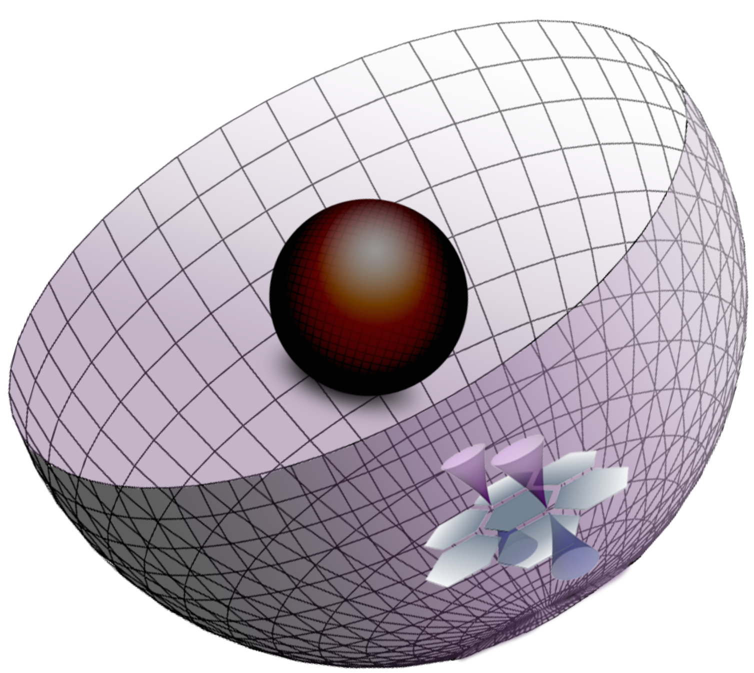
Ginzberg-Landau-Wilson theory for Flat band, Fermi-arc and Fermi-surface for Strongly interacting system.



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Contents

- Introduction
- Observation
- Can it happens? How it happens?
- What it means?

I. Introduction: Holography and GL

- Reformulate strongly interacting system as weakly interacting system in one higher dimension.

- Based on the Universality (=info. Loss) and the similarity of BH and QCP.

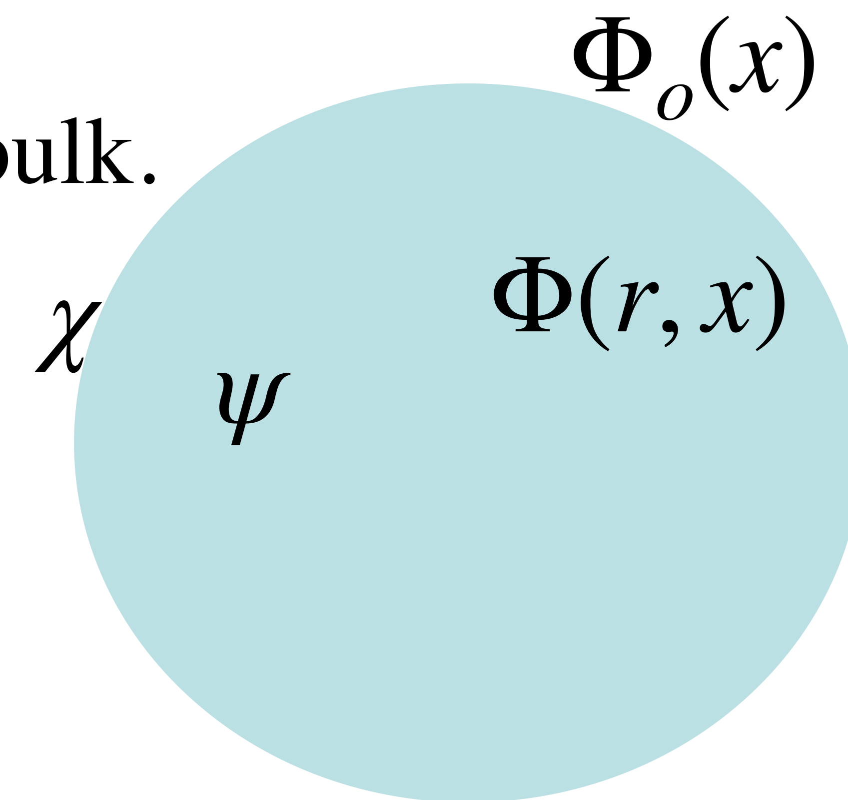
- Q: Can we consider it as a LG theory?

What is Order Parameter in this theory?

Let Φ_0 be the source field of $\bar{\chi}\Gamma^I\chi$,
$\Phi(r,x)$ is the extension of Φ_0 to the bulk.

Order parameter= $\Phi(r,x)$.

Interaction : $\Phi_I \cdot \bar{\chi}\Gamma^I\chi$



Flat spacetime spectrum

$$S = S_\chi + S_\Phi + S_{int},$$

$$S_\chi = \int d^3x \sum_{j=1}^{2} i\bar{\chi}_j\gamma^\mu\mathcal{D}_\mu\chi_j - im(\bar{\chi}_1\chi_1 - \bar{\chi}_2\chi_2),$$

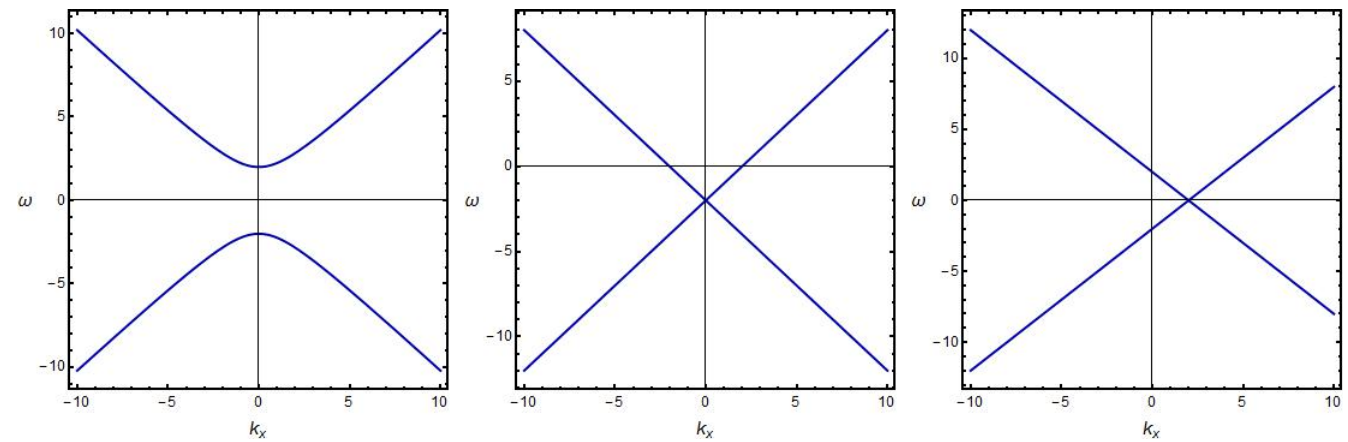
$$S_\Phi = \int d^3x \left((D_\mu\Phi_I)^2 - m_\Phi^2\Phi_I\Phi^I),\right)$$

$$S_{int} = p_1 \int d^3x \,(\bar{\chi_1}\,\Phi\cdot\gamma\,\chi_1 + h.c) + p_2\sum_{j=1}^{2}\int d^3x\,(\bar{\chi_1}\,\Phi\cdot\gamma\,\chi_2 + h.c)$$

2+1 spectrum in flat space

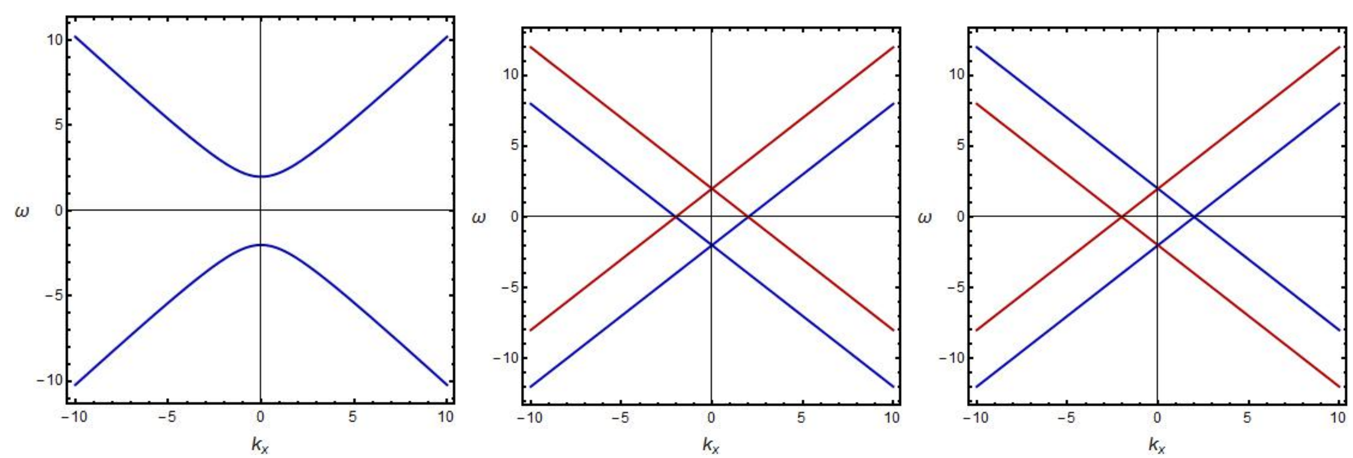
Interaction : $\Phi_{\text{I}} \cdot \bar{\chi}\Gamma^{I}\chi$

1-flavor

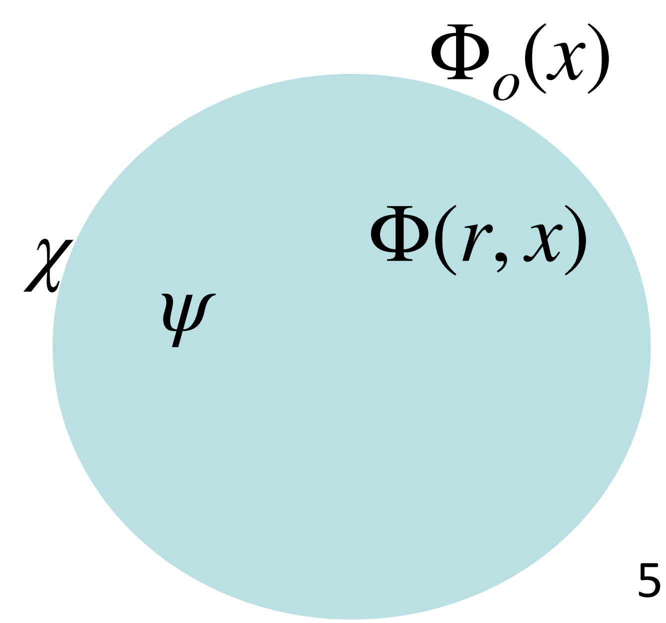


- (a) Interaction with Φ (b) Interaction with B_t (c) Interaction with B_i

2-flavor



- (a) Interaction with Φ (b) Interaction with B_t (c) interaction with B_i



AdS4 space

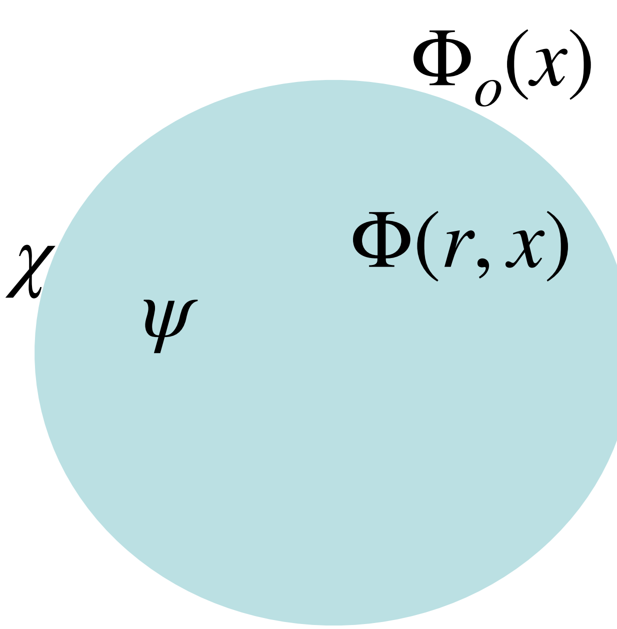
$$S_\psi = \int d^4x \sum_{j=1}^{2} i\bar{\psi}_j \gamma^\mu \mathcal{D}_\mu \psi_j - im(\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2), \tag{3.9}$$

$$S_{bdry} = \frac{1}{2}\int_{bdry} d^3x \; i(\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2), \tag{3.10}$$

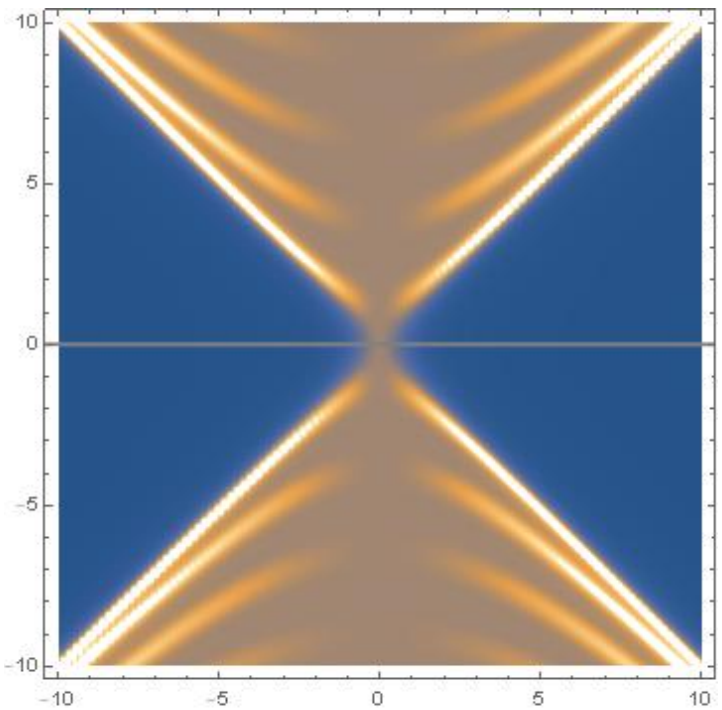
$$S_\Phi = \int d^4x\sqrt{-g}\left(|D_\mu \Phi_I|^2 - m_\Phi^2 \Phi_I^* \Phi^I\right), \tag{3.11}$$

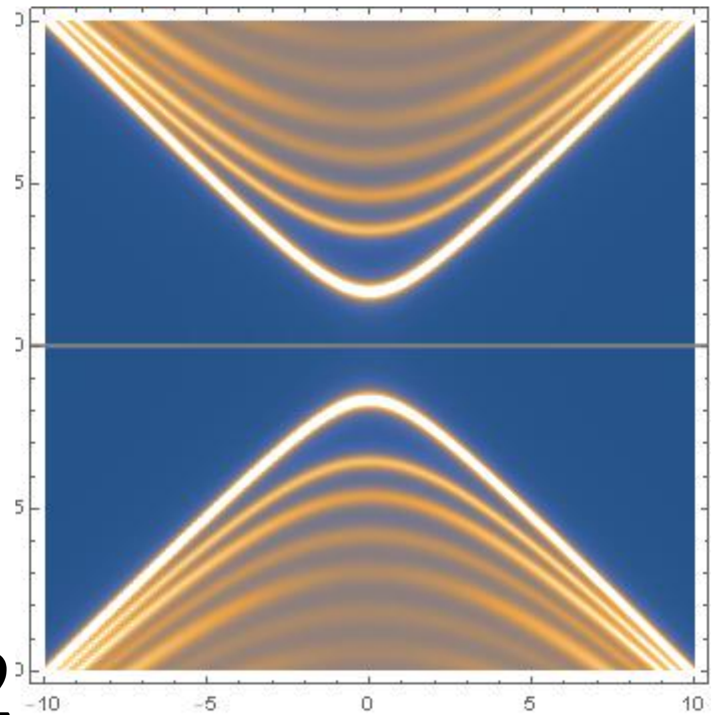
$$S_{int} = p_{2f}\sum_{j=1}^{2}\int d^4x \left(\bar{\psi}_1\,\Phi\cdot\gamma\,\psi_2 + h.c\right) + p_{1f}\int d^4x\left(\bar{\psi}_1\,\Phi\cdot\gamma\,\psi_1\right), \tag{3.12}$$

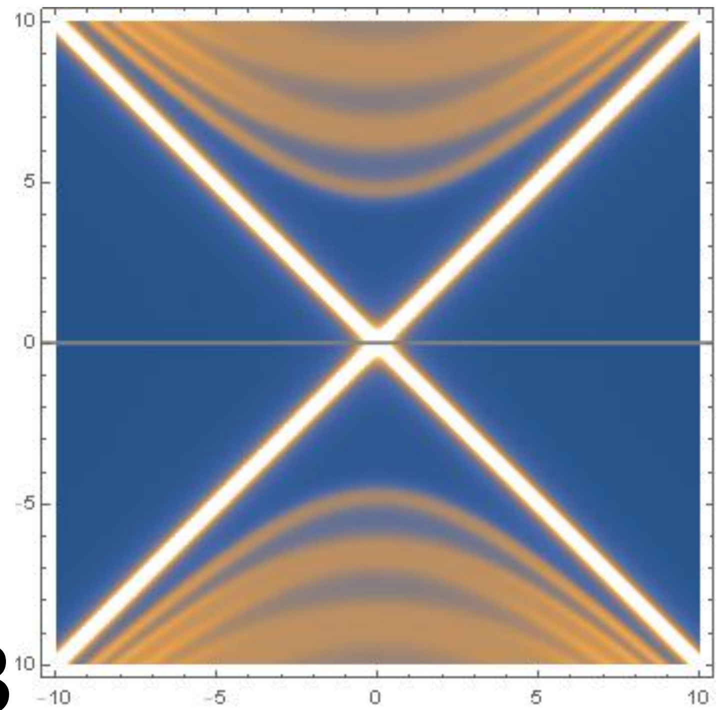
-
- Source vs Condensation
- Scalar $\Phi = \Phi_0 z^{d-\Delta-p} + \langle O_\Delta\rangle z^{\Delta-p}.$
- Vector/tensor $B_{\mu\nu} = B_{\mu\nu}^{(-1)} z^{-1} + B_{\mu\nu}^{(0)}$



Emergence of weak out of strong by symmetry breaking

- 1 

2 

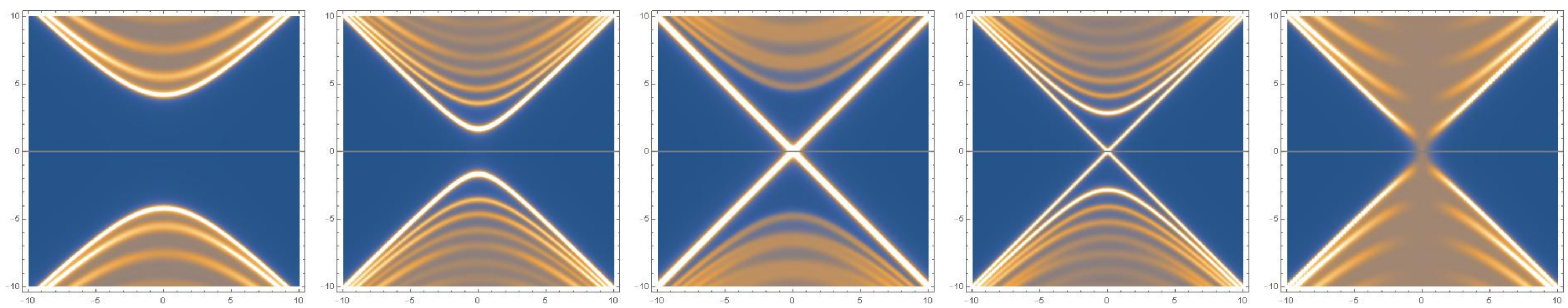
3 

1)Typical spect. of fermion with strong correlation

2,3) After symmetry breaking

Summary of spectral features

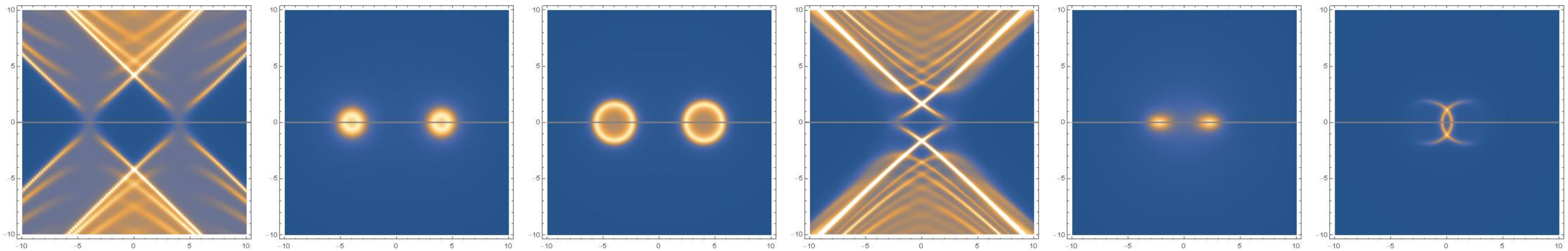
- P(inversion) breaking scalar —> Gap. (very is rare!)
- P preserving scalar —> zero mode



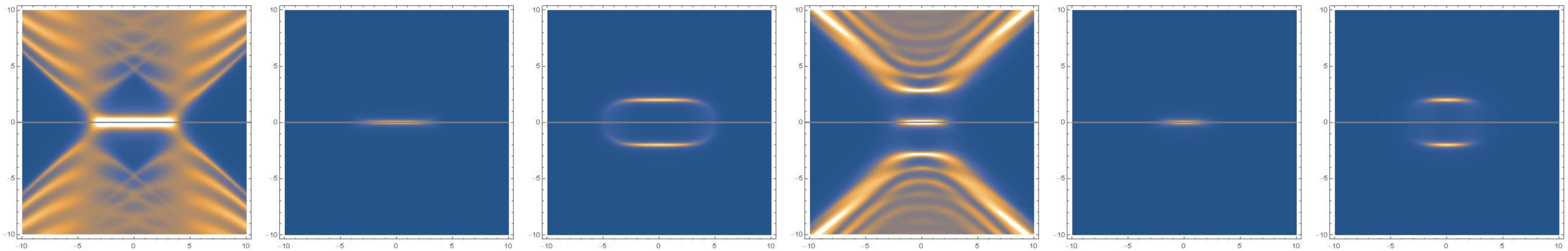
(a) M_0 (b) M (c) M_5^0 (d) M_5 (e) B_r, B_{5r} s c

Figure 4: Spectral Function(SF) (a,b) with parity breaking scalar. (a) with source only. Gap $\Delta \sim M_0$. (b) with condensation only, $\Delta \sim \sqrt{M}$; (c,d) with parity conserving source. Notice the zero modes. (c) source only (d) condensation only. (e) B_r, B_{rt} shows the spectrum of zero coupling due to the gamma matrix structrure.

Vector, pseudo-vector



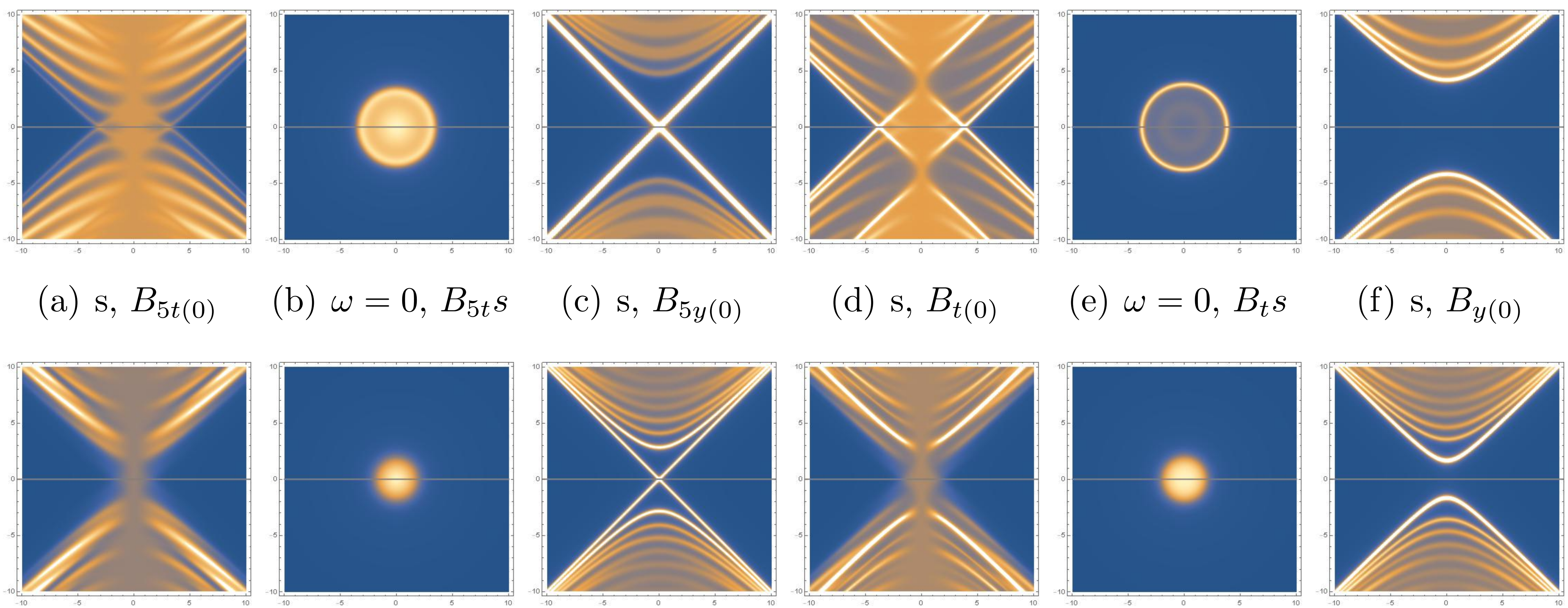
(a) B_x s (b) $\omega = 0$, B_x s (c) $\omega = 2$, B_x s (d) B_x c (e) $\omega = 0$, B_x c (f) $\omega = 2$, B_x c



(g) B_{5x} s (h) $\omega = 0$, B_{5x} s (i) $\omega = 2$, B_{5x} s (j) B_{5x} c (k) $\omega = 0$, B_{5x} c (l) $\omega = 2$, B_{5x} c

Figure 7: (adgj)Spectral Functions of B_x and B_{5x} with additional sliced view in k_x, k_y plane of $\omega = 0, 2$ slices. Notice that source splits the degenerated Dirac cones. B_x has zero modes nut B_{5x} does not. In all figures, we used $B_* = 4$.

Pseudo vectors

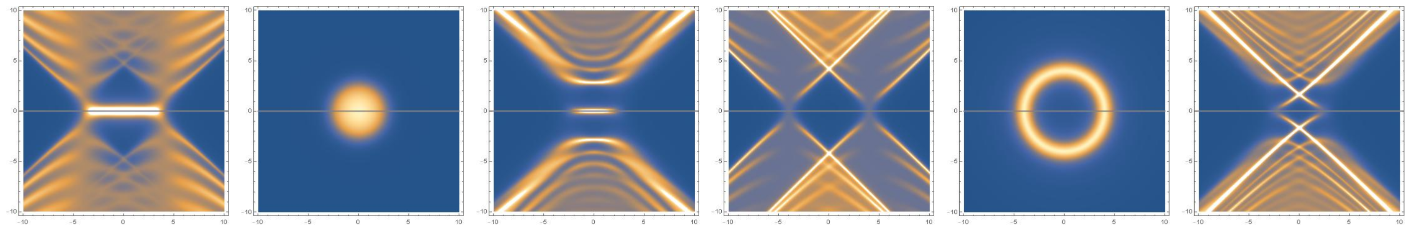


(a) s, $B_{5t(0)}$ (b) $\omega = 0$, $B_{5t}s$ (c) s, $B_{5y(0)}$ (d) s, $B_{t(0)}$ (e) $\omega = 0$, $B_{t}s$ (f) s, $B_{y(0)}$

(g) c, $B_{5t(0)}$ (h) $\omega = 0$, $B_{5t}c$ (i) c, $B_{5y(0)}$ (j) c, $B_{t(0)}$ (k) $\omega = 0$, $B_{t}c$ (l) c, $B_{y(0)}$

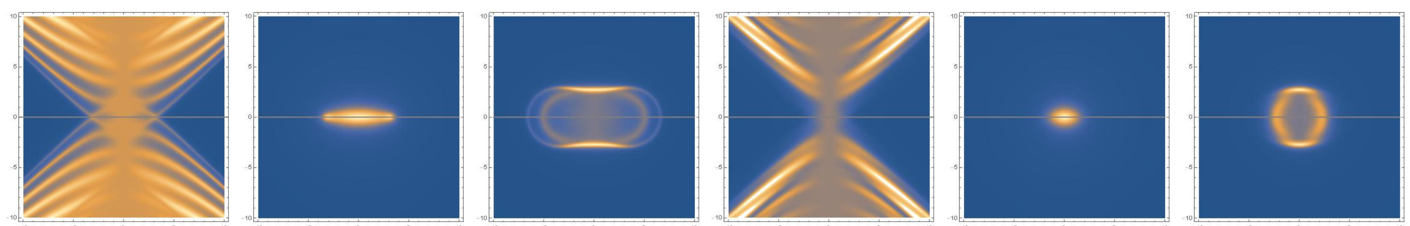
- **Figure 8**: Spectral function with (pseudo) vector source interactions (a-f), and SF with (pseudo)-vector condensation (g-l). s means source and c means condensation.

2-tensors

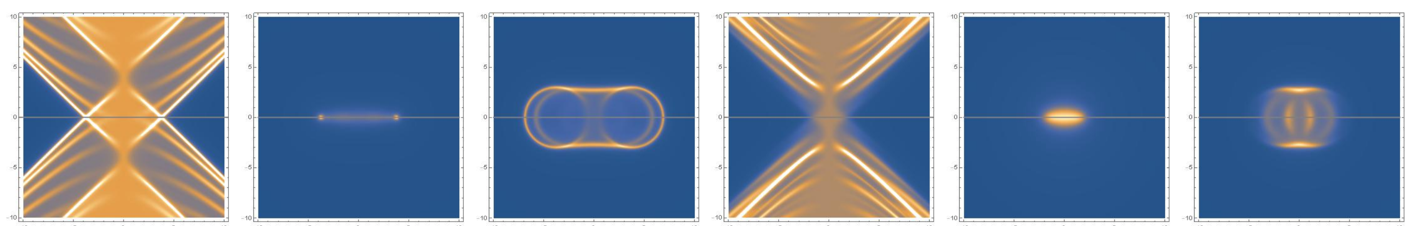


(a) $B_{xy(-1)}$ s (b) $B_{xy(-1)}$ s (c) $B_{xy(0)}$ c (d) $B_{tr(-1)}$ s (e) $B_{tr(0)}$ c (f) $B_{tr(0)}$ c

Figure 9: Spectral function with various types tensor interaction, which is decomposed into 2+1 radial vector $B_{\mu r}$'s (ab) and 2-tensor B_{xy} (cd). Notice the zero mode Disk in B_{xy}. There are rotational symmetry in B_{tr}, B_{xy}.



(a) $B_{ty(0)}$ s (b) $\omega = 0$ B_{ty} s (c) $\omega = 2$ B_{ty} s (d) c, $B_{ty(0)}$ (e) $\omega = 0$ B_{ty} c (f) $\omega = 2$ B_{ty} c

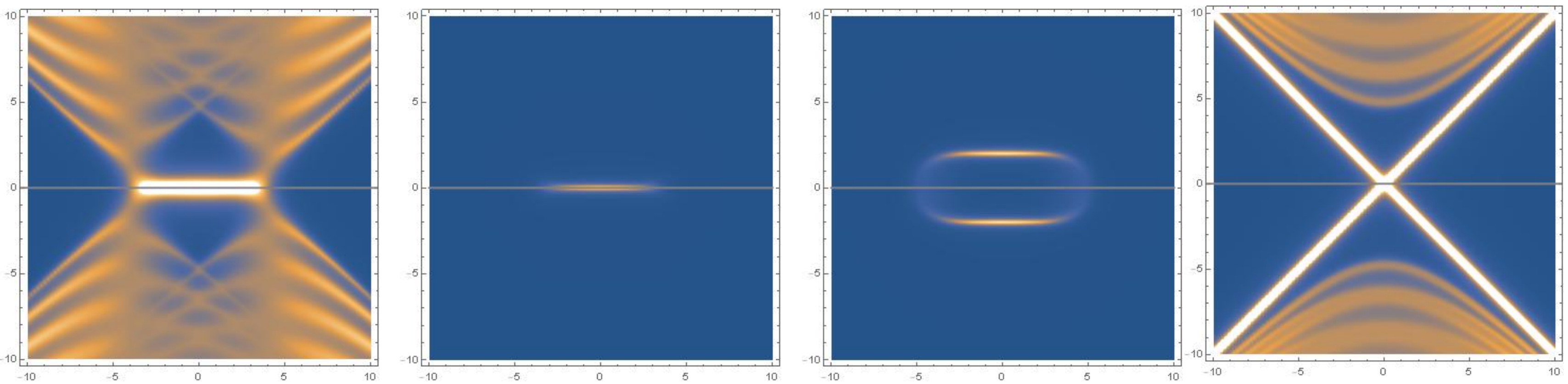


(g) $B_{rx(0)}$ s (h) $\omega = 0$ B_{rx} s (i) $\omega = 2$ B_{rx} s (j) c, $B_{rx(0)}$ (k) $\omega = 0$ B_{rx} c (l) $\omega = 2$ B_{rx} c

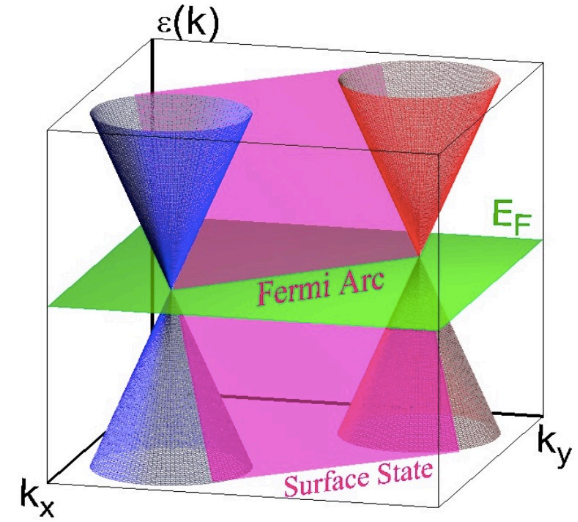
Figure 10: Spectrum of B_{rx} and B_{ty} with sliced views k_x-k_y at $\omega = 0, 2$ slices, without which these spectra are ambiguous. Notice the zero modes and Ribbons connecting the two split cones.

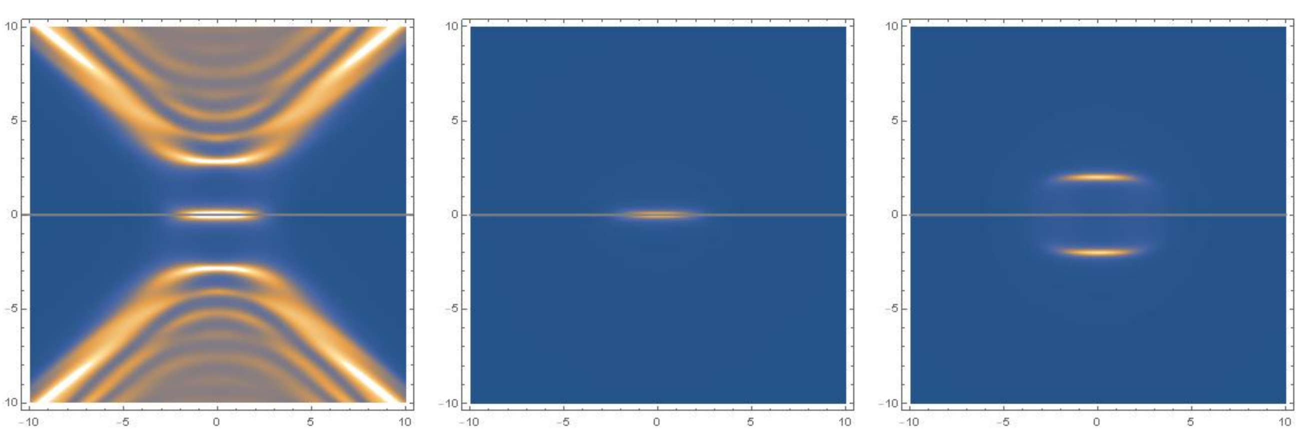
feature1) Fermi-arc: B_{5i}, B_{ri}, B_{ti} P inv

- For vector B_μ, the role of source term is to generate the splited Dirac cones there exist a spectral line connecting the tips of two Dirac cones at $\omega = 0$



(g) B_{5x} s (h) $\omega = 0$, B_{5x} s (i) $\omega = 2$, B_{5x} s (c) s, $B_{5y(0)}$

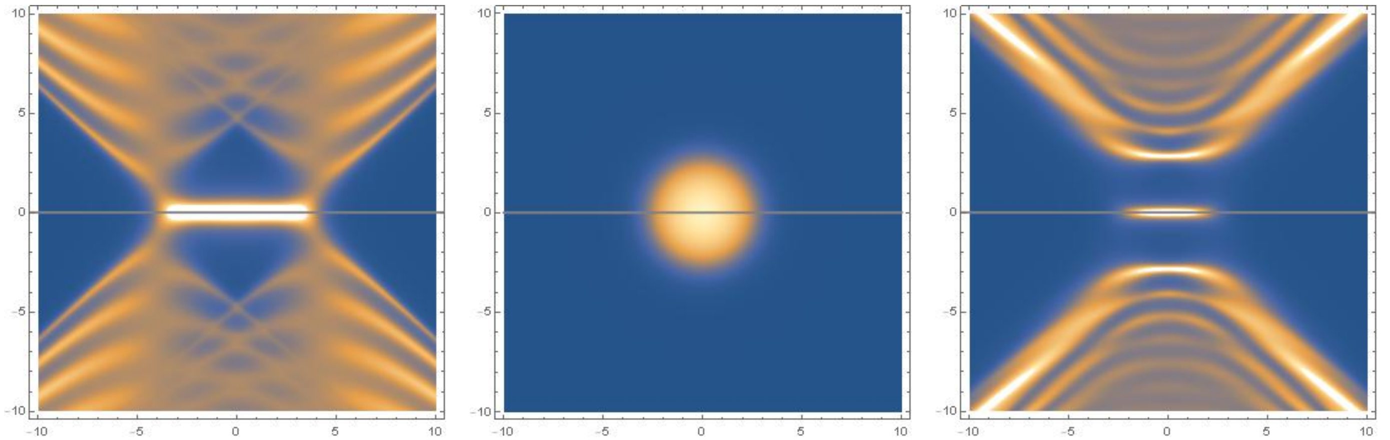




(j) B_{5x} c (k) $\omega = 0$, B_{5x} c (l) $\omega = 2$, B_{5x} c

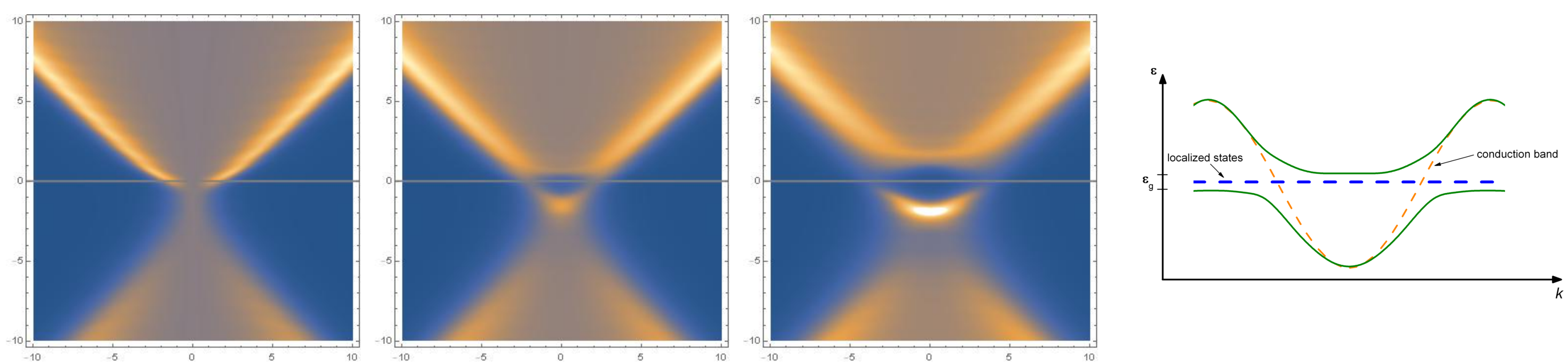
- Topological Matters

Feature) Flat band - B_{xy}



- (a) $B_{xy(-1)}$ s (b) $B_{xy(-1)}$ s (c) $B_{xy(0)}$ c

- Possible application: Twisted graphene, Kondo lattice



(a) $B_{xy,c} = 0$ (b) $B_{xy,c} = 5$ (c) $B_{xy,c} = 10$ (d) Kondo lattice

Figure 4: (a-c) Formation process of bent flat band by as we change the stregnth of the coupling. From the left to right $B_{xyc} = 0, 5, 10$. The chemical potential is fixed to be $\mu = 2\sqrt{3}$. (d) Formation of flat band by hybridization of localized state and conducting state.

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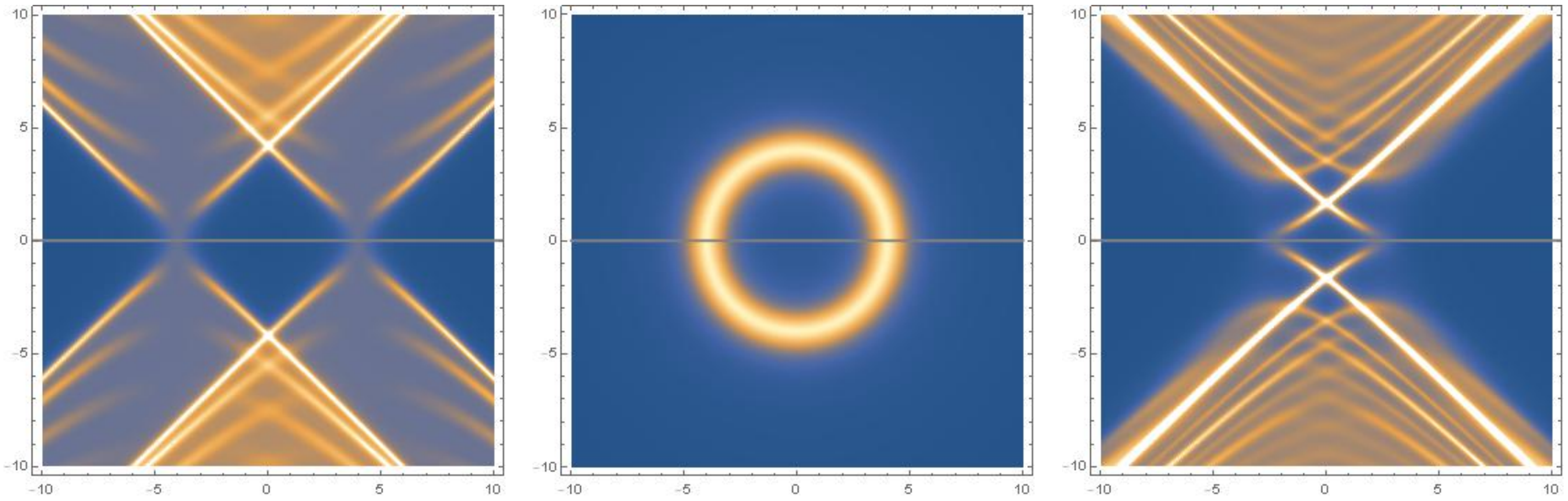
Emergent Fermi surfaces Brt



(d) $B_{tr(-1)}$ s (e) $B_{tr(0)}$ c (f) $B_{tr(0)}$ c

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Duality

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- **Duality** If we change the boundary term to $S_{bdry} = \frac{1}{2}\int_{bdry} d^3x\ i(\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2)$, then the spectrum of dual pairs are exchanged. By the dual pair, we mean one of following set of pairs:

$$(\Phi, \Phi_5), (B_\mu, B_{5\mu}), (B_{\mu\nu}, \epsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}),$$

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Exact calculation of green function for the scalar order

$$\left[\partial_z + \left(iK_\mu\Gamma^\mu + \frac{m+\Phi}{z}\right)\Gamma^r\right]\phi = 0, \text{ with } K_\mu = \left(-\frac{\omega + gA_t}{\sqrt{f}}, k_x, k_y\right)$$

m *is not a mass!If* $\Phi = M_0 z + Mz^2, M_0$ *is the mass?*

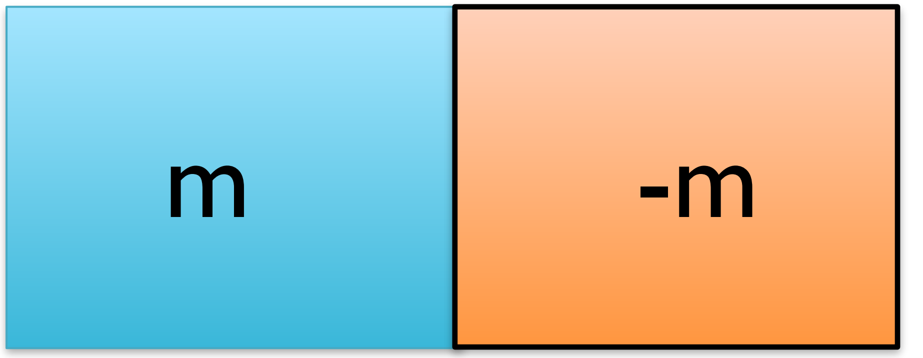
- $$G_{S\pm} = \begin{cases} \dfrac{M^{-\frac{1}{2}+m}(k_1 \mp w)\Gamma(\frac{1}{2}-m)\Gamma(m+\frac{1}{2}+\frac{k_1^2-w^2}{4M})}{2\Gamma(\frac{1}{2}+m)\Gamma(1+\frac{k_1^2-w^2}{4M})} & \text{if } M > 0 \\ \dfrac{2(-M)^{\frac{1}{2}+m}\Gamma(\frac{1}{2}-m)\Gamma(1-\frac{k_1^2-w^2}{4M})}{(k_1 \pm w)\Gamma(\frac{1}{2}+m)\Gamma(-m+\frac{1}{2}-\frac{k_1^2-w^2}{4M})} & \text{if } M < 0 \end{cases}$$

Change the sign of the Φ

$$G_{S\pm}(k,\omega;-M,-m) = 1/G_{S\pm}(k,\omega;M,m)$$

Jackiw-Rebbi fermion soliton

- If mass changes across the domain wall(DM), Fermion has a soliton zero mode with 1/2 fermion number.



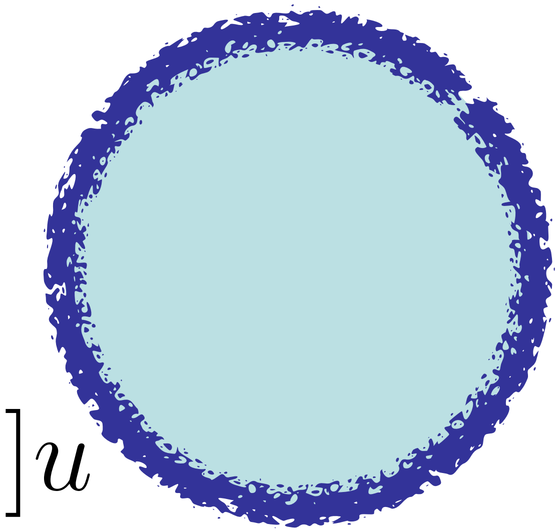
- in a topological insulator,
  DM=real boundary of matter.
  zero mode = the edge mode with 0 E.

- Q: is our zero mode an edge state, I.e, Fermion localized at the AdS boundary?

Holographic Jackiw-Rebbi

- Dirac equation with For scalar order, and in Poincare coordinate,

$$\left[\partial_z + \left(iK_\mu\Gamma^\mu + \frac{m+\Phi}{z}\right)\Gamma^r\right]\phi = 0, \text{ with } K_\mu = (-\frac{\omega + gA_t}{\sqrt{f}}, k_x, k_y),$$



- so set $k = \omega = \mu = 0, \ \Phi = M_0 z + M z^2$.

$$\phi \approx \exp\left[-(m \log z + M_0 z + \frac{1}{2} M z^2)\Gamma^r\right] u$$

Normalizable upper component ψ_+ , non-normalized down component.

- $$\Gamma^r = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

- The rest is the survival game: Is the normalizable mode survived under the projection determined by the boundary fermion action?

Spectral classification: Decompose ads4 spec to 2+1d -> spectral similarity

- $$\mathbf{16} = \mathbf{1}(\text{scalar}) + \mathbf{4}(\text{vector}) + \mathbf{6}(\text{tensor}) + \mathbf{4}(\text{axial vector}) + \mathbf{1}(\text{pseudo scalar}),$$

  - 4 scalars: $1, \Gamma^5, \Gamma^r, \Gamma^{r5} = \sigma^A \otimes \mathbb{1}$ with $\sigma^A = (\mathbb{1}, \sigma^2, \sigma^3, -i\sigma^1)$.
  - 3 types of vectors $\Gamma^\mu = \sigma^1 \otimes \gamma^\mu$, $\Gamma^{\mu 5} = i\sigma^3 \otimes \gamma^\mu$, $\Gamma^{r\mu} = \imath\sigma^2 \otimes \gamma^\mu$,
  - 3 tensors $\Gamma^{\mu\nu} = \epsilon^{\mu\nu\alpha}\mathbb{1} \otimes \gamma_\alpha$, where index runs 0, 1, 2.

Time reversal symmetry: $\mathscr{T} = \Gamma^0 K$

Under the $\psi(t) \to \psi'(t') = \mathcal{T}\psi(t) = T\psi^*(-t)$,

$$\bar{\psi}_1 \Gamma^I \psi_2 \to \bar{\psi}_2 \Gamma^{I\dagger} \psi_1.$$

Therefore the invariant Hermitian quadratic form are follwoing 8 terms:

$$\Gamma^I = \Gamma^5, \Gamma^r, \Gamma^{5t}, \Gamma^i, \Gamma^{ti}, \Gamma^{tr}.$$

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Discrete symmetry and zero modes

- $$\mathcal{L}_{int} = \Phi_I \bar{\psi}_1 \Gamma^I \psi_2 + \Phi_I^* \bar{\psi}_2 \Gamma^I \psi_1$$ $\Gamma^I = i\mathbb{1}, \Gamma^\mu, \Gamma^{5\mu}, \Gamma^{\mu\nu}$ with $\mu, \nu = t, x, y, r.$
- • The time reversal operation is given by $\mathcal{T} = TK$, $T = \Gamma^1\Gamma^2\Gamma^3.$

the invariant Hermitian bilinears correspond to following 8

$$\Gamma^I = \Gamma^5, \Gamma^{5r}, \Gamma^t, \Gamma^{5i}, \Gamma^{ti}, \Gamma^{tr}.$$

- Charge conjugation: C=1.K
- The parity symmetry $(t, x, y, z) \to (t, -x, -y, -z)$ with $z = 1/r$.

invariant Hermitian quadratic forms correspond to following 8

$$\Gamma^I = i\mathbb{1}, \Gamma^{5r}, \Gamma^t, \Gamma^{5i}, \Gamma^{ri}, \Gamma^{xy}.$$

P controls the existence of the zero mode.

- Chiral symmetry: complement to Parity symmetry. Controls the zero modes in AS quantization.

Zero mode and Parity sym.

In the presence of the background field B_I with coupling $B_I \bar{\psi} \Gamma^I \psi$, the spectrum shows the zero modes if the quadratic form is time reversal invariant.

$$\Gamma^I = \Gamma^5, \Gamma^r, \Gamma^{5t}, \Gamma^i, \Gamma^{ti}, \Gamma^{tr}$$

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New route to topological matter

- Two routes to topological matter.
  I) 0 ee interaction+lattice configuration
  ii) strong interaction+ symmetry breaking.

Order without symmetry breaking

- **Order without rotational symmetry breaking** The presence of the non-vanishing order parameter field mean the breaking of the some rotational or Lorentz symmetry. However, since the radial direction is not geometric from the boundary point of view, the orders involving r-index, like B_{rt}, can be interpreted as order without the rotational symmetry breaking.

Possible applications

Order p./ Fig.#		Gap	zero mode	feature	possible Phy.Sys.
Φ	s/4(a)	○	×	gap	RS(real Φ) SC(complex Φ)
	c/4(b)	○			
Φ_5	s/4(c)	×	○	Dirac cone	Dirac semi-metal
	c/4(d)	×			
B_r	s/4e	×	○	Rot. Sym	∼ Non-coupling
	c/4e	×			
B_r^5	s/4e	×	×	Rot. Sym	∼ Non-coupling
	c/4e	×			
B_i	s/6abc	×	○	Split cones Fermi arc	Top. Ins.
	c/6def	×			
B_i^5	s/6ghi	×	×	Split cones	no gap
	c/6jkl	×			
B_t	s/7abc	×	×	Rot. Sym	
	c/7ghi	×			
B_t^5	s/7def	×	○	Disk flat band	
	c/7jkl	×			
B_{rt}	s/8a	△	○	Disk flat band	twisted bi-layer graphene
	c/8b	△			
B_{ri}	s/9abc	×	×	Split cones, Fermi-arc	Top. Ins.
	c/9def	×			
B_{xy}	s/8c	×	×	Marginal gap	Kondo lattice
	c/8d	×			
B_{ti}	s/9ghi	×	○	Split cones, Fermi-arc	Top. Ins.
	c/9jkl	×			

- **Table 1**: In the table of "Gap", ○ denotes gap at the fermi-level, △ represents gap off the fermi level and × is gapless. SC=superconductivity, RS=Random Singlet. $A(k_x, \omega)$ means we consider the spectral function A as the function of k_x and ω. Under $k_x \leftrightarrow k_y$ those with one spatial index are assymetric. All others are symmetric. NC2=not classified in 2+1 dimension.

감사합니다