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Introduction to Quantum Computation for Particle Physicists

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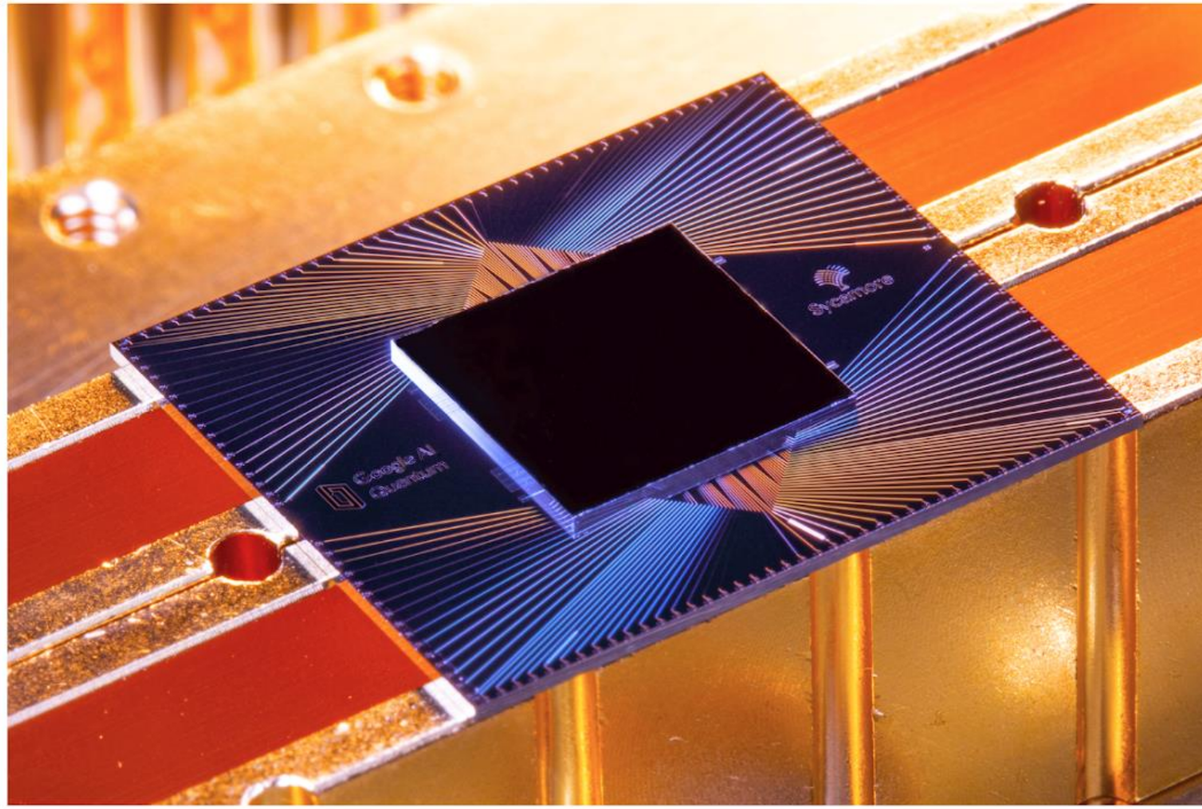
APCTP workshop “Quantum Matter and Quantum Information with Holography”

Quantum computer sounds growing well…



Article

Quantum supremacy using a programmable superconducting processor

https://doi.org/10.1038/s41586-019-1666-5

Frank Arute[1], Kunal Arya[1], Ryan Babbush[1], Dave Bacon[1], Joseph C. Bardin[1,2], Rami Barends[1],

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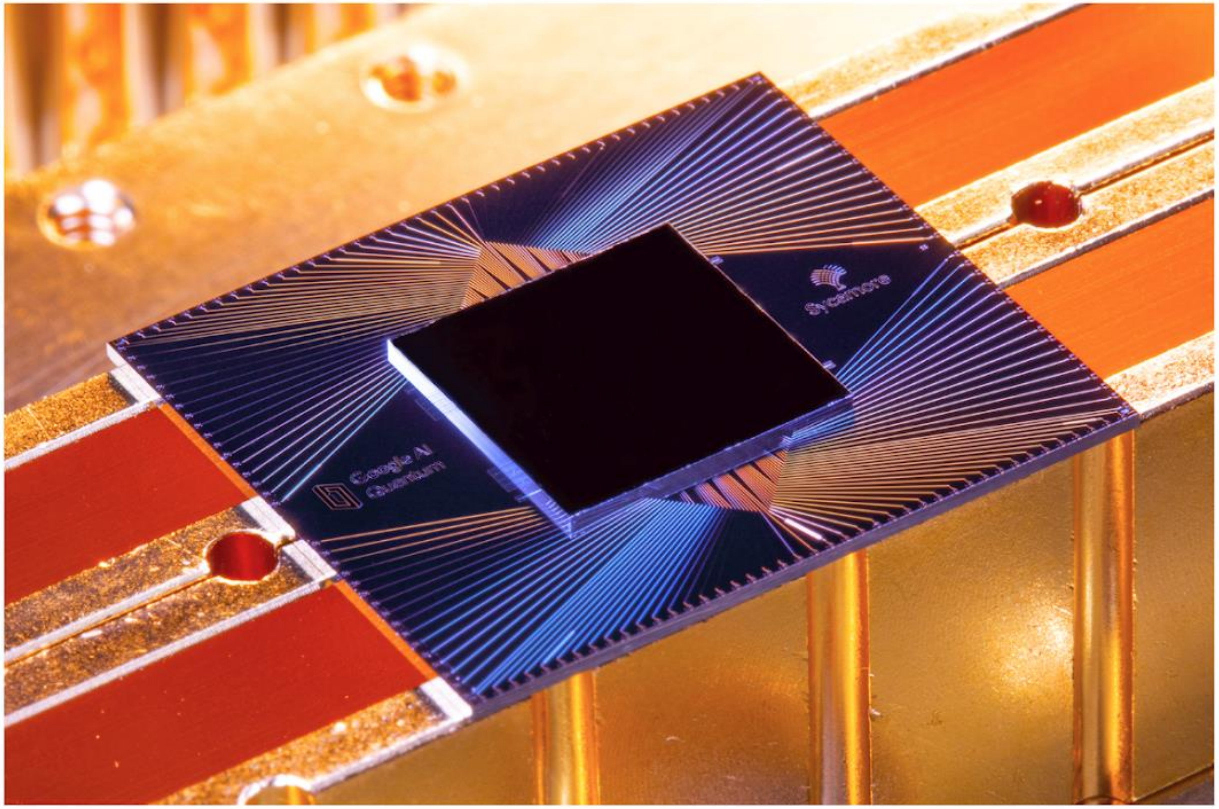

Article

Quantum supremacy using a programmable superconducting processor

This lecture = How can we use it for particle physics?

This lecture is on

Application of Quantum Computation
to
Quantum Field Theory (QFT)

- <u>Generic motivation:</u>

  simply would like to use powerful computers?

- <u>Specific motivation:</u>

This lecture is on

Application of Quantum Computation
to
Quantum Field Theory (QFT)

- <u>Generic motivation:</u>

  simply would like to use powerful computers?

- <u>Specific motivation:</u>

Quantum computation is suitable for Hamiltonian formalism

$\longrightarrow$ Liberation from infamous sign problem in Monte Carlo?

(next slide)

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

(this point is explained to give a motivation & isn't essential to understand main contents of the lectures)

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi\ \mathcal{O}(\phi) e^{-S[\phi]} \quad \Longrightarrow \quad \int d\phi\ \mathcal{O}(\phi) e^{-S(\phi)}$$

② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a probability:

$$\langle \mathcal{O}(\phi) \rangle \simeq \frac{1}{\sharp(\text{samples})} \sum_{i \in \text{samples}} \mathcal{O}(\phi_i)$$

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \ \mathcal{O}(\phi) e^{-S(\phi)}$$

probability

problematic when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " *much worse*

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \; \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

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Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " *much worse*

In Hamiltonian formalism,

sign problem is absent from the beginning

($^{\exists}$various approaches within framework of path integral formalism but I'll skip it)

Cost of Hamiltonian formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

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Quantum computers do this job?

Should we care now as "users"?

Quantum computers don't have sufficient powers yet.

Shouldn't we start to care after quantum supremacy comes?

I personally think:

$\exists$ Many things to do even now in various contexts

(numerical/analytic/purely algorithmic/lat/th/ph)

For instance,

- we haven't established
  - how to put QCD efficiently on quantum computers
  - how to efficiently pick up various real time physics

(e.g. scattering/dynamical hadronization)

- $\exists$ only 1 example so far to take a serious continuum limit

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

Some good news…

- If you have google or facebook account,
  you can immediately use IBM's quantum computer

- Algorithms for simulating quantum system are much easier than ones for generic purpose (e.g. Shor's algorithm for prime factorization)

- Simple code can be made by drug & drop in IBM's website and serious code is made by python

- I am beginner of both python and quantum computation (started on last June)

- It's fun!!

Plan

0. Introduction

1. Qubits and gates

2. Some demonstrations in IBM Q Experience

3. Quantum simulation of Spin system

4. QFT as qubits (mapping to spin system)

5. Summary

Qubit = Quantum Bit

Qubit = Quantum system w/ 2 dim. Hilbert space

Basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

"computational basis"

Generic state:

$$\alpha|0\rangle + \beta|1\rangle \qquad \text{w/} \quad |\alpha|^2 + |\beta|^2 = 1$$

Ex.) Spin 1/2 system:

$$|0\rangle = |\uparrow\rangle, \qquad |1\rangle = |\downarrow\rangle$$

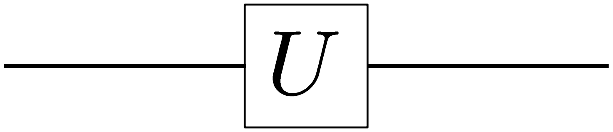
(We don't need to mind how it is realized as "users")

Single qubit operations

- **Acting unitary operator:** $|\psi\rangle \rightarrow U|\psi\rangle$ (multiplying 2x2 unitary matrix)

In quantum circuit notation,

$|\psi\rangle$



$U|\psi\rangle$

$= \alpha|0\rangle + \beta|1\rangle$

$= \alpha U|0\rangle + \beta U|1\rangle$

Single qubit operations

- <u>Acting unitary operator:</u> $|\psi\rangle \rightarrow U|\psi\rangle$ (multiplying 2x2 unitary matrix)

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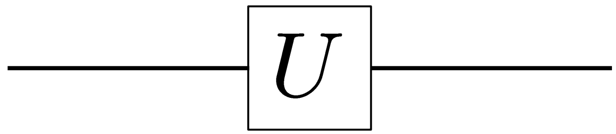
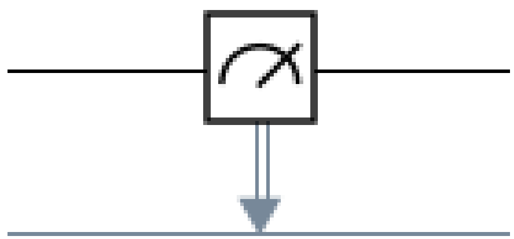
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$U|\psi\rangle = \alpha U|0\rangle + \beta U|1\rangle$$

- <u>Measurement:</u>

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



c (classical number)

$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

Single qubit gates used here

X, Y, Z gates : (just Pauli matrices)

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

X is "NOT": $X|0\rangle = |1\rangle, \; X|1\rangle = |0\rangle$

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R_X, R_Y, R_Z gates :

$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \qquad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \qquad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

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$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |+\rangle, \quad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \equiv |-\rangle$$

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T gate :

$$T = e^{\frac{\pi i}{8}} R_Z\left(\frac{\pi}{4}\right) = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Multiple qubits

2 qubits – 4 dim. Hilbert space:

$$|\psi\rangle = \sum_{i,j=0,1} c_{ij}|ij\rangle, \qquad |ij\rangle \equiv |i\rangle \otimes |j\rangle$$

$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \qquad |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \qquad |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \qquad |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

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N qubits – 2^N dim. Hilbert space:

$$|\psi\rangle = \sum_{i_1,\cdots i_N=0,1} c_{i_1\cdots i_N}|i_1\cdots i_N\rangle,$$

$$|i_1 i_2 \cdots i_N\rangle \equiv |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

Two qubit gates used here

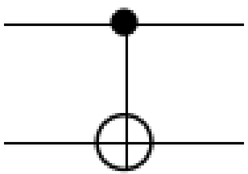
Controlled X (NOT) gate:

$$\begin{cases} CX|00\rangle = |00\rangle, & CX|01\rangle = |01\rangle, \\ CX|10\rangle = |11\rangle, & CX|11\rangle = |10\rangle \end{cases}$$

or equivalently

$$CX|0\rangle \otimes |\psi\rangle = |0\rangle \otimes |\psi\rangle, \qquad CX|1\rangle \otimes |\psi\rangle = |1\rangle \otimes X|\psi\rangle$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



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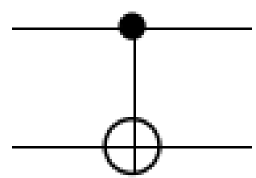
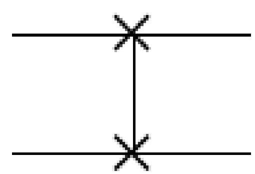


SWAP gate:

$$\text{SWAP}|\psi\rangle \otimes |\phi\rangle = |\phi\rangle \otimes |\psi\rangle$$

$$\text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

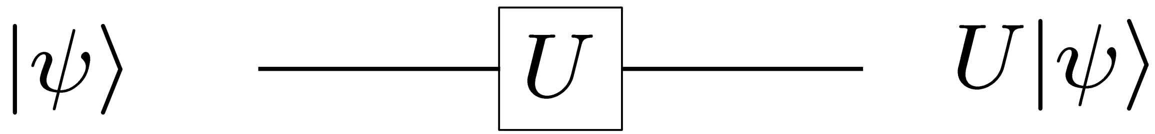


We'll see this is useful to compute Renyi entropy

Rule of the game

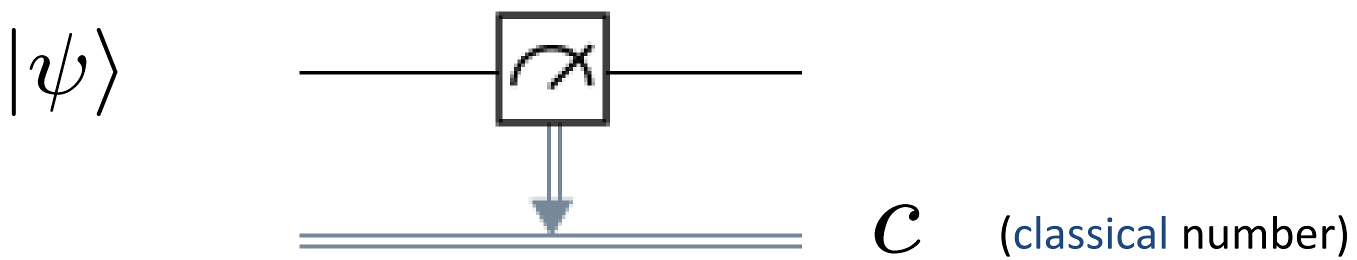
Do something interesting by a combination of

action of Unitary operators:



&

measurements:

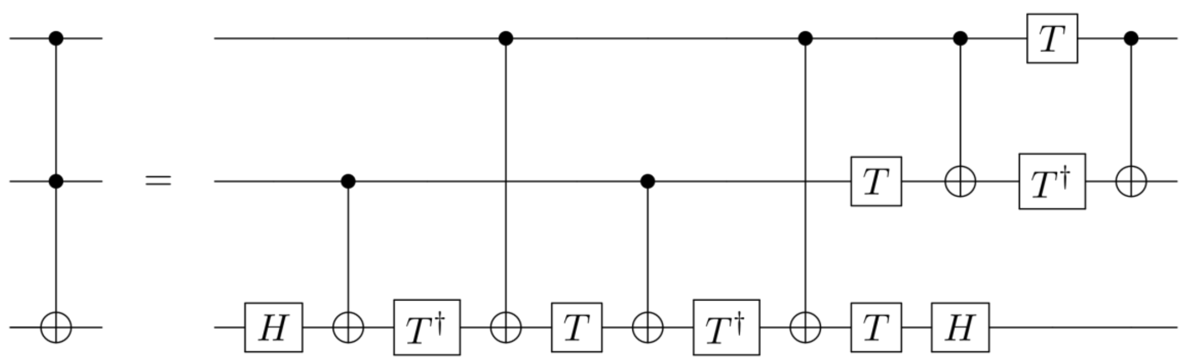


Universality

- Any unitary gate is a combination of single qubit gates & CX ("Single qubit gates & CX are universal")

Ex.) Toffoli

(controlled-controlled-NOT)



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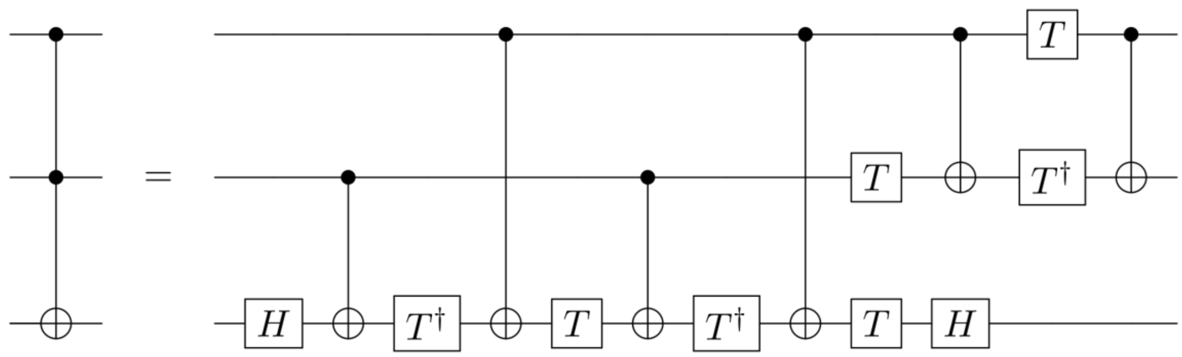

- Any single qubit gate is approximated by a combination of H & T in arbitrary precision

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

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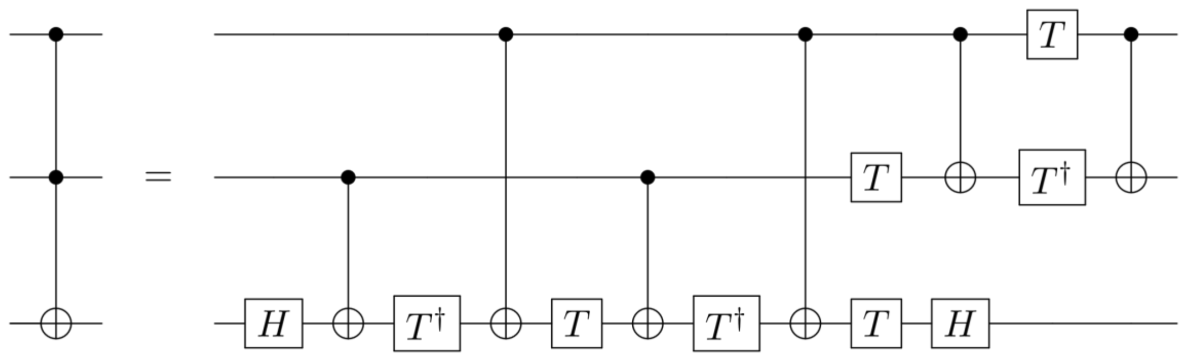

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- H, T & CX are universal

Errors in Quantum computer

In real quantum computer,

Qubits in quantum circuit ≠ isolated system

➡ Interactions w/ environment cause errors/noises

We need to include "quantum error corrections" which seem to require a huge number of qubits

(~ major obstruction of the development)

This lecture won't discuss quantum error corrections but it can be taken into account in an independent way of details of algorithm

(Classical) simulator for Quantum computer

Quantum computation ⊂ Linear algebra

The same algorithm can be implemented in classical computer but w/o speed-up (1 quantum step = many classical steps)

Simulator = Tool to simulate quantum computer by classical computer

- Doesn't have errors → ideal answers

  (More precisely, classical computer also has errors but its error correction is established)

- The same code can be run in quantum computer w/ speed-up

Useful to test algorithm & estimate computational resources

(~# of qubits, gates)

Short summary

- Qubit = Quantum bit
- Important gates:

$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \quad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \quad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |+\rangle, \quad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \equiv |-\rangle$$

$$CX|0\rangle \otimes |\psi\rangle = |0\rangle \otimes |\psi\rangle, \quad CX|1\rangle \otimes |\psi\rangle = |1\rangle \otimes X|\psi\rangle$$

- Do something interesting by a combination of acting unitary op. & measurement
- H, T & CX are universal

$$T = e^{\frac{\pi i}{8}} R_Z\left(\frac{\pi}{4}\right) = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

- Real quantum computer has errors
- Simulator = Tool to simulate quantum computer by classical computer

Some demonstrations in IBM Quantum Experience

um Network Technology Resources

IBM Quantum Experience is quantum on the cloud

Accelerate your research and applications with the next generation of the leading quantum cloud services and software platform.

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Powerful software for the most powerful hardware

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Create a circuit →

Qiskit Notebooks

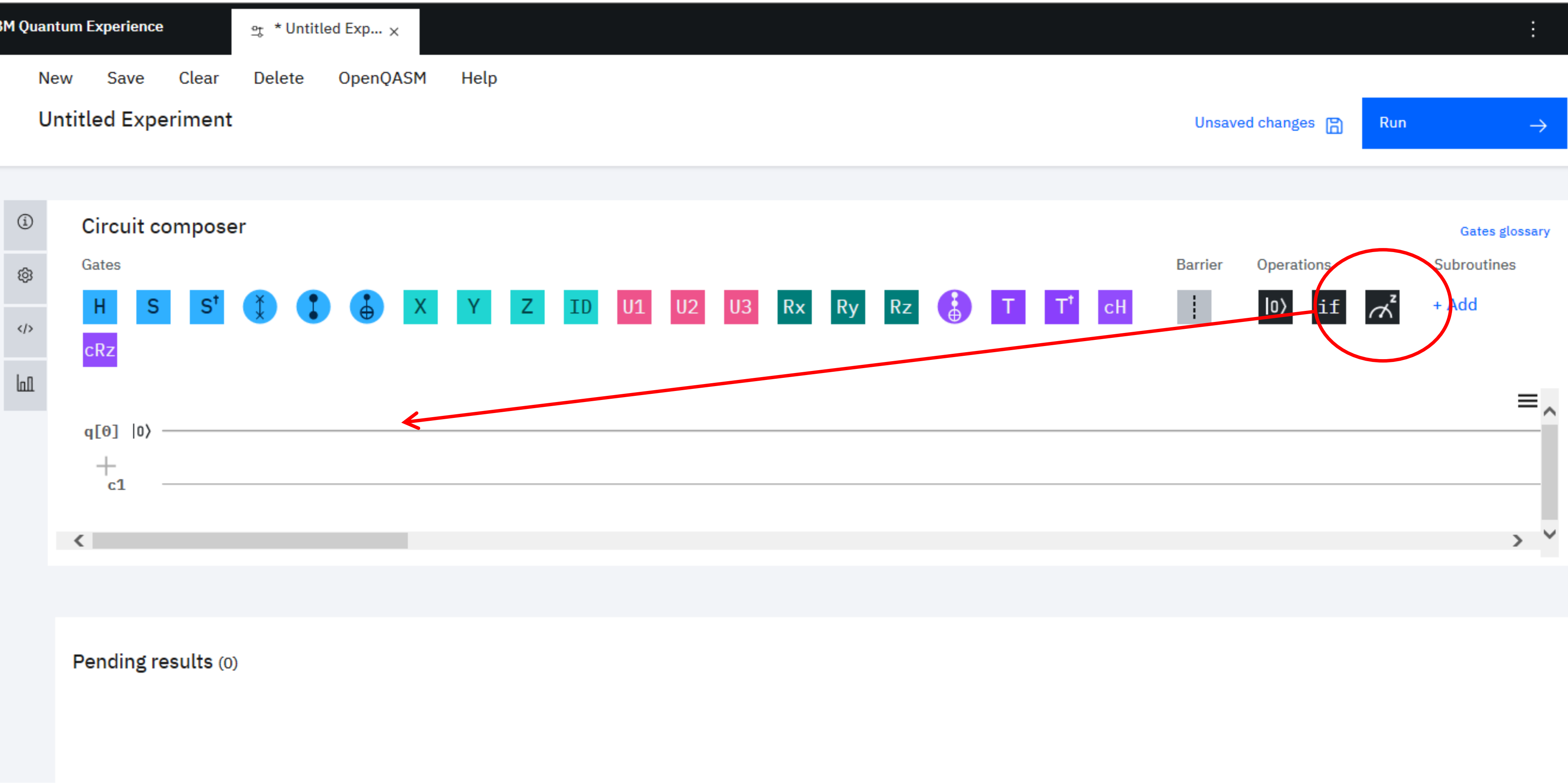
Create your first notebook and start using Qiskit

Create a notebook →

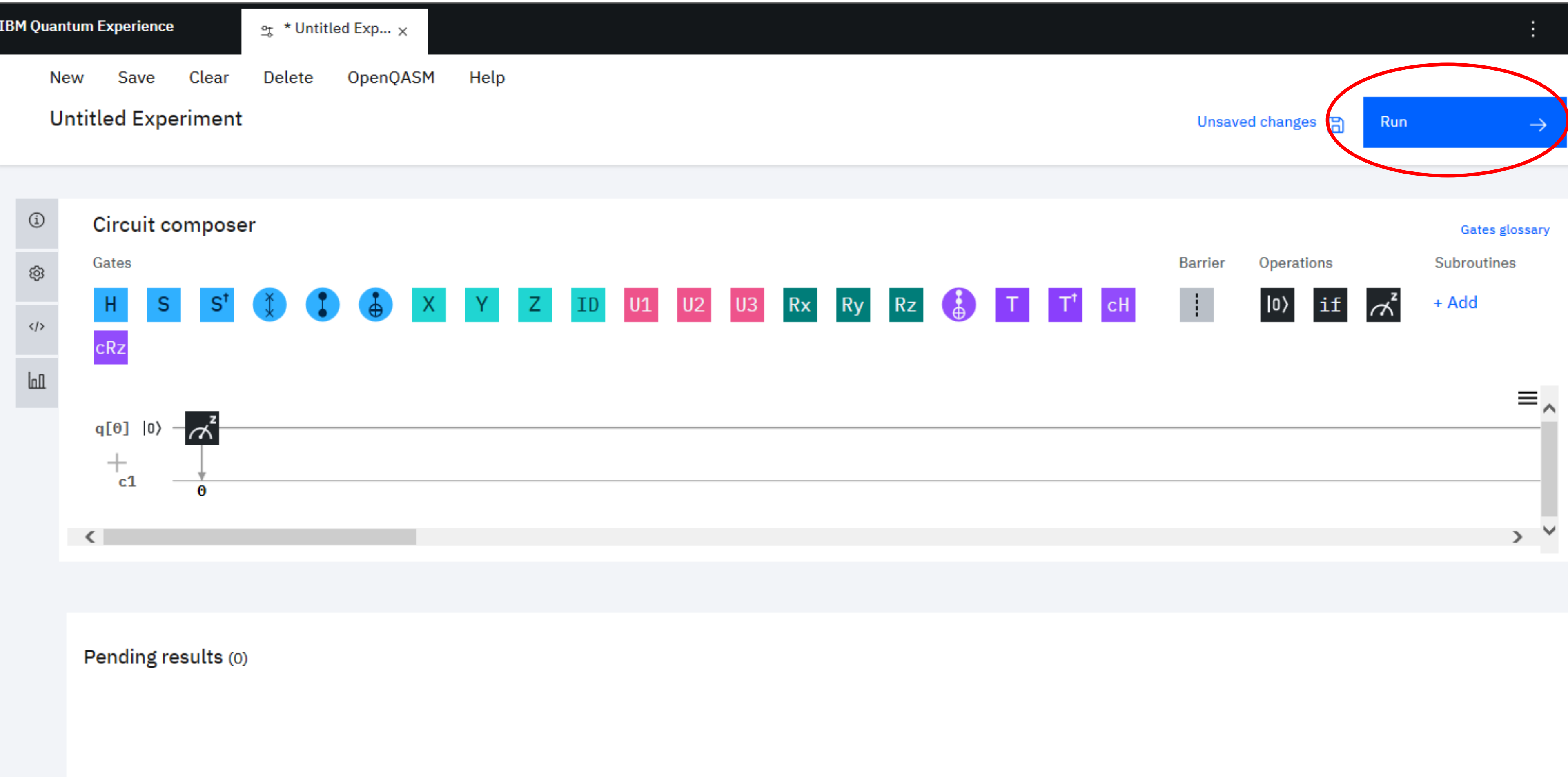
Pending results (0)

You have no experiment runs in the queue.

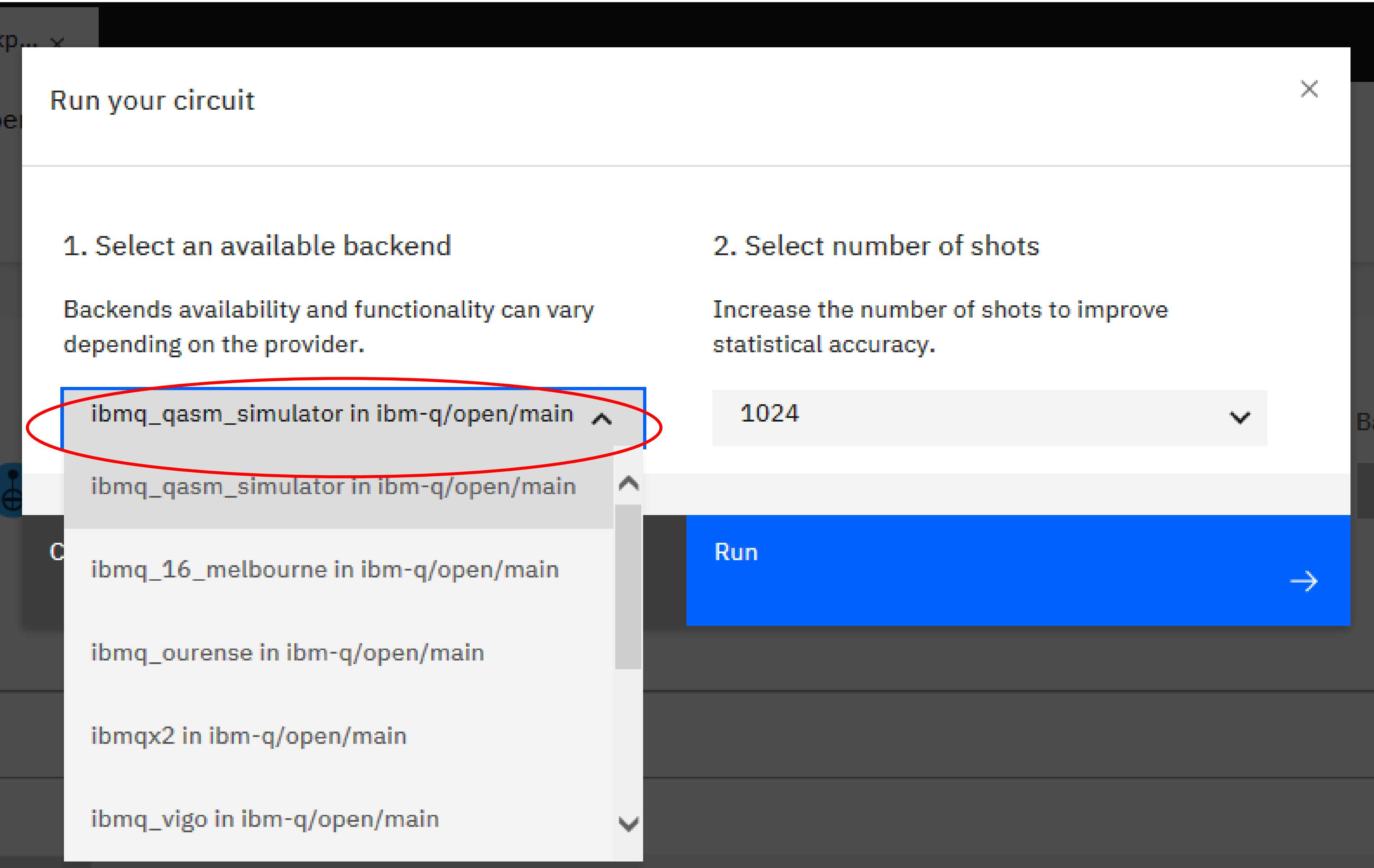
A trivial problem: measure |0⟩



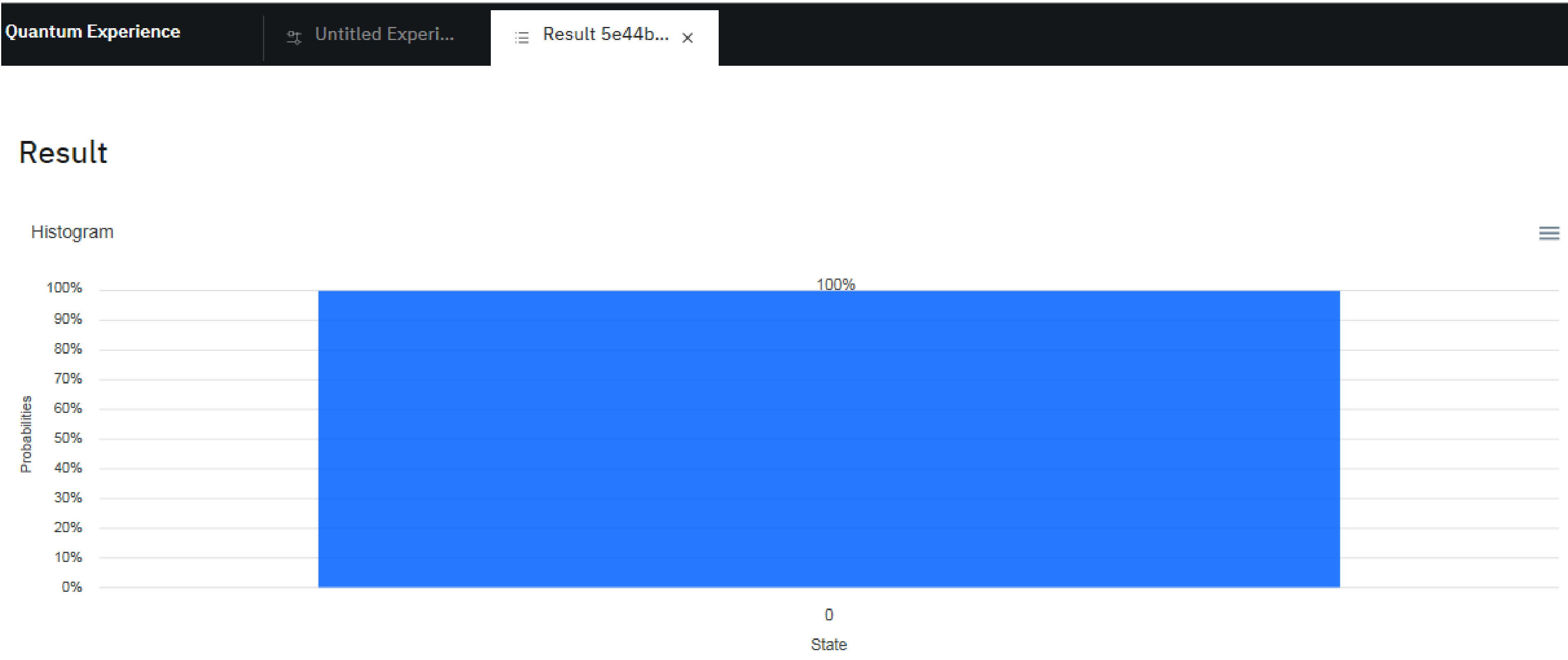
A trivial problem: measure $|0\rangle$ (Cont'd)



Measure 1024 times in simulator

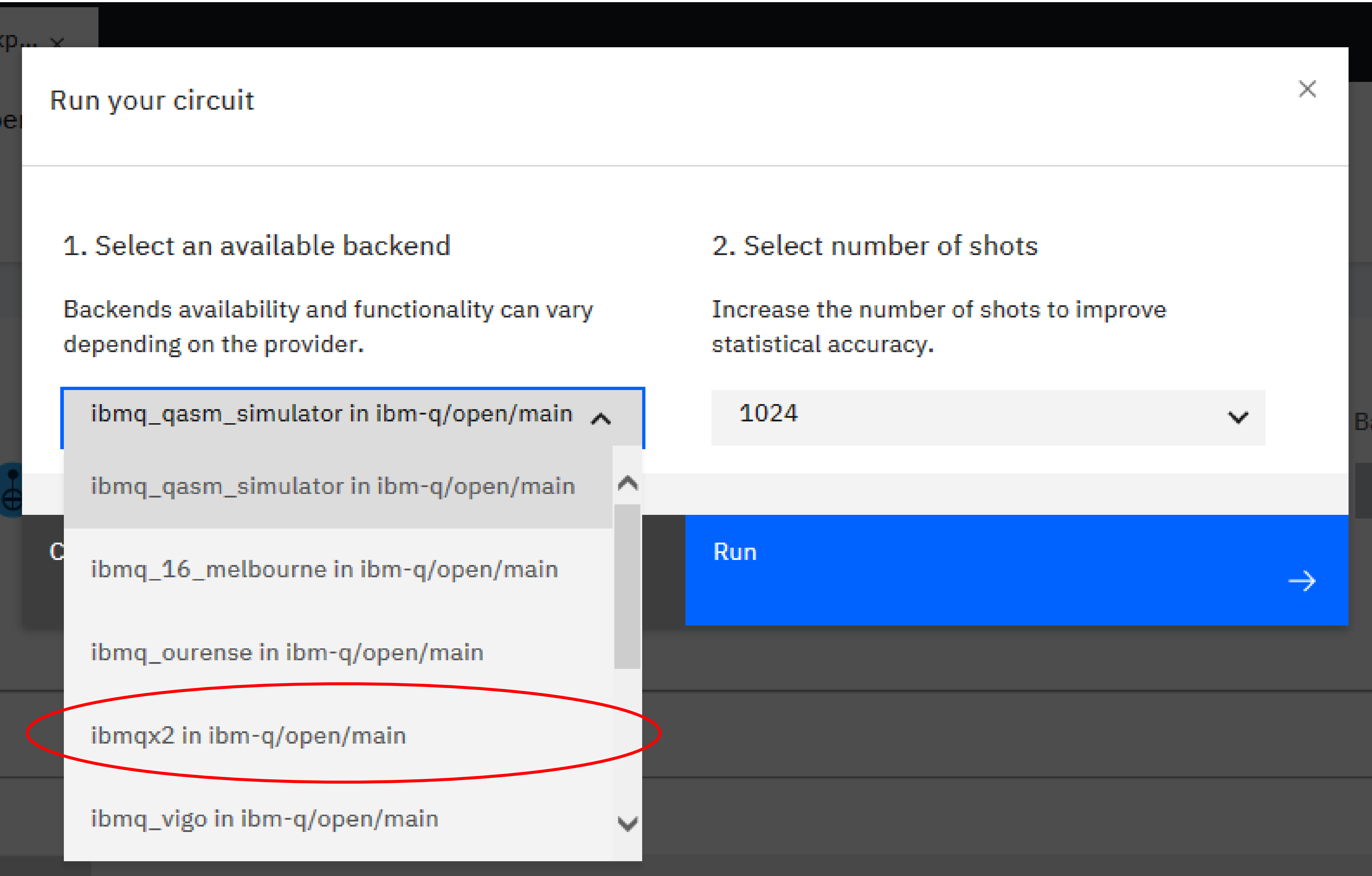


Trivial result



Of Course!

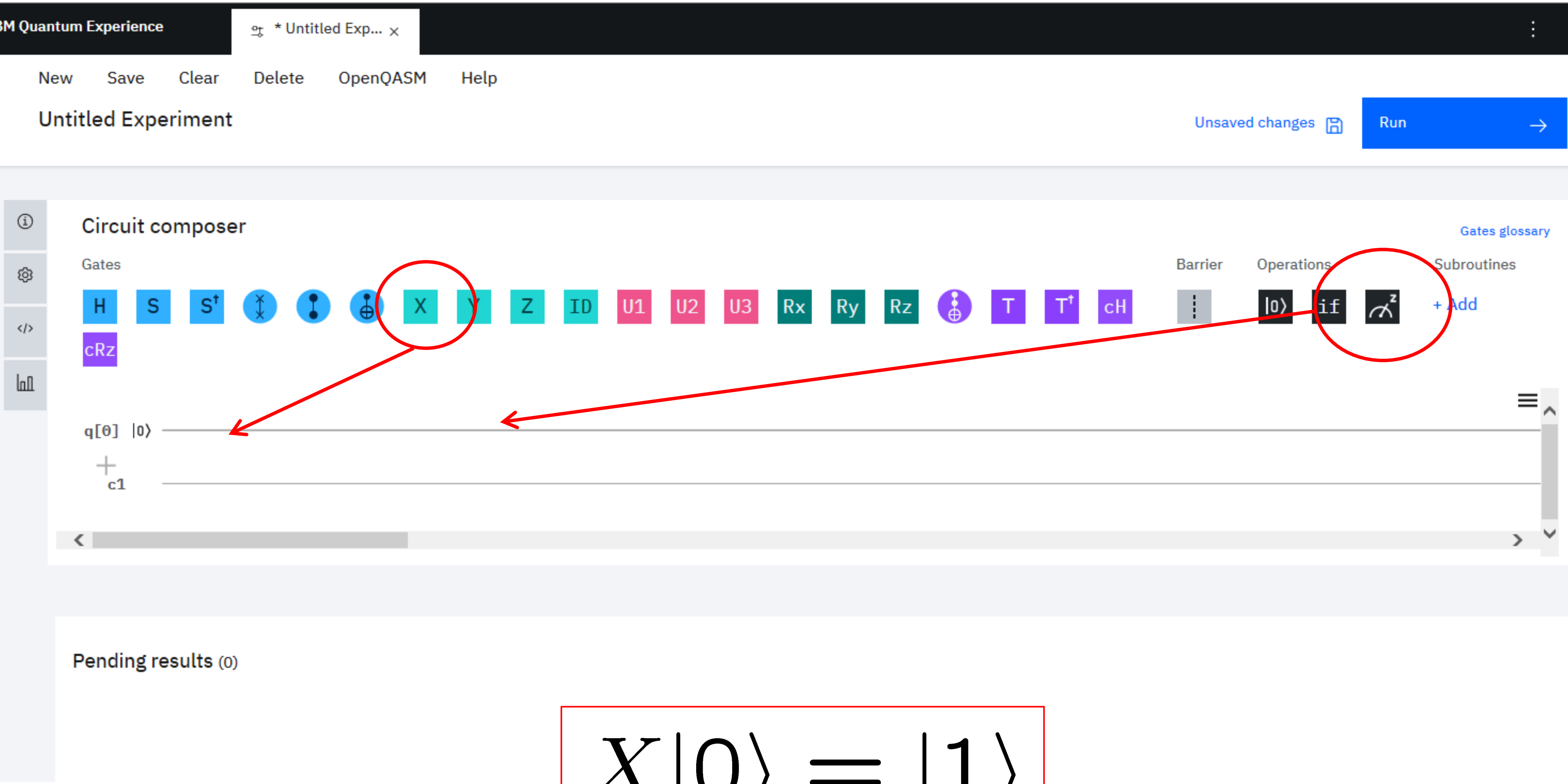
Measure 1024 times in quantum computer



Result of quantum computer?

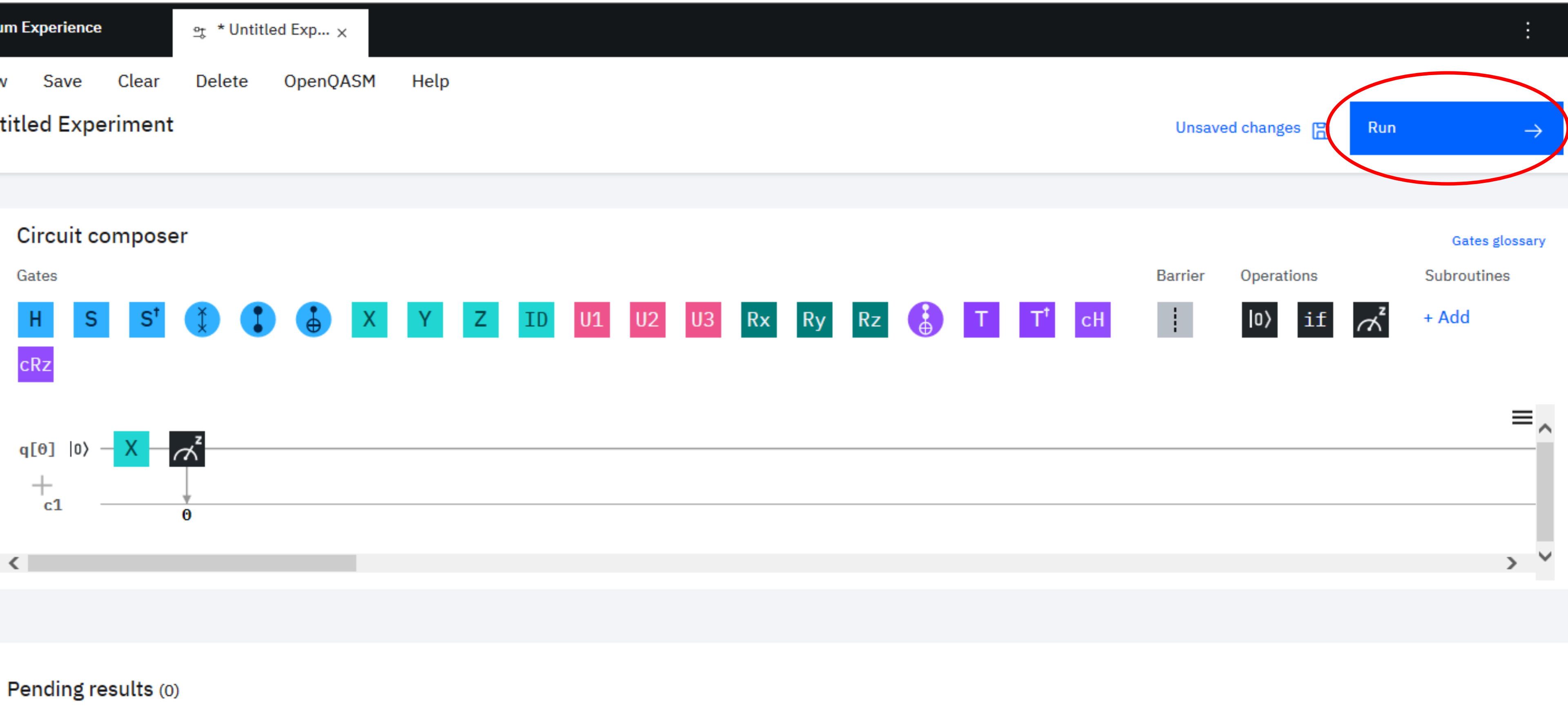
This is the error!

A trivial problem2: measure $|1\rangle$

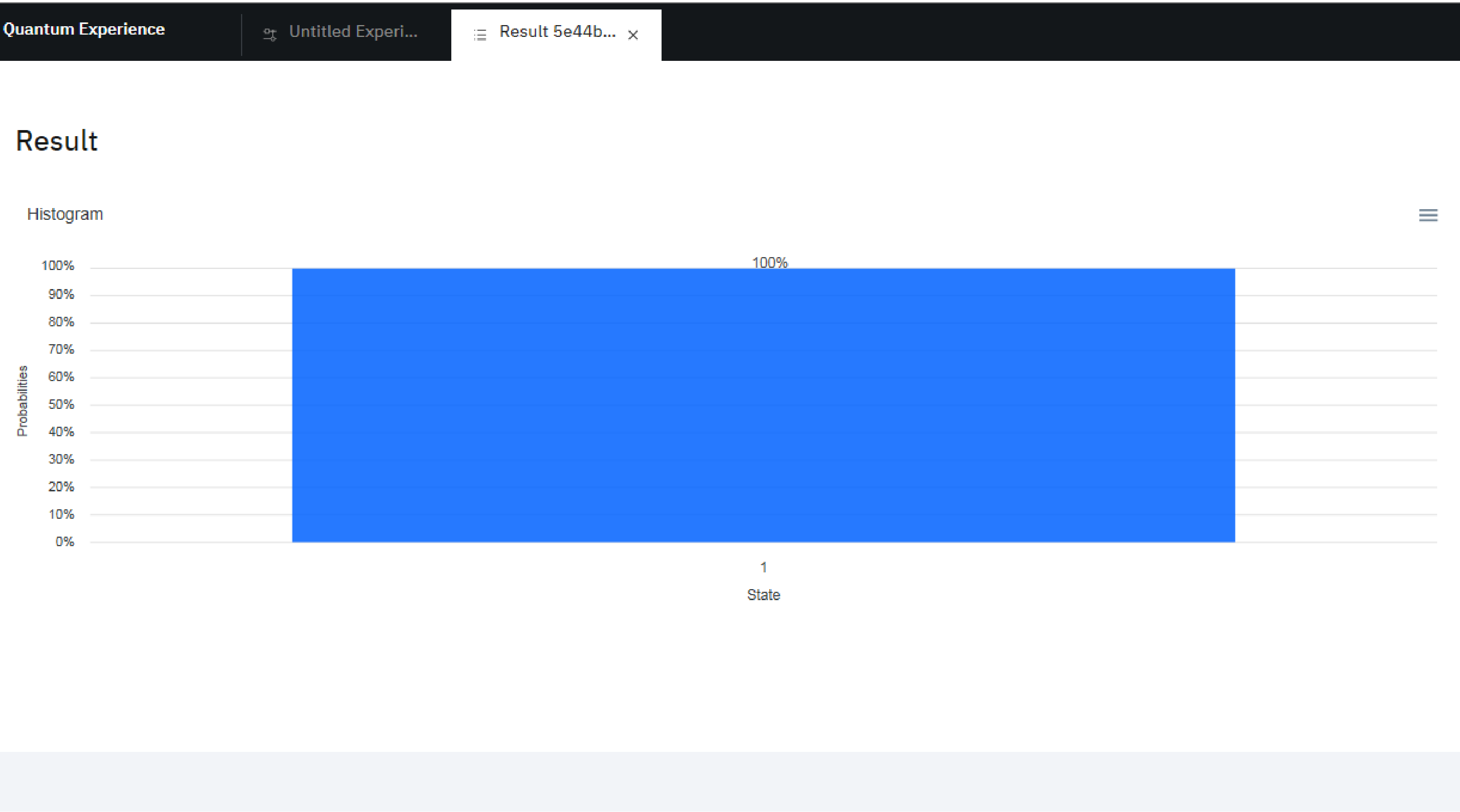


$$X|0\rangle = |1\rangle$$

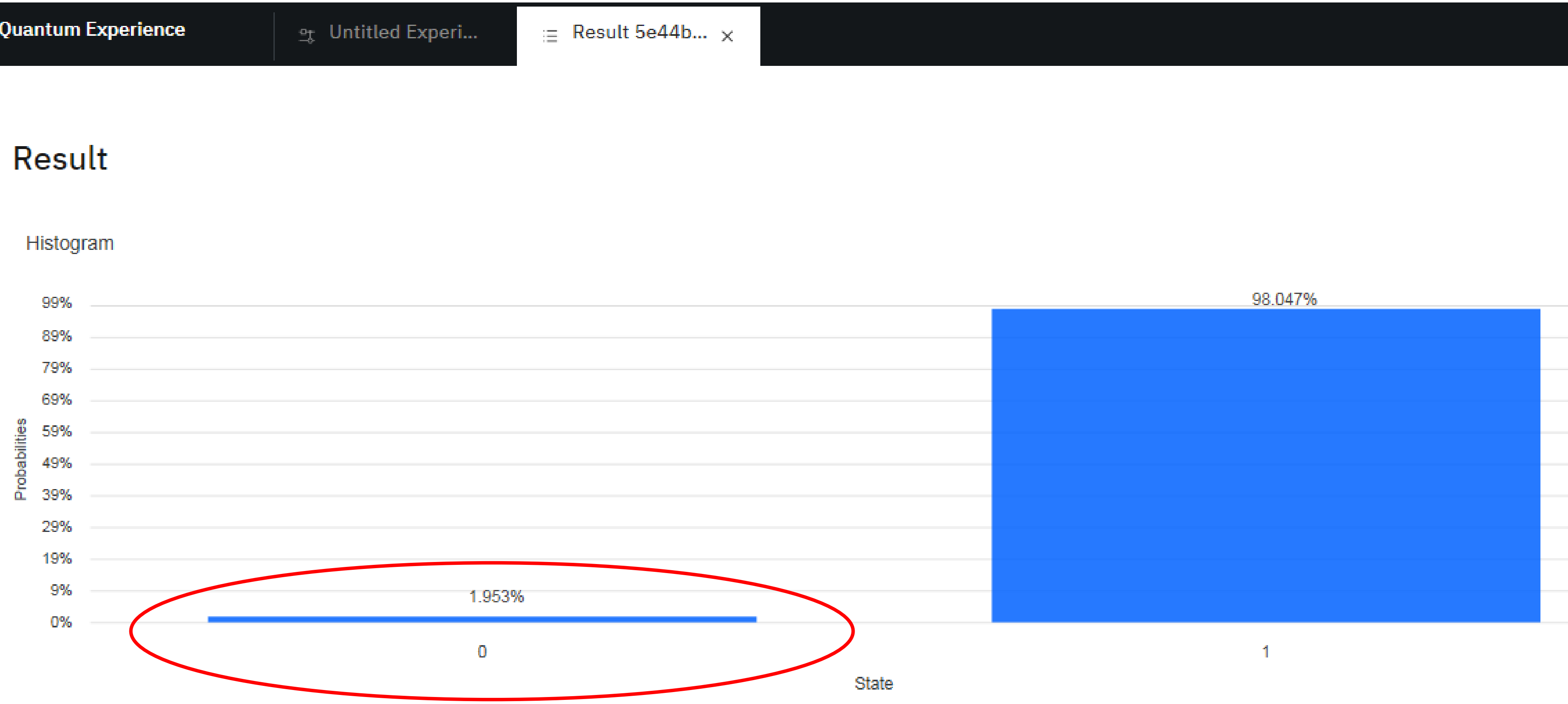
A trivial problem2: measure |1⟩ (Cont’d)



Result of simulator (1024 shots)

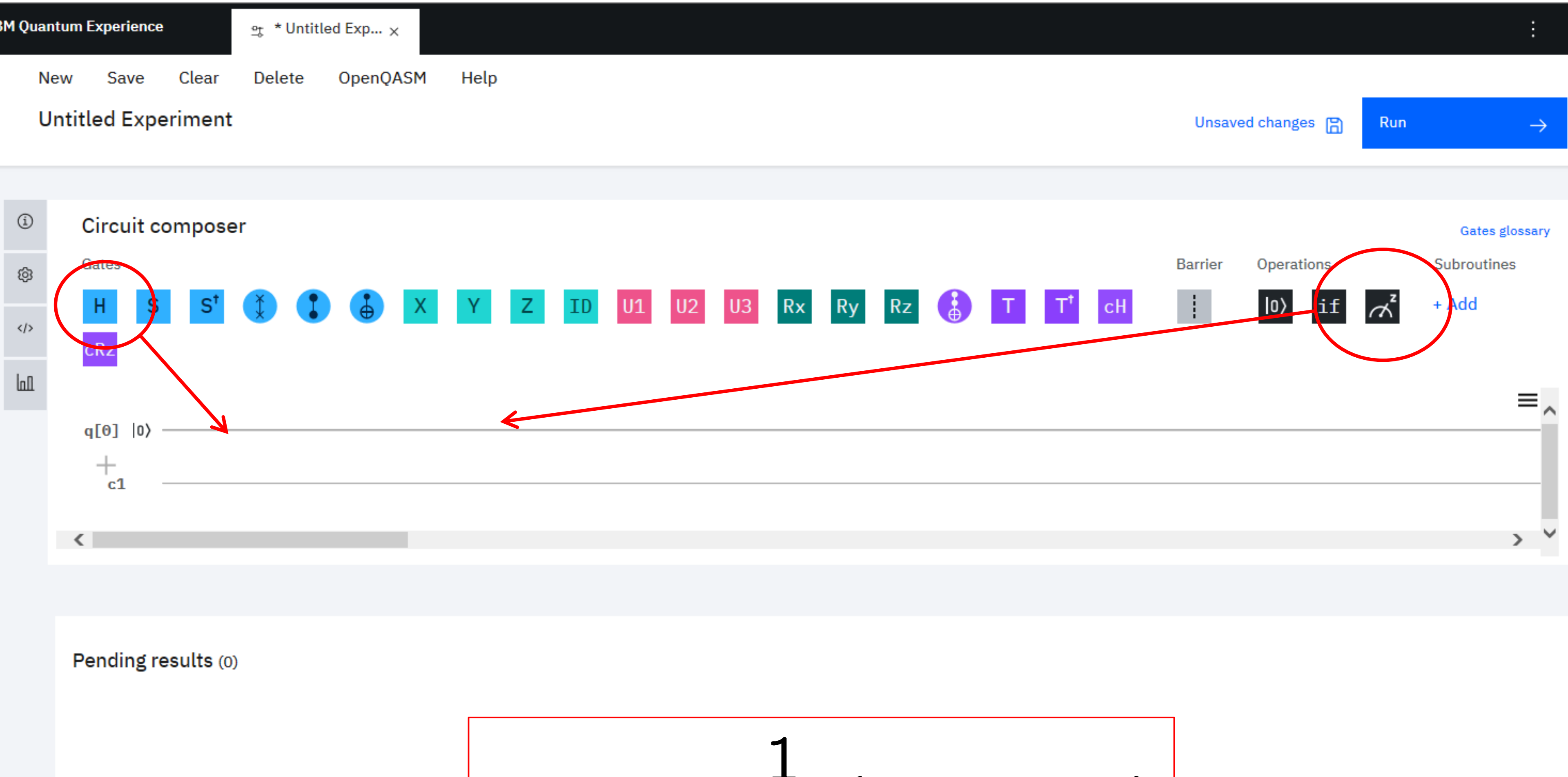


Result of quantum computer (1024shots)



Errors again

The simplest nontrivial problem: Hadamard gate

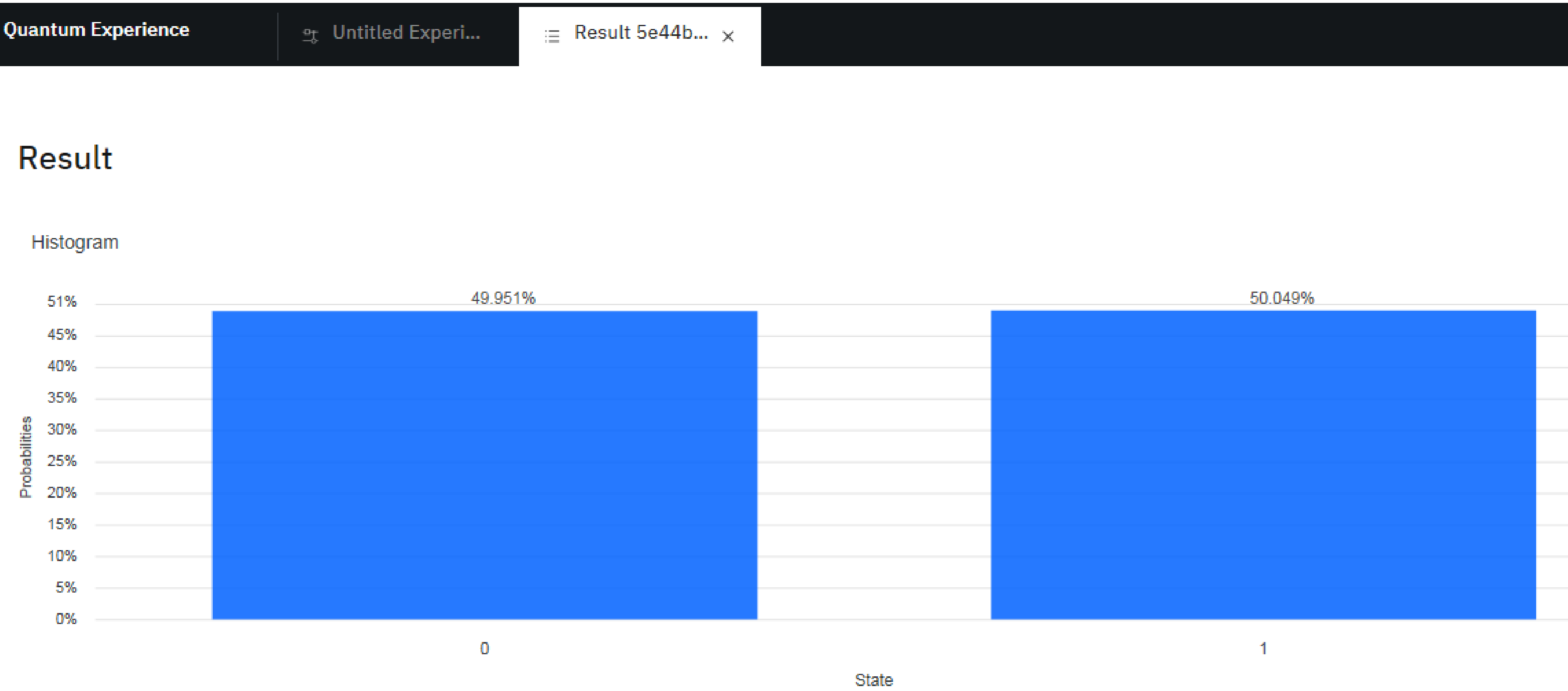


$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Result of simulator (1024 shots)

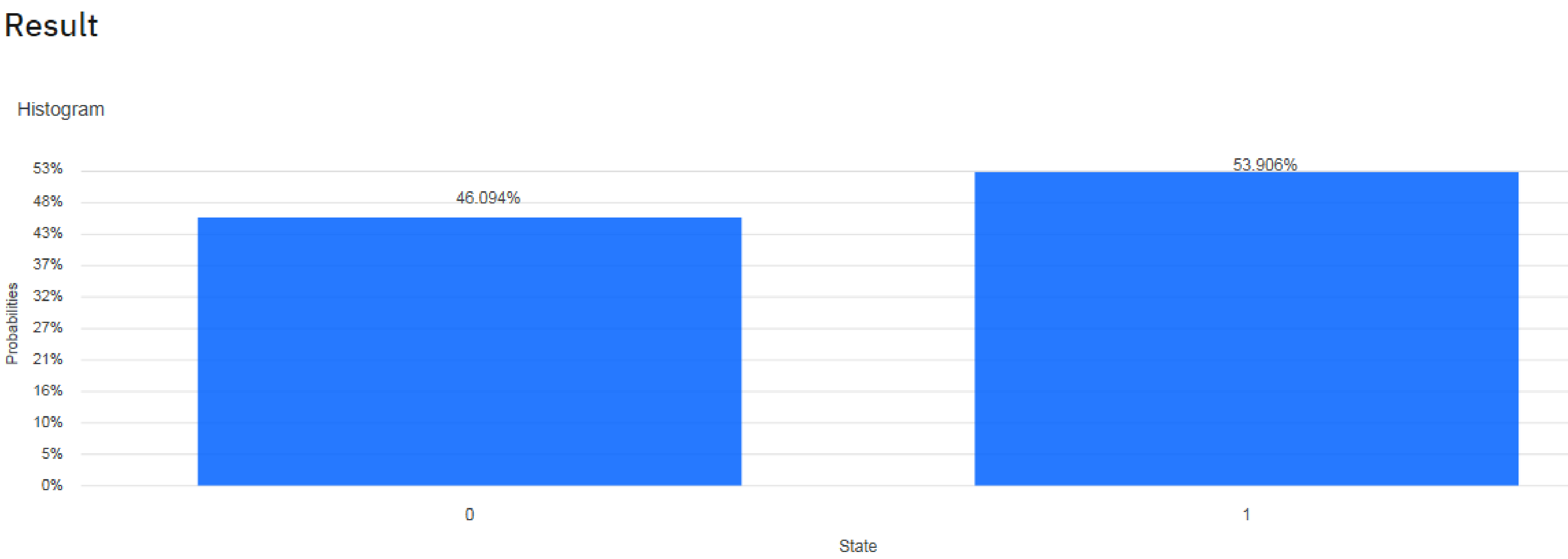
Not 50:50 because of statistical errors

Result of simulator (8192 shots)



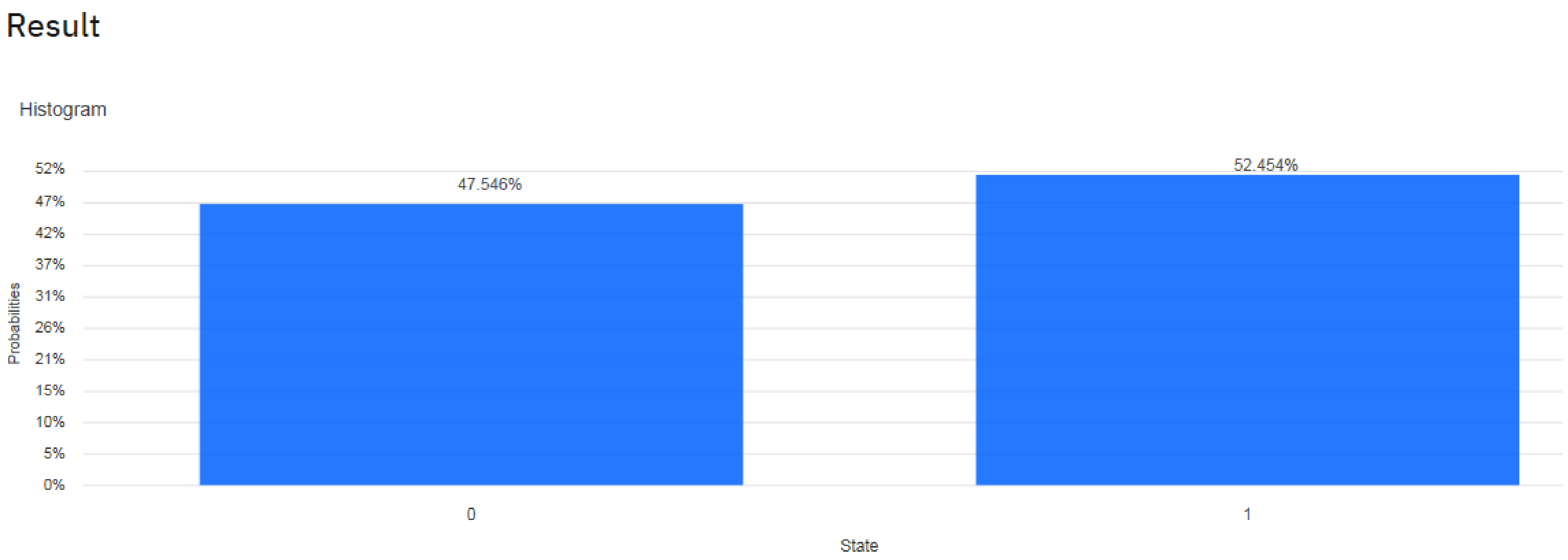
Improved!

Result of quantum computer (1024 shots)



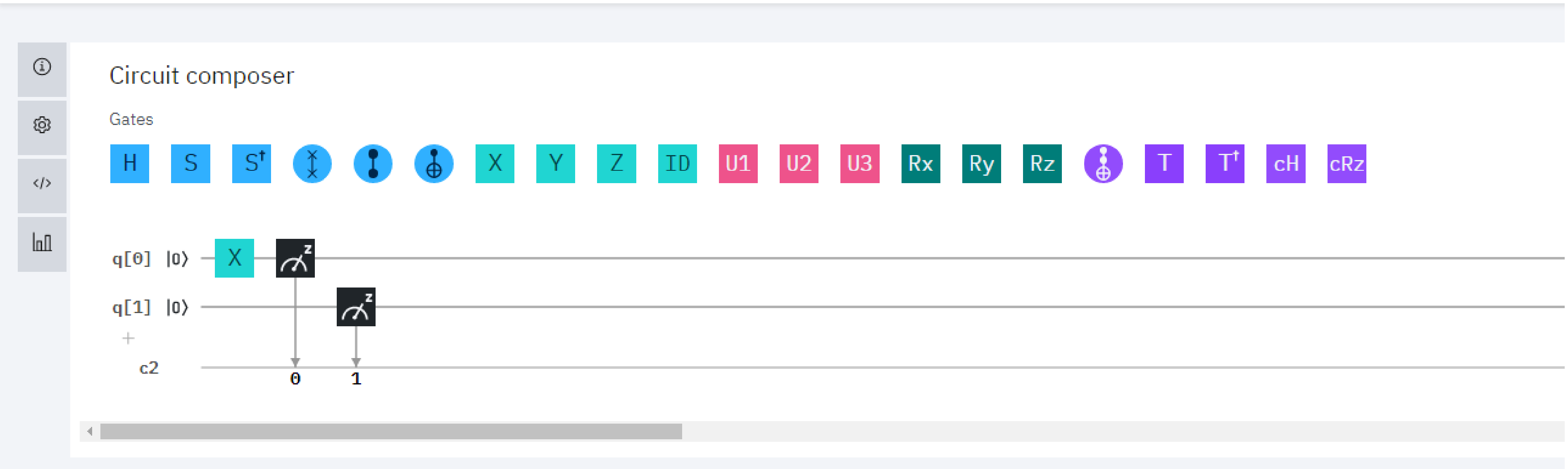
∃ Both errors & statistical errors

Result of quantum computer (8192 shots)



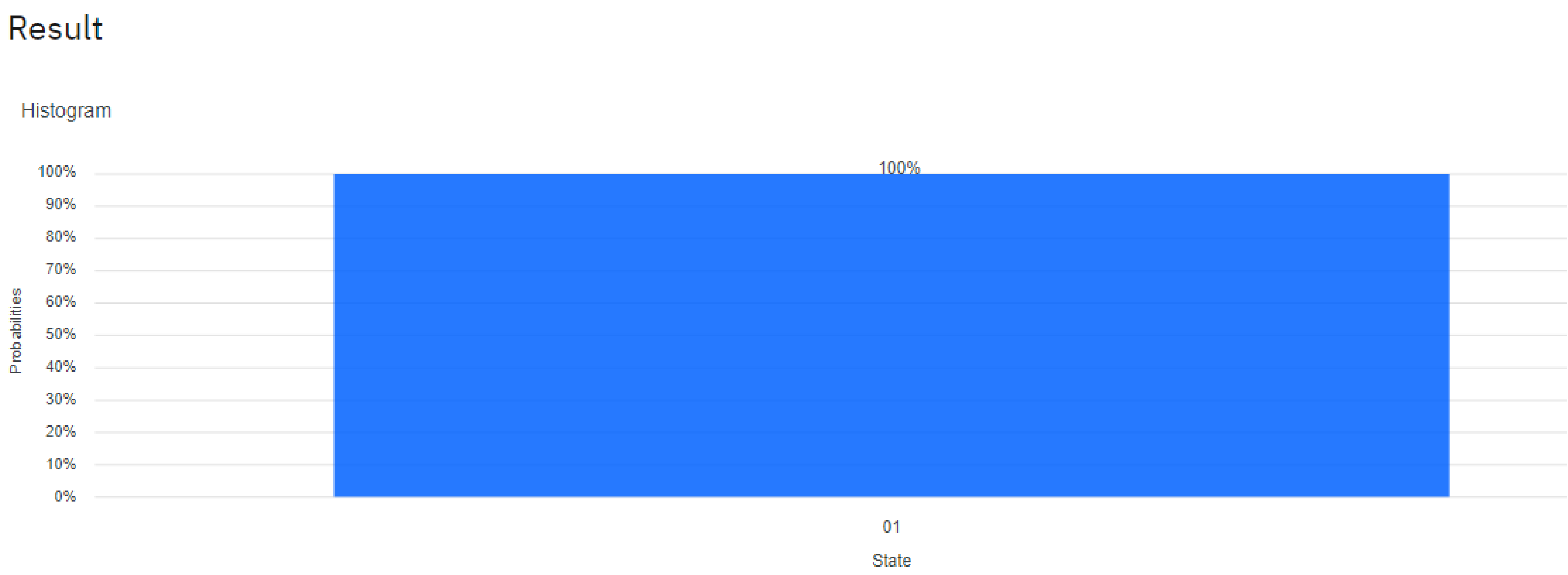
Statistical errors are improved

A trivial problem for 2 qubits



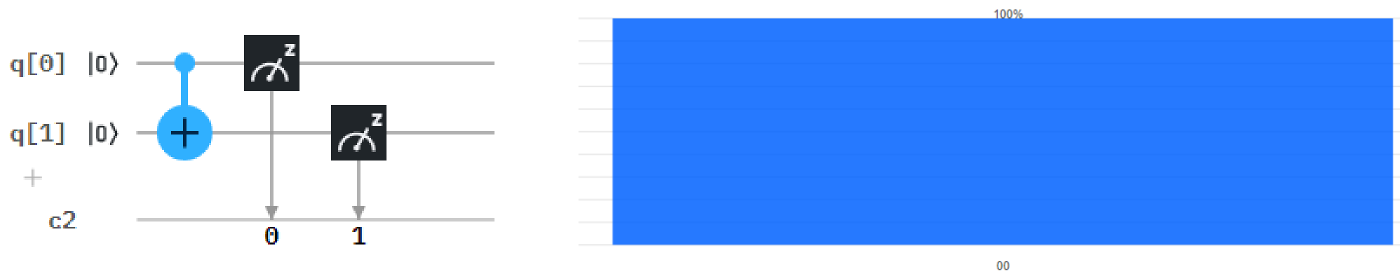
$$X_1|00\rangle = |10\rangle$$

Result of simulator (1024 shots)

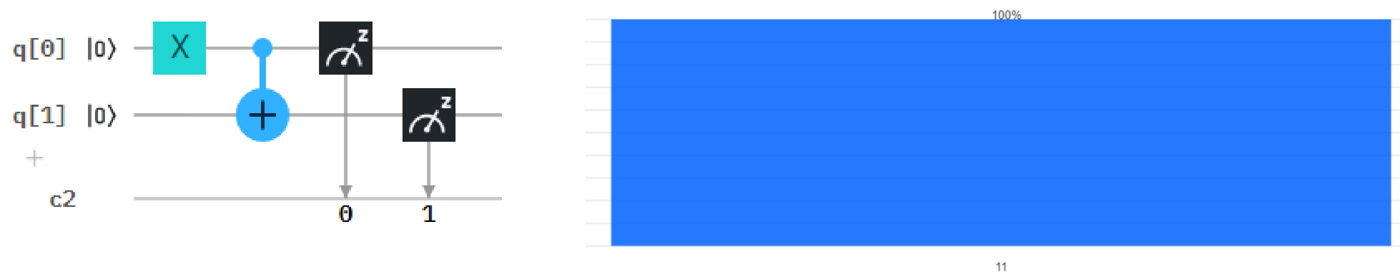


Note that notation is different from the ket notation

2 qubit operation by simulator

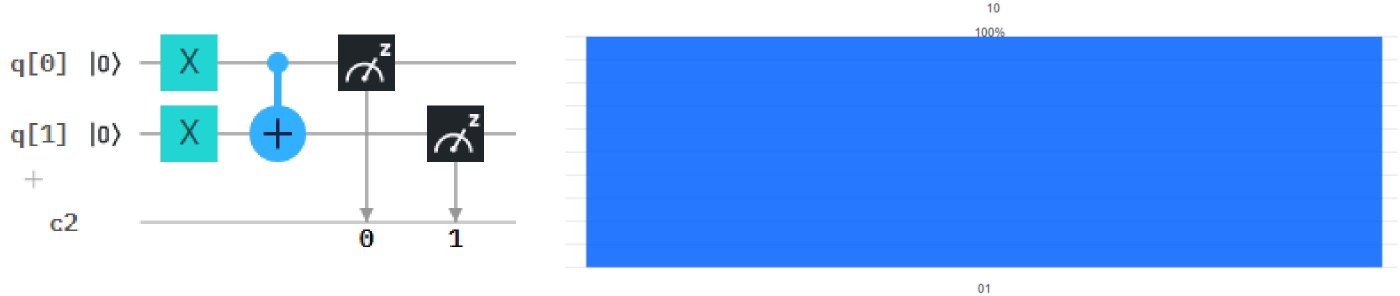


$$CX|00\rangle = |00\rangle$$



$$CX|10\rangle = |11\rangle$$

$$CX|01\rangle = |01\rangle$$



$$CX|11\rangle = |10\rangle$$

2 qubit operation by quantum computer (1024 shots)

$$CX|00\rangle = |00\rangle$$

$$CX|10\rangle = |11\rangle$$

$$CX|01\rangle = |01\rangle$$

$$CX|11\rangle = |10\rangle$$

Quantum simulation of Spin system

Warm up: 2-site transverse Ising model

$$\hat{H} = -JZ_1Z_2 - h(X_1 + X_2)$$

We are going to

- construct time evolution operator
- obtain vacuum state
- compute vacuum expectation values
- compute Renyi entropy

Time evolution operator

Time evolution of any state is studied by acting the operator

$$e^{-i\hat{H}t} = e^{-i(H_X + H_{ZZ})t}$$

where

$$H_X = -h(X_1 + X_2), \quad H_{ZZ} = -JZ_1Z_2$$

How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc…)

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How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc…)

Step 1: Suzuki-Trotter decomposition:

($\exists$ higher order improvements)

$$e^{-i\hat{H}t} = \left(e^{-i\hat{H}\frac{t}{M}}\right)^M$$

(M: large positive integer)

$$\simeq \left(e^{-iH_X\frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}}\right)^M + \mathcal{O}(1/M)$$

Time evolution operator (Cont'd)

$$e^{-i\widehat{H}t} \simeq \left(e^{-iH_X\frac{t}{M}}e^{-iH_{ZZ}\frac{t}{M}}\right)^M$$

Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}}\right)^M$$

The 1st one is trivial:

acting on qubit 2 acting on qubit 1

$$e^{-iH_X \frac{t}{M}} = e^{-i\frac{ht}{M}X_2} e^{-i\frac{ht}{M}X_1} = R_X^{(2)}\left(\frac{2ht}{M}\right) R_X^{(1)}\left(\frac{2ht}{M}\right)$$

Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}}\right)^M$$

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The 2nd one is nontrivial:

$$e^{-iH_{ZZ}\frac{t}{M}} = e^{-i\frac{Jt}{M}Z_1 Z_2} = \cos\frac{Jt}{M} - iZ_1 Z_2 \sin\frac{Jt}{M}$$

Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X\frac{t}{M}}e^{-iH_{ZZ}\frac{t}{M}}\right)^M$$

The 1st one is trivial:

acting on qubit 2 acting on qubit 1

$$e^{-iH_X\frac{t}{M}} = e^{-i\frac{ht}{M}X_2}e^{-i\frac{ht}{M}X_1} = R_X^{(2)}\left(\frac{2ht}{M}\right)R_X^{(1)}\left(\frac{2ht}{M}\right)$$

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$$e^{-iH_{ZZ}\frac{t}{M}} = e^{-i\frac{Jt}{M}Z_1Z_2} = \cos\frac{Jt}{M} - iZ_1Z_2\sin\frac{Jt}{M}$$

One can show (see next slide)

$$e^{-i\frac{Jt}{M}Z_1Z_2} = CXR_Z^{(2)}\left(\frac{2Jt}{M}\right)CX$$

Time evolution operator (Cont'd)

$$e^{-icZ_1Z_2} = CXR_Z^{(2)}(2c)CX$$

Proof:

$$CXR_Z^{(2)}(2c)CX|0\rangle \otimes |\psi\rangle$$

$$= CXR_Z^{(2)}(2c)|0\rangle \otimes |\psi\rangle = CX|0\rangle \otimes R_Z(2c)|\psi\rangle$$

$$= |0\rangle \otimes R_Z(2c)|\psi\rangle = \cos c|0\rangle \otimes |\psi\rangle - i\sin c\ Z|0\rangle \otimes Z|\psi\rangle$$

$$CXR_Z^{(2)}(2c)CX|1\rangle \otimes |\psi\rangle$$

$$= CXR_Z^{(2)}(2c)|1\rangle \otimes X|\psi\rangle = CX|1\rangle \otimes R_Z(2c)X|\psi\rangle = |1\rangle \otimes XR_Z(2c)X|\psi\rangle$$

$$= \cos c|1\rangle \otimes XX|\psi\rangle - i\sin c\ |1\rangle \otimes XZX|\psi\rangle$$

$$= \cos c|1\rangle \otimes |\psi\rangle - i\sin c\ Z|1\rangle \otimes Z|\psi\rangle$$

Thus,

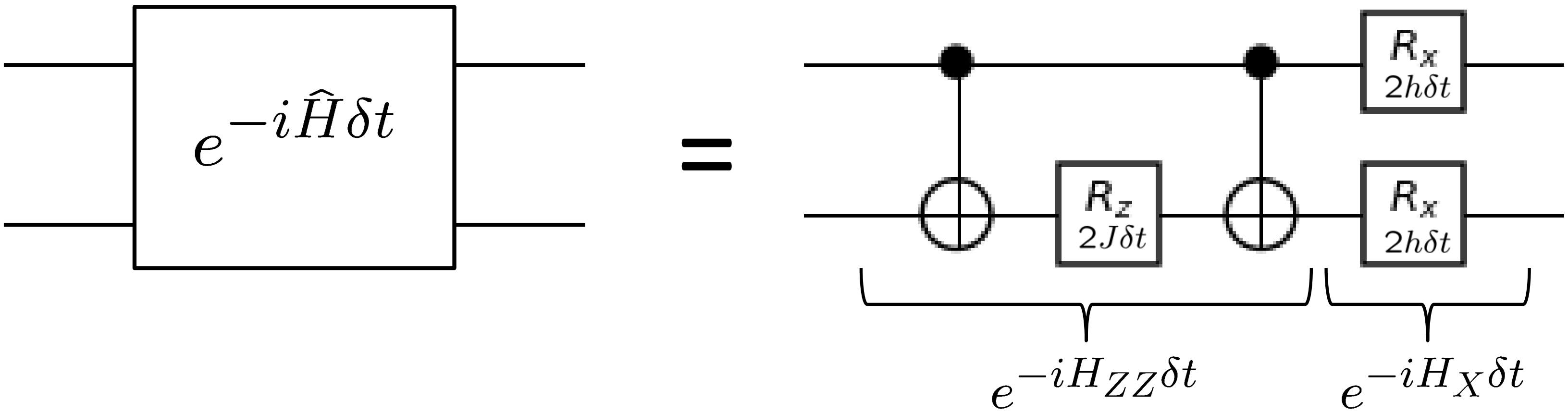
$$CXR_Z^{(2)}(2c)CX|\varphi\rangle \otimes |\psi\rangle = \cos c|\varphi\rangle \otimes |\psi\rangle - i\sin c\ Z|\varphi\rangle \otimes Z|\psi\rangle$$

$$= e^{-icZ_1Z_2}|\varphi\rangle \otimes |\psi\rangle$$

Quantum circuit for time evolution op.

$$H_X = -h(X_1 + X_2), \qquad H_{ZZ} = -JZ_1Z_2$$

$$\delta t = \frac{t}{M} \ll 1$$



$$+\mathcal{O}(\delta t)$$

Survival probability of free vacuum

For J=0, ground state is

$$\hat{H}|_{J=0} = -h(X_1 + X_2)$$

$$|++\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}} = H^{(2)} H^{(1)} |00\rangle$$

We can compute survival probability of the free vacuum:

$$P(t) = \left| \langle ++ | e^{-i\hat{H}t} | ++ \rangle \right|^2$$

Toy version of
Schwinger effect

$$= \left| \langle 00 | H^{(2)} H^{(1)} e^{-i\hat{H}t} H^{(2)} H^{(1)} |00\rangle \right|^2$$

Survival probability of free vacuum

For J=0, ground state is

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We can compute survival probability of the free vacuum:

$$P(t) = \left|\langle ++|e^{-i\hat{H}t}|++\rangle\right|^2$$

Toy version of Schwinger effect

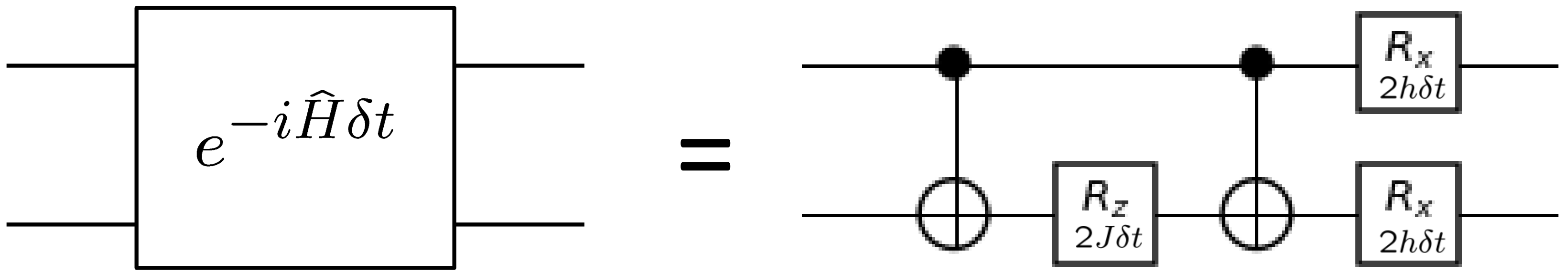
$$= \left|\langle 00|H^{(2)}H^{(1)}e^{-i\hat{H}t}H^{(2)}H^{(1)}|00\rangle\right|^2$$

Measure the probability having $|00\rangle$ inside the state

$$H^{(2)}H^{(1)}e^{-i\hat{H}t}H^{(2)}H^{(1)}|00\rangle$$

Demonstration for the survival probability

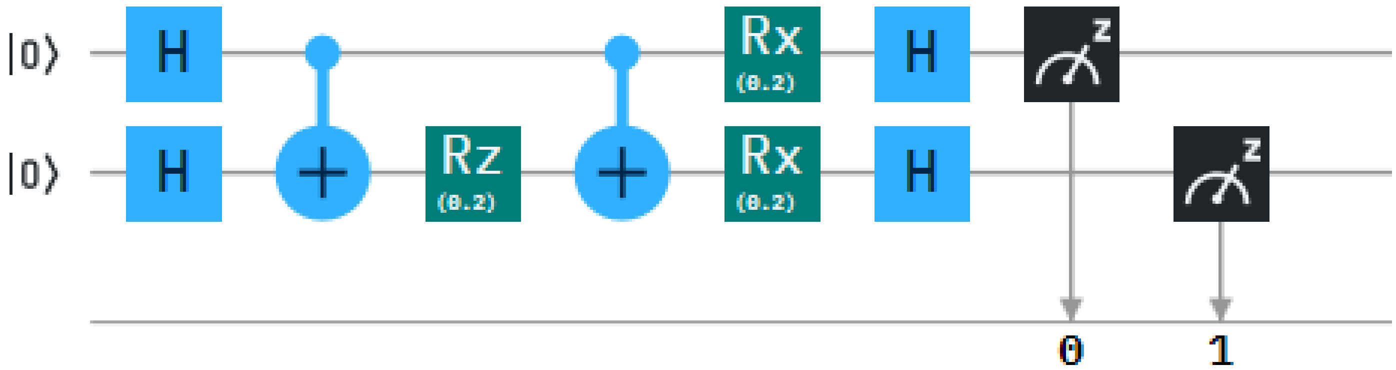
$$P(t) = \left| \langle + + | e^{-i\hat{H}t} | + + \rangle \right|^2 = \left| \langle 00 | H^{(2)} H^{(1)} e^{-i\hat{H}t} H^{(2)} H^{(1)} | 00 \rangle \right|^2$$



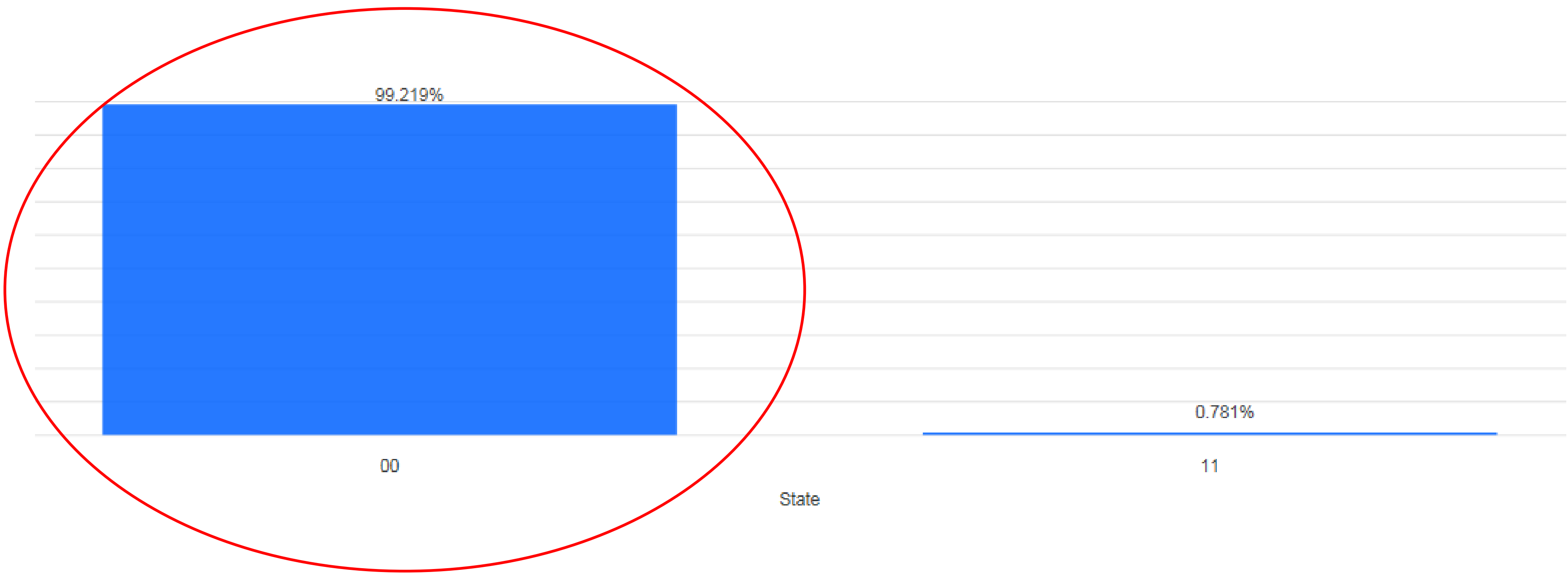
Let's compute it for $J = 1, h = 1, t = 0.1, M = 1$

$$\delta t = \frac{t}{M}$$

Demonstration for the survival probability (Cont'd)

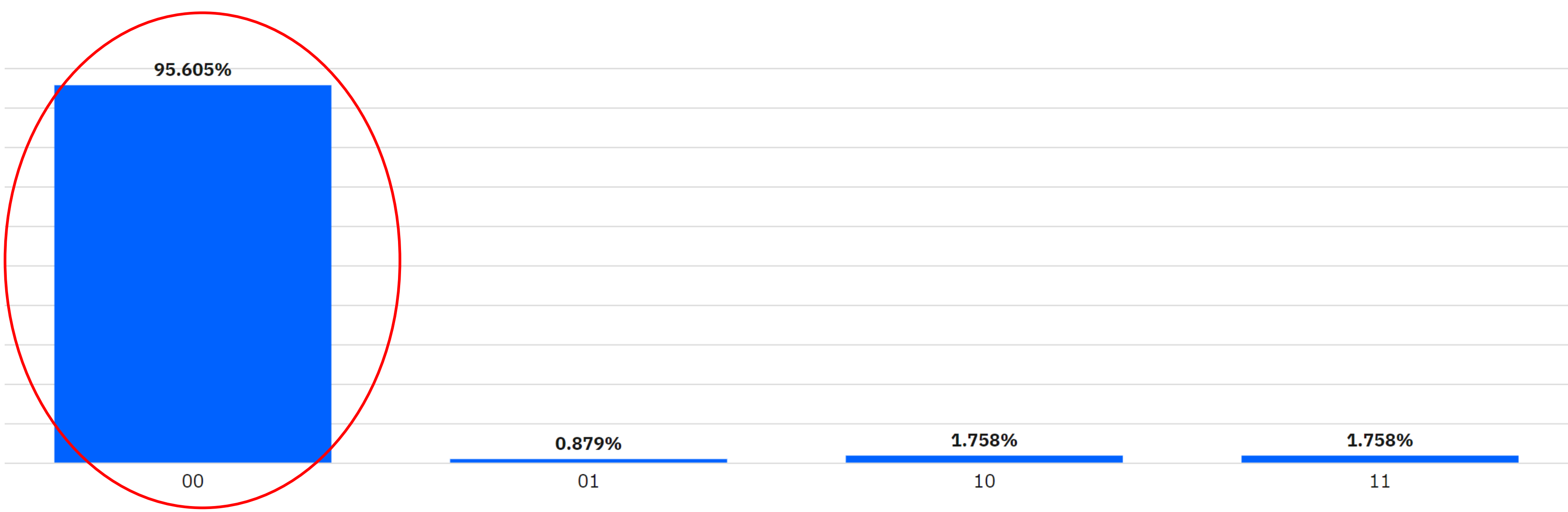


Result by simulator w/ 1024 shots:



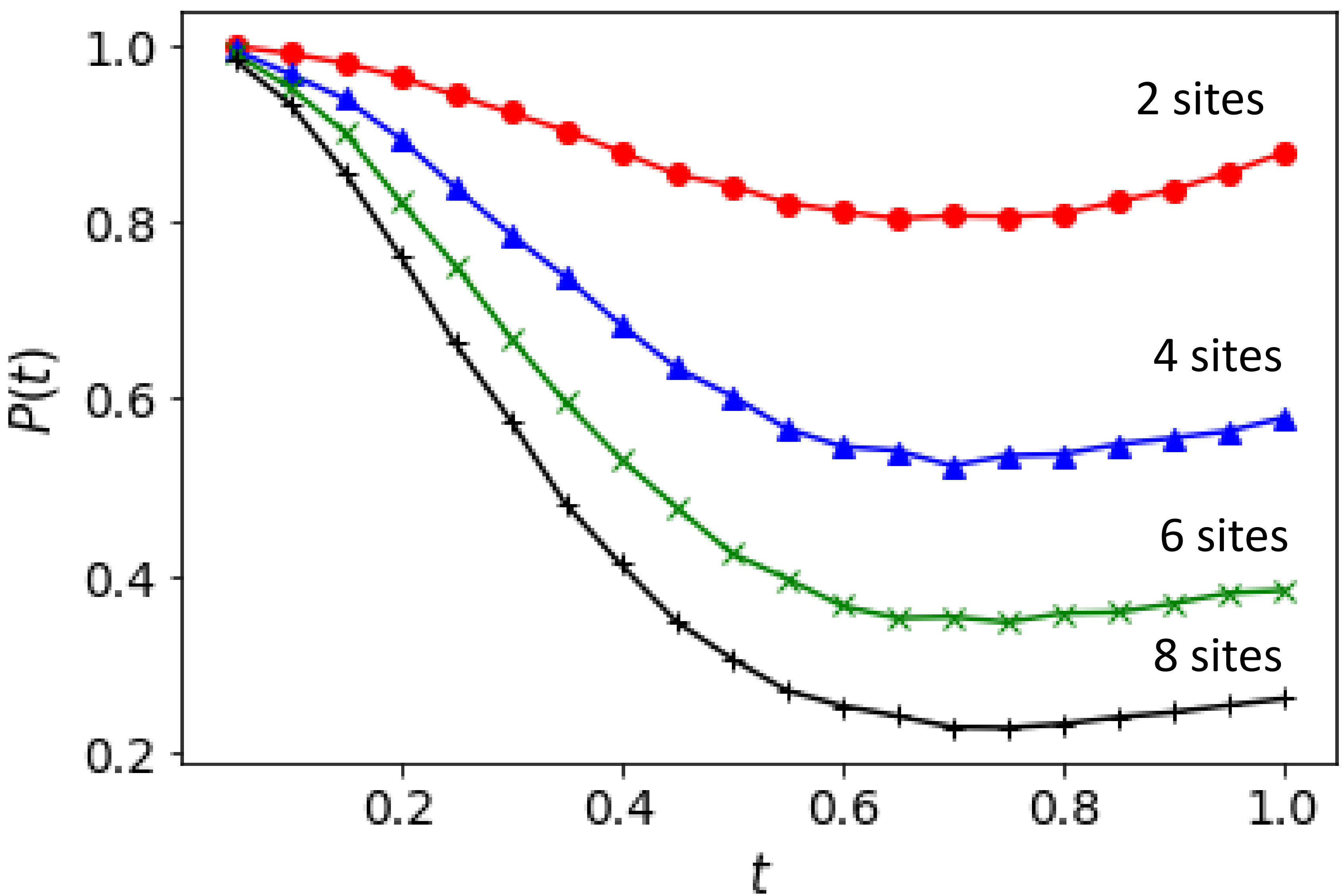
Result of simulator (1024 shots):

Result of quantum computer (1024 shots):



More serious computation

$J = 1, h = 1, t = 1, M = 100,$ 10000 shots



Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2: Consider the time evolution

$$\mathcal{T}\exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0> \quad \text{w/} \quad H_A(0)=H_0,\ H_A(T)=\hat{H}$$

Step 3: Use the adiabatic theorem

If the system w/ the Hamiltonian $H_A(t)$ has a unique gapped vacuum, then the desired ground state is obtained by

$$|\mathrm{vac}>=\lim_{T\to\infty}\mathcal{T}\exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0>$$

For transverse Ising model

$$\hat{H} = -J \sum_{n=1}^{N-1} Z_n Z_{n+1} - h \sum_{n=1}^{N} X_n - m \sum_{n=1}^{N} Z_n$$

Choose

$$\begin{cases} H_0 = -h \sum_{n=1}^{N} X_n \quad \Rightarrow \quad |\mathrm{vac}_0\rangle = |+\cdots+\rangle \\ H_A(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} \hat{H} \end{cases}$$

For transverse Ising model

$$\hat{H} = -J \sum_{n=1}^{N-1} Z_n Z_{n+1} - h \sum_{n=1}^{N} X_n - m \sum_{n=1}^{N} Z_n$$

Choose

$$\begin{cases} H_0 = -h \sum_{n=1}^{N} X_n \quad \Rightarrow \quad |\mathrm{vac}_0\rangle = |+\cdots+\rangle \\ H_A(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T}\hat{H} \end{cases}$$

Discretize the integral:

$$\mathcal{T}\exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0 > \simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\mathrm{vac}_0 >$$

where

$$U(t) = e^{-iH_A(t)\delta t}, \ \ \delta t = \frac{T}{M} \ll 1$$

Magnetization

Once we get the vacuum, we can compute VEV of operators:

$$\langle \mathrm{vac}|\mathcal{O}|\mathrm{vac}\rangle$$

It is easiest to compute magnetization:

$$\frac{1}{N}\langle \mathrm{vac}|\sum_{n=1}^{N} Z_n|\mathrm{vac}\rangle = \frac{1}{N}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}\langle \mathrm{vac}|Z_n|i_1\cdots i_N\rangle\langle i_1\cdots i_N|\mathrm{vac}\rangle$$

$$= \frac{1}{N}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}(-1)^{i_n}\,|\langle i_1\cdots i_N|\mathrm{vac}\rangle|^2$$

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$$= \frac{1}{N}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}(-1)^{i_n}\,|\langle i_1\cdots i_N|\text{vac}\rangle|^2$$

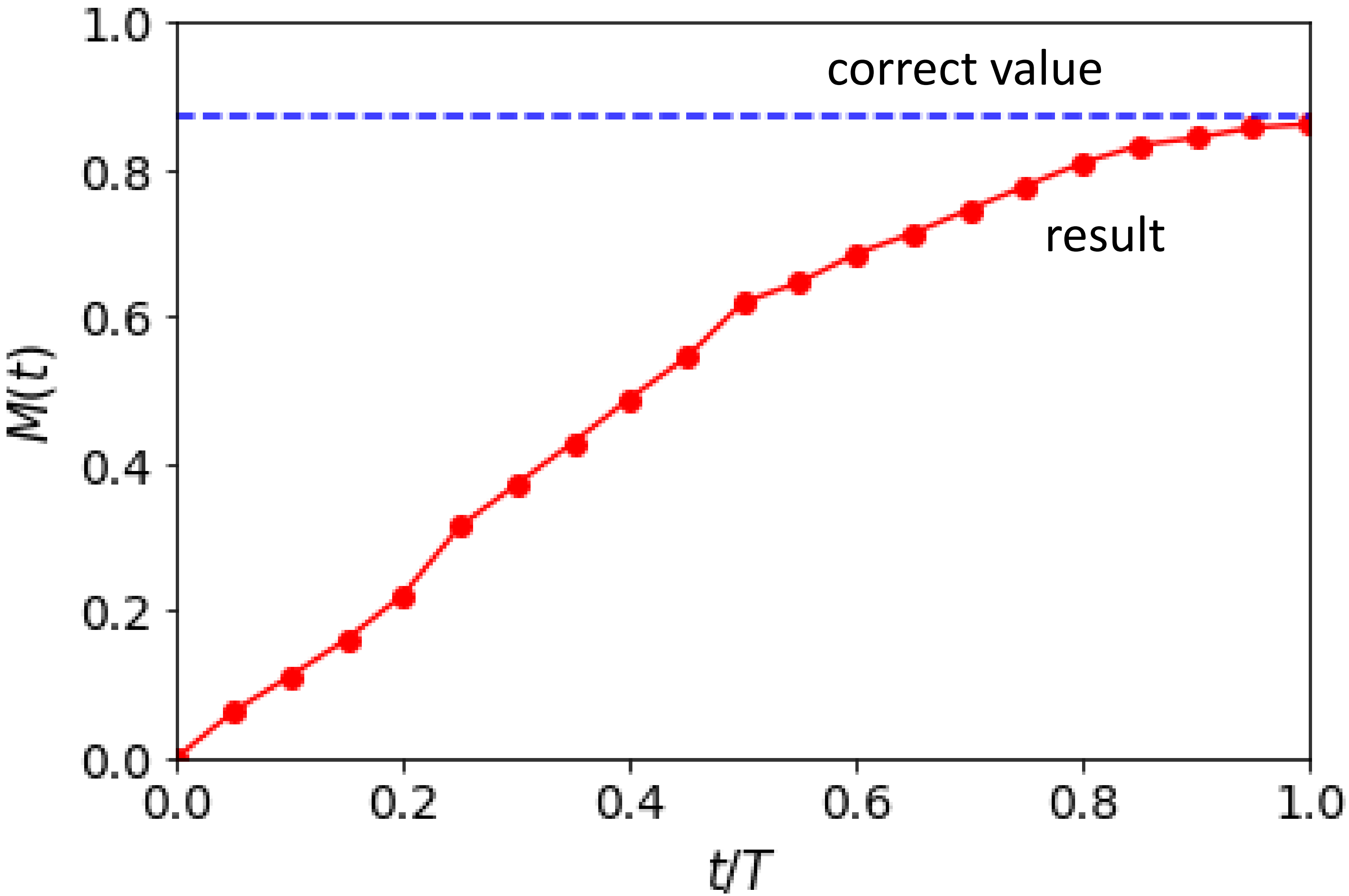
Transverse one is a bit more tricky:

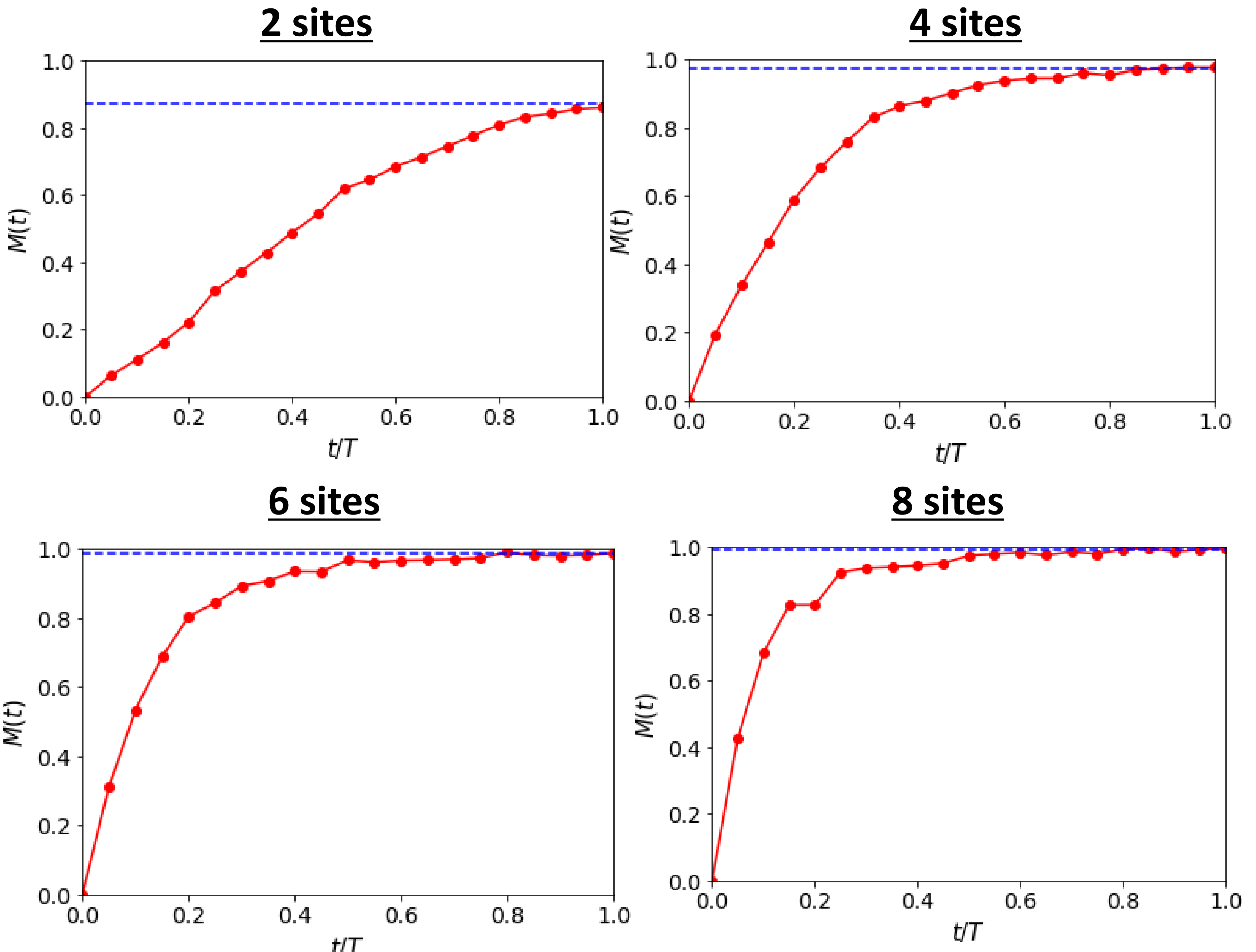
$$\frac{1}{N}\langle \text{vac}|\sum_{n=1}^{N} X_n|\text{vac}\rangle = \frac{1}{N}\langle \text{vac}|\sum_{n=1}^{N} H^{(n)}Z_nH^{(n)}|\text{vac}\rangle$$

$$= \frac{1}{N}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}(-1)^{i_n}\left|\langle i_1\cdots i_N|H^{(n)}|\text{vac}\rangle\right|^2$$

Result by simulator (10000 shots)

2 sites, $J = 1, h = 1, m = 1, T = 100, \delta t = 0.05,$ 2000 time steps




2 sites
4 sites
6 sites
8 sites
M(t)
t/T
0.0
0.2
0.4
0.6
0.8
1.0

Renyi entropy

Dividing total Hilbert space as

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_A \otimes \mathcal{H}_B,$$

reduced density matrix is defined as

$$\rho_A = \text{tr}_{\mathcal{H}_B}(\rho_{\text{tot}})$$

Entanglement entropy:

$$S_A = -\text{tr}_{\mathcal{H}_A}(\rho_A \log \rho_A)$$

n-th Renyi entropy:

$$S_n = \frac{1}{1-n} \log \text{tr}_{\mathcal{H}_A}(\rho_A^n)$$

Quantum algorithm for 2nd Renyi entropy

Consider $(N_A + N_B)$-qubit system and the density matrix

$$\rho_{N_A+N_B} = |\Psi\rangle\langle\Psi|$$

Let's divide the system into two systems: $\mathcal{H}_{N_A+N_B} = \mathcal{H}_{N_A} \otimes \mathcal{H}_{N_B}$
& consider the 2nd Renyi entropy

$$S_2 = -\log \mathrm{tr}_{\mathcal{H}_{N_A}}\left(\rho_A^2\right), \quad \rho_A = \mathrm{tr}_{\mathcal{H}_{N_B}}\left(\rho_{N_A+N_B}\right)$$

Quantum algorithm for 2nd Renyi entropy

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& consider the 2nd Renyi entropy

$$S_2 = -\log \mathrm{tr}_{\mathcal{H}_{N_A}} \left(\rho_A^2\right), \quad \rho_A = \mathrm{tr}_{\mathcal{H}_{N_B}} \left(\rho_{N_A+N_B}\right)$$

One can show (next slide) [Hastings-Gonzalez-Kallin-Melko'10]

$$\mathrm{tr}_{\mathcal{H}_{N_A}} \left(\rho_A^2\right) = \langle\Psi| \otimes \langle\Psi| \ \mathrm{SWAP}_A \ |\Psi\rangle \otimes |\Psi\rangle$$

SWAP_A : Exchange of $A-$part in $|\Psi\rangle \otimes |\Psi\rangle$

(For $|\Psi\rangle = \sum_{i,j} c_{ij} |i_1 \cdots i_{N_A} j_1 \cdots j_{N_B}\rangle$,

$$\mathrm{SWAP}_A |\Psi\rangle \otimes |\Psi\rangle \equiv \sum_{i,j,i',j'} c_{ij} c_{i'j'} |i'_1 \cdots i'_{N_A} j_1 \cdots j_{N_B}\rangle \otimes |i_1 \cdots i_{N_A} j'_1 \cdots j'_{N_B}\rangle$$

)

Quantum algorithm for 2nd Renyi entropy (Cont'd)

$$\mathrm{tr}_{\mathcal{H}_{N_A}}\left(\rho_A^2\right) = \langle\Psi| \otimes \langle\Psi|\ \mathsf{SWAP}_A\ |\Psi\rangle \otimes |\Psi\rangle$$

Proof:

$$\langle\Psi| \otimes \langle\Psi|\ \mathsf{SWAP}_A\ |\Psi\rangle \otimes |\Psi\rangle$$

$$= \sum_{k,\ell,k',\ell'} \bar{c}_{k\ell}\bar{c}_{k'\ell'}\langle\{k'\}\{\ell'\}|\otimes\langle\{k\}\{\ell\}| \sum_{i,j,i',j'} c_{ij}c_{i'j'}|\{i'\}\{j\}\rangle\otimes|\{i\}\{j'\}\rangle$$

$$= \sum_{i,j,i',j'} c_{ij}\bar{c}_{i'j}c_{i'j'}\bar{c}_{ij'}$$

$$(\rho_A)_{ii'} = \sum_j \langle\{i\}\{j\}|\ \rho_{N_A+N_B}\ |\{i'\}\{j\}\rangle = \sum_j c_{ij}\bar{c}_{i'j}$$

$$= \sum_{i,i'} (\rho_A)_{ii'}(\rho_A)_{i'i} = \mathrm{tr}_{\mathcal{H}_{N_A}}\left(\rho_A^2\right)$$

Demonstration: 2nd Renyi entropy of Bell state

Bell state:

$$|B\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Reduced density matrix:

$$\rho_{\mathrm{red}} = \mathrm{tr}_2 |B\rangle\langle B| = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$$

2nd Renyi entropy:

$$\mathrm{tr}\rho_{\mathrm{red}}^2 = \mathrm{tr}\begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix} = \frac{1}{2}$$

$$S_2 = -\log \mathrm{tr}\rho_{\mathrm{red}}^2 = \log 2$$

Let’s reproduce it in IBM Q Experience

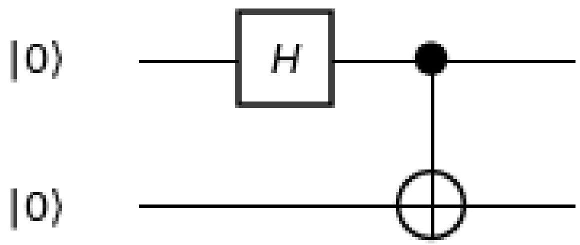
Demonstration: 2nd Renyi entropy of Bell state (Cont'd)

We know

$$|B\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\mathrm{tr}\rho_{\mathrm{red}}^2 = \langle B| \otimes \langle B| \ \mathrm{SWAP}^{(1,3)} \ |B\rangle \otimes |B\rangle$$

The Bell state is written as



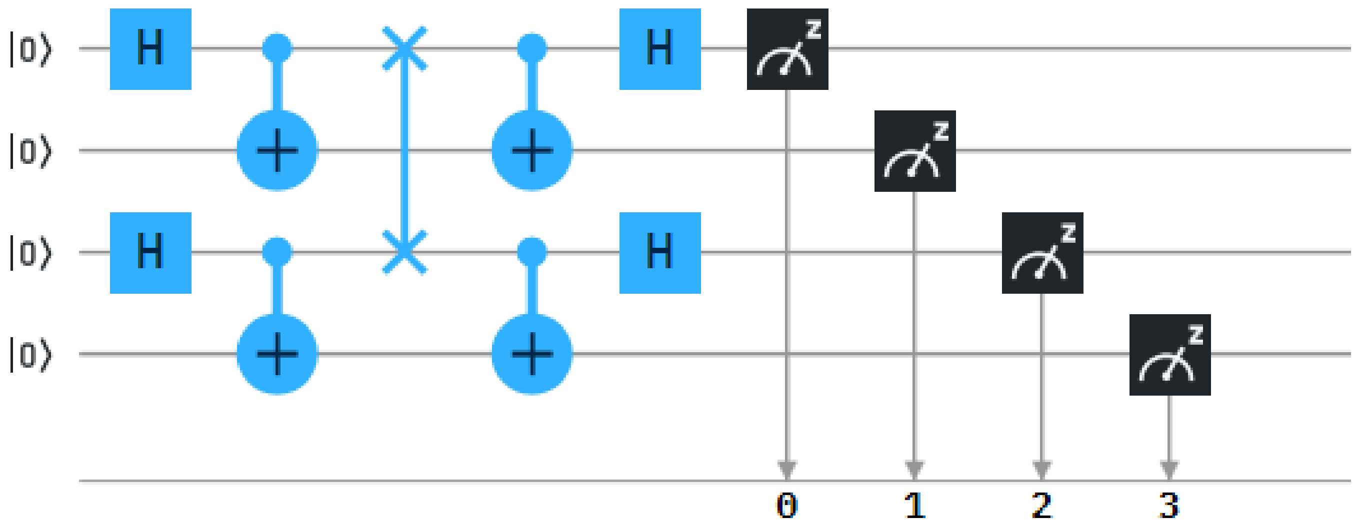
$$= |B\rangle$$

Therefore,

$$\mathrm{tr}\rho_{\mathrm{red}}^2 = \langle 0000| \ U^\dagger \mathrm{SWAP}^{(1,3)} U \ |0000\rangle \quad (|B\rangle \otimes |B\rangle \equiv U|0000\rangle)$$

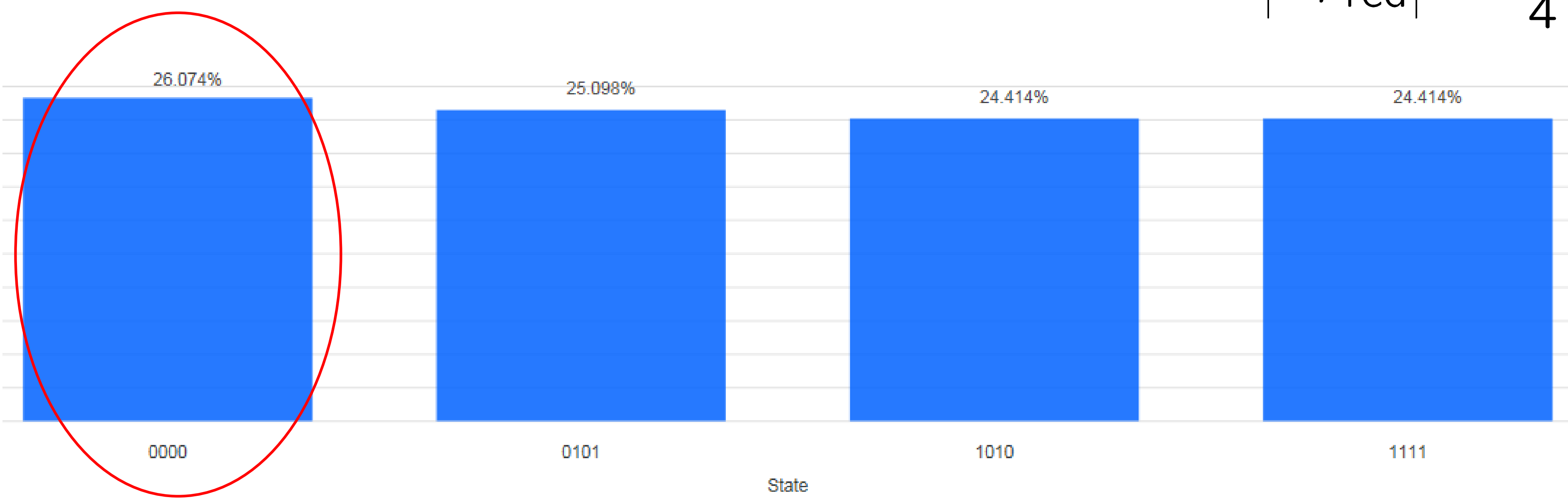
$$\left|\mathrm{tr}\rho_{\mathrm{red}}^2\right|^2 = \left|\langle 0000| \ U^\dagger \mathrm{SWAP}^{(1,3)} U \ |0000\rangle\right|^2$$

Demonstration: 2nd Renyi entropy of Bell state (Cont'd)

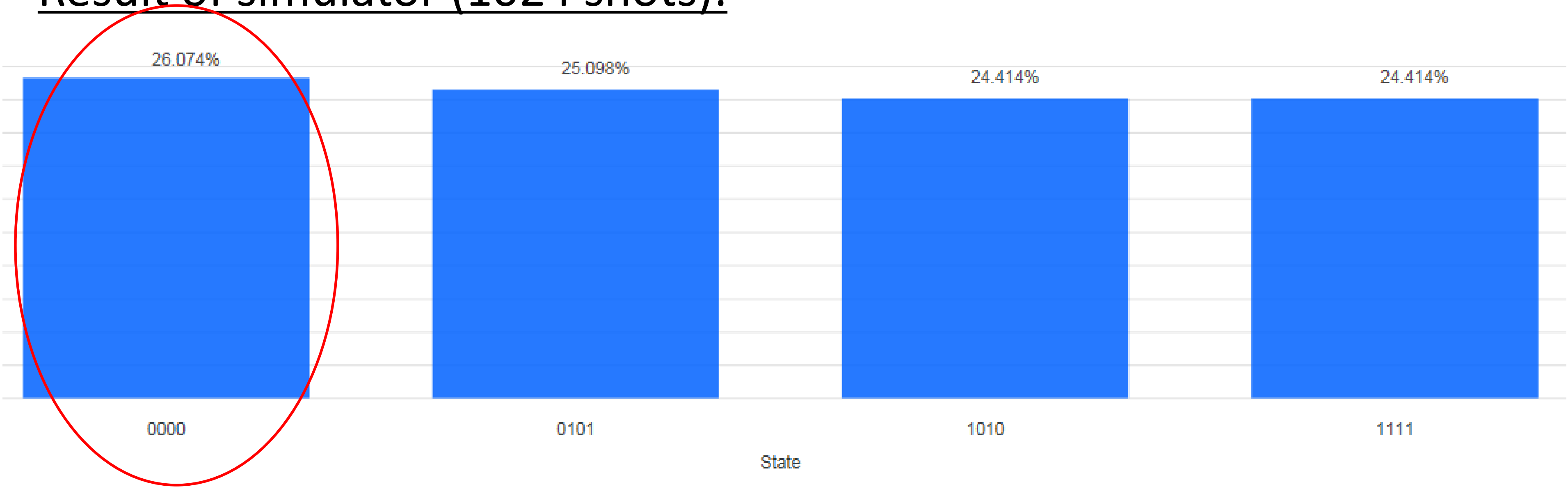


Result of simulator (1024 shots):

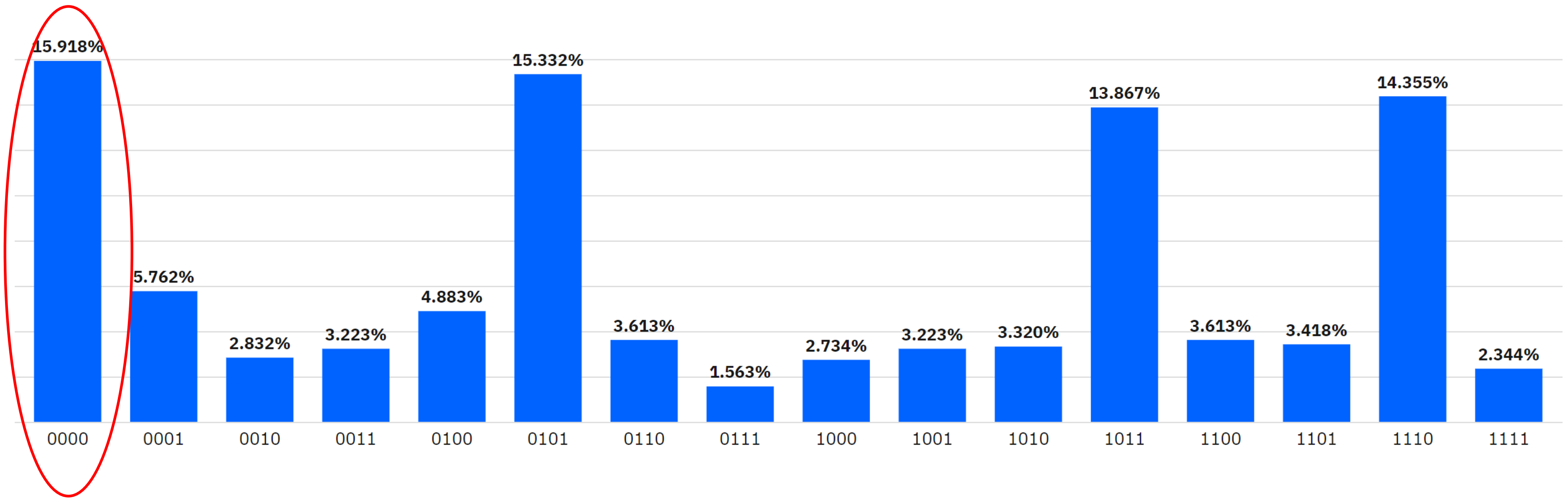
$$\left|\mathrm{tr}\rho_{\mathrm{red}}^2\right|^2 = \frac{1}{4}$$



Result of simulator (1024 shots):



Result of quantum computer (1024 shots):



More direct way?

We've directly computed

$$\left|\mathrm{tr}\rho_{\mathrm{red}}^2\right|^2 = \left|\langle 0000|\ U^\dagger \mathrm{SWAP}^{(1,3)} U\ |0000\rangle\right|^2$$

rather than itself:

$$\mathrm{tr}\rho_{\mathrm{red}}^2 = \langle 0000|\ U^\dagger \mathrm{SWAP}^{(1,3)} U\ |0000\rangle$$

Can we directly compute it?

—— Yes, there is a way to compute expectation value of unitary op. under any state: (next slide)

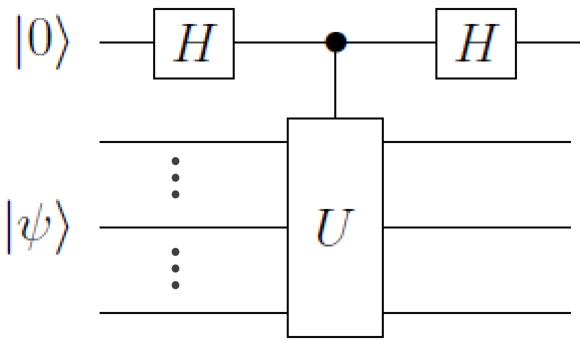
$$\langle\psi|\ \mathrm{U}\ |\psi\rangle$$

"Hadamard test": standard way to compute $\langle\psi|$ U $|\psi\rangle$

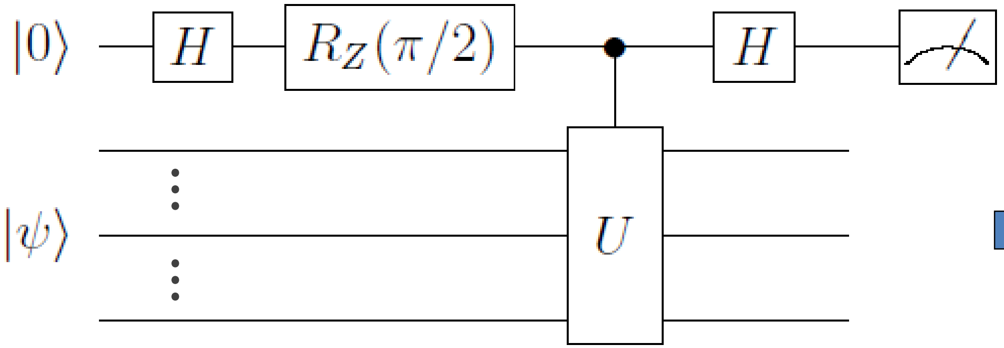
① Extend Hilbert space & consider the state

$$\underbrace{|0\rangle}_{\text{"ancillary qubit"}} \otimes |\psi\rangle$$

② We can compute $\langle\psi|\,U\,|\psi\rangle$ by using the 2 circuits: (next slide)



$\Rightarrow$ $\mathrm{Re}(\langle\psi|U|\psi\rangle)$

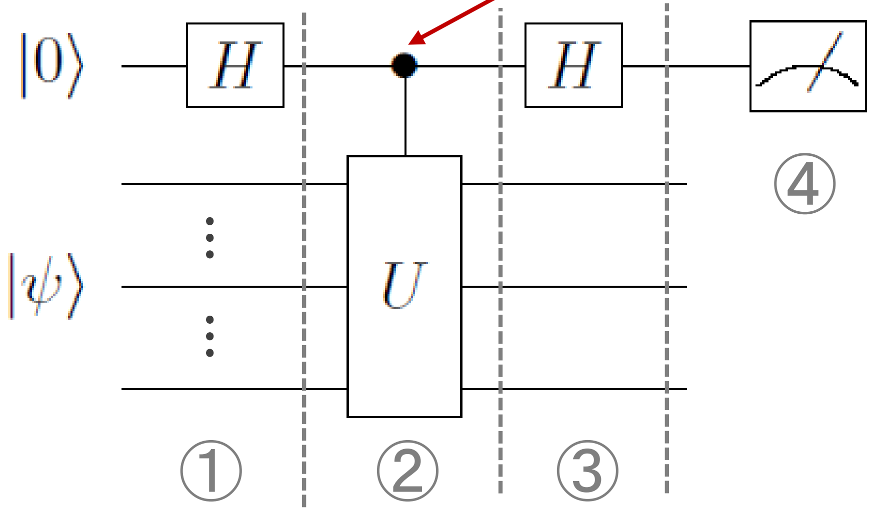


$\Rightarrow$ $\mathrm{Im}(\langle\psi|U|\psi\rangle)$

"Hadamard test": standard way to compute $\langle\psi|\ U\ |\psi\rangle$ (Cont'd)

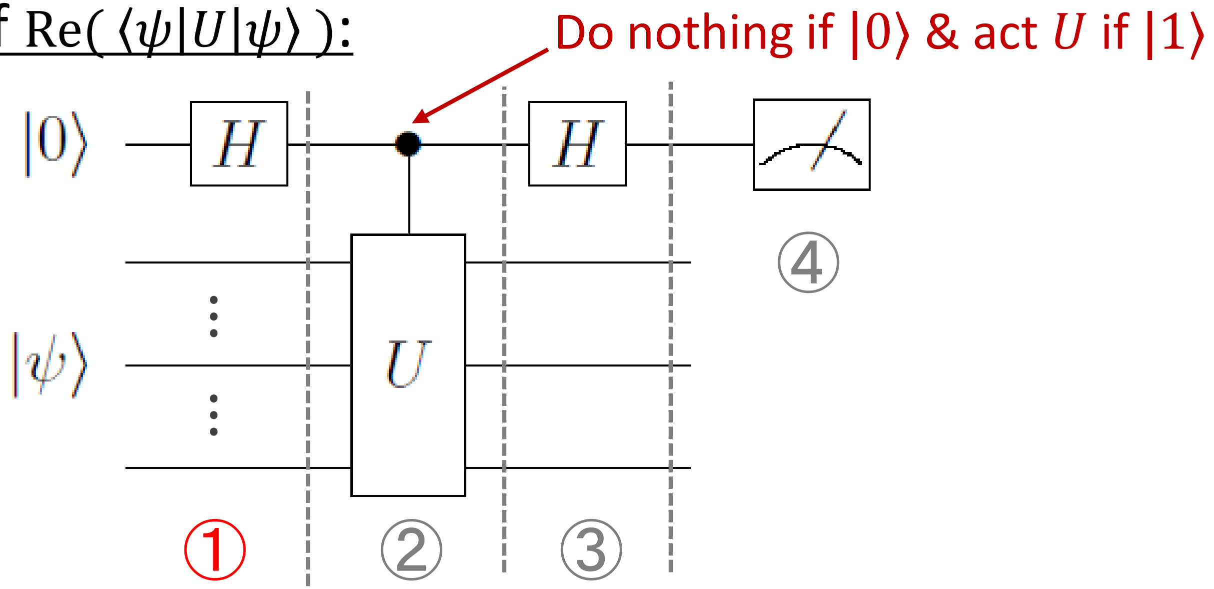
Computation of $\mathrm{Re}(\langle\psi|U|\psi\rangle)$:

Do nothing if $|0\rangle$ & act U if $|1\rangle$



"Hadamard test": standard way to compute $\langle\psi|\,U\,|\psi\rangle$ (Cont'd)

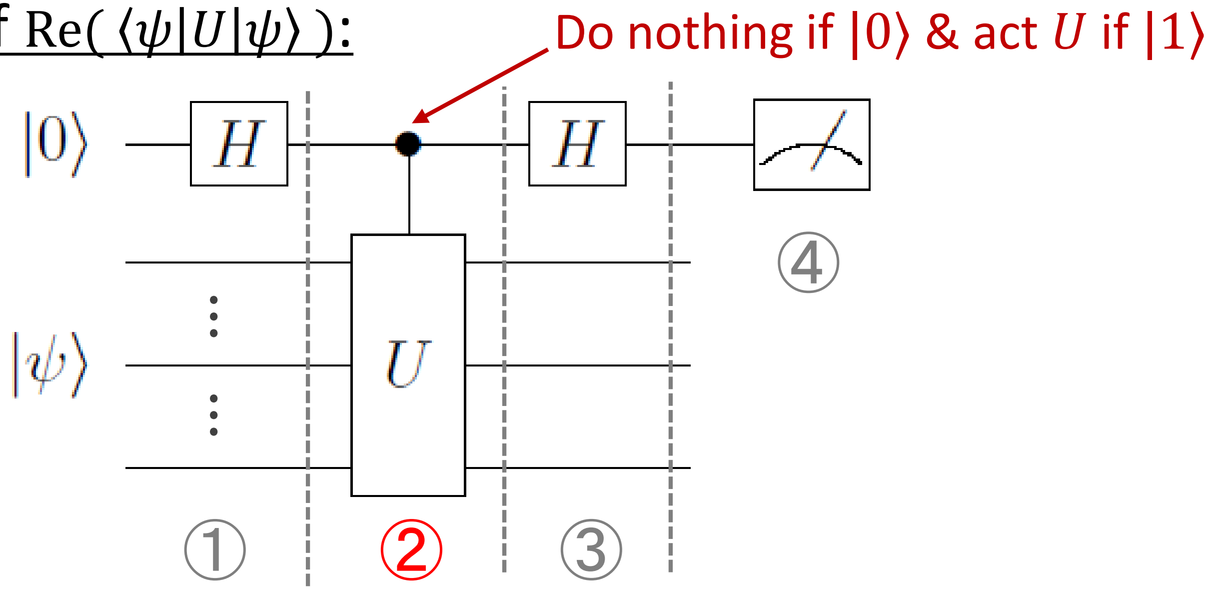
Computation of $\mathrm{Re}(\langle\psi|U|\psi\rangle)$:



① $H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$

"Hadamard test": standard way to compute $\langle\psi|\, U\, |\psi\rangle$ (Cont'd)

Computation of $\mathrm{Re}(\langle\psi|U|\psi\rangle)$:



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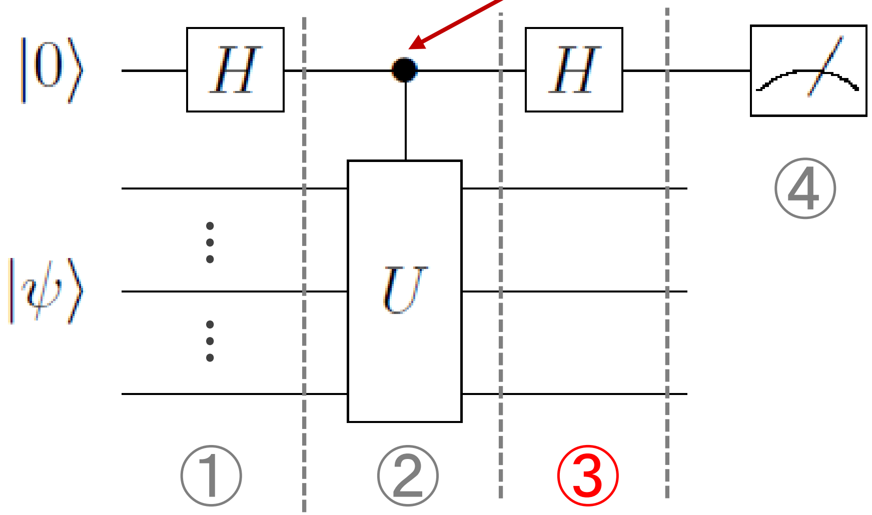
② $\frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$

"Hadamard test": standard way to compute $\langle\psi|\ U\ |\psi\rangle$ (Cont'd)

Computation of $\mathrm{Re}(\langle\psi|U|\psi\rangle)$:

Do nothing if $|0\rangle$ & act U if $|1\rangle$



① $H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$

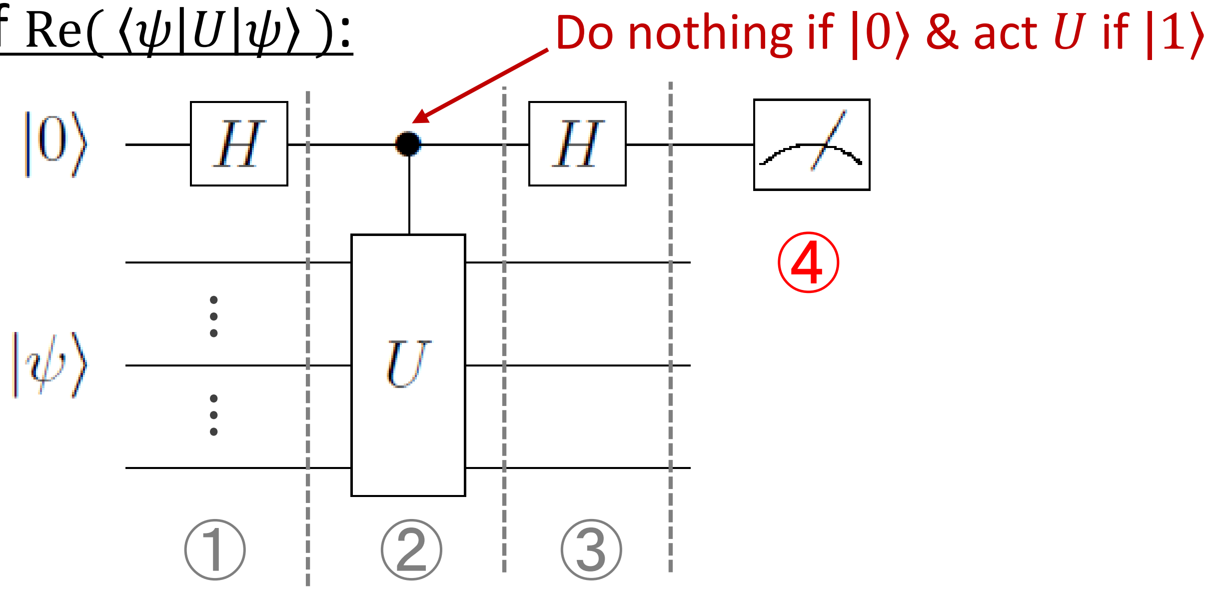
② $\frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$

③ $\frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + \frac{1}{2}(|0\rangle - |1\rangle) \otimes U|\psi\rangle$

$= \frac{1}{2}|0\rangle \otimes (1 + U)|\psi\rangle + \frac{1}{2}|1\rangle \otimes (1 - U)|\psi\rangle$

"Hadamard test": standard way to compute $\langle\psi|$ U $|\psi\rangle$ (Cont'd)

Computation of $\mathrm{Re}(\langle\psi|U|\psi\rangle)$:



① $H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$

② $\frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$

③ $\frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + \frac{1}{2}(|0\rangle - |1\rangle) \otimes U|\psi\rangle$

$= \frac{1}{2}|0\rangle \otimes (1+U)|\psi\rangle + \frac{1}{2}|1\rangle \otimes (1-U)|\psi\rangle$

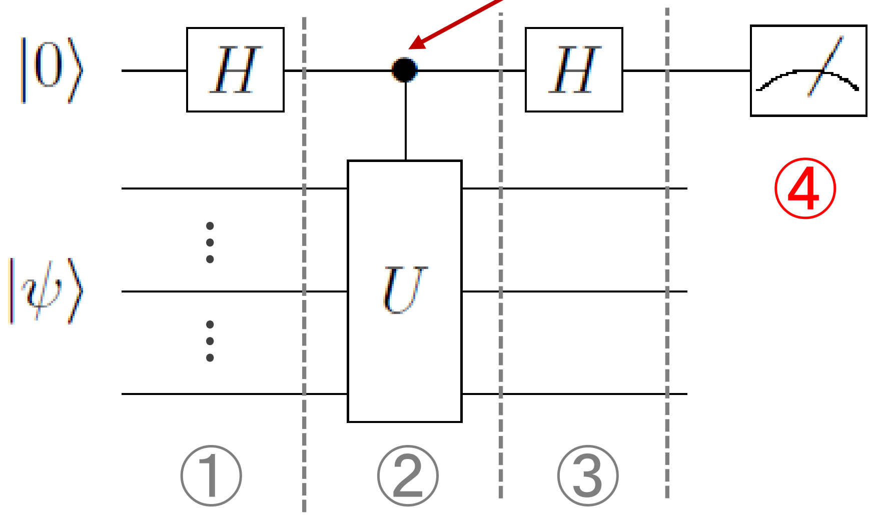
④ $P_0 = \frac{1}{4}|(1+U)|\psi\rangle|^2 = \frac{1}{2}(1 + \mathrm{Re}\langle\psi|U|\psi\rangle)$

$P_1 = \frac{1}{4}|(1-U)|\psi\rangle|^2 = \frac{1}{2}(1 - \mathrm{Re}\langle\psi|U|\psi\rangle)$

"Hadamard test": standard way to compute $\langle\psi|\ U\ |\psi\rangle$ (Cont'd)

Computation of $\mathrm{Re}(\ \langle\psi|U|\psi\rangle\ )$:

Do nothing if $|0\rangle$ & act U if $|1\rangle$



① $H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle \ + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$

② $\frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle \ + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$

③ $\frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle \ + \frac{1}{2}(|0\rangle - |1\rangle) \otimes U|\psi\rangle$

$= \frac{1}{2}|0\rangle \otimes (1 + U)|\psi\rangle \ + \frac{1}{2}|1\rangle \otimes (1 - U)|\psi\rangle$

④ $P_0 = \frac{1}{4}|(1 + U)|\psi\rangle|^2 \ = \frac{1}{2}(\ 1 + \mathrm{Re}\langle\psi|U|\psi\rangle\ )$

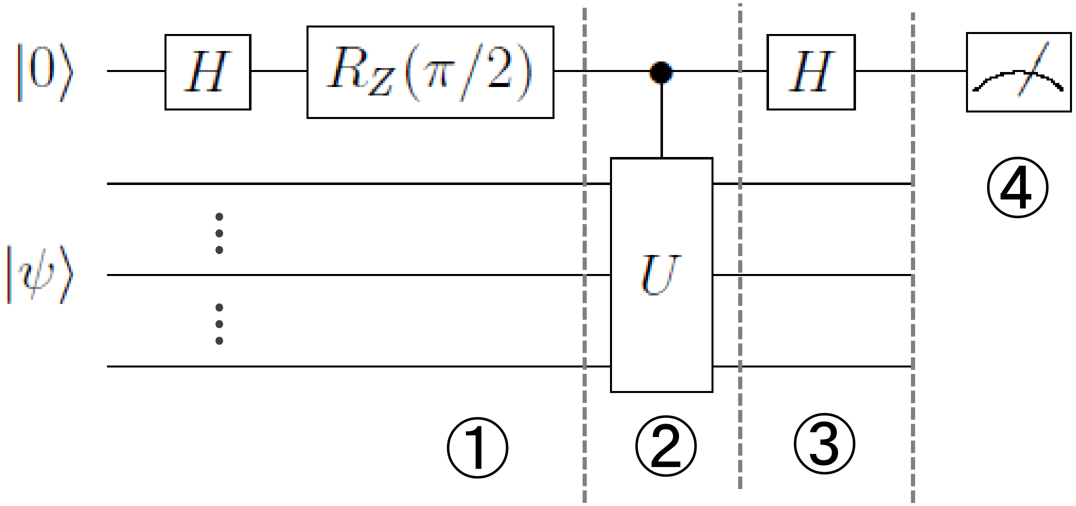
$P_1 = \frac{1}{4}|(1 - U)|\psi\rangle|^2 \ = \frac{1}{2}(\ 1 - \mathrm{Re}\langle\psi|U|\psi\rangle\ )$

$\mathrm{Re}\langle\psi|U|\psi\rangle = P_0 \ - P_1$

"Hadamard test": standard way to compute $\langle\psi|\ U\ |\psi\rangle$ (Cont'd)

Computation of $\mathrm{Im}(\ \langle\psi|U|\psi\rangle\ )$:

$$\left(R_Z(\theta) = e^{-\frac{i\theta}{2}Z} \right)$$



① $\mathrm{R_Z}(\pi/2)H|0\rangle \otimes |\psi\rangle = \frac{e^{-\frac{\pi i}{4}}}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{e^{+\frac{\pi i}{4}}}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$

② $\frac{e^{-\frac{\pi i}{4}}}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{e^{+\frac{\pi i}{4}}}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$

③ $\frac{e^{-\frac{\pi i}{4}}}{2}|0\rangle \otimes (1 + iU)|\psi\rangle + \frac{e^{-\frac{\pi i}{4}}}{2}|1\rangle \otimes (1 - iU)|\psi\rangle$

④ $P_0 = \frac{1}{4}|(1 + iU)|\psi\rangle|^2 = \frac{1}{2}(\ 1 - \mathrm{Im}\langle\psi|U|\psi\rangle\ )$

$P_1 = \frac{1}{4}|(1 - iU)|\psi\rangle|^2 = \frac{1}{2}(\ 1 + \mathrm{Im}\langle\psi|U|\psi\rangle\ )$

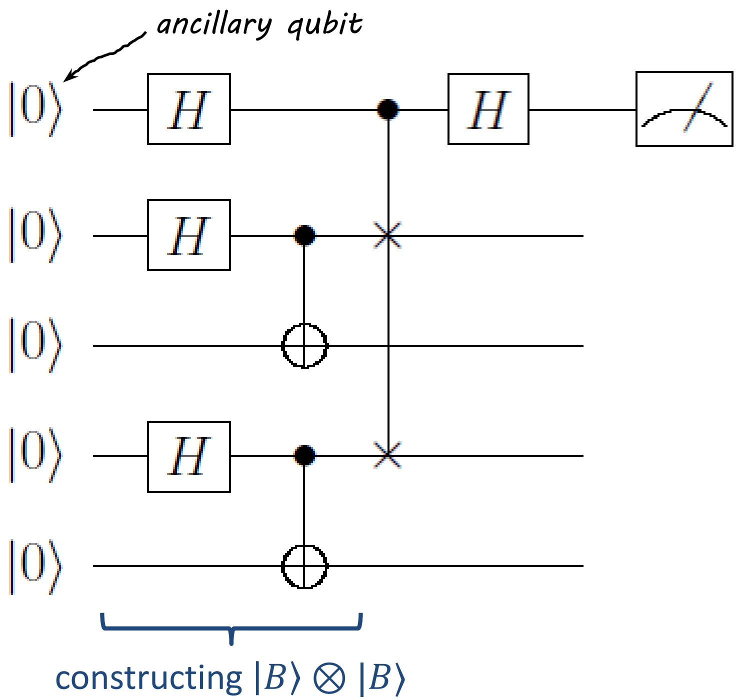
$$\mathrm{Im}\langle\psi|U|\psi\rangle = P_1 - P_0$$

Coming back to the Renyi entropy of Bell state

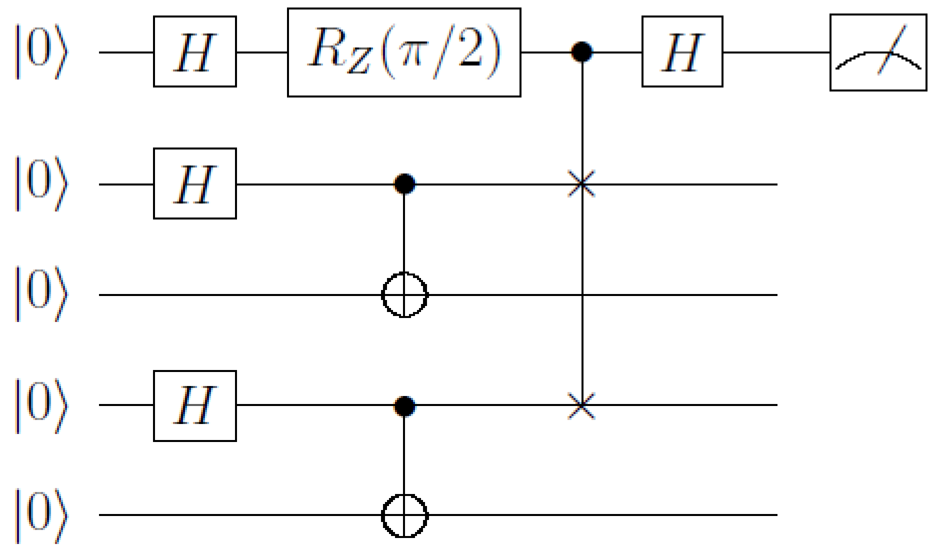
Taking $|\psi\rangle = |B\rangle \otimes |B\rangle$ & $U = \mathrm{SWAP}^{(1,3)}$, we can directly compute

$$\mathrm{tr}\rho_{\mathrm{red}}^2 = \langle B| \otimes \langle B|\ \mathrm{SWAP}^{(1,3)}\ |B\rangle \otimes |B\rangle$$

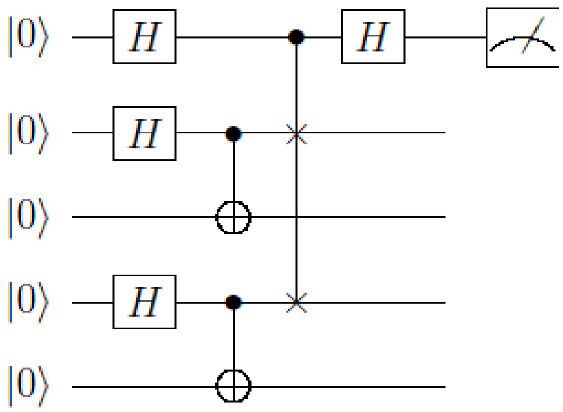
Real part:

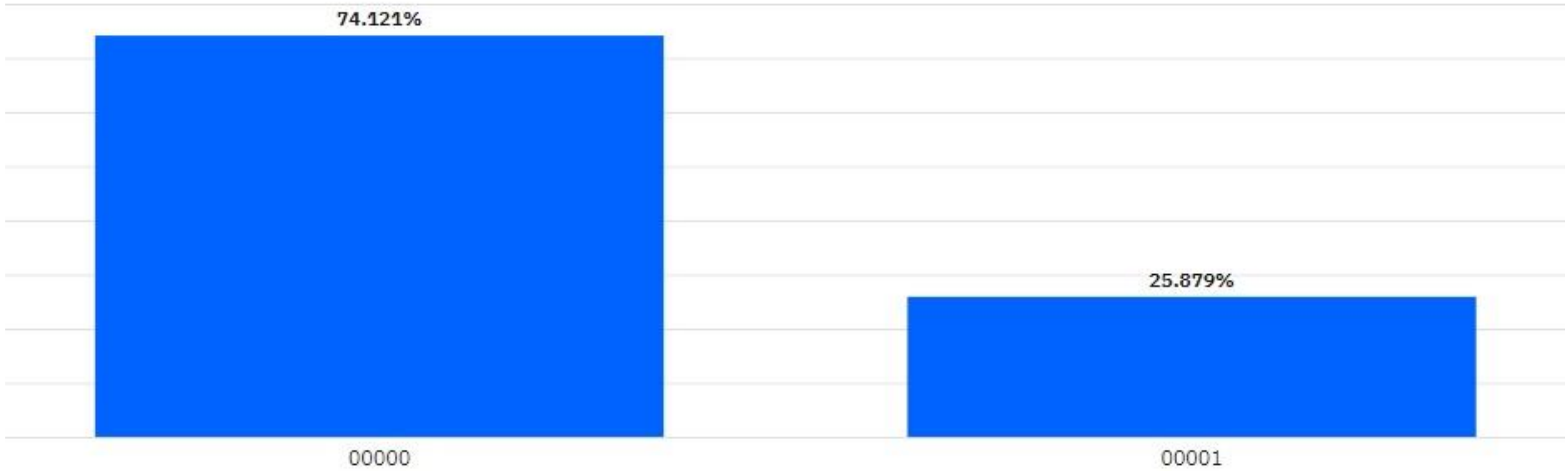


Imaginary part:



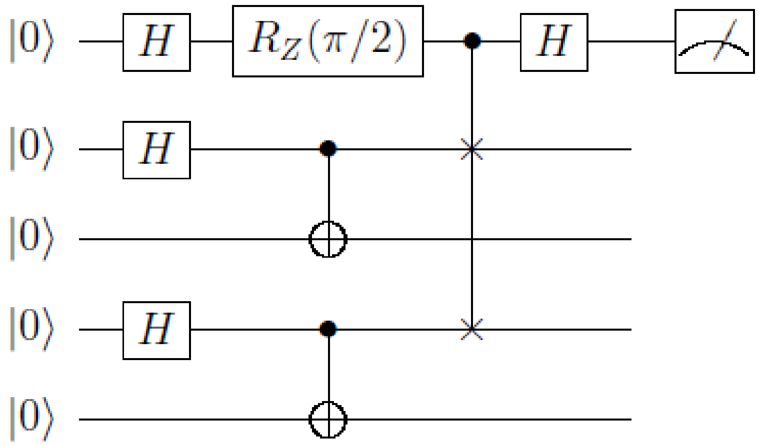
Result of simulator (real part, 1024 shots)

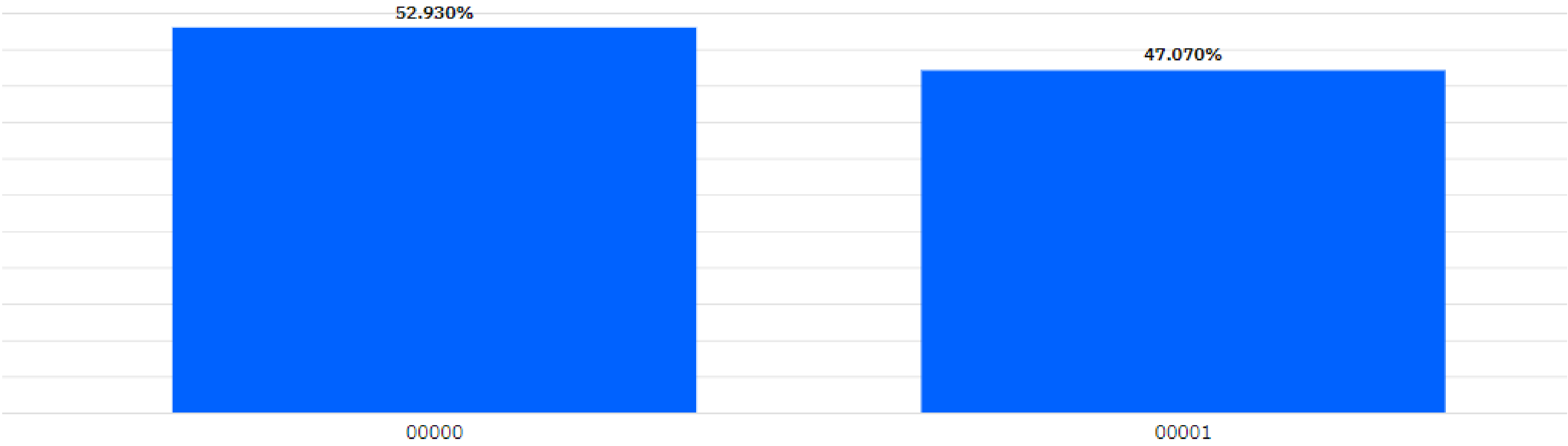




Expectation: $P_0 - P_1 = \mathrm{Re}\,\mathrm{tr}\rho_{\mathrm{red}}^2 = \frac{1}{2}$

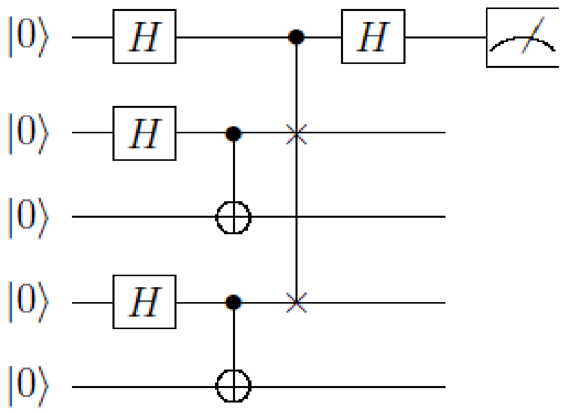
Result of simulator (imaginary part, 1024 shots)

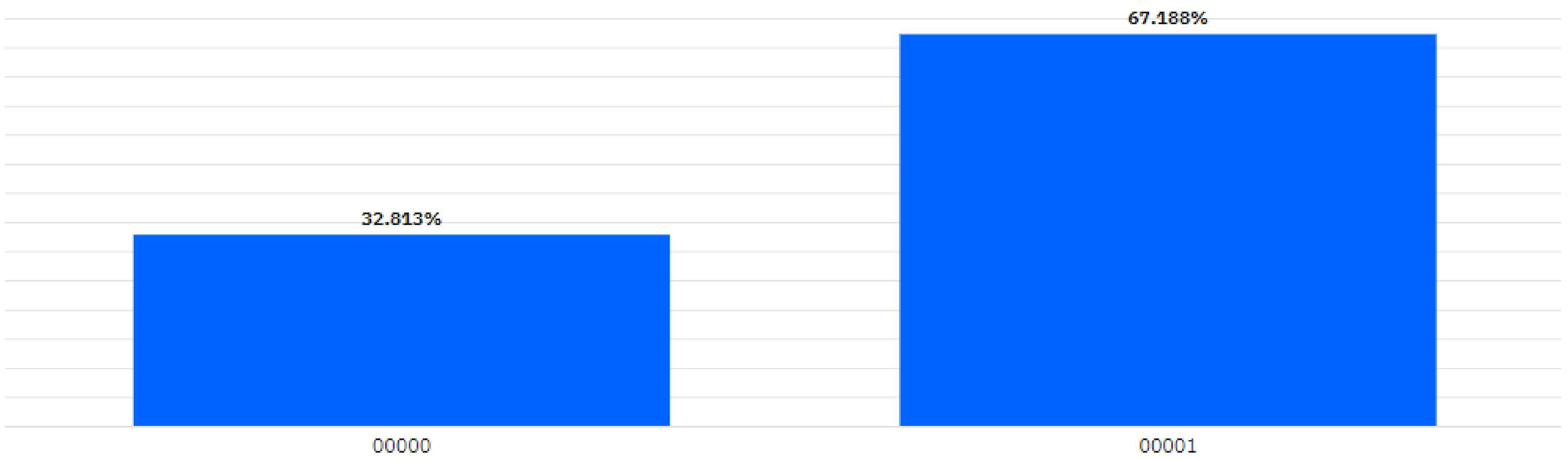




Expectation: $P_1 - P_0 = \mathrm{Im}\,\mathrm{tr}\rho_{\mathrm{red}}^2 = 0$

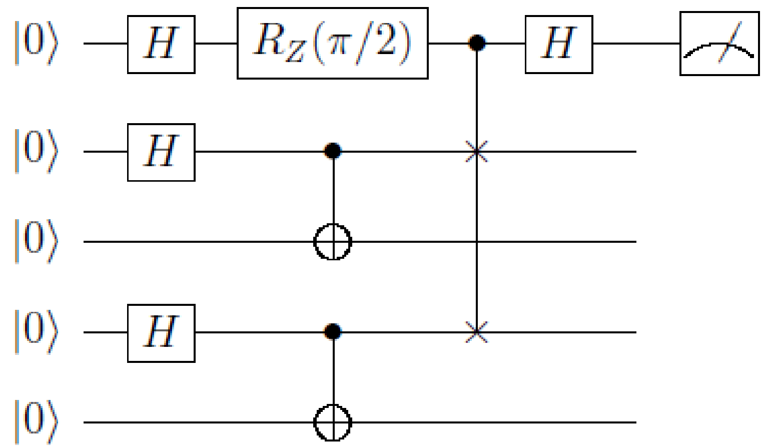
Result of quantum computer (real part, 1024 shots)

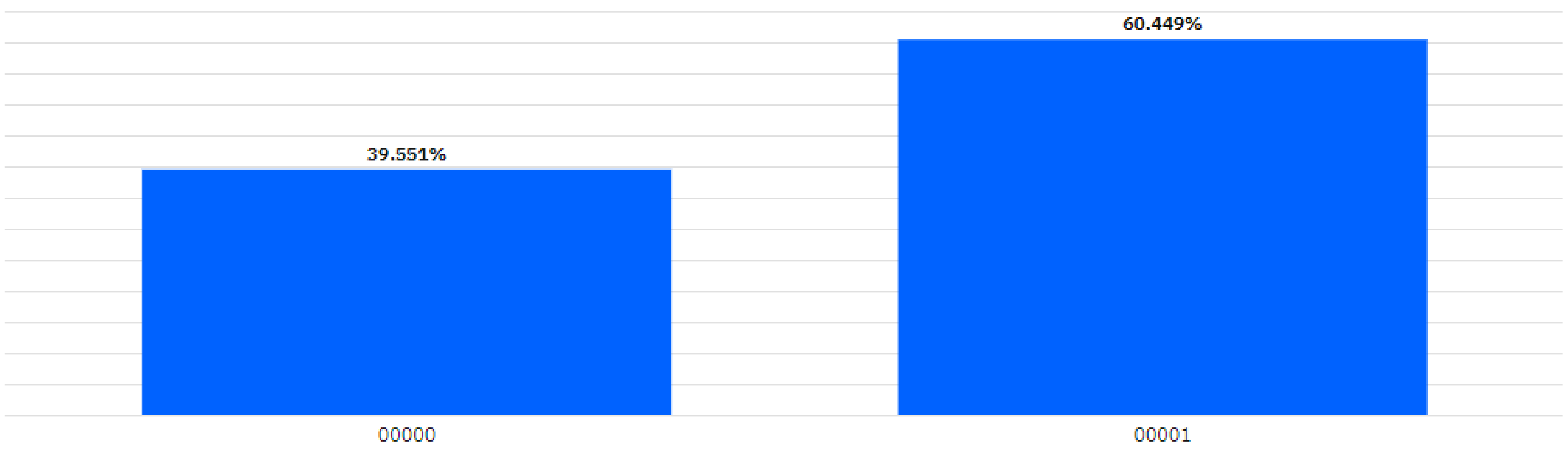




Expectation: $P_0 - P_1 = \mathrm{Re}\,\mathrm{tr}\rho_{\mathrm{red}}^2 = \frac{1}{2}$

Result of quantum computer (imaginary part, 1024 shots)





Expectation: $P_1 - P_0 = \mathrm{Im}\ \mathrm{tr}\rho_{\mathrm{red}}^2 = 0$

QFT as qubits

(mapping to spin system)

“Regularization” of Hilbert space

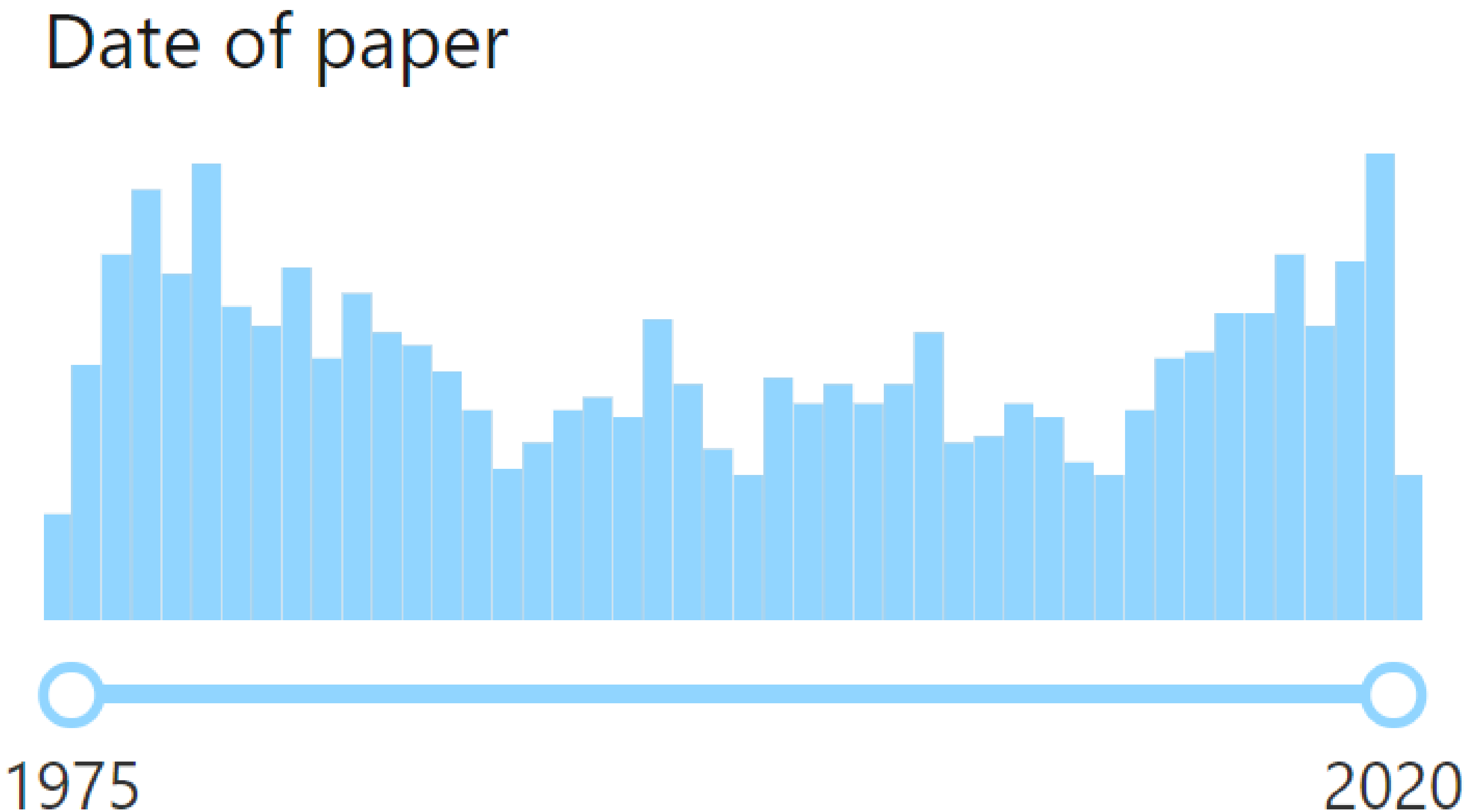
Hilbert space of QFT is typically ∞ dimensional

$\longrightarrow$ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)
  — Putting on spatial lattice, Hilbert sp. is finite dimensional
- scalar
  — Hilbert sp. at each site is ∞ dimensional
  (need truncation or additional regularization)
- gauge field (w/ kinetic term)
  — no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
  — ∞ dimensional Hilbert sp. in higher dimensions

Citation history of “Hamiltonian Formulation of Wilson's Lattice Gauge Theories” by Kogut-Susskind

(totally 1832 at this moment)



Free Dirac fermion in 1+1D

Continuum:

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

$$\longrightarrow \hat{H} = \int dx \left[-i\bar{\psi}\gamma^{1}\partial_{1}\psi + m\bar{\psi}\psi \right]$$

Lattice (w/ N sites and spacing a):

For staggered fermion: $\frac{\chi_n}{\sqrt{a}} \leftrightarrow \begin{cases} \psi_u(x) & n : \text{even} \\ \psi_d(x) & n : \text{odd} \end{cases}$ $\psi(x) = \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix}$

$$\hat{H} = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\chi_n^{\dagger}\chi_{n+1} - \text{h.c.}\right) + m\sum_{n=1}^{N}(-1)^n \chi_n^{\dagger}\chi_n$$

(anti-)commutation relation:

$$\{\chi_n^{\dagger}, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0$$

Jordan-Wigner transformation

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \;\; \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

[Jordan-Wigner'28]

$$\chi_n = \left(\prod_{\ell<n} iZ_\ell \right) \frac{X_n - iY_n}{2}$$

Then the system is mapped to the spin system:

$$\hat{H} = \frac{w}{2} \sum_{n=1}^{N-1} \left(X_n X_{n+1} + Y_n Y_{n+1} \right) + \frac{m}{2} \sum_{n=1}^{N} (-1)^n Z_n$$

We can apply quantum algorithms to QFT!

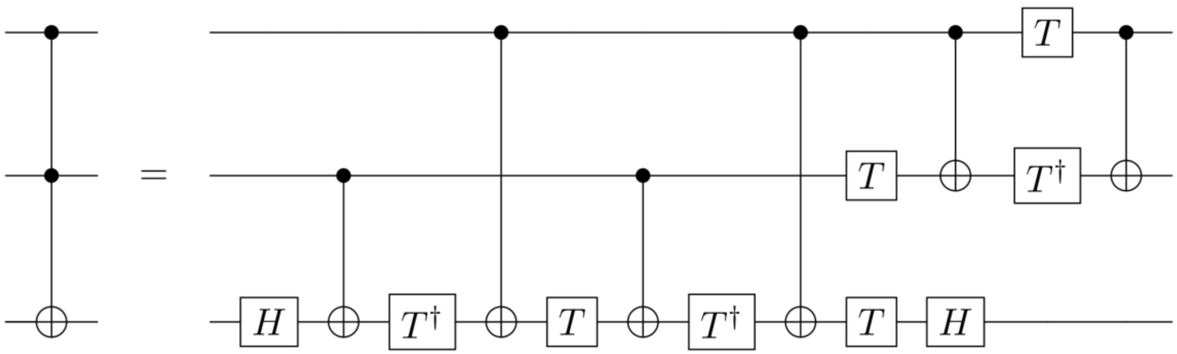
Here is the end of the 1st day

Universality

- Any unitary gate is a combination of single qubit gates & CX ("Single qubit gates & CX are universal")

Ex.) Toffoli

(controlled-controlled-NOT)



- Any single qubit gate is approximated by a combination of H & T in arbitrary precision

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

- H, T & CX are universal

Approximation of single qubit gate by H & T

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$

① Get a rotation with angle $2\pi \times$ (irrational):

$$THTH = e^{\frac{i\pi}{4}} R_{\vec{n}}(\theta) \qquad \text{with } R_{\vec{n}}(\theta) \equiv e^{-\frac{i}{2}\vec{n}\cdot\vec{\sigma}}$$

where

$$\vec{n} = \frac{1}{\sqrt{1+\cos^2(\pi/8)}}\begin{pmatrix} \cos(\pi/8) \\ \sin(\pi/8) \\ \cos(\pi/8) \end{pmatrix} \quad \& \quad \cos(\theta/2) \equiv \cos^2(\pi/8)$$

$2\pi \times (\text{irrational})!$

② Use Weyl's uniform distribution theorem:

$\theta\boldsymbol{Z}$ is uniformly distributed mod 1 ⇒ approximate $R_{\vec{n}}(\alpha)$ for $\forall\alpha$

③ Construct rotation around another axis:

$$HR_{\vec{n}}(\alpha)H = R_{\vec{m}}(\alpha) \qquad \text{with } \vec{m} = \frac{1}{\sqrt{1+\cos^2(\pi/8)}}\begin{pmatrix} \cos(\pi/8) \\ -\sin(\pi/8) \\ \cos(\pi/8) \end{pmatrix}$$

④ Approximate ∀single qubit gate: $R_{\vec{n}}(\alpha)R_{\vec{m}}(\beta)\,R_{\vec{n}}(\gamma)$

What if we replace T by something else?

$$T = e^{\frac{i\pi}{8}} R_Z(\pi/4) \quad \Rightarrow \quad T' \equiv R_Z(\phi) \text{ ??}$$

We have the identity:

$$T'HT'H = R_{\vec{n}}(\theta)$$

where

$$\vec{n} = \frac{1}{\sqrt{1 + \cos^2(\phi/2)}} \begin{pmatrix} cos(\phi/2) \\ \sin(\phi/2) \\ cos(\phi/2) \end{pmatrix} \quad \& \quad \cos(\theta/2) \equiv \cos^2(\phi/2)$$

We can approximate any single qubit gate

by combining H & T' if $\theta/2\pi$ is irrational

Advantage of using discrete gates

- To get approximation w/ precision ϵ,
  we need to use $\mathcal{O}(1/\epsilon)$ discrete gates

- But it is useful for quantum error correction

- If we use continuous gates,
  we have to consider algorithms to correct errors
  for ∞ gates

The beginning of the 2nd day (3rd slot)

Schwinger model w/ topological term

Continuum:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

Using "chiral anomaly", the same physics can be studied by

[Fujikawa'79]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}e^{i\theta\gamma^5}\psi$$

Taking temporal gauge $A_0 = 0$, $\Pi = \dot{A}^1$

$$\hat{H} = \int dx \left[-i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}e^{i\theta\gamma^5}\psi + \frac{1}{2}\Pi^2 \right]$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1 \Pi - g\bar{\psi}\gamma^0\psi$$

Sign problem in path integral formalism

In Minkowski space,

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \right] + \frac{g\theta}{4\pi} \int F \in \boldsymbol{R}$$

$$\langle \mathcal{O} \rangle = \frac{\int DAD\psi D\bar{\psi}\ \mathcal{O}\ e^{iS}}{\int DAD\psi D\bar{\psi}\ e^{iS}}$$

highly oscillating

In Euclidean space,

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \right] + i\,\frac{g\theta}{4\pi} \int F \in \boldsymbol{C}$$

$$\langle \mathcal{O} \rangle = \frac{\int DAD\psi D\bar{\psi}\ \mathcal{O}\ e^{-S}}{\int DAD\psi D\bar{\psi}\ e^{-S}}$$

highly oscillating for non-small θ

Accessible region by analytic computation

- Massive limit:

The fermion can be integrated out

&

the theory becomes effectively pure Maxwell theory w/ θ

- Bosonization:

[Coleman '76]

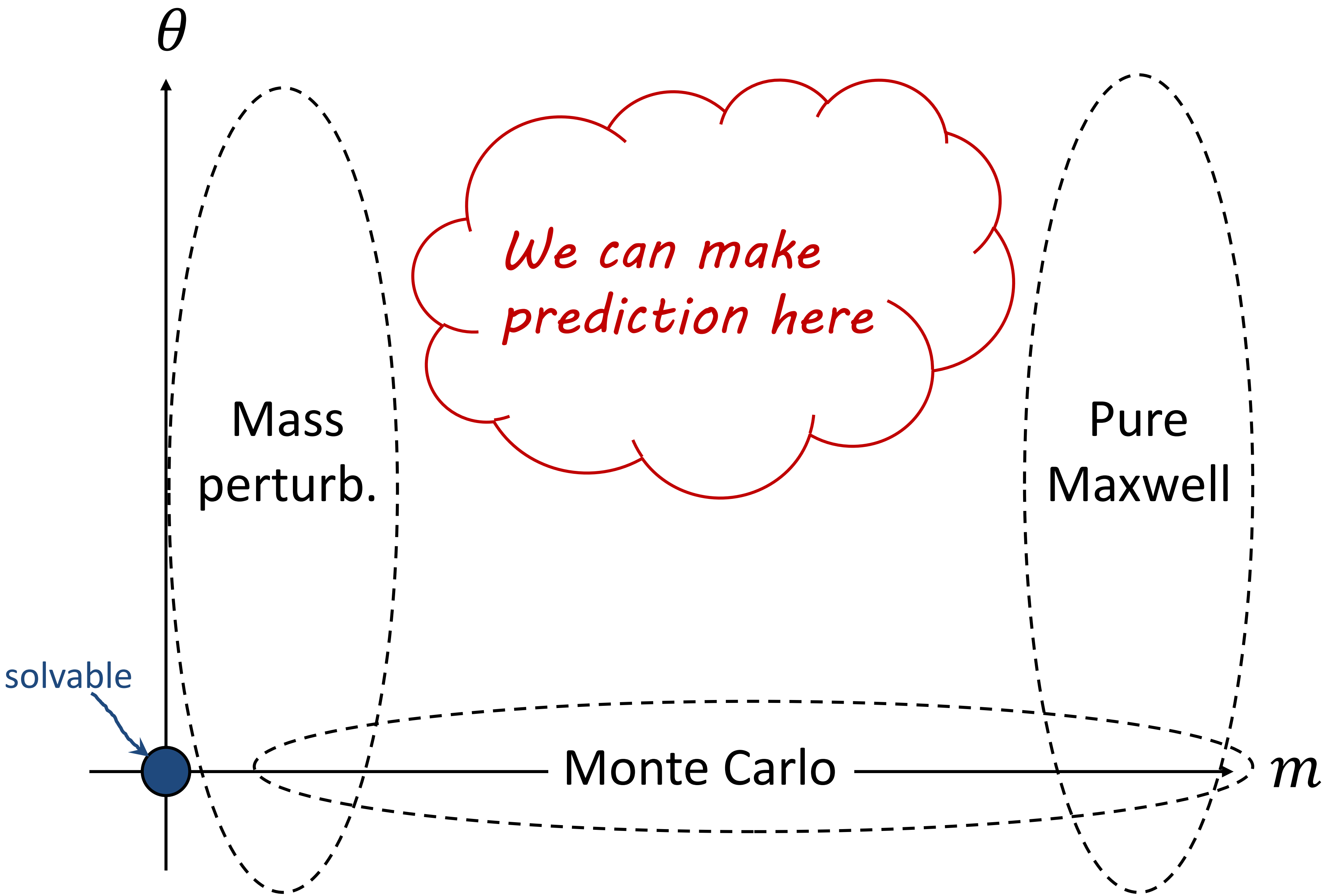
$$\mathcal{L} = \frac{1}{8\pi}\left(\partial_\mu \phi\right)^2 - \frac{g^2}{8\pi^2}\phi^2 + \frac{e^\gamma g}{2\pi^{3/2}} m \cos(\phi + \theta)$$

exactly solvable for $m = 0$

&

small m regime is approximated by perturbation

Map of accessibility/difficulty



Put the theory on lattice

- **Fermion** (on site):

"Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

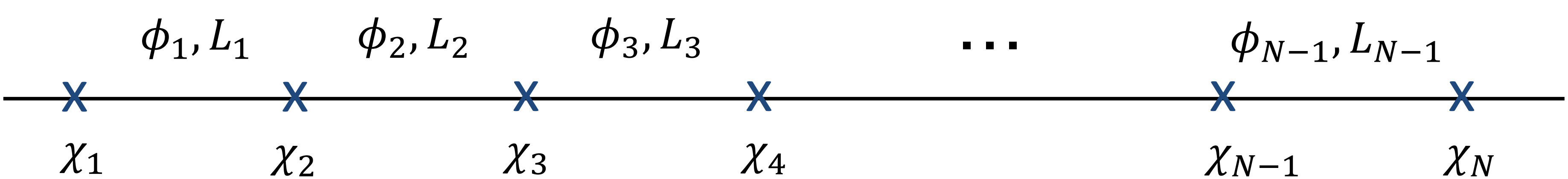
a: *lattice spacing*

ψ_u → odd site

ψ_d → even site

- **Gauge field** (on link):

$$\phi_n \leftrightarrow -agA^1(x), \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$



Lattice theory w/ staggered fermion

Hamiltonian:

$$\hat{H} = -i \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[\chi_n^\dagger e^{i\phi_n} \chi_n - \text{h.c.} \right]$$

$$+ m \cos\theta \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n + J \sum_{n=1}^{N-1} L_n^2 \qquad \left(w = \frac{1}{2a}, J = \frac{g^2 a}{2} \right)$$

Commutation relation:

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0, \ [\phi_n, L_m] = i\delta_{mn}$$

Gauss law:

$$L_n - L_{n-1} = \chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2}$$

Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = \sum_{\ell=1}^{n-1} \left[\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right] \qquad \text{(took } L_0 = 0\text{)}$$

2. Redefine fermion to absorb ϕ_n:

$$\chi_n \to \prod_{\ell<n} \left[e^{-i\phi_\ell} \right] \chi_n$$

Then,

$$\hat{H} = -i \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[\chi_n^\dagger \chi_{n+1} - \text{h.c.} \right] + m\cos\theta \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n$$
$$+ J \sum_{n=1}^{N-1} \left[\sum_{\ell=1}^{n-1} \left(\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right) \right]^2$$

This acts on finite dimensional Hilbert space

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \;\; \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\chi_n = \left(\prod_{\ell<n} iZ_\ell \right) \frac{X_n - iY_n}{2}$$

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\chi_n = \left(\prod_{\ell<n} iZ_\ell\right) \frac{X_n - iY_n}{2}$$

Now the system is purely a spin system:

$$\hat{H} = H_{ZZ} + H_{\pm} + H_Z$$

$$\begin{cases} H_{ZZ} = \frac{J}{2}\sum_{n=2}^{N-1}\sum_{1\le k<\ell\le n} Z_k Z_\ell, \\ H_{\pm} = \frac{1}{2}\sum_{n=1}^{N-1}\left(w - (-1)^n \frac{m}{2}\sin\theta\right)\left[X_n X_{n+1} + Y_n Y_{n+1}\right], \\ H_Z = \frac{m\cos\theta}{2}\sum_{n=1}^{N}(-1)^n Z_n - \frac{J}{2}\sum_{n=1}^{N-1}(n \bmod 2)\sum_{\ell=1}^{n} Z_\ell \end{cases}$$

Qubit description of the Schwinger model !!

Time evolution operator

Suzuki-Trotter decomposition:

$$e^{-i\widehat{H}t} = \left(e^{-i\widehat{H}\frac{t}{M}}\right)^M$$

(M: large positive integer)

$$\simeq \left(e^{-iH_Z\frac{t}{M}}e^{-iH_{ZZ}\frac{t}{M}}e^{-iH_{XX}\frac{t}{M}}e^{-iH_{YY}\frac{t}{M}}\right)^M + \mathcal{O}(1/M)$$

$$\begin{cases} H_Z = \dfrac{m\cos\theta}{2}\displaystyle\sum_{n=1}^{N}(-1)^n Z_n - \frac{J}{2}\sum_{n=1}^{N-1}(n \bmod 2)\sum_{\ell=1}^{n} Z_\ell \\ H_{ZZ} = \dfrac{J}{2}\displaystyle\sum_{n=2}^{N-1}\sum_{1\le k<\ell\le n} Z_k Z_\ell, \\ H_{XX} = \dfrac{1}{2}\displaystyle\sum_{n=1}^{N-1}\left(w - (-1)^n\frac{m}{2}\sin\theta\right) X_n X_{n+1} \\ H_{YY} = \dfrac{1}{2}\displaystyle\sum_{n=1}^{N-1}\left(w - (-1)^n\frac{m}{2}\sin\theta\right) Y_n Y_{n+1} \end{cases}$$

Can we express it in terms of elementary gates?

Time evolution operator (cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_Z\frac{t}{M}}e^{-iH_{ZZ}\frac{t}{M}}e^{-iH_{XX}\frac{t}{M}}e^{-iH_{YY}\frac{t}{M}}\right)^M$$

The 1st one is trivial:

$$e^{-icZ} = R_Z(2c)$$

The 2nd one appeared in Ising model:

$$e^{-icZ_1Z_2} = CXR_Z^{(2)}(2c)CX$$

The 3rd one (see next slide):

$$e^{-icX_1X_2} = CXR_X^{(1)}(2c)CX$$

The 4th one:

$$e^{-icY_1Y_2} = R_Z^{(1)}\left(-\frac{\pi}{2}\right)R_Z^{(2)}\left(-\frac{\pi}{2}\right)e^{-icX_1X_2}R_Z^{(2)}\left(\frac{\pi}{2}\right)R_Z^{(1)}\left(\frac{\pi}{2}\right)$$

Time evolution operator (Cont'd)

$$e^{-icX_1X_2} = CXR_X^{(1)}(2c)CX$$

Proof:

$$CXR_X^{(1)}(2c)CX|0\rangle \otimes |\psi\rangle$$

$$= CXR_X^{(1)}(2c)|0\rangle\otimes|\psi\rangle = CX\left[\cos c|0\rangle\otimes|\psi\rangle - i\sin c|1\rangle\otimes|\psi\rangle\right]$$

$$= \cos c|0\rangle\otimes|\psi\rangle - i\sin c|1\rangle\otimes X|\psi\rangle = \cos c|0\rangle\otimes|\psi\rangle - i\sin c\, X|0\rangle\otimes X|\psi\rangle$$

$$CXR_X^{(1)}(2c)CX|1\rangle \otimes |\psi\rangle$$

$$= CXR_X^{(1)}(2c)|1\rangle\otimes X|\psi\rangle = CX\left[\cos c|1\rangle\otimes X|\psi\rangle - i\sin c|0\rangle\otimes X|\psi\rangle\right]$$

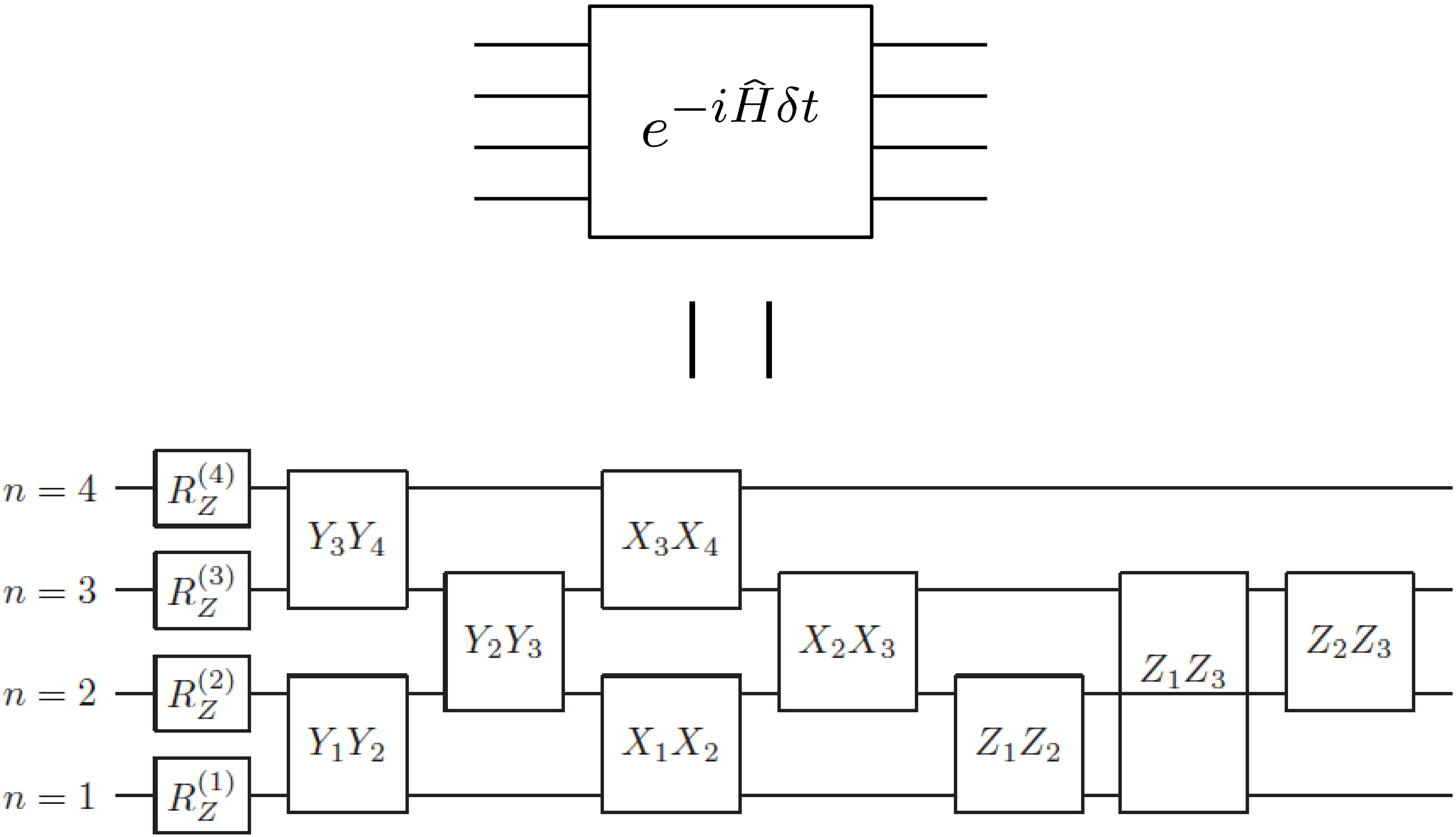
$$= \cos c|1\rangle\otimes|\psi\rangle - i\sin c|0\rangle\otimes X|\psi\rangle = \cos c|1\rangle\otimes|\psi\rangle - i\sin c\, X|1\rangle\otimes X|\psi\rangle$$

Thus,

$$CXR_X^{(1)}(2c)CX|\varphi\rangle\otimes|\psi\rangle = \cos c|\varphi\rangle\otimes|\psi\rangle - i\sin c\, X|\varphi\rangle\otimes X|\psi\rangle$$

$$= e^{-icX_1X_2}|\varphi\rangle \otimes |\psi\rangle$$

Quantum circuit for time evolution op. (N=4)



Improvement of Suzuki-Trotter decomposition

The leading order decomposition:

$$e^{-i(H_1+H_2)\delta t} = e^{-iH_1\delta t}e^{-iH_2\delta t} + \mathcal{O}(\delta t^2)$$

The 2nd order improvement:

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The 2nd order improvement:

$$e^{-i(H_1+H_2)\delta t} = e^{-iH_1\frac{\delta t}{2}}e^{-iH_2\delta t}e^{-iH_1\frac{\delta t}{2}} + \mathcal{O}(\delta t^3)$$

(cf. Baker-Campbell-Hausdorff formula:

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{12}[A,[A,B]]+\cdots}$$

)

This increases the number of gates at each time step
but we can take larger δt (smaller M) to achieve similar accuracy.
Totally we save the number of gates.

Survival probability of massive vacuum

[cf. Martinez etal. **Nature** 534 (2016) 516-519]

The ground state in the large mass limit is

$$(\text{mass term}) \propto m \sum_{n=1}^{N} (-1)^n Z_n$$

$$|\text{massive}\rangle = |0101\cdots01\rangle$$

Survival probability:

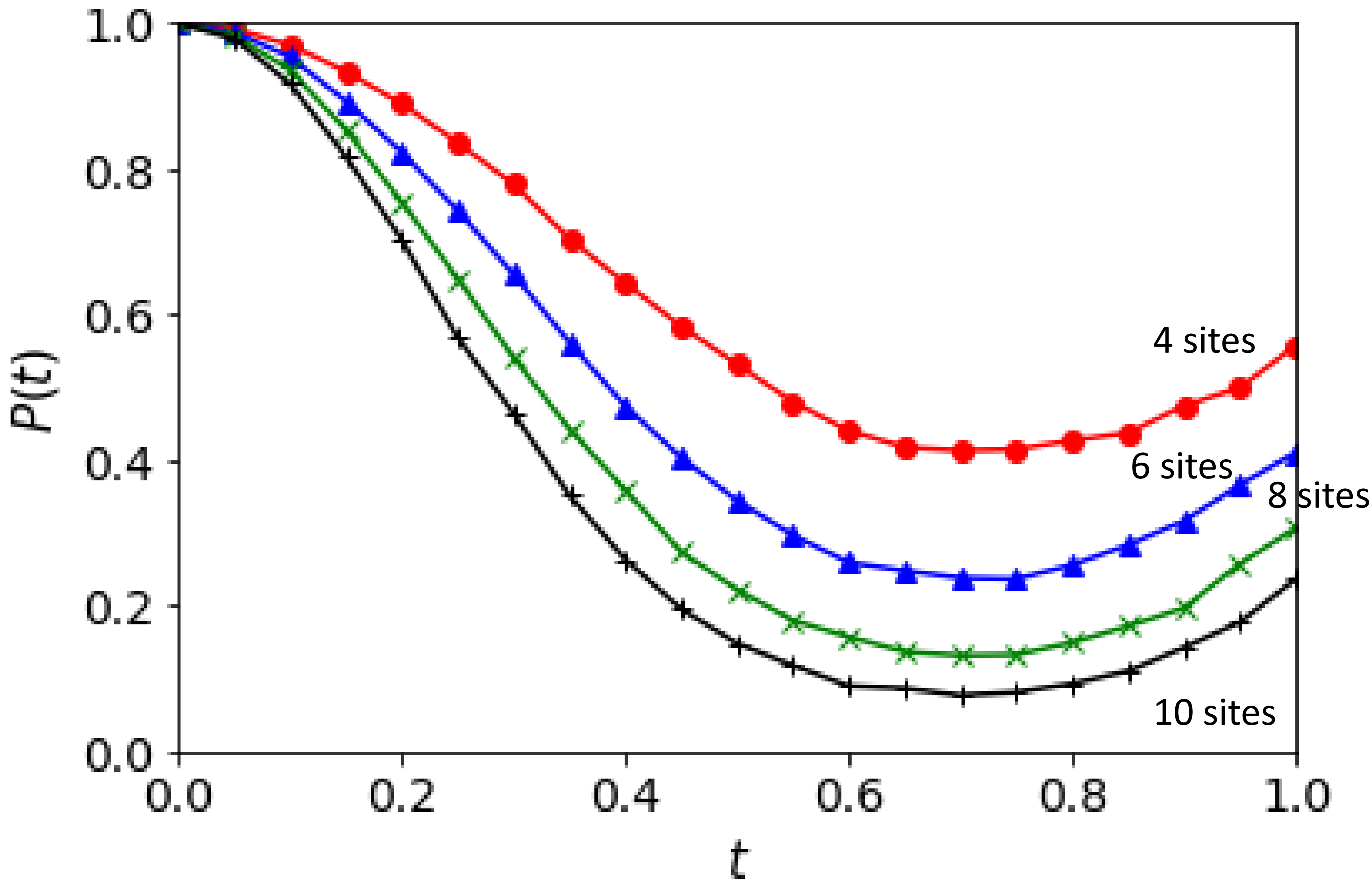
$$P(t) = \left| \langle \text{massive}| \; e^{-i\hat{H}t} \; |\text{massive}\rangle \right|^2$$

"Schwinger effect" in massive limit

$$= \left| \langle 00\cdots0| \; X_N \cdots X_4 X_2 e^{-i\hat{H}t} X_2 X_4 \cdots X_N \; |00\cdots0\rangle \right|^2$$

Result of simulator (10000 shots)

$J = 1,\ w = 1,\ m = 1,\ \theta = 0,\ \delta t = 0.01,$ 100 time steps



VEV of mass operator (chiral condensation)

$$\langle \bar{\psi}(x)\psi(x)\rangle = \langle \mathrm{vac}|\bar{\psi}(x)\psi(x)|\mathrm{vac}\rangle$$

Instead of the local op., we analyze the average over the space:

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle$$

Once we get the vacuum, we can compute the VEV as

$$\frac{1}{2Na}\langle \mathrm{vac}|\sum_{n=1}^{N}(-1)^n Z_n|\mathrm{vac}\rangle = \frac{1}{2Na}\sum_{n=1}^{N}(-1)^n \sum_{i_1\cdots i_N=0,1}\langle \mathrm{vac}|Z_n|i_1\cdots i_N\rangle\langle i_1\cdots i_N|\mathrm{vac}\rangle$$

$$= \frac{1}{2Na}\sum_{n=1}^{N}\sum_{i_1\cdots i_N=0,1}(-1)^{n+i_n}\,|\langle i_1\cdots i_N|\mathrm{vac}\rangle|^2$$

Adiabatic state preparation of vacuum

$$|\mathrm{vac}> = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0>$$

$$\simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\mathrm{vac}_0>$$

$$\left(U(t) = e^{-iH_A(t)\delta t}\right)$$

Here we choose

$$\begin{cases} H_0 = H_{ZZ} + H_Z|_{m\to m_0,\theta\to 0} \quad \Rightarrow \quad |\mathrm{vac}_0\rangle = |0101\cdots 01\rangle \\ H_A(t) = \hat{H}\Big|_{w\to w(t),\theta\to\theta(t),m\to m(t)} \\ w(t) = \frac{t}{T}w,\ \theta(t) = \frac{t}{T}\theta,\ m(t) = \left(1-\frac{t}{T}\right)m_0 + \frac{t}{T}m \end{cases}$$

m_0 can be any positive number in principle
but it is practically chosen to have small systematic error

Massless case

For massless case,

θ is absorbed by chiral rotation ⇨ $\theta = 0$ w/o loss of generality

No sign problem

Nevertheless,

it's difficult in conventional approach because computation of fermion determinant becomes very heavy

$\exists$ Exact result:

[Hetrick-Hosotani '88]

$$\langle \bar{\psi}(x)\psi(x) \rangle = -\frac{e^{\gamma}}{2\pi^{3/2}} g \simeq -0.160 g$$

Can we reproduce it?

Estimation of systematic errors

Approximation of vacuum:

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

$$|\mathrm{vac}> \simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\mathrm{vac}_0> \equiv |\mathrm{vac}_A\rangle$$

Approximation of VEV:

$$\langle \mathcal{O} \rangle \equiv \langle \mathrm{vac}|\mathcal{O}|\mathrm{vac}\rangle \simeq \langle \mathrm{vac}_A|\mathcal{O}|\mathrm{vac}_A\rangle$$

Introduce the quantity

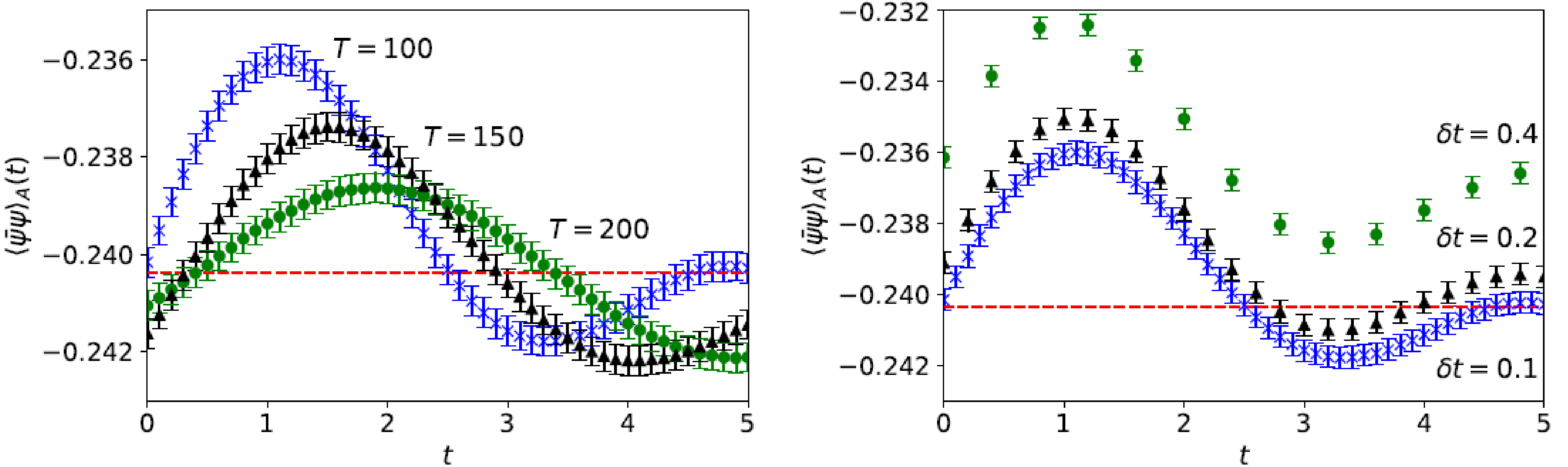
$$\langle \mathcal{O} \rangle_A(t) \equiv \langle \mathrm{vac}_A|e^{i\hat{H}t}\mathcal{O}e^{-i\hat{H}t}|\mathrm{vac}_A\rangle$$

- independent of t if $|\mathrm{vac}_A\rangle = |\mathrm{vac}\rangle$
- dependent on t if $|\mathrm{vac}_A\rangle \neq |\mathrm{vac}\rangle$

This quantity describes intrinsic ambiguities in prediction

⇒ Useful to estimate systematic errors

Estimation of systematic errors (Cont'd)



Oscillating around the correct value

➡ Define central value & error as

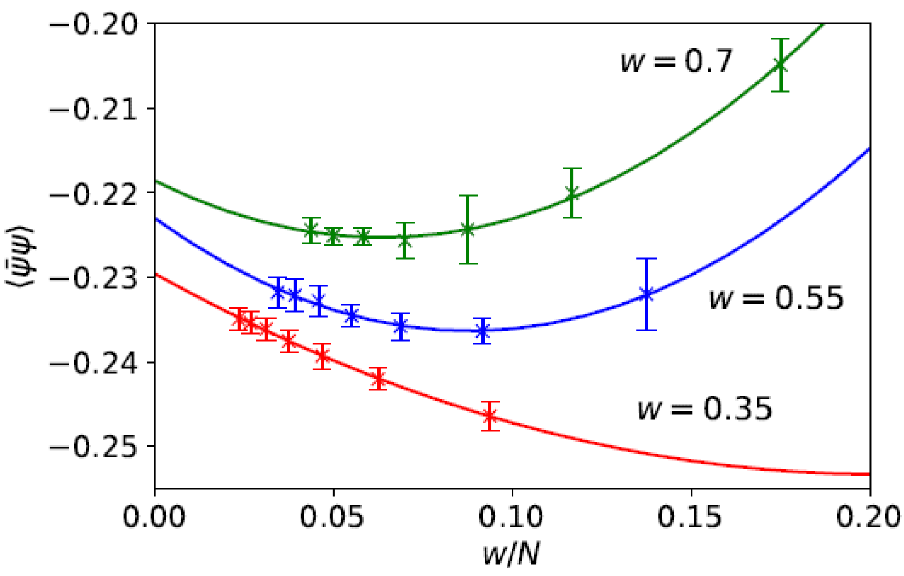
$$\frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)+\min\langle\mathcal{O}\rangle_A(t)\right) \quad \& \quad \frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)-\min\langle\mathcal{O}\rangle_A(t)\right)$$

Thermodynamic & Continuum limit

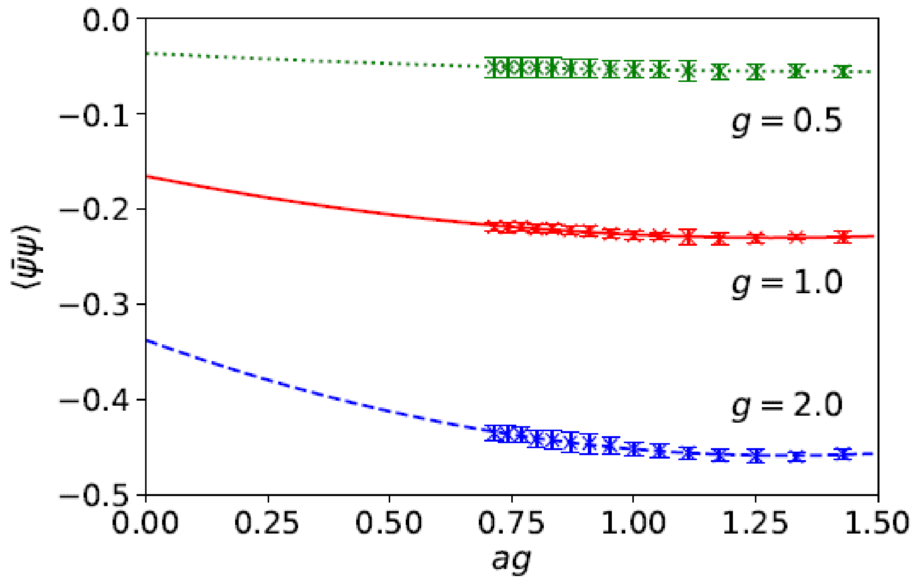
$g = 1, m = 0, N_{\text{max}} = 16, T = 100, \delta t = 0.1, 1M$ shots

#(measurements)

Thermodynamic limit:



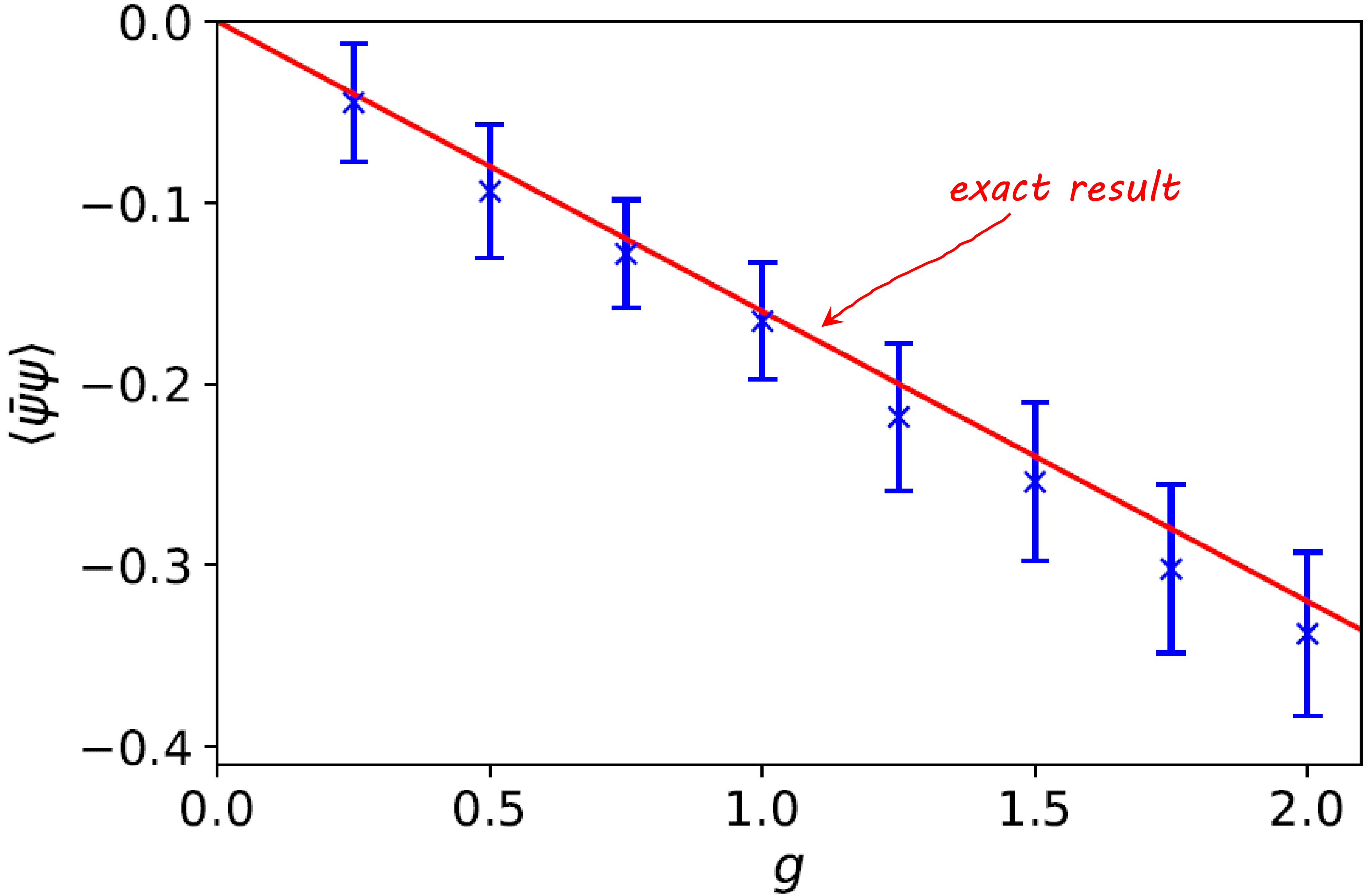
Continuum limit:



Result for massless case (after continuum limit)

$T = 100, \delta t = 0.1, N_{\text{max}} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]



Massive case

Result of mass perturbation theory: [Adam '98]

$$\langle \bar{\psi}(x)\psi(x)\rangle \sim -0.160g + 0.322m\cos\theta$$

However,

$\exists$ Subtlety in comparison: this quantity is UV divergent

$(\sim m \log \Lambda)$

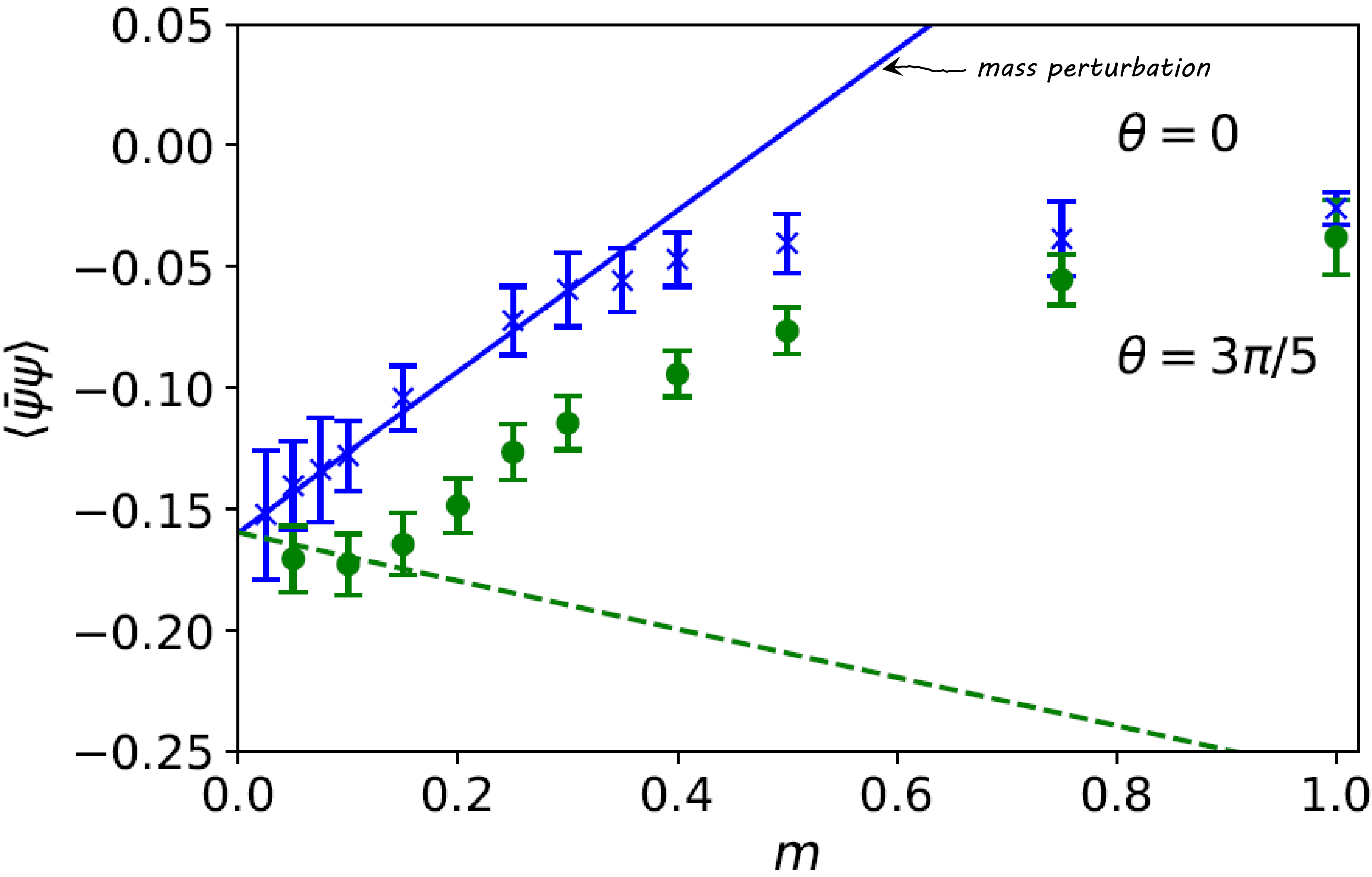
→ Use a regularization scheme to have the same finite part

Here we subtract free theory result before taking continuum limit:

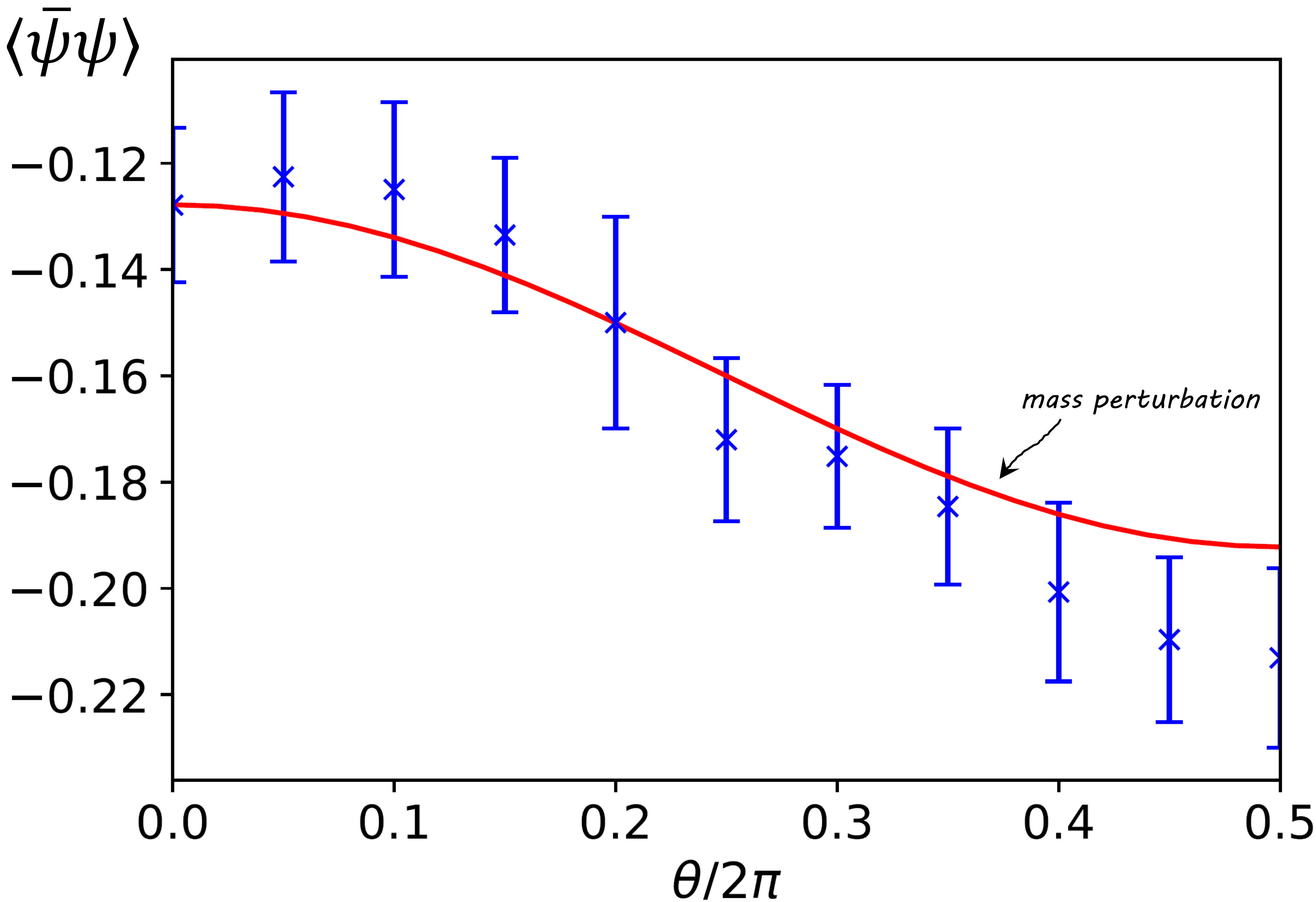
$$\lim_{a\to 0}\left[\langle\bar{\psi}\psi\rangle - \langle\bar{\psi}\psi\rangle_{\text{free}}\right]$$

Result for massive case at $g = 1$

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]



θ dependence at $m = 0.1$ & $g = 1$



Summary

Summary

fun & $^{\exists}$many things to do even now

- Quantum computation is suitable for Hamiltonian formalism which is free from sign problem

- Instead we have to deal with huge vector space. Quantum computers in future may do this job.

- We haven't established how to put many QFTs efficiently on quantum computers yet

- Quantum error correction is important

What I didn't cover

- Quantum error correction
- How to put "bosonic" QFTs on quantum computers
- Other ways to prepare vacuum
- Classical/quantum hybrid algorithm
- Finite temperature & Real time
- Confinement/screening [work in progress, MH-Itou-Kikuchi-Nagano-Okuda]
- Searching critical point [work in progress, Chakraborty-MH-Kikuchi-Izubuchi-Tomiya]
- Matrix QM & (non-)SUSY QFTs [work in progress, Buser-Gharibyan-Hanada-MH-Liu]

Thanks!