Thermodynamics of Non-Dirac materials with Strong interaction

Yunseok Seo(GIST)

August 27, 2020

Based on:
JHEP06(2020)128 with Xian-hui Ge, Geunho Song and Sang-Jin Sin
And on-going work with Xian-hui Ge, Geunho Song and Sang-Jin Sin

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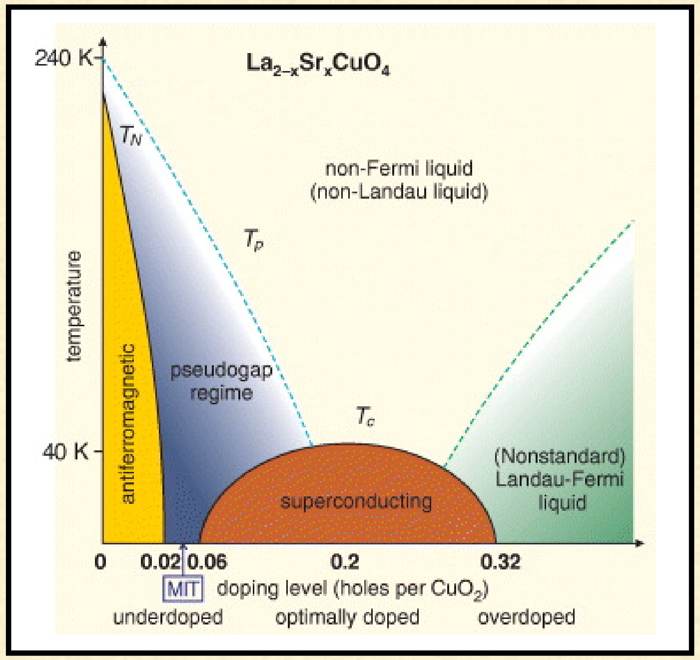
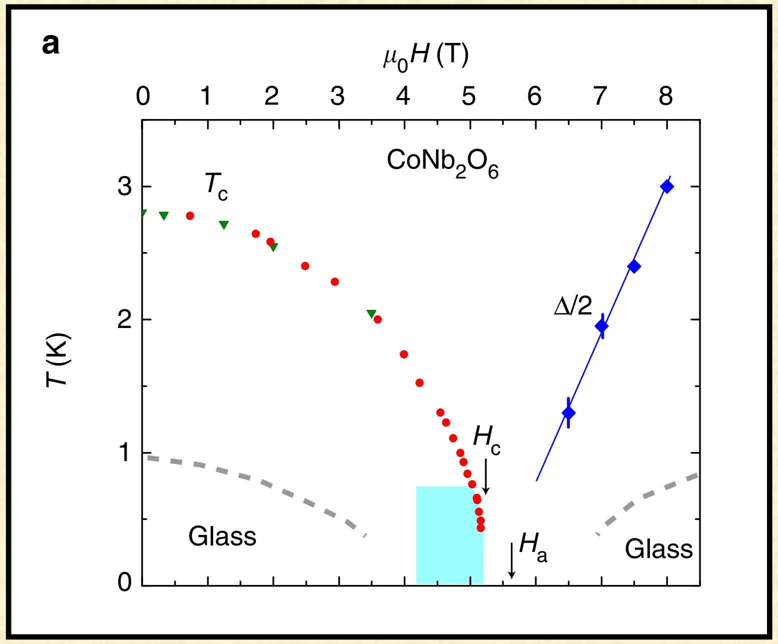
Motivation

- Strongly interacting materials





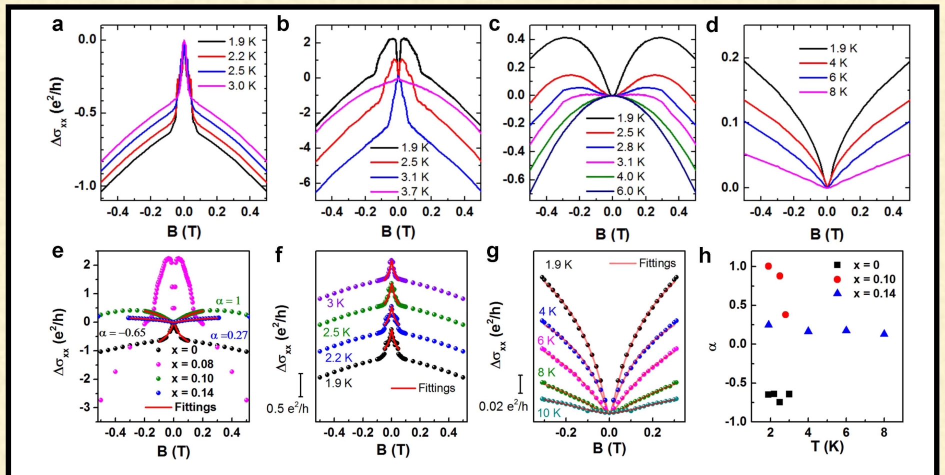
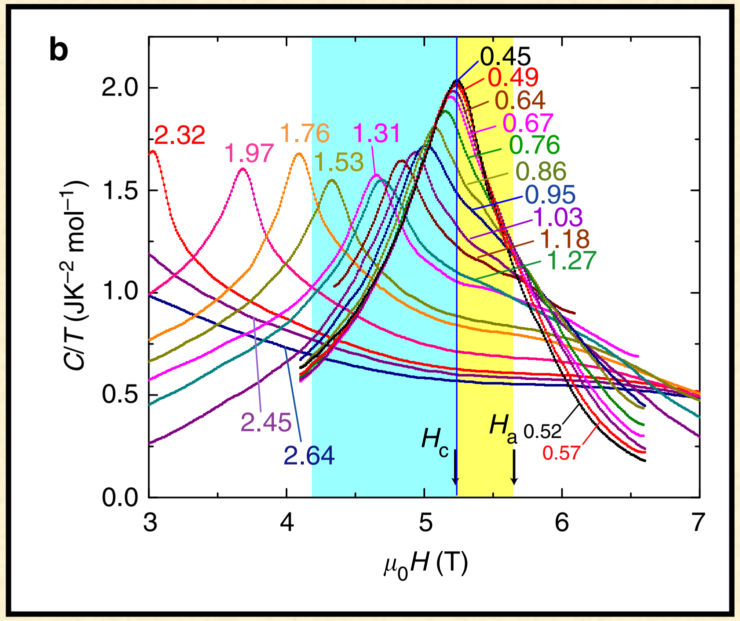


Figure 2 | Crossover of quantum corrections of magnetoconductance (MC) with increasing Cr content in $Cr_xBi_{2-x}Te_3$ thin films ($x \leq 0.14$). (a) MC curves of pure Bi_2Te_3 thin films, showing the negative MC features of WAL. (b) MC curves of $Cr_{0.08}Bi_{1.92}Te_3$ thin film, indicating a non-monotonic behavior with sharp downward cusp at low temperatures (< 3.1 K) and re-presence of WAL at higher temperatures (3.1 and 3.7 K). (c) MC curves of $Cr_{0.10}Bi_{1.90}Te_3$ thin film, showing a crossover from downward cusp feature to parabolic dependence with increasing temperatures. (d) MC curves of $Cr_{0.14}Bi_{1.86}Te_3$ thin film shows a WL dominated behavior. (e) HLN model fitting of MC curves with different Cr content at temperature of 1.9 K. (f) and (g) HLN model fitting of MC curves of pure Bi_2Te_3 and heavily doped $Cr_{0.14}Bi_{1.86}Te_3$ thin films, showing that both WAL and WL can be fitted well to the HLN model. (h) Pre-factor of α in HLN model of thin films with different Cr concentrations.



Motivation

- Quantum critical models can be characterized by
  - Dynamical critical exponent z: relative scaling of time and space $(t \to \lambda t, \quad x \to \lambda^z x)$
  - Hyperscaling-violation exponent. θ: deviation of the scaling of the low energy critical degrees of freedom from original space(effective theory live in $d-\theta$ dimension)

- Dirac materials
  - $z = 1, \quad \theta = 0$
  - Boundary geometry: AdS
  - Transport coefficient(2018), Two current model(2017), Spontaneous magnetization(2018, 2019)

- Non-Dirac materials(with magnetism)
  - $z \neq 1, \quad \theta \neq 0$
  - What is background geometry?
  - What is the role of each exponent to the transport coefficient and other thermodynamic observables?

Background Geometry

- Action

$$S_{tot} = \int_{\mathcal{M}} d^4x \left(\mathcal{L}_0 + \mathcal{L}_{int}\right)$$

$$\mathcal{L}_0 = \sqrt{-g}\left(R + \sum_{i=1}^{2} V_i e^{\gamma_i \phi} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\sum_{i=1}^{2} Z_i(\phi) F_{(i)}^2 - \frac{1}{2}Y(\phi)\sum_{i,I}^{2}(\partial\chi_I^i)^2\right)$$

$$\mathcal{L}_{int} = -\frac{q_\chi}{16}\sum_{I=1,2}(\partial\chi_I^{(2)})^2 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^{(2)} F_{\rho\sigma}^{(2)},$$

- Background solution

$$A_1 = a_1(r)dt, A_2 = a_2(r)dt + \frac{1}{2}H(xdy - ydx),$$

$$\chi_I^{(1)} = (\alpha x, \alpha y), \quad \chi_I^{(2)} = (\lambda x, \lambda y),$$

$$ds^2 = r^{-\theta}\left(-r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2(dx^2 + dy^2)\right),$$

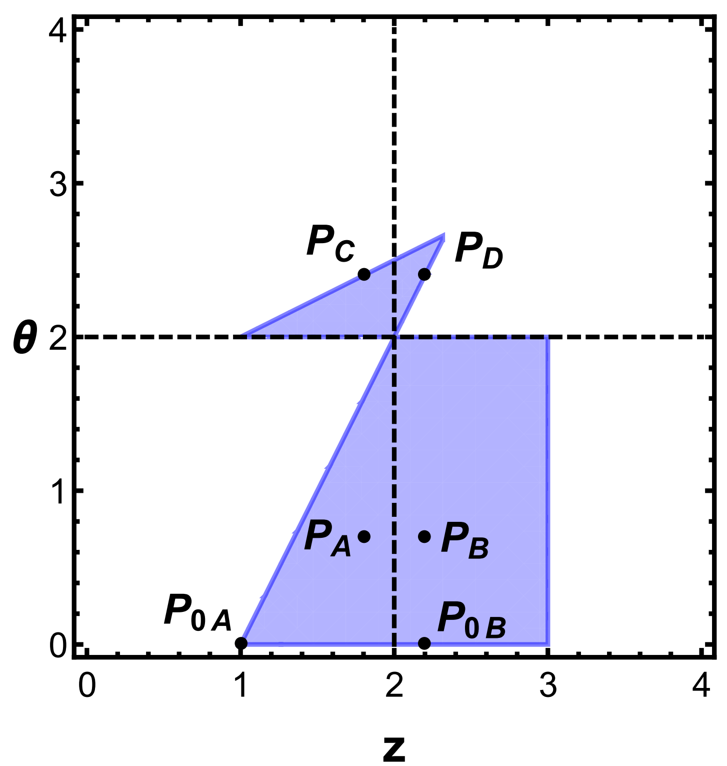
$$f(r) = 1 - m r^{\theta - z - 2} - \frac{\beta^2}{(\theta-2)(z-2)} r^{\theta - 2z} + \frac{q_2^2(\theta - z) r^{2\theta - 2z - 2}}{2(\theta - 2)} + \frac{H^2 r^{2z-6}}{4(z-2)(3z-\theta-4)} + \frac{\lambda^4 H^2 q_\chi^2 c_3}{r^{6+2z-4\theta}} - \frac{\lambda^2 H q_2 q_\chi c_2}{r^{4+2z-3\theta}},$$

$$a_1(r) = \frac{-q_1}{2+z-\theta}(r_{\rm H}^{2+z-\theta} - r^{2+z-\theta}), \quad a_2(r) = \left(\mu - q_2 r^{\theta - z}\right) - \frac{\lambda^2 H q_\chi c_4}{r^{z-2\theta+2}},$$

Background Geometry

- Validity regime
  - Charge reality condition
  - Null energy condition
  - Asymptotic HSV geometry

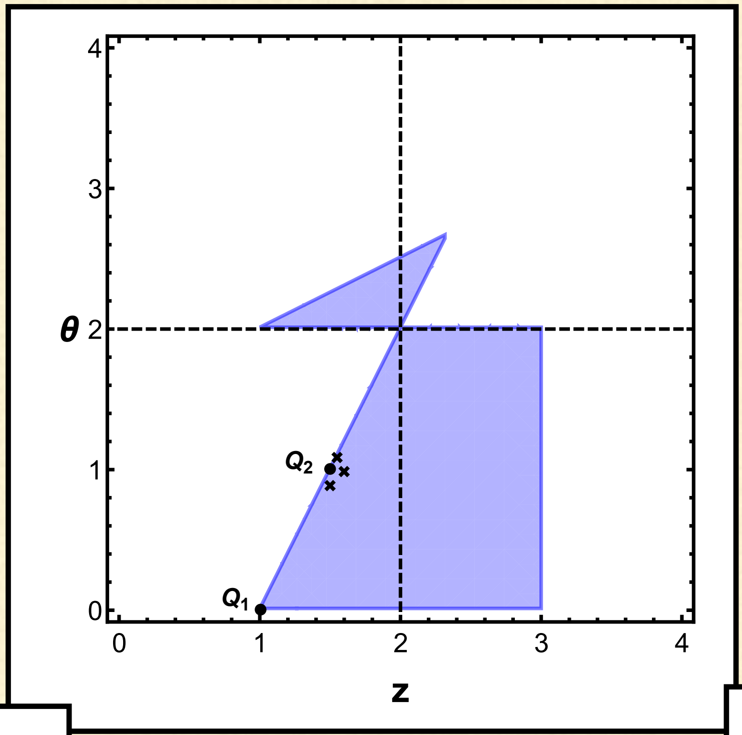
$$
\begin{aligned}
&A_1 = a_1(r)dt, A_2 = a_2(r)dt + \frac{1}{2}H(xdy - ydx),\\
&\quad \chi_I^{(1)} = (\alpha x, \alpha y), \quad \chi_I^{(2)} = (\lambda x, \lambda y),\\
&ds^2 = r^{-\theta}\Big(- r^{2z} f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2(dx^2 + dy^2)\Big),\\
&f(r) = 1 - mr^{\theta-z-2} - \frac{\beta^2}{(\theta-2)(z-2)} r^{\theta-2z} + \frac{q_2^2(\theta - z)r^{2\theta-2z-2}}{2(\theta-2)}\\
&\qquad + \frac{H^2 r^{2z-6}}{4(z-2)(3z-\theta-4)} + \frac{\lambda^4 H^2 q_\chi^2 c_3}{r^{6+2z-4\theta}} - \frac{\lambda^2 H q_2 q_\chi c_2}{r^{4+2z-3\theta}},\\
&a_1(r) = \frac{-q_1}{2+z-\theta}(r_{\mathrm{H}}^{2+z-\theta} - r^{2+z-\theta}), \;\; a_2(r) = \big(\mu - q_2 r^{\theta-z}\big) - \frac{\lambda^2 H q_\chi c_4}{r^{z-2\theta+2}},
\end{aligned}
$$

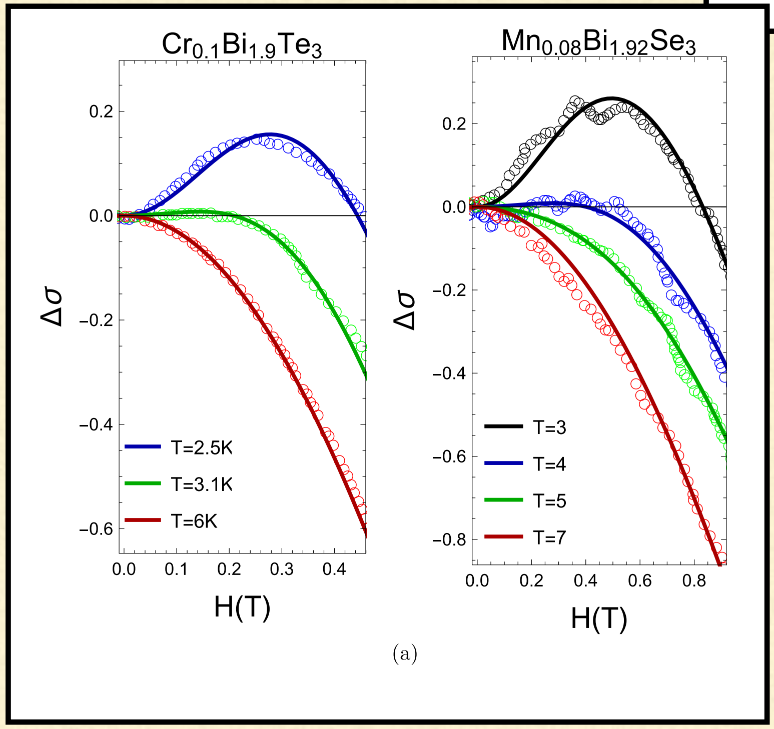


(b) Validity regime

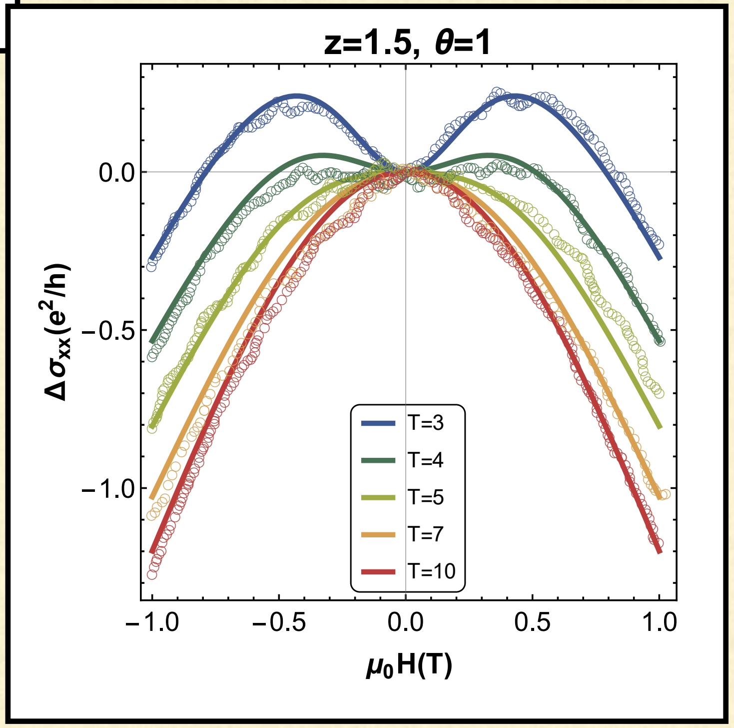
Transport coefficient

- Surface state of Topological Insulator





Phys. Rev. B **96**, 041104(2017): YS, G Song, S-J Sin



JHEP **06** (2020)128: X Ge, YS, G Song, S-J Sin

Transport coefficient

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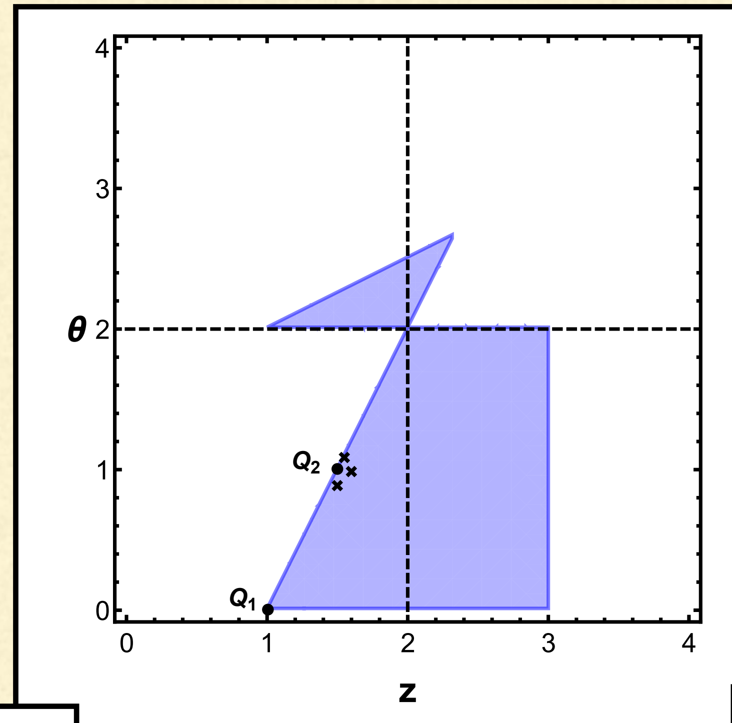

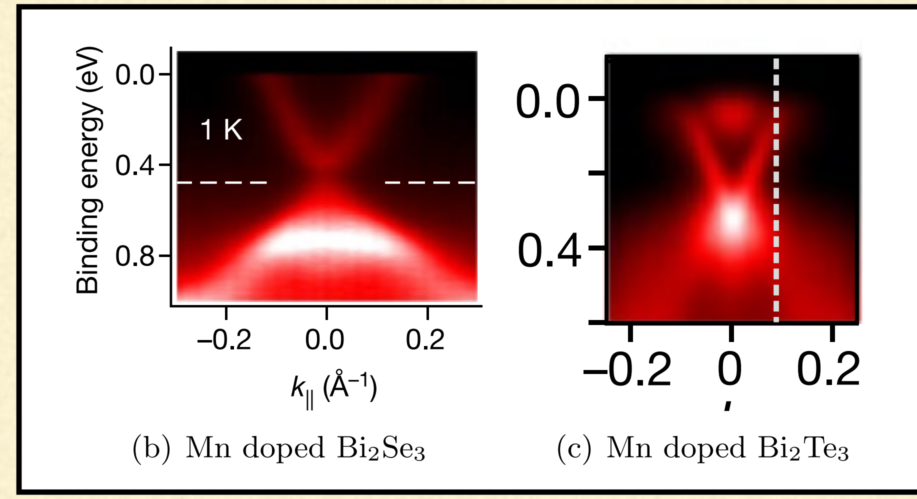

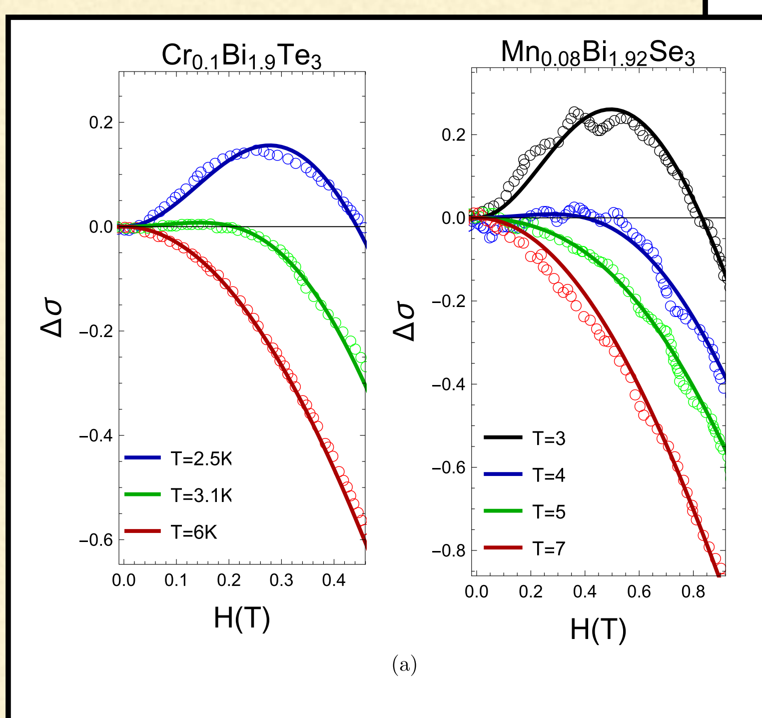

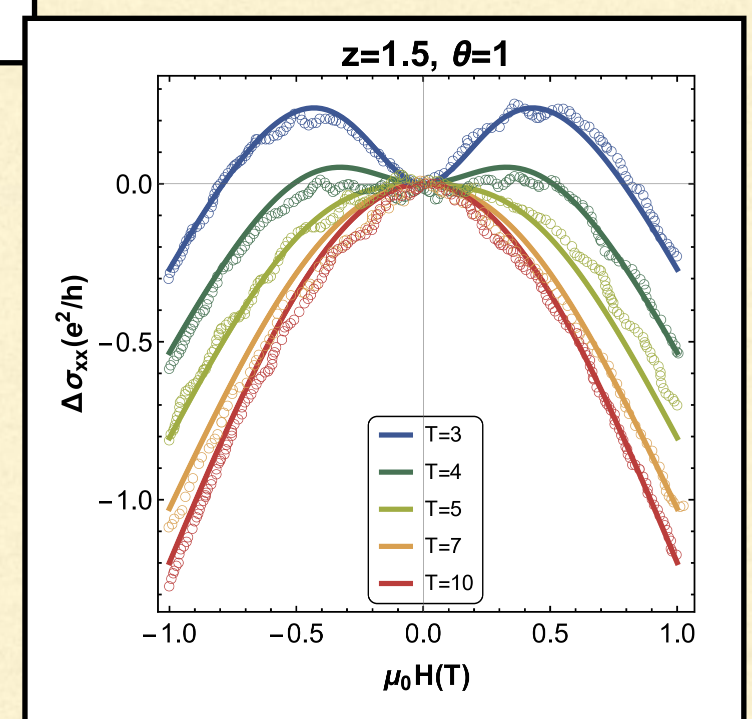

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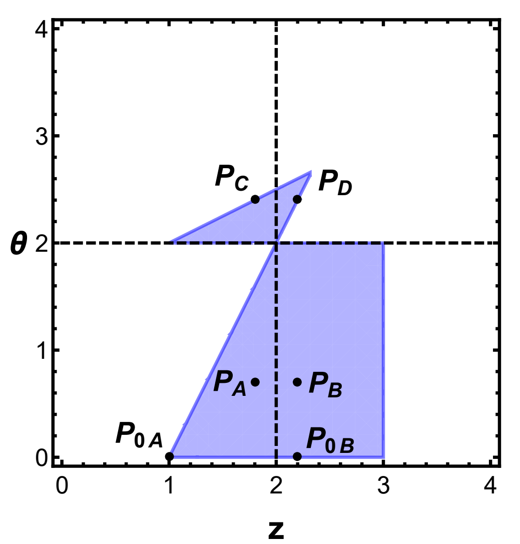
Holographic Renormalization

- The integration of the on-shell action is divergent

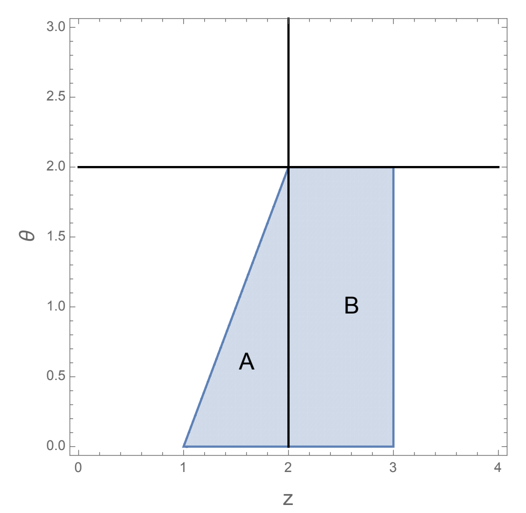
$$
\begin{aligned}
&S_{bulk} = \\
&S_0 + S_{int} = (2-\theta)\Lambda^{2+z-\theta} - \frac{1}{2}(\theta - z)q_2^2\Lambda^{\theta-z} + \frac{4z-\theta-6}{4(z-2)(3z-\theta-4)}H^2\Lambda^{-4+3z-\theta} \\
&\quad - \frac{\theta - z}{z-2\theta+2}q_\chi H q_2^2 \Lambda^{-2-z+2\theta} - \frac{3}{2(z-3\theta+4)}q_\chi^2\beta^4 H^2 \Lambda^{-4-z+3\theta},
\end{aligned}
$$

- Validity regime



(b) Validity regime

- Finite chemical potential condition



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Holographic Renormalization

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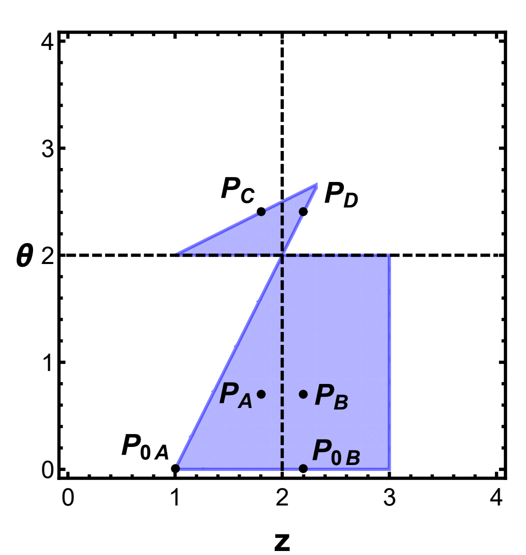
&\quad - \frac{\theta - z}{z-2\theta+2} q_\chi H q_2^2 \Lambda^{-2-z+2\theta} - \frac{3}{2(z-3\theta+4)} q_\chi^2 \beta^4 H^2 \Lambda^{-4-z+3\theta},
\end{aligned}
$$

- Validity regime



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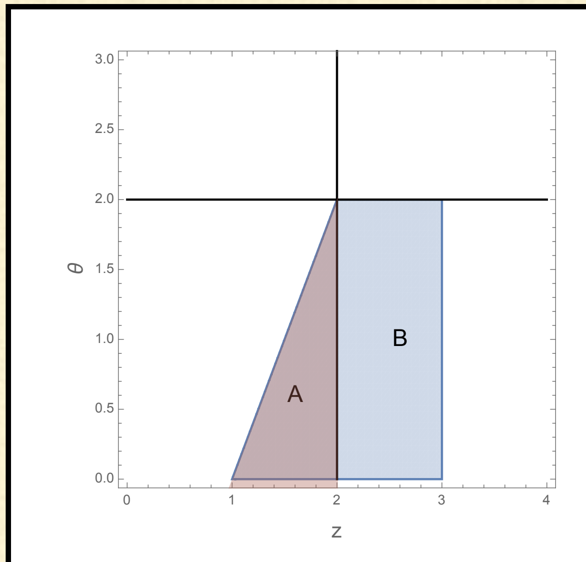

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&\qquad - \frac{\theta - z}{z - 2\theta + 2} q_\chi H q_2^2 \Lambda^{-2-z+2\theta} - \frac{3}{2(z-3\theta+4)} q_\chi^2 \beta^4 H^2 \Lambda^{-4-z+3\theta},
\end{aligned}
$$

- Boundary counter terms

$$
\begin{aligned}
S_b = &-\int d^3x \sqrt{-\gamma} \Big(-2K - (4z - 2\theta) e^{-\xi\phi} + (\mathcal{V}_1 e^{\xi\phi}) V_2 e^{\gamma_2 \phi} + \bar{\Pi}_1^t A_1^t \\
&+ \mathcal{V}_2 e^{\xi\phi} \sum_{I=1}^{2} \bar{\Pi}_\chi^\mu (\partial_\mu \chi_I) \Big)
\end{aligned}
$$

$$
ds_B^2 = dr^2 + \gamma_{ij}(r,t) dx^i dx^j
$$

$$
\begin{aligned}
K_{ij} &= -\frac{1}{2}\partial_r \gamma_{ij} \\
\bar{\Pi}_1^i &= -Z_1\, \gamma^{ij} F_{rj}^1 \\
\bar{\Pi}_{\chi_I}^\mu &= -Y\, \partial^\mu \chi_I .
\end{aligned}
$$

$$
\begin{aligned}
\xi &= -\frac{\theta}{2\nu} \\
\mathcal{V}_1 &= \frac{z-\theta-3}{(2z-\theta-2)(3z-\theta-4)} \\
\mathcal{V}_2 &= \frac{z+\theta-3}{2(z-2)(\theta-2)},
\end{aligned}
$$

'18: Cremonini, Cvetic, Papadimitriou

Holographic Renormalization

- Renormalized on-shell action

$$S^E = (1-\theta)m + (\theta-2)v_H^{2+z-\theta} + \frac{1}{2}(\theta-z)v_H^{\theta-z}q_2^2 + \frac{(4z-\theta-6)H^2}{4(z-2)(3z-\theta-4)}v_H^{-4+3z-\theta}$$
$$-\frac{\theta-z}{z-2\theta+2}q_\chi H q_2^2 v_H^{-2-z+2\theta} - \frac{3}{2(z-3\theta+4)}q_\chi^2\beta^4 H^2 v_H^{-4-z+3\theta},$$

- Boundary counter terms

$$S_b = -\int d^3x\sqrt{-\gamma}\Big(-2K-(4z-2\theta)e^{-\xi\phi}+(\mathcal{V}_1 e^{\xi\phi})V_2 e^{\gamma_2\phi}+\bar{\Pi}_1^t A_1^t + \mathcal{V}_2 e^{\xi\phi}\sum_{I=1}^{2}\bar{\Pi}_\chi^\mu(\partial_\mu\chi_I)\Big)$$

$$ds_B^2 = dr^2 + \gamma_{ij}(r,t)dx^i dx^j$$

$$\xi = -\frac{\theta}{2\nu}$$
$$\mathcal{V}_1 = \frac{z-\theta-3}{(2z-\theta-2)(3z-\theta-4)}$$
$$\mathcal{V}_2 = \frac{z+\theta-3}{2(z-2)(\theta-2)},$$

$$K_{ij} = -\frac{1}{2}\partial_r\gamma_{ij}$$
$$\bar{\Pi}_1^i = -Z_1\,\gamma^{ij}F_{rj}^1$$
$$\bar{\Pi}_{\chi_I}^\mu = -Y\,\partial^\mu\chi_I.$$

'18: Cremonini, Cvetic, Papadimitriou

Thermodynamics

- Standard AdS/CFT
  - Energy density: Boundary energy momentum tensor
  - Pressure: Negative renormalized on-shell action
  - Entropy: Horizon area
  - Temperature: Hawking temperature of BH

$$\epsilon = T_{00}$$
$$\mathcal{P} = -S^E$$

- Smarr relation

$$\epsilon + \mathcal{P} = s\,T + \mu Q.$$

- Thermodynamic First Law

$$\delta\epsilon = T\delta s + \mu\delta Q.$$

Guidline

Thermodynamics

- Thermodynamic relation

$$\epsilon = z\tilde{S}^E + s\,T + \frac{(1+z-\theta)}{(2-\theta)}\mu Q = (2-\theta)m,$$

$$\tilde{S}^E \equiv S^E + \frac{\theta(1-z)\beta^2}{z(2-z)(\theta-2)} v_H^{2-z}.$$

$$Q = (z-\theta)q_2$$

- Thermodynamic First Law

$$\delta\epsilon = T\,\delta s + \mu\delta Q - M\delta H,$$

$$M = \frac{(\theta-2)v_H^{-4+3z-\theta}}{2(z-2)(3z-4-\theta)}H + \frac{q_\chi\beta^2 Q\, v_H^{-2-z+2\theta}}{2+z-2\theta} - \frac{q_\chi^2\beta^4\, v_H^{-4-z+3\theta}}{4+z-3\theta}H.$$

Thermodynamics

- Specific heat

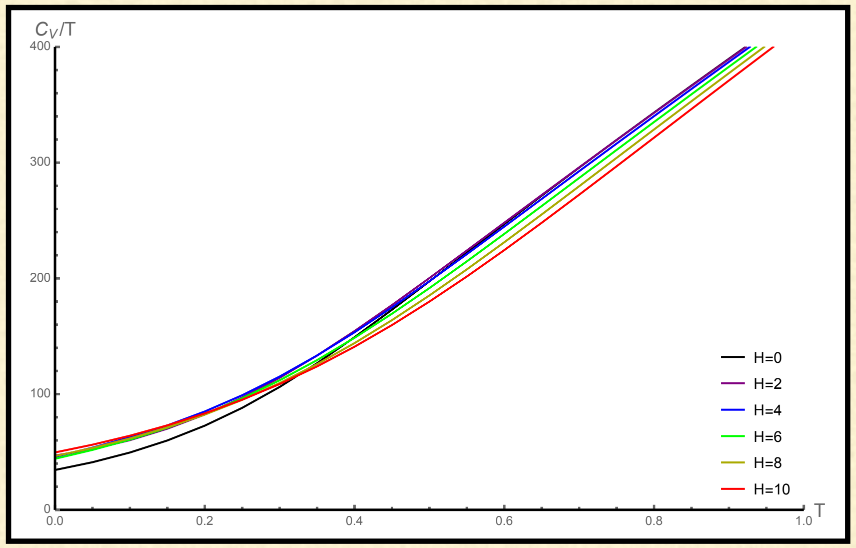
$$c_V = T\frac{\partial s}{\partial T} = \frac{\partial \epsilon}{\partial T}$$

- High temperature behavior

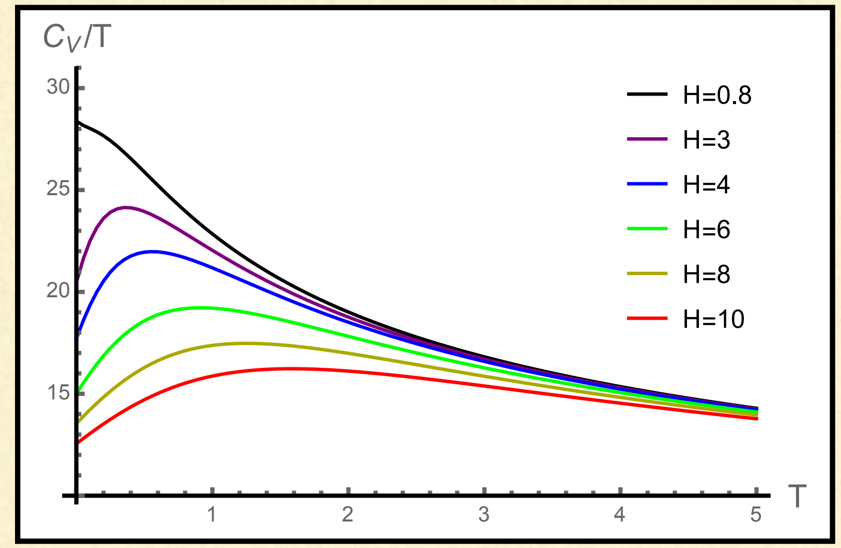
$$c_V \sim T^{\frac{2-\theta}{z}} \rightarrow \begin{array}{l} T^2 \ (z=1,\theta=0) \\ T^{2/3} \ (z=\frac{3}{2},\theta=1) \\ T \ (z=\frac{4}{3},\theta=\frac{2}{3}) \end{array}$$

Thermodynamics

- Numerical results
  - (z=1, θ=0)

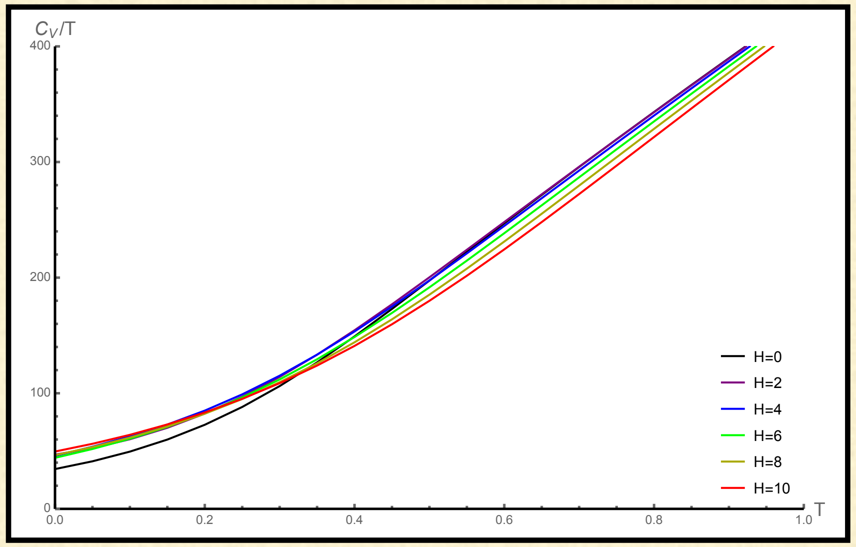


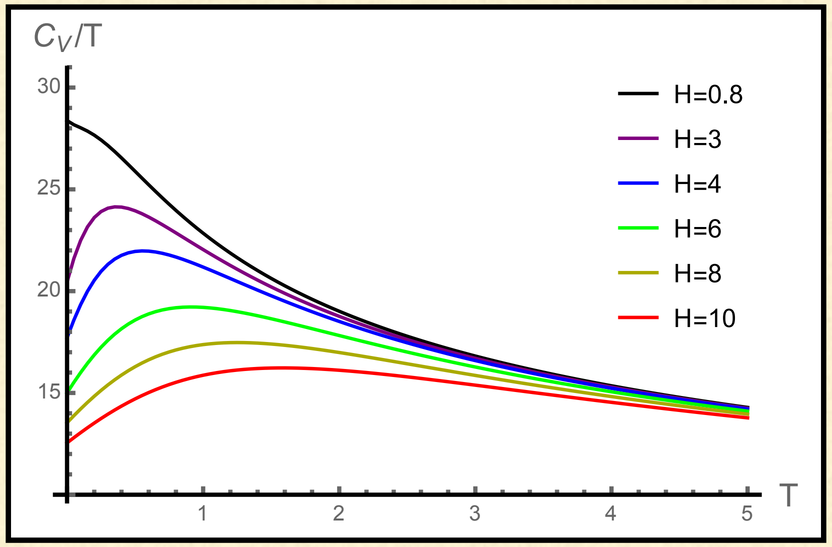
  - (z=3/2, θ=1)

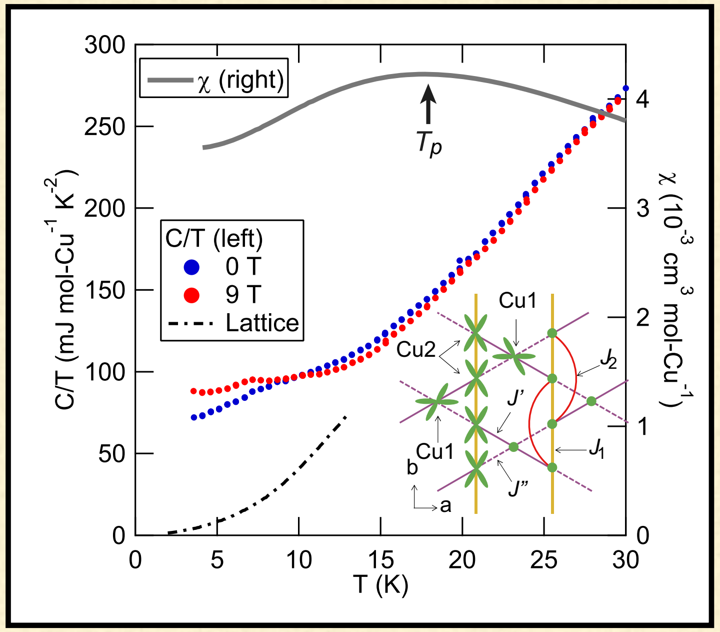


Thermodynamics

- Numerical results
  - (z=1, θ=0)
  - (z=3/2, θ=1)

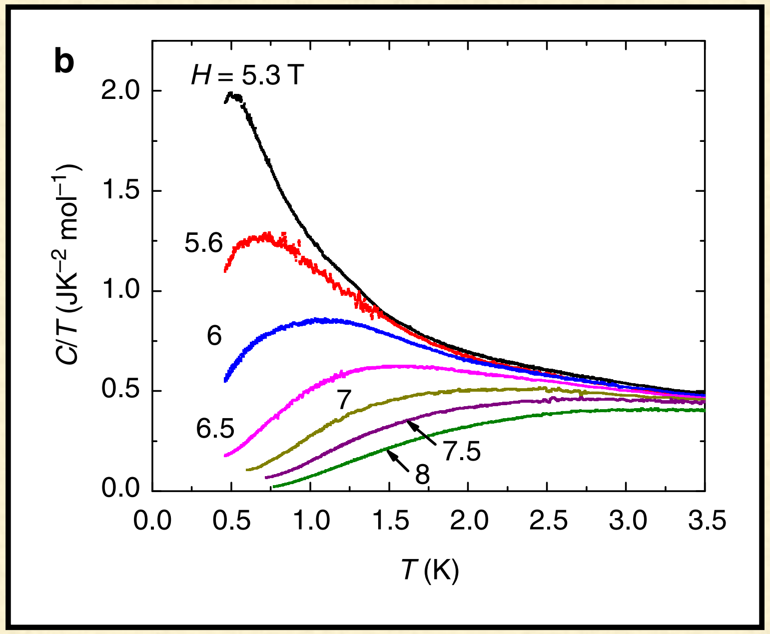






Emergence of nontrivial magnetic excitations in a spin-liquid state of kagomé volborthite

www.pnas.org/cgi/doi/10.1073/pnas.1524076113



Heat capacity peak at the quantum critical point of the transverse Ising magnet $CoNb_2O_6$

NATURE COMMUNICATIONS | 6:7611 | DOI: 10.1038/ncomms8611 |

Conclusion

- We study transport coefficient of strongly interacting non-Dirac materials via holography
- We construct renormalized on-shell action in HSV geometry
- We construct modified thermodynamic relation such that the thermodynamic 1st law of the boundary theory satisfied

- Role of (z, θ) in the thermodynamic relation: Difference of scaling behavior between time and space
- Role of extra term in pressure: Breaking diffeomorphism invariance by β

- Future direction
    - Holographic renormalization for other (z, θ) region
    - Comparing other transport coefficient(thermal conductivity…)
    - AC conductivities

Thank you!

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Thermodynamics

- Schwartzshcild HSV $(q_2 = 0,\ H = 0,\ \beta = 0,\ q_\chi = 0)$

$$S^E = -v_H^{2+z-\theta}$$
$$s = 4\pi v_H^{2-\theta}$$
$$T = \frac{(2+z-\theta)}{4\pi} v_H^z .$$

- Naive Smarr relation

$$\epsilon = S^E + s\,T = (1+z-\theta)v_H^{2+z-\theta} = (1+z-\theta)m,$$ $\longrightarrow$ $\delta\epsilon \neq T\,\delta s$

- Modified thermodynamic relation

$$\epsilon = zS^E + s\,T = (2-\theta)v_H^{2+z-\theta} = (2-\theta)m.$$ $\longrightarrow$ $\delta\epsilon = T\delta s.$

Thermodynamics

- RN HSV $(q_2 \neq 0,\ H = 0,\ \beta = 0,\ q_\chi = 0)$

$$S^E = -v_H^{2+z-\theta} + \frac{(z-\theta)}{2(\theta-z)} q_2^2\, v_H^{\theta-z}$$
$$s = 4\pi v_H^{2-\theta}$$
$$T = \frac{(2+z-\theta)}{4\pi} v_H^z + \frac{(z-\theta)^2 q_2^2}{8\pi(\theta-2)} v_H^{-2-z+2\theta}.$$

- Modified thermodynamic relation

$$\epsilon = zS^E + s\,T + \frac{(z-\theta)(1+z-\theta)}{(2-\theta)} \mu q_2 = (2-\theta)m.$$

$$\longrightarrow \quad \delta\epsilon = T\,\delta s + \mu\delta Q,$$

$$Q = (z-\theta)q_2$$

Thermodynamics

- Schwartzscild with momentum relaxation $(q_2 = 0,\ H = 0,\ \beta \neq 0,\ q_\chi = 0)$

$$S^E = -v_H^{2+z-\theta} + \frac{(\theta-1)\beta^2}{(z-2)(\theta-2)} v_H^{2-z}$$

$$s = 4\pi v_H^{2-\theta}$$

$$T = \frac{(2+z-\theta)}{4\pi} v_H^z - \frac{\beta^2}{4\pi(2-\theta)} v_H^{-z+\theta}.$$

- Modified thermodynamic relation

$$\epsilon = z\,S^E + s\,T \neq (2-\theta)m, \quad \longrightarrow \quad \delta\epsilon \neq T\,\delta s$$

- Modified pressure

$$\tilde{S}^E \equiv S^E + \frac{\theta(1-z)\beta^2}{z(2-z)(\theta-2)} v_H^{2-z}. \quad \longrightarrow \quad \begin{aligned} \epsilon &= z\,\tilde{S}^E + s\,T = (2-\theta)m, \\ \delta\epsilon &= T\delta s. \end{aligned}$$

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Thermodynamics

- Final thermodynamic relation

$$\epsilon = z\tilde{S}^E + s\,T + \frac{(1+z-\theta)}{(2-\theta)}\mu Q,$$

$$\tilde{S}^E \equiv S^E + \frac{\theta(1-z)\beta^2}{z(2-z)(\theta-2)} v_H^{2-z}.$$

- Thermodynamic First Law

$$\delta\epsilon = T\,\delta s + \mu\delta Q - M\delta H,$$

$$M = \frac{(\theta-2)v_H^{-4+3z-\theta}}{2(z-2)(3z-4-\theta)}H + \frac{q_\chi\beta^2 Q\, v_H^{-2-z+2\theta}}{2+z-2\theta} - \frac{q_\chi^2\beta^4\, v_H^{-4-z+3\theta}}{4+z-3\theta}H.$$