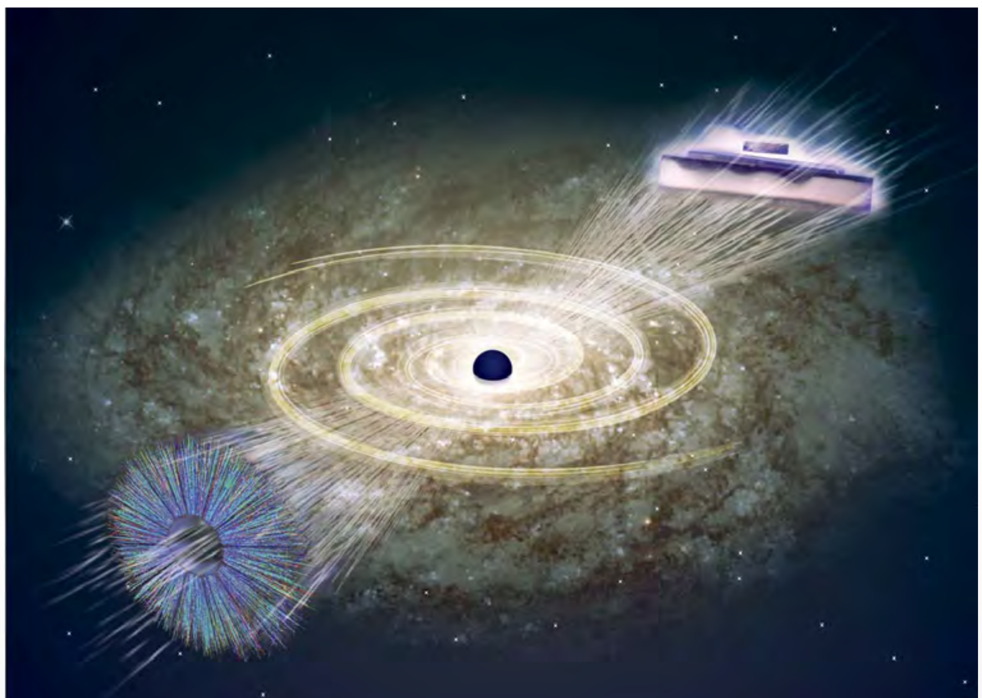
Magnetophonon and Magnetotransport in Holography

Li Li (李理)

Institute of Theoretical Physics
Chinese Academy of Sciences



arXiv.org > hep-th > arXiv:2005.01725

High Energy Physics – Theory

[Submitted on 4 May 2020]

Magnetophonons & type-B Goldstones from Hydrodynamics to Holography

Matteo Baggioli, Sebastian Grieninger, Li Li

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High Energy Physics – Theory

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Magnetotransport and Complexity of Holographic Metal-Insulator Transitions

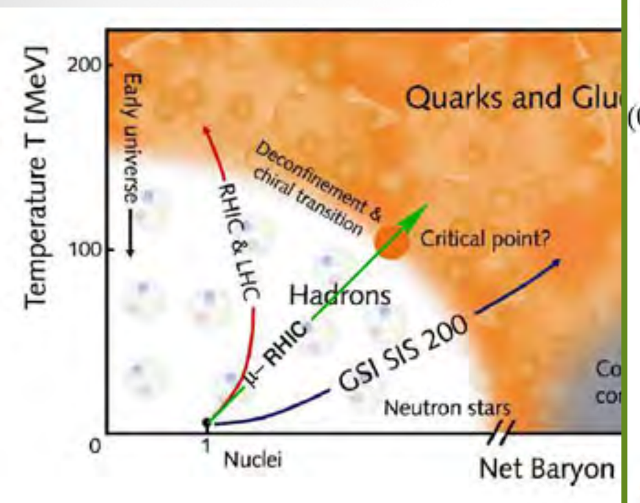
Yu-Sen An, Teng Ji, Li Li

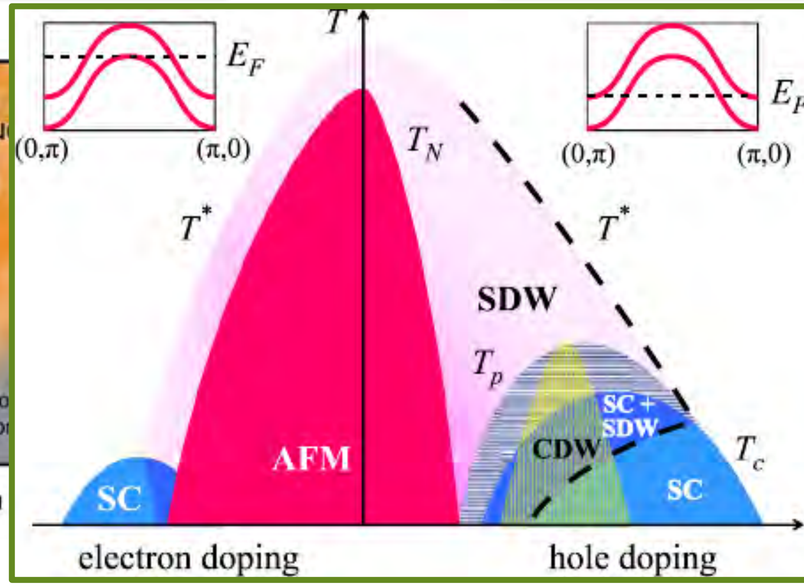
In collaboration with **Matteo Baggioli** (Marid, IFT->TDLI.)
Sebastian Grieninger (Jena U.->IFT)
Yu-Sen An (ITP-CAS)
Teng Ji (ITP-CAS)

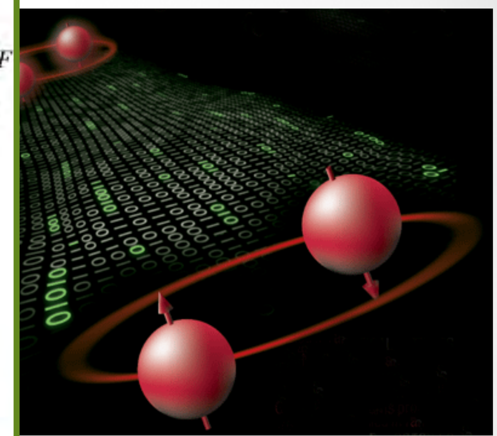
Holography as a Theoretical Laboratory

Quantum field theory
at strong coupling

Theory of gravitation
at weak coupling







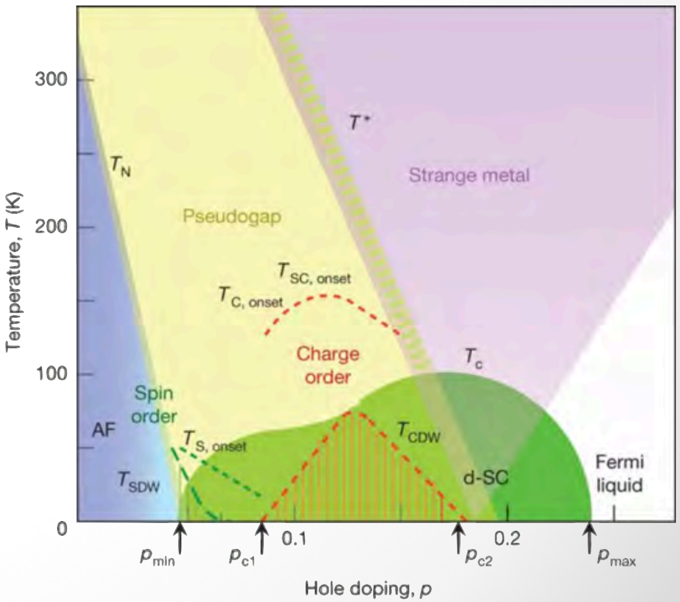
Applied holography:

QGP and QCD (drag force, jet quenching, confinement/deconfinement,…),
Condensed matter (quantum criticality, strange metal, superconductivity,…),
Quantum Entanglement, Non-equilibrium dynamics…

(See talks by Sang-Jin Sin, Yi Ling and Matteo Baggioli today)

Challenges for Strongly Coupled Quantum Phases of Matter

- Breakdown of Fermi-liquid theory, quantum matter without quasiparticles
- An intrinsically complex phase diagram exhibiting a variety of orders
- Segmented Fermi surfaces (`Fermi arcs')
- Anomalous transport
- Planckian dissipation
- Long-range entanglement
- …



Keimer et al, Nature (2015)

Strategy:
Study **solvable models that may be in the same universality class** as strongly correlated phases of interest

Goal:
Draw qualitative and quantitative lessons – universal features?
Shed light on **basic mechanisms** underlying the dynamics

Solvable often implies working with overly **simplified bottom-up toy models**

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Guiding principle :

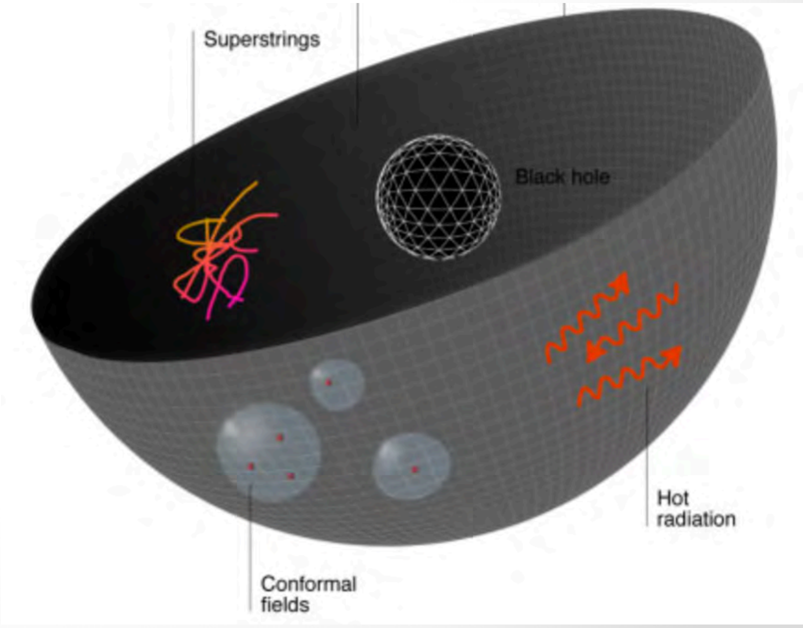
Symmetry, **Causality**, **Instability**

Outline

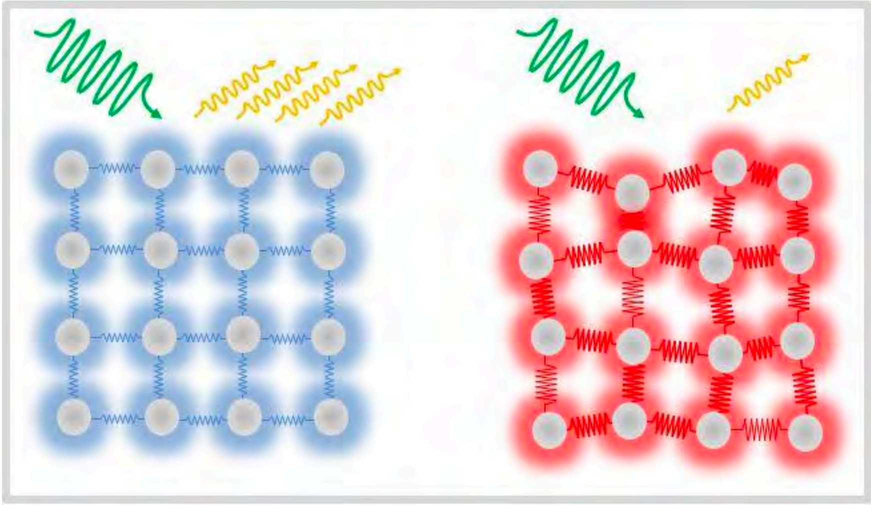
➢Magnetophonons and type-B Goldstones

➢Magnetotransport and Metal-Insulator Transition

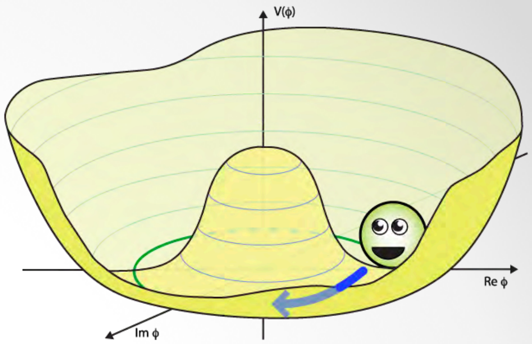
➢Summary and Discussion



Magnetophonons and type-B Goldstones from Hydrodynamics to Holography



Goldstone theorem:



The breaking of an internal continuous global symmetry guarantees the existence of gapless modes;
The number of modes coincides with the number of broken symmetry generators;
The dispersion relation is linear w(k)~k.

With the assumption:

Poincare Invariance
Internal continuous global symmetries
Non dissipative systems

The number of Goldstone modes is given by the **dimension of the coset space** G/H (G : the broken group, H: the preserved one)

$$n_{GB} = \dim G/H = \dim G - \dim H$$

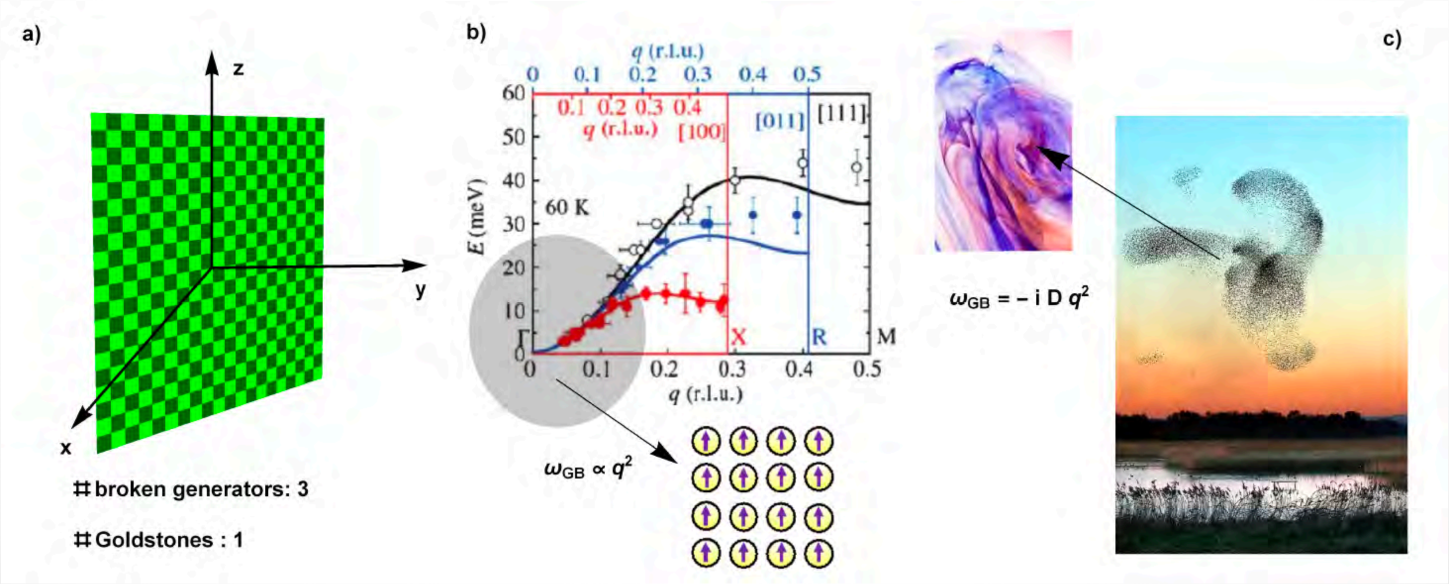
counting of the "**flat directions**" of fluctuations of the order parameters.

There are many systems which do not satisfy the above requirements. This leads to very interesting phenomena:

a)The number of Goldstone modes appearing is less than that of broken generators.

b)The dispersion relation of the Goldstone modes is not linear.

c)The Goldstone modes are not propagating but rather diffusive.



a) A **reduced number of Goldstones** is due to the fact that the **broken generators** Qs commute with the Hamiltonian H, but not with one another.

$$[H, Q_\alpha] = 0, \quad \left[H, Q_\beta\right] = 0, \quad \text{but} \quad \left[Q_\alpha, Q_\beta\right] \neq 0$$

thus the symmetries cannot be thought of as independent.

EXAMPLE Simultaneous breaking of spacetime rotations and translations.

$$[J_m\,, P_n] = i\,\epsilon_{mnk}\,P_k$$

Poincare algebra

Do not observe any Goldstone mode for rotations in a crystal!

b) Goldstone bosons have a **nonlinear dispersion relation**, which is typical of non-relativistic systems

$$\omega(k) \sim k^n \, , \quad \text{with} \quad n \neq 1$$

The fundamental point in this discussion is that the effective low energy description can be written [1402.7066]

$$\mathcal{L} = \frac{1}{2}\rho_{ab}\,\partial_t\pi^a\pi^b + \frac{1}{2}\bar{g}_{ab}\,\partial_t\pi^a\partial_t\pi^b - \frac{1}{2}g_{ab}\,\nabla\pi^a\cdot\nabla\pi^b + \ldots$$

π: Goldstone fluctuations

ρ_{ab}: anti-symmetric matrix $\quad \rho_{ab} = -i\,[Q_a, Q_b]$ **Watanabe-Brauner matrix**

The rank of ρ_{ab} determines the number of the different types of Goldstone bosons

$$\omega(k) = k^{2n-1}\,, \qquad \text{TYPE A}$$

$$\omega(k) = k^{2n}\,, \qquad \text{TYPE B}$$

$$n = n_{GB} - \frac{1}{2}\,\text{rank}\,\rho\,, \quad n_B = \frac{1}{2}\,\text{rank}\,\rho\,, \quad n_A = n_{GB} - \text{rank}\,\rho$$

Ferromagnet: break SO(3) -> SO(2) ~~two standard linear Goldstones~~

Only one Goldstone mode –the magnon – which is quadratic !

c) Diffusive Goldstones: in **dissipative systems** (e.g. open systems) diffusive Goldstone modes can appear

$$\omega = -i D k^2$$

which has been observed in several physical systems[Toner, Tu, PRL, 1995] and holographic models [1812.08118,1905.00398,2001.05737].
More **exotic subdiffusive** modes (e.g. $\omega = -iDk^4$) can arise within the hydrodynamic theory of **fractons**[2003.09429].

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We aim at studying the dynamics of **magnetophonon** resonances – **Goldstone bosons** which appear in systems with **spontaneously broken translations** at **finite magnetic field**.

Study the magnetophonons using holography

Motivation:

These modes are interesting per se because they are another example of **type B Goldstone modes** with dispersion relation $\omega \sim k^2$.
The hybridization between the **two linearly propagating Goldstone bosons** – the longitudinal and transverse phonons

$$\omega_{\|,\perp} = v_{\|,\perp}\, k$$

to a **single quadratic mode** – the magnetophonon.

From a condensed matter perspective, the physics of magnetophonon resonances is particularly appealing in the presence of **small explicit breaking of translations**. The dynamics of the magnetophonon peak as a function of the magnetic field can reveal the **fundamental nature of the "disorder"** responsible for its pinning.

Fundamental aspects of magnetophonon resonances

At finite magnetic field $$[P_i, P_j] = -i\,\epsilon_{ij}\,B\,Q$$

electric charge operator

At the level of the **effective action** for the Goldstone fluctuations π, the presence of B allows the appearance of a new term

$$\mathcal{L} = \epsilon^{ij}\,\pi_i\,\partial_t\pi_j + \ldots$$

Watanabe-Brauner matrix ρ_{ab} is non-trivial and has **rank(ρ)=2** in two dimensions. Using the counting rule, we have

$$n_A = 0\,, \quad n_B = 1$$

magnetophonon

The **two linear propagating sound modes** – Goldstones of translations – combine into a type-B mode, the **magnetophonon**, and a **gapped mode** sometimes referred to as the **magnetoplasmon**.

Fundamental aspects of magnetophonon resonances

Hydrodynamic description:

At **finite magnetic field**, the transverse and longitudinal phonons couple together, in a way that the resulting frequencies become

$$\omega_{\pm}^2 = \frac{1}{2}\left(\omega_c^2 + \omega_{\parallel}^2 + \omega_{\perp}^2\right) \pm \frac{1}{2}\sqrt{\left(\omega_c^2 + \omega_{\parallel}^2 + \omega_{\perp}^2\right)^2 - 4\,\omega_{\perp}^2\,\omega_{\parallel}^2}$$

$\omega_c(\sim B)$: the cyclotron frequency
$\omega_{\parallel,\perp}$: the frequencies of the longitudinal (∥) and transverse (⊥) phonons.

$$\omega_{\perp,\parallel} = v_{\perp,\parallel}\,k$$

The **quadratic behavior** at small momentum

$$\mathrm{Re}\,[\omega_+] = \omega_c + \frac{(v_{\parallel}^2 + v_{\perp}^2)}{2\,\omega_c}k^2 + \ldots, \qquad \mathrm{Re}\,[\omega_-] = \frac{v_{\perp}\,v_{\parallel}}{\omega_c}k^2 + \ldots$$

gapped **magnetoplasmon**

massless type-B **magnetophonon**

[Fukuyama, Lee, 1978; Chen, Princeton U. 2005]

Fundamental aspects of magnetophonon resonances

Hydrodynamic description:

In the presence of **small explicit breaking** (e.g. impurities), the dispersion relation of the **magnetonphonon** gets modified into

$$\mathrm{Re}\left[\omega_-(k)\right] = \frac{\sqrt{\left(\omega_0^2 + \omega_\perp^2(k)\right)\left(\omega_0^2 + \omega_\parallel^2(k)\right)}}{\omega_c}$$

This mode acquires a **finite gap**

$$\omega_-(k = 0) \equiv \omega_{pk}.$$

ω_0 : pinning frequency

The dependence of ω_{pk} with respect to B is the easiest to measure accurately and can give important information on "the type" of disorder in the material.

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Classical hydrodynamics suggests $\omega_{pk} = \frac{\omega_0^2}{\omega_c} \sim \frac{1}{B}$

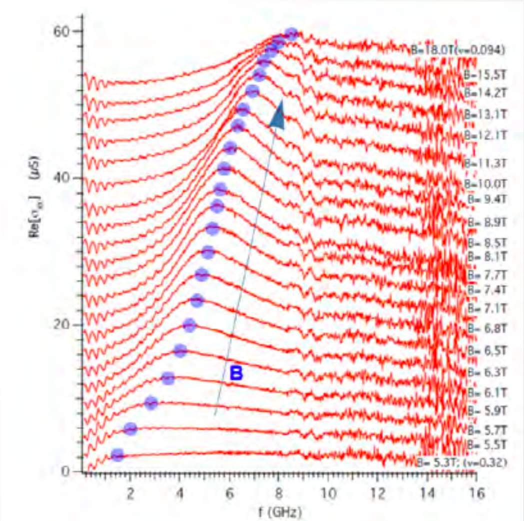
Go beyond classical treatment !

magnetic length $l_b = \sqrt{\hbar/eB}$ $\qquad l_b \gg \xi \qquad$ quantum regime ,

disorder correlation length ξ. $\qquad l_b \ll \xi \qquad$ classical regime .

A model for dilute disorder with specific corrections predicts[Fertig, PRB, 1999]:

$$\omega_{pk} \sim B^\gamma, \qquad \text{with} \qquad 0 < \gamma < 1$$



Holographic Model

$$S = M_P^2 \int d^4x \sqrt{-g}\left[\frac{R}{2} + \frac{3}{\ell^2} - V(X) - \frac{1}{4}F^2\right]$$

$$X \equiv \frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi^I\partial_\nu\phi^I$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The **dyonic black brane** solutions

$$ds^2 = \frac{1}{u^2}\left[-f(u)\,dt^2 - 2\,dt\,du + dx^2 + dy^2\right]$$

$$A_t = \mu - \rho\, u\,, \quad A_x = -\frac{B}{2}\,y\,, \quad A_y = \frac{B}{2}\,x \qquad \phi^I = \kappa\, x^I$$

$$f(u) = u^3\int_u^{u_h} dv\left[\frac{3}{v^4} - \frac{V(\kappa^2 v^2)}{v^4} - \frac{(\rho^2 + B^2)}{2}\right]$$

chemical potential

magnetic field

charge density

disorder (momentum dissipation)

event horizon

The temperature

$$T = -\frac{f'(u_h)}{4\pi} = \frac{6 - 2V\left(\kappa^2 u_h^2\right) - (\rho^2 + B^2)\,u_h^4}{8\pi\, u_h}$$

Different types of breaking translational invariance

For the **polynomial potential**:

$$\phi^I = \kappa\, x^I$$

$$V(X) = \underbrace{\alpha X}_{explicit} + \underbrace{\beta X^3}_{spontaneous}$$

explicit breaking
of translational invariance

spontaneous breaking
of translational invariance

pseudo-spontaneous breaking, where the breaking is mostly spontaneous

$$\text{pseudo-spontaneous regime:} \quad \alpha \ll 1\,, \quad \beta \gg \alpha\,.$$

(See Matteo's lecture for more details !)

[1311.5157, 1711.03100, 1904.05785, ….]

Hydrodynamic warm-up:

Investigate the **hydrodynamic description** of homogeneous holographic model with **spontaneously broken** translations at finite charge density

Previous hydrodynamic description was **lacking fundamental terms** to match the holographic results.

The **improved hydrodynamic** description was proposed[2001.07357], introducing **crystal pressure**.

To check explicitly if the improved hydrodynamic description matches the holographic results from our model in the presence of **finite charge density**.

Hydrodynamic warm-up:

$$\text{transverse sector:} \quad \omega_\perp = v_\perp k - \frac{i}{2}\Gamma_\perp k^2 ,$$

$$\text{longitudinal sector:} \quad \omega_\parallel = v_\parallel k - \frac{i}{2}\Gamma_\parallel k^2 , \quad \omega^\parallel_{1,2} = -i D^\parallel_{1,2} k^2 .$$

2 sound modes+2 diffusive modes

$$v_\perp^2 = \frac{G}{\chi_{\pi\pi}} , \quad \Gamma_\perp = \frac{\eta}{\chi_{\pi\pi}} + \frac{G\,\Pi_f^2}{\sigma\,\chi_{\pi\pi}^2} ,$$

$$v_\parallel^2 = \frac{1}{2} + v_\perp^2 , \quad \Gamma_\parallel = \frac{\eta}{\chi_{\pi\pi}} + \frac{\Pi_f^2\,G^2}{\sigma\,\chi_{\pi\pi}^3\,v_L^2} ,$$

$$\left(D - \frac{\Pi_f^2}{\sigma}\frac{G + \mathfrak{B} - \mathcal{P}}{2\chi_{\pi\pi} v_\parallel^2 (\Pi_f + \Pi_l)}\right)\left(\frac{\Xi D}{2(\Pi_f + \Pi_l)} - \frac{\sigma_q}{T^2}\right) =$$

$$= \frac{D}{\sigma}\left(\frac{s_f q_l - q_f s_l}{\Pi_f + \Pi_l} + \frac{\gamma}{T}\right)\left(\frac{s_f q_l - q_f s_l}{\Pi_f + \Pi_l} - \frac{\gamma'}{T}\right) .$$

$$\tilde{\sigma}_q = \sigma_q + \frac{1}{\sigma}\left(\frac{q_f \mathcal{P}}{\chi_{\pi\pi}} - \gamma\right)\left(\frac{q_f \mathcal{P}}{\chi_{\pi\pi}} + \gamma'\right) ,$$

$$\tilde{\gamma} = \frac{\Pi_f}{\sigma}\left(\frac{\gamma}{\chi_{\pi\pi}} - \frac{q_f \mathcal{P}}{\chi_{\pi\pi}^2}\right) ,$$

$$\tilde{\gamma}' = \frac{\Pi_f}{\sigma}\left(\frac{\gamma'}{\chi_{\pi\pi}} + \frac{q_f \mathcal{P}}{\chi_{\pi\pi}^2}\right) .$$

crystal pressure

$$p = -\Omega , \quad \epsilon = \langle T^{tt} \rangle ,$$

$$\mathcal{P} = \langle T^{xx} \rangle - p , \quad \chi_{\pi\pi} = \langle T^{tt} \rangle + \langle T^{xx} \rangle$$

$$s_f = \frac{\partial p}{\partial T} , \quad s_l = \frac{\partial \mathcal{P}}{\partial T} , \quad q_f = \frac{\partial p}{\partial \mu} , \quad q_l = \frac{\partial \mathcal{P}}{\partial \mu}$$

$$\Pi_f = \epsilon + p = s_f T + q_f \mu$$

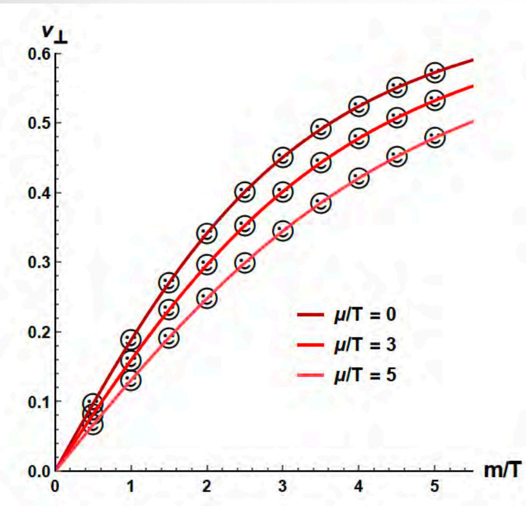
$$T\frac{\partial \mathcal{P}}{\partial T} + \mu \frac{\partial \mathcal{P}}{\partial \mu} = 3\mathcal{P} - 2\mathfrak{B} , \quad \Pi_l = \epsilon_l + \mathcal{P} = s_l T + q_l \mu$$

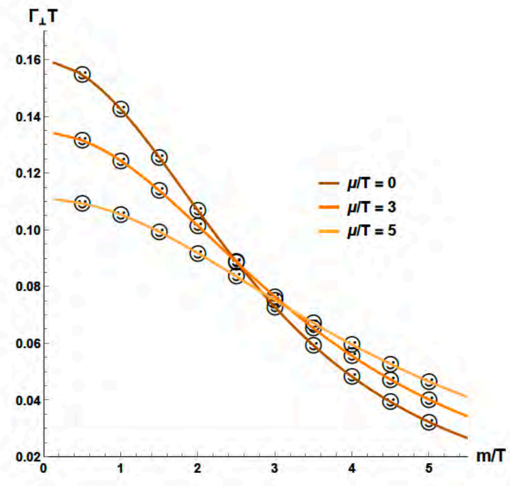
$$\Xi = \frac{\partial s_f}{\partial T}\frac{\partial q_f}{\partial \mu} - \frac{\partial s_f}{\partial \mu}\frac{\partial q_f}{\partial T}$$

[Armas and Jain, 2001.07357]

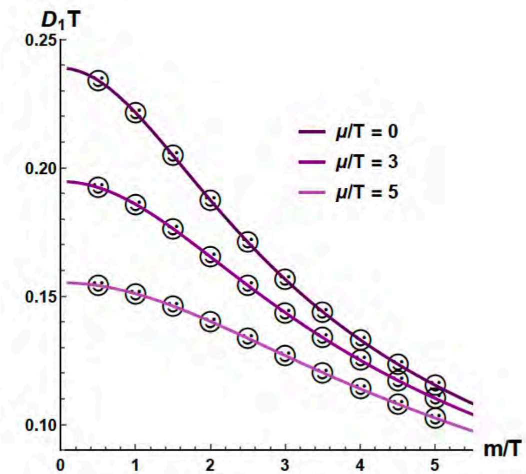
Hydrodynamic warm-up:

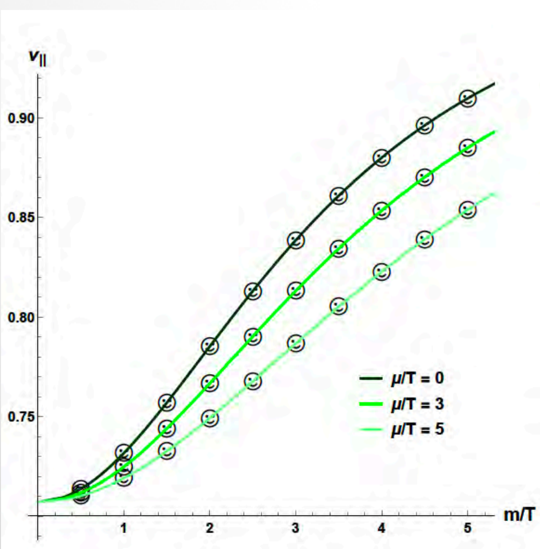
a perfect agreement between the hydrodynamic framework and the holographic results !

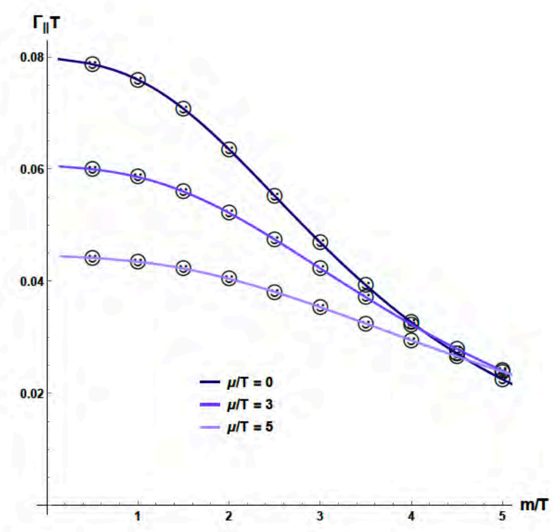


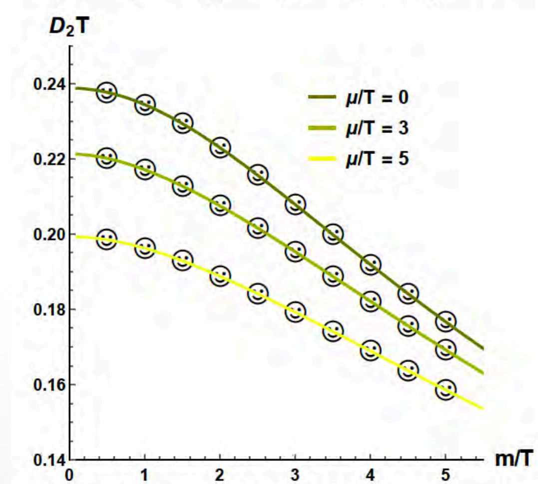


$$V(X) = m^2 X^3 , \quad \text{with} \quad \phi^I = x^I$$









Magnetophonon as a type-B Goldstone

Add **a finite magnetic field** to the setup and continue to consider the case in which translations are **spontaneously.**

At finite B one expects the presence of a **type-B mode** and a **gapped partner**

$$\mathrm{Re}[\omega] = \mathcal{C} + \mathcal{B}k^2\,, \qquad \text{magnetoplasmon}\,,$$
$$\mathrm{Re}[\omega] = \mathcal{A}k^2\,, \qquad \text{magnetophonon}\,,$$

Expectations from **hydrodynamics** and field theory

$$\mathrm{Re}\,[\omega_+] = \omega_c + \frac{(v_\parallel^2 + v_\perp^2)}{2\,\omega_c}k^2 + \dots\,,$$
$$\mathrm{Re}\,[\omega_-] = \frac{v_\perp\, v_\parallel}{\omega_c}k^2 + \dots\,, \qquad \omega_c = \frac{\rho\, B}{\chi_{\pi\pi}}$$

The **magnetoplasmon** mode ω_+ displays a **damping term**

$$\mathrm{Im}\,[\omega_+] = -\gamma_B\,, \qquad \textit{with} \quad \gamma_B = \frac{\tilde{\sigma}_q\, B^2}{\chi_{\pi\pi}}$$

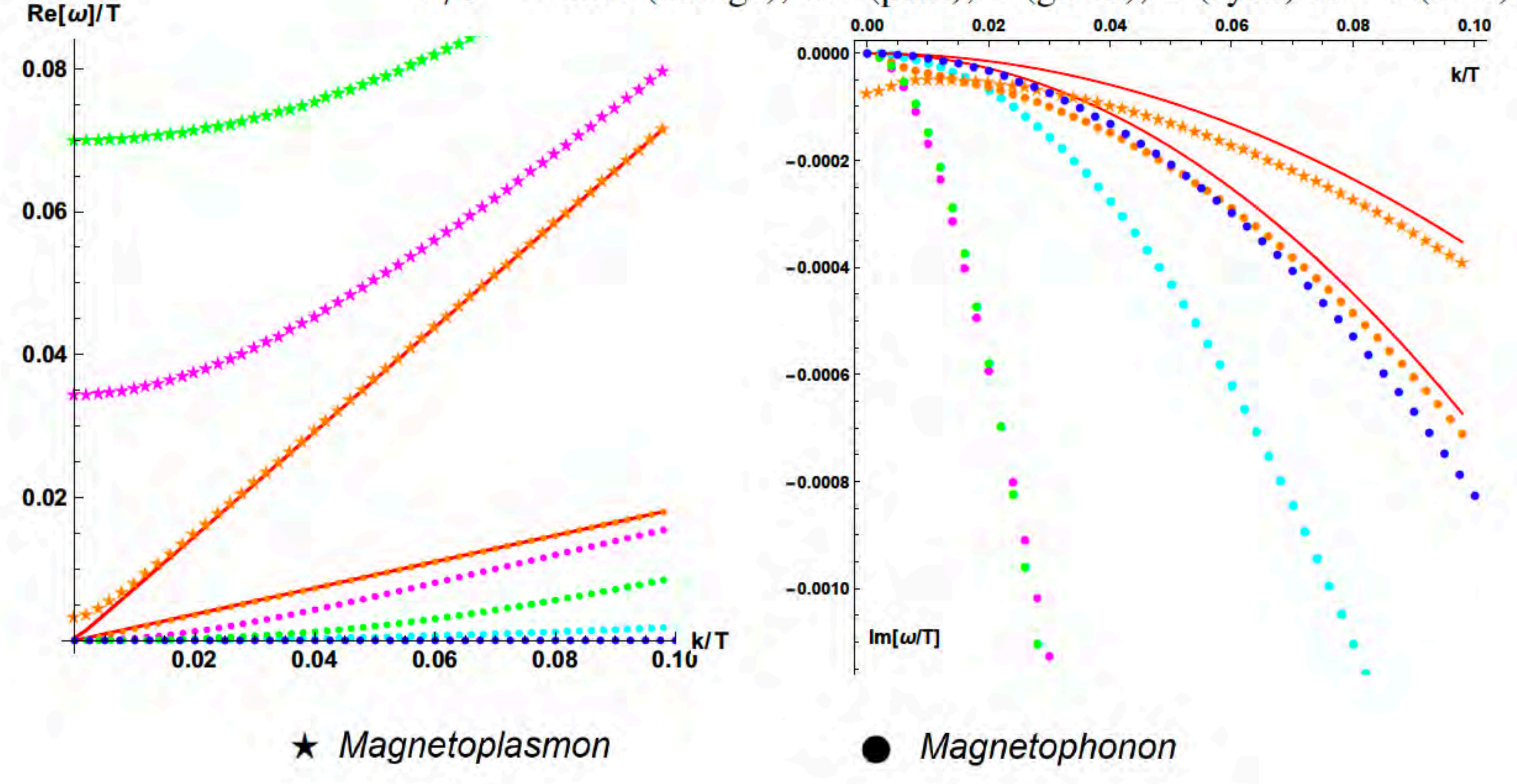
valid in the limit of **small magnetic field** $B/T^2 \ll 1$.

A **complete hydrodynamic theory** in the presence of lattice pressure, spontaneously broken translations, finite charge density and magnetic field **has not been built** yet.

Magnetophonon as a type-B Goldstone

$$V(X) = m^2 X^3 , \quad \phi^I = x^I$$

$B/T^2 = 0.01$ (orange), 0.1 (pink), 1 (green), 2 (cyan) and 4 (blue).

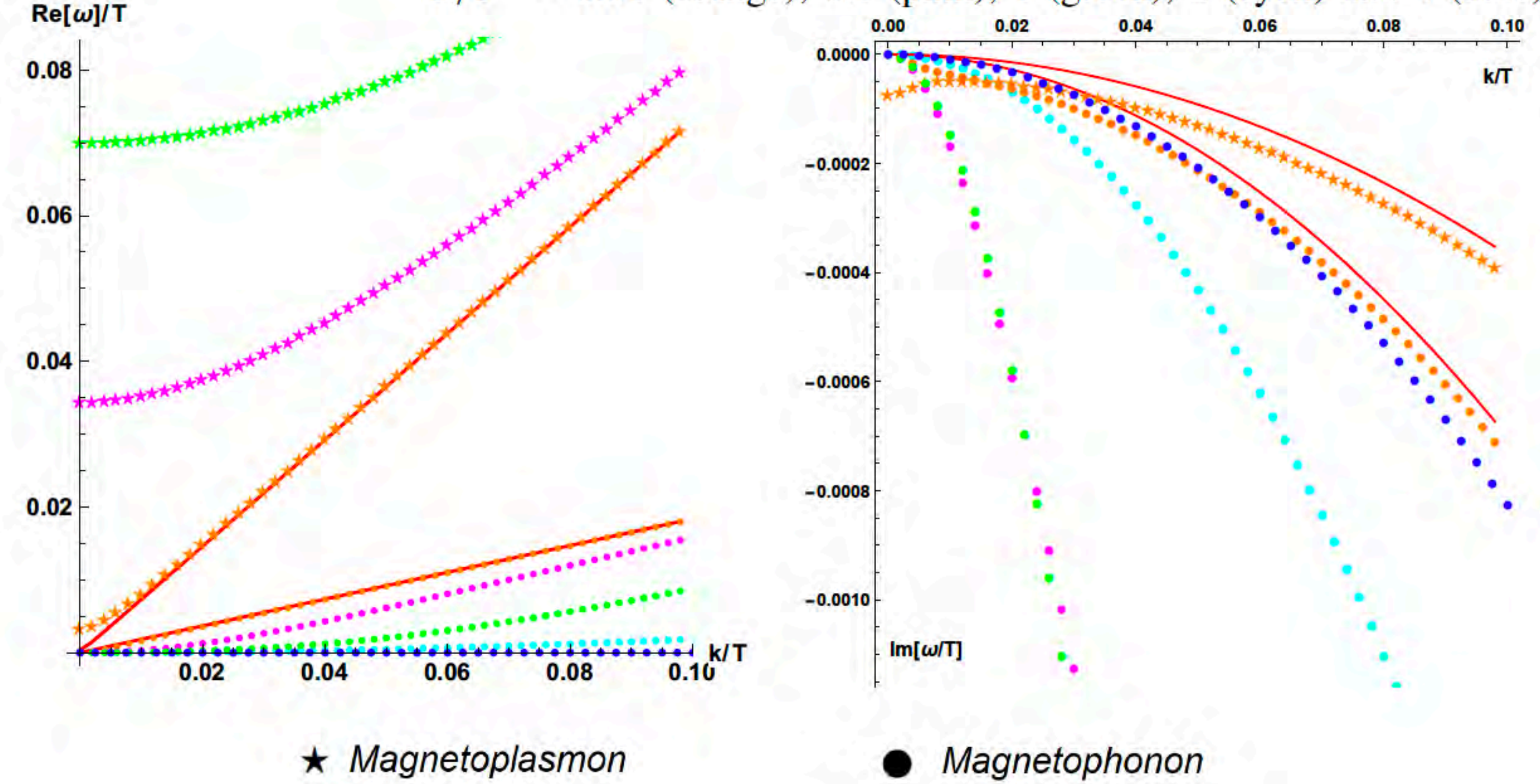


- Red lines show the two **linear sound** modes at B = 0.
- The modes combine forming the gapped **magnetoplasmon** (stars) and the quadratic type-B **magnetophonon** (circles).
- The **gap** of the magnetoplasmon **grows** with B, while the coefficient of the k^2 scaling of the magnetophonon **decreases** with it.

Magnetophonon as a type-B Goldstone

$$V(X) = m^2 X^3 , \quad \phi^I = x^I$$

$B/T^2 = 0.01$ (orange), 0.1 (pink), 1 (green), 2 (cyan) and 4 (blue).



The **dispersion relation** of the type-B Goldstone mode at **small momentum** is of the type:

$$\omega_{TYPE-B} = \mathcal{A} k^2 - i \mathcal{D} k^2 + \ldots$$

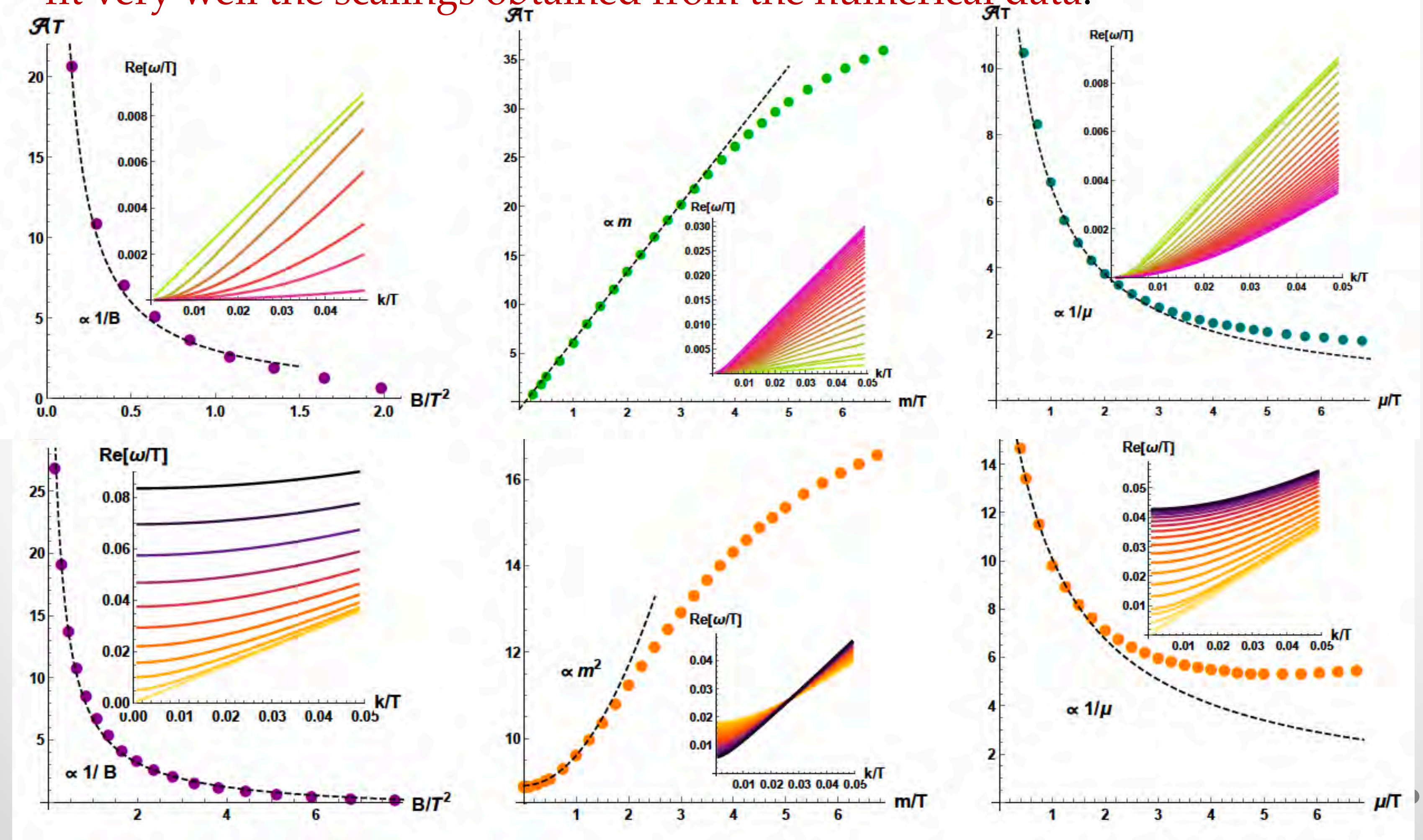
A **diffusive damping** for type-B Goldstone is not envisaged from EFT methods. EFT predicts a k^4 imaginary part: the **quasiparticle** nature of the excitation.

Magnetophonon as a type-B Goldstone

We have **confirmed numerically** that the hydrodynamic formula

$$\mathcal{B} = \frac{(v_\parallel^2 + v_\perp^2)}{2\,\omega_c}, \qquad \mathcal{A} = \frac{v_\perp\, v_\parallel}{\omega_c}, \qquad v_\parallel^2 = \frac{1}{2} + v_\perp^2, \qquad \mathcal{C} = \omega_c$$

fit very well the scalings obtained from the numerical data.



Magnetophonon Peak and Effects of Magnetic Field

Introduce a **small external source** of explicit breaking and consider the **pseudo-spontaneous regime**: $\alpha \ll 1\,, \quad \beta \gg \alpha\,.$

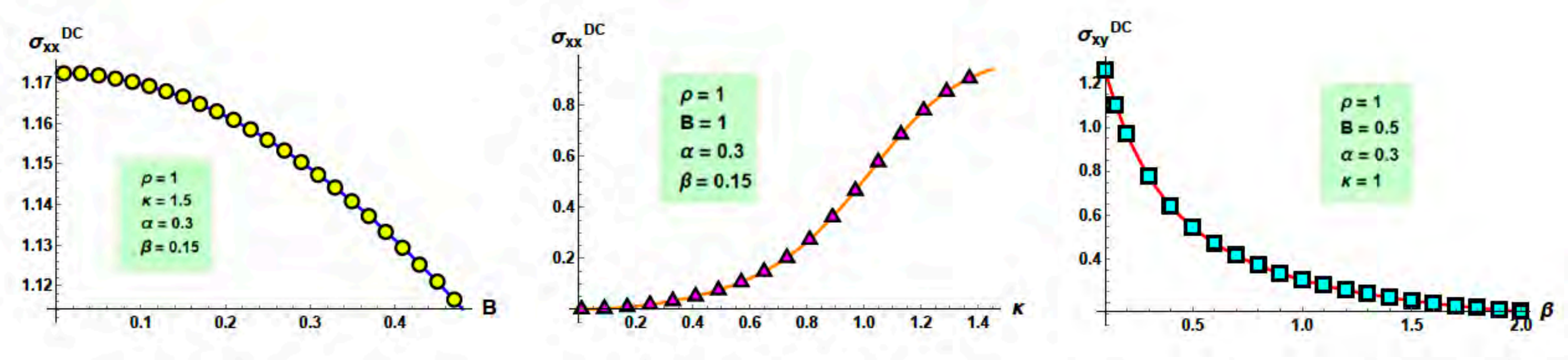
$$V(X) = \underbrace{\alpha X}_{explicit} + \underbrace{\beta X^3}_{spontaneous}$$

Kubo formulas

$$\sigma_{xx}(\omega) = \frac{i}{\omega}\langle \mathcal{J}_x \mathcal{J}_x \rangle\,, \quad \sigma_{xy}(\omega) = \frac{i}{\omega}\langle \mathcal{J}_x \mathcal{J}_y \rangle\,.$$

The DC values of such conductivities can be obtained using **horizon data**:

$$\sigma_{xx}^{DC} = \frac{\kappa^2 V' g_{xx}\left(B^2 + \kappa^2 V' g_{xx} + \rho^2\right)}{B^2\rho^2 + (B^2 + \kappa^2 g_{xx} V')^2}\Bigg|_{u_h}, \quad \sigma_{xy}^{DC} = B\rho\,\frac{\left(B^2 + 2\kappa^2 V' g_{xx} + \rho^2\right)}{B^2\rho^2 + (B^2 + \kappa^2 g_{xx} V')^2}\Bigg|_{u_h}$$



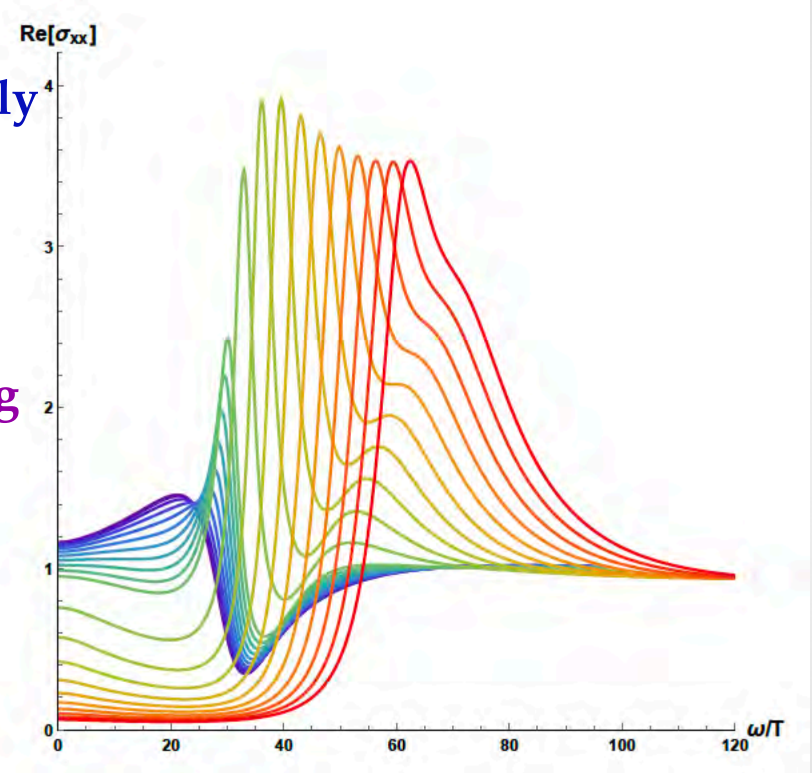
Comparison between the numerical conductivities at $\omega = 0$ (markers) and the DC formulas

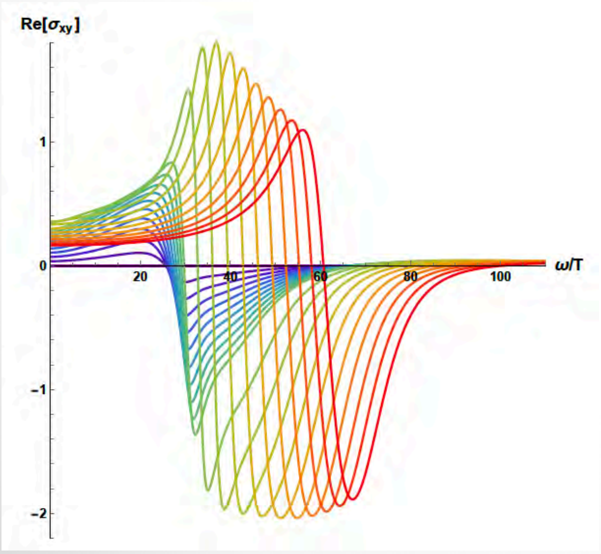
Magnetophonon Peak and Effects of Magnetic Field

The evolution of the **pseudo-phonon** peak increasing the magnetic field.

The position of the peak **increases monotonically** with the magnetic field B.

The width of the peak, which determines the lifetime of the associated resonance, becomes **first sharper** and then starts **increasing again** at very large magnetic fields.

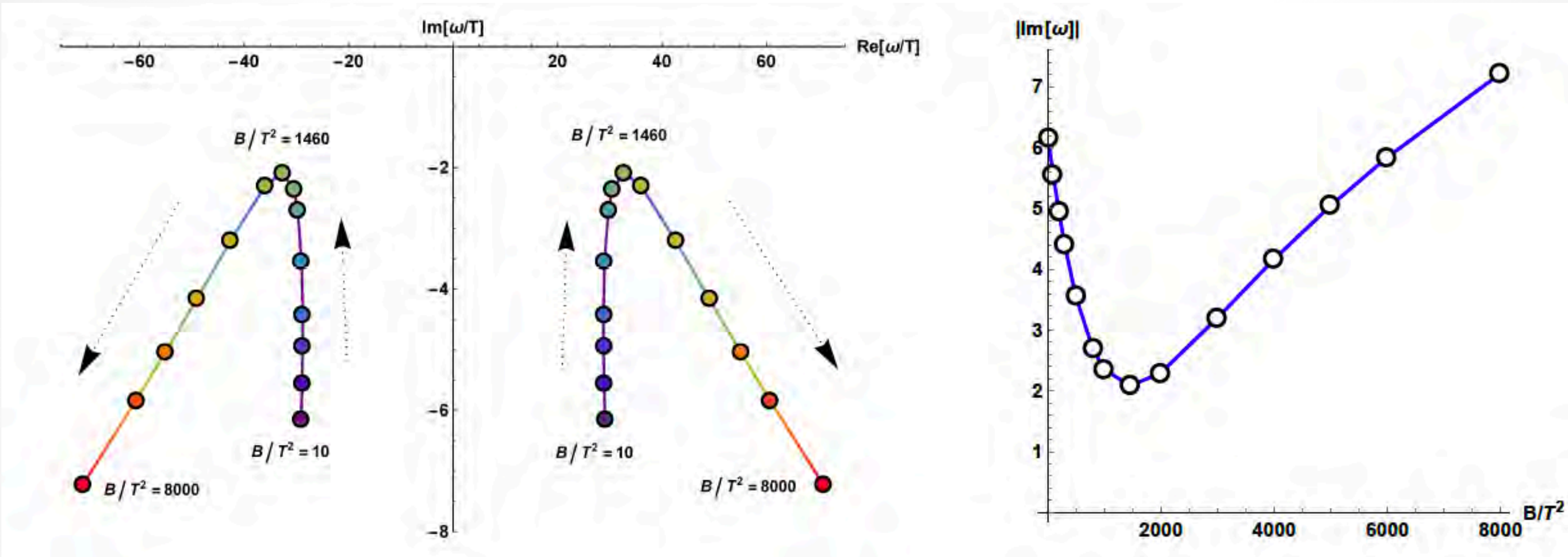




move the magnetic field $\frac{B}{T^2} \in [0,5\times10^3]$ (from blue to red)

Magnetophonon Peak and Effects of Magnetic Field

Quasinormal modes at finite charge density and magnetic field



The **real part** of QNM **increases monotonically** with the strength of the magnetic field.

The **imaginary part** is **non-monotonic**: the lifetime of the QNMs first becomes longer as a function of B, and then decreases at larger values of the magnetic field.

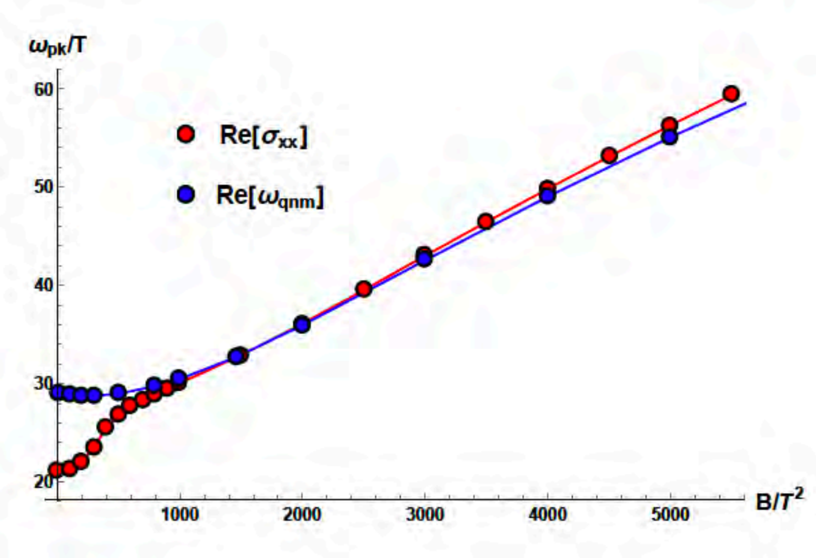
The motion of the QNMs is **consistent with** what already found in the conductivities.

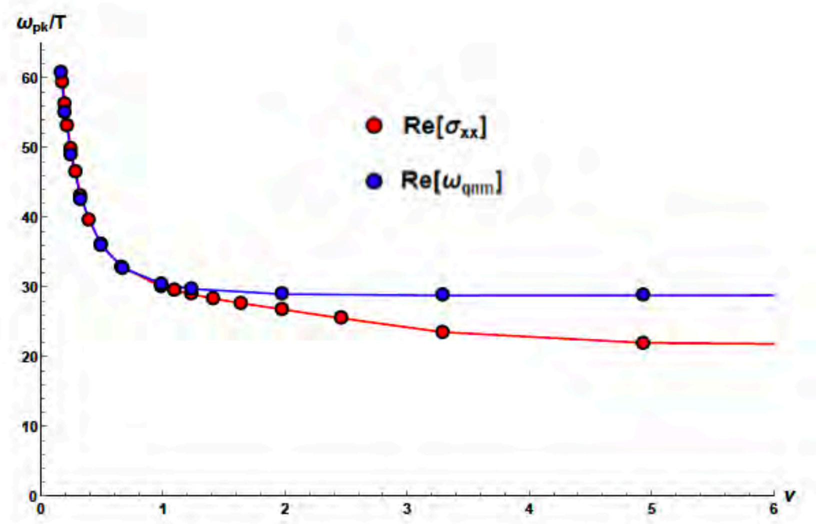
Magnetophonon Peak and Effects of Magnetic Field

The precise numerical values of peaks extracted from the maximum of $\mathrm{Re}[\sigma_{xx}]$ and the real part of the lowest QNM do not match.

$$\omega_{pk} \neq \mathrm{Re}[\omega_{qnm}]$$

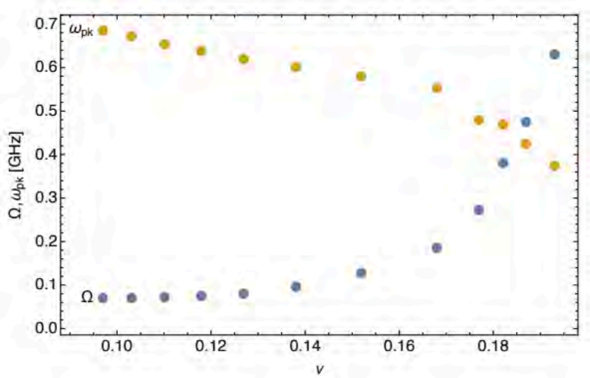
At large magnetic field, the two almost coincide.





The position of the peak seems to saturate to a constant value for large values of the filling fraction $\nu = \rho/B$.

Qualitatively, the results are in agreement with the experimental fits obtained in [1904.04872].

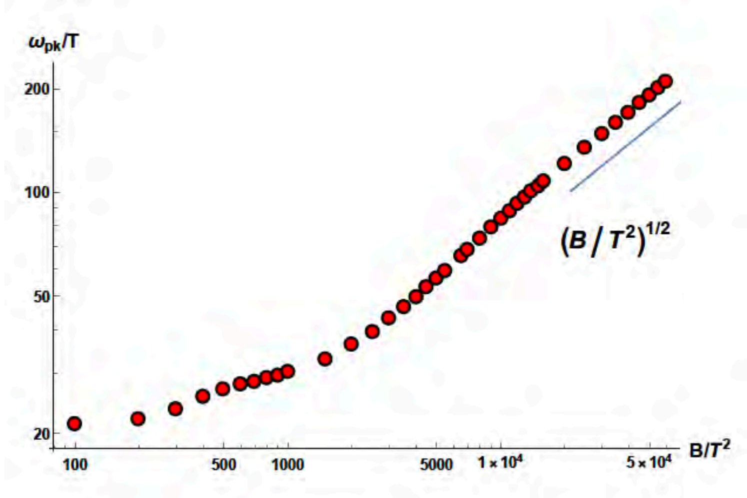


Magnetophonon Peak and Effects of Magnetic Field

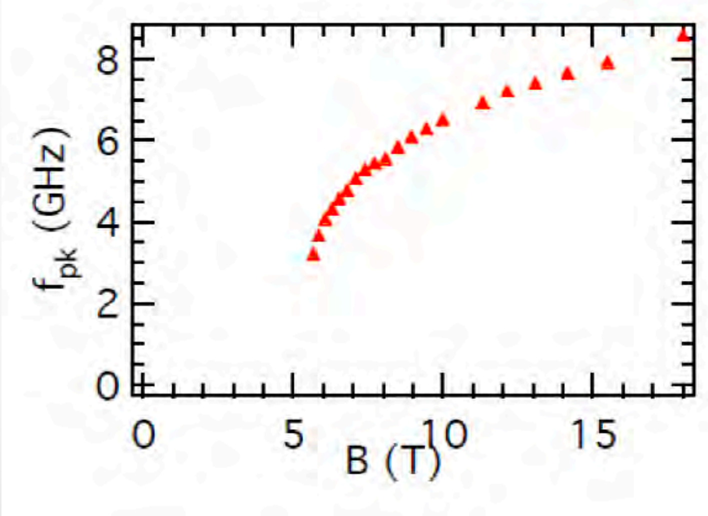
A quite **robust scaling** at large magnetic field

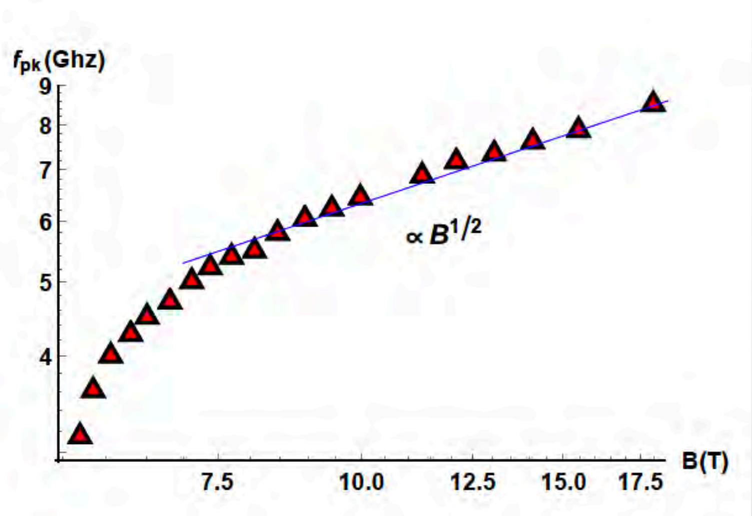
$$\omega_{pk} \sim B^{1/2}$$

It is incompatible with the idea that at large B the **magnetophonon resonance becomes light**, despite the presence of strong explicit breaking.



Our results are in **agreement with certain experimental results**, and suggest a precise interpretation of the nature of the "disorder" mimicked by these homogeneous holographic models.





Experimental data for a wide AlGaAs/GaAs/AlGaAs QW sample [Chen, Princeton U. 2005]

Remarks:

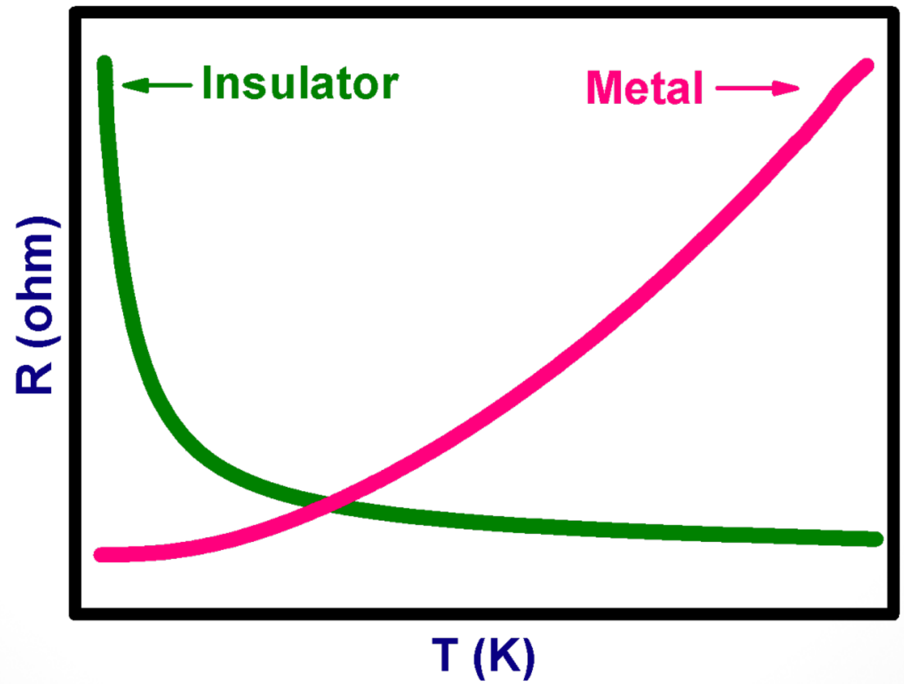
We discuss the **hydrodynamic description** of our holographic model in presence of the **spontaneous breaking** of translations and **finite charge density** and **zero magnetic field**.

We analyze the **dispersion relation** of the magnetophonons **in the absence of pinning**, and in particular we focus on their **type-B nature**.

We study electric conductivities and the dynamics of the **pinned magnetophonons** in the presence of a **small source** of explicit breaking of translations

Our study has revealed the presence of **interesting features** both from a theoretical and phenomenological point of view.

Magnetotransport and Holographic Metal-Insulator Transition



Background:

The mechanism of metal-insulator transition is one of the **oldest**, yet one of the **fundamentally least understood** problems in condensed matter physics.

A good metal and a good insulator are very different physical systems, and can be characterized by quite **different elementary excitations**.

Many theories have been proposed to understand the metal-insulator transition, such as MIT as a critical point, Scaling theories of disorder-driven transitions, Order-parameter approaches to interaction-localization. Mechanisms toward the MIT remain controversial and somewhat incomplete [1112.6166].

In the spirit of EFT, a minimal holographic model of a disorder-driven MIT was proposed [1601.07897,1602.01067]. There are still some issues that have not been understood well. Given the rich phenomenological features of this setup, it is worth understanding the theory further and uncovering some generic features.

Holographic model

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^I$$

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa_N^2}(R - 2\Lambda) - \frac{1}{4e^2} Y(X) F_{\mu\nu} F^{\mu\nu} - m^2 V(X) \right]$$

The **consistency** of a theory imposes some constraints on the couplings [1601.07897]

$$V'(X) > 0, \quad Y(X) > 0, \quad Y'(X) < 0$$

and $\boldsymbol{Y'(X) < 0}$ plays a key role in **triggering a metal-insulator transition**.

The background solutions:

$$ds^2 = \frac{1}{u^2}\left[-f(u) dt^2 + \frac{1}{f(u)} du^2 + dx^2 + dy^2 \right],$$

$$A_\mu dx^\mu = A_t(u)\, dt + \frac{1}{2} B(x\, dy - y\, dx),$$

$$\phi^I = \alpha\, x^I \qquad (x^1 = x, x^2 = y),$$

$$f(u) = u^3 \int_{u_h}^{u} \frac{1}{2} \left(\frac{B^2 \kappa_N^2 Y(\alpha^2 \xi^2)}{e^2} + \frac{\kappa_N^2 \rho^2 e^2}{Y(\alpha^2 \xi^2)} - \frac{6}{L^2 \xi^4} + \frac{2\kappa_N^2 m^2 V(\alpha^2 \xi^2)}{\xi^4} \right) d\xi,$$

$$A_t(u) = e^2 \rho \int_u^{u_h} \frac{1}{Y(\xi^2 \alpha^2)} d\xi,$$

$$\Lambda = -3/L^2$$

DC Transport and Constraint

DC conductivity and resistivity in terms of **horizon data**

$$\sigma_{xx} = \sigma_{yy} = \frac{\Omega Y[\Omega + Y(B^2Y^2+\rho^2)u_h^2]}{(\Omega + B^2Y^3u_h^2)^2 + B^2\rho^2Y^4u_h^4},$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{B\rho Y^3 u_h^2[2\Omega + Y(B^2Y^2+\rho^2)u_h^2]}{(\Omega + B^2Y^3u_h^2)^2 + B^2\rho^2Y^4u_h^4}$$

$$\Omega = \alpha^2[m^2V'Y^2 + \frac{u_h^4}{2}(B^2Y^2-\rho^2)Y']$$

$$R_{xx} = R_{yy} = \frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{yy}^2} = \frac{\Omega[\Omega + Y(B^2Y^2+\rho^2)u_h^2]}{Y[(\Omega+\rho^2Yu_h^2)^2 + B^2\rho^2Y^4u_h^4]},$$

$$R_{xy} = -R_{yx} = -\frac{\sigma_{xy}}{\sigma_{xx}^2+\sigma_{yy}^2} = -\frac{B\rho Y u_h^2[2\Omega + Y(B^2Y^2+\rho^2)u_h^2]}{(\Omega+\rho^2Yu_h^2)^2 + B^2\rho^2Y^4u_h^4}$$

Consider the **clean limit** $\alpha \to 0$ and parametrize the couplings Y and V:

$$Y(X) = 1 - k\,X + \mathcal{O}(X^2), \quad V(X) = \frac{1}{2m^2}X + \mathcal{O}(X^2) \qquad X \to 0$$

At 0th order:

$$\boxed{\sigma_{xx} = 0, \quad \sigma_{xy} = \frac{\rho}{B}}$$

DC Transport and Constraint

Include the **leading correction** coming from momentum dissipation

$$\sigma_{xx} = \frac{u_h^2}{2}[-k + B^{-2}(u_h^{-4} + \rho^2 k)]\alpha^2 + \mathcal{O}(\alpha^4)$$

$\sigma_{xx} \geq 0$ → $-k + B^{-2}(u_h^{-4} + \rho^2 k) \geqslant 0 \Rightarrow u_h^{-4} \geqslant (B^2 - \rho^2)k$

$T \geq 0$ → $\frac{3}{4\pi u_h} - \frac{(B^2+\rho^2)u_h^3}{8\pi} \geqslant 0 \Rightarrow u_h^{-4} \geqslant \frac{B^2+\rho^2}{6}$

$$T = \frac{3}{4\pi u_h} - \frac{(B^2+\rho^2)u_h^3}{8\pi} + \mathcal{O}(\alpha^2)$$

$$\frac{B^2+\rho^2}{6} \geqslant (B^2 - \rho^2)k$$

for a general choice of B and ρ.

$$0 \leqslant k \leqslant 1/6 \quad \Rightarrow -1/6 \leqslant Y'(0) \leqslant 0$$

Give a **generic constraint** on Y, without referring to the non-linear details of the coupling functions !

MIT and Scaling Behavior

Metal $\frac{dR_{xx}}{dT} > 0$ $\frac{dR_{xx}}{dT} < 0$ Insulator

Consider high T limit

Y′(X)<0 (k>0) has a dramatic impact on MIT

$$R_{xx} = 1 - \frac{2\,u_h^2}{\alpha^2}(\rho^2 - B^2 - \frac{k}{2}\alpha^4) + \mathcal{O}(u_h^4) = 1 - \frac{9}{8\pi^2\alpha^2}(\rho^2 - B^2 - \frac{k}{2}\alpha^4)T^{-2} + \mathcal{O}(T^{-4})$$

A critical charge density: $\rho_c = \sqrt{B^2 + k\alpha^4/2}$

✧$\rho < \rho_c$: R_{xx} decreases with T increased, displaying **insulating** behavior

✧$\rho > \rho_c$: R_{xx} inecreases with T increased, displaying **metallic** behavior

MIT by dialing charge density

MIT can be also triggered by increasing **magnetic field *B*** and **disorder strength *α***

MIT and Scaling Behavior

The resistivity **scales with a single parameter**:

$$R_{xx} \approx 1 - \frac{9}{8\pi^2\alpha^2}(\rho^2 - \rho_c^2)\frac{1}{T^2} = 1 \pm \frac{T_0^2(\rho)}{T^2}$$

$+$: insulating behavior
$-$: metallic behavior

with the **scaling parameter** T_0: $$T_0 = \frac{3}{2\sqrt{2}\pi\alpha}|\rho^2 - \rho_c^2|^{1/2}$$

The $R_{xx}(T)$ curves for different ρ can be **made to overlap** by T_0 along the T axis, yielding **a collapse of the data onto two curves**:
an insulating branch for $\rho < \rho_c$ and a metallic branch for $\rho > \rho_c$.

Near MIT: $$T_0 = \frac{3\sqrt{\rho_c}}{2\pi\alpha}|\rho - \rho_c|^{1/2} = C\,|\rho - \rho_c|^{1/2}, \qquad C = \frac{3}{2\pi}\left(\frac{B^2}{\alpha^4} + \frac{k}{2}\right)^{1/4}$$

The metallic and insulating curves are **mirror symmetry**:

$$R_{xx}(\rho - \rho_c, T) = 1/R_{xx}(\rho_c - \rho, T)$$

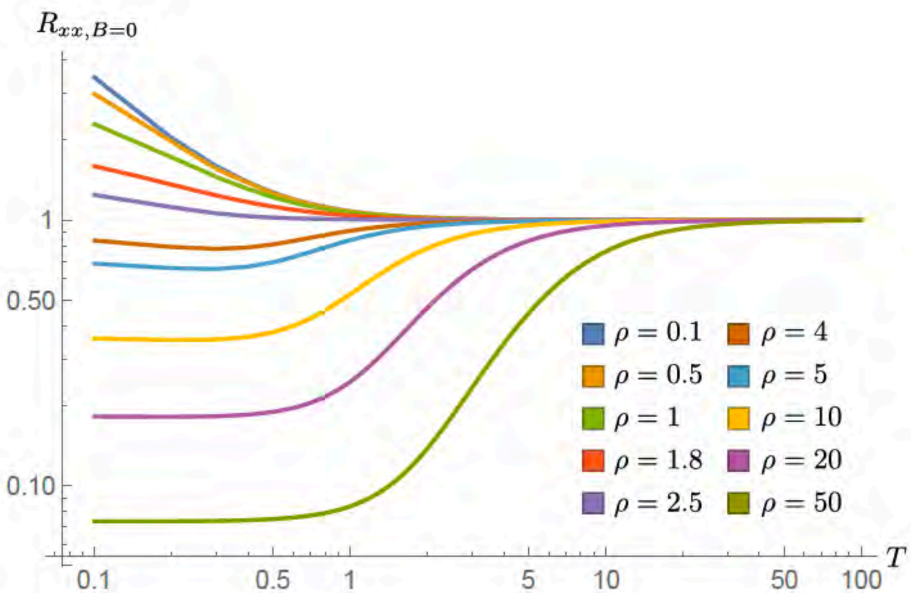
mechanism responsible for transport in insulating and metallic phases are related.

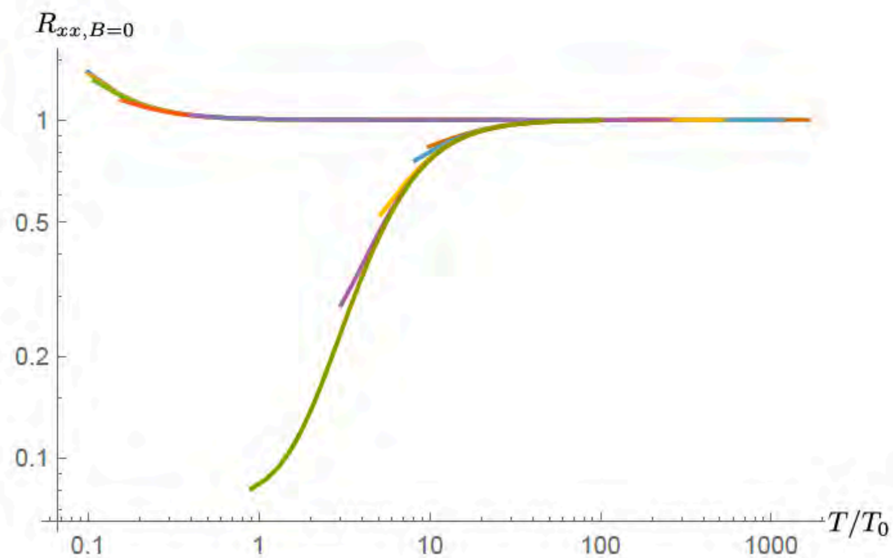
MIT and Scaling Behavior

The **collapse of resistivity** data into two separated curves holds over a broad interval of temperatures.

$\mathcal{K} = -1/6 \quad \alpha = 3$

$$Y = 1 + \mathcal{K}\, X, \quad V(X) = \frac{1}{2m^2} X$$





$$T_0 \approx 0.36 |\rho - \rho_c|^{1/2}$$

MIT by dialing the **charge density at B=0**.

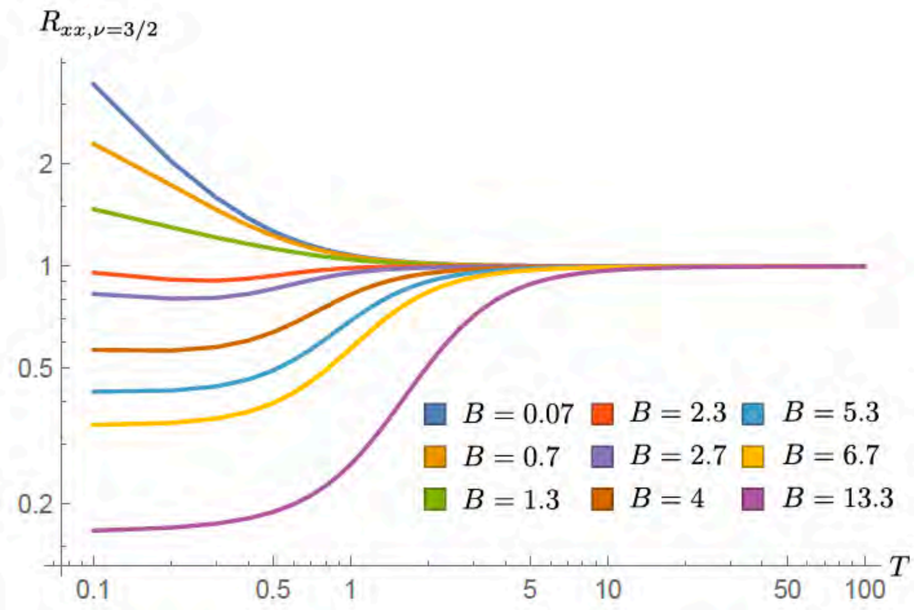
$R_{xx}(T)$ curves for **different ρ** can be made to overlap along the T axis with the scaling parameter T_0.

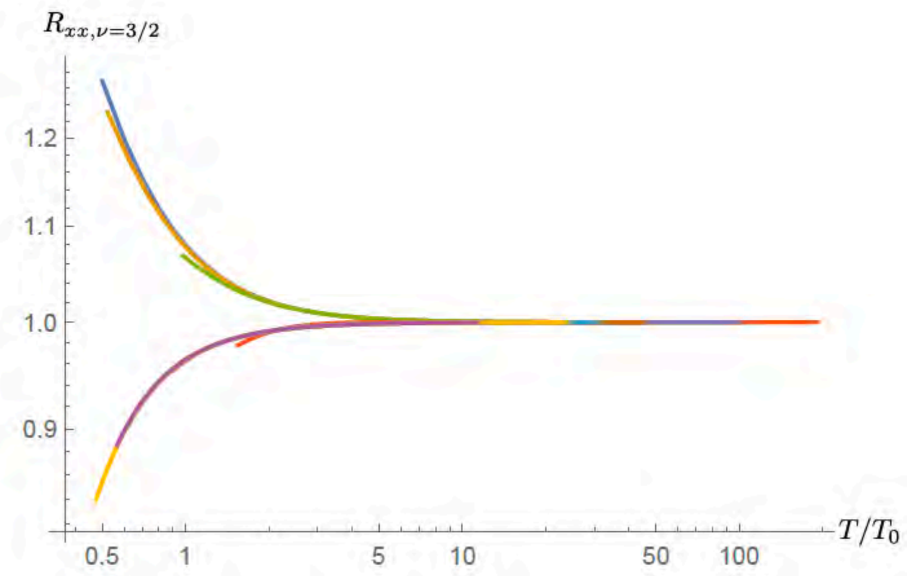
MIT and Scaling Behavior

The **collapse of resistivity** data into two separated curves holds over a broad interval of temperatures.

$\mathcal{K} = -1/6 \quad \alpha = 3$

$$Y = 1 + \mathcal{K}\,X, \quad V(X) = \frac{1}{2m^2}X$$





MIT by dialing the **magnetic field at Landau-Level filling factor** $\nu = \frac{\rho}{B} = 3/2$.

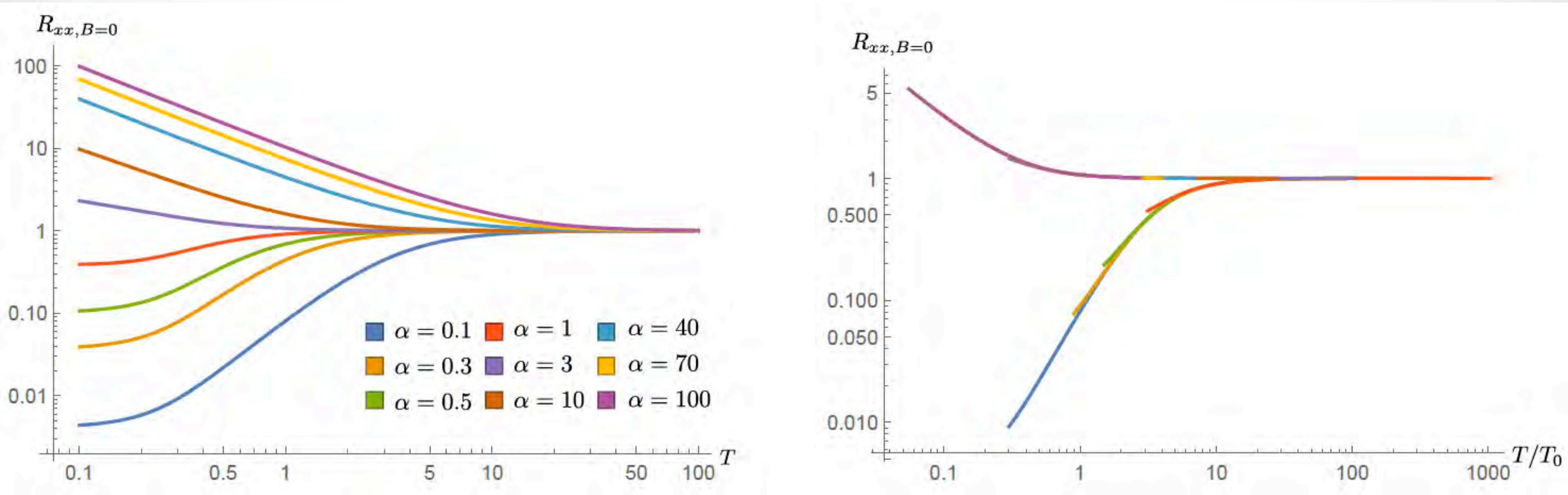
$R_{xx}(T)$ curves for **different B** can be made to overlap along the T axis with the scaling parameter T_0.

MIT and Scaling Behavior

The **collapse of resistivity** data into two separated curves holds over a broad interval of temperatures.

$\mathcal{K} = -1/6 \quad \rho = 1$

$$Y = 1 + \mathcal{K}\, X, \quad V(X) = \frac{1}{2m^2} X$$



MIT by dialing the **disorder strength at B=0**.

$R_{xx}(T)$ curves for **different α** can be made to overlap along the T axis with the scaling parameter T_0.

MIT and Scaling Behavior

The metal-insulator transition induced by charge density, magnetic field and disorder.

The collapse of resistivity data into two separated curves holds over a broad interval of temperatures.

Our holographic results agree qualitatively with the experimental observation in some two dimensional samples and materials.

Scaling of an anomalous metal-insulator transition in a two-dimensional system in silicon at B=0

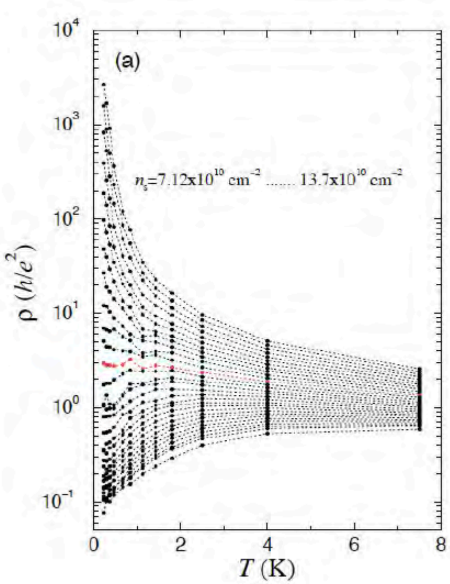
S. V. Kravchenko, Whitney E. Mason, G. E. Bowker, J. E. Furneaux, V. M. Pudalov, and M. D'Iorio
Phys. Rev. B **51**, 7038 – Published 15 March 1995

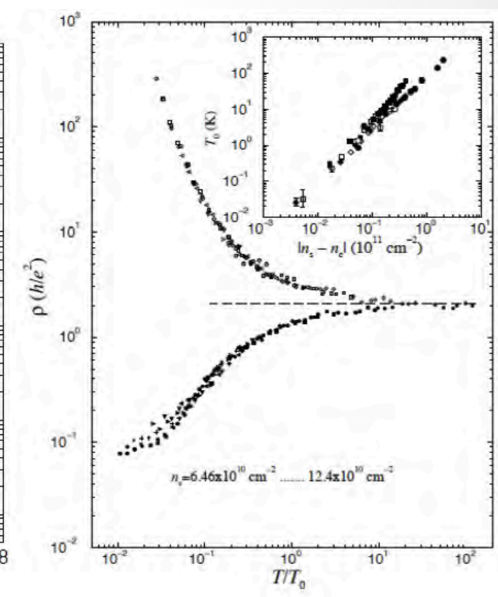
Metal-Insulator Transition in Two Dimensions: Effects of Disorder and Magnetic Field

Dragana Popović, A. B. Fowler, and S. Washburn
Phys. Rev. Lett. **79**, 1543 – Published 25 August 1997

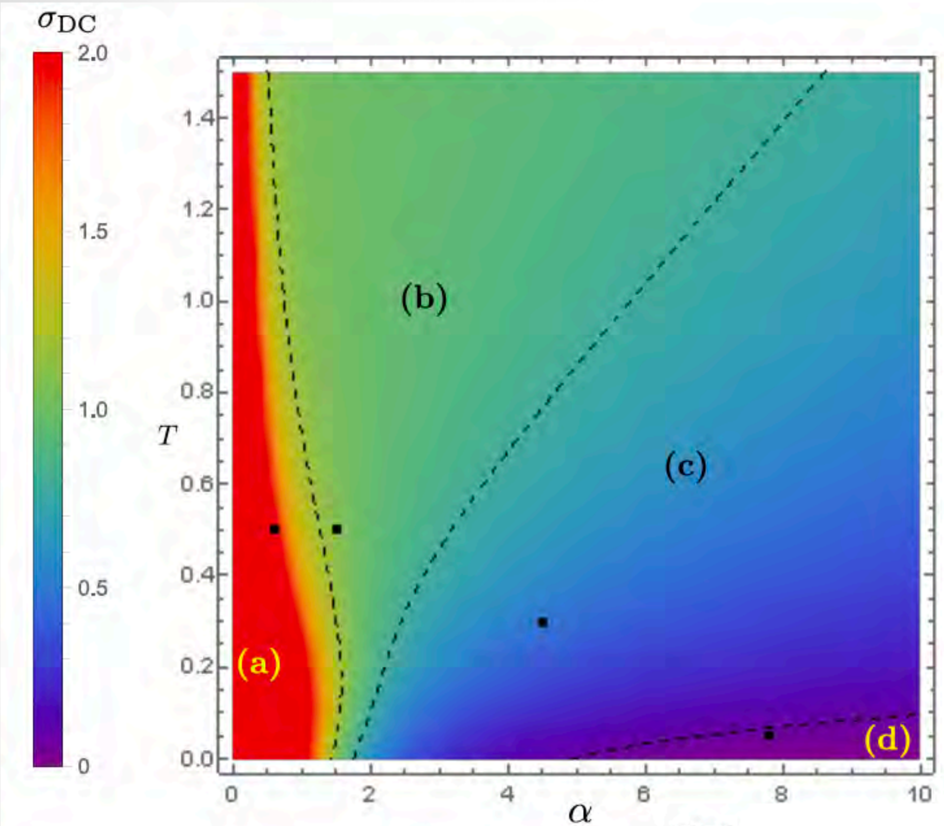
Scaling Theory of Two-Dimensional Metal-Insulator Transitions

V. Dobrosavljević, Elihu Abrahams, E. Miranda, and Sudip Chakravarty
Phys. Rev. Lett. **79**, 455 – Published 21 July 1997





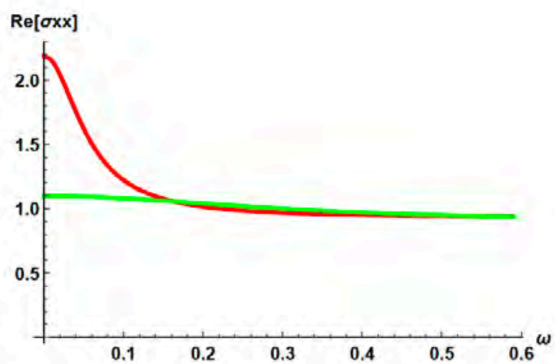
Phase Diagram

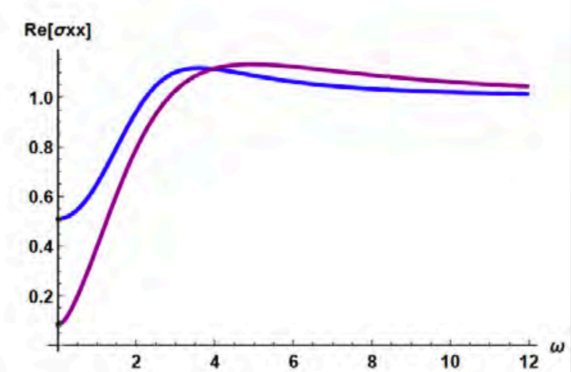


All phases share the **same symmetries** of the underlying theory, and thus beyond a simple Ginzburg-Landau description.

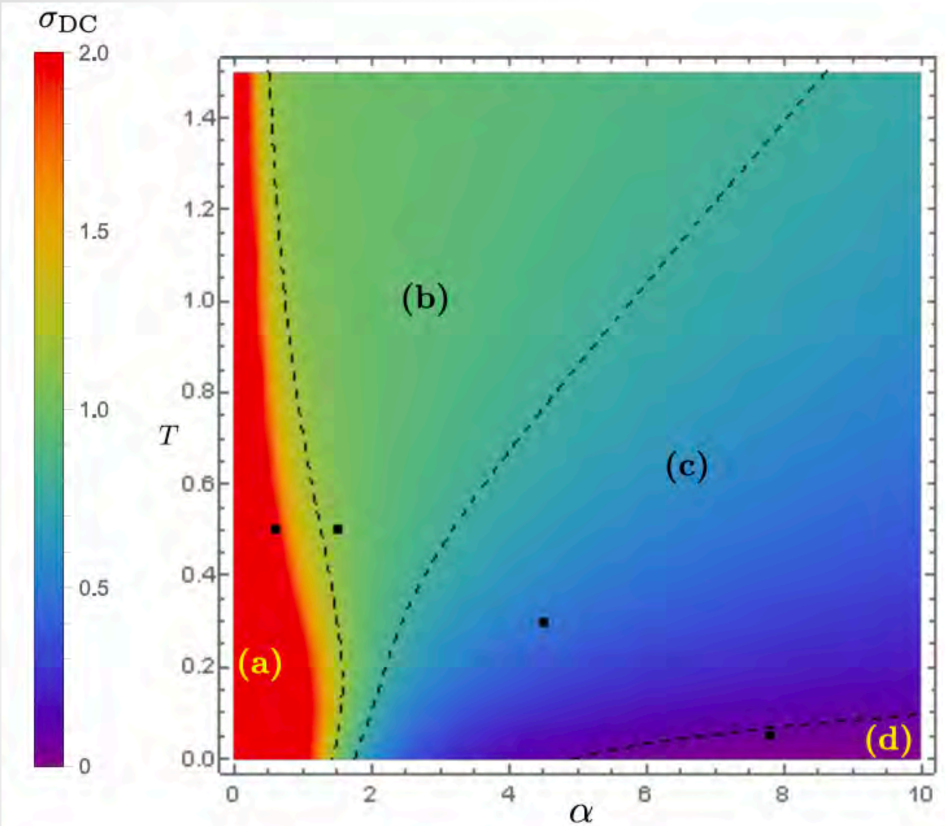
Is there is any other probe that is able to characterize different phases?

(a) good metal
(b) incoherent metal
(c) bad insulator
(d) good insulator



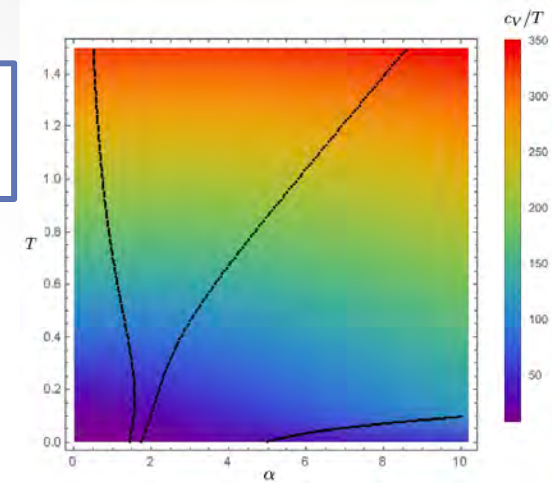


Phase Diagram

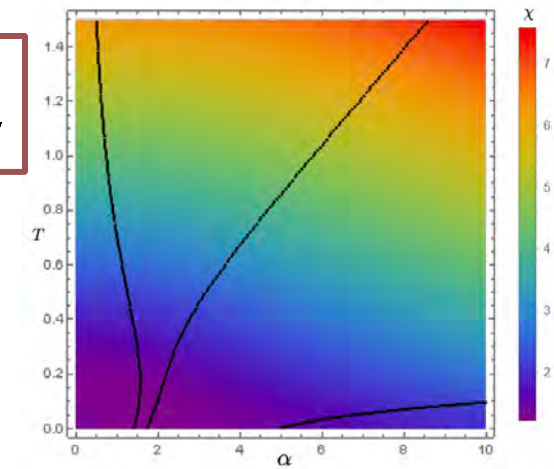


(a) good metal
(b) incoherent metal
(c) bad insulator
(d) good insulator

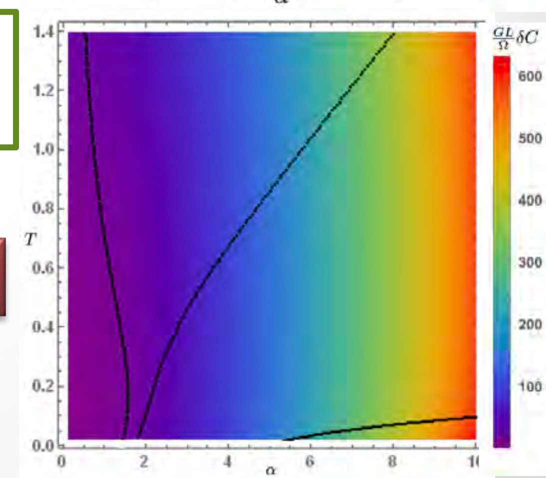
specific heat



charge susceptibility

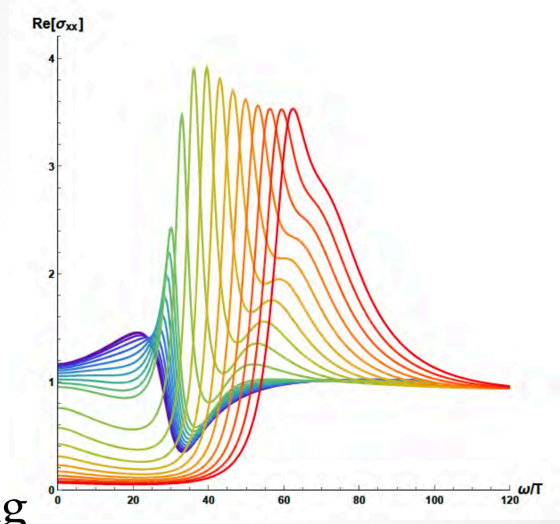


complexity of formation



No good probe has been found !

Conclusion



Holography as a Theoretical Laboratory:

(a). The **mean field** linear axion models display interesting phenomenology which is **relevant to** experimental observations.

(b). The **dispersion relations** of the hydrodynamic modes is discussed, and **magnetophonon** and its gapped partner-**magnetoplasmon** are identified.

(c). The **pinning frequency** of the magneto-resonance peak **increases with B**, in agreement with experimental data, revealing the **quantum nature** of the holographic pinning mechanism.

(d). The holographic **MIT** induced by **charge density, magnetic field and disorder**. Universal **scaling behavior** is uncovered, which agrees qualitatively with the experimental observation in some two dimensional samples and materials.

Conclusion

Holography as a Theoretical Laboratory:

➤Our study has revealed the presence of interesting features both from a theoretical and phenomenological point of view.

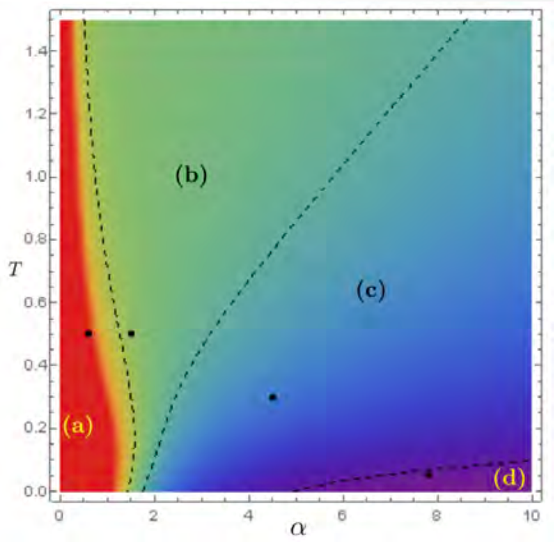
➤It represents a new step towards the understanding of the homogeneous holographic models with broken translations and their application to strange metals and transport in the absence of quasiparticles.

A strong sense of non-Fermi liquid

There are no quasi-particles

Strongly interacting "soup" with symmetry breaking

Possible relevant for unconventional quantum matter



Open questions:

- To extend our studies to more complicated holographic systems which break translations **without retaining the homogeneity** of the background.
- So far we limited ourselves to the electric conductivity, it is also worth studying the **thermal response** and the **mechanical response**.
- Despite a lot of work on this model and generalizations, the **physical nature** of the dual field theories is still **not well understood**.
- To which extent these simpler homogeneous models can **be trusted**, which features they concretely **differ from** the inhomogeneous setups (e.g. commensurability) and which phases of matter they are **actually describing**?
- ……

Map the bulk theory to the real word system?

Thank you !