Pole-skipping phenomena in CFTs

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Reminder: pole-skipping points

Intersections between lines of poles and lines of zeros in momentum space of Green's function

- $G(\omega, k) = \dfrac{\omega + k}{\omega - k}$ has a pole-skipping point at $\omega = k = 0$.

- At pole-skipping points,
  $G(\omega_*, k_*)$ is not well-defined: $G(\omega_*, k_*) \sim \dfrac{0}{0}$.

I will talk pole-skipping points in CFTs.

Pole-skipping point's study in CFTs

[F. M. Haehl, W. Reeves, M. Rozali, 2019] calculated pole-skipping points in two point function of T_{00} on $S^1 \times \mathbb{H}^{d-1}$.

- They showed that a pole-skipping point is related to $\lambda_L = 2\pi/\beta, v_B = 1/(d-1)$ in holographic CFTs.

$$\frac{\langle V(t,\mathbf{d})W(0,0)V(t,\mathbf{d})W(0,0)\rangle}{\langle V(t,\mathbf{d})V(t,\mathbf{d})\rangle\langle W(0,0)W(0,0)\rangle} \sim 1 - \varepsilon e^{\lambda_L(t-t_s-\mathbf{d}/v_B)} + \cdots$$

holographic CFTs [E. Perlmutter, 2016] [N. A-Jeddi et al. 2017]

pole-skipping point of $T_{\mu\nu}$ $\sim$ (universal in any CFTs) conformal block with $T_{\mu\nu}$ exchange

Our expectation

Even in fields other than $T_{\mu\nu}$,
the late time behavior of conformal blocks
and pole-skipping points are related.

- Is it general in various spin?

- We can **analytically** compute conformal two point function on $S^1 \times \mathbb{H}^{d-1}$.

- We check our expectation in scalar and vector cases by computing pole-skipping points on hyperbolic space.

Outline

1. Late time behavior of conformal block in OTOC

2. Pole-skipping analysis of two point functions

3. Summary

Conformal map between $S^1 \times \mathbb{H}^{d-1}$ and $\mathbb{R}^d$

$$x^\mu = (\rho \sin \tau, \rho \cos \tau, x_\perp^i)$$

Period of Euclidean time τ

$$\beta = 2\pi$$

$$ds^2_{S^1 \times \mathbb{H}^{d-1}} = d\tau^2 + \frac{1}{\rho^2}\left(d\rho^2 + dx_\perp^i dx_{\perp i}\right) = \frac{1}{\rho^2}\delta_{\mu\nu}dx^\mu dx^\nu$$

By this transformation, thermal CFT's correlation functions in the hyperbolic space can be computed by ones in $\mathbb{R}^d$.

[H. Casini, M. Huerta, R. C. Myers, 2011]

Analytic continuation of $\langle V(x_1)V(x_2)W(x_3)W(x_4)\rangle$ to OTOC $\langle V(t,\mathbf{d})W(0,0)V(t,\mathbf{d})W(0,0)\rangle$

introduction of real time t_a $\quad \tau_a = it_a + \delta_a$

real time of V $\quad t_1 = t_2 = t$

real time of W $\quad t_3 = t_4 = 0$

Spatial distance between V and W $\quad \mathbf{d}(1,3) = \mathbf{d}$

Spatial distance between the same operators

$$\mathbf{d}(1,2) = \mathbf{d}(3,4) = 0$$

For OTOC, we need to impose

$$\delta_1 > \delta_3 > \delta_2 > \delta_4.$$

Analytic continuation of Euclidean conformal block $G_{\Delta}^{(\ell)}(u,v)$ for OTOC

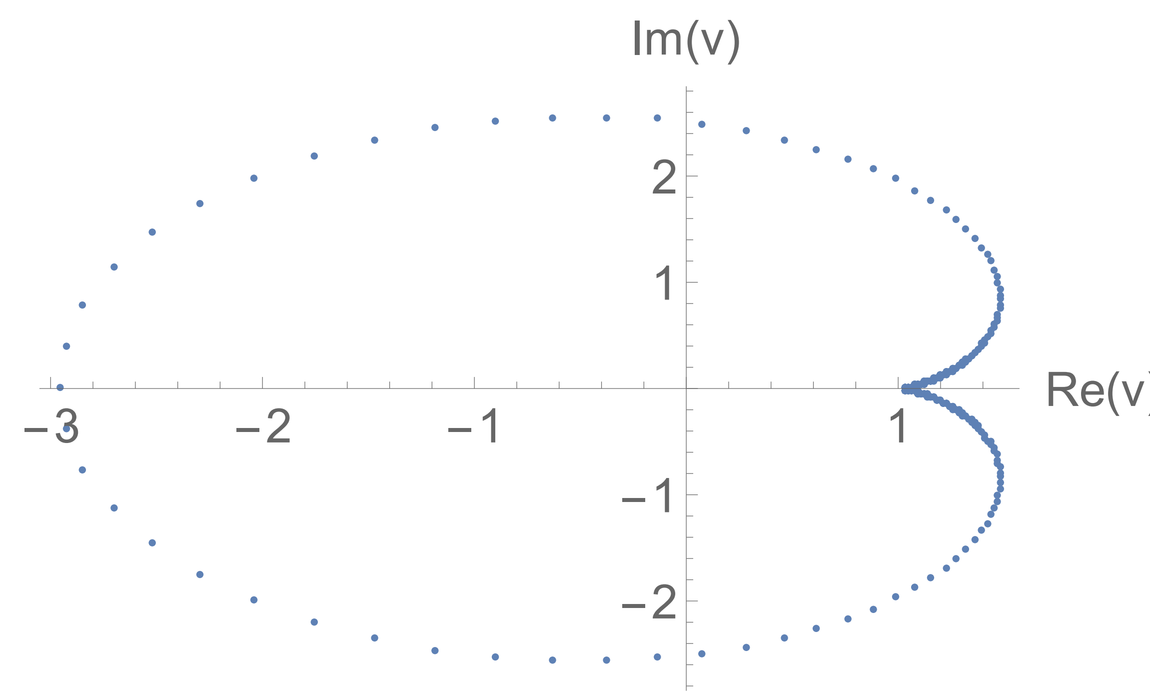
Four point functions can be expanded in terms of conformal blocks $G_{\Delta}^{(\ell)}(u,v)$.

$$\langle V(x_1)V(x_2)W(x_3)W(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_V}x_{34}^{2\Delta_W}}\sum_{\mathcal{O}} C_{VV\mathcal{O}}C_{WW\mathcal{O}}G_{\Delta}^{(\ell)}(u,v)$$

We need analytic continuation of

$$u := \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v := \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Under the time evolution, v rotates around $v=0$ clock-wisely.



Late time behavior of conformal block $G^{(\ell)}_{\Delta}(u, v \to e^{-2\pi i} v)$ in OTOC

Since $G^{(\ell)}_{\Delta}(u, v)$ is singular at $v = 0$, we should consider $G^{(\ell)}_{\Delta}(u, v \to e^{-2\pi i} v)$ in OTOC at late time.

At late time $\quad t \gg \mathbf{d} \gg 1$

$$u \sim e^{-2t}, \qquad v \sim 1 + e^{-t+\mathbf{d}}$$

Late time behavior

$$G^{(\ell)}_{\Delta}(u, v \to e^{-2\pi i} v) \sim u^{\frac{1}{2}(\Delta - \ell)}(1-v)^{1-\Delta} \sim e^{(\ell-1)t-(\Delta-1)\mathbf{d}}$$

e.g. energy momentum tensor $(\Delta = d, \ell = 2)$

$$e^{t-(d-1)\mathbf{d}}$$

This behavior is consistent with holographic computation.

[Y. Ahn, V. Jahnke, H.-S. Jeong, K.-Y. Kim, 2019]

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CFT analysis of two point functions

Scalar two point function on $S^1 \times \mathbb{H}^{d-1}$

$$\mathcal{G}^{\Delta}(P_1, P_2) = \frac{1}{(2\cosh \mathbf{d}(1,2) - 2\cos(\tau_1 - \tau_2))^{\Delta}}$$

Laplacian on $S^1 \times \mathbb{H}^{d-1}$

$$\Box_{S^1 \times \mathbb{H}^{d-1}} = \partial_\tau^2 + \rho^2 \partial_\rho^2 - (d-3)\rho\partial_\rho + \rho^2 \Box_{\mathbb{R}^{d-2}}$$

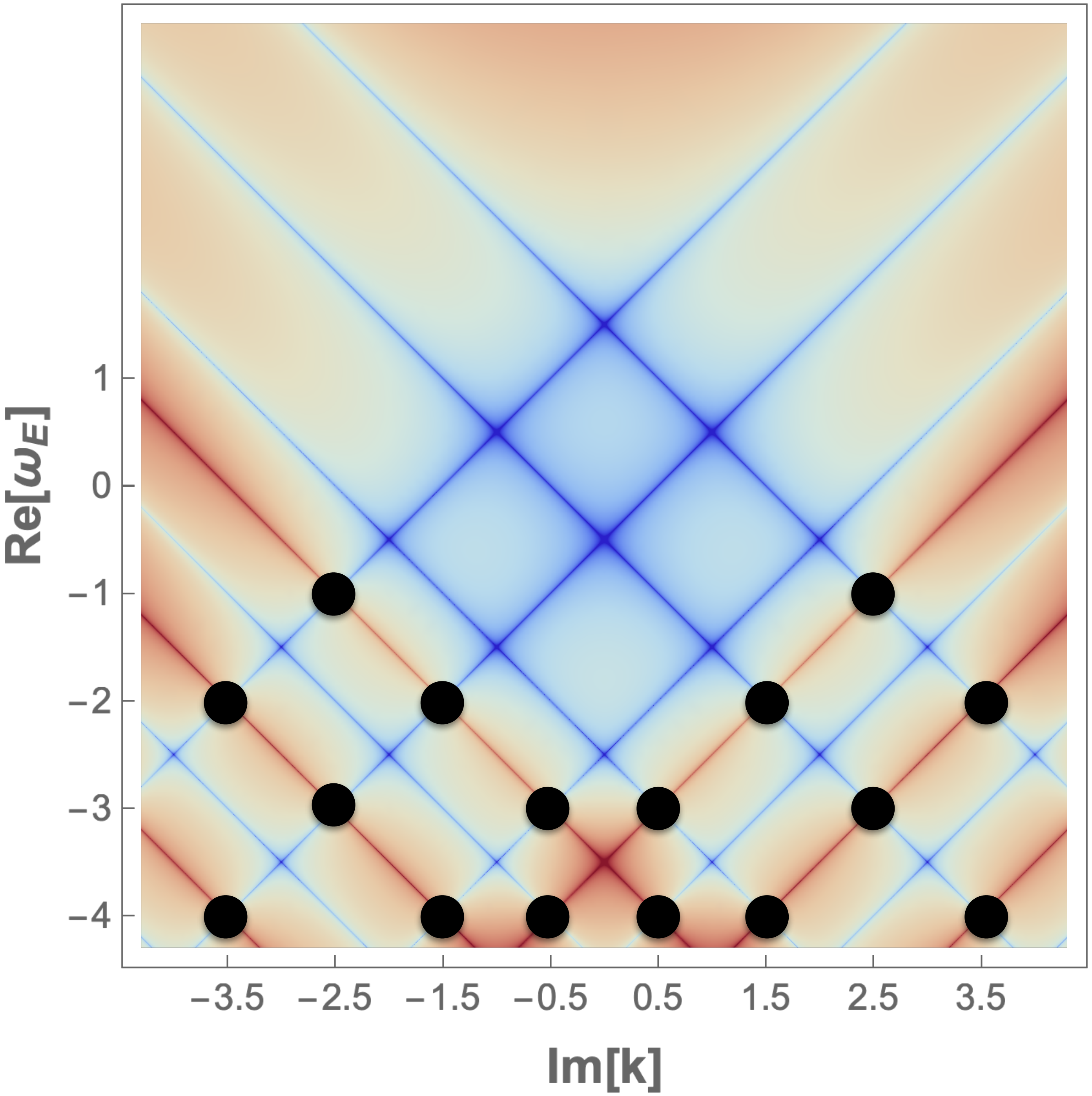
eigenfunction

$$f(P; \omega_E, k, \vec{p}_\perp) \propto \rho^{\frac{d-2}{2}} K_{ik}(|p_\perp|\rho) e^{i(\omega_E \tau + \vec{p}_\perp \cdot \vec{x}_\perp)}$$

We compute momentum two point function $\mathcal{G}^{\Delta}(\omega_E, k)$ of $\mathcal{G}^{\Delta}(P_1, P_2)$ for the eigenmode $f(P; \omega_E, k, \vec{p}_\perp)$.

[S. Ohya, 2016], [F. M. Haehl, W. Reeves, M. Rozali, 2019]

$$\mathcal{G}^{\Delta}(\omega_E, k) \propto \frac{\Gamma(\frac{1}{2}(\omega_E + ik + \Delta - d/2 + 1))\Gamma(\frac{1}{2}(\omega_E - ik + \Delta - d/2 + 1))}{\Gamma(\frac{1}{2}(\omega_E + ik - \Delta + d/2 + 1))\Gamma(\frac{1}{2}(\omega_E - ik - \Delta + d/2 + 1))}$$



$\log|\mathcal{G}^{\Delta}(\omega_E, k)|$

with $d = 4, \Delta = 4.5$

Red lines: poles

Blue lines: zeros

Black dots:
pole-skipping points

There are the leading pole-skipping points
at $\omega_{E*} = -1, k_* = \pm i(\Delta - d/2)$.

Eigenfunctions at the leading pole-skipping points $\omega_{E*} = \ell - 1, k_* = \pm i(\Delta - d/2)$

Eigenfunction's behavior at small $\rho \sim e^{-\mathbf{d}}$

$$e^{\omega_E t} e^{\left(-\frac{d-2}{2} + ik\right)\mathbf{d}}$$

At the leading pole-skipping points,

$$e^{\omega_{E*} t} e^{\left(-\frac{d-2}{2} + ik_*\right)\mathbf{d}} = \begin{cases} e^{(\ell-1)t-(\Delta-1)\mathbf{d}} \\ e^{(\ell-1)t-(d-\Delta-1)\mathbf{d}} \end{cases}$$

They agree with conformal block
and shadow conformal block's behavior.

Summary

- We study the pole-skipping points of scalar and vector thermal two point functions in CFTs on hyperbolic space.

- We show that the leading pole-skipping points are related to the late time behavior of conformal block in four point OTOCs.