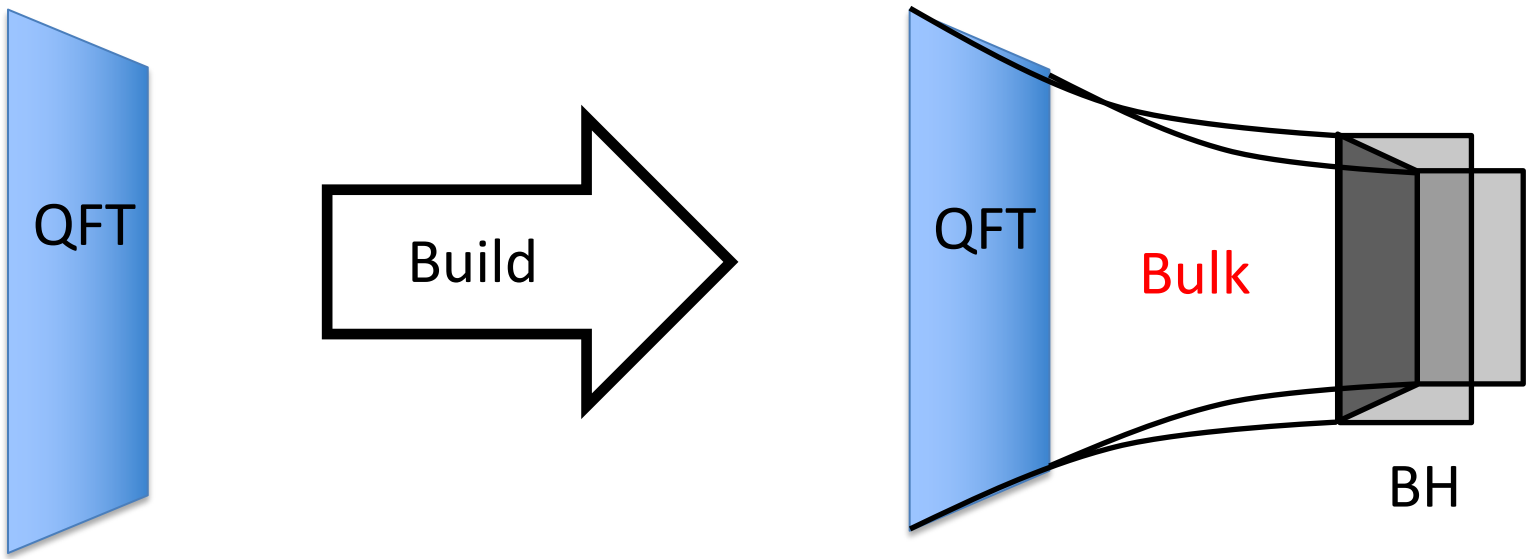
APCTP focus workshop “Quantum Matter and Quantum Information with Holography”, 29 Aug 2020

Building Bulk from Wilson Loops

Koji Hashimoto (Osaka u)

ArXiv:2008.10883



$$ds^2 = -f(\eta)dt^2 + g(\eta)d\vec{x}^2 + d\eta^2$$

Building Bulk from Wilson Loops

[KH 2008.10883]

1. Comparison of methods
2. My formulas
3. Rebuilding AdS
4. Confinement theorem

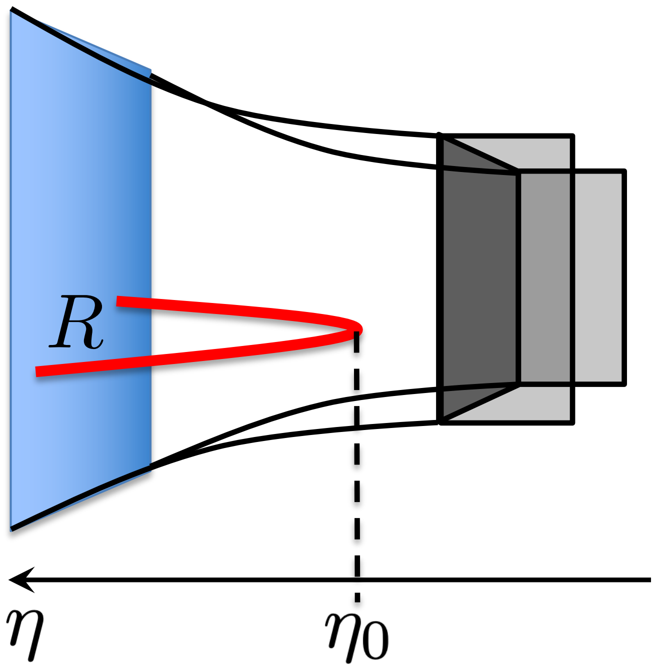
1. Comparison of methods

Wilson loops work better?

Bulk building method	No use of bulk EOM	Lattice input
Holographic renormalization [deHaro Solodukhin Skenderis 00]		✓
Entanglement [Hammersley 07] [Bilson 08]…	✓	
Correlators [Hammersley 06] [Hubeny Liu Rangamani 06]	✓	
AdS/DL [KH Tanaka Tomiya Sugishita 18]		✓
Wilson loop [KH 20]	✓	✓

2. My formulas

Invert the 1998 results!



Bulk metric in string frame

$$ds^2 = -f(\eta)dt^2 + g(\eta)d\vec{x}^2 + d\eta^2$$

$$f(\infty) = g(\infty) = \infty$$

[Maldacena 98] [Rey Yee 98]

Nambu-Goto string solution

$$E(\eta_0) = \frac{1}{\pi\alpha'}\int_{\eta_0}^{\infty} d\eta\, \sqrt{f(\eta)}\sqrt{\frac{f(\eta)g(\eta)}{f(\eta)g(\eta) - f(\eta_0)g(\eta_0)}}$$

$$R(\eta_0) = 2\int_{\eta_0}^{\infty} d\eta\, \frac{1}{\sqrt{g(\eta)}}\sqrt{\frac{f(\eta_0)g(\eta_0)}{f(\eta)g(\eta) - f(\eta_0)g(\eta_0)}}$$

Quark potential $E(R)$, $E_s(R_s)$

2. My formulas

T = 0 : Input is quark potential

Zero temperature case

Given a quark potential $E(R)$, solve

$$f_0 = 2\pi\alpha' \frac{dE(R)}{dR}$$

to get R as a function of f_0. Then substitute it to the following differential equation

$$\frac{d\eta(f)}{df} = \frac{1}{\pi}\sqrt{f}\frac{d}{df}\int_\infty^f df_0 \frac{R(f_0)}{\sqrt{f_0^2 - f^2}} \,.$$

Integrate this to find $\eta = \eta(f)$. Finally, invert it to find a bulk metric $f(\eta)$.

Cf. Entanglement entropy [Bilson 08]

2. My formulas

T > 0 : Input is temporal/spatial Wilson loops

Non-zero temperature case

Given a potential $E_s(R_s)$ for a spatial Wilson loop and $E(R)$ for a temporal Wilson loop, solve

$$g_0 = 2\pi\alpha' \frac{dE_s(R_s)}{dR_s}, \quad h_0 = 2\pi\alpha' \frac{dE(R)}{dR},$$

to get $R_s(g_0)$ and $R(h_0)$. First, substitute $R_s(g_0)$ to the differential equation

$$\frac{d\eta(g)}{dg} = \frac{1}{\pi}\sqrt{g}\frac{d}{dg}\int_\infty^g dg_0 \frac{R_s(g_0)}{\sqrt{g_0^2 - g^2}}.$$

Integrate it to find $\eta = \eta(g)$. Invert it to find a bulk metric component $g(\eta)$. Then substitute the explicit $g(\eta)$ and also $R(h_0)$ to the differential equation

$$\frac{d\eta(h)}{dh} = \frac{1}{\pi}\sqrt{g(\eta(h))}\frac{d}{dh}\int_\infty^h dh_0 \frac{R(h_0)}{\sqrt{h_0^2 - h^2}}.$$

Solve this to find $\eta(h)$, which is inverted to $h(\eta)$. Then obtain another component of the bulk metric as $f(\eta) = h(\eta)^2/g(\eta)$.

3. Rebuilding AdS

Partially reproducing near-horizon of D-branes

CFT at T=0 : $E(R) = -\frac{c}{2\pi\alpha'}\frac{1}{R}$

$\Rightarrow$ AdS spacetime w/ AdS radius $L = \frac{2\Gamma(5/4)\sqrt{c}}{\Gamma(3/4)\sqrt{\pi}}$

$$ds^2 = Ae^{2\eta/L}(-dt^2 + d\vec{x}^2) + d\eta^2 + (\text{internal space})$$

Power-law potential at T=0 : $E(R) = -\frac{c}{2\pi\alpha'}\frac{1}{R^{n-1}}$

$\Rightarrow$ Near-horizon geometry of Dp-branes

$$ds^2 = \eta^{\frac{2(7-p)}{p-3}}\left(-dt^2 + \sum_{i=1}^{p}(dx^i)^2\right) + d\eta^2 + (\text{internal space})$$

$n = (7-p)/(5-p)$

4. Confinement theorem

Confinement $\Rightarrow$ an IR bottom

Theorem 1. *Assume the linear confinement: at large R, the quark potential is given by*

$$\frac{dE(R)}{dR} = \sigma + \frac{c}{R^n} + (\textit{higher in } 1/R)\,.$$

Here $\sigma(>0)$ is the confining string tension. The second term (with $c > 0$ and $n > 0$) is the leading correction.
Then the bulk metric function $f(\eta) = g(\eta)$ in (1) has an IR bottom: $f(\eta)$ approaches a minimum $f = 2\pi\alpha'\sigma$ at which the gradient $df/d\eta$ vanishes. The location of the IR bottom in the η coordinate is

- *at a finite value of η, when $n > 2$ (or when the correction vanishes faster than the power-law).*
- *at $\eta = -\infty$, when $2 \geq n > 0$.*

Cf: IR bottom $\Rightarrow$ confinement

[Kinar Schreiber Sonnenschein 98]

Building Bulk from Wilson Loops

[KH 2008.10883]

1. Comparison of methods
2. My formulas
3. Rebuilding AdS
4. Confinement theorem

Issues to discuss…

1. Falsification of gravity dual?

2. Lüscher term?

3. Near-horizon metric of BH?