QCD in medium: Effective models and lattice QCD

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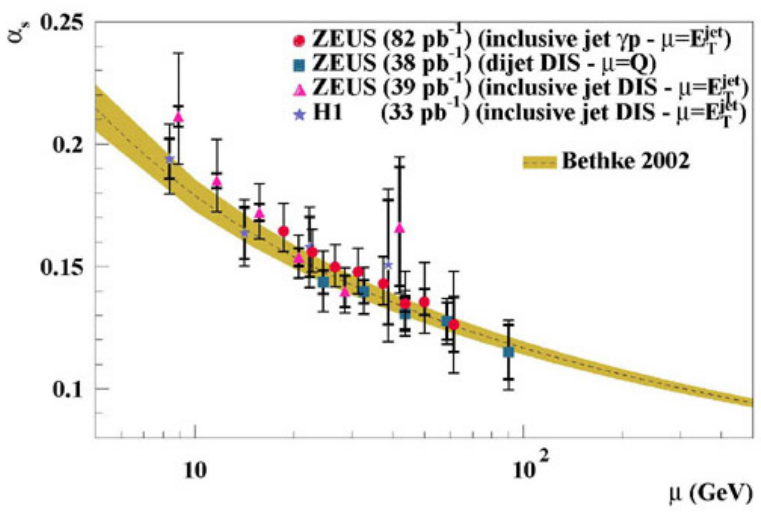
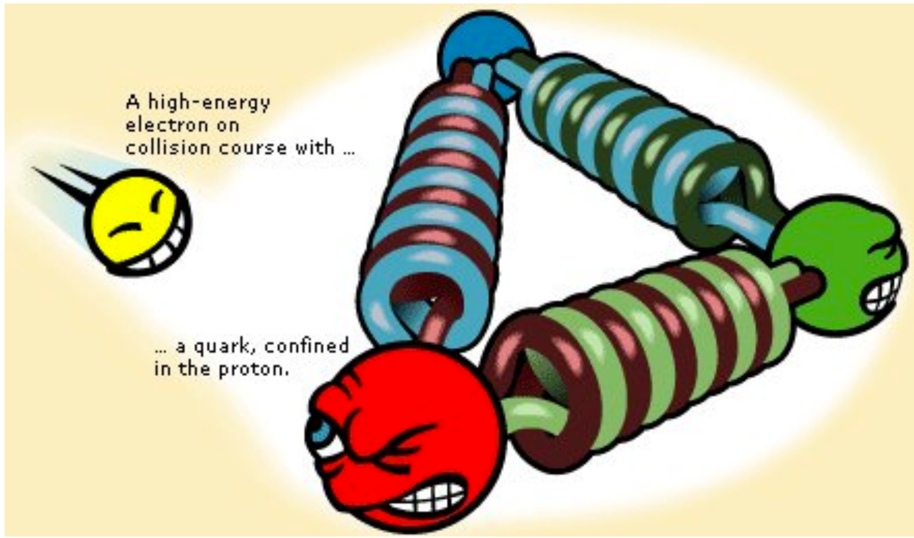
QCD in medium: Effective models

1.Introduction: QCD and effective models

- Strongly interacting particles such as quarks and gluons are governed by QCD

- Nonlinear interactions of quarks and gluons leads to nontrivial feature of QCD: Asymptotic freedom and Confinement





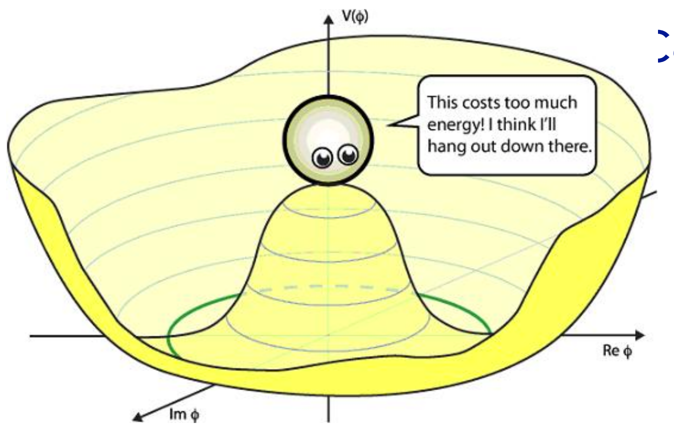
Nonperturbative QCD ← perturbative QCD →

No free quarks observed yet

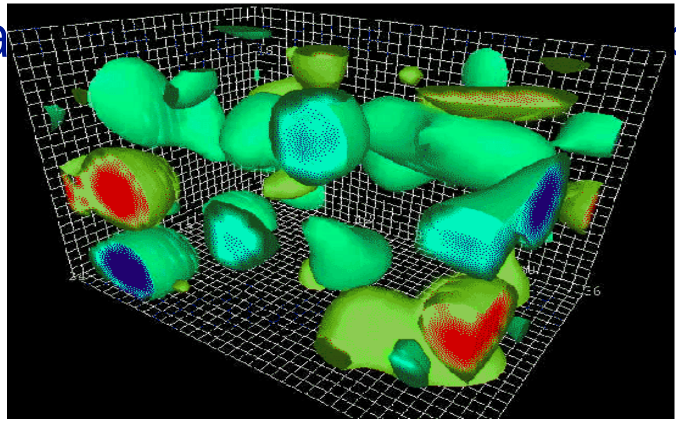
1.Introduction: QCD and effective models

- QCD has accumulated many successful interpretations for hadrons, strongly-interacting vacuum, quark matters, perturbative QCD, etc.

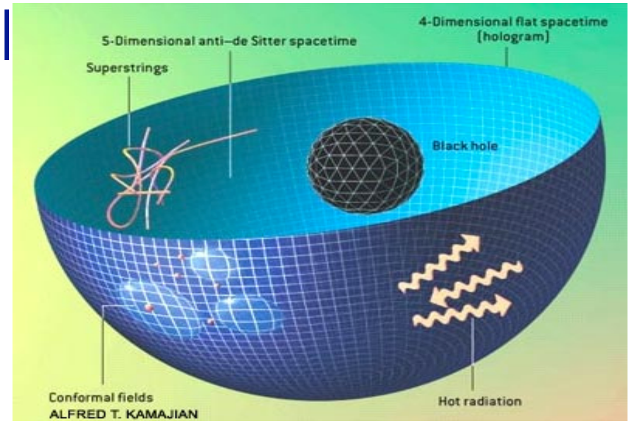
- Insufficient understandings on low-energy QCD:
  Mass gap of YM theory



Relevant symmetries
Effective QCD-like models



Embedding on computer
Lattice QCD



Dualities in space-time
Holographic QCD

1.Introduction: QCD and effective models

- Effective models of QCD based on relevant symmetries and their dynamical breakdowns

- Being motivated by the superconducting theory, Nambu and Jona-Lasinio suggested an effective model of QCD: "NJL model"

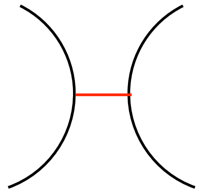
$$L = \bar{\psi}\left[\mathrm{i}\gamma_\mu \partial^\mu - \left(m_c + \eta\right)\right]\psi$$
$$-\delta\eta\bar{\psi}\psi + \delta G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right],$$

- Spontaneous Chiral Symmetry Breaking (SCSB) leads to emergence of pion, dynamical mass for quarks, finite low-energy constant, etc.

1.Introduction: QCD and effective models

According to SCSB, QCD mutates at low-energy region as

Quark-gluon interactions (QCD)



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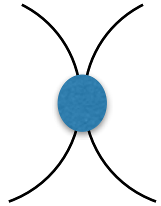
Four-quark (local) interactions (NJL)



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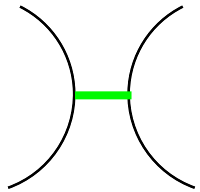
Quark-pion interactions (ChQM)



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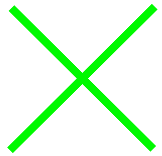
Pure-pion interactions (ChPT)



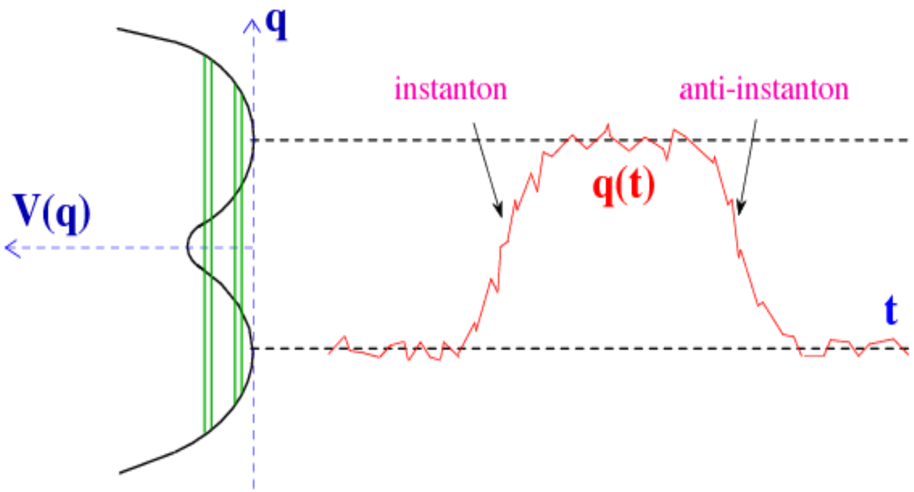
1.Introduction: QCD and effective models

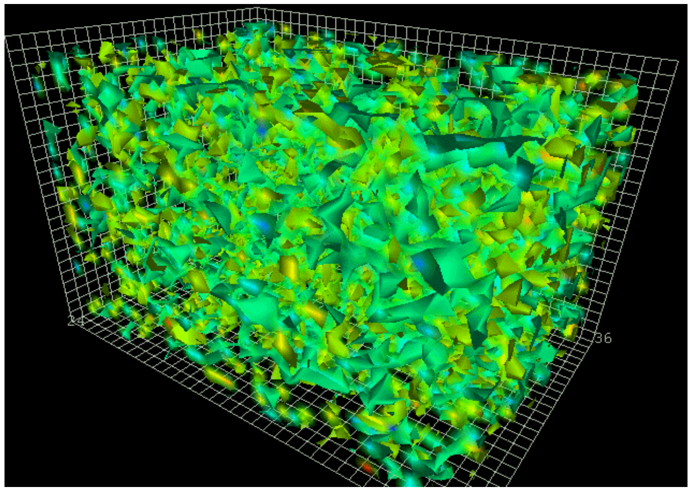
A sophiscated QCD-like model: Liquid-Instanton Model (LIM)

Instanton: A semi-classical solution which minimize YM action

Simpler understanding of instanton: Tunneling path of vacua

Or, instanton is a low-energy effective-nonperturbative gluon



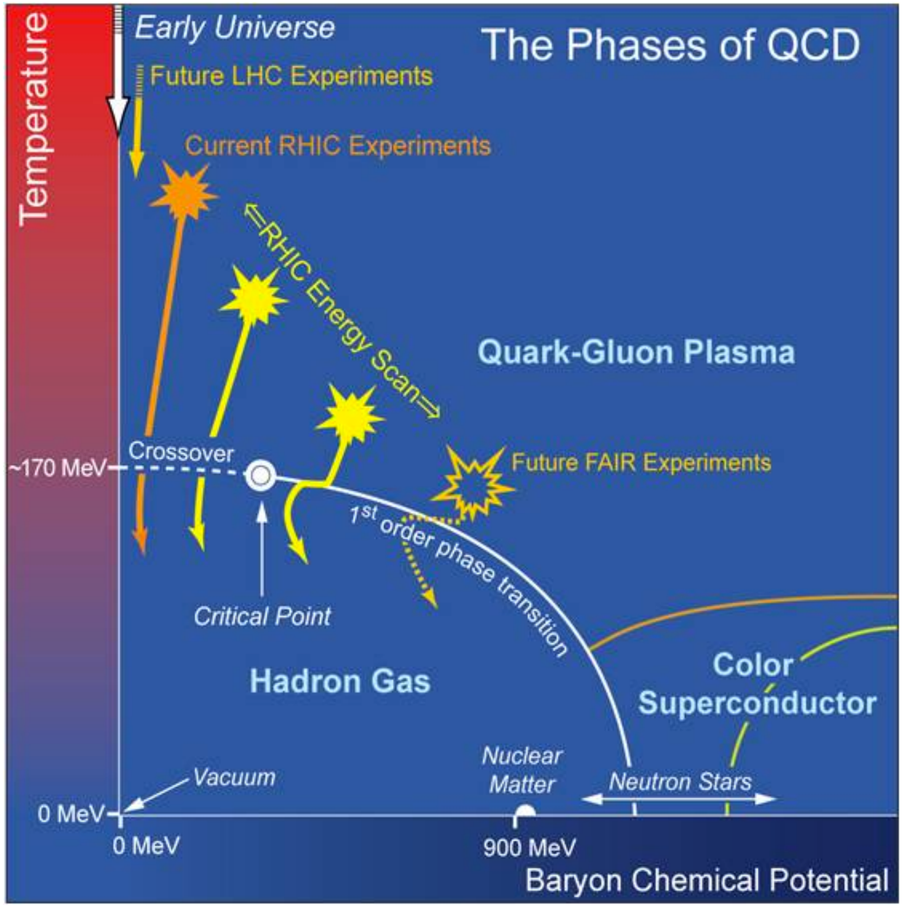


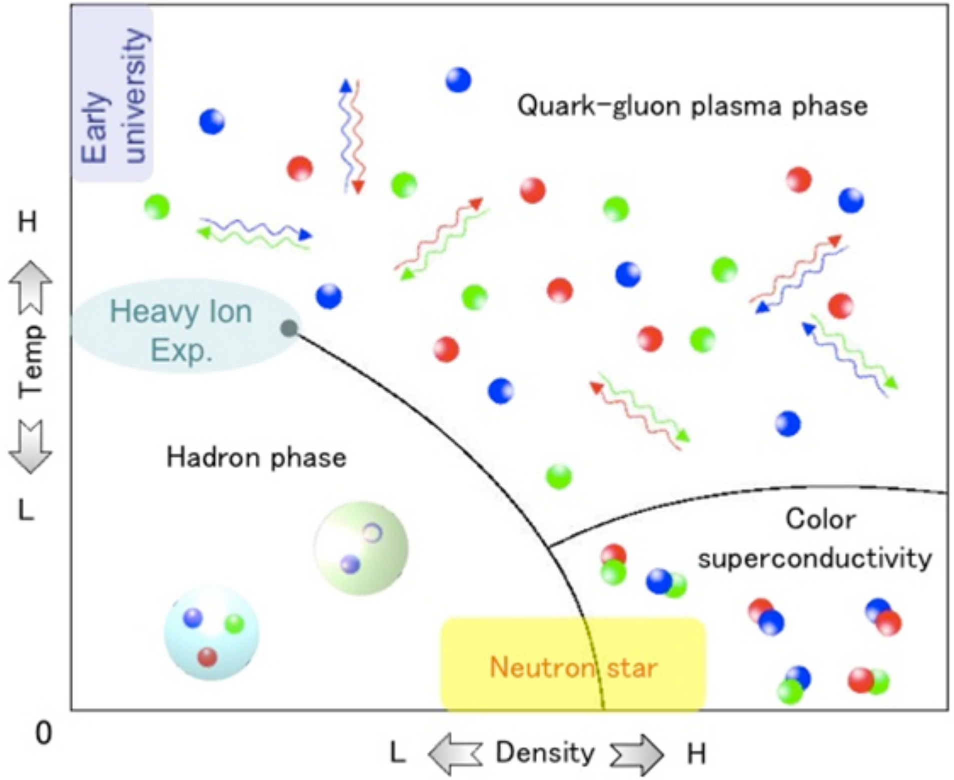
1.Introduction: QCD and effective models

- Instanton interprets well the spontaneous chiral symmetry breaking (SCSB) and U(1) axial anomaly (Witten-Veneziano theorem), etc.
- Technically, it has only two model parameters for light-flavor sector in the large Nc limit: Average instant on size & inter-instanton distance
- Unfortunately, there is NO confinement!!!
- Some suggestions for the confinement with instanton physics: Dyon, nontrivial-holonomy caloron, etc.
- It has been believed that confinement is not so relevant in ground-state hadron spectra, in contrast to resonances, Regge behavior, Hagedorn spectrum, etc.

2. QCD at extreme conditions

QCD has complicated phase structure
as a function of temperature and density





2. QCD at extreme conditions

I. Each QCD phases defined by its own order parameters

II. Behavior of order parameters governed by dynamics of symmetry

III. Symmetry and its breakdown governed by vacuum structure

Chiral symmetry ⬌ Quark (chiral) condensate: Hadron or not?

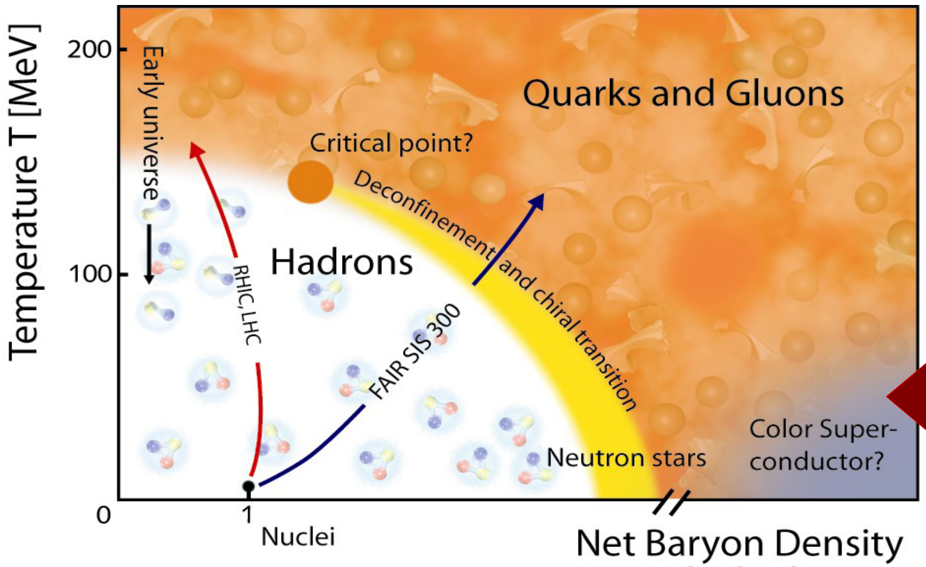
Center symmetry ⬌ VEV of Polyakov loop: Confined or not?

Color symmetry ⬌ Diquark condensate: Superconducting or not?

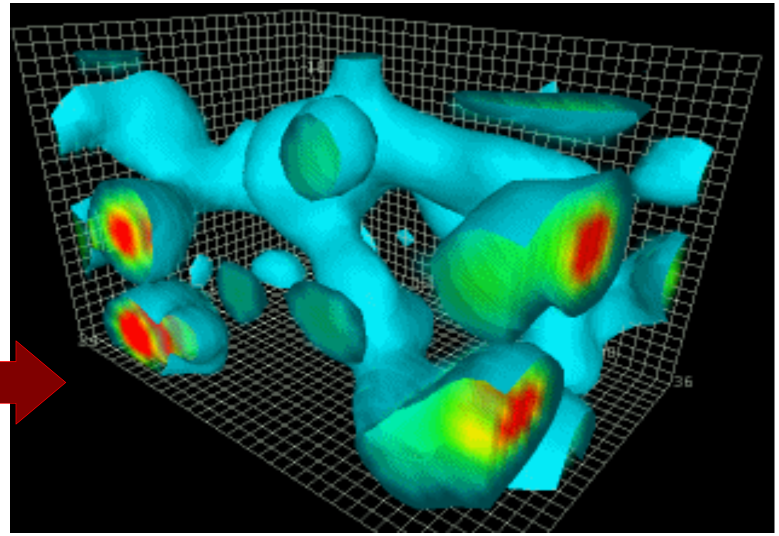
Color-flavor symmetry (locking) ⬌ Diquark condensate at high density

QCD phase ⬌ Symmetries of QCD ⬌ QCD vacuum

Why are heavy-ion collision experiments special for QCD?



Phase structure of QCD



Nontrivial QCD vacuum

Color symmetry

C/P/T-invariance

Z(N) center symmetry

Chiral symmetry

Flavor symmetry

Symmetries and their breakdown

2. QCD at extreme conditions

- SCSB results in nonzero chiral (quark) condensate due to nonzero effective quark mass even in the chiral limit, i.e. m=0

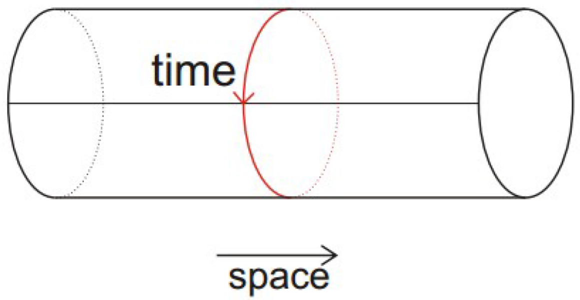
$$-\langle\bar{\psi}\psi\rangle_{\text{Mink}} = i\langle\psi^\dagger\psi\rangle_{\text{Eucl}} = 4N_c \int \frac{d^4p}{(2\pi)^4} \frac{M(p)}{p^2 + M^2(p)}$$

- Nonzero <qq> indicates hadron (Nambu-Goldstone) phase, whereas
zero <qq> does non-hadronic phase, not meaning deconfinement
- Thus, <qq> is an order parameter for chiral symmetry
- In the real world with nonzero quark current mass ~ 5 MeV, at low density, there appears crossover near T ~ 0, and it becomes 1st-order phase transition as density increases
- In the vicinity of critical density, there are various and complicated phases, such as color-superconducting, quarkyonic phase, etc.

2. QCD at extreme conditions

Dynamical (spontaneous) breakdown of center symmetry results in nonzero Polyakov-loop condensate <L>

$$
\begin{aligned}
e^{-\beta F} &= Tr[e^{-H/T}] = \sum_n \langle n | e^{-\tau \cdot H} | n \rangle_{\tau=\beta=1/T} = \\
&= \sum_n e^{-\beta E_n} = \\
&= \sum_\psi \sum_U e^{-S_{FG}} Tr\, \psi_\tau^+ U_\tau U \ldots U_0 \psi_0 = \\
&= \sum_U e^{-S_G} Tr[UU \ldots U]_{0\tau}
\end{aligned}
$$



$$\sum_U e^{-S_G} \underbrace{Tr[UU \ldots U]_{0\tau}}_{\langle Tr\, P e^{ig\int_0^\beta A_0(\vec{x}) d\tau} \rangle_G} \rightarrow \langle L(\vec{x}) \rangle$$

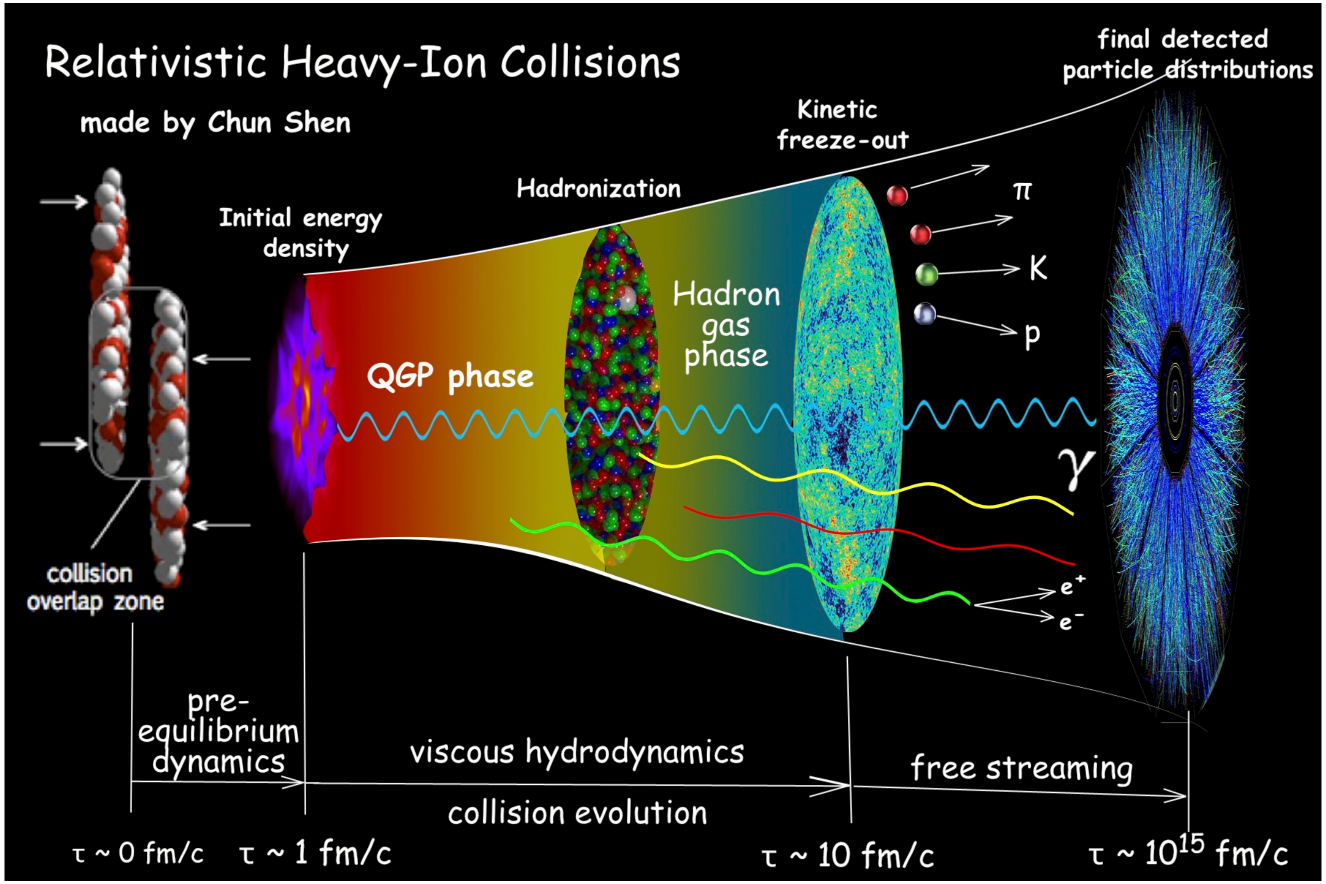
$S_{o\tau} = \psi_\tau^+ U_\tau U \ldots U_0 \psi_0$: quark propagator $0 \rightarrow \tau$

Considering Exp[-F/T]~<L>, where F is quark free energy, "<L>=0" means that F is infinity, so that quarks are confined

If <L> nonzero, F is finite to separate the quarks apart, i.e. deconfined

2. QCD at extreme conditions

Heavy-ion collision (HIC) experiments enable us to investigate hot and dense QCD matter ~ early Universe



2. QCD at extreme conditions

Theory can help to understand HIC experiments

Equation of state of QCD matter: Lattice QCD, Effective models

Evolution of QGP: (Viscous) Hydrodynamics

Hadronization: Transport models

We want to focus on the following subjects:
Critical behaviors, transport coefficients, Effects of external B fields…

For this purpose, we want to modify the effective models in terms of temperature (as well as density)

Polyakov-loop NJL model & T-modified LIM

3. Medium-modified Effective models

- We start from the effective Lagrangian of NJL, resulting in effective thermodynamic potential Ω, which gives EoS of QCD matter

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - \underline{m})\psi + G\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right)$$

- We expand the four-quark interaction in terms of SBCS

$$\bar{\psi}\psi = \langle\bar{\psi}\psi\rangle_{NJL} + \delta(\bar{\psi}\psi)$$

- Finite chiral condensate considered as an effective quark mass

$$M = m - 2G\langle\bar{\psi}\psi\rangle_{NJL}$$

- Finally, we arrive at an effective Lagrangian manifesting SBCS

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - M\right)\psi - \frac{(M-m)^2}{4G}$$

Constant potential via SBCS

Free quark with effective mass M

3. Medium-modified Effective models

- Employing *Matsubara formula* to convert the action S ~ [∫ d^{4}x Lagrangian] into thermodynamic potential

$$i\int \frac{d^4k}{(2\pi)^4}\, f(k) \longrightarrow -T\sum_n \int \frac{d^3k}{(2\pi)^3}\, f(i\omega_n+\mu,\vec{k})$$

with fermionic Matsubara frequencies $\omega_n = (2n+1)\pi T$

- We arrive at an effective thermodynamic potential

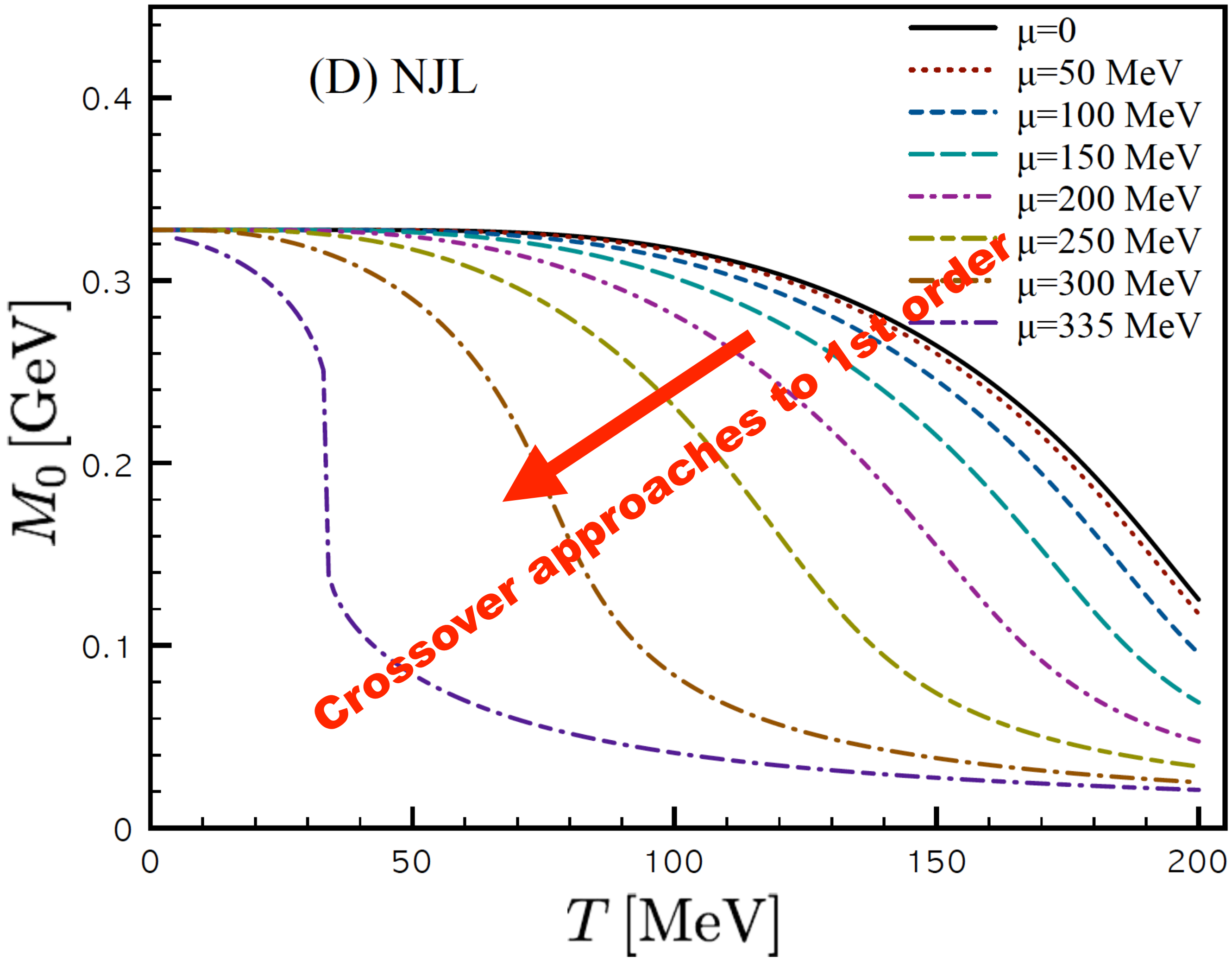
$$\Omega_{\rm NJL} = \frac{(M_0-m_q)^2}{4G} - 2N_cN_f\int_0^\Lambda \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left\{E_{\boldsymbol{k}0} + T\ln\left[\left(1+e^{-\frac{E_{\boldsymbol{k}0}-\mu}{T}}\right)\left(1+e^{-\frac{E_{\boldsymbol{k}0}+\mu}{T}}\right)\right]\right\}$$

- Computing gap equation, giving phase diagram for SBCS

$$\frac{\partial\Omega_{\rm NJL}}{\partial M_0} = \frac{M_0-m_q}{2G} - 2N_cN_f\int_0^\Lambda \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{M_0}{E_{\boldsymbol{k}0}}\left[1-\frac{e^{-\frac{E_{\boldsymbol{k}0}-\mu}{T}}}{1+e^{-\frac{E_{\boldsymbol{k}0}-\mu}{T}}}-\frac{e^{-\frac{E_{\boldsymbol{k}0}+\mu}{T}}}{1+e^{-\frac{E_{\boldsymbol{k}0}+\mu}{T}}}\right]=0$$

3. Medium-modified Effective models

QCD phase diagram as a function of T and μvia NJL model



3. Medium-modified Effective models

- K. Fukushima develop a modified NJL with Polyakov loop, i.e **pNJL**

- Identifying the imaginary quark chemical potential as Polyakov line,

$$\Omega/V = V_{\text{glue}}[L] + \frac{1}{2G}(M - m_q)^2 - 2N_c N_f \int \frac{d^3p}{(2\pi)^3} \Big\{ E_p + T\frac{1}{N_c} \times \mathrm{Tr_c} \ln\big[1 + L e^{-(E_p-\mu)/T}\big] + T\frac{1}{N_c}\mathrm{Tr_c}\ln\big[1 + L^\dagger e^{-(E_p+\mu)/T}\big]\Big\},$$

$$L(\vec{x}) = \mathcal{T}\exp\Big[-i\int_0^\beta dx_4\, A_4(x_4, \vec{x})\Big]$$

$$V_{\text{glue}}[L] \cdot a^3/T = -2(d-1)e^{-\sigma a/T}|\mathrm{Tr_c}\, L|^2 - \ln\big[-|\mathrm{Tr_c}\, L|^4 + 8\,\mathrm{Re}(\mathrm{Tr_c}\, L)^3 - 18|\mathrm{Tr_c}\, L|^2 + 27\big]$$

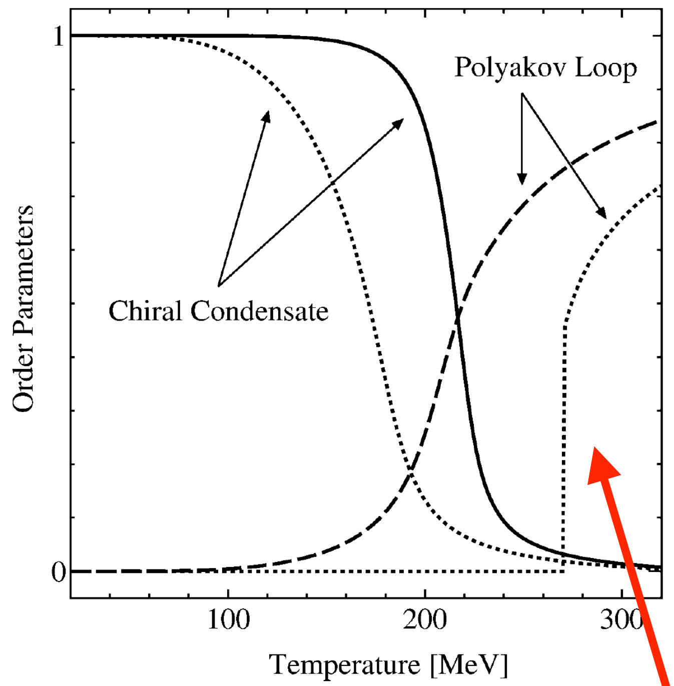
- V_{glue}[L] constructed by $Z(N_c)$ symmetry and lattice QCD information

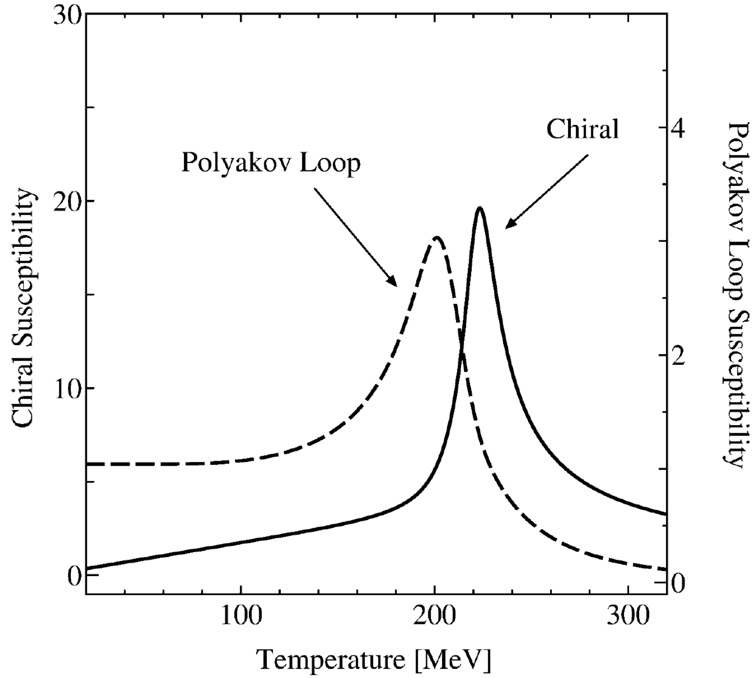
$$\Omega^{\phi}_{\text{eff}} = -T^4\left[\frac{b_2(T)}{2}(\phi\,\phi^*) + \frac{b_3}{6}(\phi^3 + \phi^{*3}) - \frac{b_4}{4}(\phi\,\phi^*)^2\right]$$

$$b_2(T) = a_0 + a_1\left[\frac{T_0}{T}\right] + a_2\left[\frac{T_0}{T}\right]^2 + a_3\left[\frac{T_0}{T}\right]^3$$

3. Medium-modified Effective models

Realization of simultaneous crossover of chiral and deconfinement phase transitions





Due to quark-L interaction, ⟨L⟩ shows crossover, rather than 1st order in pure-glue theory

3. Medium-modified Effective models

- T-modified LIM:(mLIM) Instanton parameters are modified with trivial-holonomy caloron solution (Not dyon, vortex, or something)

- Caloron is an instanton solution for periodic in Euclidean time, i.e temperature, but no confinement

- Distribution func. via trivial-holonomy (Harrington-Shepard) caloron

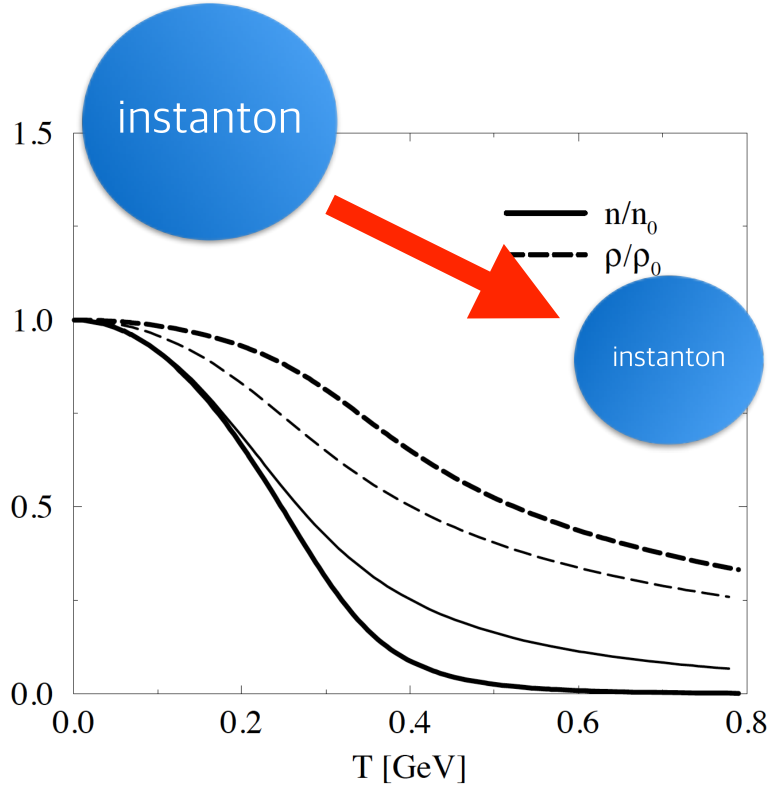
$$d(\rho,T) = \mathcal{C}\,\rho^{b-5}\exp\left[-\mathcal{F}(T)\rho^2\right], \quad \mathcal{F}(T) = \frac{1}{2}A_{N_c}T^2 + \left[\frac{1}{4}A^2_{N_c}T^4 + \nu\bar{\beta}\gamma n\right]^{\frac{1}{2}}$$

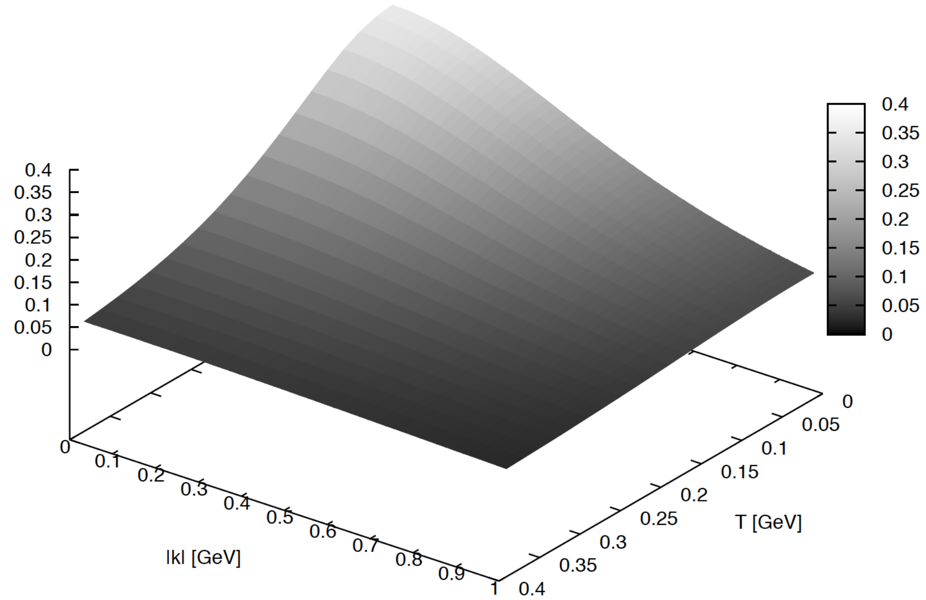
$$A_{N_c} = \frac{1}{3}\left[\frac{11}{6}N_c - 1\right]\pi^2, \quad \gamma = \frac{27}{4}\left[\frac{N_c}{N_c^2-1}\right]\pi^2, \quad b = \frac{11N_c - 2N_f}{3}.$$

- Using this, we modify the two instanton parameters as functions of T

3. Medium-modified Effective models

mLIM parameters (left) and effective quark mass M (right)



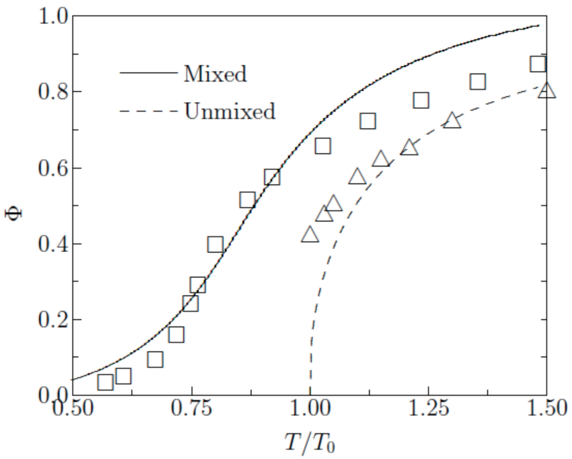


Hence, effective quark mass plays the role of UV regulato

3. Medium-modified Effective models

Finally, we arrive at an effective thermodynamic potential
via instanton and Polyakov loop

$$\begin{aligned}\Omega_{\text{eff}} &= \Omega_{\text{eff}}^{\text{q}+\Phi} + \Omega_{\text{eff}}^{\Phi} = 2\sigma^2 - 2N_f\left[N_c\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}E_{\boldsymbol{k},T}\right. \\ &+ T\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\ln\left[1+N_c\left(\Phi+\bar{\Phi}\,e^{-\frac{E_{\boldsymbol{k},T}}{T}}\right)e^{-\frac{E_{\boldsymbol{k},T}}{T}}+e^{-\frac{3E_{\boldsymbol{k},T}}{T}}\right] \\ &+ \left. T\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\ln\left[1+N_c\left(\bar{\Phi}+\Phi\,e^{-\frac{E_{\boldsymbol{k},T}}{T}}\right)e^{-\frac{E_{\boldsymbol{k},T}}{T}}+e^{-\frac{3E_{\boldsymbol{k},T}}{T}}\right]\right] \\ &- T^4\left[\frac{b_2(T)}{2}(\Phi\,\bar{\Phi})+\frac{b_3}{6}(\Phi^3+\bar{\Phi}^3)-\frac{b_4}{4}(\Phi\,\bar{\Phi})^2\right],\end{aligned}$$



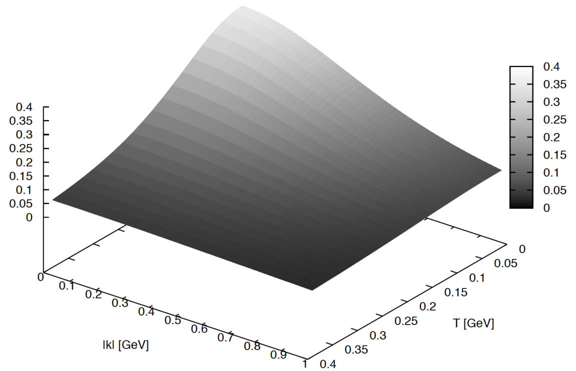
Basically, we have similar results with pNJL results

In detail, positions for critical T and ρ , structure of phase shift, etc.
are different quantitatively

Medium-modified Effective model

Momentum-dependent effective quark mass

$$M = M_0(\mu, T)\left[\frac{2}{2+\bar{\rho}^2\,\boldsymbol{k}^2}\right]^{\mathcal{N}}$$



Gap (saddle-point) equations for LIM and NJL

$$\frac{NN_f}{VM_0} = 2N_cN_f\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{(m+M)F^{\mathcal{N}}}{E}\left[\frac{(1-XY)}{(1+X)(1+Y)}\right],$$
$$\frac{\mathcal{M}-m}{2G} = 2N_cN_f\int^{\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{(m+\mathcal{M})}{\mathcal{E}}\left[\frac{1-\mathcal{X}\mathcal{Y}}{(1+\mathcal{X})(1+\mathcal{Y})}\right],$$

Parameterization of instanton packing fraction in medium

$$\frac{N}{V}\to\frac{N}{V}\left[\frac{M_0}{M_{0,\mathrm{vac.}}}\right]^2$$

Medium-modified Effective model

Standard representations for thermodynamic properties of QCD matter

$$p(T,\mu) = -(\Omega - \Omega_{\text{vac.}}), \;\; n(T,\mu) = -\frac{\partial \Omega}{\partial \mu},$$

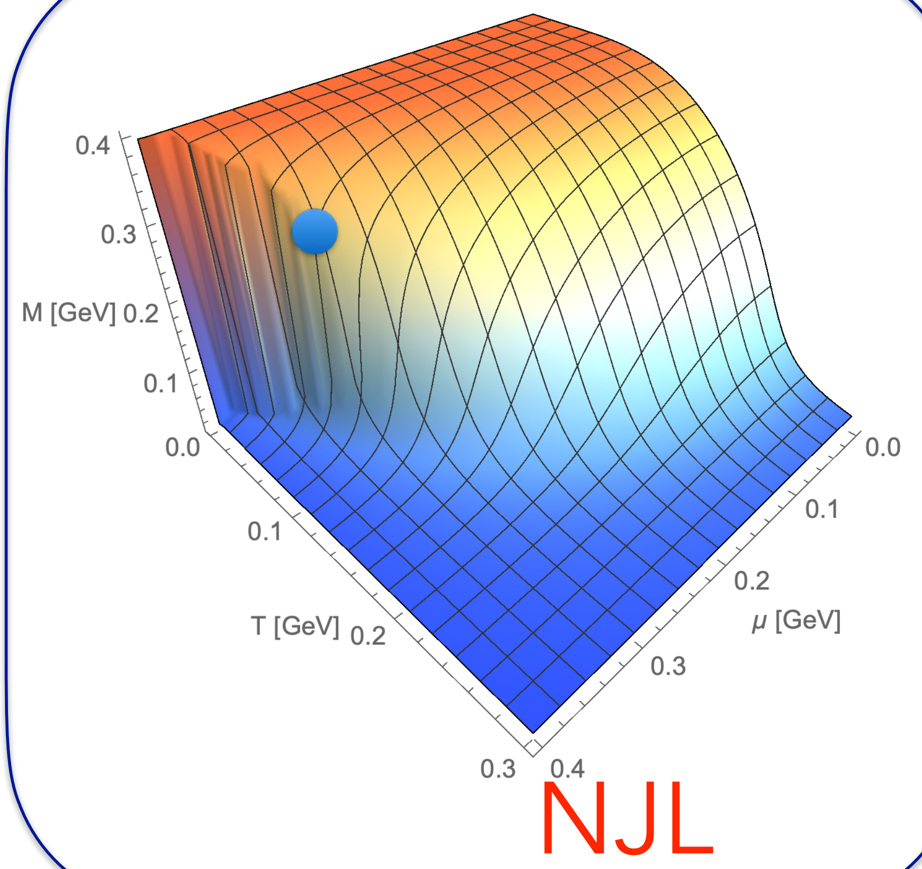
$$s(T,\mu) = -\frac{\partial \Omega}{\partial T}, \;\; \epsilon(T,\mu) = T\, s(T,\mu) + \mu\, n(T,\mu) - p(T,\mu),$$

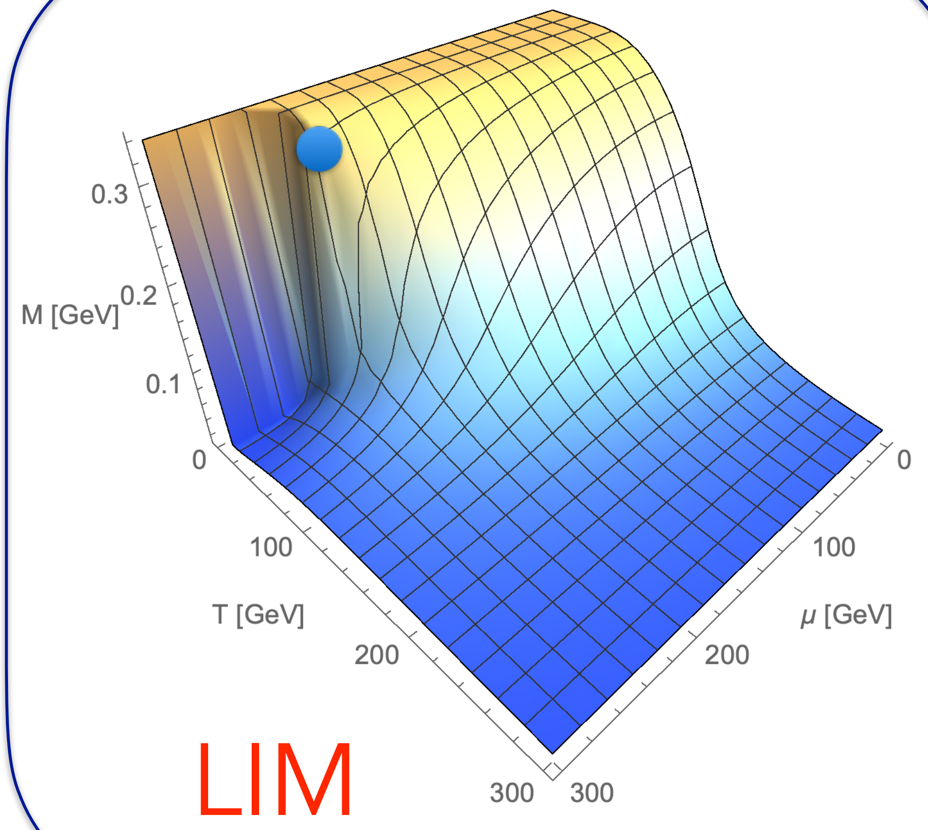
Thermodynamic properties of QCD matter for LIM and NJL

$$\begin{aligned}
p_{\text{NJL}} &= -(\Omega^{\text{NJL}}_{\text{eff}} - \Omega^{\text{NJL}}_{\text{eff,vac.}}), \\
n_{\text{NJL}} &= 2N_f N_c \int^{\Lambda} \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[\frac{\mathcal{E}(\mathcal{Y}-\mathcal{X}) + (1-\mathcal{X}\mathcal{Y})\mathcal{M}\mathcal{M}^{(\mu)}}{\mathcal{E}(1+\mathcal{X})(1+\mathcal{Y})} \right] - \frac{(\mathcal{M}-m)\mathcal{M}^{(\mu)}}{2G}, \\
s_{\text{NJL}} &= 2N_f N_c \int^{\Lambda} \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[\ln\left[(1+\mathcal{X})(1+\mathcal{Y})\right] + \frac{\mathcal{E}[\mathcal{E}_-(1+\mathcal{X})\mathcal{Y} + \mathcal{E}_+(1+\mathcal{Y})\mathcal{X}] + T(1-\mathcal{X}\mathcal{Y})\mathcal{M}\mathcal{M}^{(T)}}{\mathcal{E}T(1+\mathcal{X})(1+\mathcal{Y})} \right] \\
&- \frac{(\mathcal{M}-m)\mathcal{M}^{(T)}}{2G}.
\end{aligned}$$

$$\begin{aligned}
p_{\text{LIM}} &= -(\Omega^{\text{LIM}}_{\text{eff}} - \Omega^{\text{LIM}}_{\text{eff,vac.}}), \\
n_{\text{LIM}} &= 2N_f N_c \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[\frac{E(Y-X) + (1-XY)MM^{\mu}}{E(1+X)(1+Y)} \right] - \frac{2M_0 M_0^{\mu}}{M^2_{0,\text{vac.}}} \frac{N}{V} \\
s_{\text{LIM}} &= 2N_f N_c \int^{\Lambda} \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[\ln\left[(1+X)(1+Y)\right] + \frac{E[E_-(1+X)Y + E_+(1+Y)X] + T(1-XY)MM^{(T)}}{ET(1+X)(1+Y)} \right] - \frac{2M_0 M_0^{(T)}}{M^2_{0,\text{vac.}}} \frac{N}{V}
\end{aligned}$$

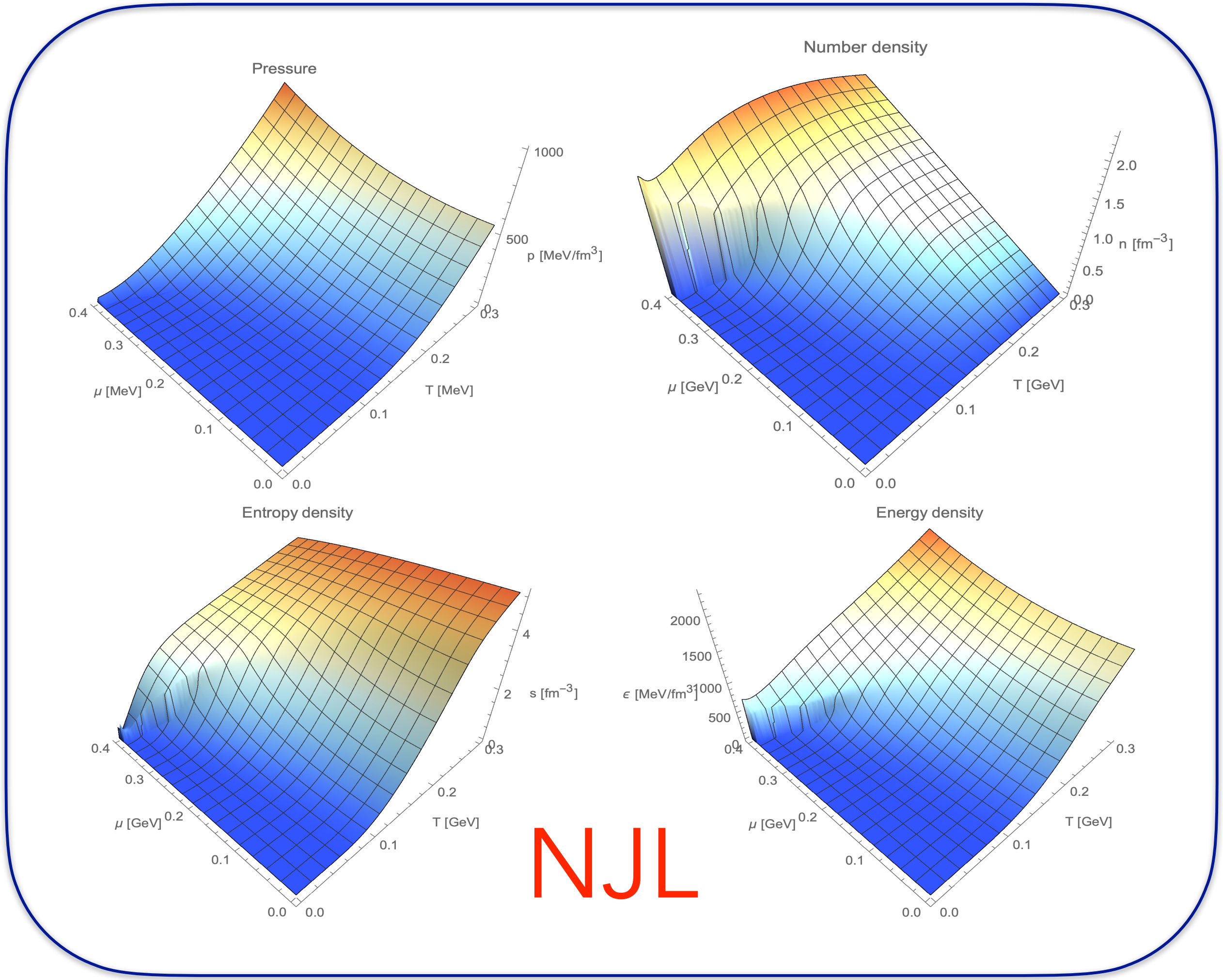
Thermodynamic properties: NJL vs. LIM

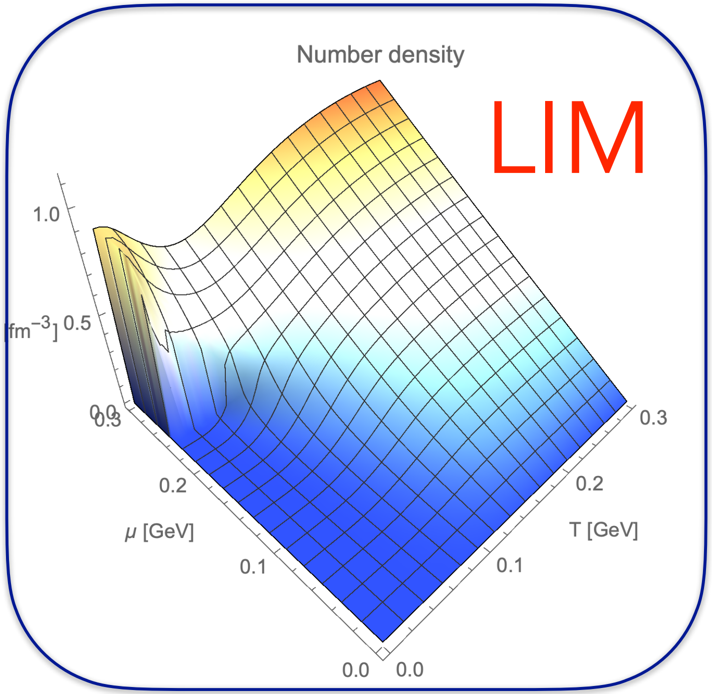




Chiral phase diagram via effective quark mass

Thermodynamic properties: NJL vs. LIM





4. Some numerical results

- Interesting subjects in hot and dense QCD (QGP) in terms of the strongly interacting quark-gluon matter

  1. Phase structure: Where are CEP and TCP?
  2. Effects of external magnetic fields: CME, CMS
  3. Transport coefficients: Viscosities, conductivities, etc.
  4. Contributions from flavors, colors, axial anomaly
  5. Various current-current correlators: Jet-quenching parameter
  6. LEC in color fields
  7...

- Very rapidly developing fields
- Much relations with lattice QCD community
- Still huge amounts of research subjects waiting for you!

4. Some numerical results

This time, I focus on

Transport coefficients under external magnetic fields

QGP and Transport coefficients:

Recent heavy-ion collision experiment showed possible evidence of QGP

Interpreted well by hydrodynamics with small viscosity ~ perfect fluid

Properties of QGP can be understood by transport coefficients:
Bulk and sheer viscosities, electrical conductivity, and so on

They can be studied using Kubo formula via linear response theory

QGP and transport coefficients

- Recent heavy-ion collision experiment showed possible evidence of QGP
- Interpreted well by hydrodynamics with small viscosity: ~ perfect fluid

J. Adams et al. [STAR Collaboration], Nucl. Phys. A, 102 (2005)

- Properties of QGP can be understood by transport coefficients:

Bulk and sheer viscosities, electrical conductivity, and so on

- They can be studied using Kubo formulae via linear response theory

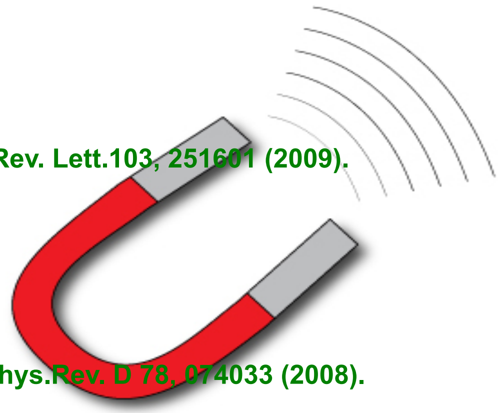
F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B 663, 217 (2008).

Strong magnetic (B) field in QGP

- RHIC experiments observed strong B field ~ (pion mass)^2
- Strong B field modify nontrivial QCD vacuum structure

B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett.103, 251601 (2009).

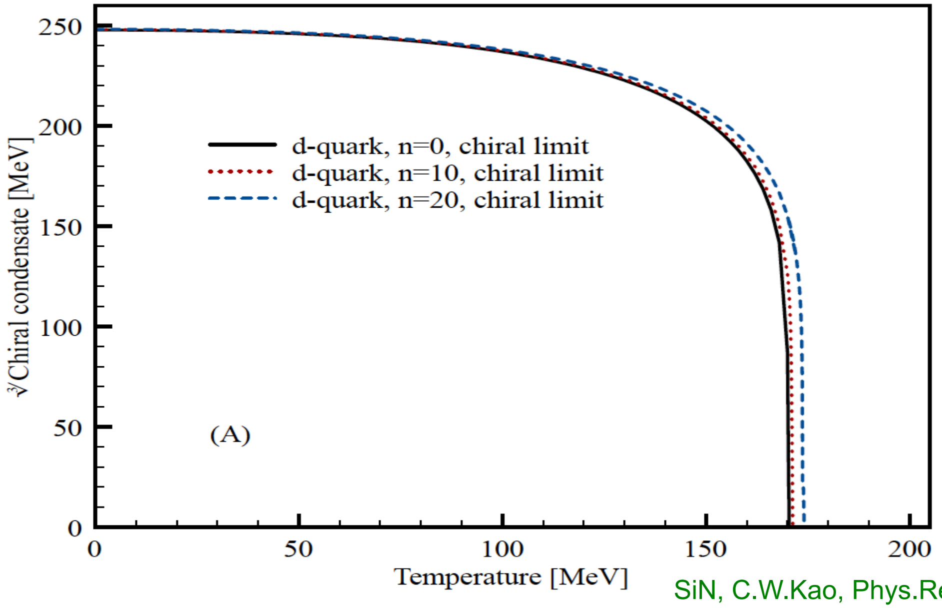
- Charged-current asymmetry: *Chiral magnetic effect (wave)*
- B field enhances SBCS: *Magnetic catalysis*

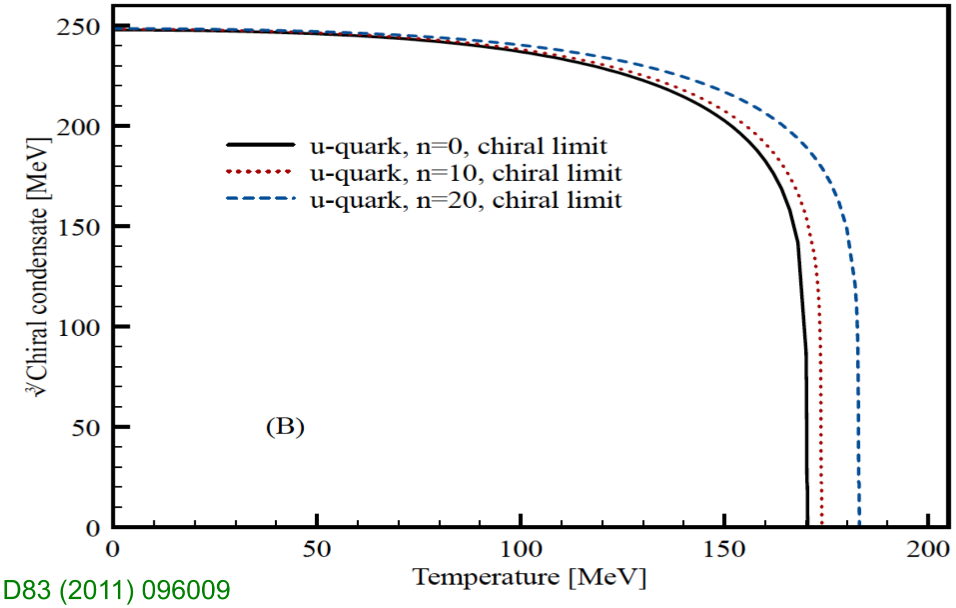


K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008).

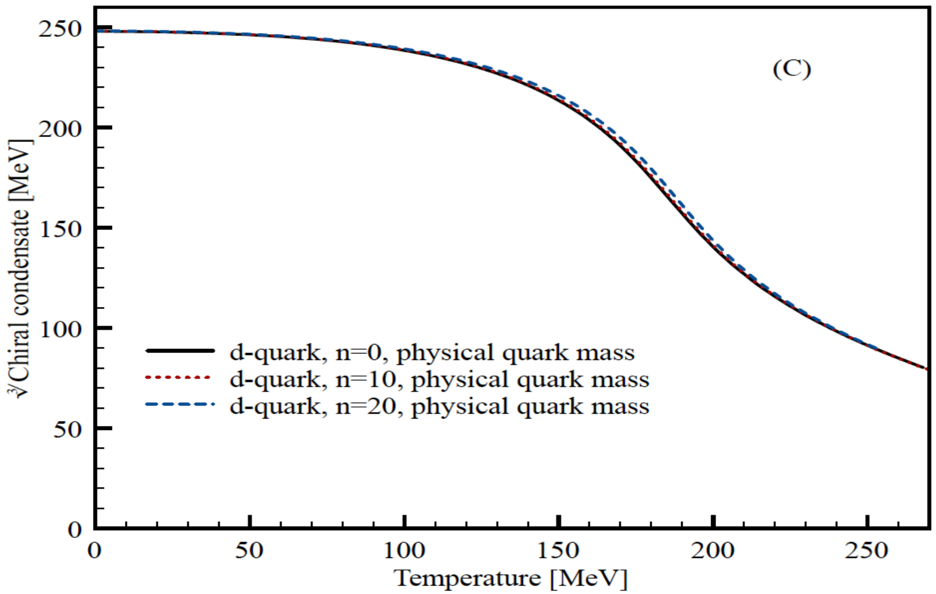
D. P. Menezes, M. Benghi Pinto, S. S. Avancini, A. Perez Martinez, and C. Providencia, Phys. Rev. C 79, 035807 (2009).

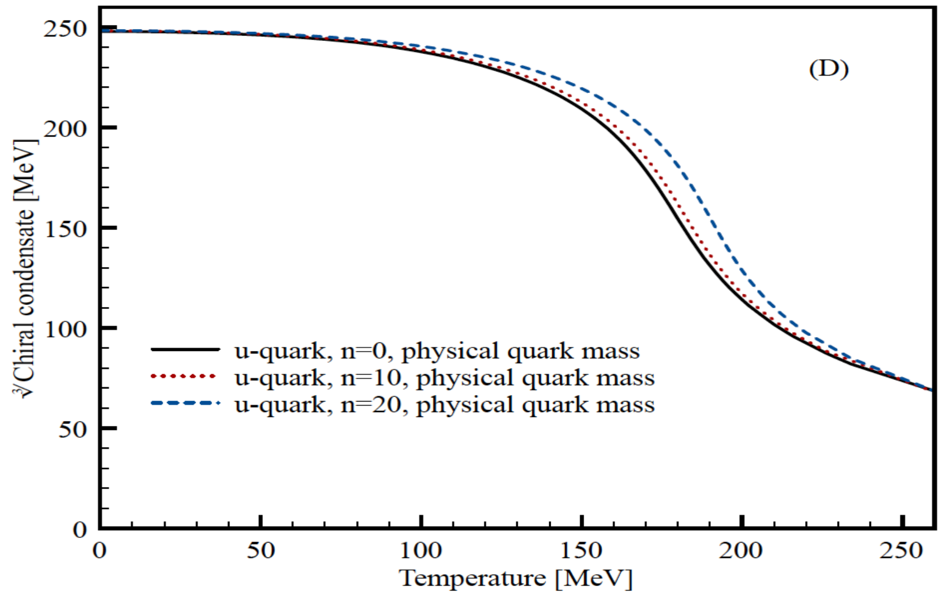
Chiral condensate for u and d flavors under B field



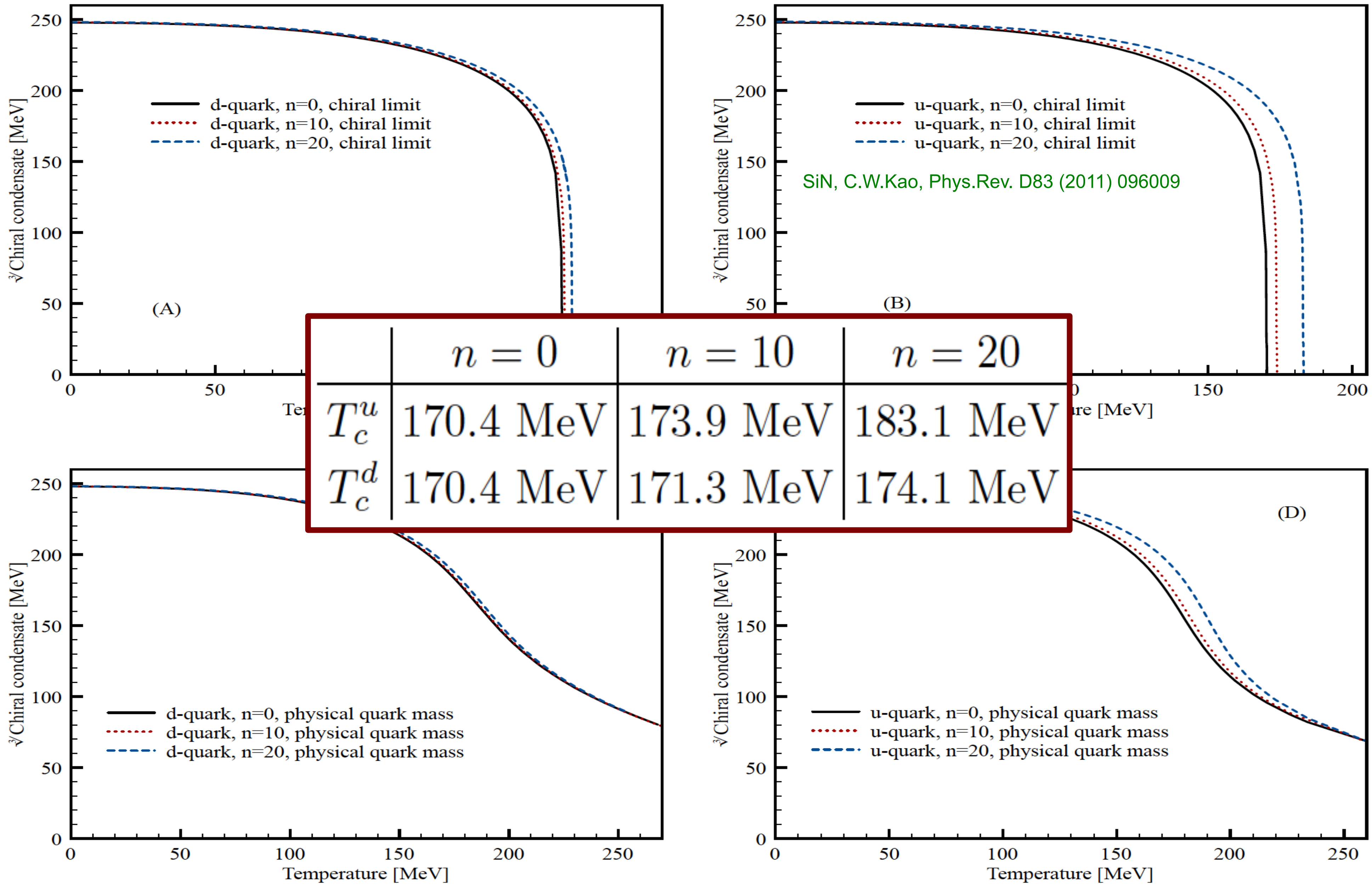


SiN, C.W.Kao, Phys.Rev. D83 (2011) 096009



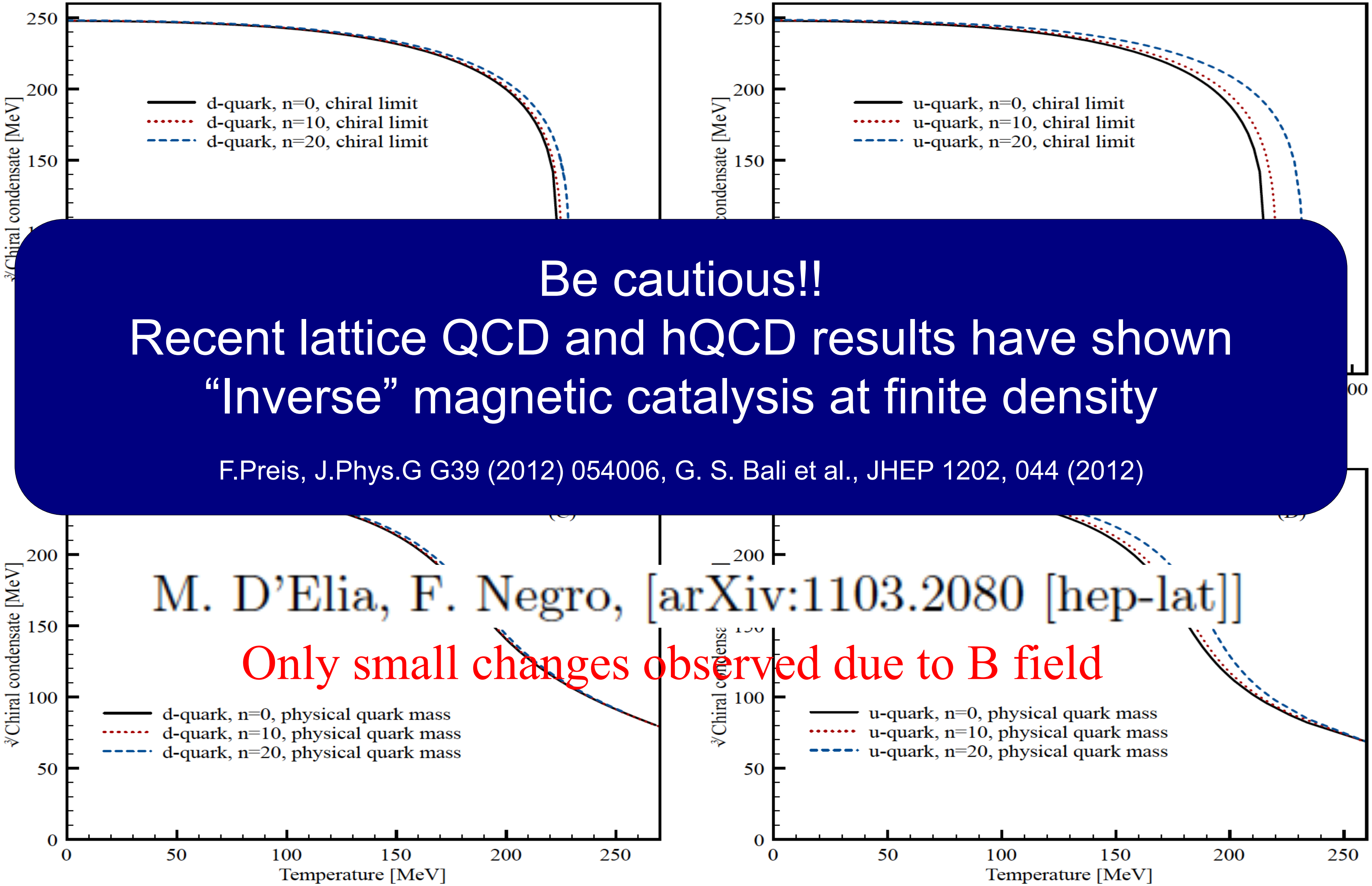


Chiral condensate for u and d flavors under B field



	$n=0$	$n=10$	$n=20$
T_c^u	170.4 MeV	173.9 MeV	183.1 MeV
T_c^d	170.4 MeV	171.3 MeV	174.1 MeV

Chiral condensate for u and d flavors under B field



Be cautious!!
Recent lattice QCD and hQCD results have shown "Inverse" magnetic catalysis at finite density

F.Preis, J.Phys.G G39 (2012) 054006, G. S. Bali et al., JHEP 1202, 044 (2012)

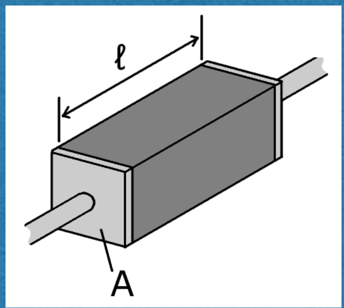
M. D'Elia, F. Negro, [arXiv:1103.2080 [hep-lat]]

Only small changes observed due to B field

4. Some numerical results

Various transport coefficients

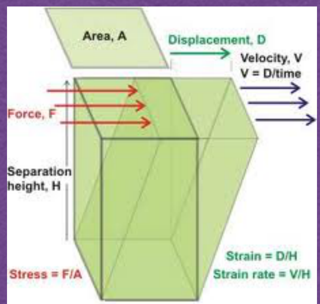
Electric conductivity



Heat conductivity

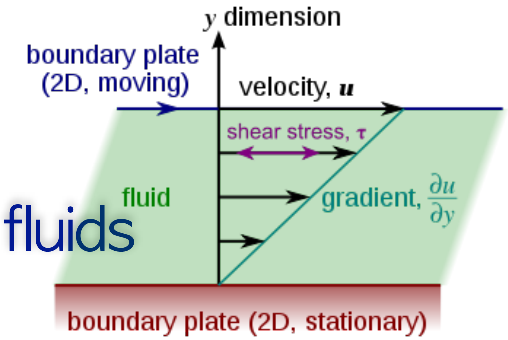
Kubo formula: Current-current correlation

Shear viscosity



Bulk viscosity

4. Some numerical results



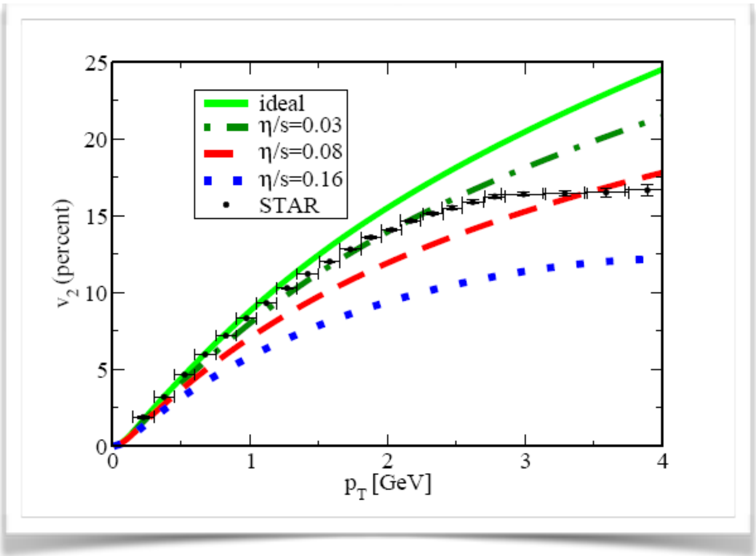
Shear viscosity (η)

η amounts strength of the shear force for fluids

$$\frac{F}{A} = \eta \frac{u}{y}$$

Small shear viscosity means "not sticky"

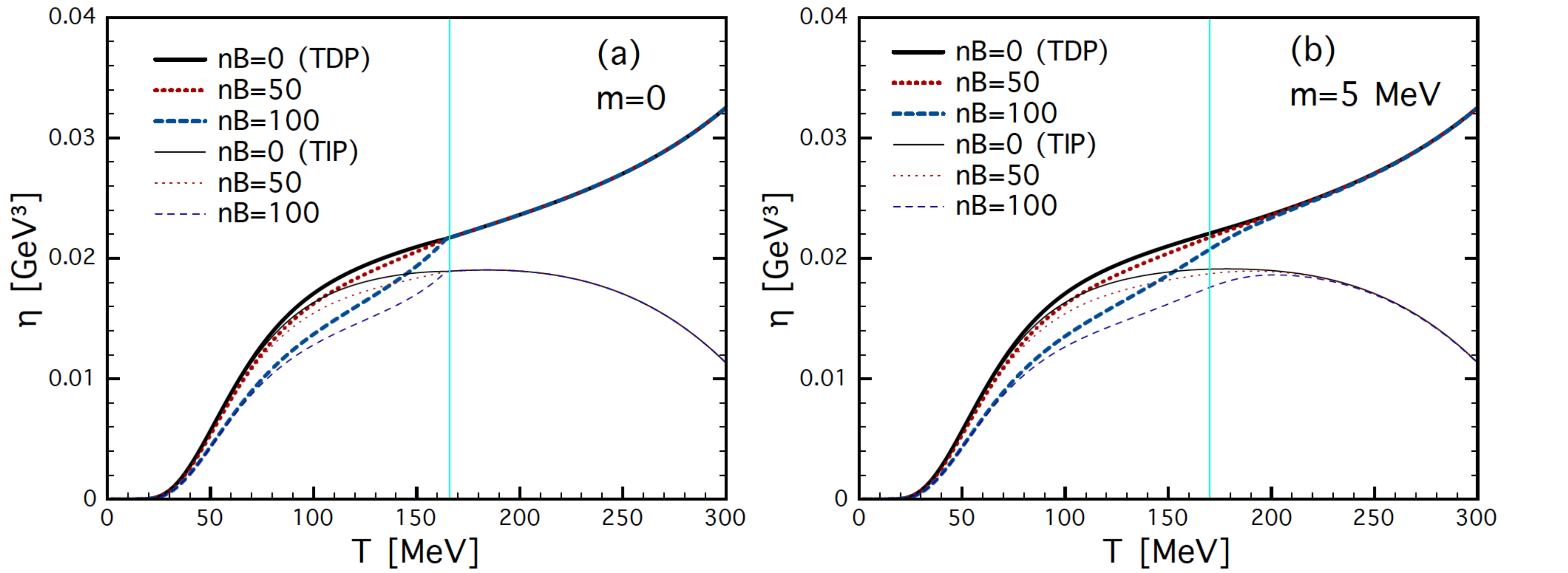
In viscous hydrodynamics simulations, η of QGP used as a parameter



Small η

Large η

4. Some numerical results

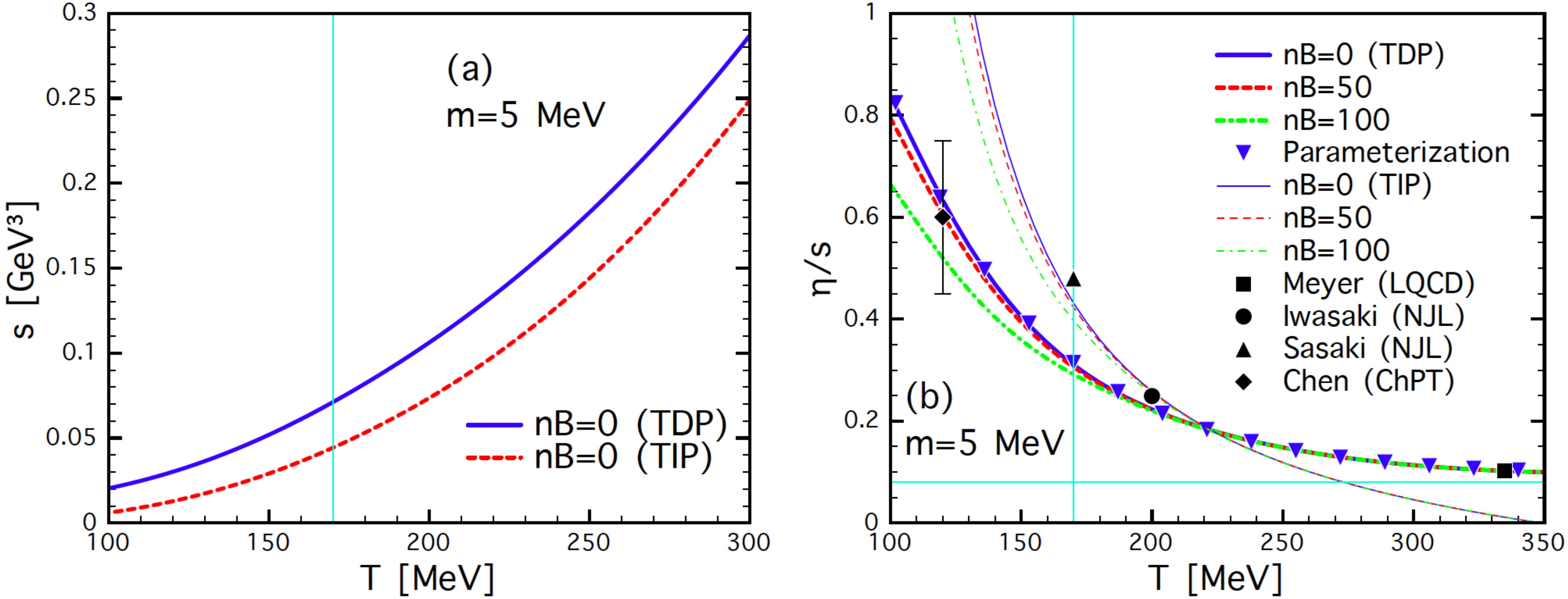


TD(I)P: Temperature (in)dependent parameters of ρ and N/V

TDP curves increase wrt T, whereas TIP ones get diminished beyond T_c

B-field effects negligible beyond T_c: Less effects on QGP

4. Some numerical results

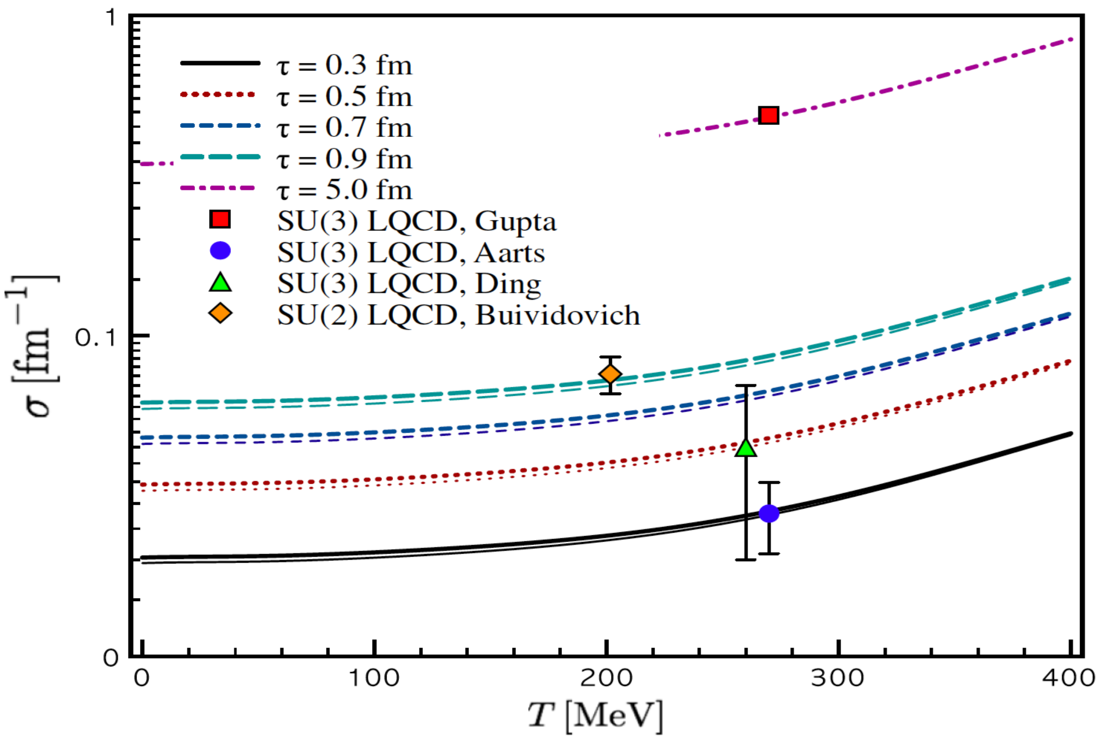


Entropy density shows increasing functions of T for TDP and TIP

Min〔η /s〕 ~ 1/(4π):KSS bound (Kovtun, Son, and, Starinets)

LQCD, NJL, and ChPT results are compatible with ours

Numerical results vs. SU(Nc) lattice QCD (LQCD)



Gupta et al., PLB597 (2004)
SU(3). Unrenormalized VC

Aarts et al., PRL99 (2007)
SU(3). Unrenormalized VC

Ding et al., PRD83 (2011): SU(3)
SU(3). Unrenormalized VC

Buividovich et al., PRL105 (2010): **SU(2)**

- The numerical results compatible with LQCD data for various τ
- Effects of B field is negligible (thick and thin lines)
- EC increases obviously beyond T ~ 200 MeV

B. Kerbikov and M. Andreichikov, arXiv:1206.6044.

5.Summary

- Along with lattice QCD and theory beyond QFT, QCD-like EFT plays a important role to understand strongly-interacting systems
- Strongly-interacting QGP believed to be created in HIC is a good place to test QCD in extreme conditions, i.e. hot and dense QCD matter
- QCD-like EFTs are modified in medium with helps of lattice QCD, Euclidean-time formula, nonperturbative gluonic correlations, etc.
- Various physical properties of QGP investigated using QCD-like EFTs, such as transport coefficients, EoS, effects of B-fields, etc.
- There are still insufficient understandings and obvious distinctions between EFTs, and they can be resolved along with lattice QCD

QCD in medium: Lattice QCD

In collaboration with Prof. M.Wakatama (Kokushikan Univ., Japan)

1.Introduction: Lattice QCD

- QCD is a first principle for strong interactions but too difficult in low-E as we have seen
- Ideas for overcoming huddles:

1) We have computing machines
2) Physics is based on CALCULUS
3) Correlations can be expressed by multiple differentiations
4) Reconstruct QCD in discret spacetime

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

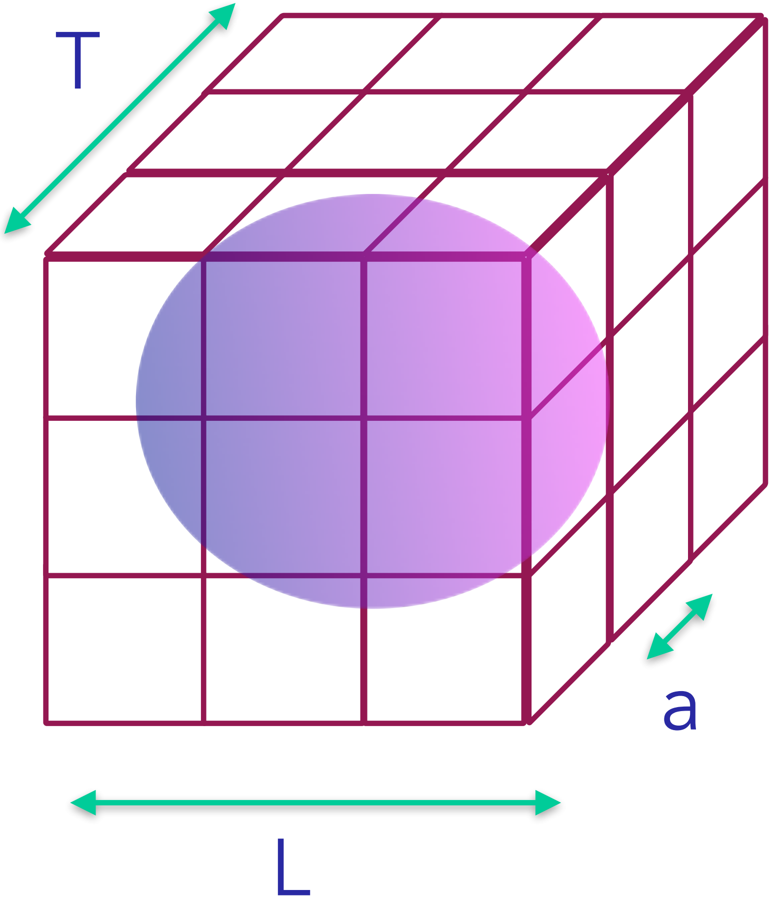
5) Using path integral for correlations and statistical methods: Why????
6) Profit!!

Kenneth G. Wilson (1936 ~ 2013)

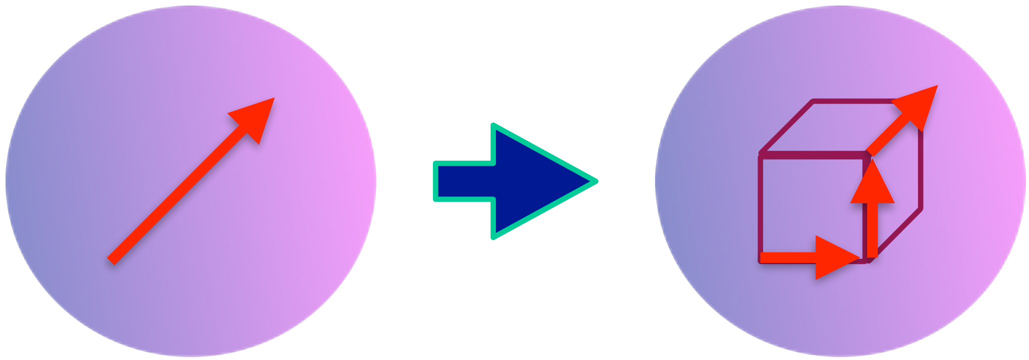
Physical Review D. **25** (10): 2649.

1.Introduction: Lattice QCD

QCD correlation functions are redefined in discretized space-time



Four-dimensional Euclidean space-time with volume L^3T



<0|O(x)O(y)|0>

In continuous limit a → 0,
it becomes our world again

1.Introduction: Lattice QCD

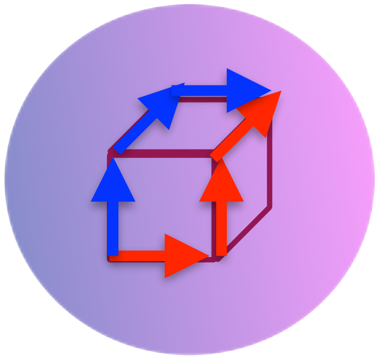
- Unfortunately, we have infinite possible paths as quantum fluctuations: Which route do I need to take?

- We have a powerful method for this: Path integral

- Ok, fine, then how to perform path integral with the discrete spacetime technically?

- Again, we have powerful method:

  Statistical Monte-Carlo simulation



Stanisław Marcin Ulam

1.Introduction: Lattice QCD

- First, we start with the path integral for this purpose for QCD

$$\langle \mathcal{O}(\bar{\psi},\psi,U)\rangle = \frac{1}{Z}\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\ \mathcal{O}(\bar{\psi},\psi,U)\ \mathrm{e}^{-S_G[U]-S_F[\bar{\psi},\psi,U]}$$

- Using external Grassmann fields to integrate out the fermion fields

$$\langle \mathcal{O}(\bar{\psi},\psi,U)\rangle = \frac{1}{Z}\int \mathcal{D}U\ \ (\det D(U))\ \ \mathrm{e}^{-S_G[U]}\ \mathcal{O}'(U)$$

Redefined operator $\mathcal{O}'(U) \equiv \mathcal{O}(-\frac{\partial}{\partial\eta},\frac{\partial}{\partial\bar{\eta}},U)\ \mathrm{e}^{\bar{\eta}D^{-1}(U)\eta}\Big|_{\eta=\bar{\eta}=0}$

1.Introduction: Lattice QCD

How to perform MC with this???

1. Generate a uniform random number i
2. Generate a gauge configuration Ui by weighting probability
   P=det[D(Ui)] exp(-SG[Ui]) to the uniform random number

Importance sampling: P and 1/P are known!!

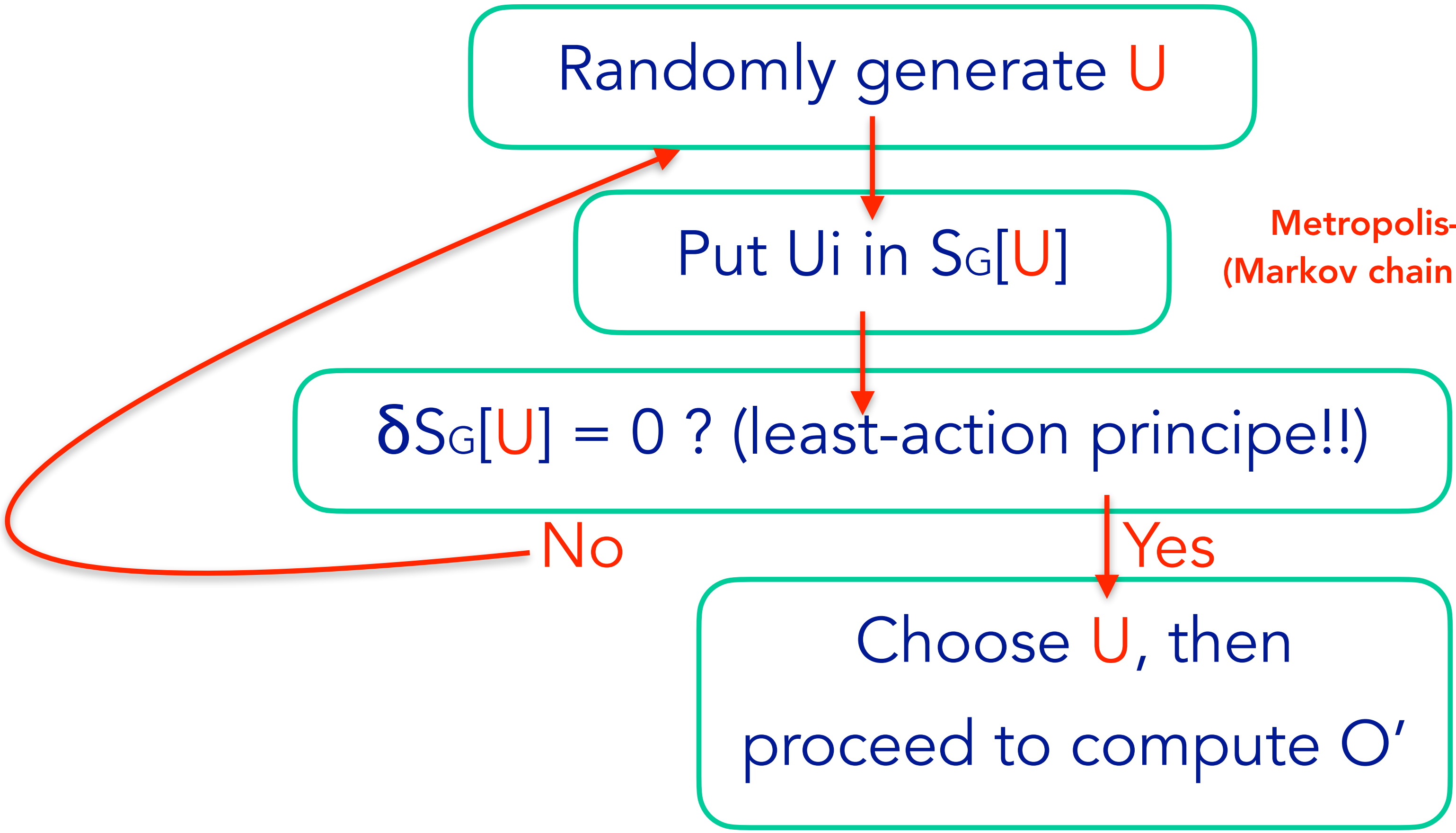
3. Calculate O'(Ui) for the obtained Ui
4. Repeat the process N times

$$\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \mathcal{O}'(U_i) = \frac{1}{Z}\int \mathcal{D}U\ \mathrm{e}^{-S_G[U]}\ \mathcal{O}'(U) = \langle \mathcal{O}(\bar{\psi},\psi,U)\rangle$$

Generating U_i with P

1.Introduction: Lattice QCD

- Sequential generating U via Markov-Chain MC
- Metropolis-Hastings algorithm: Certain probability of $U_i \rightarrow U_j$



Metropolis–Hastings algorithm
(Markov chain Monte Carlo (MCMC))

Quenched!

1.Introduction: Lattice QCD

- Make things easy! : Quenched approximation

- There are infinite sea (virtual) quarks in Dirac sea: Quark loops

- Decoupling sea quarks by making sea-quark mass infinite

$$\frac{\int \mathcal{D}U \ (\det D(U)) \ \mathrm{e}^{-S_G[U]} \ \mathcal{O}'(U)}{\int \mathcal{D}U \ (\det D(U)) \ \mathrm{e}^{-S_G[U]}} \sim \frac{\int \mathcal{D}U \ (\det \not{D}(U)) \ \mathrm{e}^{-S_G[U]} \ \mathcal{O}'(U)}{\int \mathcal{D}U \ (\det \not{D}(U)) \ \mathrm{e}^{-S_G[U]}}$$

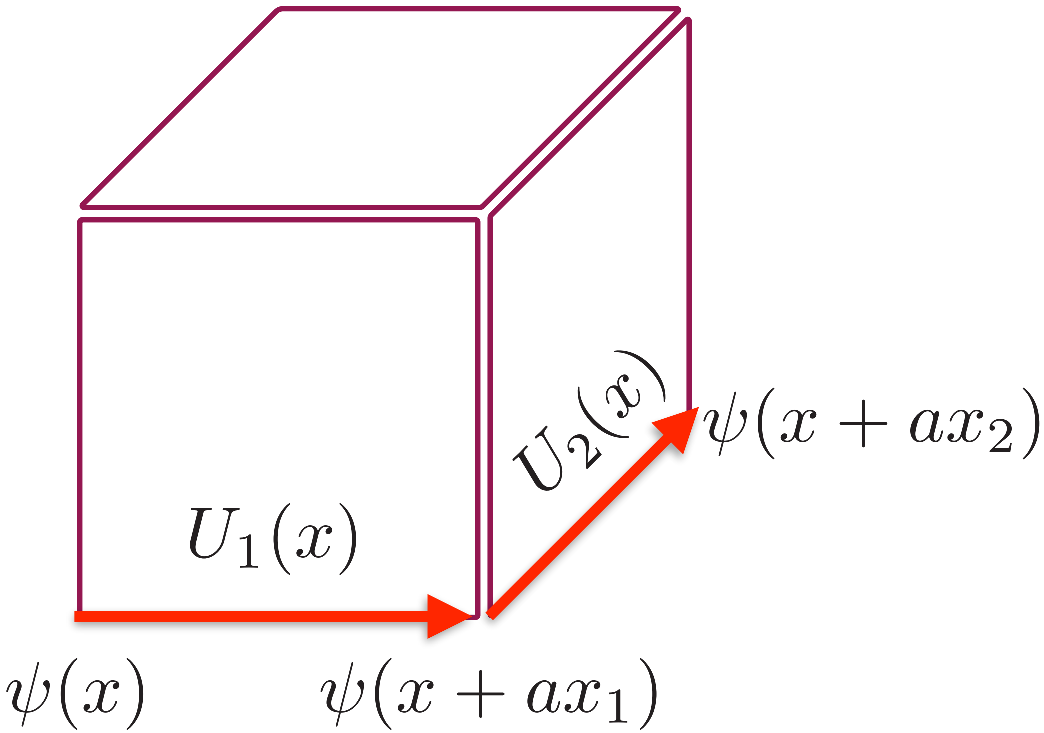
- This treatment is the same with det D(U) =1

- Due to this, "P" becomes local (without derivatives) and simple!!!!

Quenched!

1.Introduction: Lattice QCD

- How to make S_G in LQCD? : Plaquette action



- Link variable U which make (anti)quark move to a next site

$$U_\mu(x) = \exp\left[iaA_\mu(x)\right]$$

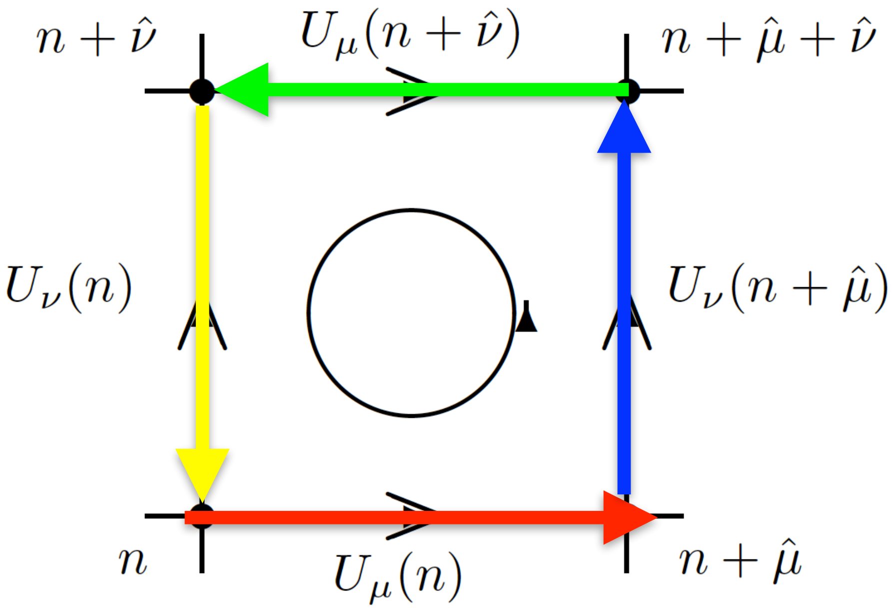
- U can be understood as a gauge link in SU(Nc)

$$G(x, y) = P \exp\left[i \int A_\mu ds^\mu\right]$$

Quenched!

1.Introduction: Lattice QCD

- What is a gauge-invariant quantity, constructed by U?
- A smallest closed loop L of multiplications of U: Plaquette



I do not prove Plaquette is gauge invariant..

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+ax_\mu)U^\dagger_\mu(x+ax_\nu)U^\dagger_\nu(x)$$

Quenched!

1.Introduction: Lattice QCD

- Constructing action with Plaquette: Wilson gauge action

$$S_G[U] = \frac{2}{g^2}\sum_x\sum_{\mu<\nu}\mathrm{ReTr}\left[1-U_{\mu\nu}(x)\right]$$

I do not prove equivalence..

- In continuous limit, it (closely) becomes usual QCD gauge action
- In SU(2), this action can be written as

$$S_P[U] \;=\; \beta\sum_x\sum_{\mu=1}^{3}\left[(4-\mu)-\frac{2}{N_c}b^0_\mu(x)\right] \quad \sum_{a=0}^{3}\left(b^a_\mu(x)\right)^2 = 1$$

- Here, we have used the SU(2) generator nature (Pauli matrix)
- After tedious calculations, we arrive at the final expression:

$$\left\langle\mathcal{O}(\bar\psi,\psi,U)\right\rangle \;=\; \frac{1}{Z}\prod_{x,\,\mu}\mathrm{e}^{-(4-\mu)}\int_{-1}^{1}d\left(\cos\theta\right)\int_0^{2\pi}d\phi\int_{-1}^{1}db^0_\mu(x)$$

$$\frac{\sqrt{1-\left(b^0_\mu(x)\right)^2}}{2}\exp\left[\frac{2\beta}{N_c}b^0_\mu(x)\right]\mathcal{O}'(U)$$

Quenched!

1.Introduction: Lattice QCD

■SU(2) Willson (plaquette) action gets simpler

$$\langle \mathcal{O}(\bar{\psi},\psi,U)\rangle = \frac{1}{Z}\prod_{x,\mu} \mathrm{e}^{-(4-\mu)} \int_{-1}^{1} d\left(\cos\theta\right) \int_{0}^{2\pi} d\phi \int_{\mathrm{e}^{-2\beta/N_c}}^{\mathrm{e}^{2\beta/N_c}} dY \quad \frac{N_c}{4\beta}\sqrt{1-\left(\frac{N_c}{2\beta}\log Y\right)^2}\ \mathcal{O}'(U)$$

$$Y = \exp\left[\frac{2\beta}{N_c} b^0_\mu(x)\right] \iff b^0_\mu(x) = \frac{N_c}{2\beta}\log Y$$

Pseudo-Heat-bath method (importance sampling)

1. Random generation of Y (~b) and 0≦ξ≦1
2. Computing P = √~ then compare it with ξ
3. If P ≧ ξ, take Y (~b), and vice versa going to 1 again
4. Computing O'(U) with obtained Y
5. Generating angles randomly then perform integration!!

2. Application Lattice QCD

Although we have a big jump….

- LQCD in finite quark chemical potential: What's wrong with this?
  - We comput $\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \mathcal{O}'(U_i)$ with P = $(\det D(U))\ \mathrm{e}^{-S_G[U]}$

Note that det[D] = DD⁺

- The quark Dirac operator with chemical potential reads

$$D(\mu_q) = \not{D} + m + \mu_q\gamma_0$$

$$D^\dagger(\mu_q) = -\not{D} + m + \mu_q^*\gamma_0 = \gamma_5 D^\dagger(-\mu_q^*)\gamma_5$$

$$\{\det[D(\mu_q)]\}^* = \det[D^\dagger(\mu_q)] = \det[\gamma_5 D(-\mu_q^*)\gamma_5] = \det[D(-\mu_q^*)]$$

- If μ is real, det[D] is not real (complex), and VICE VERSA

det[D] must be real, since it is probability P!!!

2. Application Lattice QCD

- In addition, if it is a complex, then we have

$$\int dU O'(U)(R+iI)e^{-S_G} \sim \int dU O'(U)e^{-S_G+i\phi}$$

- It's oscillation to cancel out the integral: Sign problem
- Notorious problem in strongly interacting fermion systems even in condensed matter, QFT, and nuclear physics as well.

- How to solve the sign problem???
- So far, there have been no cures (NP-hard problem)
- Many indirect and approximated methods developed

2. Application Lattice QCD

■Canonical approach developed!!

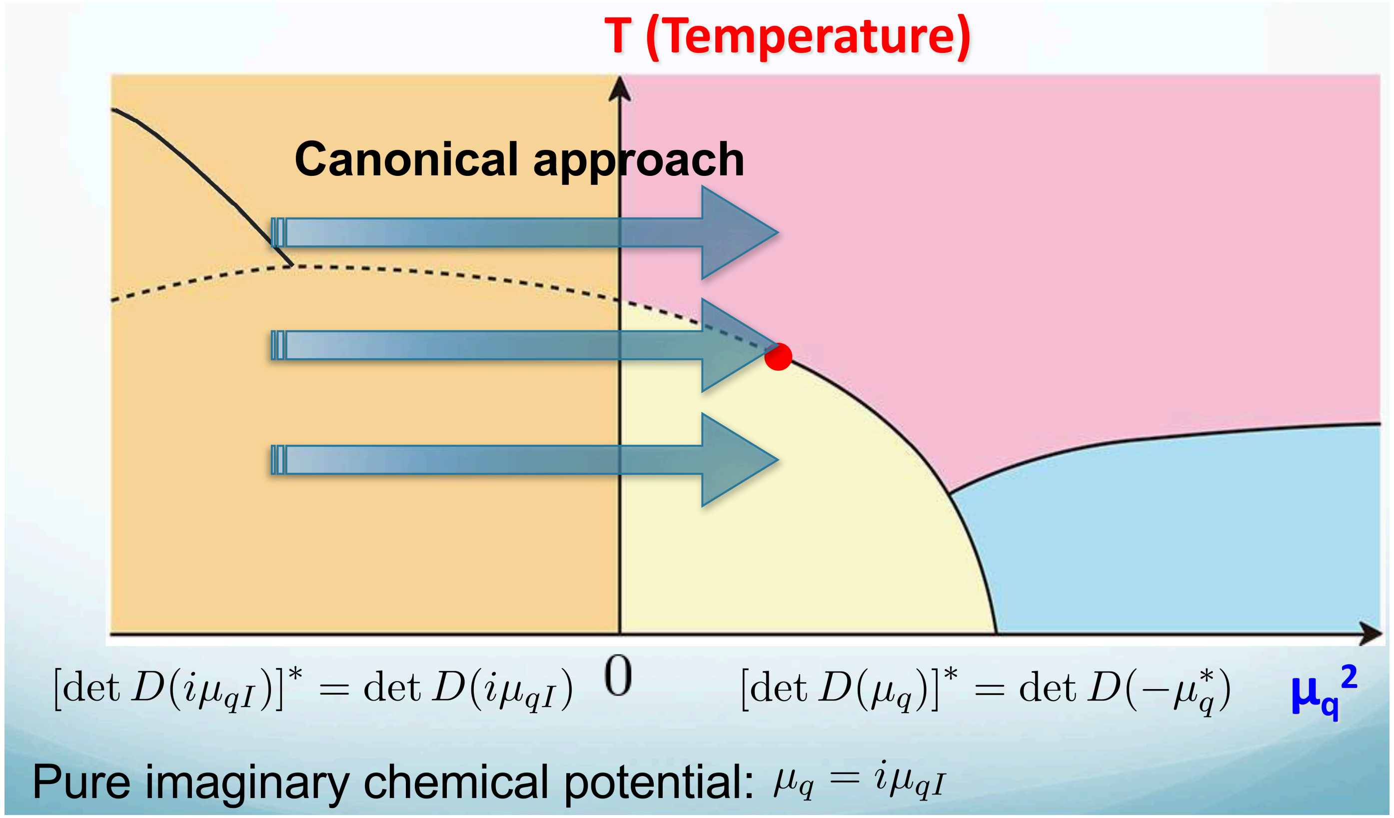
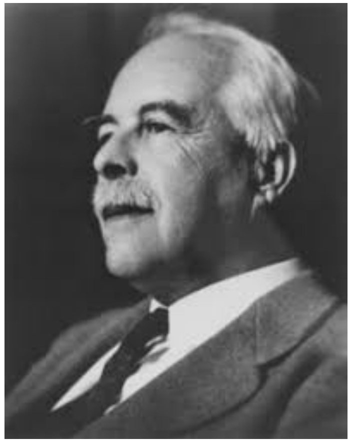


Figure by Dr. Wakayama

2. Application Lattice QCD


Gilbert Newton Lewis

- Fugacity expansion of grand canonical partition function

$$Z_{GC}[\mu_q, T, V] = \sum_n Z_C[n, T, V]\xi^n, \quad \xi = e^{\mu_q/T}$$

Fugacity

- Obtain canonical function partition function by Fourier transform

$$Z_C[n, T, V] = \int_0^{2\pi} \frac{\mu_{qI}/T}{2\pi} e^{-n\mu_{qI}/T} Z_{GC}[\mu_{qI}, T, V]$$

- For imaginary chemical potential, there is no SIGN problem

One can do MCMC or Metropolis-Hastings MC

Then, we obtain Z_{GC} on LQCD

2. Application Lattice QCD

Canonical approach developed

Lattice QCD

$$n_q(\mu_q = i\mu_{qI}, T, V)$$

Number density formulation
V. Bornyakov et al., PRD95(2017)

$$\frac{n_q}{T^3} = \frac{1}{VT^2}\frac{\partial}{\partial \mu_q} \ln Z_{\rm GC}$$

Like Hohenberg-Kohn theorem??

$$Z_{\rm GC}(\mu_q = i\mu_{qI}, T, V)$$

Fourier transform

$$Z(n, T, V)$$

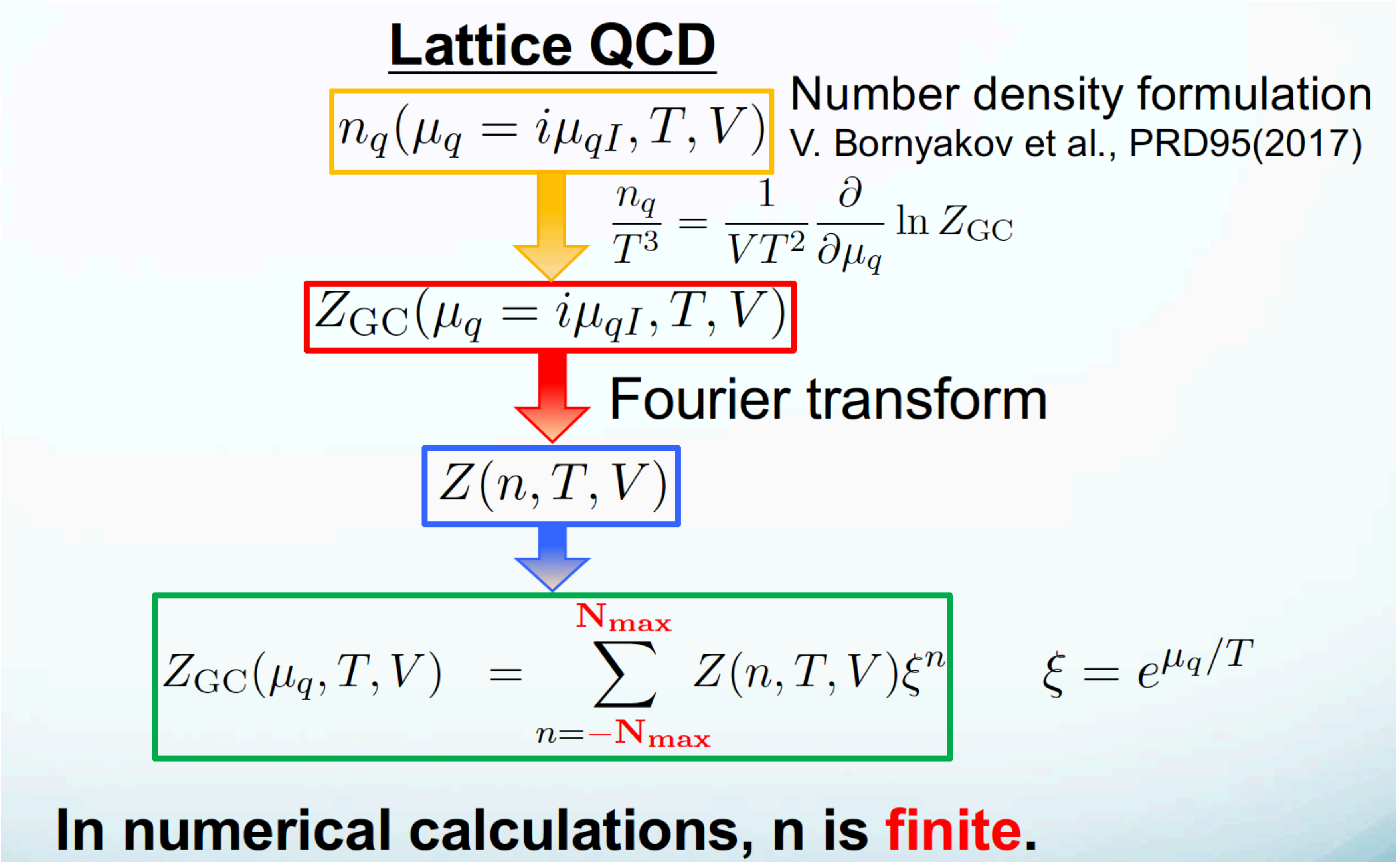
$$Z_{\rm GC}(\mu_q, T, V) = \sum_{n=-\infty}^{\infty} Z(n, T, V)\xi^n \qquad \xi = e^{\mu_q/T}$$

If we get Z_n for all n, we can search at ANY density!

Slide by Dr. Wakayama

2. Application Lattice QCD

Canonical approach



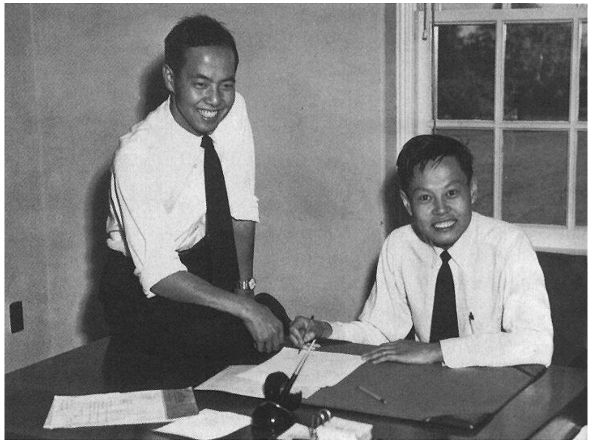
Slide by Dr. Wakayama

2. Application Lattice QCD

▪Application of canonical method: **Lee-Yang zeros**

▪Zeros of Z_{GC} so-called Lee-Yang Zeros (LYZ) contain a valuable information on the phase transitions of a system.

T.D. Lee & C.N. Yang, Phys. Rev. 87, 404&410 (1952)



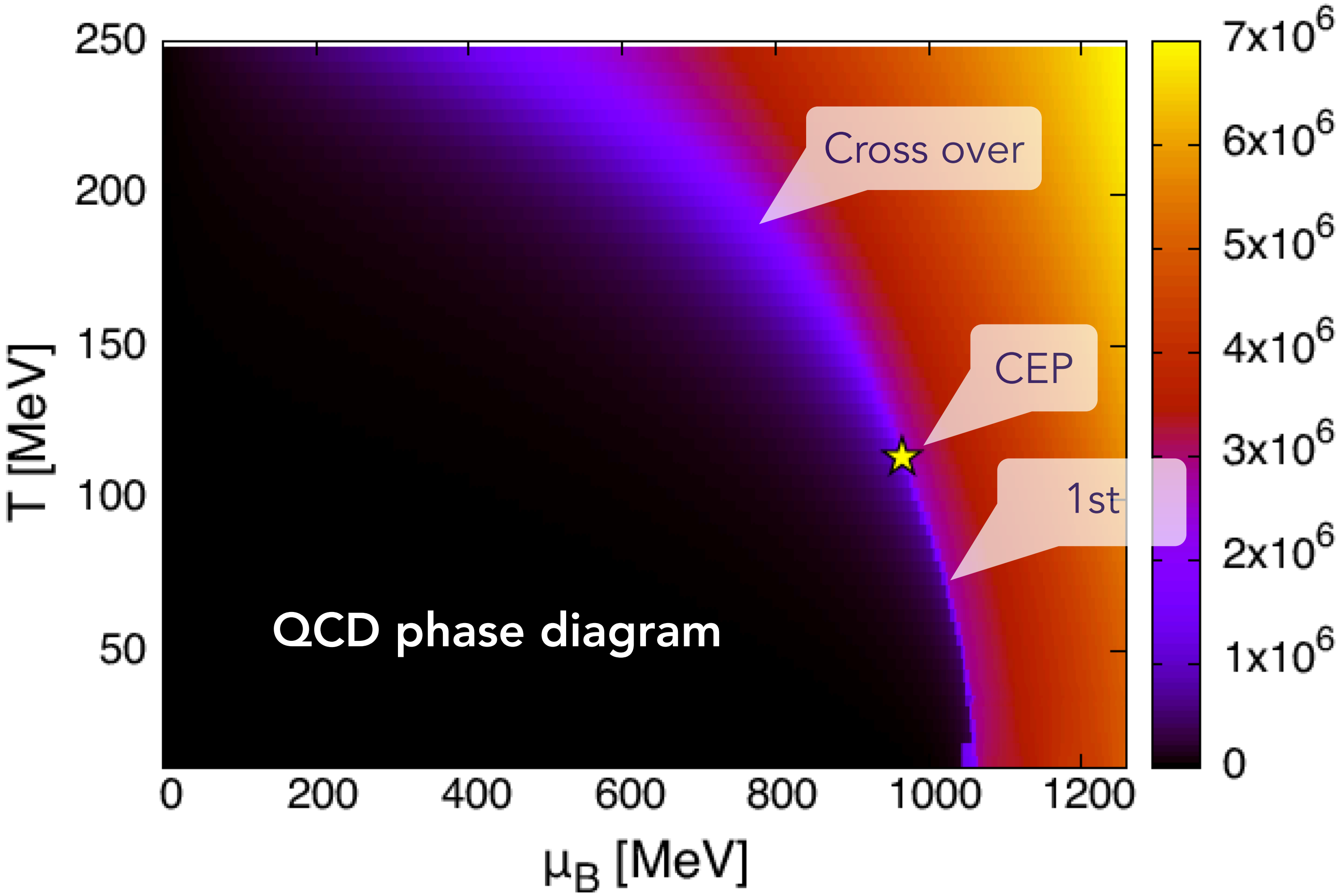
$$Z_{\rm GC}(\mu_q, T, V) = \sum_{n=-N_{\rm max}}^{N_{\rm max}} Z_{\rm c}(n, T, V)\xi^n = 0$$

Physically, at LYZ, critical-end point (CEP) appears!!

2. Application Lattice QCD

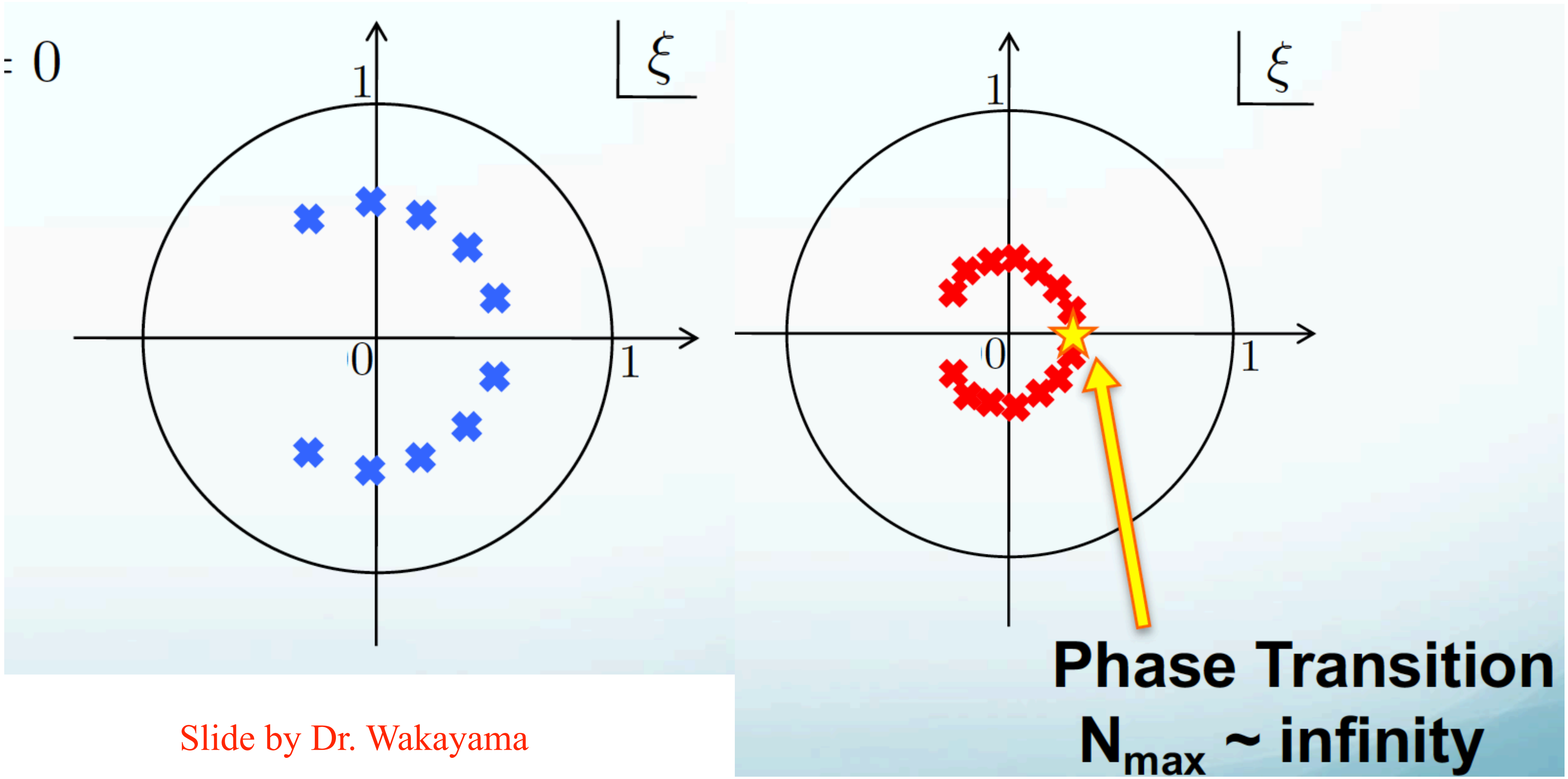
Application of canonical method: **Lee-Yang zeros**

What is critical-end point (CEP)??



2. Application Lattice QCD

- Application of canonical method: **Lee-Yang zeros**
- There are $2N_{max}$ LYZs in complex fugacity plane



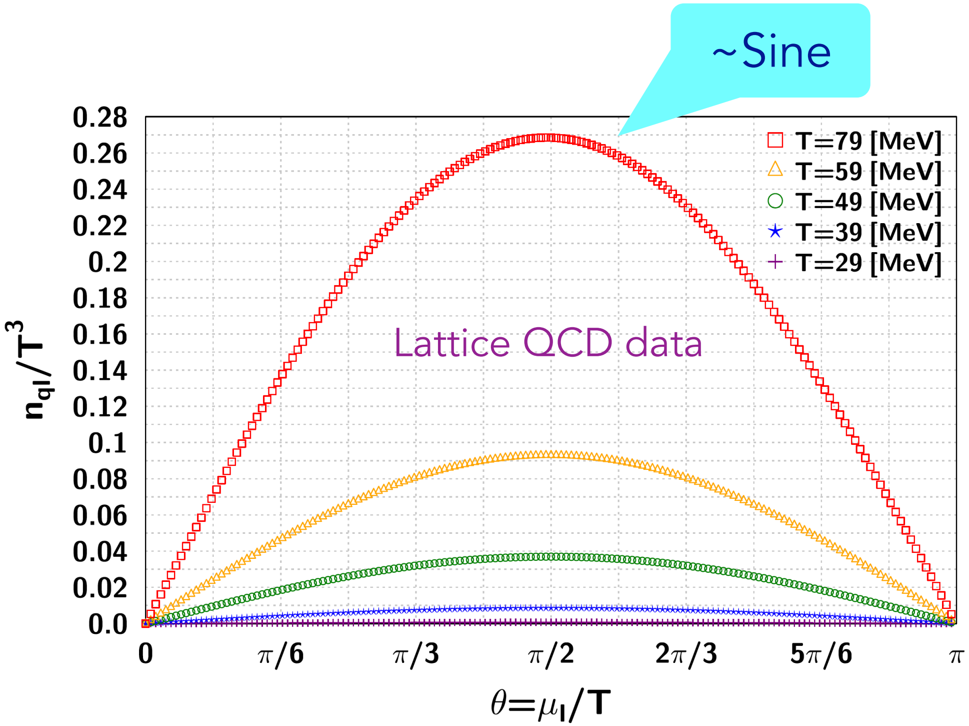
Slide by Dr. Wakayama

2. Application Lattice QCD

- Application of canonical method: **Lee-Yang zeros**
- First, we parameterize number density with sine function for more reliable numerical treatment in lattice QCD

$$\frac{n_{qI}}{T^3}(\theta) = \sum_{k=1}^{N_{\sin}} f_k \sin(k\theta)$$

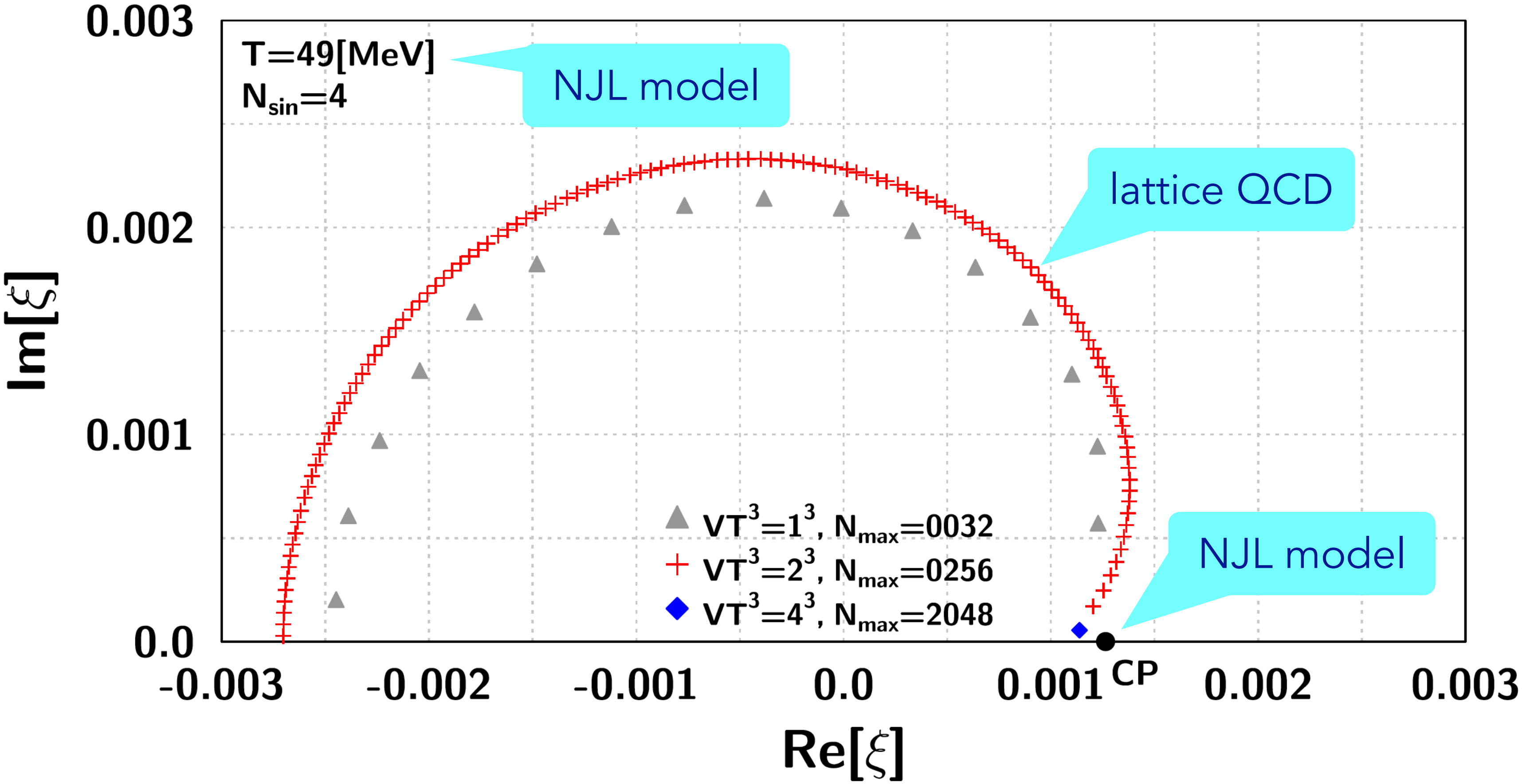
$$\begin{aligned} Z_{GC}(i\mu_I, T, V) &= C \exp\left\{-V \int_0^\theta d\theta' \, n_{qI}(\theta')\right\} \\ &= C \exp\left\{VT^3 \sum_{k=1}^{N_{\sin}} \frac{f_k}{k} \cos(k\theta)\right\}, \end{aligned}$$



Wakayama and Hosaka, PRD (2019)

2. Application Lattice QCD

- Application of canonical method: **Lee-Yang zeros**
  - We observe LYZs cross the Im[ξ]=0 line: CEP



Wakayama and Hosaka, PRD (2019)

2. Application Lattice QCD

- Application of canonical method: **QCD phase structure**
- This method is not full lattice QCD but mimics it closely
- Then, can we describe QCD phase diagram???: Yes!!!
- Before doing lattice QCD with canonical method,
- we test it in effective models: NJL and PNJL

Thermodynamic potential of PNJL

Quark

Quark-Gluon

$$\omega = \frac{1}{2G}(M-m_q)^2 - 2N_cN_f\int\frac{d^3p}{(2\pi)^3}E_p \; -2N_fT\int\frac{d^3p}{(2\pi)^3}\left\{\mathrm{Tr}_c\ln\left[1+Le^{-\frac{E_p-\mu}{T}}\right]\right.$$

$$\left.+\mathrm{Tr}_c\ln\left[1+L^\dagger e^{-\frac{E_p+\mu}{T}}\right]\right\}+T^4\left[-\frac{b_2(T)}{2}\ell\bar{\ell}-\frac{b_3}{6}\left(\ell^3+\bar{\ell}^3\right)+\frac{b_4}{4}\left(\ell\bar{\ell}\right)^2\right]$$

Mass gap

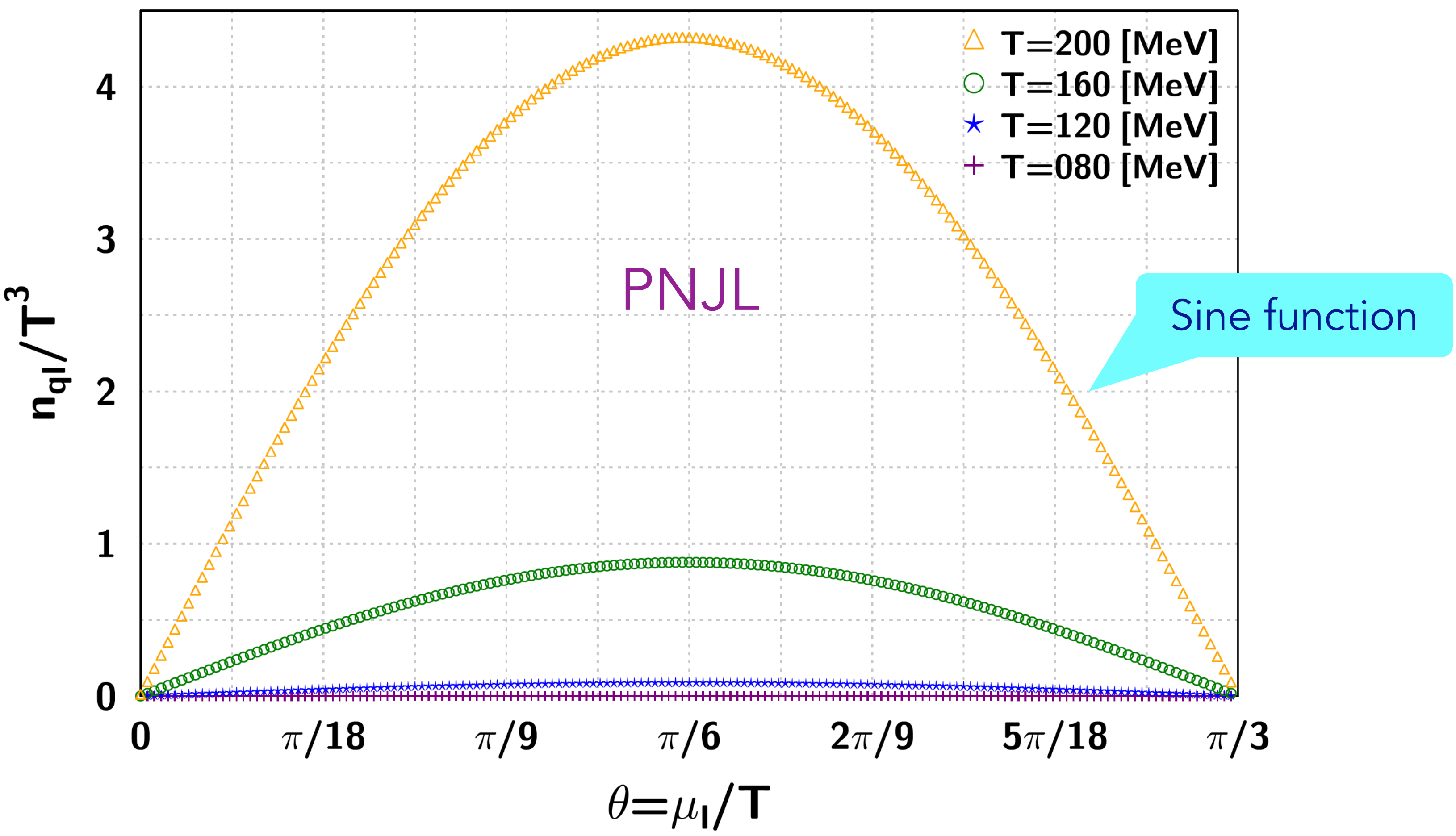
Quark-Gluon

Gluon ~ Z(Nc)

Wakayama, Nam, and Hosaka, PRD (2020)

2. Application Lattice QCD

Application of canonical method: **QCD phase structure**

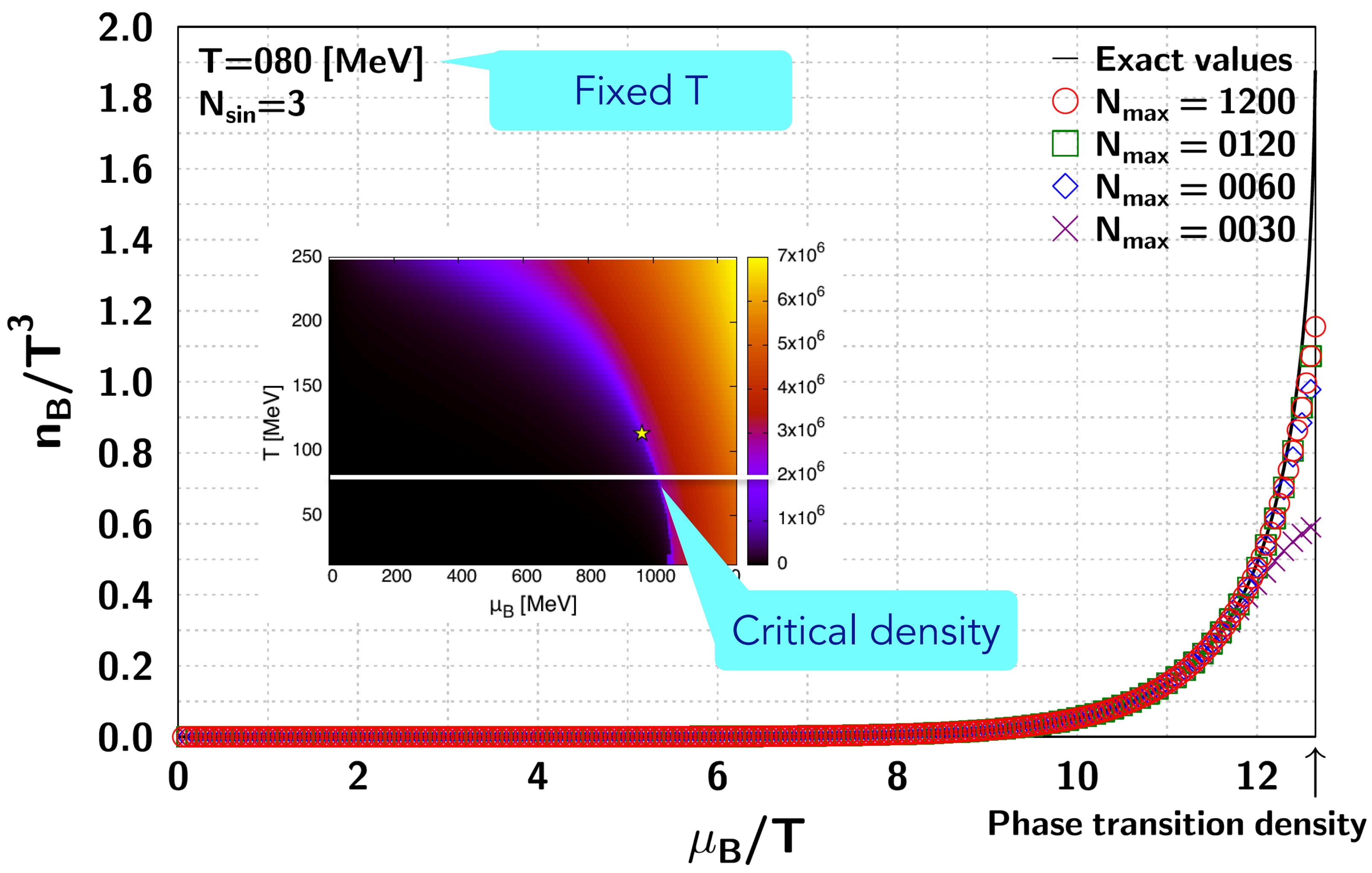


Wakayama, Nam, and Hosaka, PRD (2020)

2. Application Lattice QCD

Wakayama, Nam, and Hosaka, PRD (2020)

Application of canonical method: **QCD phase structure**



2. Application Lattice QCD

Wakayama, Nam, and Hosaka, PRD (2020)

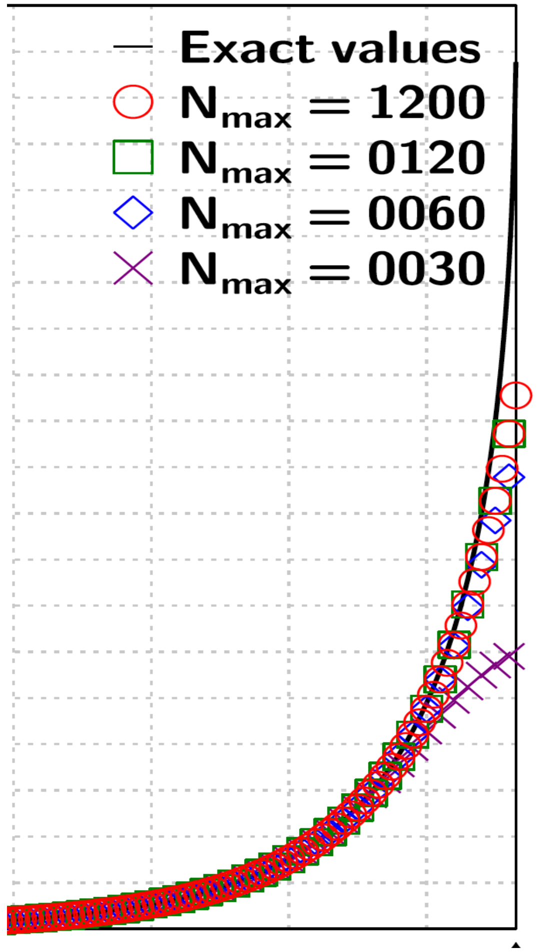
▪Application of canonical method: **QCD phase structure**

▪As N_{max} increases, results from canonical method reaches to exact value

▪Nonetheless, canonical method does not coincide with exact one: limitation of the method…

▪Then, how do we quantify phase transition in this method?: Taking tolerance

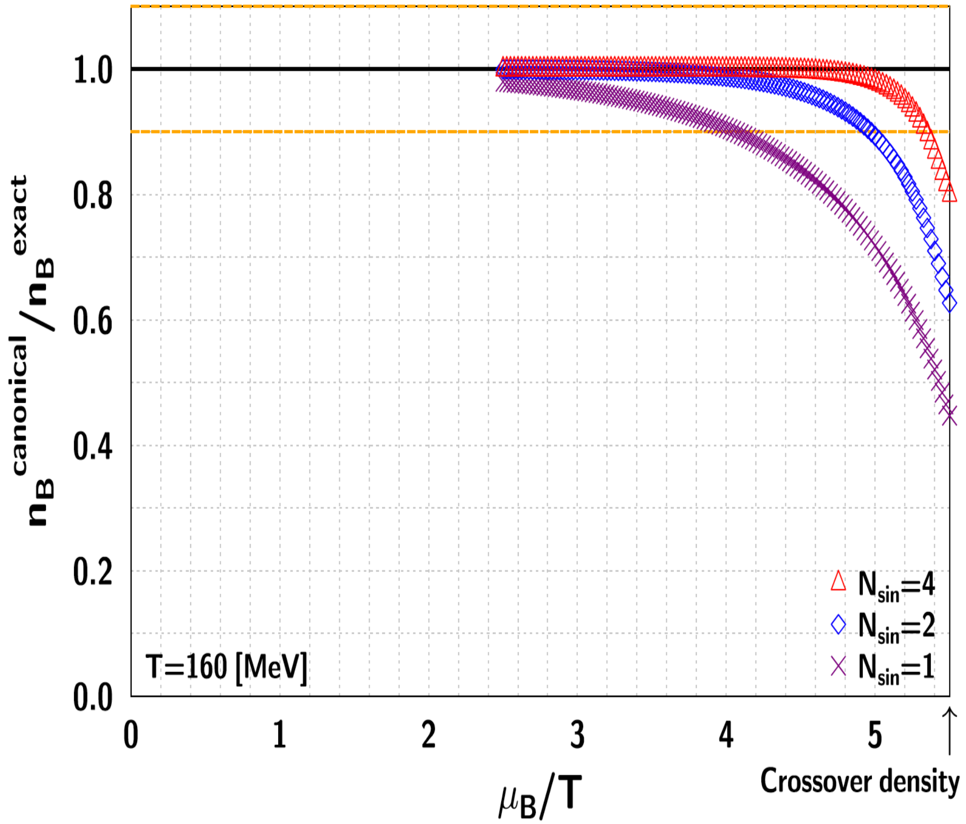
$$\frac{n_B^{\mathrm{PNJL}}}{n_B^{\mathrm{Canonical}}} < 10\%$$

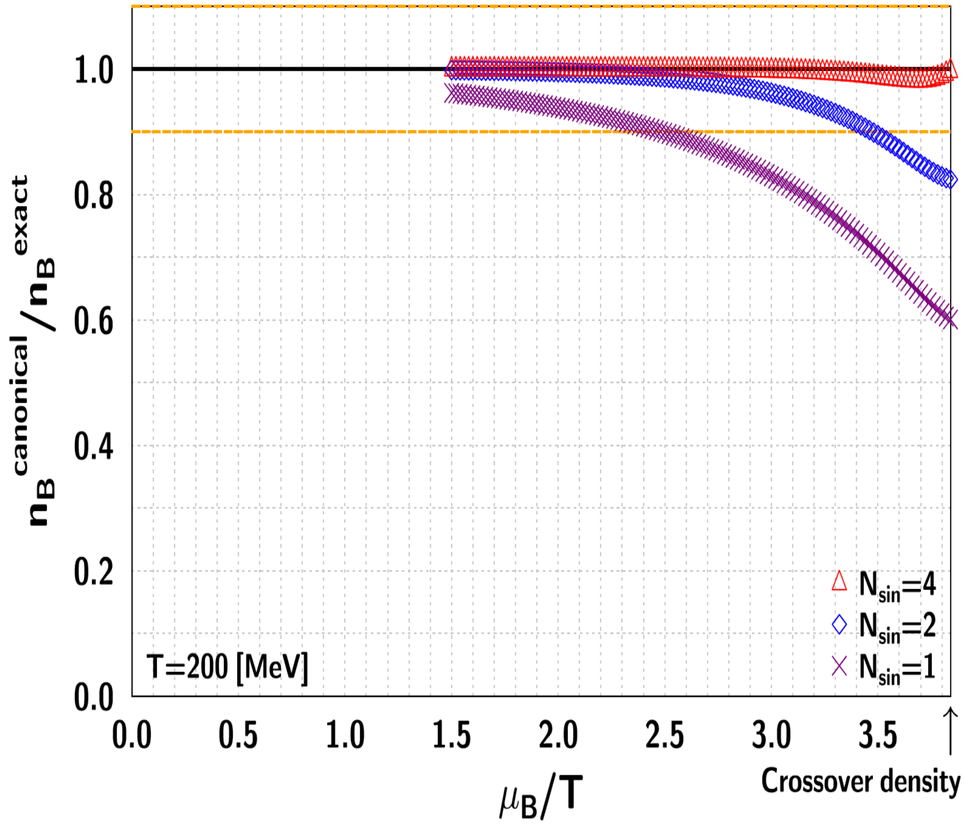


2. Application Lattice QCD

Wakayama, Nam, and Hosaka, PRD (2020)

▪Application of canonical method: **QCD phase structure**

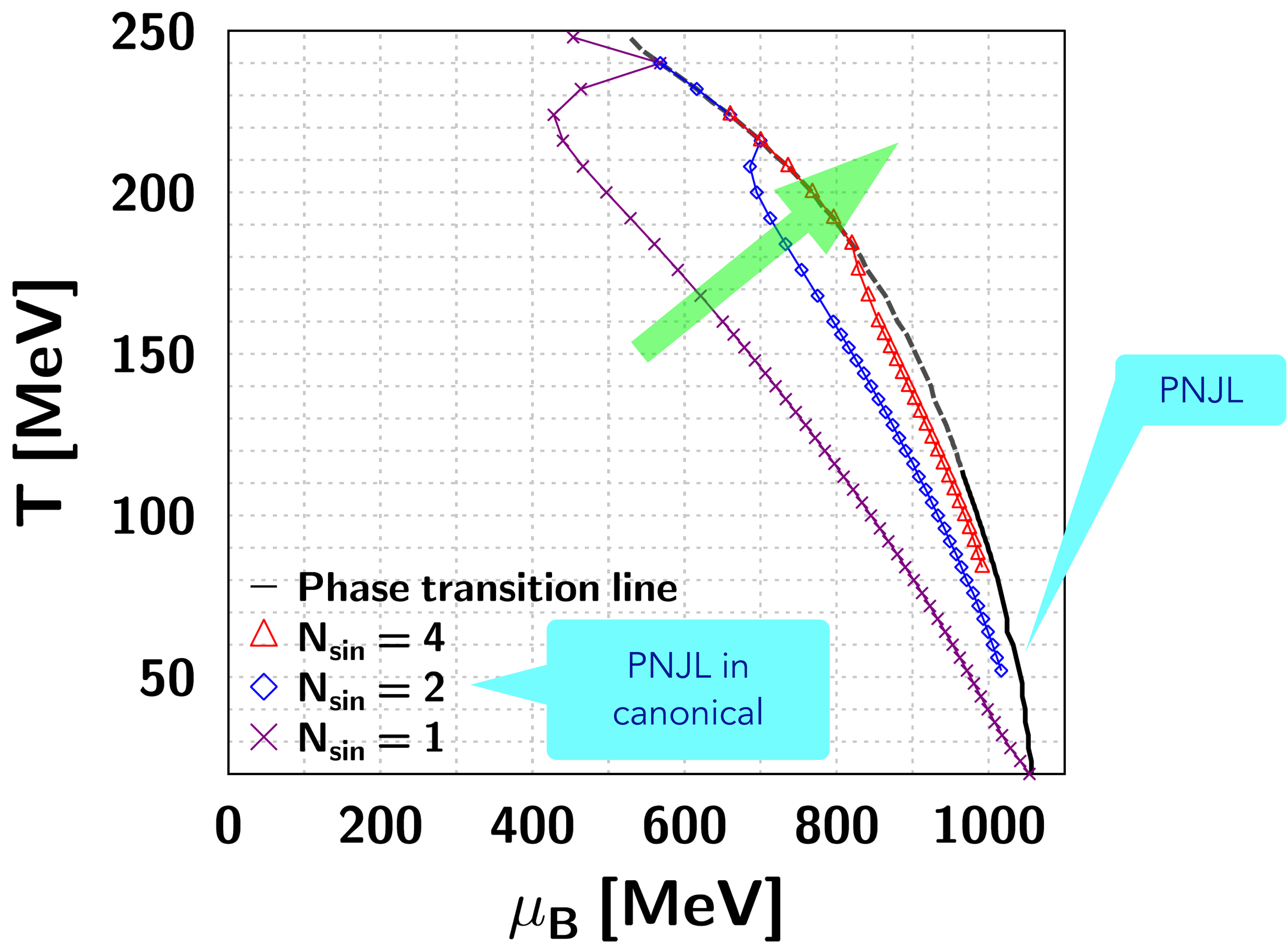




$$\frac{n_B^{\mathrm{PNJL}}}{n_B^{\mathrm{Canonical}}} < 10\%$$

2. Application Lattice QCD

Wakayama, Nam, and Hosaka, PRD (2020)



Thank you for your attention!!