Quantum chaos in topologically massive gravity

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based on: 2005.08508 (with Avinash Raju)

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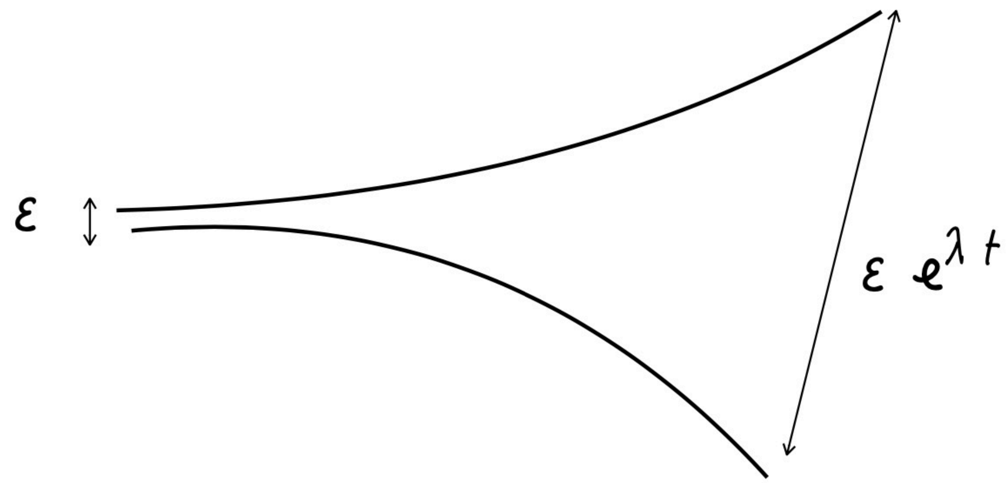
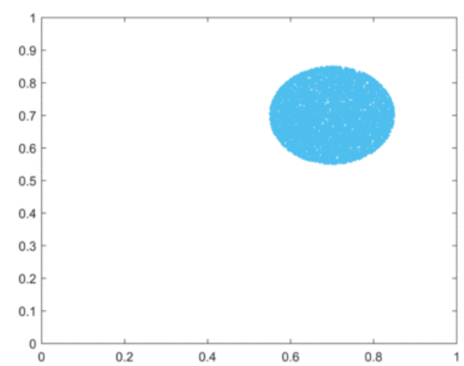
B E I H A N G U N I V E R S I Y T

Classical Chaos

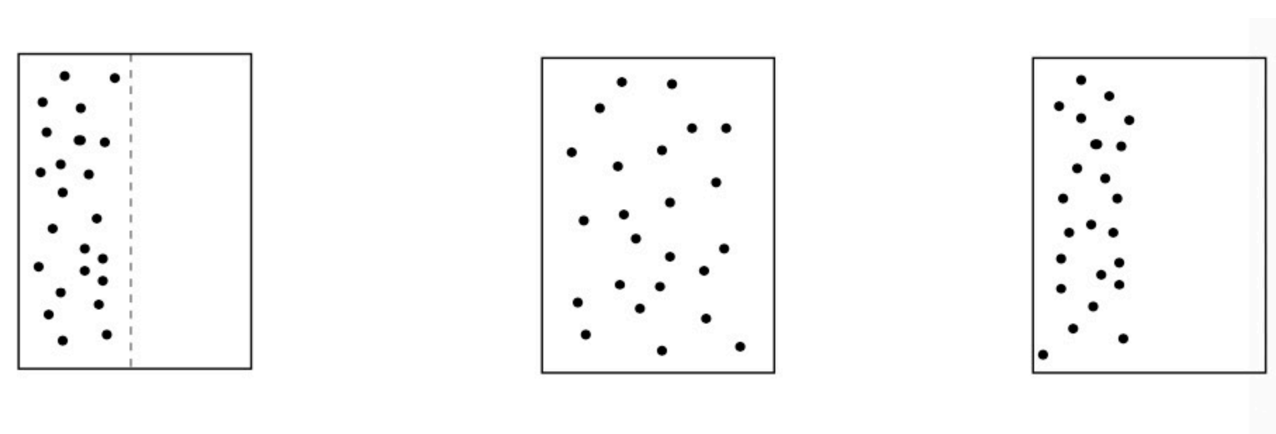
- Early time: exponential sensitivity of phase-space trajectories to the initial conditions



- Long time: mixing



- Very late time: Poincare recurrences

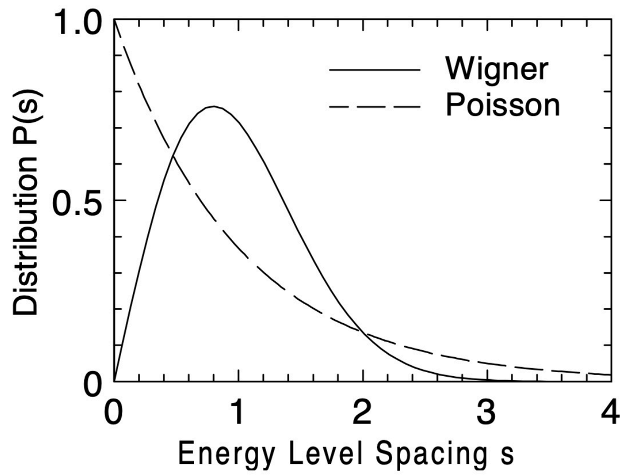


Classical Chaos vs Quantum Mechanics

- In quantum systems, the 'classical' coarse-graining is set by $\hbar$
- The picture above might not be useful (after Ehrenfest time scale) $t_{\text{Ehrenfest}} \sim \frac{1}{\lambda} \log \left(\int_{\Sigma} p dq / \hbar \right)$
- How the previous discussion should be modified due to QM
- Classical chaos for nonzero $\hbar$? Quantum chaos?

Quantum Chaos

- Quantize the classical chaotic system: chaotic systems have statistical level repulsion characteristic of random matrices [Review by D'Alessio, Kafri, Polkovikov, Rigol, 1509.06411]



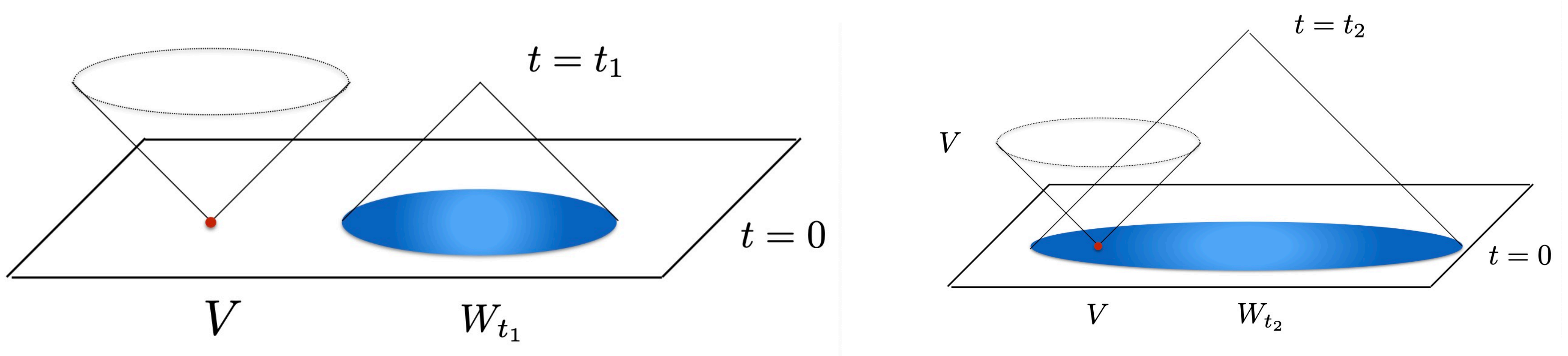
- Local chaotic behavior can be generalized on the short time scale (semi-classical intuition)

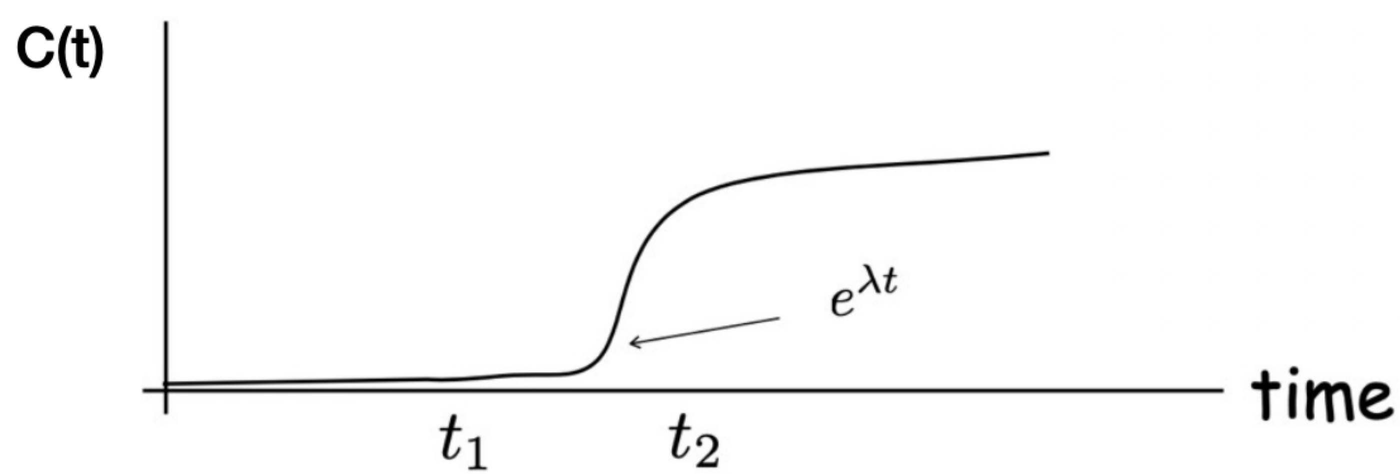
$$\frac{\partial q(t,p_0)}{\partial p_0} = \{q(t), p_0\}_{\mathrm{PB}} \quad \Rightarrow \quad \langle [V, W_t]^2 \rangle \sim e^{\lambda t}$$

Expectation value of the commutators

[Larkin, Ovchinnikov, JETP (1969); Shenker, Stanford, 1306.0622; Maldcena, Shenker, Stanford, 1503.01409]

$$C(t, \mathbf{x}) = -\langle [W(t, \mathbf{x}), V(0)]^\dagger [W(t, \mathbf{x}), V(0)] \rangle_\beta$$





Out-of-Time-Ordered Correlators (OTOC)

[Larkin, Ovchinnikov, JETP (1969); Shenker, Stanford, 1306.0622; Maldcena, Shenker, Stanford, 1503.01409]

- $C(t,\mathbf{x}) = -\langle [W(t,\mathbf{x}), V(0)]^\dagger [W(t,\mathbf{x}), V(0)] \rangle_\beta$
- Probing the sensitivity of a system to the initial conditions

$$C_2 = C_1 - C(t,\mathbf{x})$$

$$C_2 = \langle \Psi_1(t) | \Psi_2(t) \rangle + \langle \Psi_2(t) | \Psi_1(t) \rangle$$ → **Out-of-time Ordered**

$$C_1 = \langle \Psi_1(t) | \Psi_1(t) \rangle + \langle \Psi_2(t) | \Psi_2(t) \rangle$$ → **Time ordered**

$$|\Psi_1(t)\rangle = V_0 W_t |\mathrm{TFD}\rangle$$

$$|\Psi_2(t)\rangle = W_t V_0 |\mathrm{TFD}\rangle$$

- Lyapunov exponents, butterfly velocities: for interacting quantum systems with many degrees of freedom

$$C_2 = 1 - \epsilon e^{\lambda_L \left(t - \frac{x}{v_B}\right)} \qquad t_r \ll t \ll t_*$$

Different (diffusive) spreading might be seen in non-maximally chaotic systems

Holographic OTOC

[Shenker, Stanford, 1306.0622; 1412.6087]

- OTOC = amplitudes for 2-to-2 scatterings of particles dual to W and V in a black hole geometry dual to the thermal state $|\mathrm{TFD}\rangle$

- In elastic eikonal gravity approximation, the dominate contribution is related to the gravitational shock waves on the horizon of a two-sided black hole

- Universal Lyapunov exponent $\lambda_L = 2\pi T$

- The butterfly velocity depends on the details of the black hole geometry

Chaos bound [Maldacena, Shenker, Stanford, 1503.01409]

- Related regulated function

$$F(t) = \mathrm{Tr}\big(yV(0)yW(t)yV(0)yW(t)\big) \sim 1 - \epsilon e^{\lambda_L t}\,, \quad y^4 = \frac{e^{-\beta H}}{Z}$$

- For systems with large hierarchy between thermalization and scrambling, analyticity in correlation functions demands

$$\lambda_L \leq 2\pi T$$

- It holds for very generic quantum many-body systems

- Black holes saturate this bound: maximal chaos

- SYK/AdS2 [Kitaev, 2015]

Chaos from hydrodynamics via pole skipping

[Grozdanov, Schalm, Scopelliti, 1710.00921; Blake, Lee, Liu, 1801.00010; Blake, Davison, Grozdanov, Liu, 1809.01169 ;…]

- Naively hydrodynamics has nothing to do with chaos
- Deep connection from EFT: Signatures of chaos in energy density two point function of $G^R_{T^{00}T^{00}}(\omega, k)$
- There exists a special point $(\omega_*, k_*) = (i\lambda_L, \frac{i\lambda_L}{v_B})$

  in $G^R(\omega, k) = \frac{B(\omega, k)}{A(\omega, k)}$ with $A(\omega_*, k_*) = B(\omega_*, k_*) = 0$
- Examples of pole skipping in many maximally chaotic systems: SYK, AdS black holes in Einstein gravity plus matter
- Pole skipping also exists for 2-pt correlators of other operators on the lower half plane

[talks by Koenraad Schalm, Keun-Young Kim, Viktor Jahnke, Mitsuhiro Nishida]

Motivation

- Connection between OTOC and pole skipping, e.g. for systems with multiple Lyapunov exponents or non-maximally chaotic systems
- What is the role of rotation in holographic chaos
- What is the role of massive graviton in holographic chaos

Why 3D gravity

- A “simple” toy model to understand quantum gravity
- We can learn much from CFT calculations
- In the following, we will talk about
  - Quantum chaos in 3D Einstein gravity
  - Quantum chaos in 3D TMG

3D Einstein gravity

- Einstein-Hilbert action

$$S_{\rm EH} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\,\sqrt{-g}\,(R-2\Lambda)$$

- BTZ Black hole solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\varphi - \frac{r_+ r_-}{\ell r^2}dt\right)^2,$$
$$f(r) = \frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2 r^2}.$$

- M, T, Ω are determined by r_+, r_-.

- The dual theory is expected to be a CFT with $\beta_\pm = \beta(1\mp\ell\Omega)$ and
$$c_+ = c_- = \frac{3\ell}{2G}$$

- The angular direction is periodic. At high temperature $\frac{\beta}{\ell}\to 0$, we can take a ``decompactification” limit (a boosted brane)

Chaos parameters from OTOC

[Jahnke, Kim, Yoon, 1903.09086; (Stikonas 2018; Poojary 2018)]

- From shock wave calculations

$$\mathrm{OTOC}(t,\varphi_{12}) \simeq 1 + \epsilon e^{\frac{2\pi}{\beta}t} h(\Omega t - \varphi) \simeq 1 + C_1 e^{\frac{2\pi}{\beta_+}(t+\ell\varphi_{12})} + C_2 e^{\frac{2\pi}{\beta_-}(t-\ell\varphi_{12})}$$

- Naively we have

$$\lambda_\pm = \frac{2\pi}{\beta(1\mp\Omega\ell)} \qquad v_\pm = \mp 1$$

- The chaos bound is violated: $\lambda_- < \frac{2\pi}{\beta} < \lambda_+$

- However, the angular coordinate is periodic, i.e. the profile of shock wave is periodic, therefore the two coefficients C_1 and C_2 are not independent [Mezei, Sarosi, 1908.03574]

$$\mathrm{OTOC}(t,\varphi_{12}) \simeq 1 + \epsilon\left[e^{\frac{2\pi}{\beta_+}(t+\ell\varphi_{12})} + \# e^{\frac{2\pi}{\beta_-}(t-\ell\varphi_{12})}\right]$$

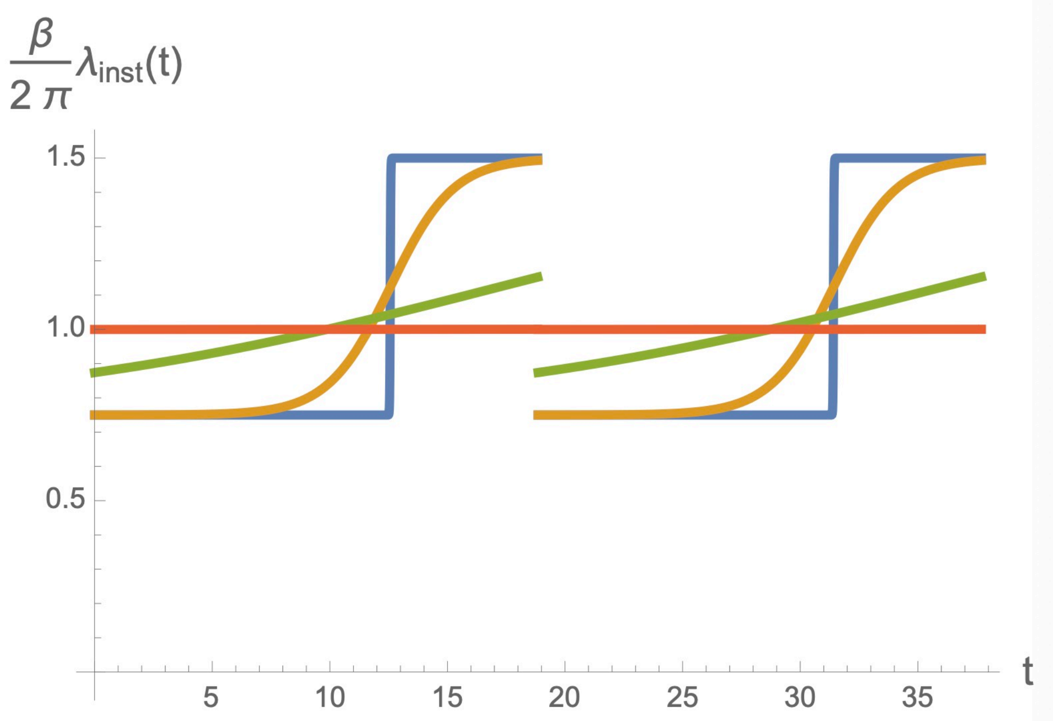
Instantaneous Lyapunov exponent

[Mezei, Sarosi, 1908.03574]

$$\mathrm{OTOC}(t,\varphi_{12}) \simeq 1+\epsilon\left[e^{\frac{2\pi}{\beta_+}(t+\ell\varphi_{12})} + \# e^{\frac{2\pi}{\beta_-}(t-\ell\varphi_{12})}\right]$$

- instantaneous Lyapunov exponent

$$\mathrm{OTOC}(t,0) \simeq 1+\epsilon e^{\lambda_{\mathrm{inst.}} t}$$



$$\beta = 0, 2\pi, 16\pi, \infty$$

- In the high temperature limit, the instantaneous Lyapunov exponents behave as step function.

- The average of instantaneous Lyapunov exponents is $\frac{2\pi}{\beta}$

Pole skipping in holography

- From EOM near the horizon $E_{vv}=0$

  Expand $h_{ab}=e^{-i\omega v+ik\phi}(r-r_+)^{\gamma}\sum_{n=0}^{\infty}\tilde{h}_{ab}^{(n)}(r-r_+)^n$

  near horizon,

  $$\left(2\pi i\omega+4\pi i\Omega k-k^2\beta(1-\Omega^2)\right)\tilde{h}_{vv}^{(0)}=-(2\pi i-\beta\omega)(1-\Omega^2)\left[2k\tilde{h}_{v\phi}^{(0)}+\omega\tilde{h}_{\phi\phi}^{(0)}\right].$$

  At $(\omega,k)=\left(\frac{2\pi i}{\beta(1\mp\Omega)},\ \pm\frac{2\pi i}{\beta(1\mp\Omega)}\right)$ both solutions are regular

  ⇒ $$\lambda_{\pm}=\frac{2\pi}{\beta(1\mp\Omega\ell)}\qquad v_{\pm}=\mp 1$$

- Correlators of energy density from holography

  $$\langle T^{\tau\tau}(\omega_E,k)T^{\tau\tau}(-\omega_E,-k)\rangle\propto\frac{\delta^2 S_{\text{ren.}}}{\delta\tilde{h}_{\tau\tau}^{(0)}\delta\tilde{h}_{\tau\tau}^{*(0)}}=\frac{k^2(4+k^2)}{2(\omega_E^2+k^2)}$$

  The pole skipping point is

  $$(\omega,k)=\left(\frac{2\pi i}{\beta(1\mp\Omega)},\ \pm\frac{2\pi i}{\beta(1\mp\Omega)}\right)$$

Pole skipping in CFT

- For CFT on cylinder

$$G_R(\omega,k)=\frac{c_L}{6}\left(\frac{2\pi}{\beta_L}\right)^3\left(\frac{2i}{\omega-k}+\pi\delta(\frac{k-\omega}{2})\right)\sinh\left[\frac{\beta_L k}{2}\right]\left|\Gamma\left(2+\frac{i\beta_L}{2\pi}k\right)\right|^2$$
$$-\frac{c_R}{6}\left(\frac{2\pi}{\beta_R}\right)^3\left(\frac{2i}{\omega+k}+\pi\delta(\frac{k+\omega}{2})\right)\sinh\left[\frac{\beta_R k}{2}\right]\left|\Gamma\left(2+\frac{i\beta_R}{2\pi}k\right)\right|^2.$$

  - From the first term $(\omega,k)=\left(\pm\frac{2\pi i}{\beta(1-\Omega)},\ \pm\frac{2\pi i}{\beta(1-\Omega)}\right)$
  - From the second term $(\omega,k)=\left(\pm\frac{2\pi i}{\beta(1+\Omega)},\ \mp\frac{2\pi i}{\beta(1+\Omega)}\right)$

[see also Haehl, Rozali, 1808.02898]

- Pole skipping is a generic feature of any CFT, including chaotic CFTs and non-chaotic CFTs.

Topologically Massive Gravity (TMG)

- A gravitational Chern-Simons deformation to Einstein gravity [Deser, Jackiw, Templeton, 1988]

$$S = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left(R + 2 + \frac{1}{2\mu}\varepsilon^{abc}\Gamma^d{}_{ae}\left(\partial_b\Gamma^e{}_{cd} + \frac{2}{3}\Gamma^e{}_{bf}\Gamma^f{}_{cd}\right)\right)$$

- Any solution of Einstein gravity is a solution of TMG

- Thermodynamics for rotating BTZ black holes [Krause, Larsen, hep-th/0508218]

$$M(\mu) = M + \frac{J}{\mu}, \qquad J(\mu) = J + \frac{M}{\mu}$$

- The angular direction and the decompactification limit

- The dual field theory for TMG on rotating BTZ is a CFT with $\beta_\pm = \beta(1 \mp \ell\Omega)$

  and $(c_L, c_R) = \frac{3\ell}{2G}\left(1 - \frac{1}{\mu}, 1 + \frac{1}{\mu}\right)$

- When $\mu\ell < 1$: negative central charge; Black hole instability [Park, hep-th/0608165]

- Chiral point $\mu\ell = 1$ [Li, Song, Stronminger, 0801.4566]

Chaos in TMG from OTOC

- Profile for shock wave and OTOC ($\mu \neq 1$)

$$h(\phi) = c_1\, e^{-\frac{2\pi\phi}{\beta(1+\Omega)}} + c_2\, e^{\frac{2\pi\phi}{\beta(1-\Omega)}} + c_3\, e^{\frac{2\pi(\Omega-\mu)\phi}{\beta(1-\Omega^2)}}$$

$$\mathrm{OTOC}(t,\varphi) = 1 - \varepsilon\, e^{\frac{2\pi}{\beta}t} h(\Omega t - \varphi)$$

- Naively, we have three Lyapunov exponents (non-maximal chaos?)

$$\lambda_\pm = \frac{2\pi}{\beta(1\mp\Omega)}, \quad \lambda_m = \frac{2\pi(1-\mu\Omega)}{\beta(1-\Omega^2)} \qquad v_\pm = \pm 1, \quad v_m = \frac{1-\mu\Omega}{\Omega-\mu}$$

- Periodicity in ϕ: $h(\phi) \to h(\phi \bmod 2\pi)$ there is a constraint equation among c_i

$$\mathrm{OTOC}(t,\varphi) = 1 - \varepsilon_{VW} e^{\frac{2\pi}{\beta}t}\Big[\alpha_1 h_1(\Omega t - \varphi) + \alpha_2 h_2(\Omega t - \varphi)\Big]$$

- There are two independent “instantaneous Lyapunov exponents”

$$\lambda_{\mathrm{I,inst.}}(t) = \frac{2\pi}{\beta} + \frac{\Omega\partial_t h_1(\Omega t)}{h_1(\Omega t)}, \quad \lambda_{\mathrm{II,inst.}}(t) = \frac{2\pi}{\beta} + \frac{\Omega\partial_t h_2(\Omega t)}{h_2(\Omega t)}$$

High T limit of instantaneous Lyapunov exponents

- When $\mu > 1$

$$\lambda_{\text{I, inst.}} = \begin{cases} \lambda_- \,, & \text{if } \; t \in \big[0, \; \frac{\pi(1+\Omega)}{\Omega}\big) \\ \lambda_+ \,, & \text{if } \; t \in \big[\frac{\pi(1+\Omega)}{\Omega}, \; \frac{2\pi}{\Omega}\big) \end{cases} \; ; \quad \lambda_{\text{II, inst.}} = \begin{cases} \lambda_m \,, & \text{if } \; t \in \big[0, \; \frac{2\pi(1+\Omega)}{\Omega(1+\mu)}\big) \\ \lambda_+ \,, & \text{if } \; t \in \big[\frac{2\pi(1+\Omega)}{\Omega(1+\mu)}, \; \frac{2\pi}{\Omega}\big) \end{cases}$$

$$\langle \lambda_{\text{I, inst.}} \rangle = \langle \lambda_{\text{II, inst.}} \rangle = \frac{2\pi}{\beta}$$

- When $\mu < 1$

$\Omega < \mu$

$$\lambda_{\text{I, inst.}} = \lambda_{\text{II, inst.}} = \begin{cases} \lambda_m \,, & \text{if } \; t \in \big[0, \; \frac{2\pi(1+\Omega)}{\Omega(1+\mu)}\big) \\ \lambda_+ \,, & \text{if } \; t \in \big[\frac{2\pi(1+\Omega)}{\Omega(1+\mu)}, \; \frac{2\pi}{\Omega}\big) \end{cases} \qquad \langle \lambda_{\text{I, inst.}} \rangle = \langle \lambda_{\text{II, inst.}} \rangle = \frac{2\pi}{\beta}$$

$\mu < \Omega$

$$\lambda_{\text{I, inst.}} = \begin{cases} \lambda_- \,, & \text{if } \; t \in \big[0, \; \frac{2\pi(\Omega-\mu)}{\Omega(1-\mu)}\big) \\ \lambda_m \,, & \text{if } \; t \in \big[\frac{2\pi(\Omega-\mu)}{\Omega(1-\mu)}, \; \frac{2\pi}{\Omega}\big) \end{cases} \; ; \quad \lambda_{\text{II, inst.}} = \lambda_m \,, \quad \text{if } \; t \in \big[0, \; \frac{2\pi}{\Omega}\big)$$

$$\langle \lambda_{\text{I, inst.}} \rangle = \frac{2\pi}{\beta} \qquad \langle \lambda_{\text{II, inst.}} \rangle = \lambda_m > \frac{2\pi}{\beta}$$

$\Omega = \mu$

$$\lambda_{\text{I, inst.}} = \begin{cases} \lambda_- \,, & \text{if } \; t \in \big[0, \; \frac{\pi(1+\Omega)}{\Omega}\big) \\ \lambda_+ \,, & \text{if } \; t \in \big[\frac{\pi(1+\Omega)}{\Omega}, \; \frac{2\pi}{\Omega}\big) \end{cases} \; ; \quad \lambda_{\text{II, inst.}} = \lambda_m \,, \quad \text{if } \; t \in \big[0, \; \frac{2\pi}{\Omega}\big) \,,$$

$$\langle \lambda_{\text{I, inst.}} \rangle = \langle \lambda_{\text{II, inst.}} \rangle = \frac{2\pi}{\beta}$$

Lyapunov exponent and butterfly velocities from OTOC

- In the high temperature limit and $|\Omega t - \varphi| \ll 1$
  - When $\mu > 1$

$$\mathrm{OTOC}(t,\varphi) = 1 - \varepsilon \begin{cases} \#\ e^{\lambda_+(t-\varphi)}\,, & \text{if } \Omega t < \varphi \\ \#_1\ e^{\lambda_-(t+\varphi)} + \#_2\ e^{\lambda_m\left(t-\frac{\varphi}{v_m}\right)}\,, & \text{if } \Omega t > \varphi \end{cases}$$

  - When $\mu < 1$

$\Omega < \mu$

$$\mathrm{OTOC}(t,\varphi) = 1 - \varepsilon \begin{cases} \#_1\ e^{\lambda_+(t-\varphi)}\,, & \text{if } \Omega t < \varphi \\ \#_2\ e^{\lambda_m\left(t-\frac{\varphi}{v_m}\right)}\,, & \text{if } \Omega t > \varphi \end{cases}$$

$\mu < \Omega$

$$\mathrm{OTOC}(t,\varphi) = 1 - \varepsilon \begin{cases} \#\ e^{\frac{4\pi^2}{\beta(1-\Omega)}})e^{\lambda_m\left(t-\frac{\varphi}{v_m}\right)}\,, & \text{if } \Omega t < \varphi \\ \#_1\ e^{\lambda_-(t+\varphi)} + \#_2 e^{\lambda_m\left(t-\frac{\varphi}{v_m}\right)}\,, & \text{if } \Omega t > \varphi \end{cases}$$

violate the chaos bound; $v_m > c$

$\Omega = \mu$

$$\mathrm{OTOC}(t,\varphi) = 1 - \varepsilon \begin{cases} \#_1 e^{\lambda_+(t-\varphi)} + \#_2 e^{\frac{2\pi}{\beta}t}\,, & \text{if } \Omega t < \varphi \\ \#_1 e^{\lambda_-(t+\varphi)} + \#_2 e^{\frac{2\pi}{\beta}t}\,, & \text{if } \Omega t > \varphi \end{cases}$$

Chaos in TMG from OTOC

- When $\mu = 1$, the profile of the shock wave is

$$h(\phi) = c_1\, e^{-\frac{2\pi}{\beta(1+\Omega)}\phi} + c_2\, e^{\frac{2\pi}{\beta(1-\Omega)\phi}} + c_3\, \phi\, e^{-\frac{2\pi}{\beta(1+\Omega)}\phi}$$

$$\lambda_{\text{I, inst.}} = \lambda_{\text{II, inst.}} = \lambda_- \,, \quad \text{for}\;\; t \in [0,\, \frac{2\pi}{\Omega})$$

$$\text{OTOC}(t,\varphi) = 1 - \varepsilon \begin{cases} \#_1\; e^{\lambda_-(t+\varphi)}\,, & \text{if}\;\; \Omega t < \varphi \\ \#_2\; e^{\lambda_-(t+\varphi)}\,, & \text{if}\;\; \Omega t > \varphi \end{cases}$$

- At high temperature, if we impose the chaos bound on the average of instantaneous Lyapunov exponent, only $\mu \geq 1$ is allowed; *If we lower the temperature and impose the chaos bound, only the chiral point is allowed*

- At high temperature, if we impose the chaos bound on the Lyapunov exponent, only $\mu \geq 1$ is allowed

Pole skipping from holography

- Pole skipping from near horizon EOM

$$e^{(0)}_{vv}h^{(0)}_{vv}+e^{(0)}_{vr}h^{(0)}_{vr}+e^{(0)}_{v\phi}h^{(0)}_{v\phi}+e^{(0)}_{r\phi}h^{(0)}_{r\phi}+e^{(0)}_{\phi\phi}h^{(0)}_{\phi\phi}+e^{(1)}_{vv}h^{(1)}_{vv}+e^{(1)}_{v\phi}h^{(1)}_{v\phi}=0$$

$$\left(\frac{2\pi i}{\beta(1\mp\Omega)}\,,\quad \mp\frac{2\pi i}{\beta(1\mp\Omega)}\right)\quad \&\quad \left(\frac{2\pi i(1-\Omega\mu)}{\beta(1-\Omega^2)}\,,\quad \frac{2i\pi(\Omega-\mu)}{\beta(1-\Omega^2)}\right)$$

- Pole skipping from holographic massive mode

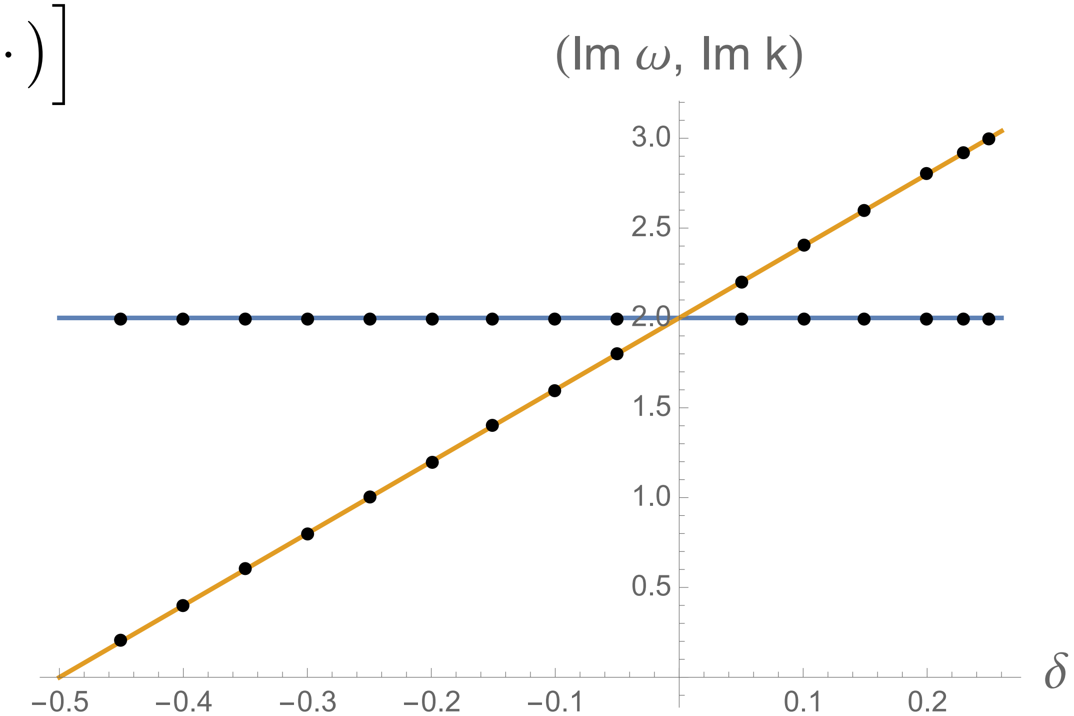
$$h_{ij}(\rho)=e^{-i\omega T+ikX}\Big[h^{(0)}_{ij}+\rho h^{(1)}_{ij}+\rho^2h^{(2)}_{ij}+\rho^{-\delta}\big(b^{(0)}_{ij}+\rho b^{(1)}_{ij}+\rho^2 b^{(2)}_{ij}+\cdots\big)$$
$$+\rho^{\delta+1}\big(c^{(0)}_{ij}+\rho c^{(1)}_{ij}+\rho^2 c^{(2)}_{ij}+\cdots\big)\Big]$$

$$\mu=2\delta+1$$

$$G_R^{t^{00}t^{00}}(\omega,k)\propto\frac{c^{(0)}_{tt}}{b^{(0)}_{tt}}$$

$$\mathrm{Im}\omega=-\mathrm{Im}k+4(1+\delta)$$

$$\mathrm{Im}\omega=\mathrm{Im}k-4\delta$$



Pole skipping from CFT

- The massive graviton is dual to an operator with conformal dimension $(2+\delta,\delta)$

- The retarded Green's function

$$G_R(\omega,k) \propto \sin\left[\delta + \frac{i\beta_R}{2\pi}\left(\frac{\omega-k}{2}\right)\right] \sin\left[2+\delta+\frac{i\beta_L}{2\pi}\left(\frac{\omega+k}{2}\right)\right] \times$$
$$\times \left|\Gamma\left(\delta+\frac{i\beta_R}{2\pi}\left(\frac{\omega-k}{2}\right)\right)\right|^2 \left|\Gamma\left(2+\delta+\frac{i\beta_L}{2\pi}\left(\frac{\omega+k}{2}\right)\right)\right|^2$$

- Pole-skipping point

$$(\omega,k) = \left(\frac{2\pi i(1-\Omega\mu)}{\beta(1-\Omega^2)}\,,\quad \frac{2i\pi(\Omega-\mu)}{\beta(1-\Omega^2)}\right)$$

Conclusion

- OTOC and pole-skipping (from near horizon dynamics, holographic correlators, CFT calculations) are two features of quantum chaos
- For rotating BTZ in 3D Einstein gravity, we find a match between the two methods in the high temperature limit
- For rotating BTZ in 3D TMG, we find a match between these two methods in the high temperature limit and $\mu \geq 1$
- $\mu \geq 1$ is also the limit that the chaos bound is satisfied

* It would be interesting to study other systems with the non-maximal chaos (from CFT or holography)

Thank you!