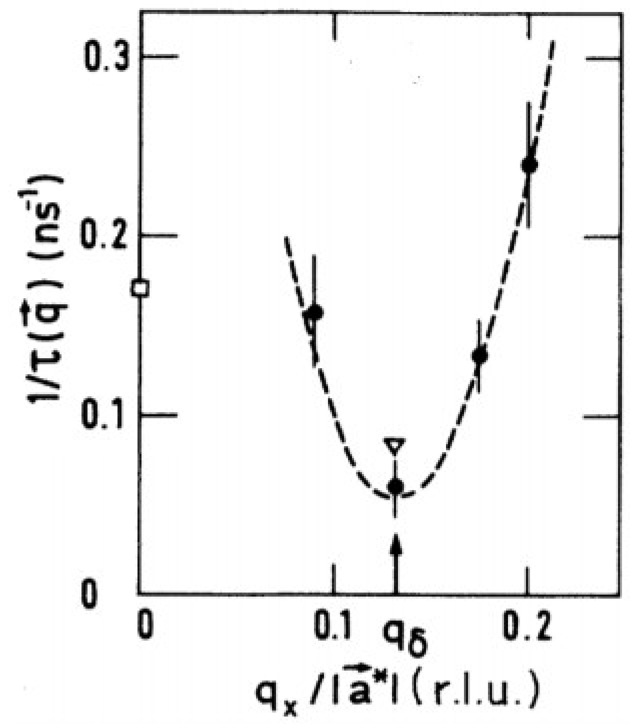
Alternatively the additional vectors can be recasted into phases (= modulation)

Aperiodic structure needs more vectors than d

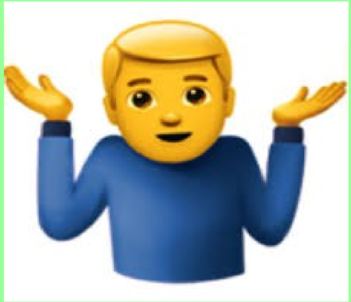
Phasons

New hydrodynamic low-energy excitation



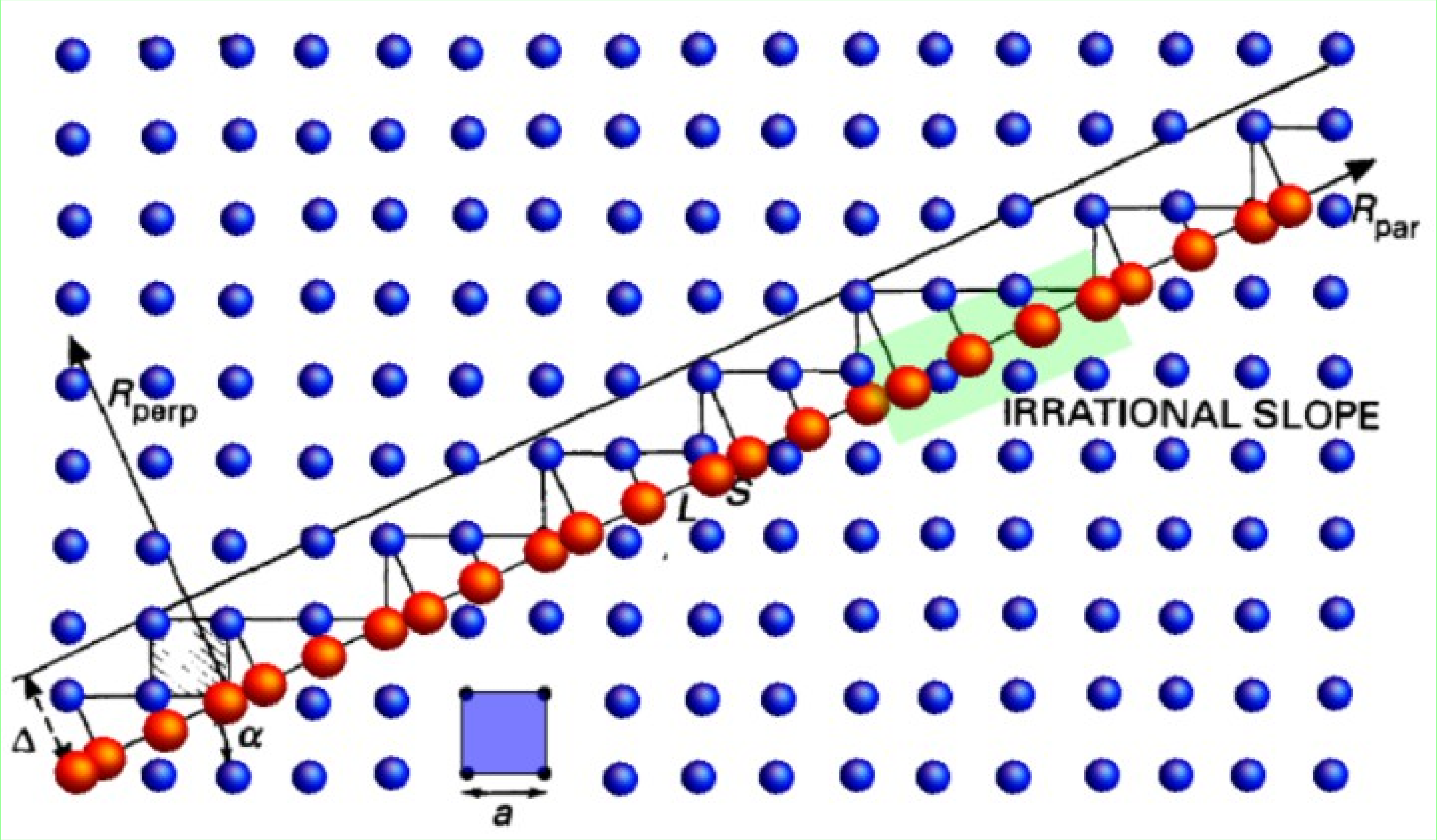


$$\omega = -i\, D_{phason}\, k^2 + \ldots$$



And propagating at large wave-vector ...

The superspace formalism

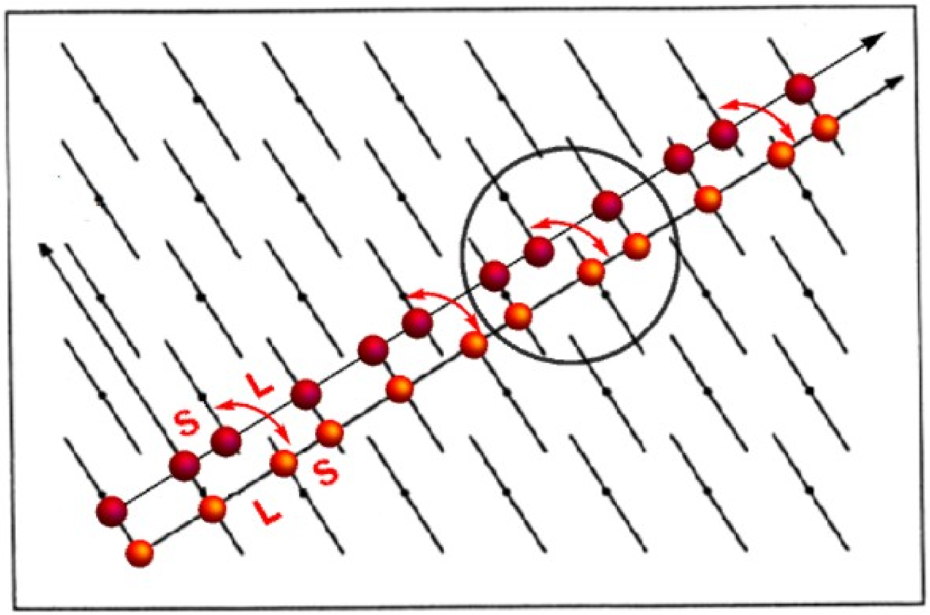


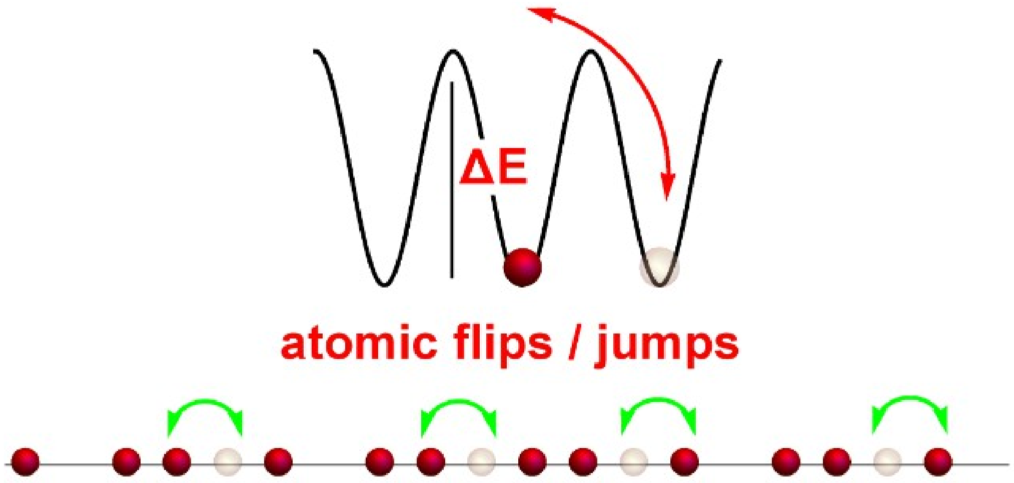
$$L = a \cos\alpha \,, \qquad S = a \sin\alpha \,.$$

$$\ldots SLLSLLLSLLS \ldots$$

Where do phasons come from ?

Rigid translations in the internal space





$$\ldots LS \ldots \quad \rightarrow \quad \ldots SL \ldots,$$

Phason jumps (only at finite T)

Why they are diffusive

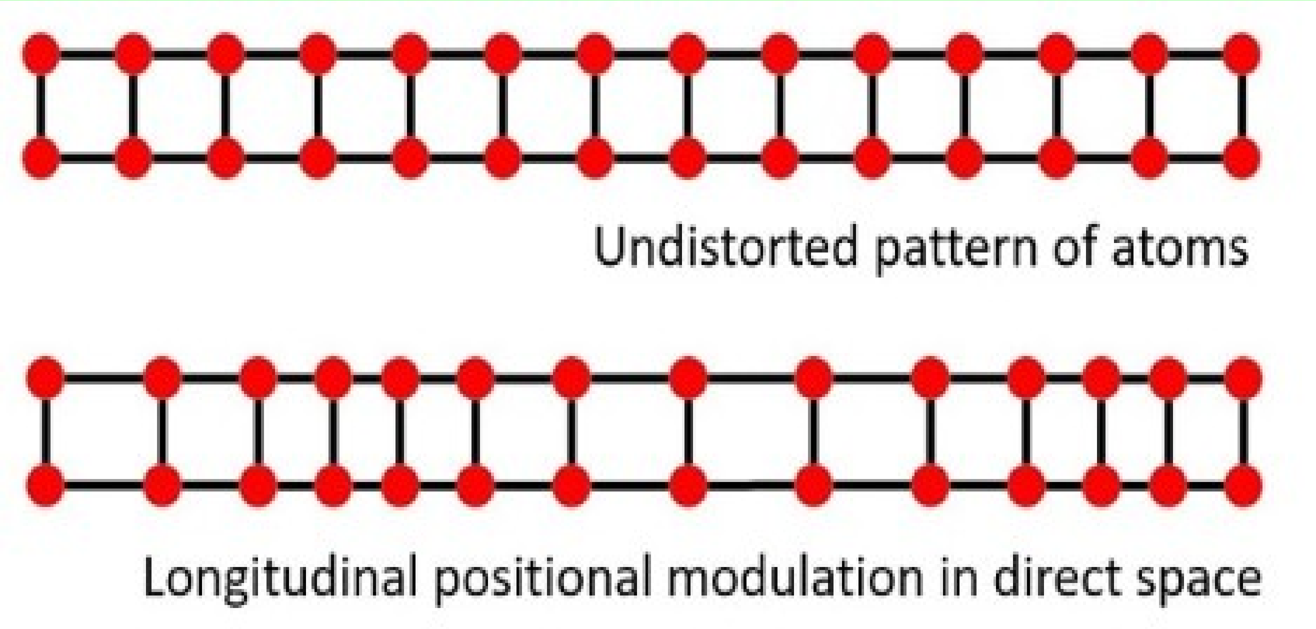
"Mode counting arguments and the Goldstone theorem lead to the prediction that phason modes are diffusive-like excitation,"

the phason shifts leave the free energy unchanged but they do not commute with the hamiltonian of the system

TASK.

Find a formal & satisfactory explanation from symmetries

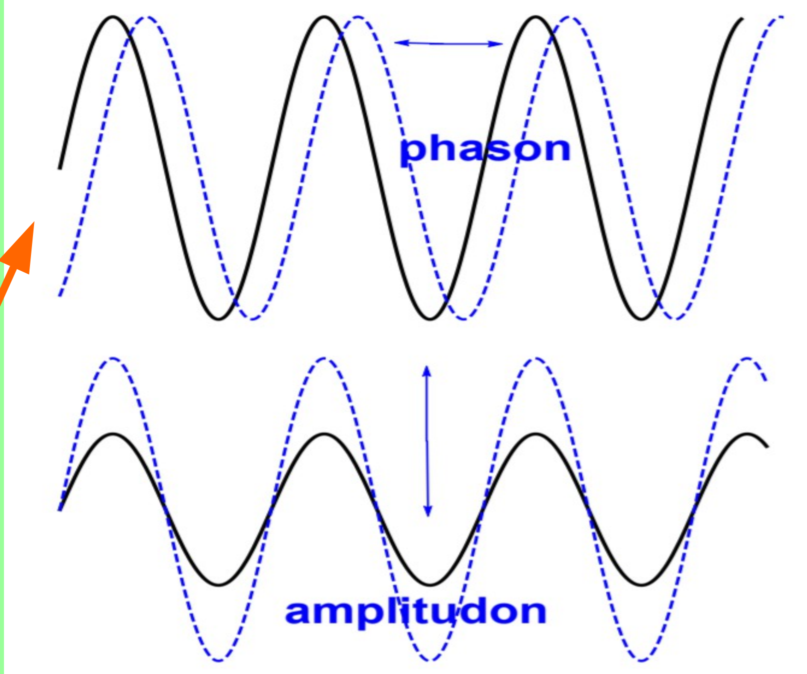
Incommensurate structures



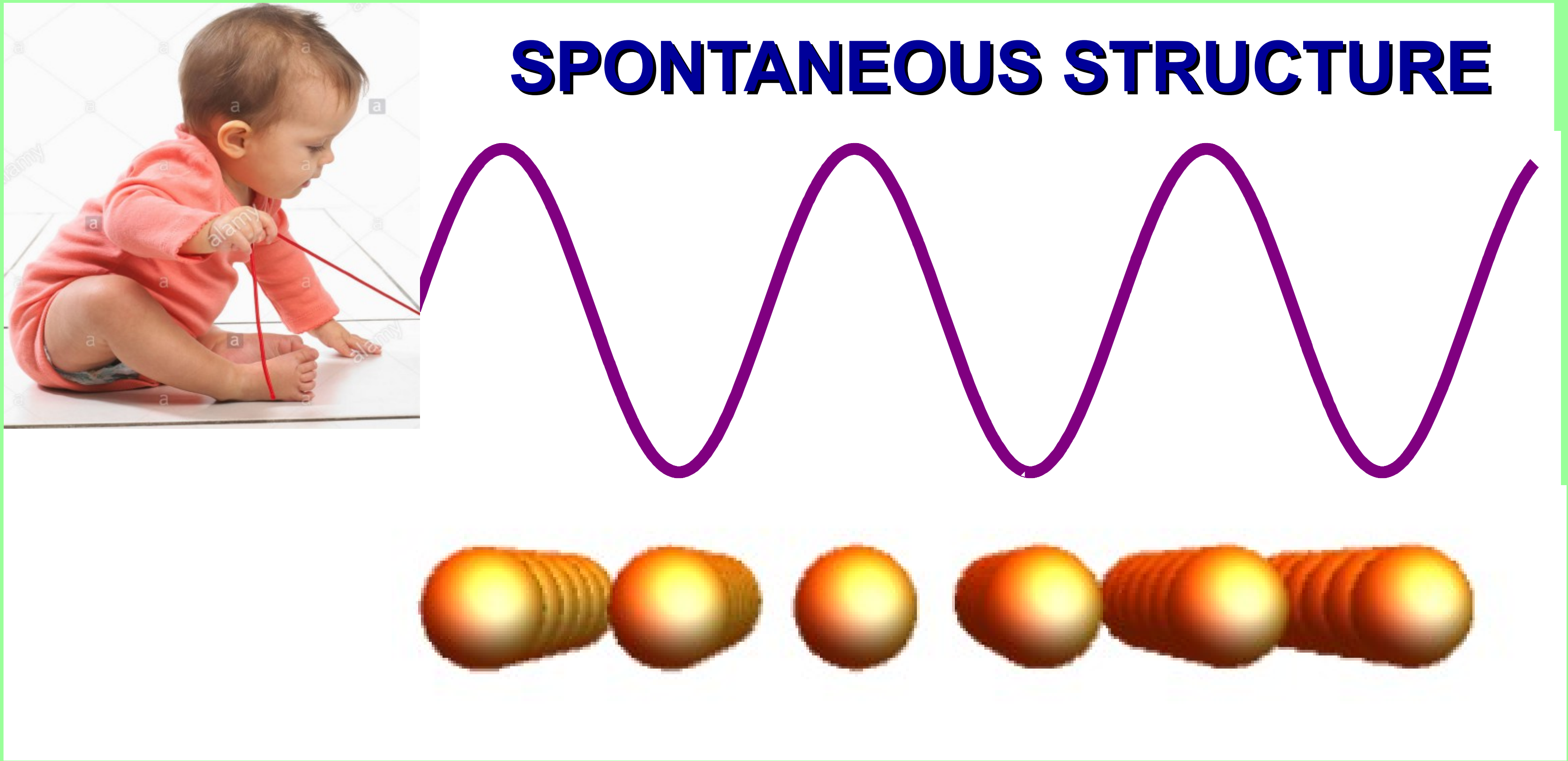
$$\rho(r) = \rho_0 + R\cos\left(k\,r + \phi(r)\right)$$

Free energy invariant under phase shifts

Hydro-massless mode: phason



Phasons (2nd point of view)

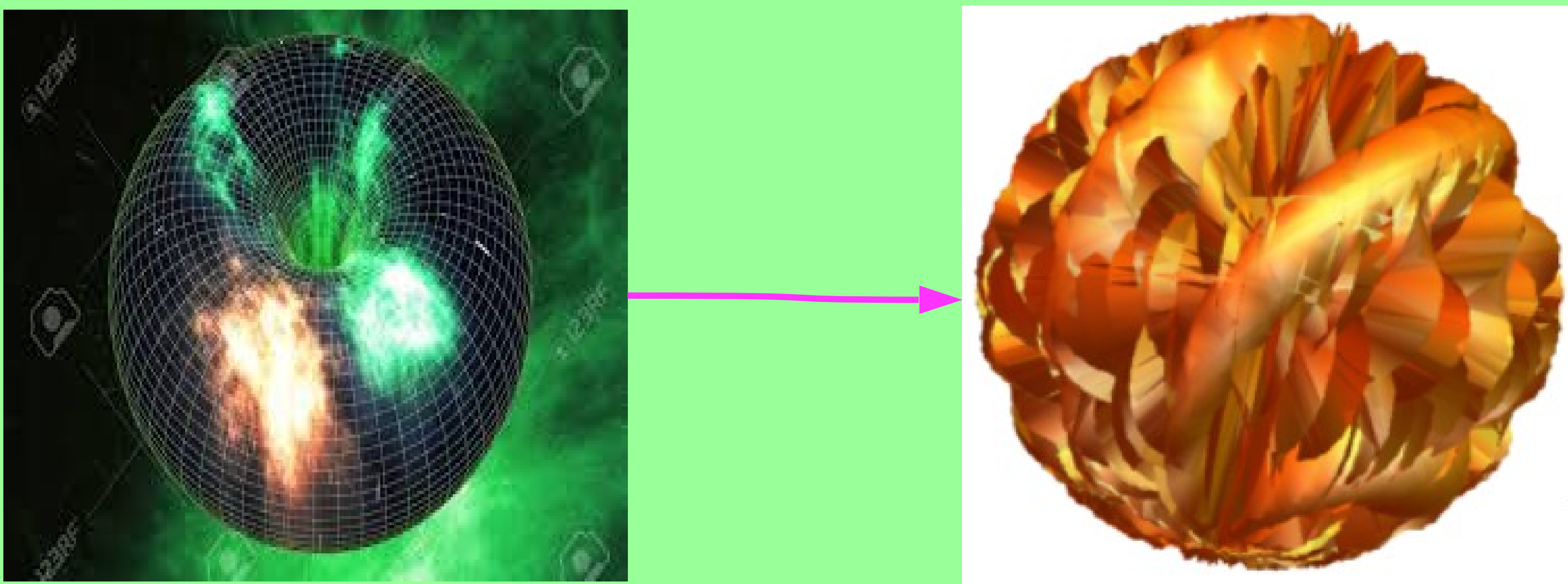


IONIC LATTICE

PHASON = free sliding of the spontaneous structure

Back to holography

Holographic spontaneous lattices
(e.g. Charge density waves)



Spontaneous inhomogeneous structures

[Donos, Gauntlett, Ooguri, Park, Lippert, Jokela, Li, Zaanen, Krikun, Andrade + …..]

Analysis of homogeneous models

Use a global symmetry in the bulk to break translations in a homogeneous way

Axions-like models
[Andrade,Withers]

$$\psi^{(0)} \propto \alpha_i x^i.$$

Shift symmetry (goldstones)

Q-lattices
[Donos,Gauntlett]

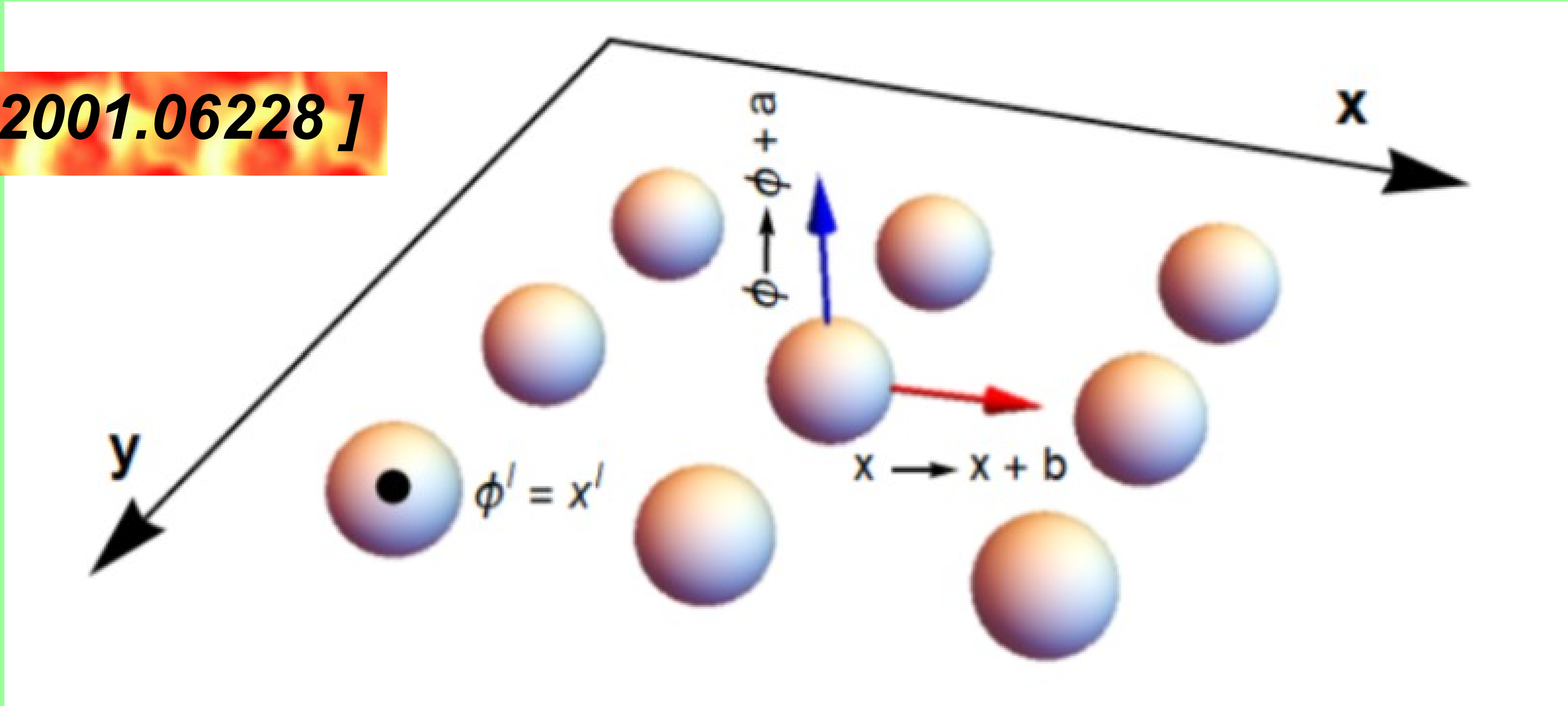
$$\phi = e^{ikx_1}\varphi(r)$$

Global U(1)

Geometry homogeneous but translations are broken !

Analysis of homogeneous models

[MB,2001.06228]



$$\phi^I = \alpha\, x^I$$

Spacetime directions

Internal directions

$$x^I \to x^I + b^I$$

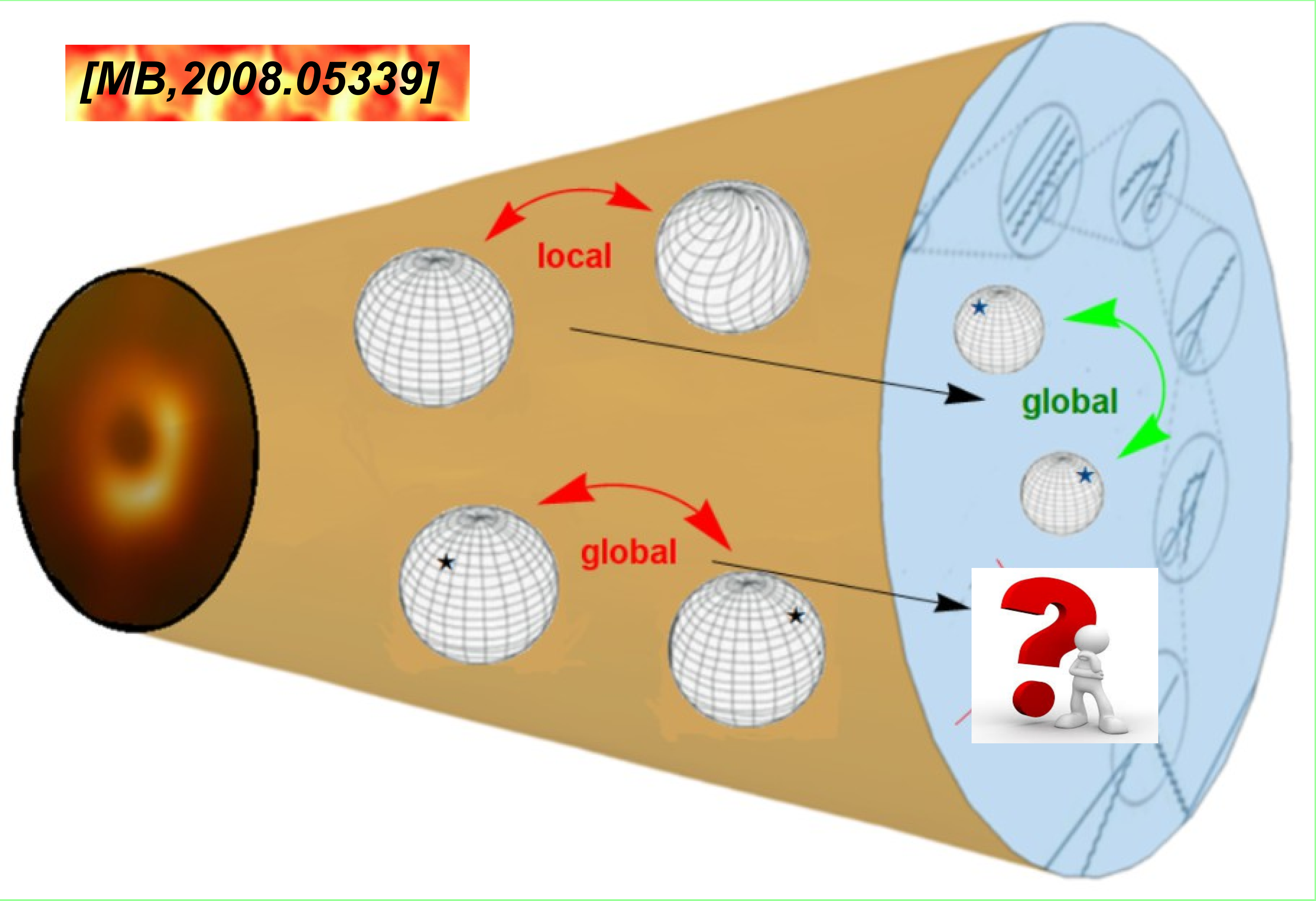
$$\phi^I \to \phi^I + a^I$$

PHONONS

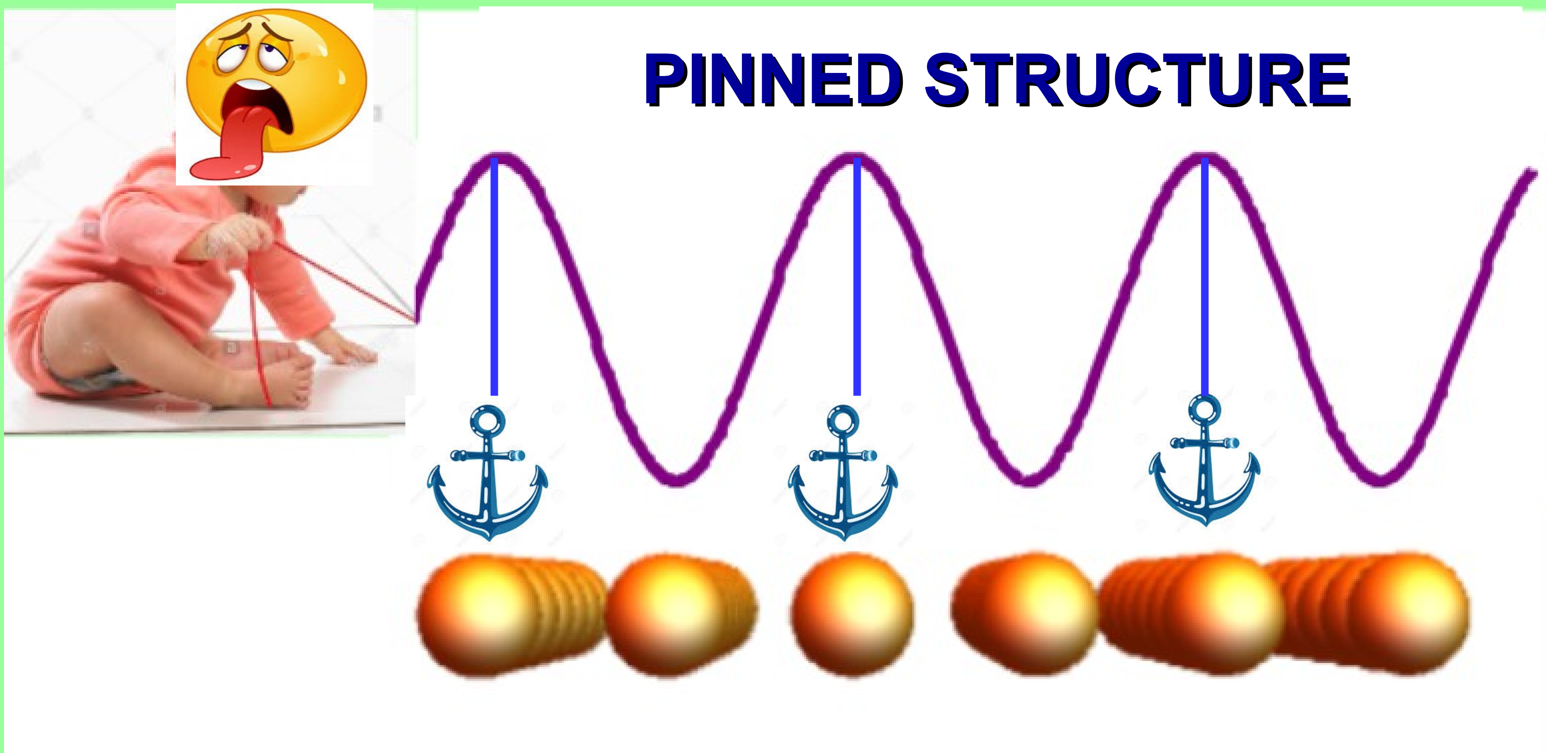
PHASONS

Analysis of homogeneous models

They are *NOT* the gravity dual of solids EFTs



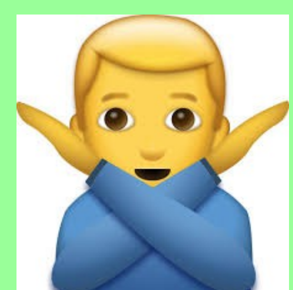
Phase relaxation



$$\omega = -i\Omega +$$

Phase is relaxed with rate Omega

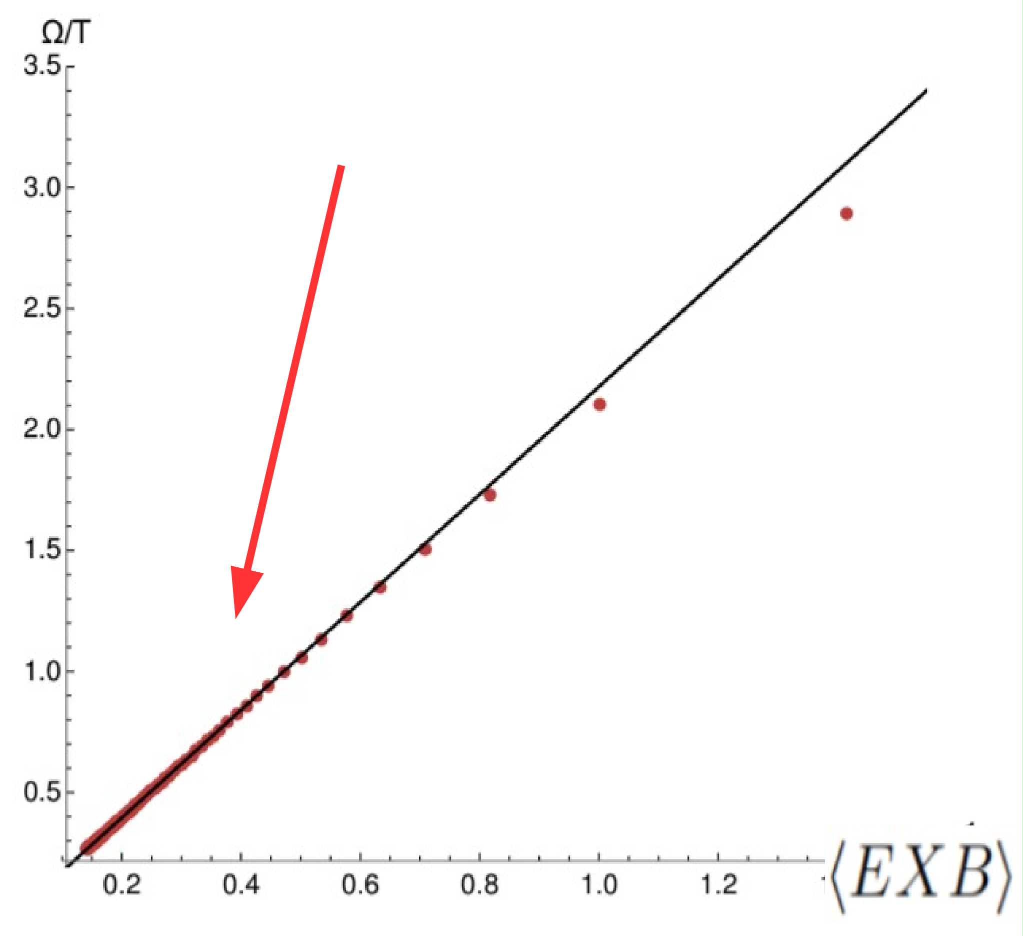
Now you cant just slide freely



The phase relaxation mistery

$$\bar{\Omega} \sim \frac{\langle EXB \rangle}{\langle SSB \rangle}$$

Induced by explicit breaking !



Different from the standard phase relaxation induced by defects (e.g. dislocations)

[MB,1904.05785]

Common to many holographic models

[I would say all]

A universal relation

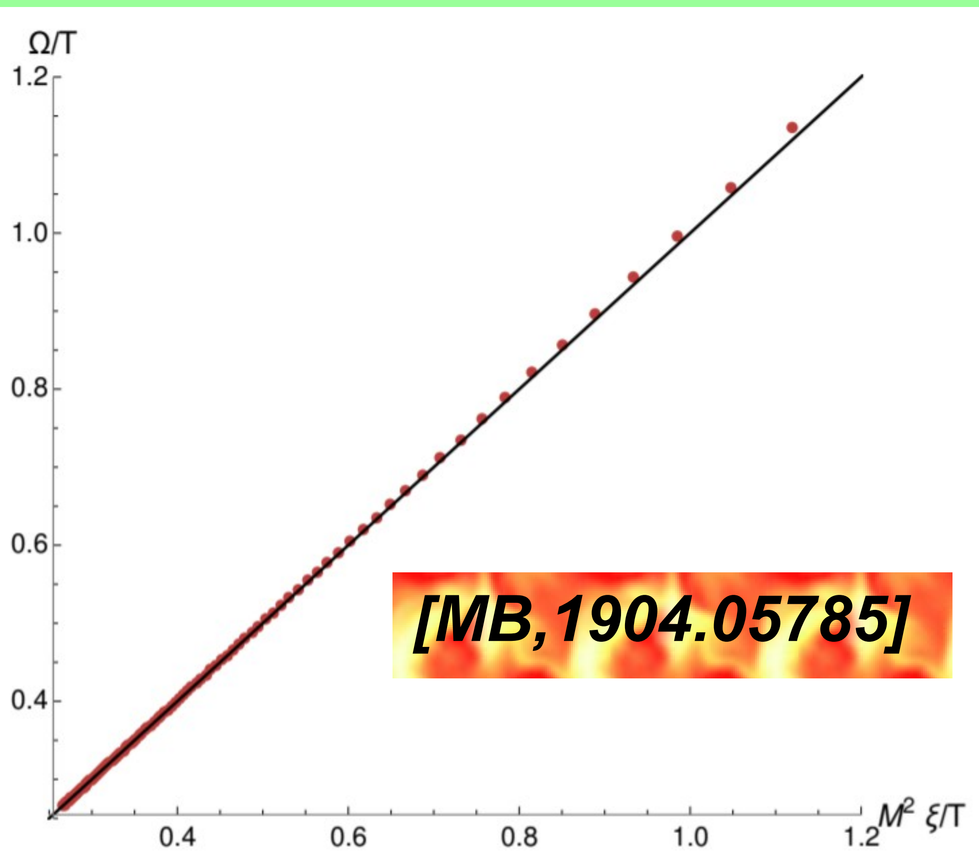
[Gouteraux et Al.]

Confirmed in several models with numerics and perturbative methods

$$\Omega = \chi_{\pi\pi}\,\omega_0^2\,\Xi$$

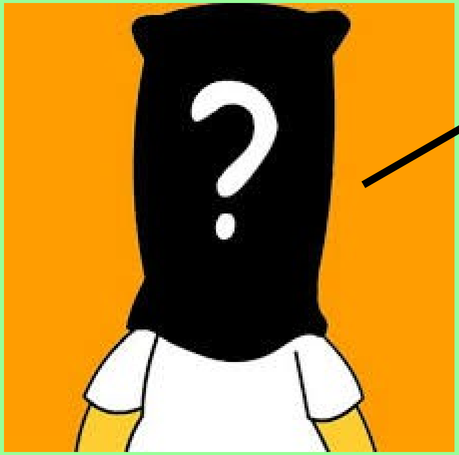
Phason diffusion

Phonon pinning frequency

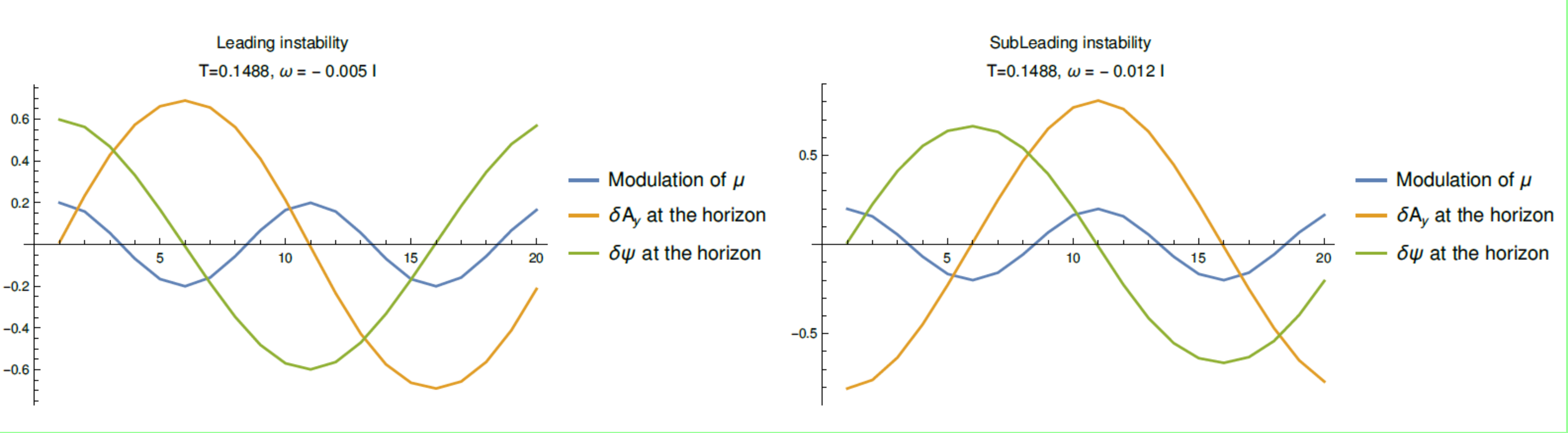


Complete formal proof still absent

Inhomogeneous models



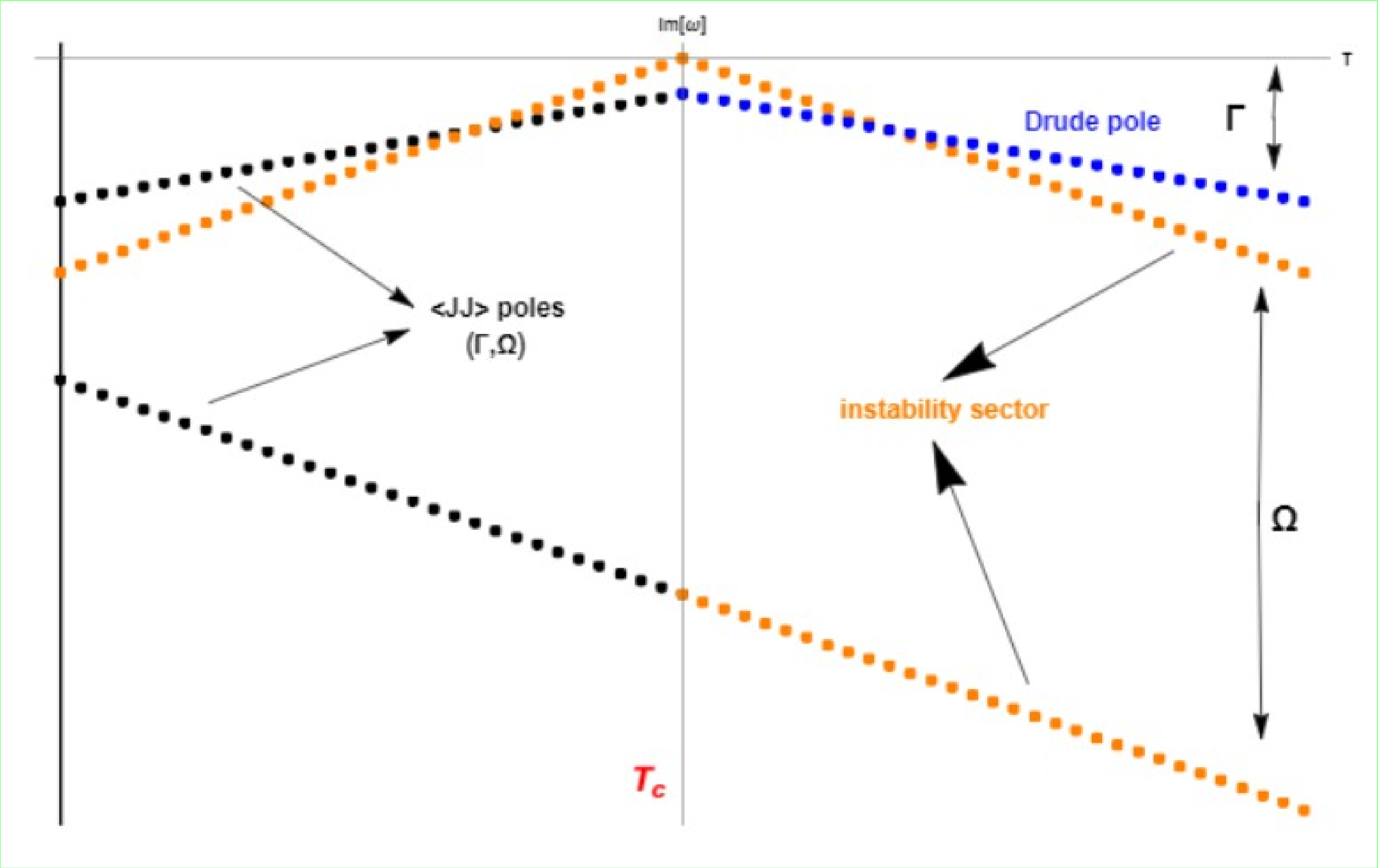
Those are toy models,
not real lattices !!
They are missing a lot of real physics
(and probably introducing a lot of
fake one ...)

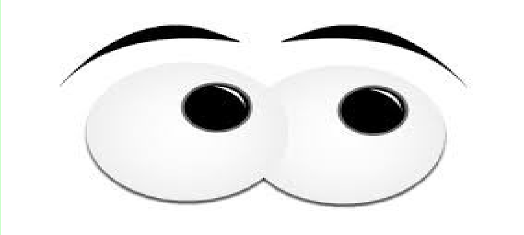


Let us check in "real" lattices !

[MB,"coming soon"]

Inhomogeneous models





This is what we get !

[MB, "coming soon"]

What did we learn ?

Below Tc the hydro spectrum of homogeneous models and the inhomogeneous ones <u>ARE IDENTICAL</u> !
& in perfect agreement with hydrodynamics

$$(\Gamma - i\omega)\,(\Omega - i\omega)\;+\;\omega_0^2\;=\;0$$

1) No matter if the system is inhomogeneous at large scale physics is always homogeneous [Nicolis et Al.]

2) Homogeneous models capture well (almost) everything



[MB,"coming soon"]

What did we learn ?

1) Physics of the model can be understood perfectly from "amplitude equation" and theory of pattern formation

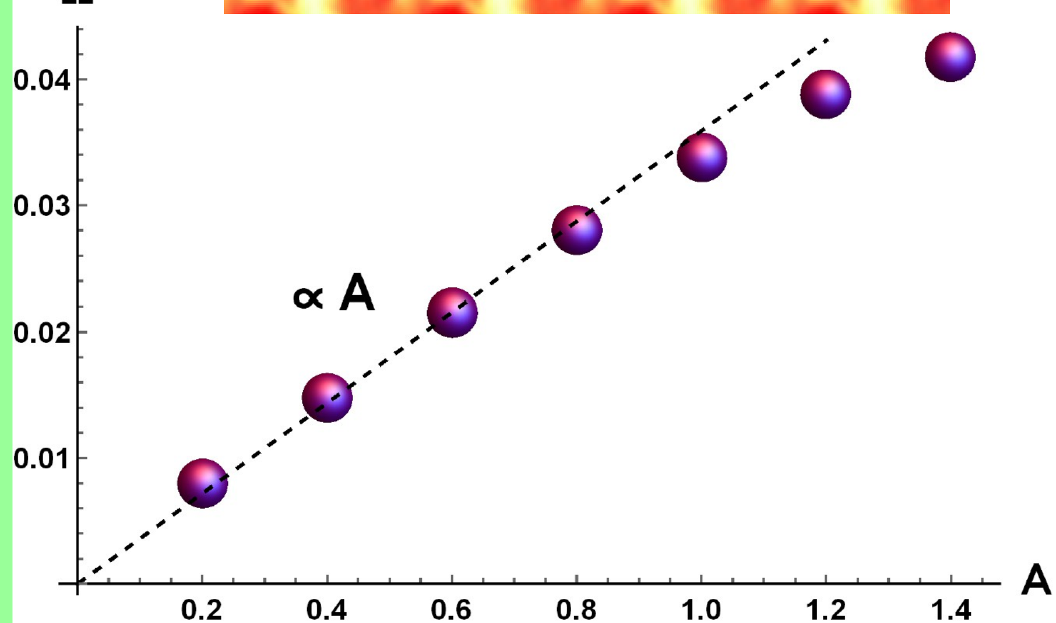
Commensurate-incommensurate transition in nonequilibrium systems

P. Coullet

Phys. Rev. Lett. **56**, 724 – Published 17 February 1986

2) Omega can be really understood as phase relaxation and identified also above Tc

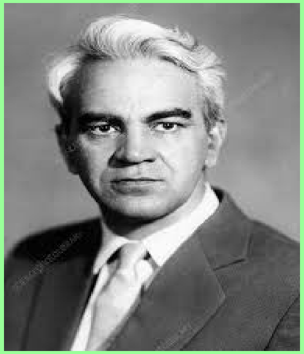
[MB,"coming soon"]



3) The Omega dependence is confirmed

EFT with dissipation

$$e^{W[J_1,J_2]} \equiv \mathrm{tr}[U(+\infty,-\infty;J_1)\,\rho\,U^\dagger(+\infty,-\infty,J_2)]$$

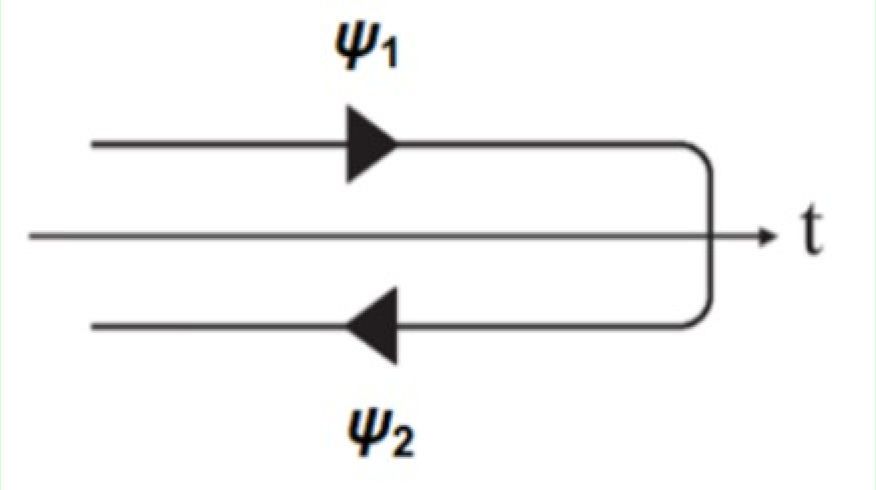


$$\int_{\mathrm{SK}} \mathcal{D}[\varphi_1\varphi_2]\, e^{iI_{\mathrm{EFT}}[\varphi_1,\varphi_2;J_1,J_2]} = e^{W[J_1,J_2]}$$

$$\varphi_r \equiv \frac{1}{2}(\varphi_1+\varphi_2)\,, \qquad \varphi_a \equiv \varphi_1 - \varphi_2\,.$$

Classical field (VEV of quantum fields)

Thermal and quantum fluctuations (dissipation)



EFT for quasicrystals

$$\psi_s^A \to \psi_s^A + \lambda^A$$

$$A = 1, 2, 3, 4$$

Superspace fields

[MB,2008.05339]

$$X_r^\mu \equiv \frac{1}{2}(X_1^\mu + X_2^\mu), \qquad X_a^\mu \equiv X_1^\mu - X_2^\mu\,, \qquad \psi_r^A \equiv \frac{1}{2}(\psi_1^A + \psi_2^A), \qquad \psi_a^A = \psi_1^A - \psi_2^A\,.$$

$$\mathcal{L}_{\text{EFT}} = T^{\mu\nu}\partial_\mu X_{a\nu} + J^{A\mu}\partial_\mu\psi_a^A + \Gamma^A\psi_a^A + \frac{i}{2}M^{AB}\psi_a^A\psi_a^B$$

Quasicrystal part

$$J^{A\mu} = F^A(\beta, Y^{AB}, Z^A)\,u^\mu + H^{AB}(\beta, Y^{AB}, Z^A)\,\partial^\mu\psi_r^B$$

EFT for quasicrystals

$$H^{44}\,\partial_\mu\partial^\mu\psi_r^4 + \frac{\partial F^4}{\partial Z^4}\,\partial_0^2\psi_r^4 - M^{44}\,\partial_0\psi_r^4 = 0\,.$$

[MB,2008.05339]

$$\omega^2 + i\,\gamma\,\omega = v^2 k^2$$

$$\gamma \equiv \frac{M^{44}}{H^{44} - \partial F^4/\partial Z^4}\,, \qquad v^2 = \frac{H^{44}}{H^{44} - \partial F^4/\partial Z^4}\,.$$

Dissipative coefficient (non hermitian part of the action)

Phason elasticity

EFT for quasicrystals

$$\partial_\mu J^{4\mu} = -\Gamma^4 \sim M^{44}$$

[MB,2008.05339]

Only allowed at finite T (with dissipation)

PHASON SHIFT =
Symmetry with no associated conserved Noether current !

$$\langle [H, \mathcal{P}_4] \rangle \sim \left\langle H^\dagger \mathcal{P}_4 - \mathcal{P}_4 H \right\rangle \sim \frac{d\langle \mathcal{P}_4 \rangle}{dt} \sim M^{44}$$

Cf. DIFFUSIVE Goldstone modes in dissipative systems [Hidaka et Al.]

Light on the (no longer) mistery

Let us introduce explicit breaking

$$\mathcal{L}_{breaking} = \omega_0^2 \, X_r^\mu \, X_r^\mu + \omega_1^2 \, \psi_r^A \, \psi_r^A - \Omega_0 \dot{X}_r^\mu X_{a\mu} - \Omega_1 \dot{\psi}_r^A \psi_a^A + \dots$$

But let us retain diagonal symmetry (as in the holographic models)

$$\mathcal{L}_{breaking} = \omega_0^2 \, \left(X_r^\mu \, X_{a\mu} + \psi_r^A \, \psi_a^A \right) - \Omega_0 (\dot{X}_r^\mu X_{a\mu} + \dot{\psi}_r^A \psi_a^A) \dots$$

[MB,2008.05339]

PHONONS

$$\omega^2 + i \, \Omega_0 \, \omega = \omega_0^2 + V^2 \, k^2$$

1) damped

2) pinned

Light on the (no longer) mistery

PHASONS

$$\omega^2 + i\,\bar{\gamma}\,\omega = v^2 k^2 + \omega_0^2,$$

[MB,2008.05339]

$$\omega \equiv -i\,\Omega + \mathcal{O}(k^2) = -i\,\frac{\omega_0^2}{\bar{\gamma}} + \mathcal{O}(k^2),$$

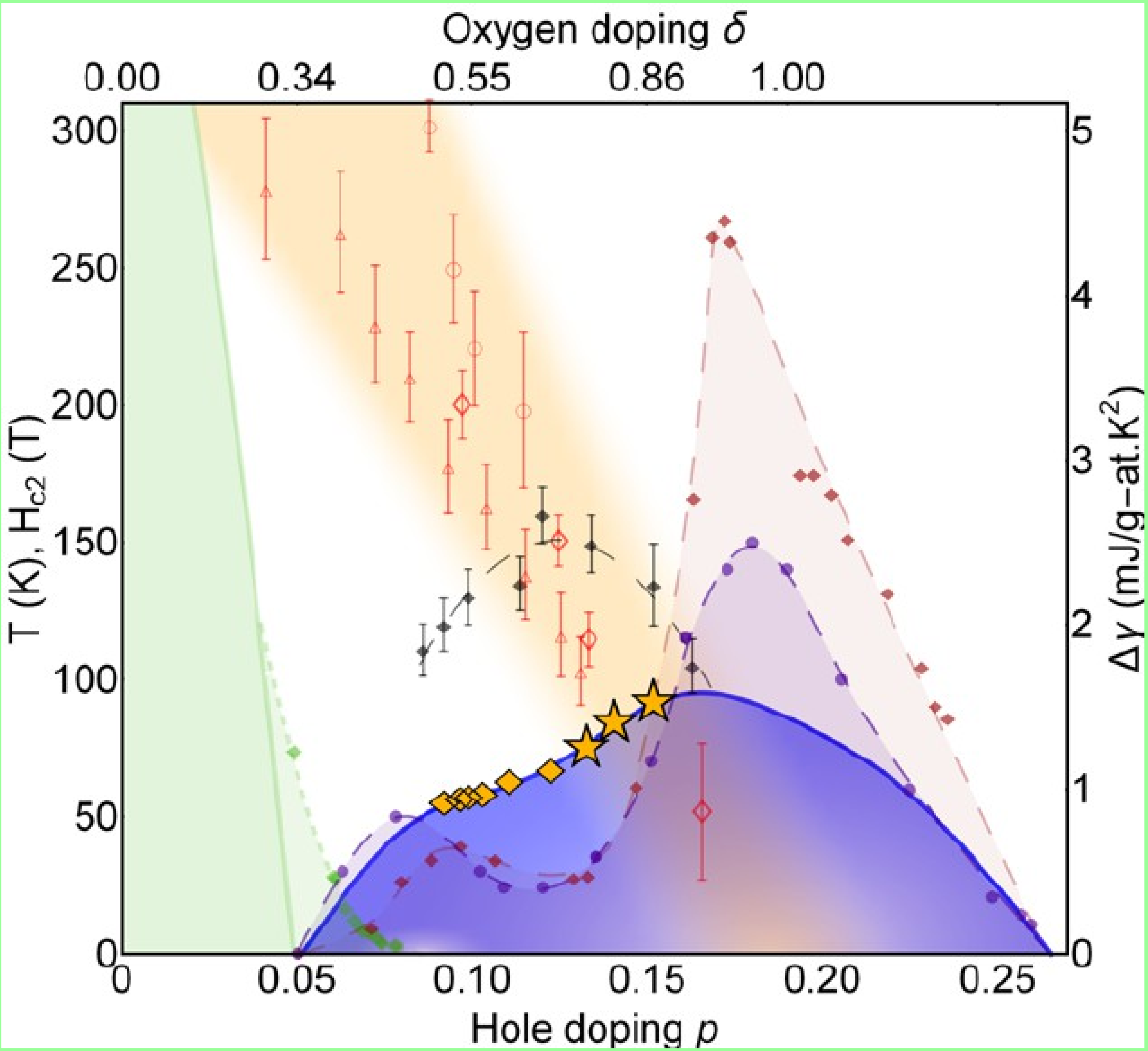
PHASE RELAXATION

$$\Omega = \frac{\omega_0^2\,D}{v^2} + \ldots,$$

$$\Omega = \chi_{\pi\pi}\,\omega_0^2\,\frac{D}{G} = \frac{\omega_0^2\,D}{v^2},$$

EXACTLY WHAT WE FIND IN HOLOGRAPHY !
IT IS SIMPLY THE SYMMETRY BREAKING PATTERN
(equivalent of GMOR relation for mass)

More advanced topics/questions



$$\omega = -i\Omega +$$

Role for phenomenology !

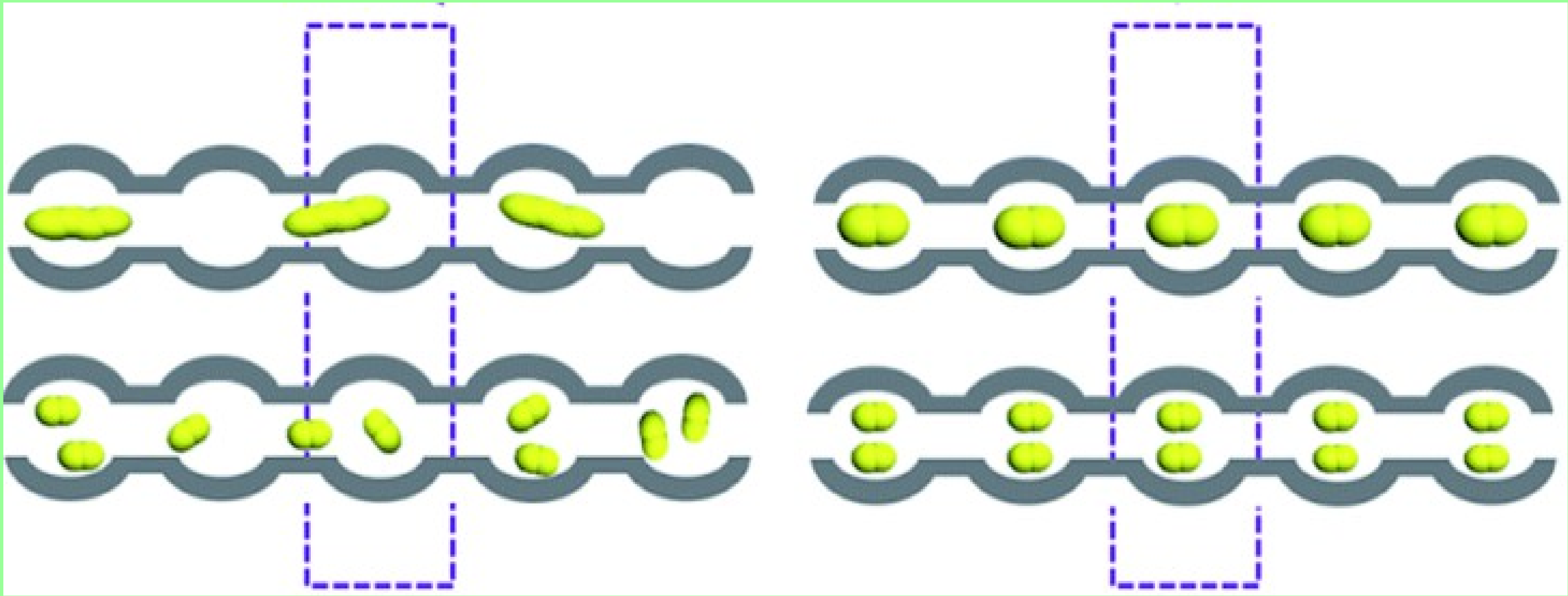
Linear in T?

High Tc ?

Optical conductivity ?

[Hartnoll et Al.]

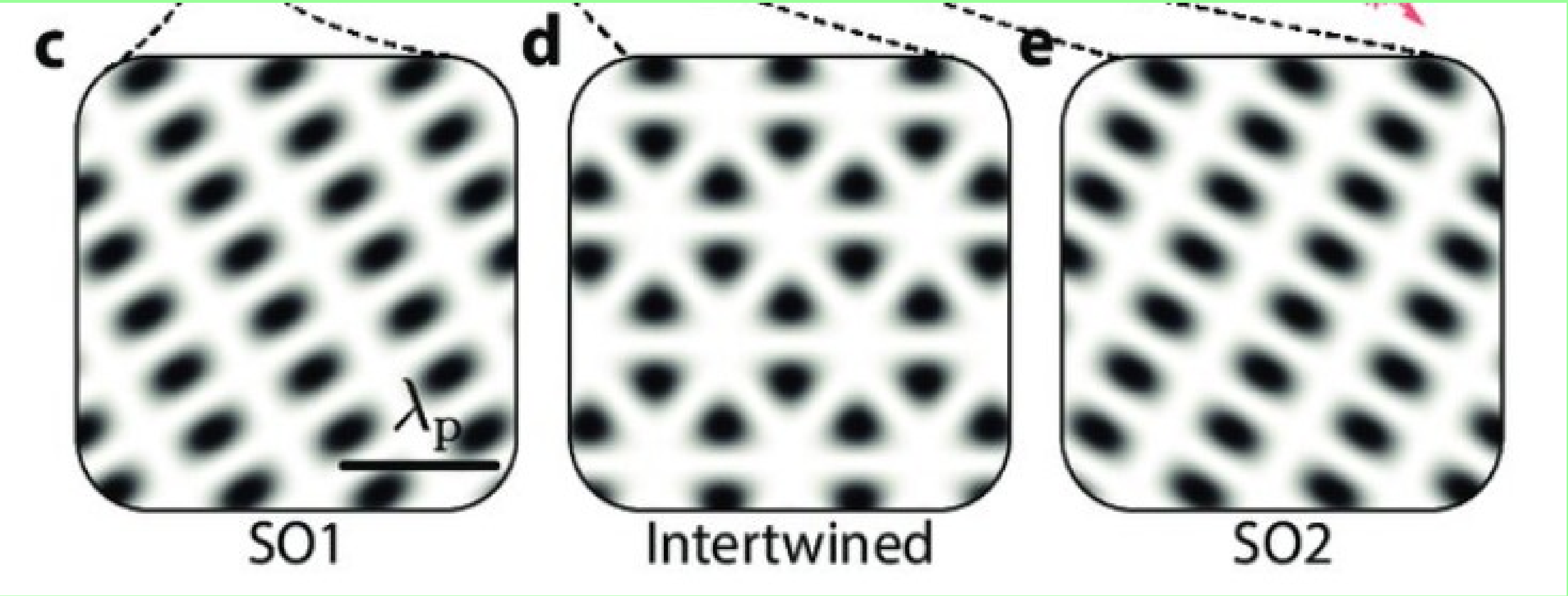
More advanced topics/questions



INCOMMENSURATE → **COMMENSURATE**

What happens to the phason ?
Properties of the phase transition ?
Holographic implementation

More advanced topics/questions



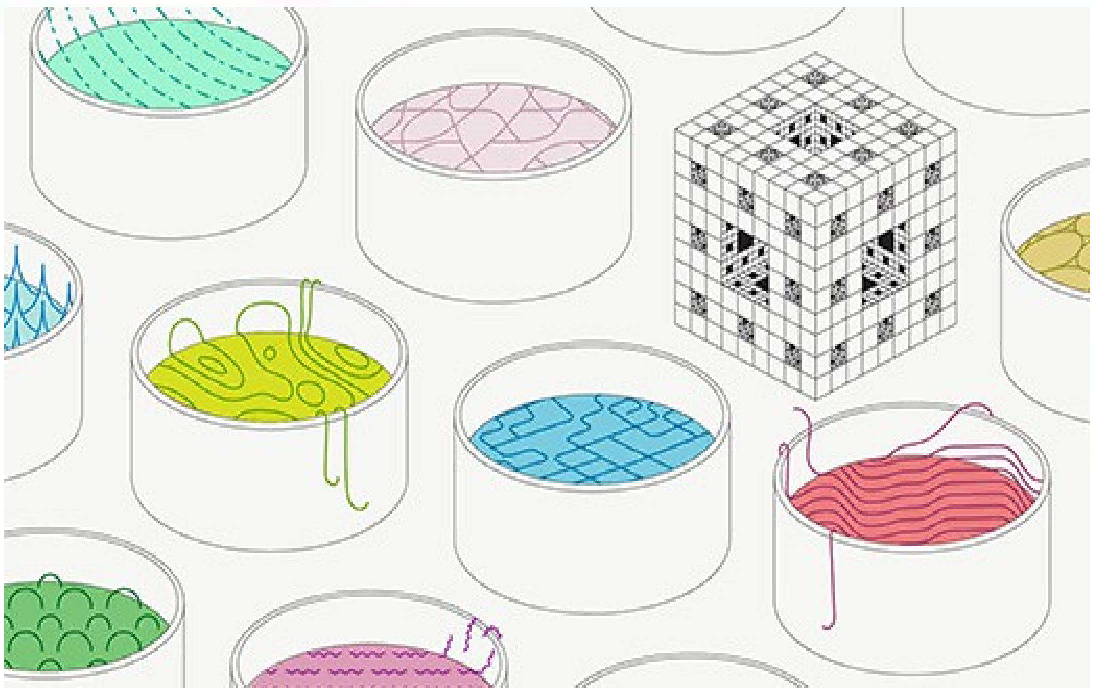
ORDER <-----------> SUPERCONDUCTIVITY

Other orders: nematicity, pair density waves, spin waves

[Holography + hydrodynamics + EFT- field theory]


GRAND FINALE!!!

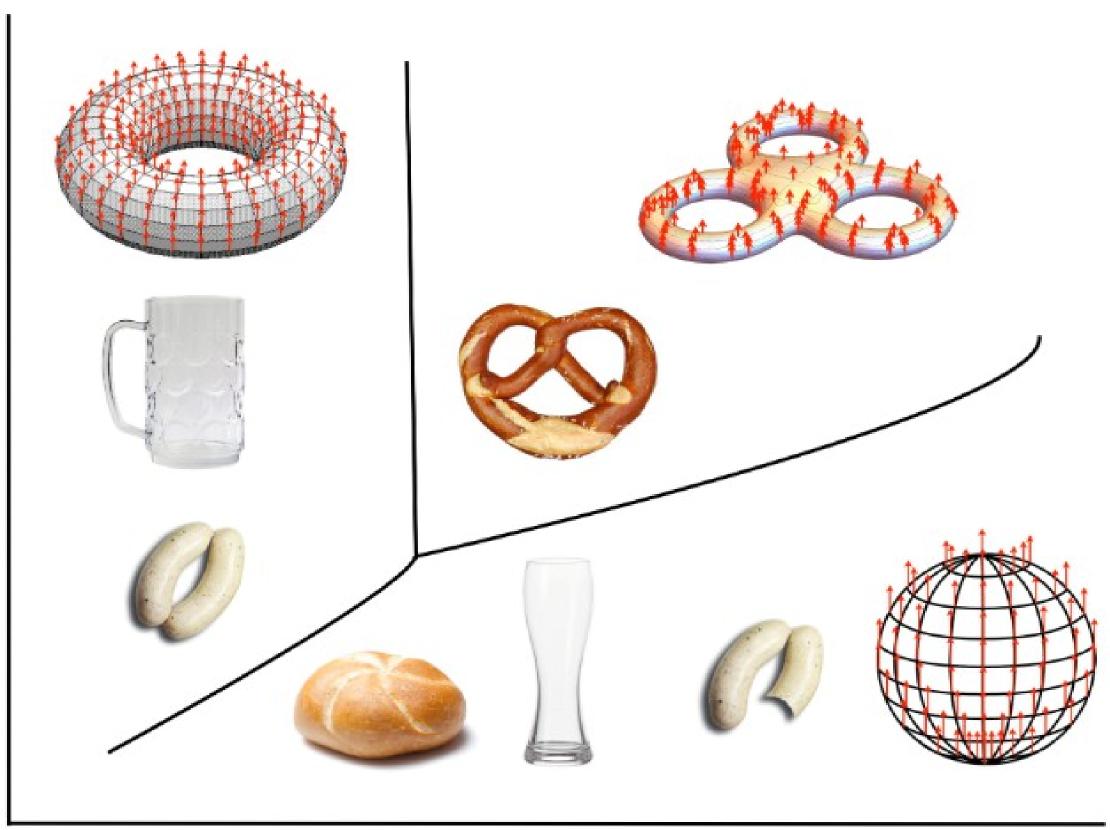
CONCLUSIONS



A lot of interesting and cool physics

CONCLUSiONS

Organizing principle: symmetries



Acknowledgments

and many more ...

ORGANIZERS

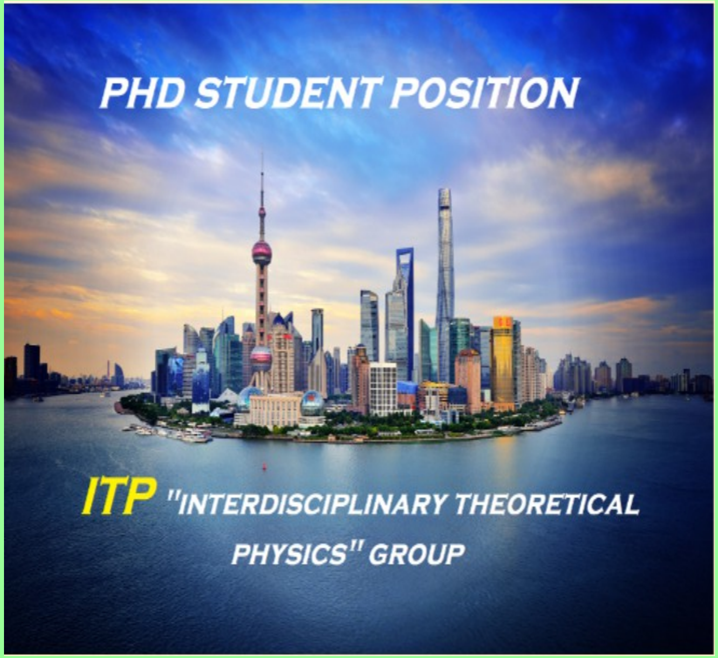
Chair Sang-Jin Sin (Hanyang U)
Co-chair Keun-Young Kim (GIST)
Koji Hashimoto (Osaka U)
Deog-Ki Hong (Pusan Nat'l U)
Ki-Seok Kim (POSTECH)
Nakwoo Kim (Kyung Hee U)
Ioannis Papadimitriou (KIAS)

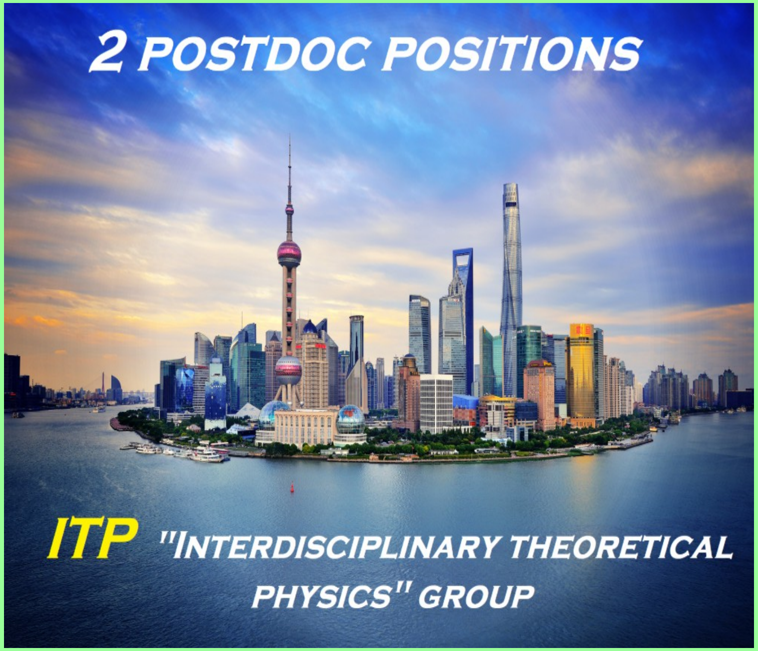
Thanks a lot

(looking forward to coming back to Korea)

Several positions available for next year !!

上海交通大學
SHANGHAI JIAO TONG UNIVERSITY





If you are interested (or you know some good candidate) please contact me at : mbaggioli@ifae.es (and share the info)

Quantum Matter and Quantum Information with Holography
August 23 (Sun), 2020 ~ August 31 (Mon), 2020

APCTP
Asia Pacific Center for Theoretical Physics

감사합니다

For any questions: mbaggioli@ifae.es


Discussion


SPEAKUP