dynamical black holes and information loss paradox

Pei-Ming Ho
National Taiwan University

information loss paradox

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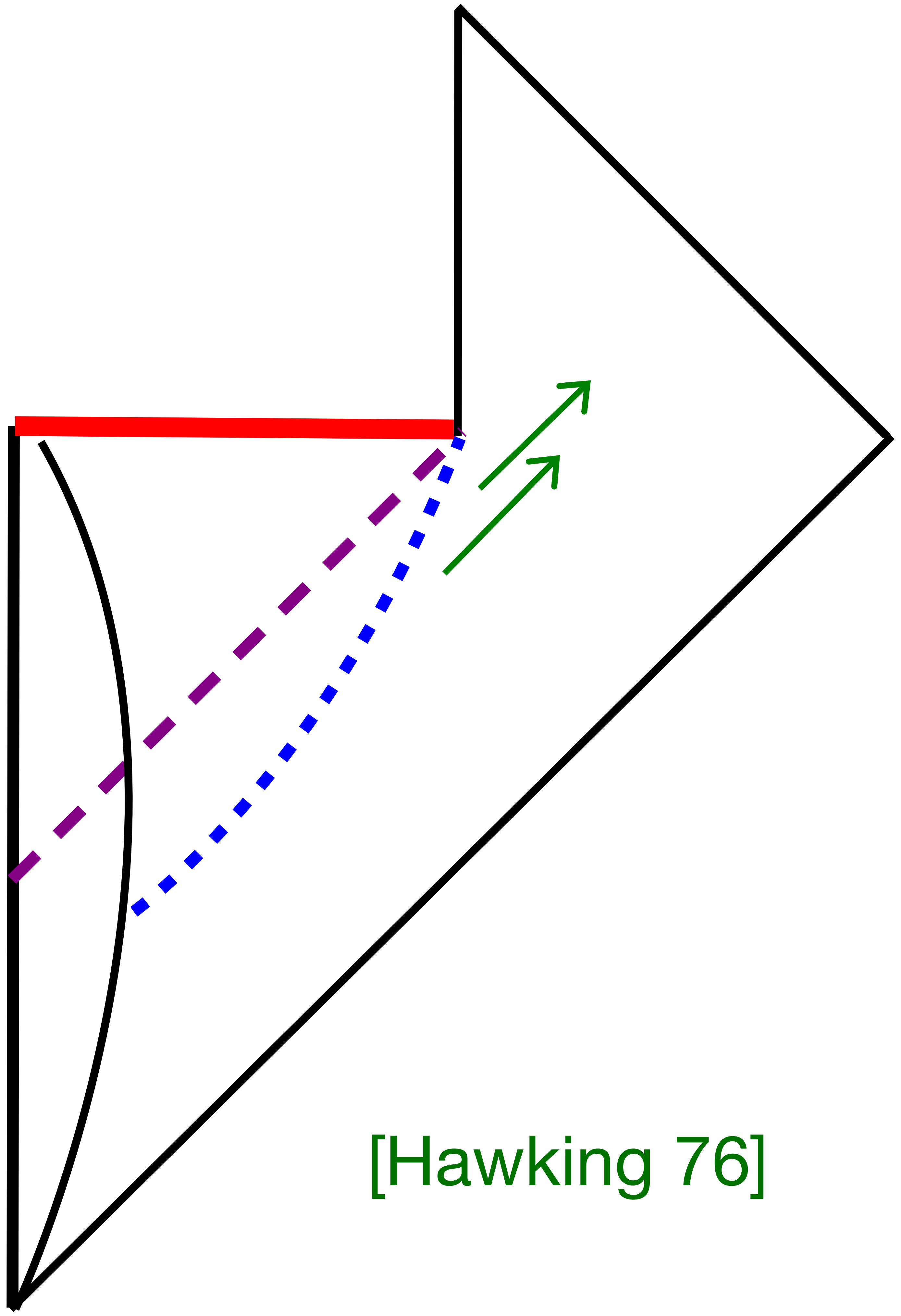


[Hawking 76]

conventional model:

For a huge black hole,

collapsing matter feels nothing

when crossing (uneventful) horizon

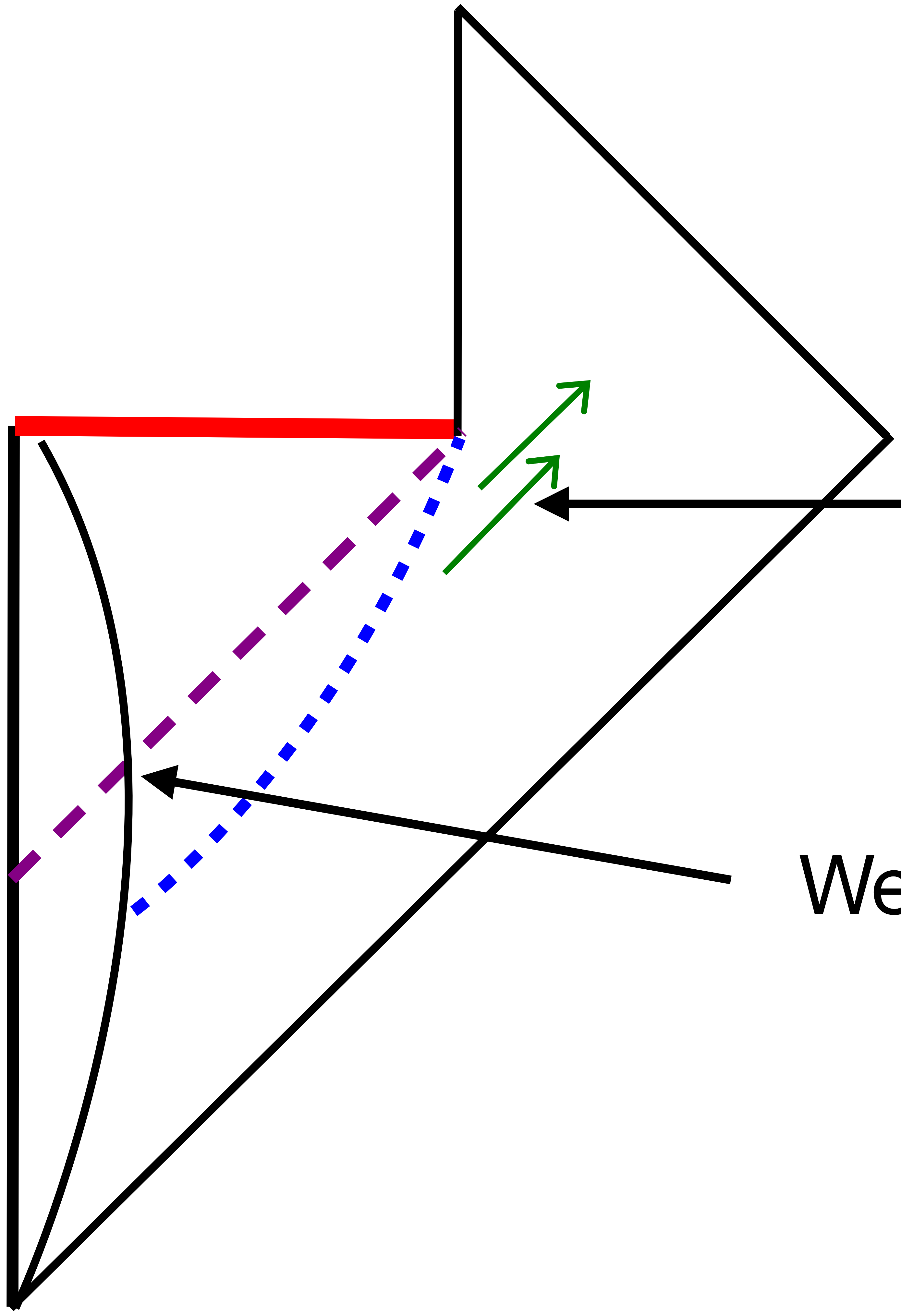
(similar to a classical black hole).

Q: How is info transferred to HR?

a question about the conventional model.

information loss paradox 2

A black hole can be arbitrarily large so that surface gravity is smaller than that on earth.



information of *EVERYTHING*

We can be passing through the horizon right now!

Astrology can be true?

Stringy effect?

information loss paradox

string theory or holographic principle:

[Strominger-Vafa 96, Maldacena 98, Witten 98, Gubser-Klebanov-Polyakov 98]

information must come out as Hawking radiation. ['t Hooft, Susskind, ...]

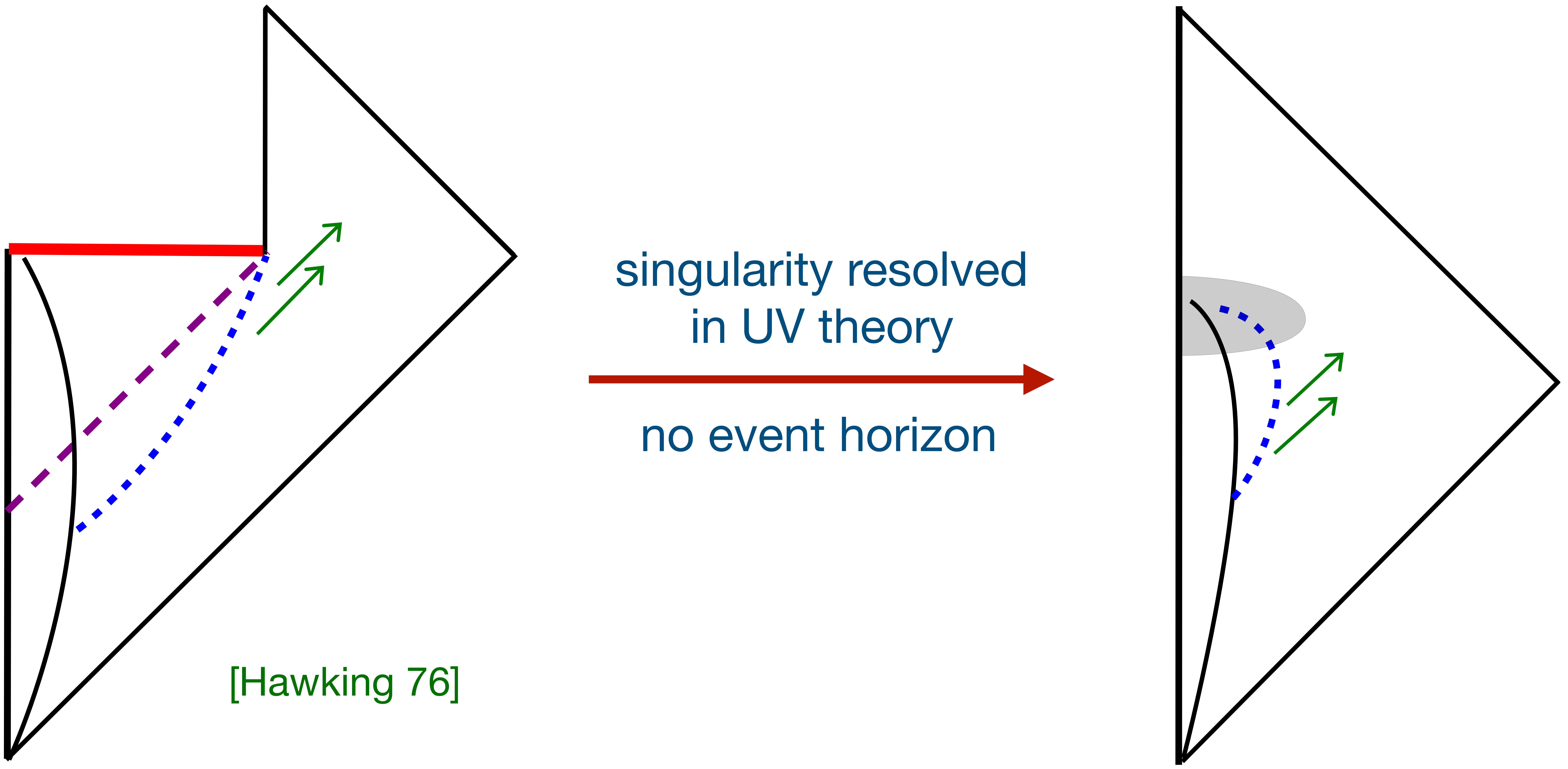
Why should people care about it? [Mathur 09, Polchinski 16, Marolf 17]

decoupling principle:

Without high-energy events, string theory is irrelevant.

$\Rightarrow$ Unitarity should hold in effective field theory (EFT).

information loss paradox



information loss paradox

Assuming uneventful horizon,

how is the info of collapsed matter transferred into Hawking radiation?

Need drama at horizon.

[Mathur 09]

argument 1 for high energy

charge conservation of global symmetries (e.g. baryon number),

Consider particles with the *largest* q/m ratio.

Given N of these particles in gravitational collapse from large distances.

→ radiation during collapse → $M < Nm$

→ Hawking radiation $M \to M - \Delta M$

→ total change: $$\frac{M - \Delta M}{M} Nq + \Delta M \frac{q}{m} < Nq$$

→ Charge conservation violation!

→ *There must be high-energy events breaking global symmetry.*

[Banks-Seiberg 11], [Kawai-Matsuo-Yokokura 13]

argument 2 for high energy

collapsing particles A & B with the same m & q and the same profile:

$\rightarrow$ same gravitational effect $\rightarrow$ *same Hawking radiation* $\rightarrow$ info lost

$\Rightarrow$ Need high-energy interactions to distinguish A from B in the UV theory.

Even pure states contain info.

BMS supertranslation charges are not enough.

argument 3 for high energy

Two books of the same weight and size thrown into a black hole

$\Rightarrow$ Need outgoing radiation of wavelengths $<$ printed letters.

Need interaction.

principles at risk

Equivalence Principle

curvature invariants $\sim 1/a^n$

$\rightarrow$ No high energy event with $E \gg 1/a$.

$\rightarrow$ Not enough to transfer info to Hawking radiation.

Either ***equivalence principle*** or ***decoupling principle*** fails?

“minimal” resolution

If decoupling principle fails, physics has no predictability.

(If no DP, maybe there is no HR.)

minimal resolution:

HR is incompatible with **uneventful horizon** in **EFT**.

HR + EFT $\Rightarrow$ high-energy events (and violation of EP)

AMPS firewall

[Almheiri-Marolf-Polchinski-Sully 13]

Postulate 1: *unitarity* S-matrix from initial state to final state

Postulate 2: semi-classical physics

Postulate 3: Bekenstein entropy for distant observers

Postulate 4: uneventful horizon for freely falling observers (X)

People claim that the arguments for firewalls can be circumvented,

but it does not means that firewalls will not happen.

counting information

entropy problem [Bekenstein 72,73, Page 93, Strominger-Vafa 96, Maldacena 98, Witten 98, Gubser-Klebanov-Polyakov 98, Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07, Lewkowycz-Maldacena 13, Penington 19, Penington-Shenker-Stanford-Yang 19, Almheiri-Engelhardt-Marolf-Maxfield 19, Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini 20]

no-cloning theorem [Susskind-Thorlacius-Uglum 93, Susskind-Thorlacius 93, Hayden-Preskill 07, Sekino-Susskind 08]

……

entropy problem

Unitarity $\Rightarrow$ pure state $\rightarrow$ pure state

$\Rightarrow$ entropy = 0 $\longrightarrow$ entropy = 0

insufficient to guarantee unitarity.

Example:

state A $\longrightarrow$ state A

state B $\longrightarrow$ state A

entropy problem 2

"pure state $\rightarrow$ pure state" is necessary but insufficient,

as pure states contain info.

Need high-energy events and interaction with matter

Q: Where are the high-energy events in EFT?

The high-energy events also changes the gravitational background and affects the calculation of the entropy in Hawking radiation.

conventional model

Assumptions:

1. semi-classical Einstein equation
2. low-energy effective QFT

$G_{\mu\nu} = \kappa \langle T_{\mu\nu} \rangle$

assumption
about the theory

3. Schwarzschild approximation
4. uneventful horizon

assumption
about the state

$\rightarrow$ Hawking radiation

Task:

understand the dynamical process of black-hole evaporation

EFT

semi-classical Einstein equation: $G_{\mu\nu} = \kappa \langle T_{\mu\nu} \rangle$.

QFT in curved background for matter field ϕ .

Guess an approximate metric $g_{\mu\nu}$

$\longrightarrow$ quantize ϕ

$\longrightarrow$ $\langle T_{\mu\nu} \rangle$ for given quantum state *hard*

$\longrightarrow$ solve $G_{\mu\nu} = \kappa \langle T_{\mu\nu} \rangle$

Q: generic low-energy initial states $\longrightarrow$ *high-energy events?*

Q: Does the back reaction of $\langle T_{\mu\nu} \rangle$ *play an important role?*

Schwarzschild approximation

Schwarzschild approximation (classical vacuum): $G_{\mu\nu} = 0.$

quantum correction $\longrightarrow$ $G_{\mu\nu} = \kappa \langle T_{\mu\nu} \rangle \propto \kappa \hbar \propto \ell_p^2$

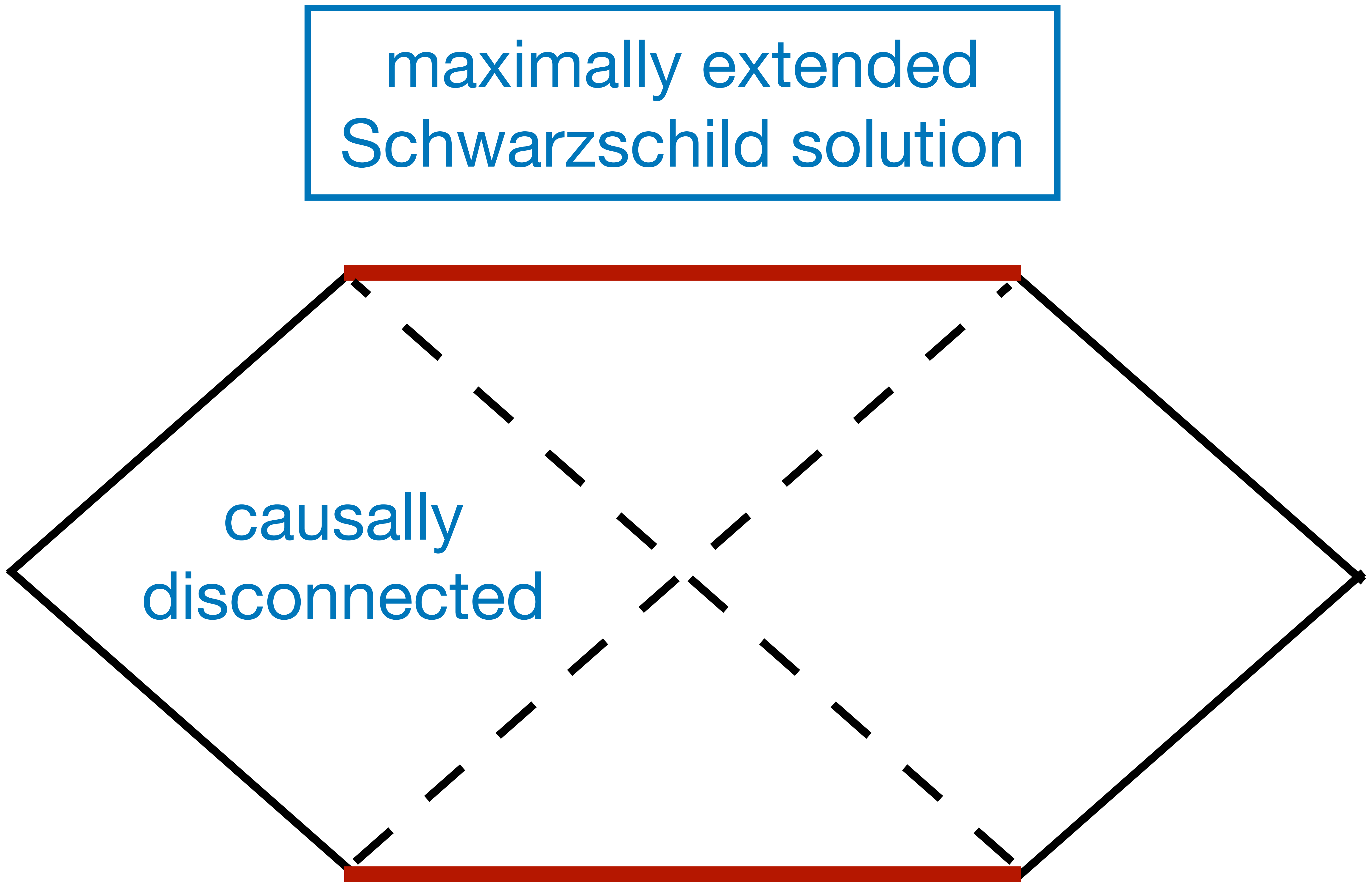
$\longrightarrow$ perturbative expansion in powers of ℓ_p^2 / a^2.

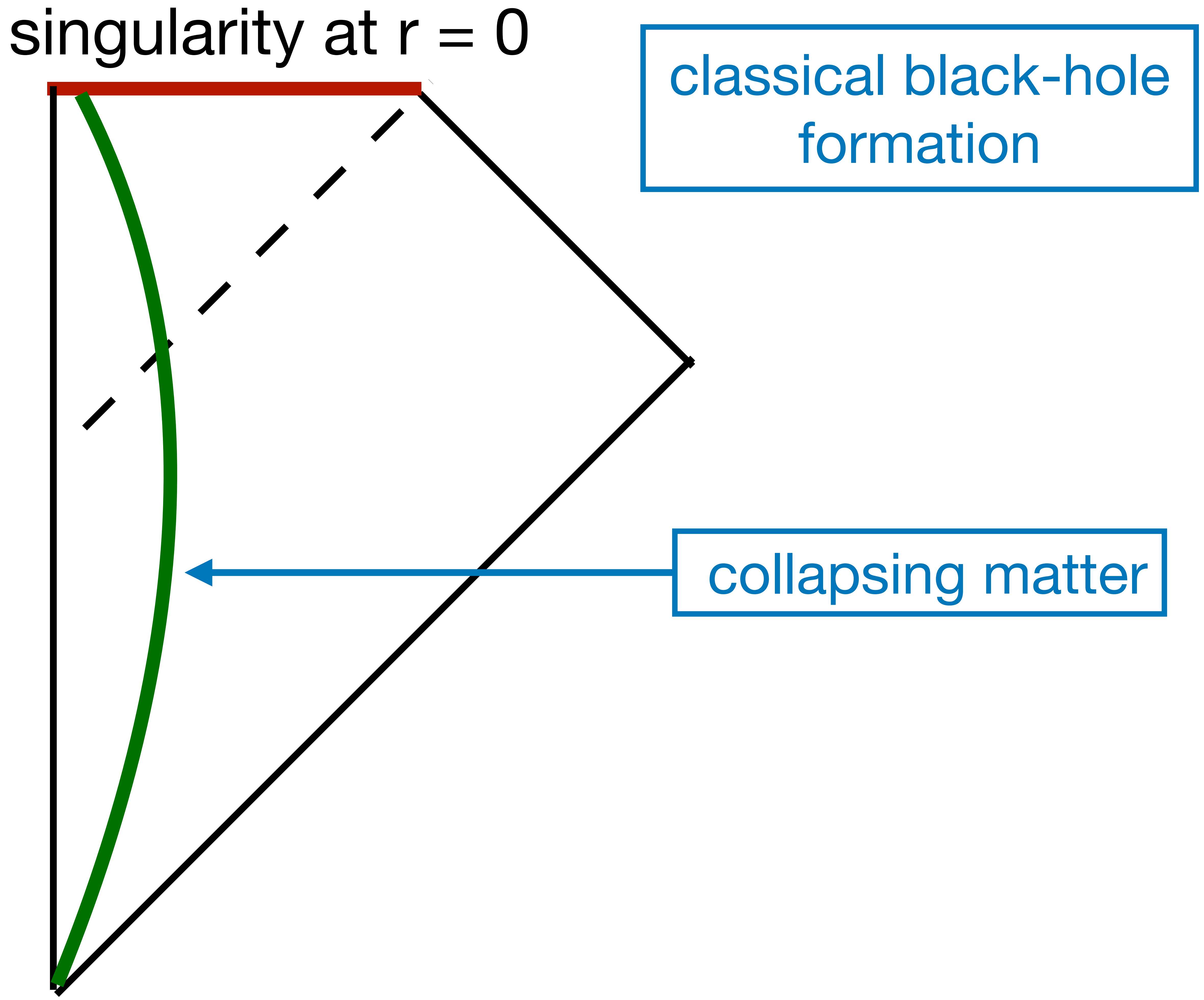
But $ds^2 = -\left(1 - \frac{a}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{a}{r}} + r^2 d\Omega^2$ $\quad (a = 2 G_N M)$

$\rightarrow$ At $r - a \sim \mathcal{O}\left(\ell_p^2 / a\right)$, we have $\left(1 - \frac{a}{r}\right) \sim \frac{\ell_p^2}{a^2}$.

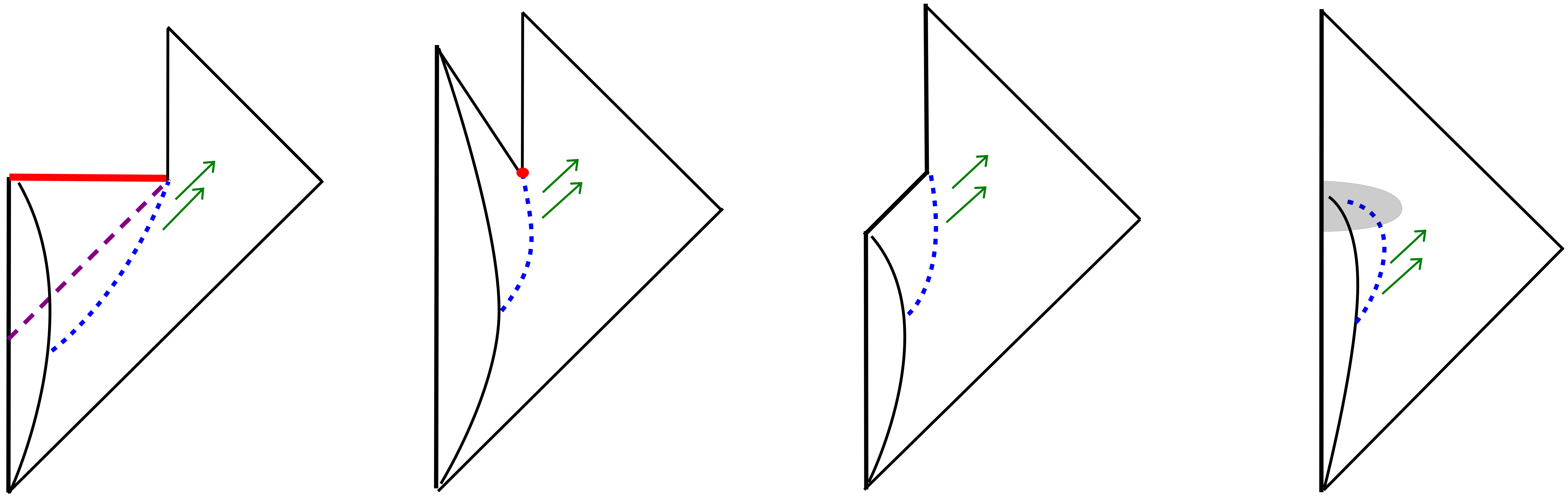
$\rightarrow$ ℓ_p^2 / a^2 introduced on the left-hand side of the EE: $G_{\mu\nu} = \kappa \langle T_{\mu\nu} \rangle$

Schwarzschild solution



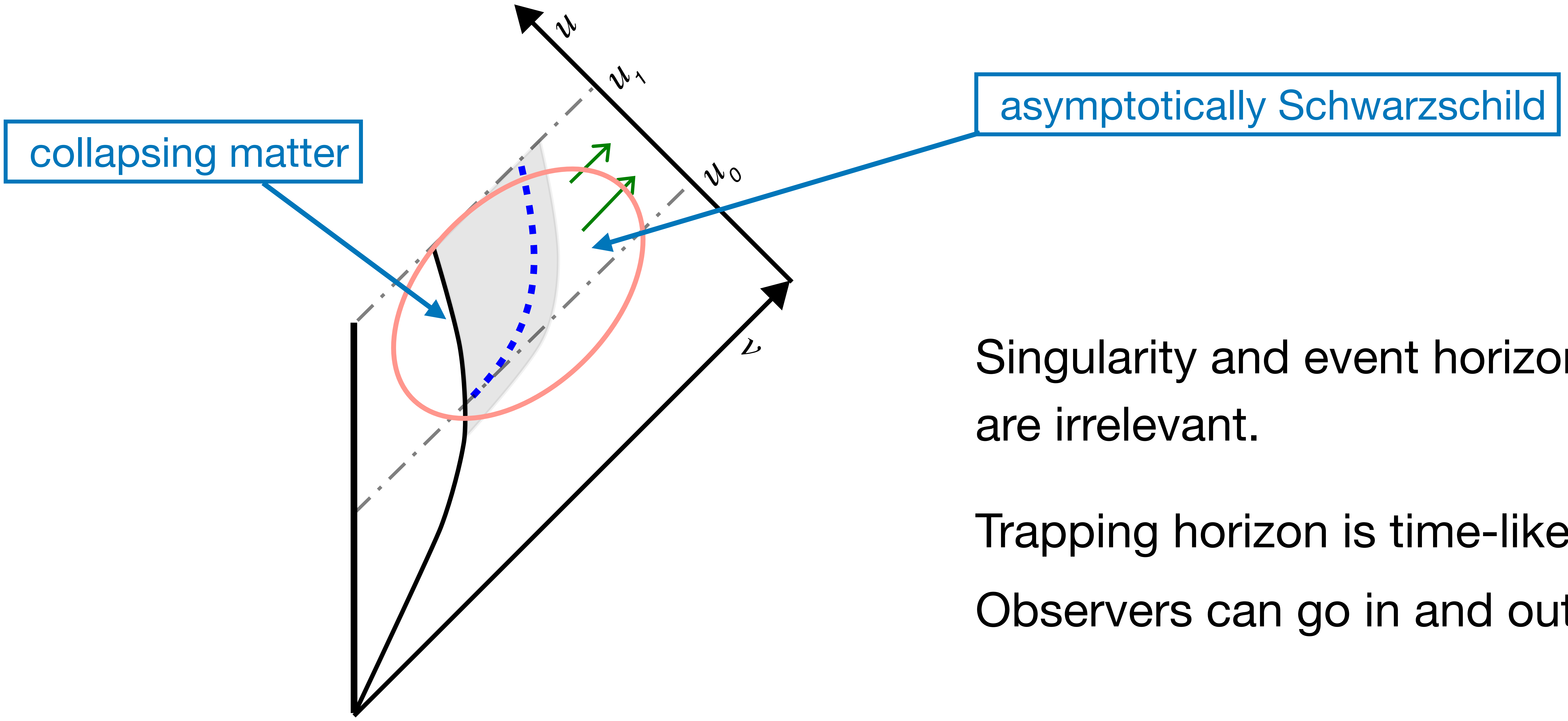


different proposals



assuming uneventful horizon

early stage of evaporation



Singularity and event horizon are irrelevant.

Trapping horizon is time-like.

Observers can go in and out.

uneventful horizon

Naively, *Equivalence Principle demands uneventful horizon.*

Freely falling observers comoving with the collapsing matter

see everything almost like an inertial frame up to the length scale $\mathcal{O}(a)$.

$$\Rightarrow \quad \langle T_{\tau\tau} \rangle \sim \langle T_{\tau\sigma} \rangle \sim \langle T_{\sigma\sigma} \rangle \sim \mathcal{O}(1/a^4) \quad \text{in vacuum.}$$

Assuming that the quantum effect does not change the length scale.
That is, the quantum effect is small.

uneventful horizon

Ansatz for metric

$$ds^2 = -C(u,v)dudv + r^2(u,v)(d\theta^2 + \sin^2\theta d\phi^2)$$

[Davies-Fulling-Unruh 76, Fulling 77, Christensen-Fulling 77]

$$\langle T_{uu}\rangle \sim \mathcal{O}(C^2/a^4), \qquad \langle T_{uv}\rangle \sim \mathcal{O}(C/a^4), \qquad \langle T_{vv}\rangle \sim \mathcal{O}(1/a^4)\,.$$

$$C \sim 0 \quad \Rightarrow \quad \langle T_{uu}\rangle \sim 0, \quad \langle T_{uv}\rangle \sim 0, \quad \langle T_{vv}\rangle \sim -(HR) < 0.$$

ingoing negative energy flux around horizon

possibilities

1. Conventional model is “correct”.

   → Decoupling Principle fails.

2. Conventional model is “wrong”.

   → What is the mistake?

   → When there is Hawking radiation,

   there are high-energy events around collapsing matter.

conventional model

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conventional model

Assumptions:

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2. low-energy effective QFT

assumption about the theory

3. Schwarzschild approximation
4. uneventful horizon

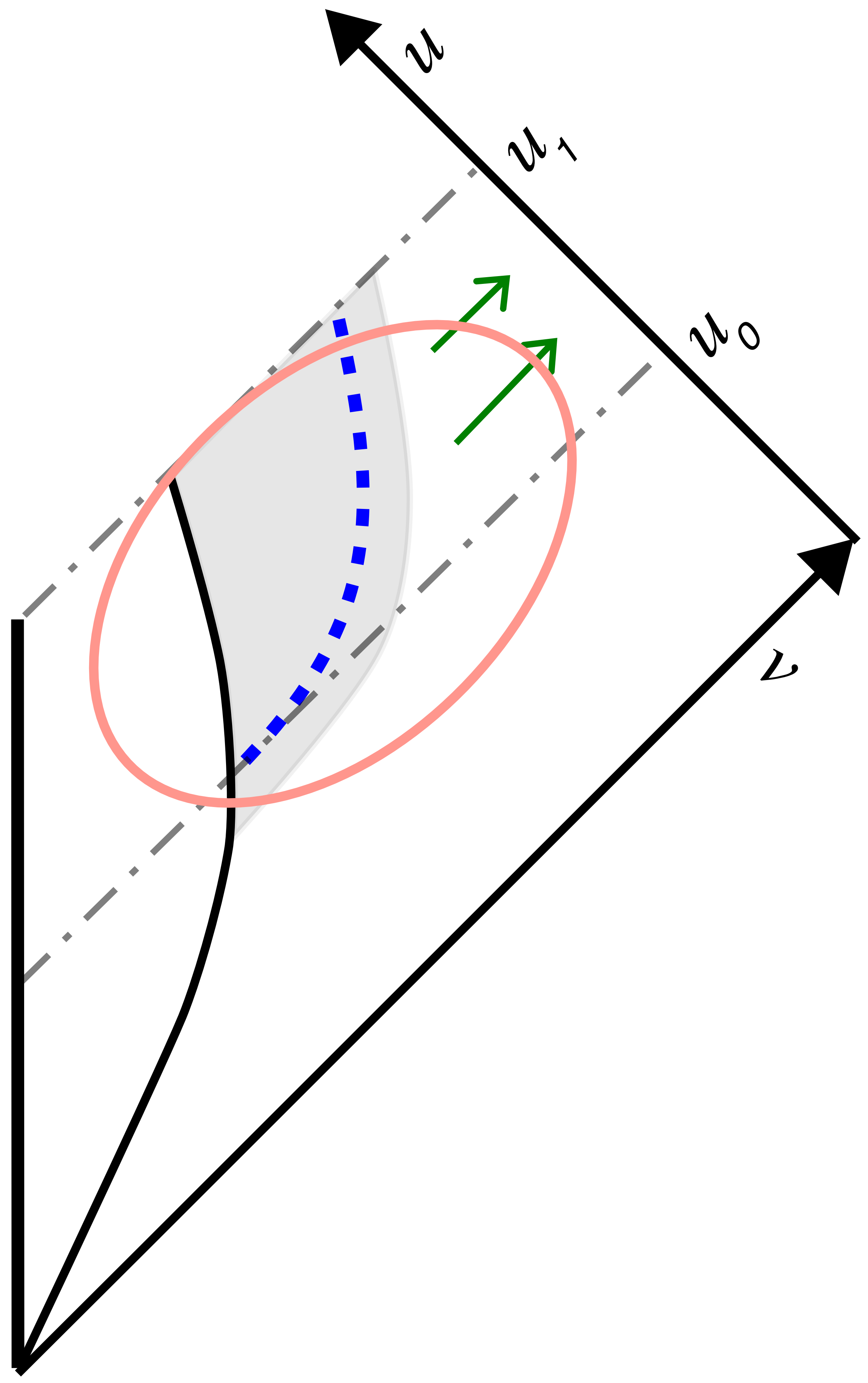
assumption about the state

$\rightarrow$ Hawking radiation

Task:

understand the dynamical process of black-hole evaporation

early stage of evaporation



1. Start with the Schwarzschild metric as the 0th-order approximation.
2. Compute $\langle T_{\mu\nu} \rangle$. *(uneventful?)*
3. Solve $G_{\mu\nu} = \kappa \langle T_{\mu\nu} \rangle$.
4. ….

metric

Spherical symmetry $\Rightarrow$

$$ds^2 \simeq -C(r_*)(dt^2 - dr_*^2) + r^2(r_*)d\Omega^2$$

conformal factor — $C(r_*)$

areal radius — $r^2(r_*)$

$$dudv = dt^2 - dr_*^2$$

$$(u = (t - r_*)/2, \quad v = (t + r_*)/2)$$

$$C = 1 - \frac{a}{r}, \quad r_* = r + a\log\left(\frac{r}{a} - 1\right)$$ for Schwarzschild solution

uneventful horizon

equivalence principle $\quad \to \quad \langle T_{\tau\tau}\rangle \sim \langle T_{\tau\sigma}\rangle \sim \langle T_{\sigma\sigma}\rangle \sim \mathcal{O}(1/a^4)$

$U \equiv \tau - \sigma, \quad V \equiv \tau + \sigma \quad \to \quad \langle T_{UU}\rangle \sim \langle T_{UV}\rangle \sim \langle T_{VV}\rangle \sim \mathcal{O}(1/a^4)$

Since $\boxed{\dfrac{dU}{du} \sim C, \quad \dfrac{dV}{dv} \sim 1,}$

$$\langle T_{uu}\rangle = \frac{dU}{du}\frac{dU}{du}\langle T_{UU}\rangle, \quad \langle T_{uv}\rangle = \frac{dU}{du}\frac{dV}{dv}\langle T_{UV}\rangle, \quad \langle T_{vv}\rangle = \frac{dV}{dv}\frac{dV}{dv}\langle T_{VV}\rangle\,.$$

$$\boxed{\langle T_{uu}\rangle \sim \mathcal{O}(C^2/a^4), \qquad \langle T_{uv}\rangle \sim \mathcal{O}(C/a^4), \qquad \langle T_{vv}\rangle \sim \mathcal{O}(1/a^4)\,.}$$

$$C \sim 0 \quad \Rightarrow \quad \langle T_{uu}\rangle \sim 0, \quad \langle T_{uv}\rangle \sim 0, \quad \langle T_{vv}\rangle \sim -(HR) < 0.$$

ingoing negative energy flux around horizon

freely falling observer

static-geometry approximation: $ds^2 \simeq -C(x)(dt^2 - dx^2) + r^2(x)d\Omega^2$

conserved energy: $E = m_0 \gamma \sqrt{C}$, $\gamma \equiv \frac{1}{\sqrt{1-\dot{x}^2}} = \frac{E/m_0}{\sqrt{C}}$

proper time: $d\tau = \sqrt{C}\,\gamma^{-1} dt = \frac{m_0}{E} C\, dt$

$\dot{x} \simeq -1$

$U \equiv \tau - \sigma, \qquad V \equiv \tau + \sigma$ $\Rightarrow$ $\frac{dU}{du} \simeq \frac{m_0}{E} C, \qquad \frac{dV}{dv} \simeq \frac{E}{m_0}$

$\Rightarrow \quad \langle T_{uu} \rangle \sim C^2 \langle T_{UU} \rangle, \qquad \langle T_{uv} \rangle \sim C \langle T_{UV} \rangle, \qquad \langle T_{vv} \rangle \sim \langle T_{VV} \rangle$

thin shell

Consider a thin shell with spherical symmetry.

curved space outside the shell: $ds^2 = -\boxed{C(u,v)}dudv + r^2(u,v)d\Omega^2$

flat space inside the shell:

$$ds^2 = -dT^2 + dr^2 + r^2 d\Omega^2 = -dUdV + r^2(U,V)d\Omega^2$$

$$T = (V+U)/2, \qquad r = (V-U)/2$$

continuity across an infinitesimally thin shell at $r = R(T)$:

$$\frac{dR}{dU} = -\frac{v}{2}, \qquad \frac{dR}{du} = -\frac{1}{2}C \qquad \Rightarrow \qquad \boxed{\frac{dU}{du} = \frac{1}{v}C}$$

$$\Rightarrow \qquad \langle T_{uu}\rangle \sim C^2\langle T_{UU}\rangle, \qquad \langle T_{uv}\rangle \sim C\langle T_{UV}\rangle, \qquad \langle T_{vv}\rangle \sim \langle T_{VV}\rangle$$

computing $\langle T_{\mu\nu} \rangle$ 1

For a 2D conformal field in the background $ds^2_{(2D)} = -C(u,v)dudv$

trace anomaly $\langle T^{\mu}_{(2D)\mu} \rangle = \frac{1}{24\pi}\mathscr{R}_{(2D)}$

energy-momentum conservation $\nabla_\nu \langle T^{\nu}_{(2D)\mu} \rangle = 0$

$$\Rightarrow \begin{cases} \langle T_{uu} \rangle = -\frac{1}{12\pi} C^{1/2}\partial_u^2 C^{-1/2} + f(u), \\ \langle T_{vv} \rangle = -\frac{1}{12\pi} C^{1/2}\partial_v^2 C^{-1/2} + \bar{f}(v). \end{cases}$$

determined by the quantum state

[Davies-Fulling-Unruh 76, Fulling 77, Christensen-Fulling 77]

computing $\langle T_{\mu\nu} \rangle$ 2

Upon a coordinate transformation $u \to u'(u), \quad v \to v'(v)$

$$f(u) \to f(u) - \frac{1}{16\pi}\{u', u\}, \quad \bar{f}(v) \to \bar{f}(v) - \frac{1}{16\pi}\{v', v\}$$

Schwarzian derivative $\{f, x\} \equiv \left(\frac{\frac{d^2 f}{dx^2}}{\frac{df}{dx}} \right)^2 - \frac{2}{3} \frac{\frac{d^3 f}{dx}}{\frac{df}{dx}}$

Let (U, V) = affine parameter defined on the infinite past.

$$\Rightarrow \quad f(u) = \frac{1}{16\pi}\{U, u\}, \quad \bar{f}(v) = \frac{1}{16\pi}\{V, v\}$$

computing $\langle T_{\mu\nu} \rangle$

Good approximation for $\Delta t \ll \mathcal{O}(a^3/\ell_p^2)$

[Davies-Fulling-Unruh 76]:

2D conformal field ϕ in dim. reduced Schwarzschild background

$$ds^2_{(2D)} = -\left(1 - \frac{a}{r}\right) dudv$$

$$\langle T_{uu} \rangle = \frac{1}{r^2} \langle T^{(2D)}_{uu} \rangle$$

Vacuum EMT computed at the leading order for a collapsing thin shell.

The interior is in the Minkowski vacuum state.

$\langle T^{(2D)}_{UU} \rangle \sim \mathcal{O}(1/a^2) \quad \Rightarrow \quad \langle T^{(2D)}_{uu} \rangle = 0$ @ event horizon.

$\langle T^{(2D)}_{uu} \rangle \to$ HR uniquely determined at large distances.

$\longrightarrow$ *uneventful horizon*

computing $\langle T_{\mu\nu} \rangle$

$\langle T_{\mu\nu} \rangle$ is different for different quantum state.

Unruh vacuum: $\langle T_{uu} \rangle \sim \frac{\#}{a^4}, \quad \langle T_{uv} \rangle \sim 0, \quad \langle T_{vv} \rangle \sim 0 \qquad (r \to \infty)$

static configurations:

Boulware vacuum: $\langle T_{uu} \rangle \sim \langle T_{uv} \rangle \sim \langle T_{vv} \rangle \sim 0 \qquad (r \to \infty)$

Firewall

Hartle-Hawking vacuum: $\langle T_{uu} \rangle \sim \frac{\#}{a^4}, \quad \langle T_{uv} \rangle \sim 0, \quad \langle T_{vv} \rangle \sim \frac{\#}{a^4}$

computing $\langle T_{\mu\nu} \rangle$

[Christensen-Fulling 76]

4D conformal field

trace anomaly + conservation law + Schwarzschild background

$\rightarrow$ a single functional degree of freedom in $\langle T_{\mu\nu} \rangle$ $\longleftarrow$ *not computed.*

2D picture extended to 4D as a consistent scenario:

uneventful horizon $\rightarrow$ Hawking radiation

Uneventful horizon is (possibly) compatible with assumption of Schwarzschild approximation.

semi-classical Einstein equation

$$G_{uu} = \frac{2\partial_u C \partial_u r}{Cr} - \frac{2\partial_u^2 r}{r} = \kappa \langle T_{uu} \rangle \sim \mathcal{O}(\ell_p^2 C/a^4),$$

$$G_{vv} = \frac{2\partial_v C \partial_v r}{Cr} - \frac{2\partial_v^2 r}{r} = \kappa \langle T_{vv} \rangle \sim \mathcal{O}(\ell_p^2/a^4),$$

$$G_{uv} = \frac{C}{2r^2} + \frac{2\partial_u r \partial_v r}{r^2} + \frac{2\partial_u \partial_v r}{r} = \kappa \langle T_{uv} \rangle \sim \mathcal{O}(\ell_p^2 C/a^4),$$

$$G_{\theta\theta} = \frac{2r^2}{C^3}\left(\partial_u C \partial_v C - C\partial_u \partial_v C\right) - \frac{4r}{C}\partial_u \partial_v r = \kappa \langle T_{\theta\theta} \rangle \sim \mathcal{O}(\ell_p^2/a^2)\,.$$

trapping horizon

First choose a foliation:

$r(u, v) = \text{const}\,.$ if there is spherical symmetry.

Normally, $\partial_u r < 0, \qquad \partial_v r > 0.$

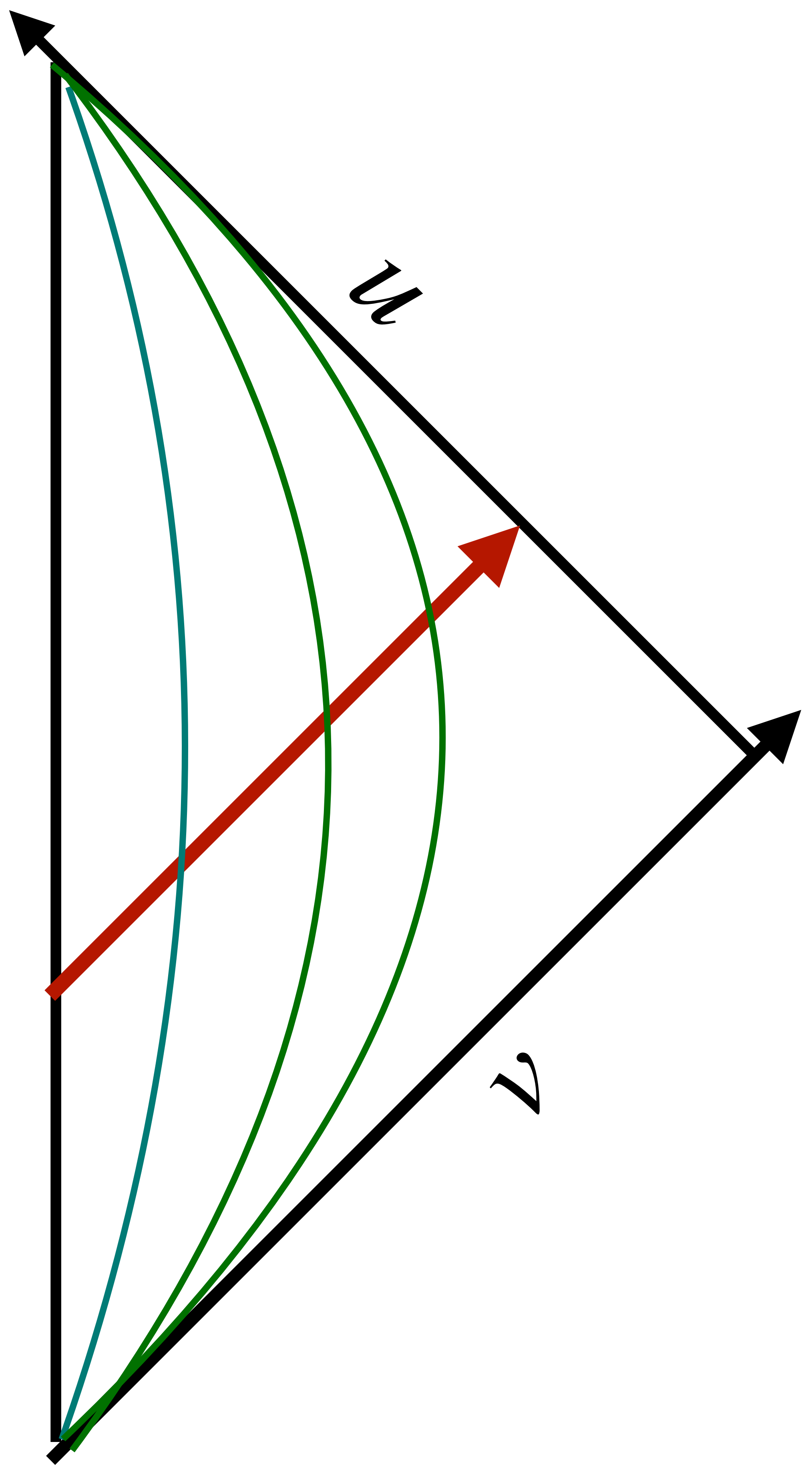
Trapped region: $\partial_u r < 0, \qquad \partial_v r < 0.$

Trapping horizon: $\boxed{\partial_v r = 0.}$

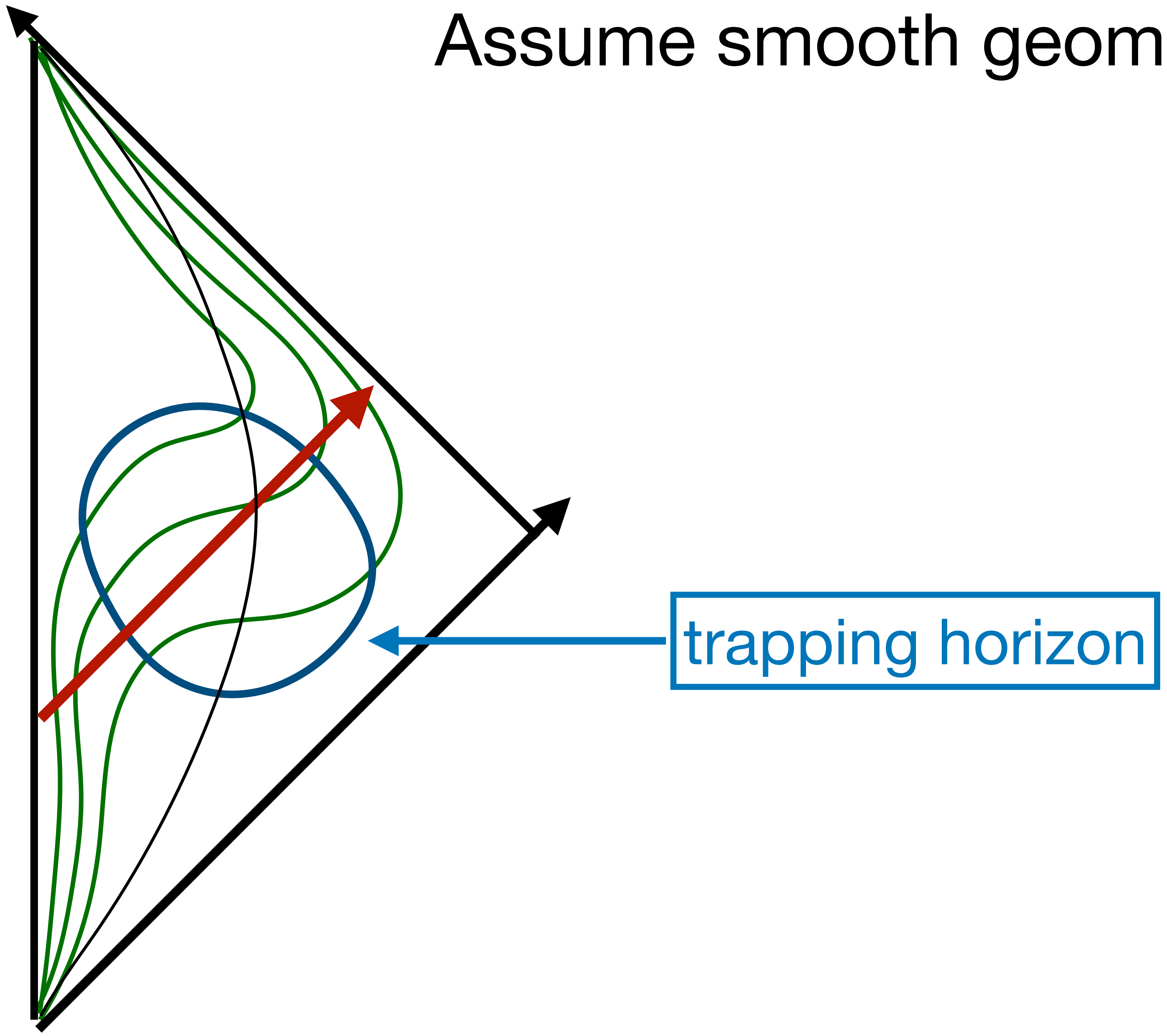
outer trapping horizon: $\partial_v^2 r > 0,$ *inner trapping horizon*: $\partial_v^2 r < 0.$

apparent horizon: a space-like slice of the trapping horizon.

trapping horizon 2



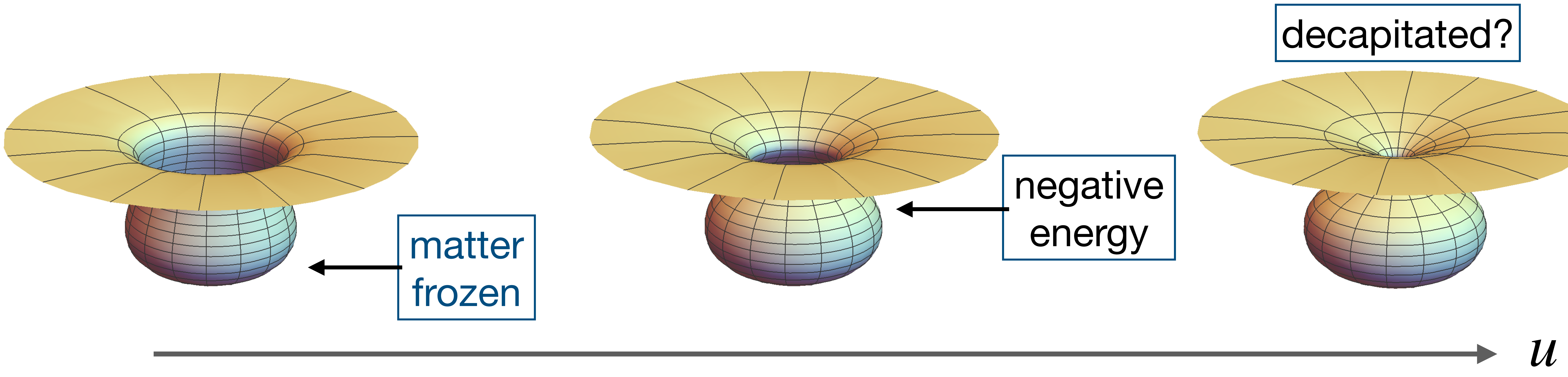
Assume smooth geometry.



wormhole-like geometry

energy of collapsed matter cancelled by negative ingoing energy

$\Rightarrow$ smaller neck



[Parentani-Piran 94, Ho-Matsuo 18]

remnant = “Wheeler’s bag of gold”, “baby universe”

dynamical horizon geometry

$$ds^2 = -C(u,v)dudv + r^2(u,v)d\Omega^2$$

around *trapping horizon* $(u, v = v_h(u))$ where $\partial_v r(u,v) = 0$

$$C(u,v) = C_0(u) + C_1(u)(v - v_h(u)) + \cdots$$

$$r(u,v) = r_0(u) + \frac{r_2(u)}{2}(v - v_h(u))^2 + \frac{r_3(u)}{6}(v - v_h(u))^3 + \cdots$$

geometry of uneventful horizon

$$G_{uu} = \frac{2\partial_u C \partial_u r}{Cr} - \frac{2\partial_u^2 r}{r} = \kappa \langle T_{uu} \rangle,$$

$$G_{vv} = \frac{2\partial_v C \partial_v r}{Cr} - \frac{2\partial_v^2 r}{r} = \kappa \langle T_{vv} \rangle, \qquad \longrightarrow \qquad -\frac{2r_2(u)}{r_0} = \kappa \langle T_{vv} \rangle < 0$$

$$G_{uv} = \frac{C}{2r^2} + \frac{2\partial_u r \partial_v r}{r^2} + \frac{2\partial_u \partial_v r}{r} = \kappa \langle T_{uv} \rangle, \qquad \longrightarrow \qquad \frac{C_0}{2r_0^2} - \frac{2r_2(u)\dot{v}_h(u)}{r_0} = \kappa \langle T_{uv} \rangle \sim \mathcal{O}\left(\frac{\kappa C}{a^4}\right)$$

$$G_{\theta\theta} = \frac{2r^2}{C^3}\left(\partial_u C \partial_v C - C \partial_u \partial_v C\right) - \frac{4r}{C}\partial_u \partial_v r = \kappa \langle T_{\theta\theta} \rangle \,.$$

$$\Rightarrow \qquad \dot{v}_h(u) > 0$$

$\langle T_{vv} \rangle < 0 \quad \Rightarrow \quad$ *trapping horizon in vacuum* $v = v_h(u)$ *is time-like.*

2 classes of $\langle T_{\mu\nu} \rangle$

event horizon $\longrightarrow$ trapping horizon

Naively, a time-dependent Schwarzschild radius a $\Rightarrow$

Schwarzschild metric $\longrightarrow$ no trapping horizon

outgoing Vaidya metric $\longrightarrow$ no trapping horizon

ingoing Vaidya metric $\longrightarrow$ trapping horizon

outgoing positive energy flux vs. *ingoing negative energy flux*

$$\langle T_{uu} \rangle > 0 \quad \text{vs} \quad \langle T_{vv} \rangle < 0$$

eventful horizon vs. uneventful horizon

classical black hole

Schwarzschild coordinates

$$ds^2 = -\left(1 - \frac{a}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{a}{r}} + r^2 d\Omega^2$$

$$dr = \left(1 - \frac{a}{r}\right) dr_*$$

tortoise coordinate

$$ds^2 = -\left(1 - \frac{a}{r}\right)(dt^2 - dr_*^2) + r^2 d\Omega^2$$

$$u = t - r_*, \qquad v = t + r_*$$

$$ds^2 = -\left(1 - \frac{a}{r}\right) dudv + r^2 d\Omega^2$$

ingoing Vaidya metric

$$ds^2 = -\left(1 - \frac{a}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

outgoing Vaidya metric

$$ds^2 = -\left(1 - \frac{a}{r}\right) du^2 - 2dudr + r^2 d\Omega^2$$

naive time-dependent solution

outgoing Vaidya metric:

$$ds^2 = -\left(1 - \frac{a(u)}{r}\right) du^2 - 2dudr + r^2 d\Omega^2$$

$$T_{uu} = -\frac{1}{8\pi G_N r^2}\frac{da(u)}{du} > 0, \qquad T_{ur} = T_{rr} = 0.$$

Trajectory $r = a(u)$ (where $\partial_u r = 0$) is space-like during evaporation:

$$ds^2\Big|_{r=a(u)} = 0 - 2duda = -2\frac{da}{du}du^2 > 0 \qquad \left(\frac{da}{du} < 0\right)$$

Trajectory $r = a(u)$ cannot be crossed by any causal trajectory.

naive time-dependent solution

ingoing Vaidya metric:

$$ds^2 = -\left(1 - \frac{\bar{a}(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

$$T_{vv} = \frac{1}{8\pi G_N r^2} \frac{d\bar{a}(v)}{dv} < 0, \qquad T_{vr} = T_{rr} = 0.$$

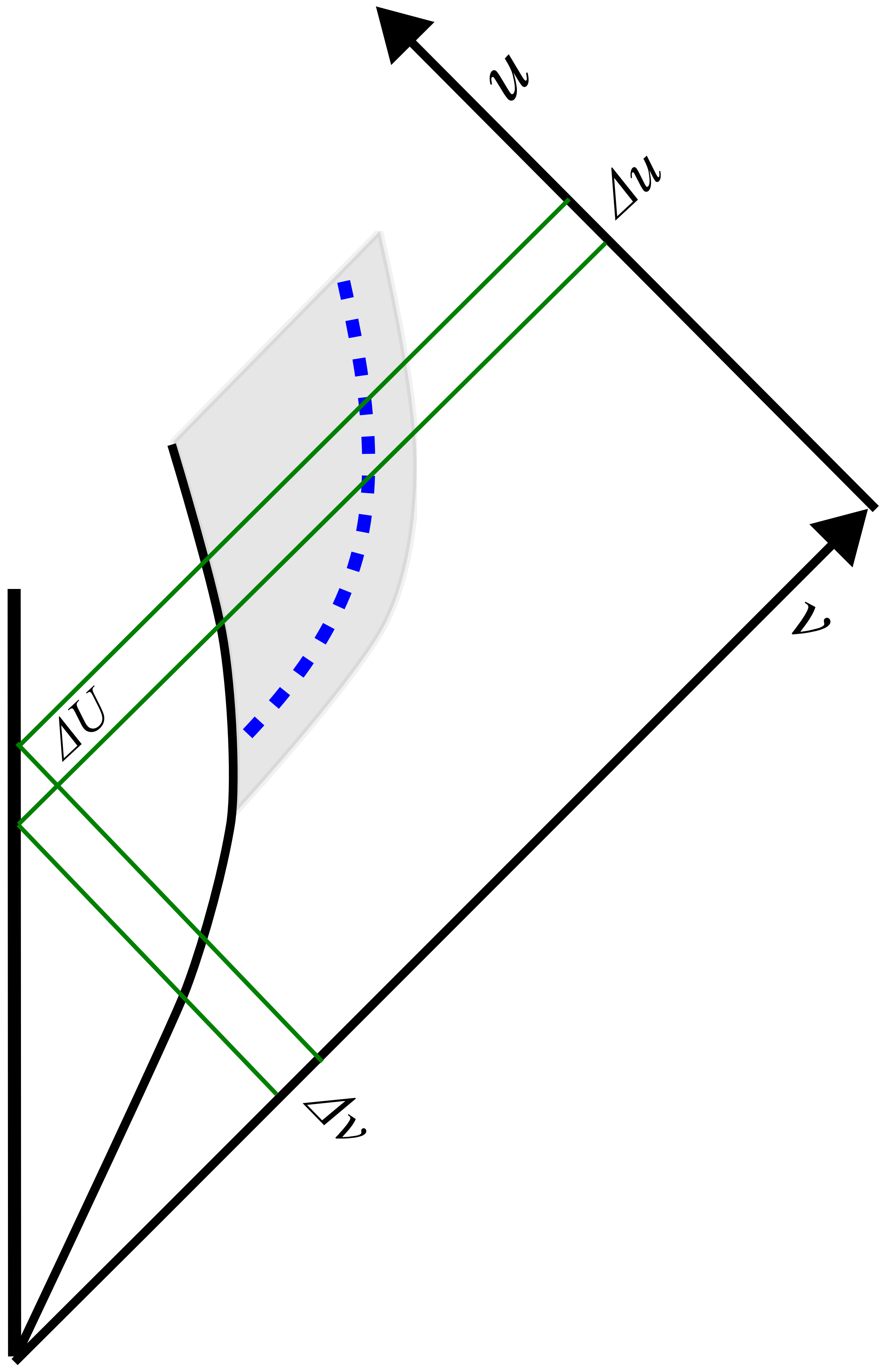
(uneventful horizon)

Trapping horizon in vacuum $r = \bar{a}(v)$ is time-like during evaporation :

$$ds^2\Big|_{r=\bar{a}(v)} = 0 + 2dvd\bar{a} = 2\frac{d\bar{a}}{dv}dv^2 < 0 \qquad \left(\frac{d\bar{a}}{dv} < 0\right)$$

This scenario has an uneventful horizon.

Hawking radiation



Hawking radiation arises when the affine parameters on the past and future null infinities are related via an approximate exponential relation. (It carries purely geometric info.)

[Visser 01, Barcelo-Liberati-Sonego-Visser 06,06,10,10]

Hawking radiation

scalar field ϕ in the black-hole background: $ds^2 = -Cdudv + r^2 d\Omega^2$

$$\nabla^2 \phi = 0 + \lambda \phi^2 + \cdots$$

$$\phi \equiv \frac{\varphi}{r} \quad \Rightarrow \quad \boxed{\partial_u \partial_v \varphi \simeq \frac{\partial_u \partial_v r}{r} \varphi + \frac{C}{4r^2} \nabla_{\Omega^2} \varphi - \frac{\lambda}{4r} C \varphi^2 \sim \mathcal{O}(C)}$$

$$\varphi \simeq \int_0^\infty d\omega \, \left[c_\omega^\dagger e^{i\omega u} + c_\omega e^{-i\omega u} + \tilde{c}_\omega^\dagger e^{i\omega v} + \tilde{c}_\omega e^{-i\omega v} \right]$$

positive/negative frequency modes $\longleftrightarrow$ creation/annihilation operators

Hawking radiation is insensitive to non-gravitational info.

Hawking radiation

In the coordinate system (U, V):

$$\varphi = \int_0^\infty d\omega \, \left[a_\omega^\dagger e^{i\omega U} + a_\omega e^{-i\omega U} + \tilde{a}_\omega^\dagger e^{i\omega V} + \tilde{a}_\omega e^{-i\omega V}\right]$$

positive/negative frequency modes $\longleftrightarrow$ creation/annihilation operators

Bogoliubov transformation:

$$c_\omega = \int_0^\infty d\omega' \left[A_{\omega\omega'} a_{\omega'} + B_{\omega\omega'} a_{\omega'}^\dagger\right], \qquad c_\omega^\dagger = \int_0^\infty d\omega' \left[B^*_{\omega\omega'} a_{\omega'} + A^*_{\omega\omega'} a_{\omega'}^\dagger\right]$$

Hawking radiation

Bogoliubov coefficients:

$$A_{\omega\omega'} \equiv \frac{1}{2\pi}\sqrt{\frac{\omega}{\omega'}}\int_{-\infty}^{\infty} du\, e^{i\omega u - i\omega' U(u)}, \qquad B_{\omega\omega'} \equiv \frac{1}{2\pi}\sqrt{\frac{\omega}{\omega'}}\int_{-\infty}^{\infty} du\, e^{i\omega u + i\omega' U(u)}.$$

freely falling observers: *vacuum* $|0\rangle$, *1-particle states* $a^\dagger_\omega |0\rangle$

spectrum of Hawking radiation $\langle 0 | c^\dagger_\omega c_{\omega'} | 0 \rangle$

✻ The state $c_\omega |0\rangle = \int_0^\infty d\omega'\, B_{\omega\omega'} a^\dagger_{\omega'} |0\rangle$ must be well-defined.

Hawking radiation

The spectrum of Hawking radiation

$\longleftarrow$ Bogoliubov coefficients

$\longleftarrow$ *$U(u) \simeq$ exponential function in u*

$\longleftarrow$ the trajectory of freely falling observers

$\mathscr{V}(\phi)$ polynomial interactions in EFT around horizon are suppressed.

How can matter pass info to Hawking radiation?

$$\frac{dU}{du} \sim C(x), \quad C(x) = 1 - \frac{a}{r} \simeq \frac{r-a}{a} \simeq \exp\left(\frac{r_* - a}{a}\right) = e^{-1} \exp\left(-\frac{u-v}{2a}\right)$$

Hawking radiation

Hawking radiation as *Unruh effect*:

In Minkowski spacetime,

the notion of particles is different for different reference frames.

vacuum for inertial frames $\longleftrightarrow$ Unruh temperature for accelerating frames

Near the horizon of a black hole,

freely falling frame $\longrightarrow$ inertial frame,

accelerating observers $\longrightarrow$ fiducial observers.

$\Rightarrow$ Fiducial observers see radiation at the Hawking temperature.

$\langle T_{\mu\nu} \rangle$ and Hawking radiation

vacuum EMT:

$\langle T_{uu} \rangle \to HR, \quad \langle T_{vv} \rangle \to 0$ at large distances.

$\langle T_{uu} \rangle \to 0, \quad \langle T_{vv} \rangle \to -HR$ around horizon.

At large distances,

both freely falling and fiducial observers

see outgoing particles as HR.

Around the horizon,

fiducial observers see the same spectrum of HR;

freely falling observers see nothing.

conventional model?

Perturbative expansion around Schwarzschild metric is consistent.

It is also consistent with an uneventful horizon.

The uneventful horizon appears to be supported by EFT.

The conventional model appears to be consistent (apart from info loss).

What's wrong?

dynamical black holes

Pei-Ming Ho
National Taiwan University

conventional model?

Assuming

- uneventful horizon
- Schwarzschild approximation
- semi-classical Einstein equation

→ better approximation of near-horizon geometry

- low-energy effective theory

→ non-renormalizable interactions

Q: Will there be high-energy events?

near-horizon geometry

spherical symmetry: $ds^2 = -C(u,v)dudv + r^2(u,v)d\Omega^2$

semi-classical Einstein equation: $G_{\mu\nu} = \kappa\langle T_{\mu\nu}\rangle$

uneventful horizon:

$$\langle T_{uu}\rangle \sim \mathcal{O}(C^2/a^4), \qquad \langle T_{uv}\rangle \sim \mathcal{O}(C/a^4), \qquad \langle T_{vv}\rangle \sim \mathcal{O}(1/a^4)\,.$$

asymptotically Schwarzschild with time-dependent Schwarzschild radius a

at $r - a \gg a/N \gg \mathcal{O}(\ell_p^2/a)$ $\quad (1/N \gg \ell_p^2/a^2)$

near-horizon geometry

[Ho-Matsuo-Yokokura 19*2, Ho-Yokokura 20]

near-horizon region:

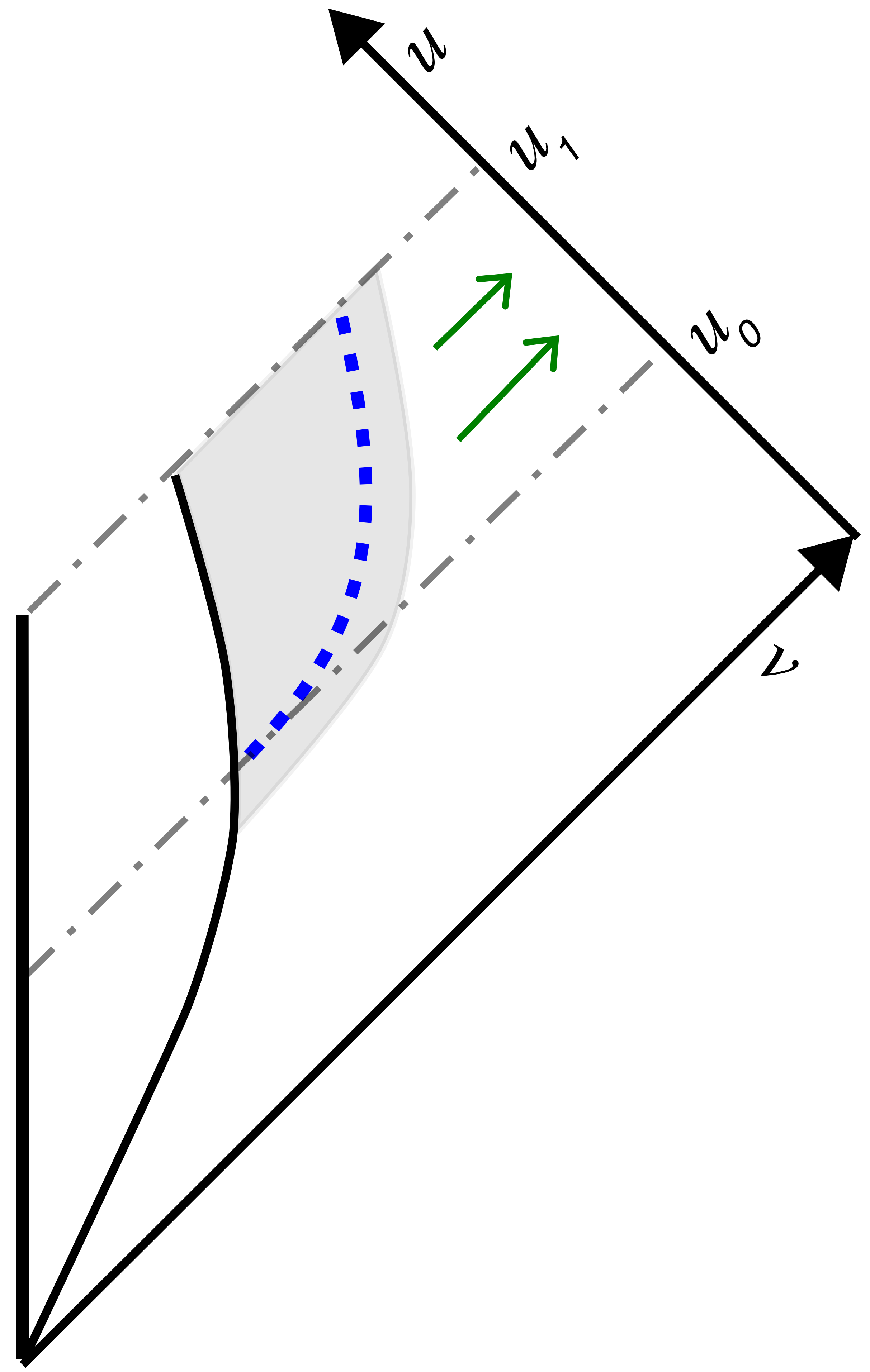
semi-classical Einstein equation ⇒

$$C = \frac{\text{const}}{r} \exp\left[-\int^{u} \frac{du'}{2a(u')} - \int_{v} \frac{dv'}{2\bar{a}(v')}\right](1 + \mathcal{O}(C))$$

$\bar{a}(v)$ = Schwarzschild radius along a constant-v slice

$a(u)$ = Schwarzschild radius along a constant-u slice

$$\frac{da}{du} \sim -\frac{\sigma \ell_p^2}{a^2}, \qquad \frac{d\bar{a}}{dv} \sim -\frac{\bar{\sigma} \ell_p^2}{\bar{a}^2}$$



near-horizon geometry

[Ho-Matsuo-Yokokura 19*2, Ho-Yokokura 20]

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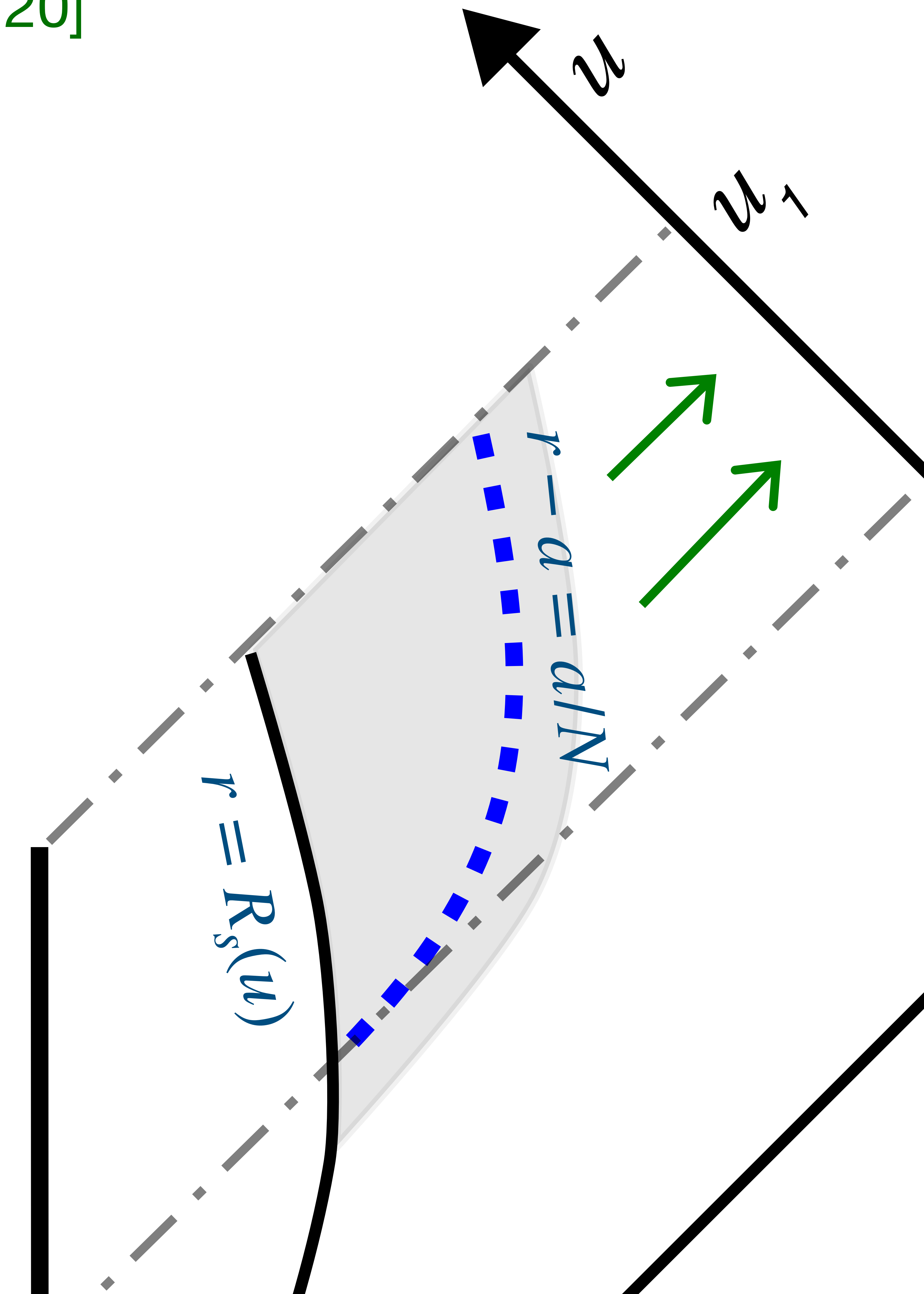


near-horizon geometry

[Ho-Matsuo-Yokokura 19*2, Ho-Yokokura 20]

Expansion in powers of $C_0(u,v)$

$$C(u,v) = C_0(u,v) + \alpha_1(u,v)C_0^2(u,v) + \alpha_2(u,v)C_0^3(u,v) + \cdots,$$

$$r(u,v) = r_0(u,v) + r_1(v)C_0(u,v) + r_2(u,v)C_0^2(u,v) + \cdots$$

with coefficients expanded in powers of $\left(\ell_p^2/a^2\right)$

$$C_0(u,v) \simeq C_* \frac{r_*}{r} \exp\left(-\int_{u_*}^{u} \frac{du'}{2a(u')} - \int_{v}^{v_*} \frac{dv'}{2\bar{a}(v')}\right)$$

$$r_0(v) \simeq \bar{a}(v), \qquad r_1(u,v) \simeq \frac{a(u)\bar{a}(v)}{r_0(v)}$$

near-horizon geometry

Due to the exponential form of $C(u, v) \simeq C_0(u, v)$,

large variation in u, v correspond to tiny proper distance.

When the black hole evaporates to $1/n$ of its initial mass,

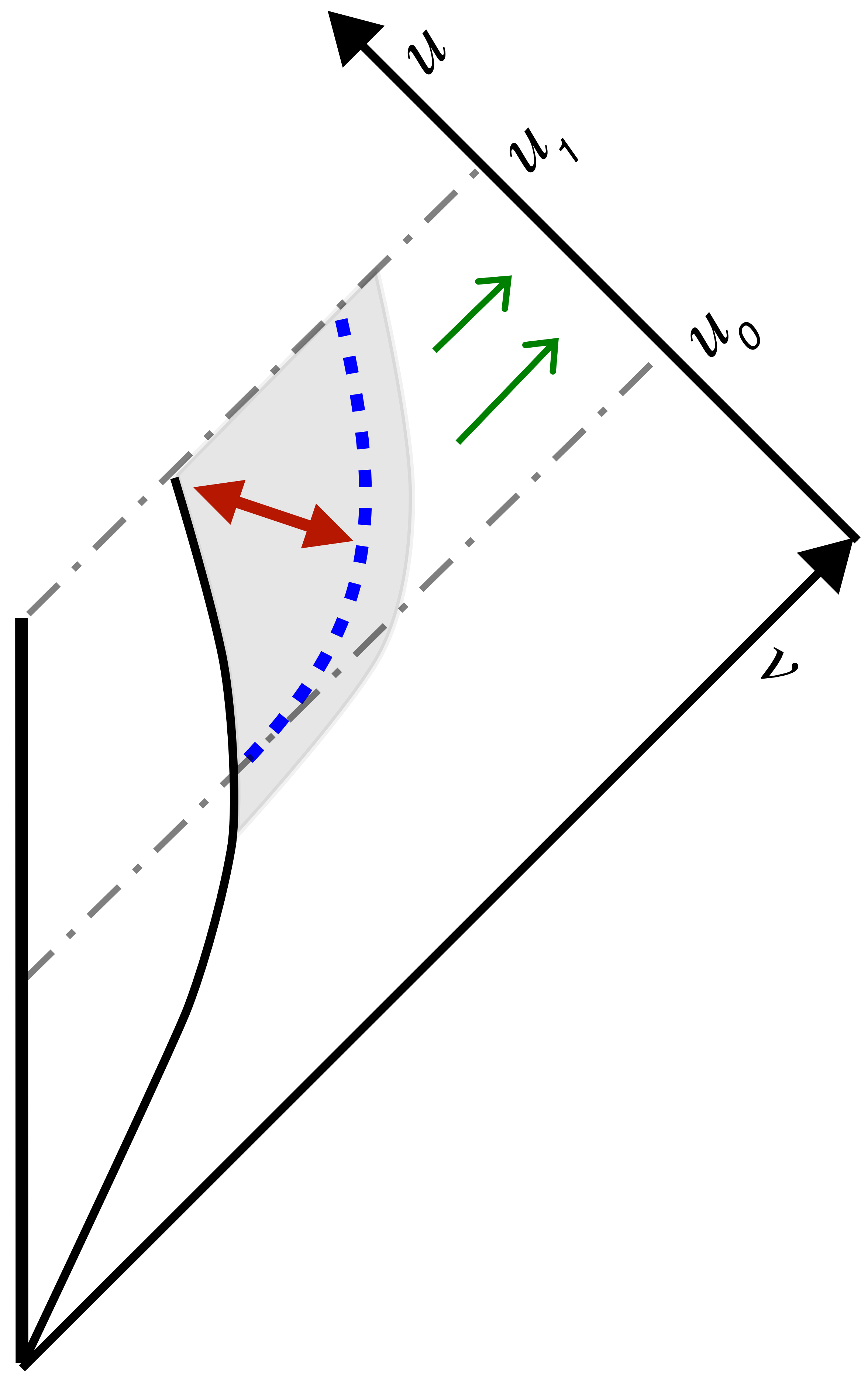
proper distance d btw collapsing matter and trapping horizon:

$$d \lesssim \mathcal{O}(n^{3/2}\ell_p)$$

as long as $n \ll (a/\ell_p)^{2/3}$.

Planck length separation within the early stage of evaporation.

[Ho-Matsuo-Yokokura 19*2, Ho-Yokokura 20]



near-horizon geometry

Lowest order approximation:

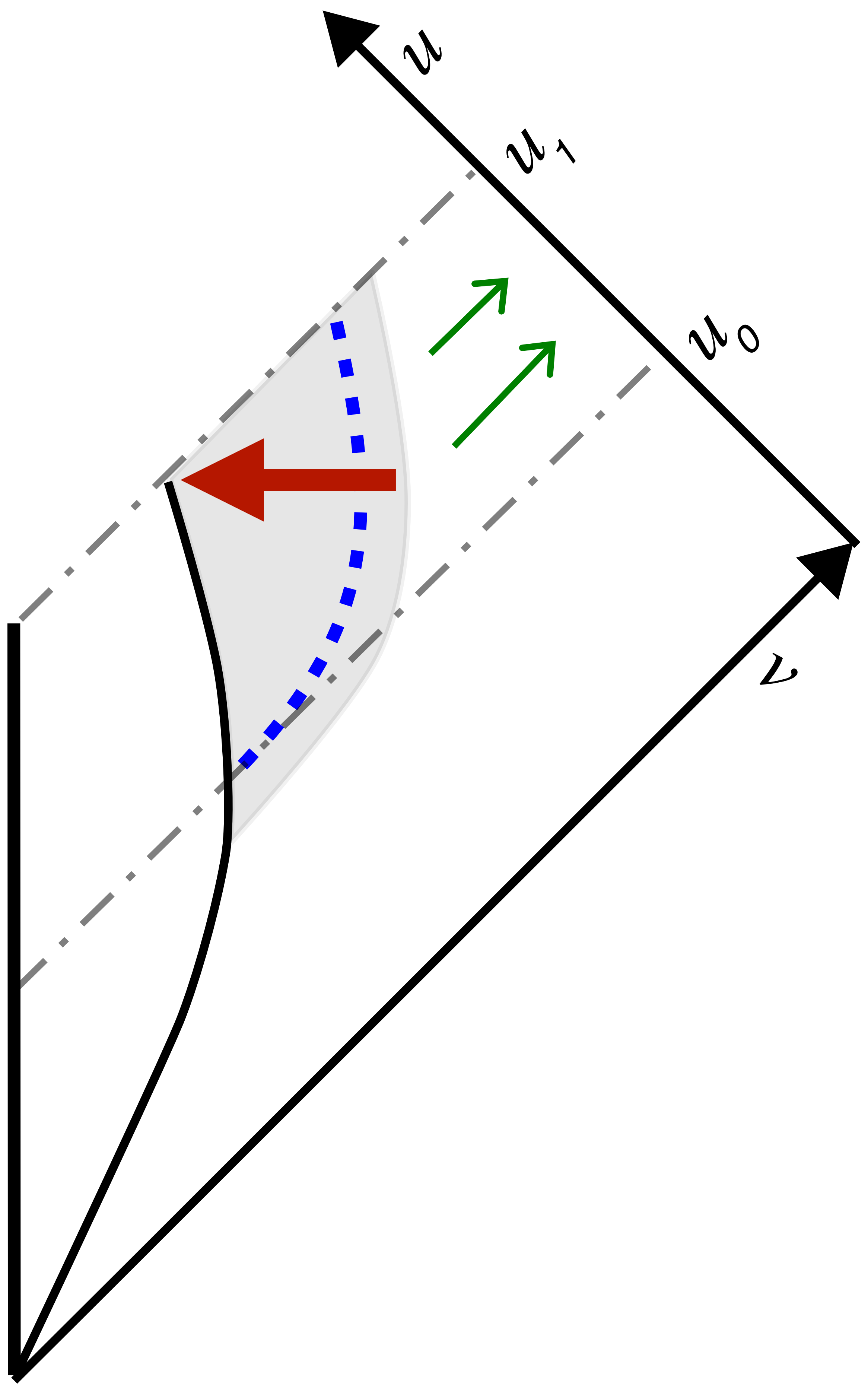
$$C(u,v) \simeq C_* \exp\left(-\frac{u - u_* + v_* - v}{2a}\right),$$

$$r(u,v) \simeq r_0(v) \simeq \bar{a}(v)$$

Exponential factor $\Rightarrow$ Hawking radiation

Dependence of areal radius r on v $\Leftarrow$ $\langle T_{vv} \rangle < 0$

[Ho-Yokokura 20, Ho 20]



interaction of vacuum & matter

$$\nabla^2 \phi = 0 + \lambda \phi^2 + \cdots$$

$$\phi \equiv \frac{\varphi}{r} \quad \Rightarrow \quad \partial_u \partial_v \varphi \simeq \frac{\partial_u \partial_v r}{r} \varphi + \frac{C}{4r^2} \nabla_{\Omega^2} \varphi - \frac{\lambda}{4r} C \varphi^2 \sim \mathcal{O}(C)$$

$\phi^4, \ (\nabla \phi)^2$ have little effect around horizon. [Unruh-Leahy 83, Giddings 06]

Higher-dimensional (non-renormalizable) operators?

effective field theory (EFT)

$$\mathcal{L}_{EFT} = \frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{m^2}{2}\varphi^2 + \frac{\lambda_0}{4}\varphi^4 + \frac{1}{16\pi G_N}\mathcal{R} + \lambda_1\mathcal{R}\varphi^2$$
$$+ \frac{\lambda_2}{M_p^2} g^{\mu_1\nu_1}g^{\mu_2\nu_2}\partial_{\mu_1}\partial_{\nu_1}\varphi\partial_{\mu_2}\partial_{\nu_2}\varphi + \frac{\lambda_3}{M_p^2} g^{\mu_1\nu_1}g^{\mu_2\nu_2}\mathcal{R}_{\mu_1\mu_2}\partial_{\nu_1}\varphi\partial_{\nu_2}\varphi$$
$$+ \cdots$$

All invariant operators included in the $1/M_p$ expansion of UV theory.

Higher-dim. (non-renormalizable) operators are suppressed by $1/M_p^n$.

EFT breaks down if higher-dim. operators are more important.

large curvature invariant? 1

effect of back-reaction:

$\langle T_{vv} \rangle$ induces through Einstein's equation an ingoing geometric deformation

reflected in the areal radius $r \simeq r_0(v)$.

$\Rightarrow$ All curvature invariants $\mathscr{R}$, $\mathscr{R}_{\mu\nu}\mathscr{R}^{\mu\nu}$, $\mathscr{R}_{\mu\nu\lambda\rho}\mathscr{R}^{\mu\nu\lambda\rho}$, … are still small

because a Lorentz boost can make the v-dependence arbitrarily fast or slow.

large curvature invariant?

The dominant effect of back-reaction resides in $r(u,v) \sim r_0(v)$.

However, under a local Lorentz boost

$$u \to u' = \sqrt{\frac{1-\upsilon}{1+\upsilon}}\, u, \qquad v \to v' = \sqrt{\frac{1+\upsilon}{1-\upsilon}}\, v$$

the v-dependence can become arbitrarily weak.

$\Rightarrow$ All curvature invariants are small $\sim \mathcal{O}(a)$.

trans-Planckian problem

$\omega_U = \left(\frac{dU}{du}\right)^{-1} \omega_u$ can be *trans-Planckian* since $\frac{dU}{du} \propto C(u, v)$.

['t Hooft 85]

Due to local Lorentz boosts,

$$u \to u' = \sqrt{\frac{1-\upsilon}{1+\upsilon}}\, u, \qquad U \to U' = \sqrt{\frac{1-\Upsilon}{1+\Upsilon}}\, U,$$

$$v \to v' = \sqrt{\frac{1+\upsilon}{1-\upsilon}}\, v \qquad V \to V' = \sqrt{\frac{1+\Upsilon}{1-\Upsilon}}\, V$$

The frequency ω_u can be arbitrarily large or small.

EFT breaks down if $\left|\omega_u\, g^{uv}\, \omega_v\right| > M_p^2$ in the absence of selection rules.

trans-Planckian problem

Generalizing conditions for the validity of EFT:

$$\left|\omega_u g^{uv}\omega'_v\right| \ll M_p^2 \quad \rightarrow \quad \left|\omega_u g^{vu}\frac{\partial_v f}{f}\right| \ll M_p^2, \quad \rightarrow \quad \left|\omega_u^n (g^{uv})^n \frac{\partial_v^n f}{f}\right| \ll M_P^{2n}$$

EFT is reliable if, for all background fields f and $\bar{f}$,

[Ho-Yokokura 20, Ho 20]

$$\left|\frac{\partial_u^n f}{f}(g^{uv})^n\frac{\partial_v^n \bar{f}}{\bar{f}}\right| \ll M_p^{2n}$$

trans-Planckian problem

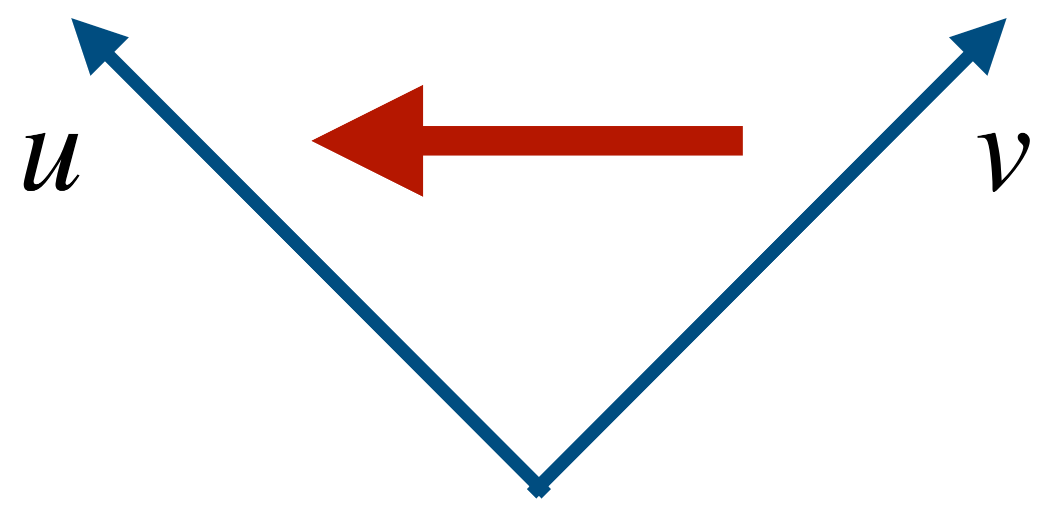
HR: $\omega_u \sim 1/a \quad \Rightarrow \quad \omega_u g^{uv} \frac{\partial_v \bar{a}}{\bar{a}} \sim \frac{1}{a} \frac{1}{C} \frac{\ell_p^2/a^2}{a} \gg M_p^2$

when $C \ll \ell_p^4/a^4$.

$\partial_v \bar{a} = 0$ for Schwarzschild metric

Since $C(u,v) \simeq C_* \exp\left(-\frac{u - u_* + v_* - v}{2a}\right)$,

with a reference point on trapping horizon, $C_* \sim \ell_p^2/a^2$



$$u - u_* + v_* - v > \boxed{2a \log(a^2/\ell_p^2)}$$

scrambling time

[Ho-Yokokura 20, Ho 20]

large invariants

$$g^{\mu_1\nu_1}\cdots g^{\mu_n\nu_n}\left(\nabla_{\mu_1}\cdots\nabla_{\mu_{2n}}\phi\right)\left(\nabla_{\nu_1}\cdots\nabla_{\nu_{n-2}}\mathscr{R}_{\nu_{n-1}\nu_n}\right)\left(\nabla_{\nu_{n+1}}\cdots\nabla_{\nu_{2n-2}}\mathscr{R}_{\nu_{2n-1}\nu_{2n}}\right)$$

$$\longrightarrow \quad \left(g^{uv}\right)^{2n}\left(\nabla_u^{2n}\phi\right)\ \left(\nabla_v^{n-2}\mathscr{R}_{vv}\right)^2$$

$$g^{\mu_1\nu_1}\cdots g^{\mu_{2n}\nu_{2n}}\left(\nabla_{\mu_1}\cdots\nabla_{\mu_{2n}}\phi_1\right)\left(\nabla_{\nu_1}\cdots\nabla_{\nu_n}\phi_2\right)\left(\nabla_{\nu_{n+1}}\cdots\nabla_{\nu_{2n}}\phi_2\right)$$

$$\longrightarrow \quad \left(g^{uv}\right)^{2n}\left(\nabla_u^{2n}\phi_1\right)\ \left(\nabla_v^{n}\phi_2\right)^2$$

$$g^{-n}\left(\nabla^{m_1}\phi\right)\cdots\left(\nabla^{m_s}\phi\right)\ \left(\nabla^{p_1}\mathscr{R}\right)\cdots\left(\nabla^{p_r}\mathscr{R}\right)$$

“minimal” resolution

If decoupling principle fails, physics has no predictability.

(If no DP, maybe there is no HR.)

minimal resolution:

HR is incompatible with **uneventful horizon** in **EFT**.

HR + EFT $\Rightarrow$ high-energy events (and violation of EP)

large transition amplitude

EFT must include the following 3 states: [Ho-Yokokura 20, Ho 20]

$|0\rangle$ Unruh vacuum,

$a^\dagger_{\omega'}|0\rangle$ with $\omega' \to 0$

$c_\omega|0\rangle = \sum_{\omega'} B_{\omega\omega'} a^\dagger_{\omega'}|0\rangle$ because the spectrum of HR is $\langle 0|c^\dagger_\omega c_\omega|0\rangle$.

→ We can rely on EFT to compute the transition amplitude $\langle f|\hat{\mathcal{O}}|i\rangle$ for

$$|i\rangle = |0\rangle \otimes |0\rangle \quad \longrightarrow \quad |f\rangle = c_\omega|0\rangle \otimes a^\dagger_{\omega'}|0\rangle \otimes \cdots$$

due to a higher-derivative interaction $\hat{\mathcal{O}}$ in the EFT.

$$\frac{\lambda}{M_p^{2n+k+1}} \int d^4x \sqrt{-g}\, \langle f | \boxed{g^{\mu_1\nu_1} \cdots g^{\mu_n\nu_n}} \varphi^k \left(\nabla_{\mu_1} \cdots \nabla_{\mu_n} \varphi \right) \left(\nabla_{\nu_1} \cdots \nabla_{\nu_{n-2}} \mathcal{R}_{\nu_{n-1}\nu_n} \right) | i \rangle$$

$$| i \rangle = | 0 \rangle \otimes | 0 \rangle \quad \longrightarrow \quad | f \rangle = c_{\omega_u} | 0 \rangle \otimes | \omega_v^{(1)}, \cdots, \omega_v^{(k)} \rangle$$

$$\to \frac{\lambda}{M_p^{2n+k+1}} \int dudv\, C(u,v) \boxed{C^{-n}(u,v)} \frac{1}{r^k(u,v)} \frac{\omega_u^n}{r(u,v)} e^{i\left[\sum_{i=1}^k \omega_v^{(i)}\right] v + i\omega_u u} \frac{\ell_p^2}{\bar{a}^{n+2}(v)}$$

$$\sim \frac{\lambda}{M_p^{2n+k+1}} \int dudv \left(\frac{\ell_p^2}{\bar{a}^2(v_*)} \boxed{e^{-(u-u_*+v_*-v)/(2\bar{a})}} \right)^{n-1} \frac{1}{\bar{a}^k(v)} \frac{\omega_u^n}{\bar{a}(v)} e^{i\left[\sum_{i=1}^k \omega_v^{(i)}\right] v + i\omega_u u} \frac{\ell_p^2}{\bar{a}^{n+2}(v)}$$

$$\sim \frac{\lambda \ell_p^{4n+k+1}}{\bar{a}^{4n+k+1}} \int dudv \boxed{e^{-(n-1)(u-u_*+v_*-v)/(2\bar{a})}} e^{i\left[\sum_{i=1}^k \omega_v^{(i)}\right] v + i\omega_u u}$$

[Ho-Yokokura 20, Ho 20]

large transition amplitude

Lorentz invariant

energy conservation?

$$\mathscr{M} \sim \left(\frac{\ell_p}{a}\right)^{\#} \int dt dx\, C^{-(n-1-\#)}\, e^{i\omega u + \cdots}$$

$\mathscr{R} \sim 1/a^2, \quad \phi \sim 1/a$

$\sqrt{-g} \sim C, \quad g^{uv} \sim C^{-1}$

$\Rightarrow$ firewall within *scrambling time* $\sim \mathcal{O}(a \log(a/\ell_p))$

(In general, larger amplitude in matter.)

[Ho-Yokokura 20, Ho 20]

higher-derivative effect

$$\nabla_u^n \varphi \quad \longrightarrow \quad \omega_u^n \sim \mathcal{O}(1/a^n), \qquad \nabla_v^n \mathcal{R} \quad \longrightarrow \quad \frac{\ell_p^2}{a^{n+4}}$$

$$\left(g^{uv}\right)^n \quad \longrightarrow \quad C^{-n}(u,v), \qquad C(u,v) \sim \frac{\ell_p^2}{\bar{a}^2(v_*)} \exp\left(-\frac{u-u_*+v_*-v}{2\bar{a}(v_*)}\right)$$

$$\nabla_U^n \varphi \quad \longrightarrow \omega_U^n \sim \left(\frac{dU}{du}\right)^{-n} \omega_u^n, \qquad \nabla_V^n \mathcal{R} \sim \partial_V^n \bar{a}^{-2}(v) \quad \longrightarrow \quad \left(\frac{dV}{dv}\right)^{-n} \frac{\ell_p^2}{a^{n+4}}$$

$$\left(g^{UV}\right)^n \quad \longrightarrow \quad 1, \qquad C(u,v) = \frac{dU}{du}\frac{dV}{dv}$$

What happened? 1

Equivalence principle is violated by higher-derivative interactions.

[Lafrance-Myers 94]

ingoing negative energy flux $\langle T_{vv} \rangle < 0$

→ ingoing deformation of geometry $r(u, v) \simeq \bar{a}(v)$

→ scattering with virtual particles with large frequencies

→ particle creation ("firewall")

saddle point approximation → $\omega_U \sim \left(\frac{dU}{du} \right)^{-1} \omega_u$

[Ho-Yokokura 20, Ho 20]

What happened?

The conventional model is consistent

only for EFT without higher-derivative interactions.

UV theory is not arbitrary:

No global symmetry.

Higher-derivative interactions in EFT.

implication

Hawking radiation and the **uneventful horizon** *cannot* be compatible

for a period of time longer than the **scrambling time**.

After the scrambling time,

EFT is no longer completely reliable.

Depending on the UV theory and the black-hole state,

either (1) Hawking radiation stops,

or (2) Hawking radiation continues with a firewall.

This is a purely EFT conclusion.

UV effect?

“Trans-Planckian problem does not affect Hawking radiation"

[Jacobson 91, Unruh 95, Brout-Massar-Parentani-Spindel 95]

“Trans-Planckian modes do not exist.” “Vacuum stays vacuum.”

Before EFT breaks down, there are created high-energy particles.

Example: a null thin shell stops radiating. [Kawai-Matsuo-Yokokura 13]

Many other possibilities…

Einstein equivalence principle

Einstein Equivalence Principle:

The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the lab's velocity and location.

Apart from the energy of the particles,
the vacuum EMT cannot be measured non-gravitationally.

The Einstein EP restricts particles' EMT,
but not vacuum EMT.

self-consistent models?

1. If Hawking-radiation stops. $\rightarrow$ classical/extremal black holes.

2. If Hawking radiation continues,

   to transfer info from matter to radiation,

   there must be outgoing particles with high-energy scatterings with matter.

   $\rightarrow$ eventful horizon with $\langle T_{uu} \rangle > 0$.

   "eventful horizon"!

   [Kawai-Matsuo-Yokokura 13, Kawai-Yokokura 14, 16, 17]

   The surface of matter may be stringy. [FuzzBall, VECRO]

conclusion

Both Hawking radiation and large transition amplitudes arise

due to the exponential form of $C(u, v)$.

$\rightarrow$ Hawking radiation and uneventful horizon cannot coexist in EFT

for a time scale much longer than the scrambling time.

Depending on the UV theory and the black-hole configuration,

either Hawking radiation stops or “firewall” arises.

(either classical black hole or “drama at horizon”).

Eventful horizon not always incompatible with equivalence principle.

It is possible to have low-energy effective description.

end