Holographic superconductor induced by CDW

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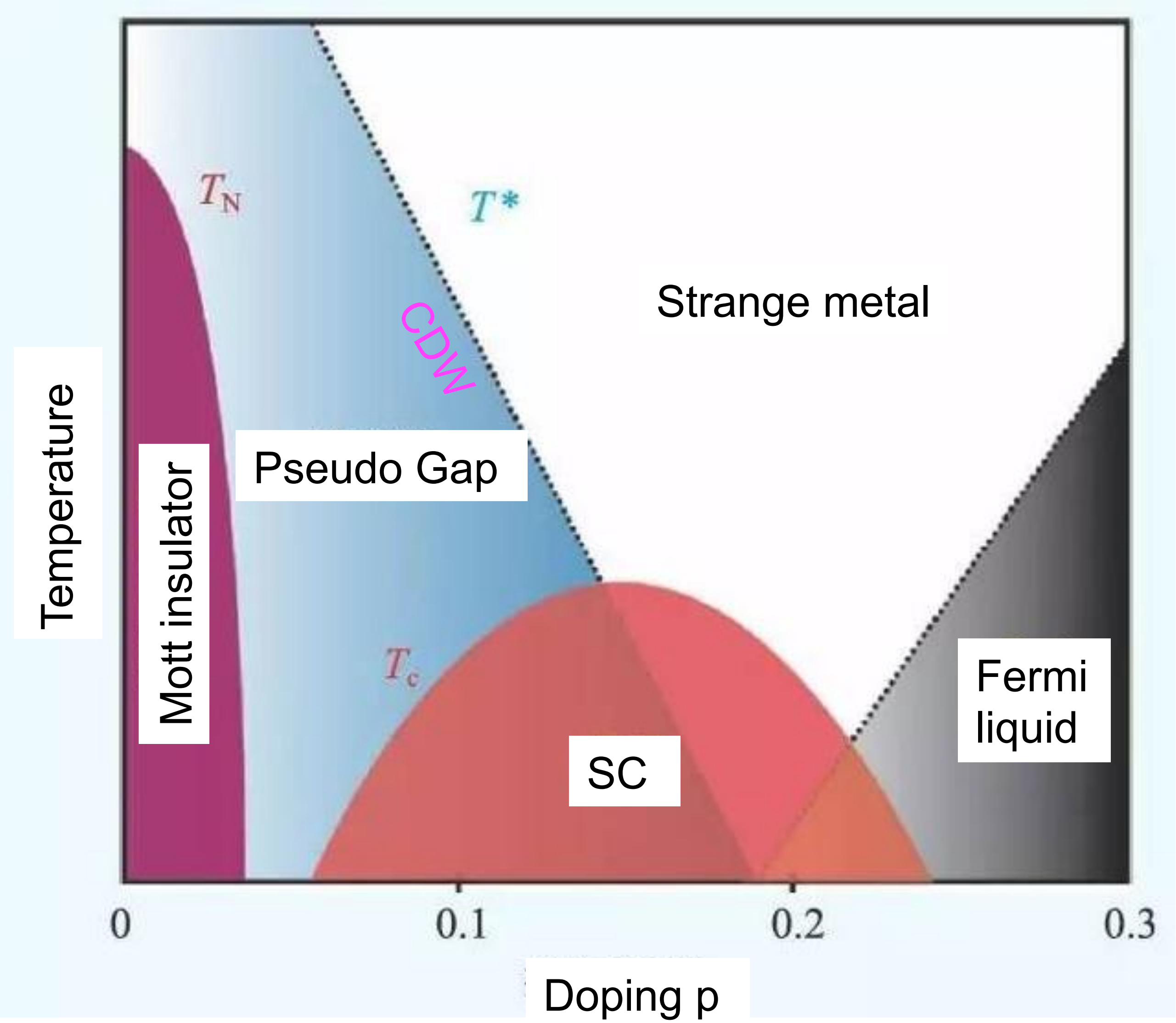
Outlines

1. The phase diagram of high temperature superconductivity
2. Holographic charge density waves
3. Holographic superconductor induced by CDW
4. Summary

Yi Ling, Peng Liu, Meng-He Wu, arXiv:1911.10368

The Phase Diagram of High Temperature Superconductivity



The Phase Diagram of High Temperature Superconductivity

The relations between CDW and SC is the key to understand the mechanism of high temperature superconductivity

1、Competitive relations

- The suppression of the critical temperature by CDW
- The competitions of order parameters
- The carriers grabbed by SC from CDW

2、Cooperative relations

- The positive correlation of the critical temperature
- CDW may improve the formation of Cooper pairs via phonons
- The formation of SC from CDW

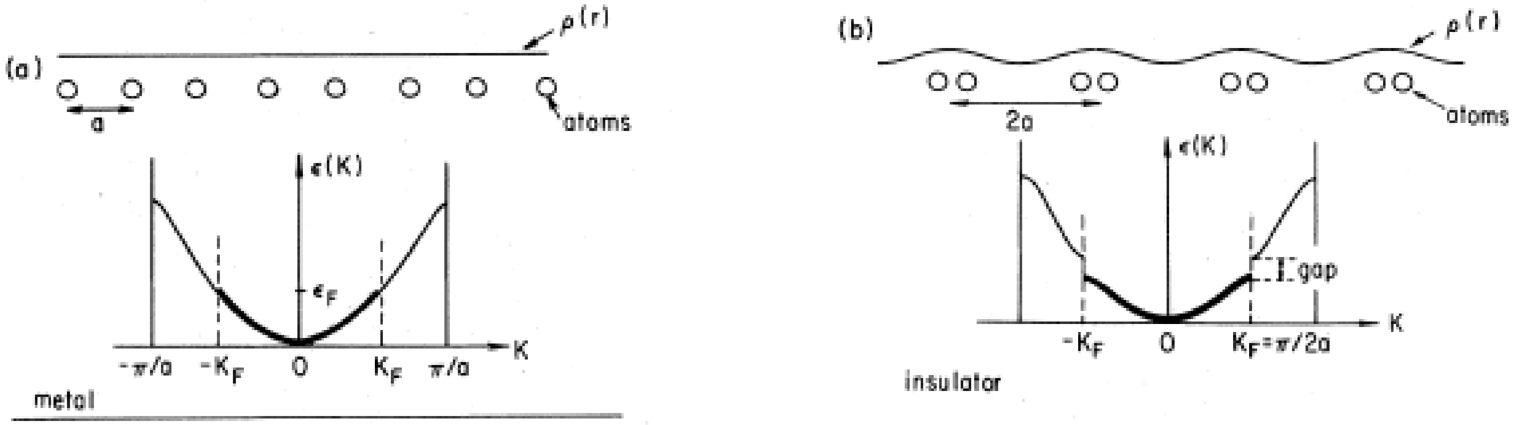
3、Signaled by strong coupling

Holographic Charge Density Waves (HCDW)

- **Charge density waves**

One dimensional case

$$\boxed{\rho(x)=\rho_0+\rho_1\cos(2k_F x+\varphi)}$$



The generation of CDW is a result of Peierls transition which is a typical metal-insulator transition.

Rev.Mod.Phys.Vol.60(1988),No.4

Holographic Charge Density Waves (HCDW)

- **CDW**

Mechanism

The generation of CDW results from the spontaneous breaking of the translational symmetry.

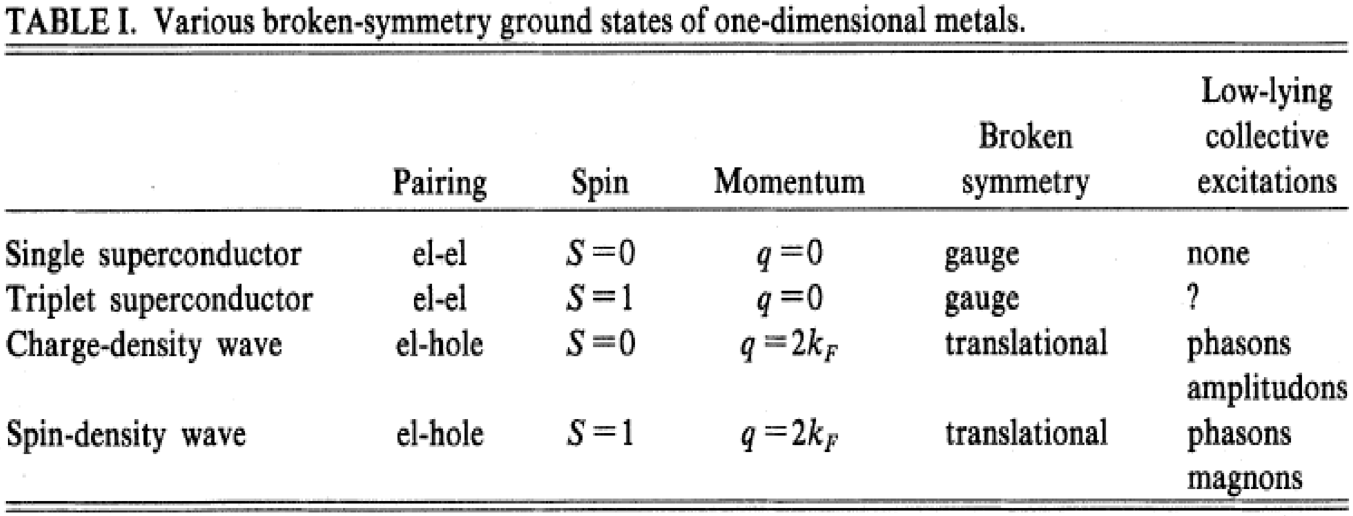


TABLE I. Various broken-symmetry ground states of one-dimensional metals.

	Pairing	Spin	Momentum	Broken symmetry	Low-lying collective excitations
Single superconductor	el-el	$S=0$	$q=0$	gauge	none
Triplet superconductor	el-el	$S=1$	$q=0$	gauge	?
Charge-density wave	el-hole	$S=0$	$q=2k_F$	translational	phasons amplitudons
Spin-density wave	el-hole	$S=1$	$q=2k_F$	translational	phasons magnons

Holographic Charge Density Waves (HCDW)

- **Perturbative instabilities of AdS2**

A. Donos and J. P. Gauntlett, JHEP 1108, 140 (2011)

$$\boxed{\begin{aligned} &T=0 \qquad AdS_2\times R^2 \\ &ds^2=-12r^2dt^2+\frac{dr^2}{12r^2}+(dx^2+dy^2) \\ &F=2\sqrt{3}dr\wedge dt \end{aligned}}$$

$$\boxed{S_{top}\sim\int d^4x\,\frac{c_1\varphi}{16\sqrt{3}}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}}$$

$$\boxed{\begin{aligned} \delta g_{ty}&=2\sqrt{3}h_{ty}(t,r)\sin(kx) \\ \delta g_{xy}&=h_{xy}(t,r)\cos(kx) \\ \delta A_y&=a(t,r)\sin(kx) \\ \delta\varphi&=w(t,r)\cos(kx) \end{aligned}} \Rightarrow \boxed{\Box_{AdS2}\vec{V}-M^2\vec{V}=0}$$

$$\vec{V}=(\phi_{xy},a,w)$$

$$M^2=\begin{pmatrix} k^2 & \frac{1}{\sqrt{3}}k & 0 \\ 24\sqrt{3} & 24+k^2 & -c_1k \\ 0 & -c_1k & k^2+m^2 \end{pmatrix}$$

AdS2 BF bound

$$\boxed{M^2\geq-3}$$

Could be violated !

Holographic Charge Density Waves (HCDW)

- **Holographic setup (D=4)**

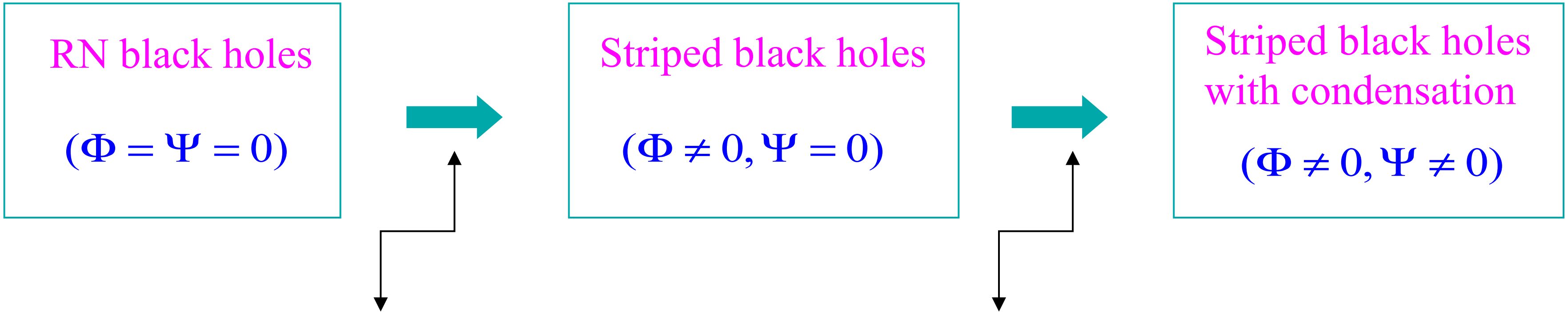
$$\boxed{\begin{aligned} S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}[R-\frac{1}{2}(\nabla\Phi)^2-V(\Phi)-\frac{1}{4}Z_A(\Phi)F^2-\frac{1}{4}G^2-\frac{1}{2}Z_{AB}(\Phi)FG \\ -|(\nabla-ieB)\Psi|^2-m_v^2\Psi\Psi^*] \end{aligned}}$$

$$F=dA, G=dB, V(\Phi)=-\frac{1}{L^2}+\frac{1}{2}m_s^2\Phi^2, Z_A(\Phi)=1-\frac{\beta}{2}L^2\Phi^2, Z_{AB}(\Phi)=\frac{\gamma}{\sqrt{2}}L\Phi$$

Φ : The order parameter of CDW

Ψ : The order parameter of SC

The logic line:



RN black holes

$(\Phi=\Psi=0)$

Striped black holes

$(\Phi\neq 0, \Psi=0)$

Striped black holes with condensation

$(\Phi\neq 0, \Psi\neq 0)$

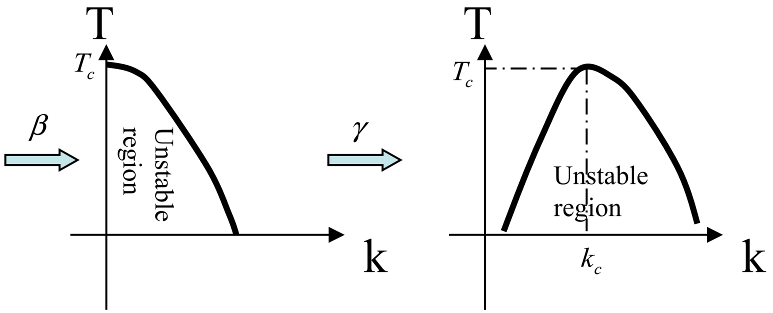
breaking the translational symmetry.

breaking U(1) gauge symmetry.

Holographic Charge Density Waves (HCDW)

- **HCDW** **Spontaneous breaking of translational invariance**

RN black brane → Striped black holes



$$\delta\phi = w(z)\cos(kx)$$

Striped black holes

$$\boxed{\begin{gathered} ds^2 = \frac{1}{z^2}\left[-(1-z)p(z)Qdt^2 + \frac{S}{(1-z)p(z)}dz^2 + T(dx+z^2Udz)^2 + Vdy^2\right] \\ A = \mu(1-z)\psi dt,\quad B = (1-z)\chi dt,\quad \Phi = z\phi,\quad \Psi = \eta e^{i\theta} \end{gathered}}$$

RN black brane $Q(x,z) = S(x,z) = T(x,z) = V(x,z) = 1, U(x,z) = 0$

$$\psi(x,z) = 1,\qquad \chi(x,z) = 0,\qquad \phi(x,z) = 0\qquad \Psi(x,z) = 0$$

$$\boxed{\frac{T}{\mu} = \frac{48-\mu^2}{16\pi\mu}}\qquad p(z) = 4(1+z+z^2-z^3\mu^2/16)$$

Holographic Charge Density Waves (HCDW)

- Charge density

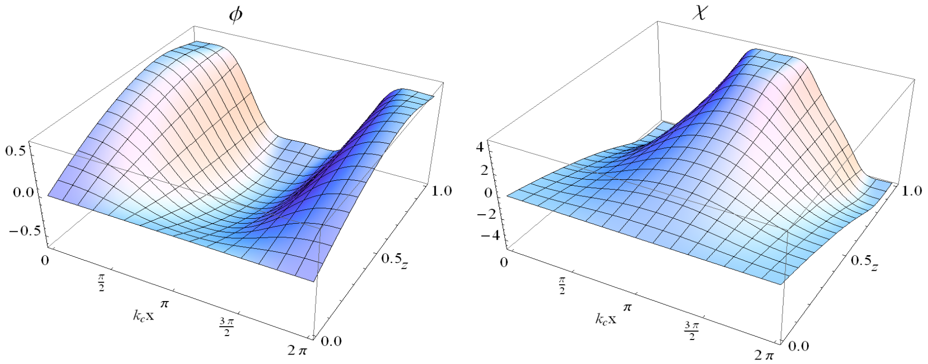
$$B_t = (1-z)\chi = -\rho(x)z + 0(z^2)$$

$$\boxed{\rho(x) = \rho_1 \cos[k_c x] + \rho_3 \cos[3k_c x] + \ldots}$$

$$\rho_0 \cong \rho_2 \cong 0\ldots, \quad \rho_1, \quad \rho_3 \quad \to CDW$$

No free electrons

- Numerical solutions



Holographic Superconductor Induced by CDW

- Condition for U(1) symmetry breaking

$$\boxed{\left(\nabla^2 - m_v^2\right)\eta = e^2 B^2 \eta}$$

$$\Psi = \eta e^{i\theta}, \quad \theta = 0$$

Necessary condition

$$\eta \neq 0 \Rightarrow B^2 = g^{tt} B_t B_t \neq 0$$

$$B_t = (1-z)\chi = -\rho(x) z + 0(z^2)$$

charges

$$\boxed{\rho(x) = \rho_0 + \ldots}$$

For all the previous models

$$\rho_0 \neq 0$$

CDW

$$\boxed{\rho(x) = \rho_1 \cos[k_c x] + \rho_3 \cos[3k_c x] + \ldots}$$

$$\boxed{\rho_0 \cong \rho_2 \cong 0\ldots, \quad \rho_1, \quad \rho_3 \quad \rightarrow CDW}$$

No free electrons

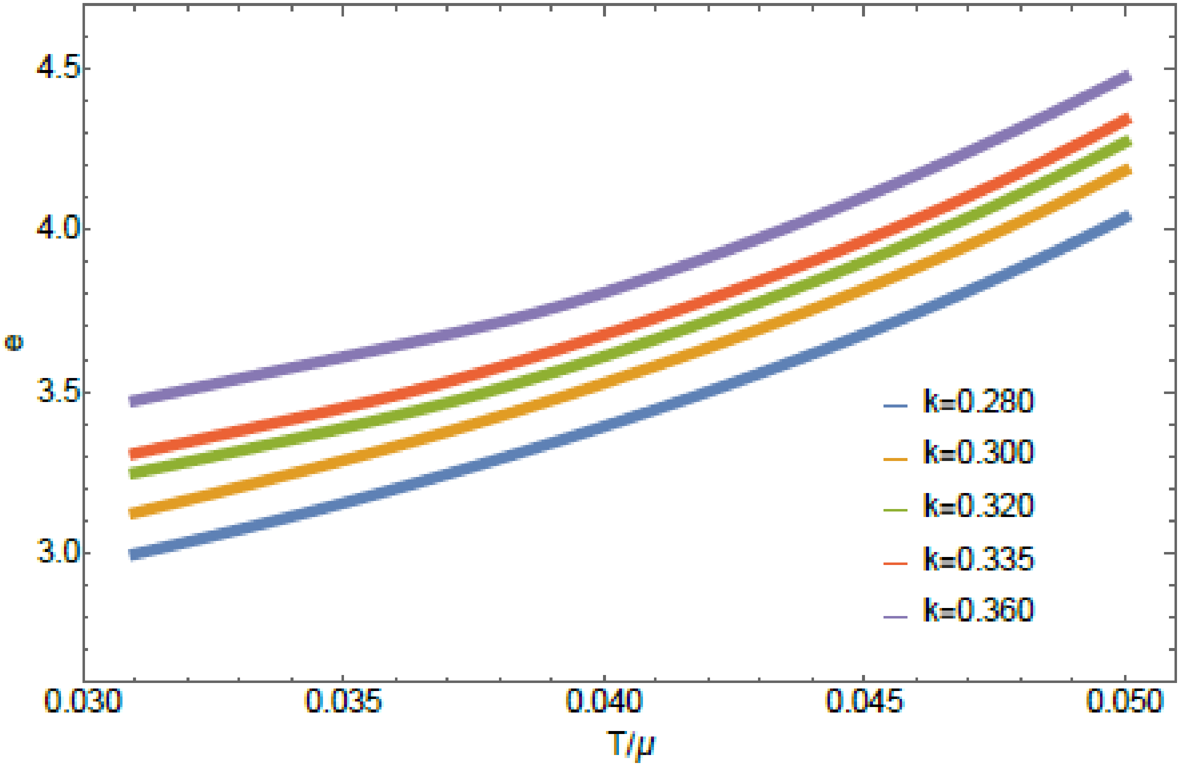
$$B_t = (1-z)\chi \neq 0$$

only due to the presence of CDW!

Holographic Superconductor Induced by CDW

- The relation between the critical temperature and the charge

$$\boxed{\left(\nabla^2 - m_v^2\right)\eta = e^2 B^2 \eta}$$

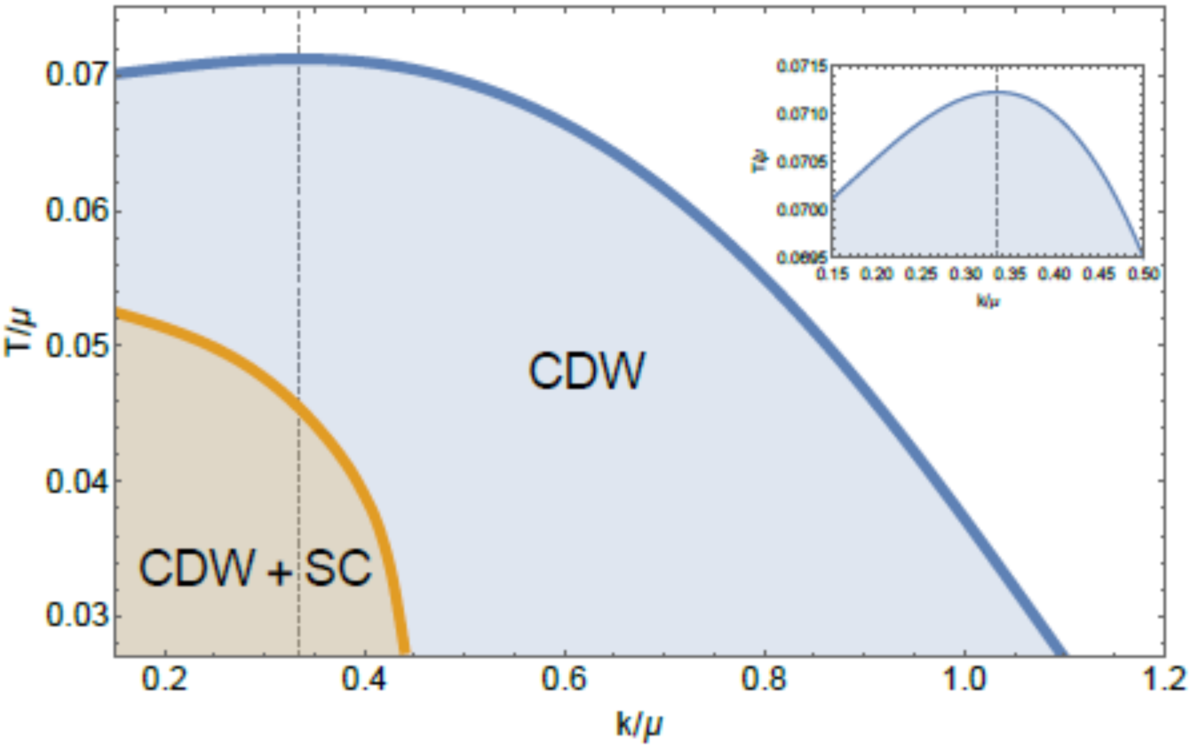


$$\boxed{\begin{array}{l} m_v^2 = m_s^2 = -8 \\ l^2 = 6L^2 = 1/4 \\ \beta = -130, \gamma = 16.6 \\ T_{CDW} = 0.071\mu \\ k_c = 0.335\mu \end{array}}$$

Holographic Superconductor Induced by CDW

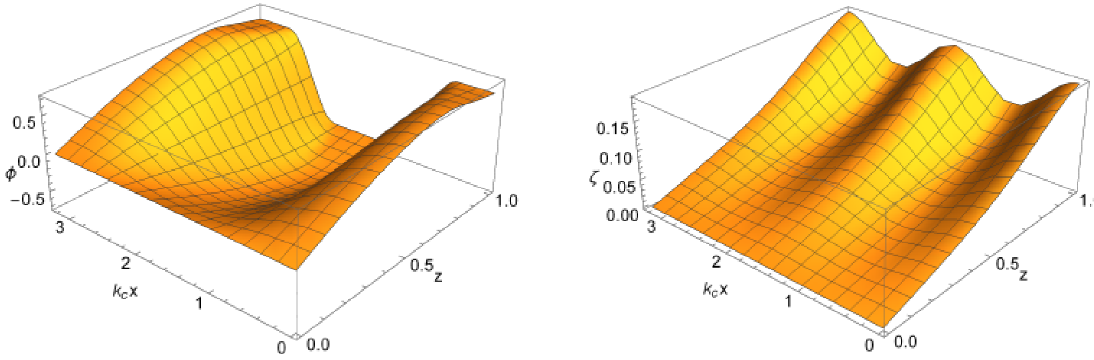
- **Phase diagram**

$\boxed{e=4}$



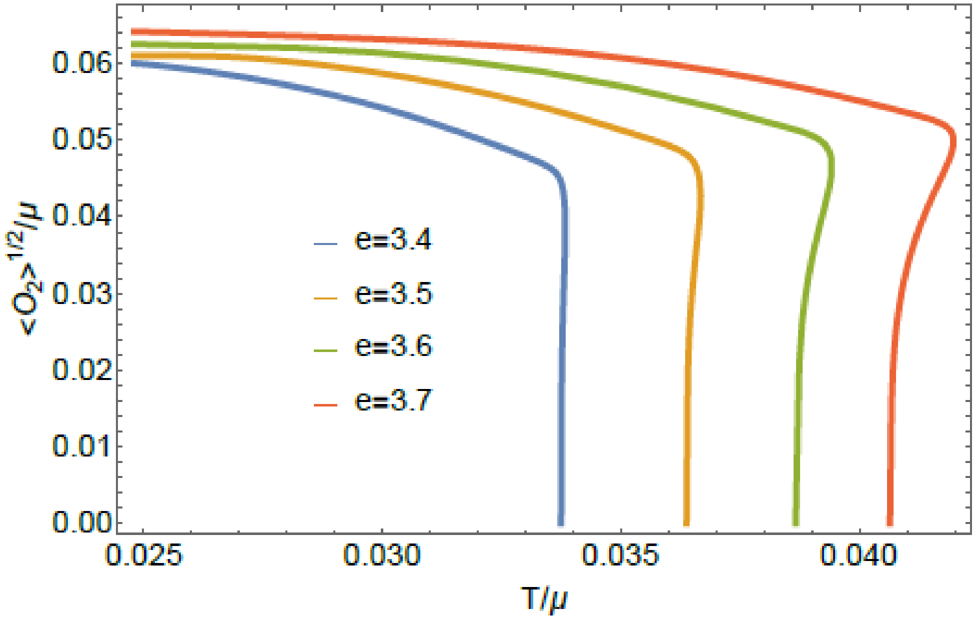
- **The solutions of background**

$\boxed{T=0.988T_c}$

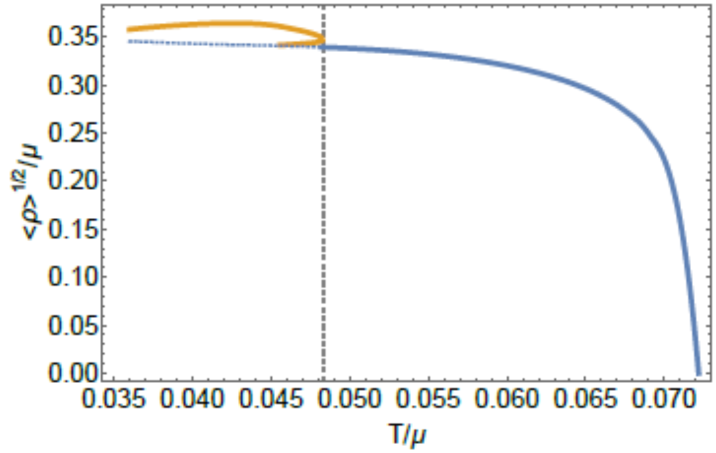


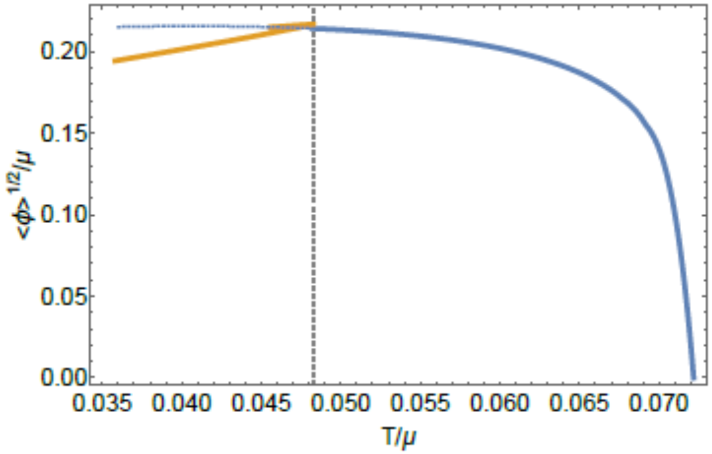
Holographic Superconductor Induced by CDW

- **The condensation of SC**



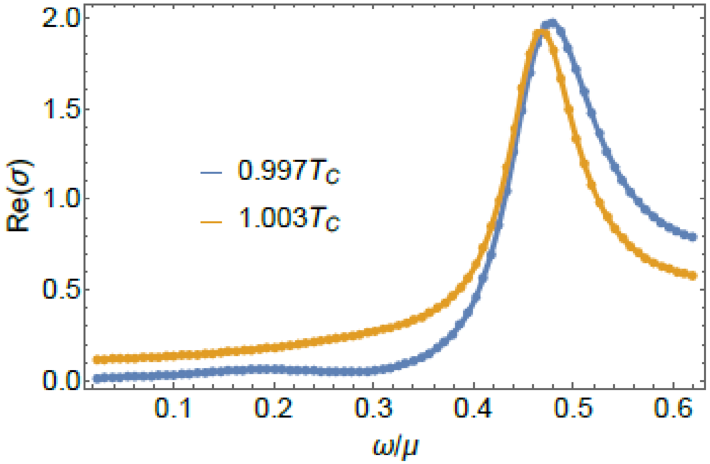
- **The leading order of the charge and the order parameter of CDW**

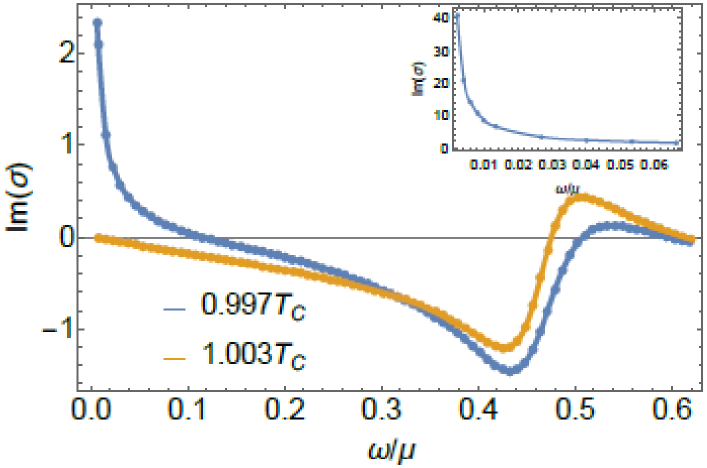




Holographic Superconductor Induced by CDW

- **The optical conductivity**





$$\boxed{\sigma(\omega) = i\frac{K_0}{\omega} + \frac{K_1\tau_1}{1 - i\omega\tau_1(1-\omega_{01}^2/\omega^2)} + \frac{K_2\tau_2}{1 - i\omega\tau_2(1-\omega_{02}^2/\omega^2)}}$$

$$b_x = (1 + j_x(x)z + \ldots)e^{-i\omega t}$$

$$\sigma(\omega/\mu) = \frac{4j_x^{(0)}}{i\omega}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, A_\mu = \bar{A}_\mu + \delta A_\mu, B_\mu = \bar{B}_\mu + \delta B_\mu,$$

$$\Phi = \bar{\Phi} + \delta\Phi, \qquad \eta = \bar{\eta} + \delta\eta, \qquad \theta = 0 + \delta\theta.$$

$$\nabla^\mu \hat{h}_{\mu\nu} = 0, \nabla^\mu a_\mu = 0, \nabla^\mu b_\mu = 0. \qquad \hat{h}_{\mu\nu} = h_{\mu\nu} - h\bar{g}_{\mu\nu}/2.$$

21 **linear equations !**

Summary

- A novel holographic model in which the role of CDW during the phase transition of superconductivity has been clearly disclosed.
- Superconductivity can form from the pre-existing CDW phase.
- The system is characterized by the coexistence of the CDW phase and the superconducting phase.
- CDW phase and superconducting phase can both cooperate and compete with each other.
- The pseudo-gap in CDW phase promotes the pre-formed pairs of carriers such that the formation of superconductivity benefits from the presence of CDW.

Next: $\rho_0 \neq 0$

Thank you !