Quantum Matter and Quantum Information with Holography
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Constraining Non-Relativistic RG Flows with Holography

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Today's talk

- Work with **Li Li, Kyle Ritchie and Yuezhang Tang → arXiv:2006.10780**
- We probe non-relativistic RG flows using holography
- Goal: identify generic properties and quantities that flow monotonically under RG

Li Li
CAS, Beijing

Kyle Ritchie
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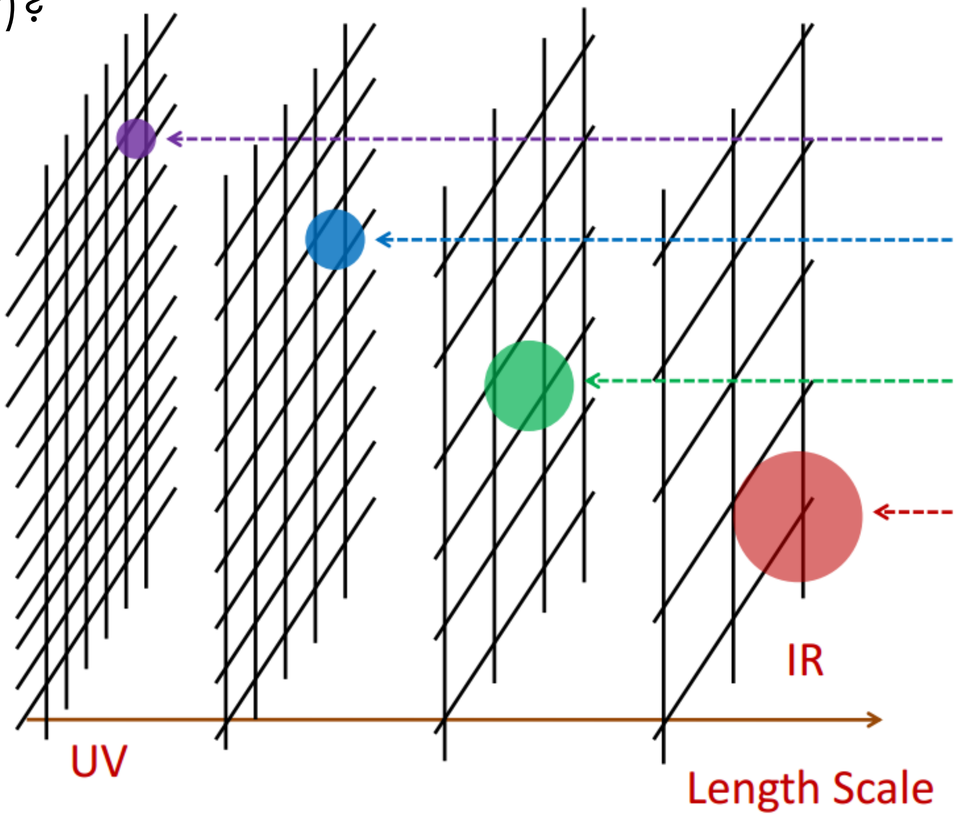
→ Berkeley for PhD

Yuezhang (John) Tang
Lehigh U.

Basic questions

- How do we **track the number of degrees of freedom (d.o.f.) in a QFT at different scales**?
- We expect to lose d.o.f. as the energy scale is lowered. How do we capture this loss in a <u>generic</u> QM system (without a lot of symmetry)?
- How does gravity encode the process of integrating out d.o.f. ?



Basic questions

- How do we **track the number of degrees of freedom (d.o.f.) in a QFT at different scales**?
- We expect to lose d.o.f. as the energy scale is lowered. How do we capture this loss in a generic QM system (without a lot of symmetry)?

- In (certain) relativistic QFTs, **c-theorems provide a measure for the # of d.o.f. → c-function decreases monotonically** from UV to IR, reproducing central charge at fixed points [Zamolodchikov, Casini/Huerta, Cardy, Komargodski/Schwimmer…] (also F-theorem and F-functions)
- Generalized c-theorems from holography, valid in any # of dimensions [Freedman/Gubser/Pilch/Warner, Girardello/Petrini/Porrati/Zaffaroni, Myers/Sinha, Myers/Singh,…]
- Such theorems **rely on Lorentz invariance** and otherwise break down → **can they be extended to non-relativistic flows (at least under some set of restrictions)?**

Can we identify any generic features?

- With X. Dong (arXiv:1311.3307): we proposed **a generalized c-function from the EE of a strip** [building on Myers/Singh] for non-relativistic systems → **generically nonmonotonic, but we identified possible constraints** that make it monotonic. Can we say more?

- **Radial Hamiltonian formalism/Hamilton-Jacobi approach** → plays key role for understanding RG flow [de Boer, Verlinde^2, Papadimitriou, Skenderis,…].

- The **fake superpotential particularly useful to characterize RG flows** (for relativistic case it's essentially the holographic c-function [Freedman et al]) and classify GR solutions.

Our plan:
adopt the superpotential formalism to study non-relativistic solutions to EMD theories and ask what we can learn about possible monotonic behaviors

Holographic flows and a c-function

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$$ds^2 = dr^2 + e^{2A(r)}(-dt^2 + d\vec{x}^2)\,, \quad \phi = \phi(r)$$

Einstein's equations

$$\dot\phi^2 + 2(d-1)\ddot{A} = 0$$

$$d(d-1)\dot{A}^2 - \frac{1}{2}\dot\phi^2 + V = 0$$

Define a fake superpotential W

$$W(\phi(r)) = -2(d-1)\dot{A}$$

To satisfy Einstein's EOM we need $\dot\phi = W_\phi$

$$\frac{1}{2}W_\phi^2 - \frac{d}{4(d-1)}W^2 = V(\Phi)$$

$$E = E_0 e^A \Rightarrow \beta(\phi) = \frac{d\phi}{d\log E} = -2(d-1)\frac{W_\phi}{W}$$

See Kiritsis, Nitti, Pimenta arXiv:1611.05493 for extensive RG flow classification

Holographic flows and a c-function

The superpotential W is monotonic and is essentially the c-function.
Easy to see here:

$$\dot{W} = W_\phi \, \dot{\phi} = \dot{\phi}^2 \geq 0$$

$$\dot{A} = -\frac{1}{2(d-1)} W$$

$$\dot{\phi} = W_\phi$$

Famous result by Freedman et al., hep-th/9904017

$$a(r) \equiv \frac{\pi^{d/2}}{\Gamma\left(d/2\right)\left(\ell_{\rm P} A'(r)\right)^{d-1}}$$

$$\text{NEC} \quad \Rightarrow \quad -2\ddot{A} \geq 0 \qquad \text{or equivalently} \quad \dot{W} \geq 0$$

- Null energy condition (NEC) ensures that the c-function is monotonic in a relativistic system (see e.g. Freedman et al.)
- Also visible from entanglement (Myers and Singh arXiv:1202.2068, EE of strip geometry)

Our Setup [arXiv:2006.10780, SC, L.Li,K. Ritchie and Y. Tang]

We want to extend this story to Einstein-Maxwell-scalar theories:

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

$$ds^2 = dr^2 + e^{2A(r)}(-f(r)dt^2 + d\vec{x}^2)\,, \quad \phi = \phi(r), \quad A_\mu dx^\mu = A_t(r)dt$$

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Characterizes deviation from Lorentz invariance
→ Dual flow is non-relativistic

We restrict ourselves to geometries that approach AdS at the boundary $f(r) \to 1$ as $r \to \infty$
→ UV is relativistic

Superpotential Formalism

We follow Lindgren, Papadimitriou, Taliotis, Vanhoof [JHEP1507 (094) 2015, arXiv:1505.04131]

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

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We introduce **a (fake) superpotential W** → now more degrees of freedom

$$\dot{A} = -\frac{1}{2(d-1)}W$$

$$\frac{\dot{f}}{f} = -\frac{1}{d-1}W_A$$

$$\dot{\phi} = W_\phi$$

$W_A = \partial_A W$

$$\frac{1}{2}W_\phi^2 - \frac{1}{4(d-1)}(d+\partial_A)W^2 = V_{eff}(\phi)$$

$$V_{eff}(\phi, A) = V(\phi) + 2Z^{-1}e^{-2(d-1)A}(\kappa^2\rho)^2$$

(Note: different formalism in Kiritsis, Niarchos, arXiv:1205.6205)

A few things to note

Recall definition of W:

$$\dot{A} = -\frac{1}{2(d-1)} W$$

$$\frac{\dot{f}}{f} = -\frac{1}{d-1} W_A$$

$$\dot{\phi} = W_\phi$$

W and W_A → **two effective degrees of freedom**

W_A encodes non-relativistic effects
(W_A = 0 for Lorentz invariant case)

$$E = E_0 \sqrt{f} e^A \Longrightarrow \beta(\phi) \equiv \frac{d\phi}{d \log E} = -2(d-1) \frac{W_\phi}{W + W_A}$$

$$ds^2 = dr^2 + e^{2A(r)}(-f(r)dt^2 + d\vec{x}^2)\,, \quad \phi = \phi(r), \quad A_\mu dx^\mu = A_t(r)dt$$

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Note that f_{UV} describes the effective speed of light $c^2{}_{\mathrm{UV}}$ in the UV theory (similarly f_{IR} gives the one in the IR)

Index of refraction in holography

The **relative speed of propagation of light-like signals in the UV and IR** can be described using an "index of refraction"   (see Gubser, Pufu, Rocha arXiv:0908.0011)

$$n = \sqrt{\frac{f_{UV}}{f_{IR}}}$$

quantifies the renormalization of scales from the UV to the IR

Also studied more recently by Donos et al. (1705.03000, 1712.08017), Hoyos et al. (2001.08218)

We examine the **radial flow** of $\boxed{n(r) = \sqrt{f}}$ → $\dot{n} = \frac{1}{2}\frac{\dot{f}}{\sqrt{f}} = \kappa^2 e^{-dA}(T\,s + A_t\,\rho)$

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Both terms are non-negative:

$$\dot{A}_t = 2\kappa^2 e^{(2-d)A}\frac{\sqrt{f}}{Z}\rho \geqslant 0$$

$$A_t(r_h) = 0 \;\text{ and }\; \dot{A}_t \geqslant 0 \quad \Rightarrow \quad A_t(r) > 0$$

(without loss of generality take $\rho > 0$)

→ $\dot{n} \geq 0$

n(r) is monotonic along RG flow in EMD theories

Superpotentials and monotonicity conditions

The monotonicity of n(r) is tied to W_A having a definite sign

$$n(r) = \sqrt{f}$$

$$\dot{n} = \frac{1}{2}\frac{\dot{f}}{\sqrt{f}} = -\frac{\sqrt{f}}{2(d-1)} W_A$$

$$\dot{f} \geq 0 \quad \text{or equivalently} \quad W_A \leq 0$$

Also easy to show that warp factor A(r) is monotonic, increasing towards the UV:

$$\dot{A} \geq 0 \quad \Rightarrow \quad W \leq 0$$

Radial flow of W:

$$\frac{dW}{dr} = \dot{\phi}^2 - \frac{W W_A}{2(d-1)}$$

→ spoils monotonicity ($W W_A \geq 0$)

NEC doesn't help →

$$\dot{W} + \frac{1}{2(d-1)} W W_A \geq 0$$

$$d\, W W_A - 2(d-1)\dot{W}_A + W_A^2 \geq 0$$

Same behavior visible from entanglement

Superpotentials and monotonicity conditions

Radial flow of W_A:

Competition between superpotential term and gauge field contribution:

$$\frac{dW_A}{dr} = \frac{d}{2(d-1)} W_A \left(W + \frac{W_A}{d} \right) - (d-1) Z(\phi) e^{-2A} \frac{\dot{A}_t^2}{f}$$

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always ≥ 0

Always opposite signs

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Focus on simpler Einstein-scalar theories

$$\frac{dW_A}{dr} = \frac{d}{2(d-1)} W_A \left(W + \frac{W_A}{d} \right) \geq 0$$

W_A is **always monotonic in Einstein-scalar theories**
BUT it becomes trivial in relativistic limit (W_A=0) → can we do better?

Superpotentials and monotonicity conditions

Einstein-scalar theory (turn off gauge field)

Interesting combination of W and W_A:

$$\frac{d}{dr}\left(W + \frac{1}{d}W_A\right) = \frac{1}{2d(d-1)}W_A^2 + \dot{\phi}^2 \geq 0$$

monotonic and reduces to relativistic result

Even with non-relativistic geometries, there is (at least) one function that behaves as a c-function (increases monotonically towards UV, reduces to known Lorentz invariant result)

Superpotentials and monotonicity conditions

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$$\frac{d}{dr}\left(W + \frac{1}{d}W_A\right) = \frac{1}{2d(d-1)}W_A^2 + \dot{\phi}^2 \geq 0$$

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Einstein-Maxwell-scalar theory

$$\frac{d}{dr}\left(W + \frac{1}{d}W_A\right) = \frac{1}{2\,d(d-1)}W_A^2 + \dot{\phi}^2 - \frac{(d-1)}{d}Z(\phi)\,e^{-2A}\,\frac{\dot{A}_t^2}{f}$$

Spoils monotonicity in general

Surprisingly, still monotonic for many known BH solutions (why?)

Explicit examples of radial flow ($W + W_A/d$)

Maximal gauged SUGRA in 4D, charged black holes in AdS_4 [Cvetic et al, hep-th/9903214]

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

	one-charge	two-charge	three-charge
$V(\phi)$	$-6\cosh(\phi/\sqrt{3})$	$-2(\cosh\phi + 2)$	$-6\cosh(\phi/\sqrt{3})$
$Z(\phi)$	$e^{\sqrt{3}\phi}$	e^{ϕ}	$e^{\phi/\sqrt{3}}$
e^{2A}	$\bar{r}^{3/2}(\bar{r}+Q)^{1/2}$	$\bar{r}(\bar{r}+Q)$	$\bar{r}^{1/2}(\bar{r}+Q)^{3/2}$
f	$1 - \frac{\bar{r}_h^2(\bar{r}_h+Q)}{\bar{r}^2(\bar{r}+Q)}$	$1 - \frac{\bar{r}_h(\bar{r}_h+Q)^2}{\bar{r}(\bar{r}+Q)^2}$	$1 - \frac{(\bar{r}_h+Q)^3}{(\bar{r}+Q)^3}$
A_t	$\frac{\sqrt{Q}\bar{r}_h}{\sqrt{\bar{r}_h+Q}}\left(1 - \frac{\bar{r}_h+Q}{\bar{r}+Q}\right)$	$\sqrt{2Q\bar{r}_h}\left(1 - \frac{\bar{r}_h+Q}{\bar{r}+Q}\right)$	$\sqrt{3Q(\bar{r}_h+Q)}\left(1 - \frac{\bar{r}_h+Q}{\bar{r}+Q}\right)$
ϕ	$\frac{\sqrt{3}}{2}\ln\left(1+\frac{Q}{\bar{r}}\right)$	$\ln\left(1+\frac{Q}{\bar{r}}\right)$	$\frac{\sqrt{3}}{2}\ln\left(1+\frac{Q}{\bar{r}}\right)$

($d = 3$)

Explicit examples of radial flow (W +W_A/d)

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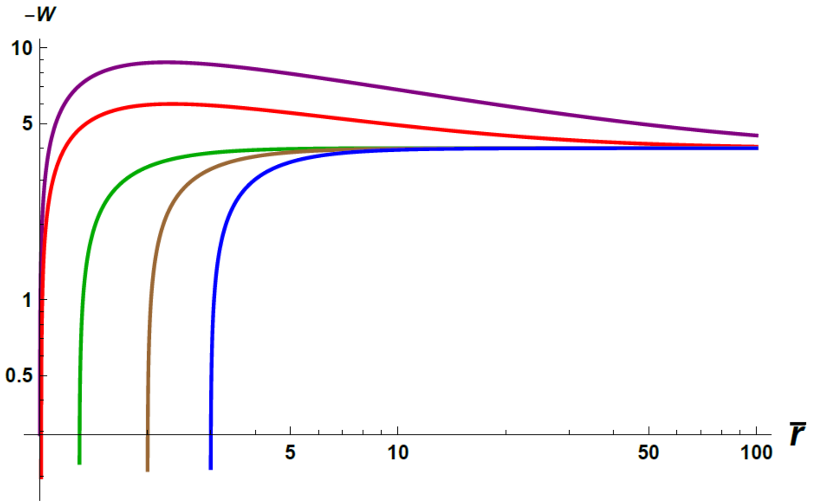
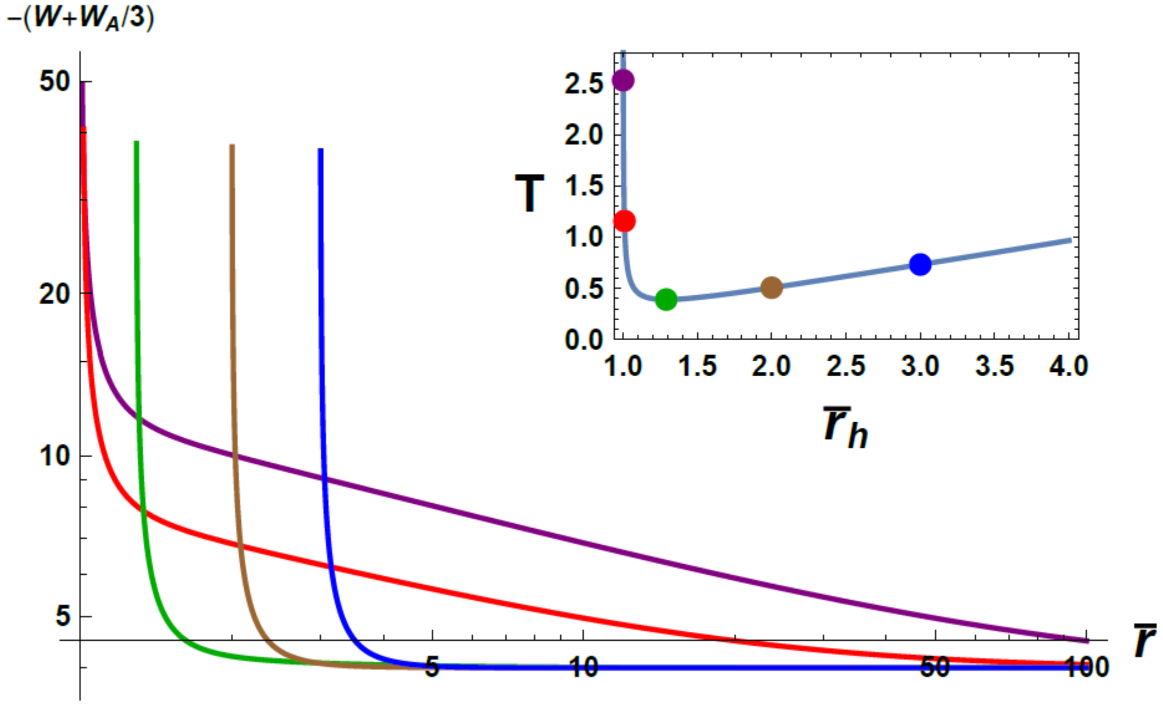
1-charge BH





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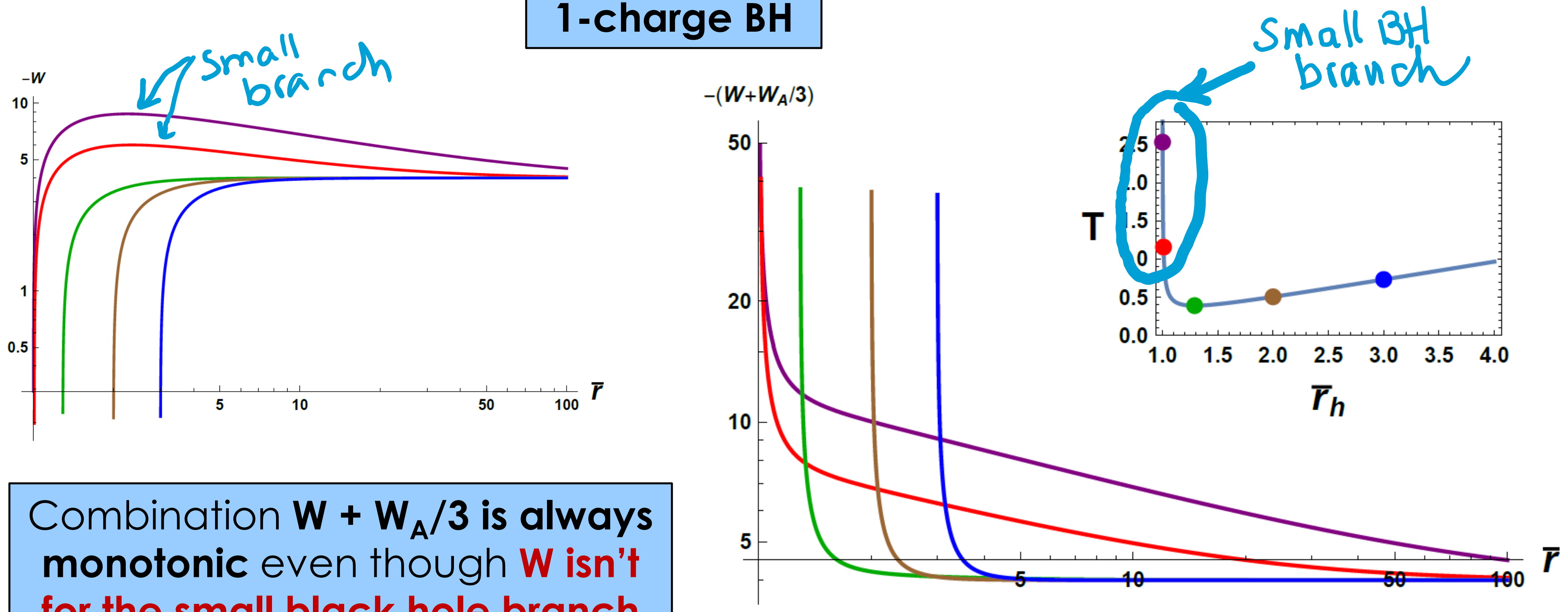

Combination **$W + W_A/3$ is always monotonic** even though **W isn't for the small black hole branch**

Explicit examples of radial flow ($W + W_A/d$)

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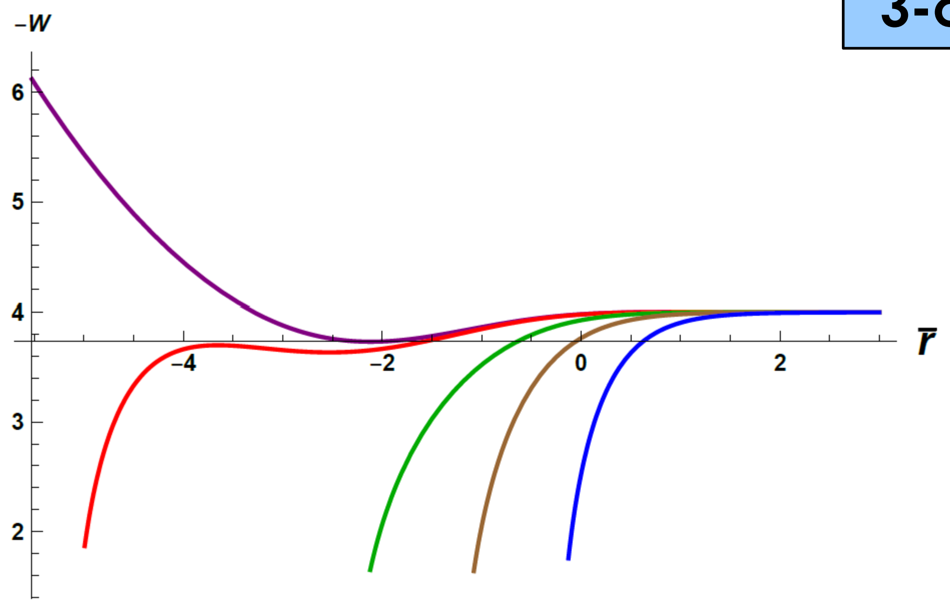
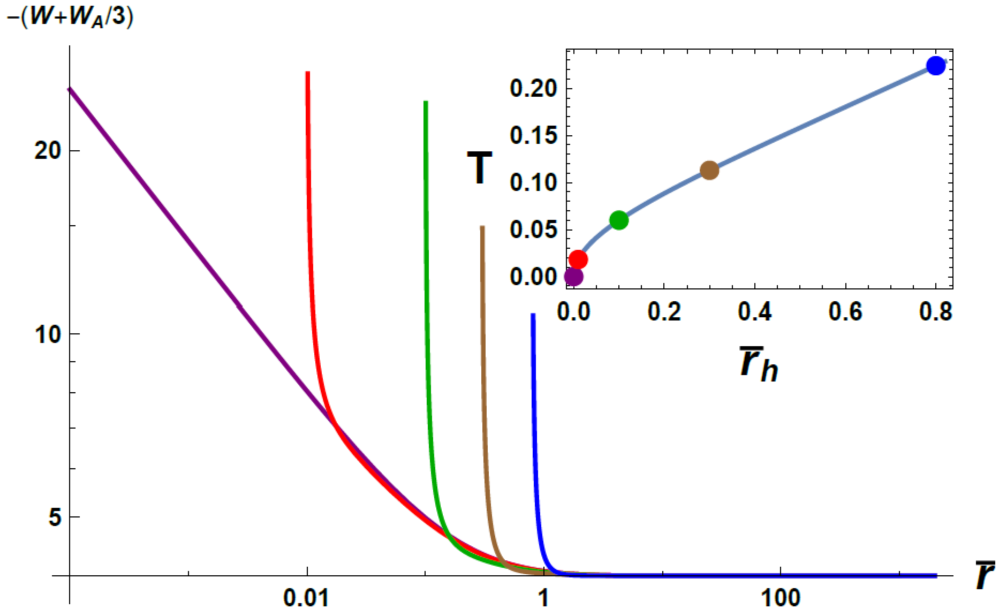
3-charge BH





Here W is not monotonic at small T or small r_h (only one branch).
$W + W_A/3$ is always monotonic

Examples in 5D (STU model)

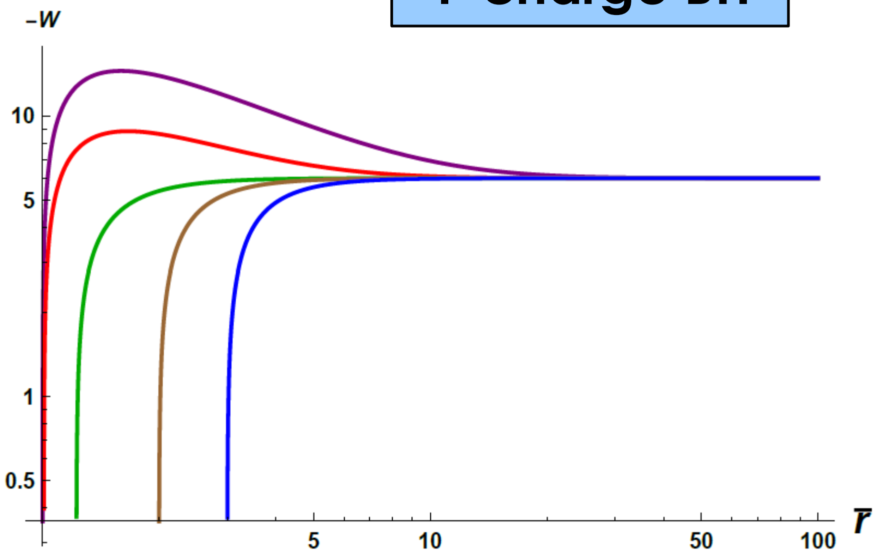
5D BH solutions to STU Model in maximal gauged SUGRA (two equal charges)
[DeWolfe,Gubser,Rosen 1207.3352]

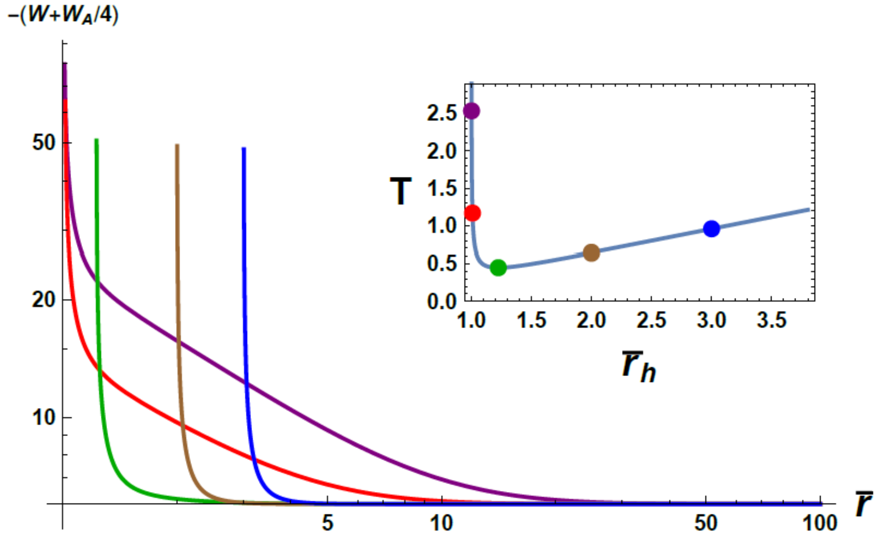
$$V(\phi) = -\frac{8}{L^2}e^{\phi/\sqrt{6}} - \frac{4}{L^2}e^{-2\phi/\sqrt{6}}$$

$$Z(\phi) = e^{-4\phi/\sqrt{6}} \quad \text{one-charge case}, \qquad Z(\phi) = e^{2\phi/\sqrt{6}} \quad \text{two-charge case}$$

$W + \frac{W_A}{4}$ monotonic for all cases (W isn't for small BH branch)

1-charge BH





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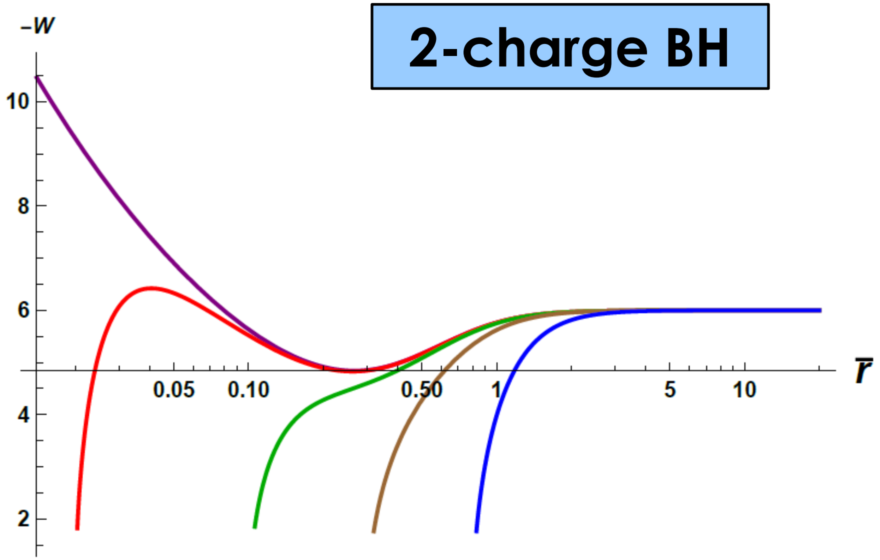
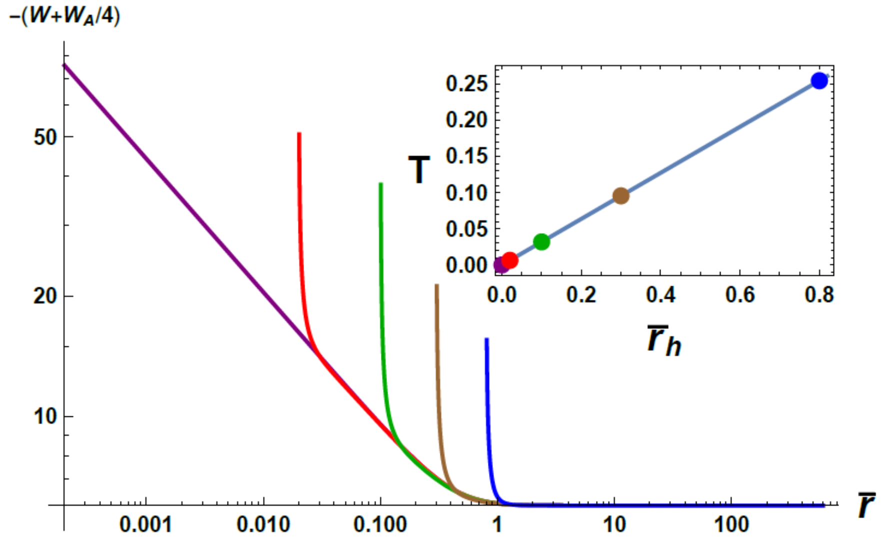
$W + \frac{W_A}{4}$ monotonic for all cases
(W only at high T)

2-charge BH





To conclude

We have identified **a few generic features and several quantities that are monotonic under RG flow** (stronger constraints for Einstein-scalar theories)

- What is their physical interpretation and fundamental origin? Can we interpret these results using properties of entanglement?
- Comparison with other approaches/first order formalisms?
- Can we extend this analysis to geometries that break more symmetries?
- Is our combination of superpotentials monotonic for boomerang RG flows?
- …

Many open questions, but it's a good sign that even without Lorentz invariance, some of the intuition of the relativistic case is present and results can be generalized.
A sign of a deeper structure?

Thank you