기초과학연구원
Institute for Basic Science

Axions with hierarchical couplings

based on K. Choi, SHI, C. S. Shin, *Ann. Rev. Nucl ###, ####* (2021) [arXiv:2012.05029]

Sang Hui Im (IBS CTPU)

Dark Matter as a Portal to New Physics, APCTP, Feb 2nd, 2021

Outline

- Axion periodicity and a natural coupling scale
- Hierarchies of axion couplings in cosmology
- Hierarchies from the axion landscape
- Conclusions

Axion / Axion-Like Particle (ALP)

i) Approximate shift symmetry : $a(x) \to a(x) + c \ \ (c \in \mathbb{R})$

ii) Periodicity : $a(x) \to a(x) + 2\pi n f_a \ \ (n \in \mathbb{Z}) \quad \left(\frac{a(x)}{f_a} = \theta(x); \text{ angle} \right)$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a}\left(c_\psi \bar{\psi}\gamma^\mu \psi + c_\Phi \Phi^\dagger i \overleftrightarrow{D}^\mu \Phi \right) + \frac{c_A}{32\pi^2}\frac{a}{f_a} F^A_{\mu\nu}\widetilde{F}^{A\mu\nu}$$

i) → Axion can be naturally light.

ii) → Natural size of axion couplings is determined by f_a.

Axion periodicity

- Axion from a complex scalar field

$$\Phi(x) = \frac{1}{\sqrt{2}}(f_a + \rho(x))e^{i\theta(x)} \quad \Rightarrow \quad \theta(x) = \frac{a(x)}{f_a}$$

$$f_a = \sqrt{2}\langle\Phi(x)\rangle$$

Canonical normalization

i) Approximate shift symmetry from an accidental global U(1): $\Phi(x) \to \Phi(x)e^{i\alpha}$

ii) Lagrangian must be invariant under $\theta(x) \to \theta(x) + 2\pi n \qquad n \in \mathbb{Z}$

$$\left(\mathcal{L} = \mathcal{L}(\Phi^q, \Phi^{*q}),\ q \in \mathbb{Z}\right)$$

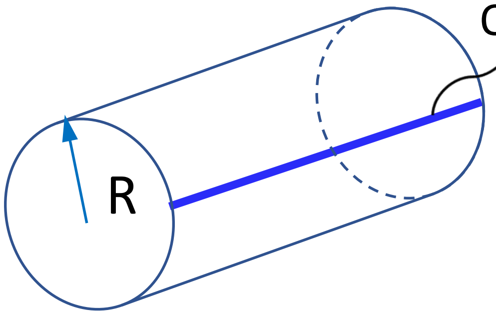
$\Leftrightarrow$ $\boxed{a(x) \to a(x) + 2\pi n f_a}$ "Exact" discrete symmetry

Axion periodicity

- Axion from a higher dimensional p-form field

$$S = \int d^5x \left(-\frac{1}{4g_5^2} F^{MN} F_{MN} + \bar{\Psi}(i\partial_M + q_\Psi A_M)\gamma^M \Psi \right)$$

$$a(x^\mu) = \frac{1}{2\pi R} \oint dx_5 A_5(x^\mu, x_5)$$



$$x_5 \cong x_5 + 2\pi R$$

i) Approximate shift symmetry from $A_M \to A_M + C_M$ (broken by q_Ψ)

ii) "Discrete" gauge symmetry

$$\begin{cases} \Psi \to \Psi e^{i q_\Psi \frac{n}{R} x_5} & (q_\Psi \in \mathbb{Z}) \\ A_5 \to A_5 + \frac{n}{R} & n \in \mathbb{Z} \end{cases}$$

$$a(x) \to a(x) + 2\pi n f_a$$

$f_a \sim R^{-1}$

"Exact" discrete symmetry

Natural coupling scale from the axion periodicity

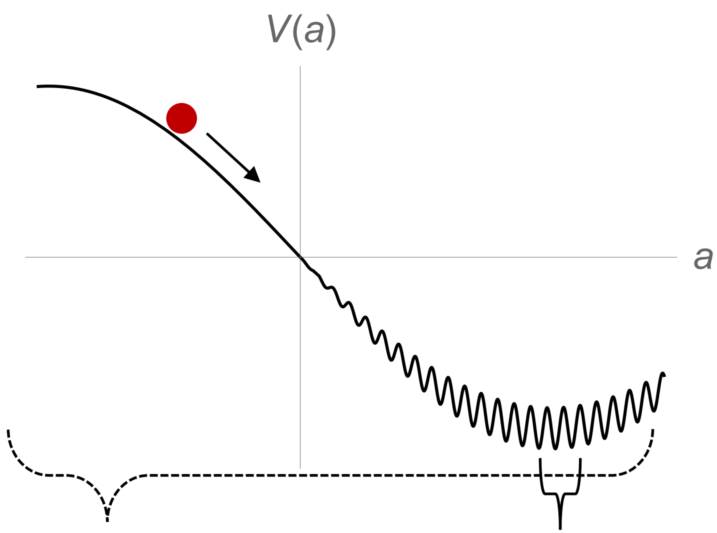
$$\frac{1}{32\pi^2}\frac{a}{f}G\widetilde{G} + \frac{1}{32\pi^2}\frac{a}{F}G_H\widetilde{G}_H$$

$$\Rightarrow \quad V(a) = \Lambda_{\rm QCD}^4 \cos\left(\frac{a}{f}\right) + \Lambda_{\rm H}^4 \cos\left(\frac{a}{F} + \delta\right)$$

$$a(x) \cong a(x) + 2\pi n f_a \quad \Rightarrow \quad f = \frac{f_a}{n_1}, \; F = \frac{f_a}{n_2} \qquad n_1, n_2 \in \mathbb{Z}$$

$$\Rightarrow \quad \boxed{F \sim f \sim f_a}$$

Hierarchical axion couplings in cosmology



Long-periodic $\Lambda_1^4 \cos\left(\frac{a}{F}\right)$

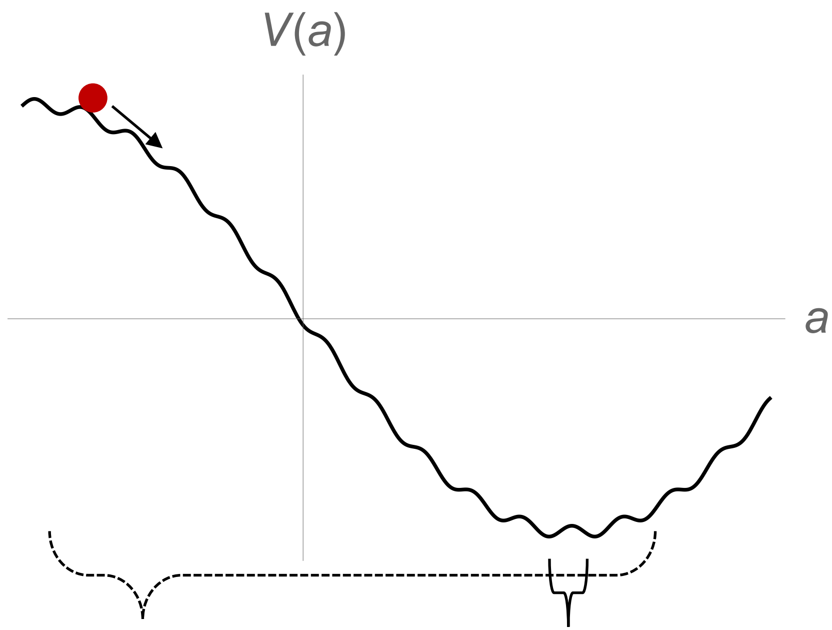
Short-periodic $\Lambda_2^4 \cos\left(\frac{a}{f} + \delta\right)$

- Natural inflation
- Ultralight axion DM / DE
- Relaxion
- …

$\Rightarrow$ $F \gg f$

Natural inflation

Freese, Frieman, Olinto '90



Long-periodic $\Lambda_1^4 \, e^{-S_1} \cos\left(\frac{a}{F}\right)$ Short-periodic $\Lambda_2^4 \, e^{-S_2} \cos\left(\frac{a}{f} + \delta\right)$

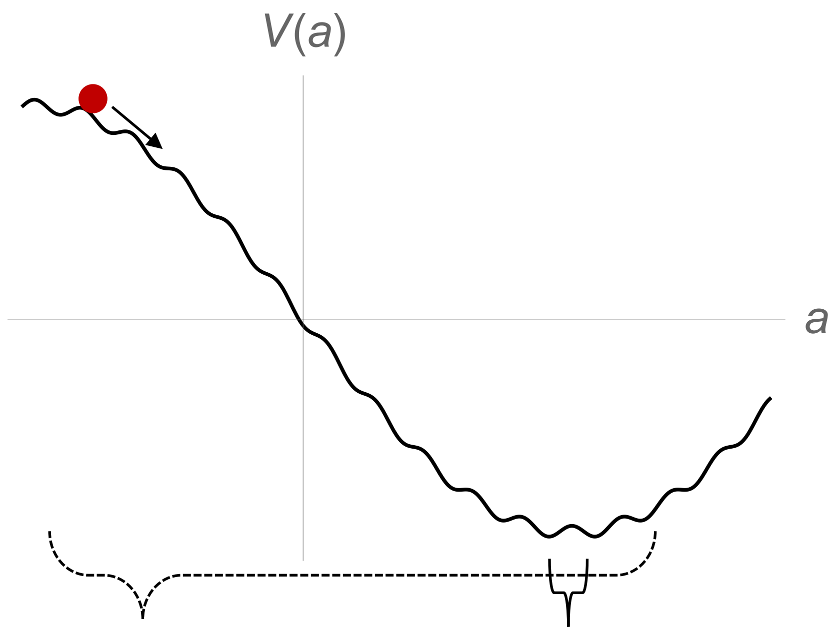
- Slow-roll inflation : $F \gtrsim \sqrt{N_e}\, M_P$
- Weak gravity conjecture : $\frac{1}{f} \gtrsim \frac{S_2}{M_P}$

$$\frac{F}{f} \gtrsim \sqrt{N_e}\, S_{\rm ins} > \mathcal{O}(100)$$

Arkani-Hamed, Motl, Nicolis, Vafa '06

$$S_{\rm ins} = \frac{8\pi^2}{g_{\rm YM}^2} > 2\pi$$

Natural inflation



The small modulation can give a substantial deviation on n_s with a better fit to CMB data:

$$\frac{\Lambda_2^4\, e^{-S_2}}{\Lambda_1^4\, e^{-S_1}} < \mathcal{O}(10^{-4})$$

K Choi, Hyungjin Kim ’15
Kapple, Nilles, Winkler ‘15

Long-periodic $\Lambda_1^4\, e^{-S_1} \cos\left(\frac{a}{F}\right)$ Short-periodic $\Lambda_2^4\, e^{-S_2} \cos\left(\frac{a}{f} + \delta\right)$

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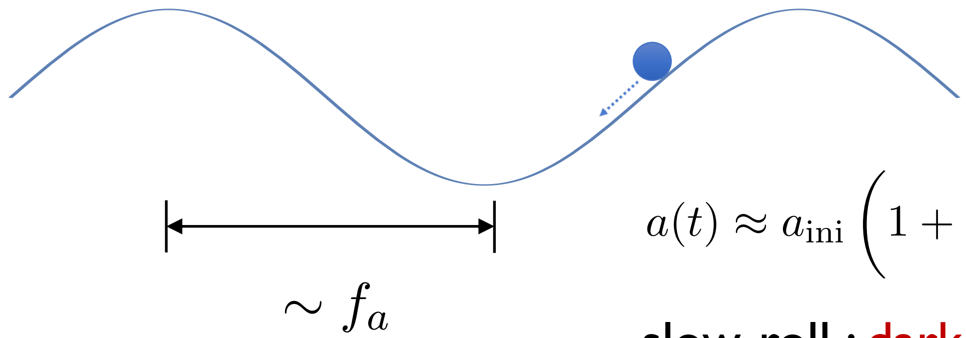
$$\frac{F}{f} \gtrsim \sqrt{N_e}\, S_{\rm ins} > \mathcal{O}(100)$$

Ultralight axion DM / DE

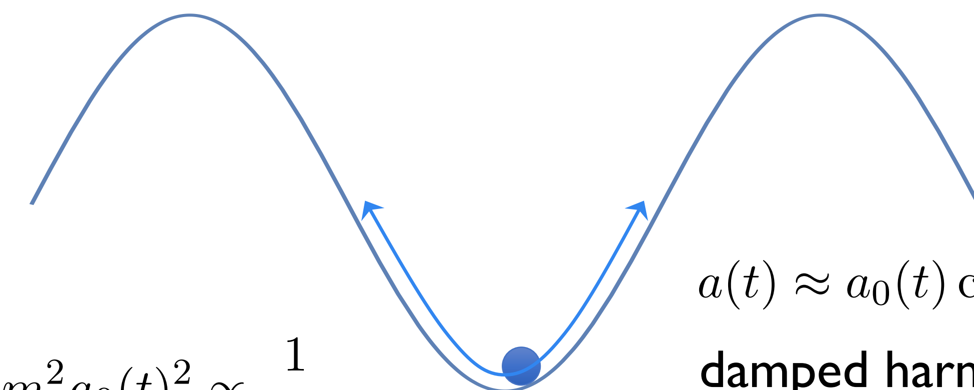
$$m_a \ll H(t)$$



$$a(t) \approx a_{\rm ini}\left(1 + \frac{1}{20}\left[\frac{m_a^2}{H_{\rm ini}^2} - \frac{m_a^2}{H(t)^2}\right]\right)$$

slow-roll : dark energy

$$m_a \gg H(t)$$



$$a(t) \approx a_0(t)\cos(m_a t)$$

damped harmonic oscillator : cold dark matter

$$\rho_a \approx \frac{1}{2} m_a^2 a_0(t)^2 \propto \frac{1}{R^3}$$

i) $m_a \lesssim H_0 \sim 10^{-33}\,\mathrm{eV}$ → DE-like (axion quintessence)

$f_a \gtrsim M_P$

ii) $10^{-33}\,\mathrm{eV} \lesssim m_a \lesssim H_{\rm eq} \sim 10^{-27}\,\mathrm{eV}$ → Early dark energy

iii) $m_a \gtrsim 10^{-27}\,\mathrm{eV}$ → DM-like

$$\Omega_a h^2 \simeq 0.1 \sqrt{\frac{m_a}{10^{-22}\,\mathrm{eV}}} \left(\frac{f_a}{10^{17}\,\mathrm{GeV}}\right)^2 \theta_{\rm ini}^2$$

$m_a \sim 10^{-22}\,\mathrm{eV}$ "Fuzzy" DM

Hu, Barkana, Gruzinov '00

Hui, Ostriker, Tremaine, Witten '17

Many works

$$\frac{\lambda_{\rm dB}}{2\pi} = \frac{\hbar}{mv} \approx 60\ \mathrm{pc}\left(\frac{10^{-22}\ \mathrm{eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$

Galaxy size : 1~10 kpc

The macroscopic wavelike property suppresses power on small scale.

⇔ $f_{\rm wgc} \lesssim \frac{M_P}{S_{\rm ins}} \sim 10^{16}\mathrm{GeV}$

$O(10)$~$O(10^2)$ separation from the axion scale implied by WGC dominant potential for axion dynamics

subdominant potential for WGC

$$V(a) = \Lambda_{\rm dyn}^4 \cos\frac{a}{f_a} + \Lambda_{\rm wgc}^4 e^{-S_{\rm ins}} \cos\left(\frac{a}{f_{\rm wgc}} + \delta\right)$$

$$\Rightarrow \quad m_a^2 \gtrsim \frac{\Lambda_{\rm wgc}^4}{f_{\rm wgc}^2} e^{-S_{\rm inst}}$$

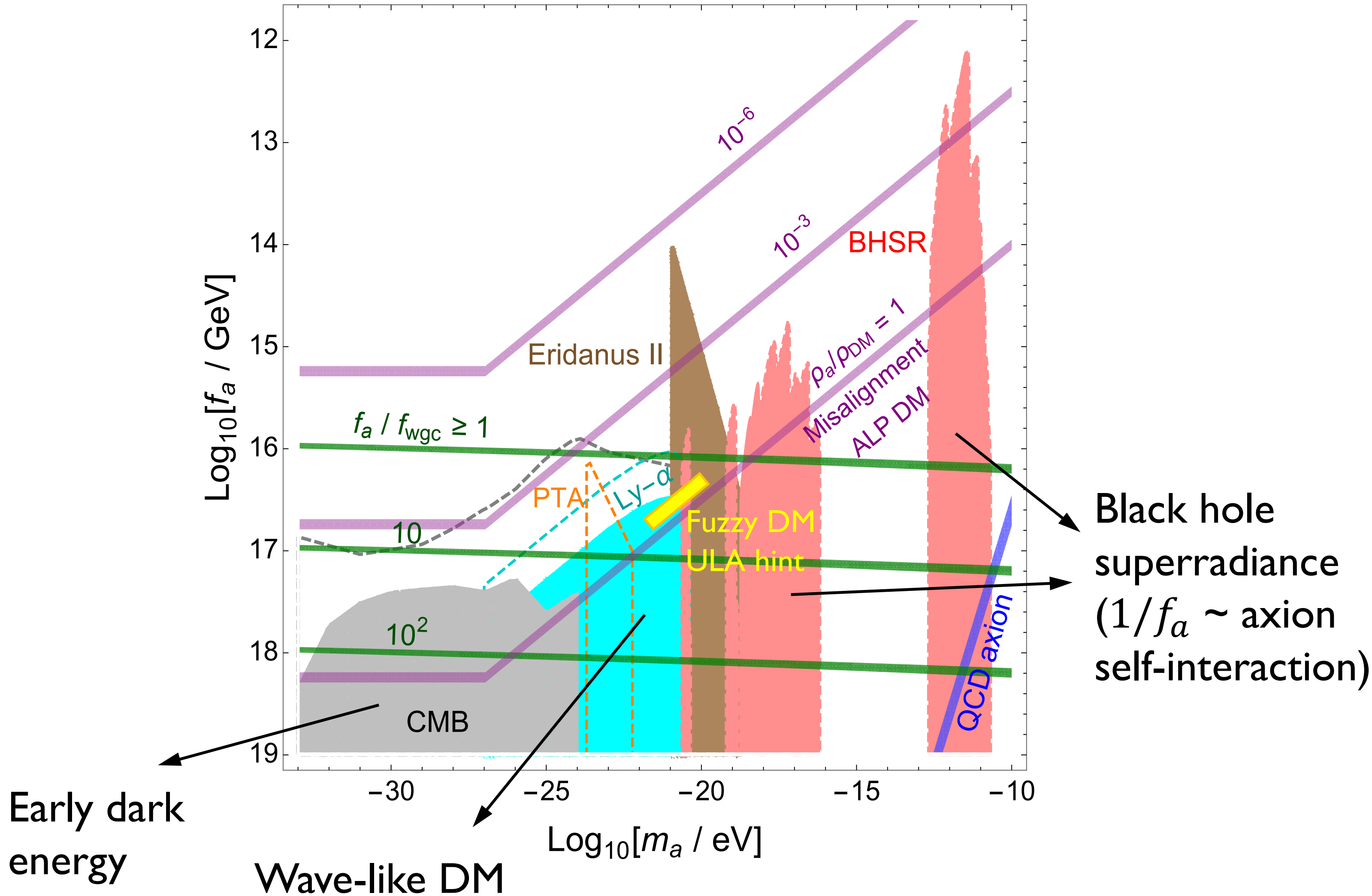
where $\Lambda_{\rm wgc}^4 \sim m_{3/2} M_P^3$

assuming that the WGC instanton gives a non-perturbative correction to the superpotential

$$\Rightarrow \quad S_{\rm ins} \gtrsim \ln(m_{3/2} M_P / m_a^2)$$

$$\Rightarrow \quad \boxed{\frac{f_a}{f_{\rm wgc}} \gtrsim S_{\rm ins} \frac{f_a}{M_P} \gtrsim \frac{f_a}{M_P} \ln\left(\frac{m_{3/2} M_P}{m_a^2}\right) \sim \frac{f_a}{M_P} \times (100 \sim 200)}$$

Gravitational probes on axion DM / DE



In most of the ULA parameter space to be probed, $\frac{f_a}{f_{wgc}} \gtrsim O(10)$

Laboratory searches for axion DM

$$\frac{g_{a\gamma}}{4} a F\tilde{F} \quad \Rightarrow \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} - g_{a\gamma}\partial_t a \vec{B}$$

$\Downarrow$

Background axion DM field

$$a \approx a_0 \cos\left[m_a(t - \vec{v}\cdot\vec{x})\right]$$

$$\rho_a = \frac{1}{2} m_a^2 a_0^2 \qquad |\vec{v}| \sim 10^{-3} c$$

Experimentally measurable quantity : $g_{a\gamma}\partial_t a = -g_{a\gamma}\sqrt{2\rho_a}\sin m_a t$

The best experimental sensitivity on $g_{a\gamma}$ is obtained when $\rho_a = \rho_{DM}$.

Laboratory searches for axion DM

$\frac{g_{a\gamma}}{4} aF\tilde{F}$ measurable quantity : $g_{a\gamma}\partial_t a = -g_{a\gamma}\sqrt{2\rho_a}\sin m_a t$

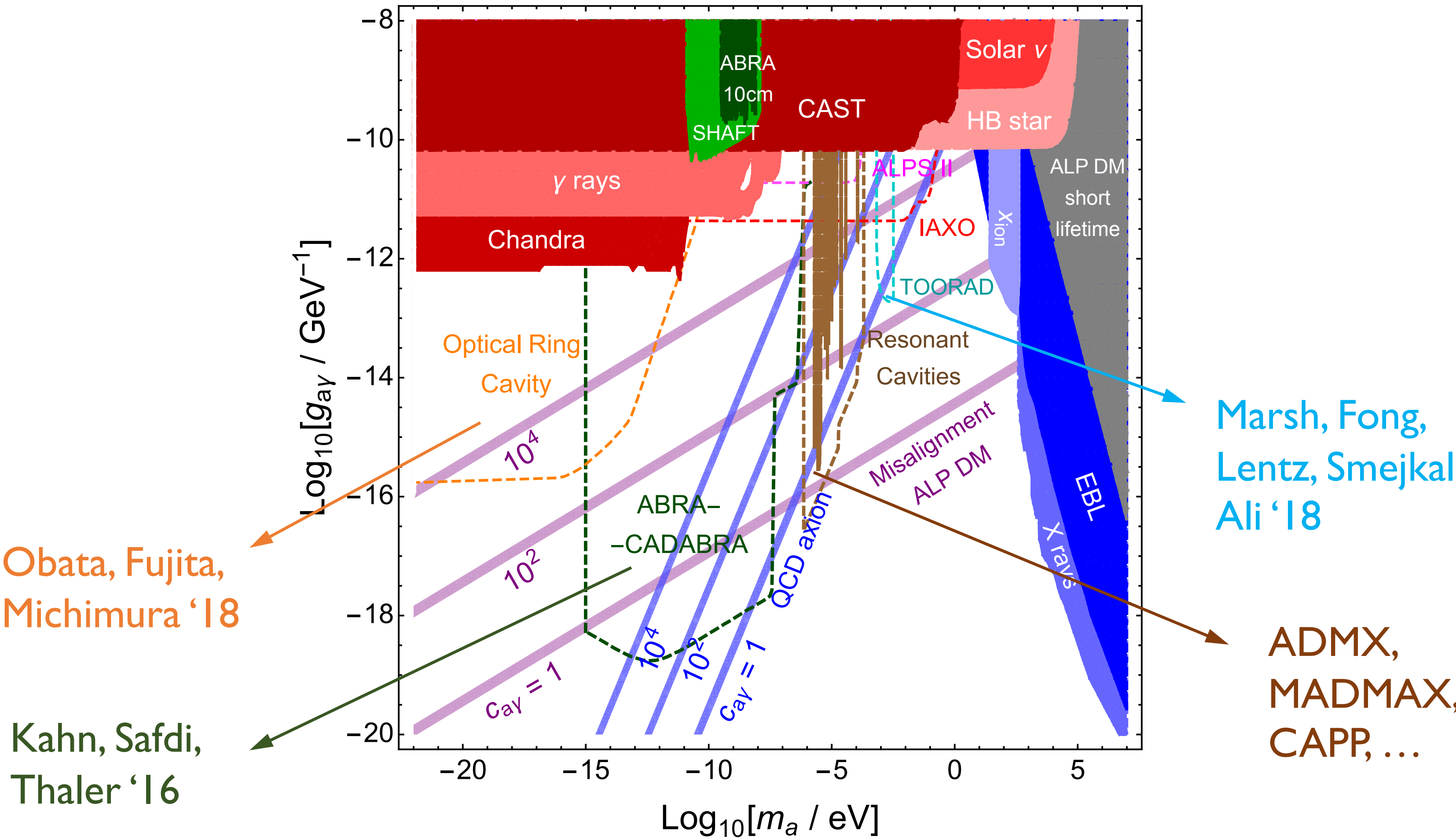
Misalignment production of the axion DM :

$$f_a \simeq 10^{17}\,\mathrm{GeV}\left(\frac{10^{-22}\,\mathrm{eV}}{m_a}\right)^{1/4}\sqrt{\frac{\rho_a}{\rho_{\mathrm{DM}}}}$$

The axion-photon coupling is quantized in terms of $1/f_a$.

$g_{a\gamma} = \frac{e^2}{4\pi^2}\frac{1}{f_a}c_{a\gamma}$ $c_{a\gamma}$: EM anomaly ($\in$ rational number)

Current and future limits on $g_{a\gamma}$



In major parameter space that can be probed in future, $c_{a\gamma} \gg 1$.

Laboratory searches for axion DM

$$g_{aN}\frac{\partial_\mu a}{2m_N}\bar{N}\gamma^\mu\gamma^5 N \quad \Rightarrow \quad \underbrace{g_{aN}\frac{\nabla a}{m_N}}_{\vec{B}_{\rm eff}}\cdot\vec{S}_N$$

Background axion DM field

$$a \approx a_0\cos\left[m_a(t-\vec{v}\cdot\vec{x})\right]$$

$$\rho_a = \frac{1}{2}m_a^2 a_0^2 \qquad |\vec{v}|\sim 10^{-3}c$$

Measurable quantity : $g_{aN}\nabla a \approx g_{aN}\sqrt{2\rho_a}\,\vec{v}\sin m_a t$

The best experimental sensitivity on g_{aN} is obtained when $\rho_a = \rho_{DM}$.

Laboratory searches for axion DM

QCD confinement

$$c_{aq}\frac{\partial_\mu a}{f_a}\bar{q}\gamma^\mu\gamma^5 q \quad \Rightarrow \quad g_{aN}\frac{\partial_\mu a}{2m_N}\bar{N}\gamma^\mu\gamma^5 N$$

c_{aq} : PQ charge of the quark

$$g_{aN} = \frac{m_N}{f_a}c_{aq} \times \mathcal{O}(1)$$

The axion-nucleon coupling is expected to be around m_N/f_a for normal PQ charges of order one.

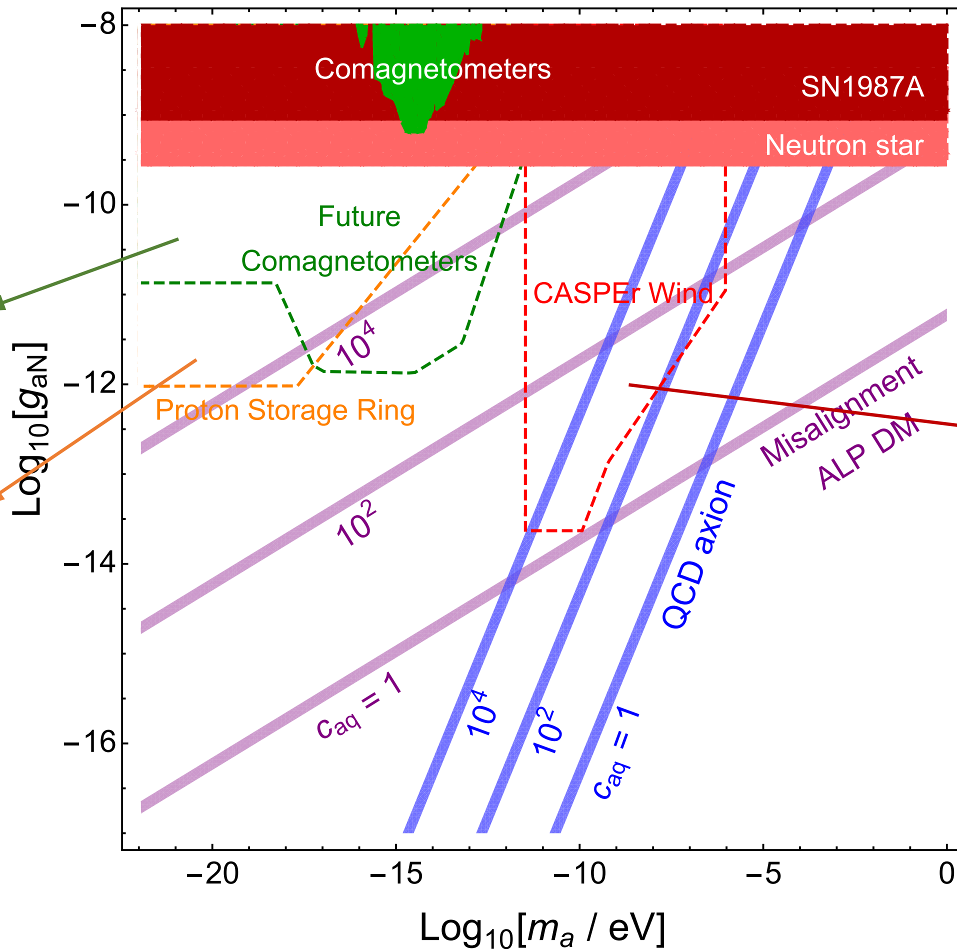
Misalignment production of the axion DM :

$$f_a \simeq 10^{17}\,\mathrm{GeV}\left(\frac{10^{-22}\,\mathrm{eV}}{m_a}\right)^{1/4}\sqrt{\frac{\rho_a}{\rho_{\mathrm{DM}}}}$$

Current and future limits on g_{aN}

Bloch, Hochberg, Kuflik, Volansky '19

Graham, Haciomeroglu, Kaplan, Omarov, Rajendran, Semertzidis '20



Kimball et al '17

In major parameter space that can be probed in future, $c_{aq} \gg 1$.

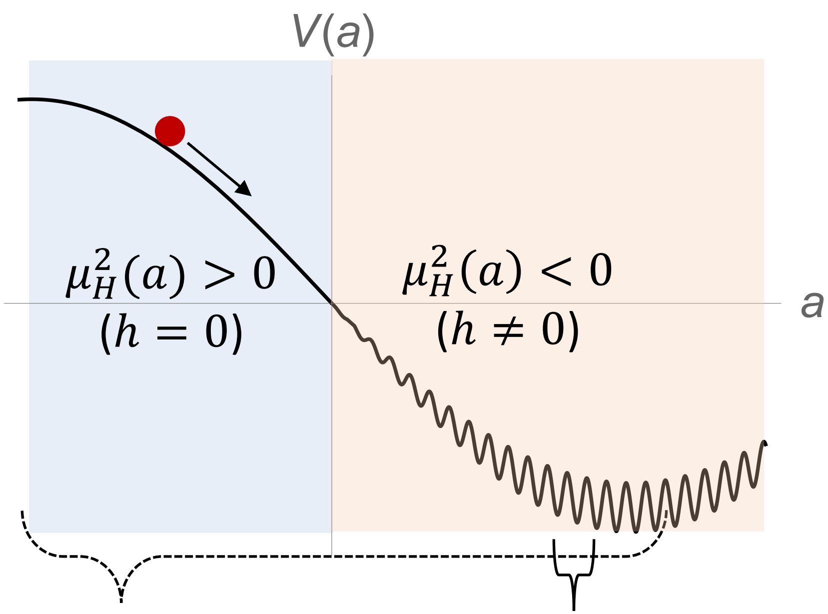
Relaxion

Graham, DE Kaplan, Rajendran '15

A brand-new solution to the weak scale hierarchy problem

$$\underbrace{\left[\Lambda^2 - \Lambda^2 \cos\left(\frac{a}{F}\right)\right] |H|^2}_{\mu_H^2 = \mu_H^2(a)} - \underbrace{\Lambda^4 \cos\left(\frac{a}{F}\right)}_{\text{Rolling potential}} - \underbrace{\Lambda_{\rm br}^2 |H|^2 \cos\left(\frac{a}{f} + \delta\right)}_{\text{Barrier}}$$

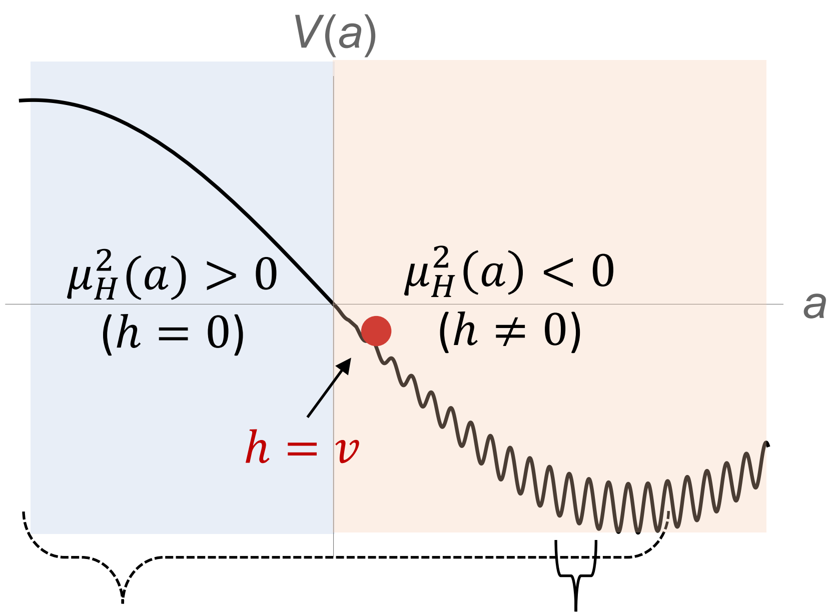
Λ : UV cutoff



Long-periodic $\Lambda^4 \cos\left(\frac{a}{F}\right)$ Short-periodic $\Lambda_{\rm br}^2 |H|^2 \cos\left(\frac{a}{f} + \delta\right)$

Relaxion

$$\underbrace{\left[\Lambda^2 - \Lambda^2 \cos\left(\frac{a}{F}\right)\right]}_{\mu_H^2 = \mu_H^2(a)} |H|^2 \underbrace{- \Lambda^4 \cos\left(\frac{a}{F}\right)}_{\text{Rolling potential}} \underbrace{- \Lambda_{\rm br}^2 |H|^2 \cos\left(\frac{a}{f} + \delta\right)}_{\text{Barrier}}$$



Long-periodic $\Lambda^4 \cos\left(\frac{a}{F}\right)$

Short-periodic $\Lambda_{\rm br}^2 |H|^2 \cos\left(\frac{a}{f} + \delta\right)$

Relaxion stabilization at $h = v$:

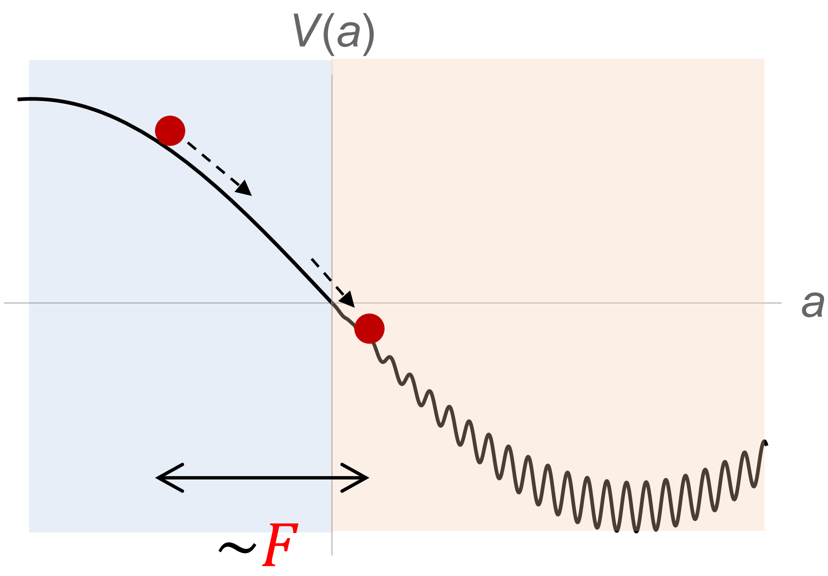
$$\frac{\partial}{\partial a}\left[\Lambda^4 \cos\left(\frac{a}{F}\right)\right] = -\frac{\partial}{\partial a}\left[\Lambda_{\rm br}^2 v^2 \cos\left(\frac{a}{f}\right)\right]$$

$$\Rightarrow \quad \boxed{\frac{F}{f} \sim \frac{\Lambda^4}{\Lambda_{\rm br}^2 v^2} \gtrsim \frac{\Lambda^4}{v^4} > \mathcal{O}(10^4)}$$

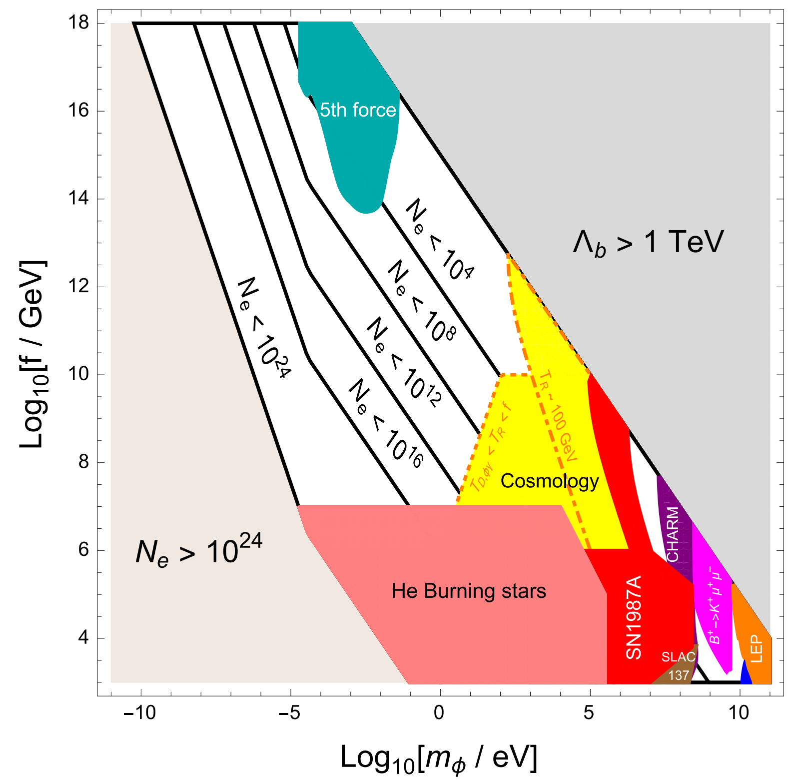
K Choi, SHI ‘15

Many e-folds of the slow-roll relaxion

In the GKR model, the relaxion slowly rolls during inflation by the Hubble friction. Typically it takes a long time to finish its long excursion.



$$N_e \sim H_I \frac{F}{\dot{a}}$$



K Choi, SHI '16

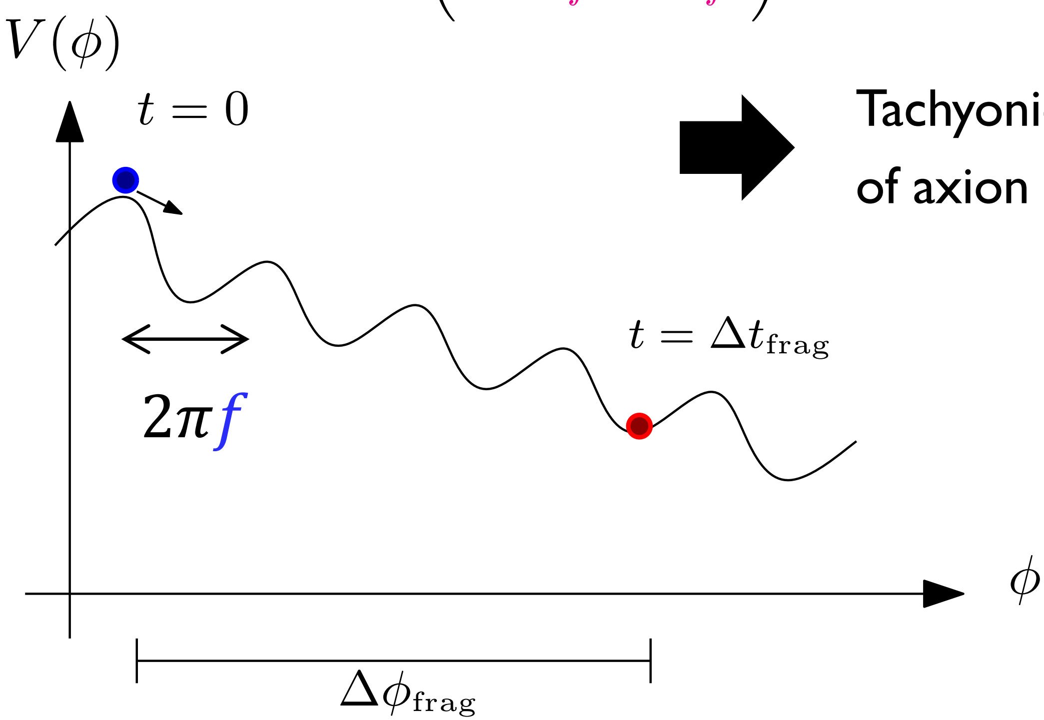
Variants: bosonic particle production as friction

"(Rel)axion fragmentation" N Fonseca, E Morgante, R Sato, G Servant '19

The wiggles = External driving force with frequency $\frac{\dot{\phi}}{f}$ on harmonic oscillator

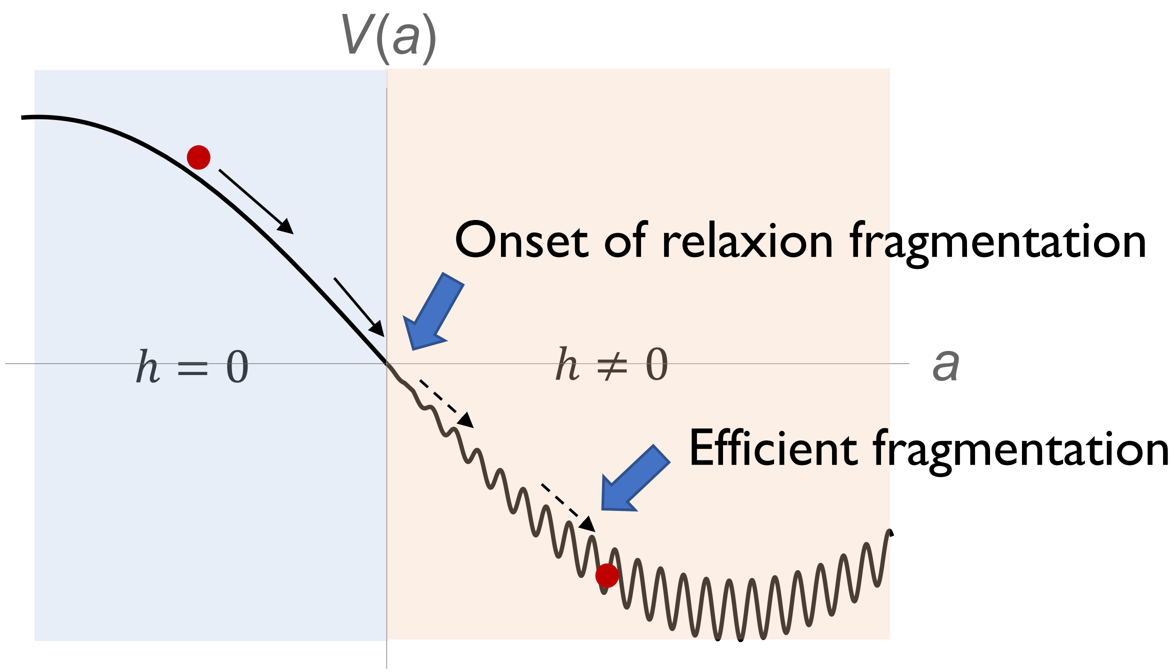
$$\delta\ddot{\phi}_k + \left(k^2 - \frac{\Lambda_{\rm br}^4}{f^2}\cos\frac{\dot{\phi}}{f}t\right)\delta\phi_k = 0$$

Non-homogeneous mode described by the Mathieu equation



Tachyonic growth (parametric resonance) of axion modes with $k \sim \frac{\dot{\phi}}{2f}$

Kinetic energy dissipation by (rel)axion particle production



Relaxion fast rolls down the potential so that it can finish the long excursion even in a post-inflationary period.

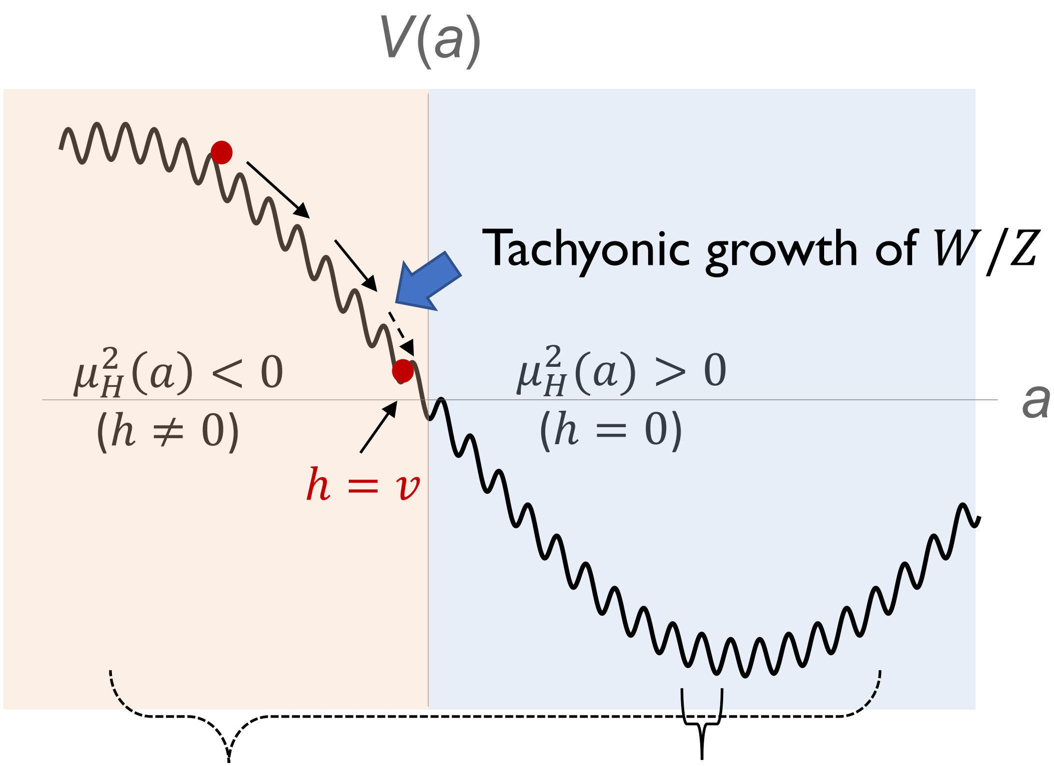
The price is a larger coupling hierarchy. (The overall slope has to be quite flat for the fragmentation to be effective.)

$$\frac{F}{f} \sim \max\left[\frac{\Lambda^4}{\Lambda_{\rm br}^8}\dot{\phi}_0^2\,,\ \ \frac{\Lambda^2}{v^2\Lambda_{\rm br}^8}\dot{\phi}_0^4\right] > \begin{cases} \frac{\Lambda^4}{v^4} > \mathcal{O}(10^4), & \dot{\phi}_0 \sim \Lambda_{\rm br}^2 \\ \frac{\Lambda^{10}}{v^{10}} > \mathcal{O}(10^{10}), & \dot{\phi}_0 \sim \Lambda^2 \end{cases}$$

Vector boson production as friction

A Hook, G Marques-Tavares '16

- Contrary to the previous models, here initially $\langle h\rangle\sim\Lambda$.
- Relaxion fast rolls scanning the Higgs mass and stops near $\langle h\rangle\sim 0$ by exponential production of light electroweak gauge bosons: needs another scale f_{WZ}



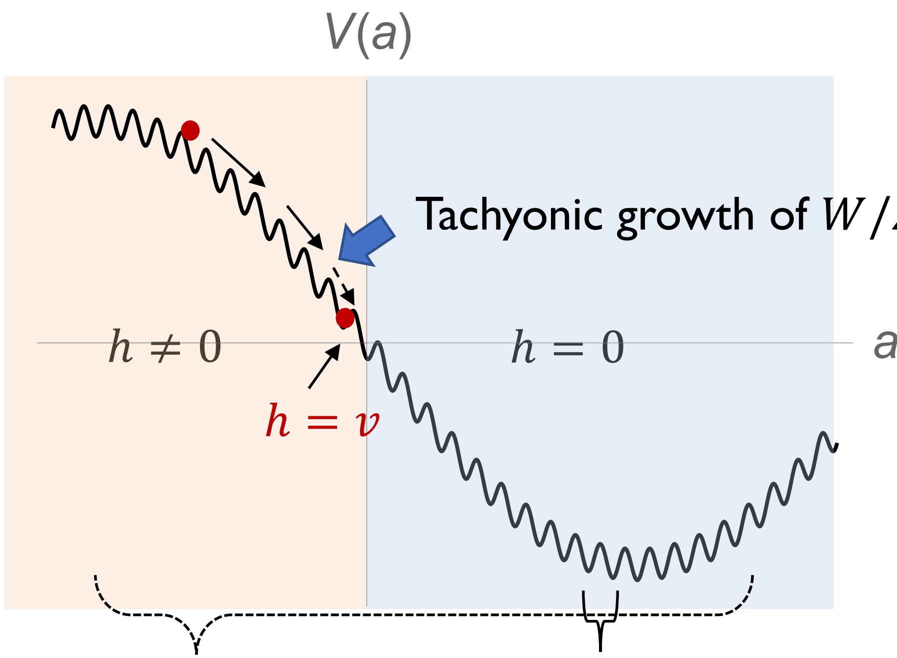
Long-periodic $\Lambda^4 \cos\left(\frac{a}{F}\right)$ Short-periodic $\Lambda_{\rm br}^4 \cos\left(\frac{a}{f} + \delta\right)$

$$\frac{a}{f_{WZ}}\left(W\widetilde{W} - B\widetilde{B}\right)$$

$$\ddot{A}_\pm + \left(k^2 + m_A^2 \pm k\frac{\dot{a}}{f_{WZ}}\right) A_\pm = 0$$

negative (tachyon) when

$$\frac{\dot{a}}{2f_{WZ}} > m_A$$



$\frac{a}{f_{WZ}}\left(W\widetilde{W} - B\widetilde{B}\right)$

Long-periodic $\Lambda^4 \cos\left(\frac{a}{F}\right)$ Short-periodic $\Lambda_{\rm br}^4 \cos\left(\frac{a}{f} + \delta\right)$

Here $\Lambda_{\rm br}^4$ is Higgs-independent, so it can be greater than the weak scale.

$$\frac{F}{f} > \max\left[\frac{\Lambda^4}{\Lambda_{\rm br}^4}, \frac{\Lambda^2}{v^2}\right] > \mathcal{O}(10^2)$$

Multiple local minima

Enough precision of the scanning

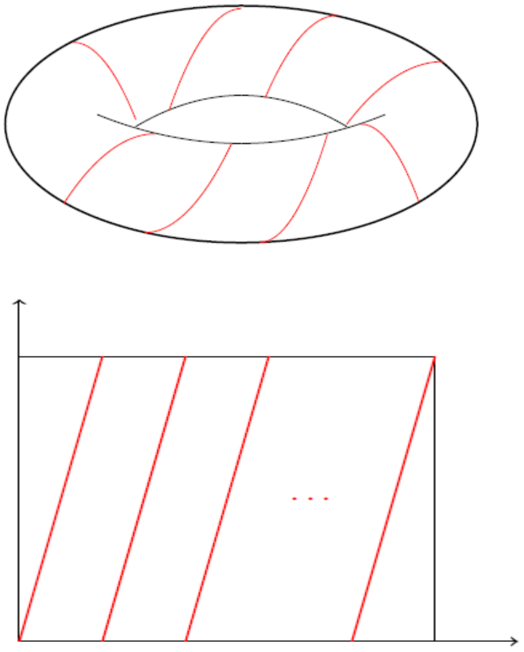
and

$$f_{WZ} \sim \frac{\dot{a}_0}{v} > \frac{\Lambda_{\rm br}^2}{v}$$

May need three hierarchically different axion scales.

Hierarchies from the axion landscape

In a landscape, a river usually winds the territory many times rather than flowing in a straight line.



Hierarchies from the axion landscape

General N axion lagrangian

$$\mathcal{L} = \frac{1}{2} f_{ij}^2 \partial_\mu \theta^i \partial^\mu \theta^j + \Lambda_X^4 \cos(q_i^X \theta^i + \delta^X) + \frac{k_{Ai}\theta^i}{32\pi^2} F^A \tilde{F}^A + c_{\psi i} \partial_\mu \theta^i \bar{\psi} \bar{\sigma}^\mu \psi + \cdots$$

$$\theta^i \equiv \frac{a_i}{f_i} \ (i = 1, 2, \ldots, N) \qquad \mathbb{Z}^N : \quad \theta^i \ \to \ \theta^i + 2\pi \ell^i \quad (\ell^i \in \mathbb{Z})$$

⬇ Integrating out heavy axions

Light axions with enhanced field ranges

KNP alignment

Jin E. Kim, Nilles, Peloso '04

$$\mathcal{L} = \Lambda^4 \cos\left(\frac{a}{f_a} - n_b \frac{b}{f_b}\right) + \frac{a}{f_a} F_1 \widetilde{F}_1 + \frac{b}{f_b} F_2 \widetilde{F}_2 \qquad n_b \in \mathbb{Z}$$

Integrating out the axion b $\quad \frac{b}{f_b} = \frac{1}{n_b}\frac{a}{f_a}$

$$\mathcal{L}_{\text{eff}} = \frac{a}{f_a} F_1 \widetilde{F}_1 + \frac{a}{n_b f_a} F_2 \widetilde{F}_2$$

Enhanced periodic field range of a : $\quad a \cong a + 2\pi n_b f_a$

But how much large n_b can be?

A model example

P Agrawal, JJ Fan, M Reece, LT Wang '17

$$\Lambda_{\rm hid}^4 \cos\left[2N\left(\frac{a}{f_a}+\frac{b}{f_b}\right)+\frac{a}{f_a}\right]+\left(\frac{a}{f_a}+\frac{b}{f_b}\right)G\widetilde{G}+(N^2-1)\frac{a}{f_a}F\widetilde{F}$$

Dynkin index of an adjoint rep of $SU(N)_{hid}$

Dimension of an adjoint rep of $SU(N)_{hid}$

Integrating out the axion b

$$\frac{1}{2N}\frac{a}{f_a}G\widetilde{G}+(N^2-1)\frac{a}{f_a}F\widetilde{F}$$

$$\frac{g_{a\gamma}}{g_{ag}} \sim N^3 = \mathcal{O}(10^2-10^3)$$

Landau pole constraint

For $SU(M)_{hid}$, the PQ scale has to be large for a large axion scale hierarchy in order to not hit a Landau pole of $U(1)_Y$ below M_P

M	m_Q (GeV)	f_a (GeV)	$r = 4M(M^2-1)$
3	1.3×10^4	6.0×10^5	96
4	5.5×10^{10}	3.3×10^{12}	240
5	3.6×10^{13}	2.7×10^{15}	480
6	1.0×10^{15}	9.1×10^{16}	840
7	7.3×10^{15}	7.5×10^{17}	1344
8	2.6×10^{16}	3.0×10^{18}	2016
9	5.9×10^{16}	7.7×10^{18}	2880
10	1.0×10^{17}	1.5×10^{19}	3960

P Agrawal, JJ Fan, M Reece, LT Wang '17

Clockwork

K Choi, Hyungjin Kim, S Yun '14
K Choi, SHI '15
Kaplan, Rattazzi '15

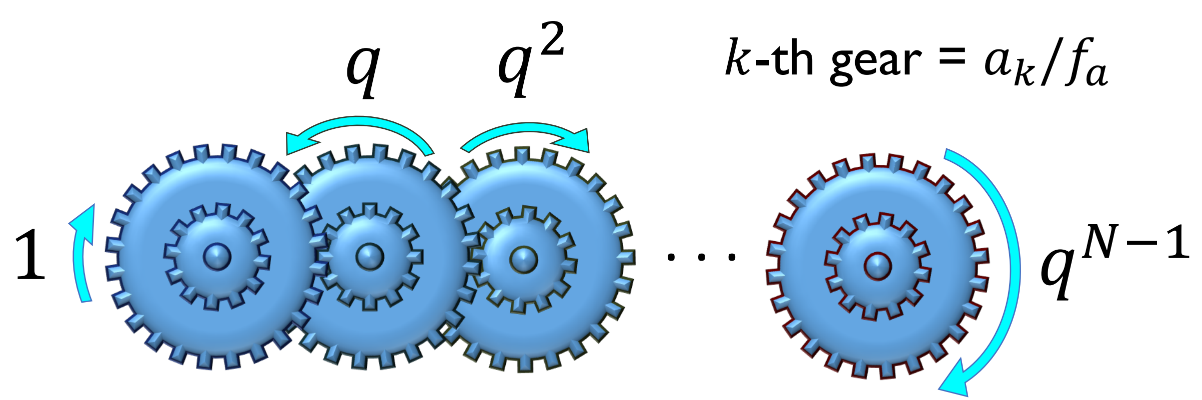
$$V(a_1, ..., a_N) = -\sum_{i=1}^{N-1} \Lambda_i^4 \cos\left(q\frac{a_i}{f_a} - \frac{a_{i+1}}{f_a} \right)$$

q : $O(1)$ integer

One flat direction $\Rightarrow$ $\frac{a_k}{f_a} \to \frac{a_k}{f_a} + q^k \alpha$

$\Rightarrow$ $a^{(0)} \approx a_N + \frac{1}{q} a_{N-1} + \cdots + \frac{1}{q^{N-1}} a_1$

k-th gear = a_k/f_a



$$\frac{1}{32\pi^2}\frac{a_N}{f_a}G\widetilde{G}+\frac{1}{32\pi^2}\frac{a_1}{f_a}G_H\widetilde{G}_H$$

$$\Downarrow \qquad a^{(0)} \approx a_N + \frac{1}{q}a_{N-1} + \cdots + \frac{1}{q^{N-1}}a_1$$

$$\frac{1}{32\pi^2}\frac{a^{(0)}}{f}G\widetilde{G}+\frac{1}{32\pi^2}\frac{a^{(0)}}{F}G_H\widetilde{G}_H \qquad f \sim f_a,\ \ F \sim q^N f_a$$

Periodic field range of $a^{(0)}$: $\quad a^{(0)} \cong a^{(0)} + 2\pi F$

The clockwork scheme provides a large integer responsible for a hierarchy between f and F without suffering the Landau pole problem.

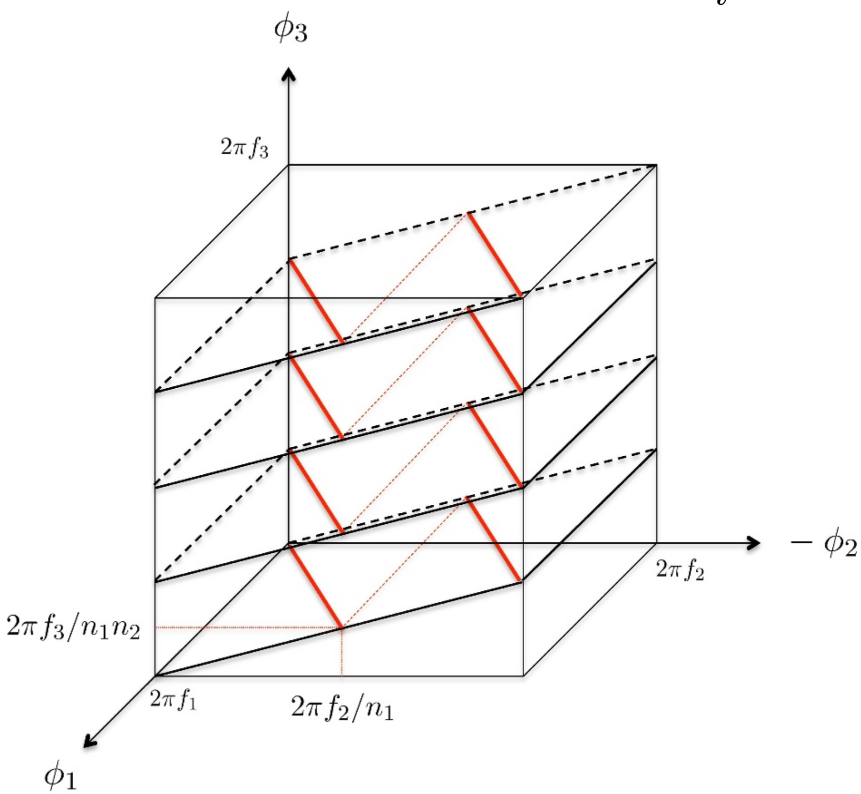
$$\frac{F}{f} = q^N \text{ (a large integer)} \gg 1$$

K Choi, Hyungjin Kim, S Yun '14

In the axion landscape, generically for $N_H \gg N_L$

Periodic length of light axions $\quad F \sim \mathcal{O}(n^{N_H/N_L})\, f_a$

$$a_L^{(k)} = \sum_i c_i^{(k)} a_i \qquad c_j^{(k)} \sim \mathcal{O}(n^{-N_H/N_L}) \text{ for some } j$$



Since the orthogonal direction to "many" heavy axions has to be "finely" determined by means of integer numbers, it inevitably involves a large integer number.

But this does not automatically guarantee a long periodic axion "potential". To obtain it, an axion potential has to be aligned to heavy axion directions.

Probability $\sim \mathcal{O}(f_a/F)$

Hierarchy from kinetic mixing

P Agrawal, JJ Fan, M Reece, LT Wang '17
K Fraser, M Reece '19

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu b)^2 + \epsilon\, \partial_\mu a \partial^\mu b + \frac{a}{f_a} G\widetilde{G} + \frac{b}{f_b}\left(G_H \widetilde{G}_H + F\widetilde{F}\right)$$

Redefining $b \to b - \epsilon a$

$$\mathcal{L} \simeq \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu b)^2 + \frac{a}{f_a} G\widetilde{G} + \left(\frac{b}{f_b} - \epsilon \frac{a}{f_b}\right)\left(G_H \widetilde{G}_H + F\widetilde{F}\right)$$

$m_b \gg m_a$

The axion b is integrated out : $\frac{b}{f_b} = \epsilon \frac{a}{f_b}$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a} G\widetilde{G}$$

$m_a \gg m_b$

Those couplings remain.

$$\boxed{\frac{g_{a\gamma}}{g_{ag}} \sim \epsilon \frac{f_a}{f_b}}$$

P Agrawal, JJ Fan, M Reece, LT Wang '17

$$\frac{g_{a\gamma}}{g_{ag}} \sim \epsilon \frac{f_a}{f_b} \quad \gg 1\ ?$$

i) Field-theoretic axion

$$\frac{1}{\Lambda^2}(\Phi_a^* \partial_\mu \Phi_a)(\Phi_b^* \partial^\mu \Phi_b) \;\Rightarrow\; \frac{f_a f_b}{\Lambda^2}\partial_\mu a \partial^\mu b$$

$$\epsilon \sim \frac{f_a f_b}{\Lambda^2} \quad \Rightarrow \quad \frac{g_{a\gamma}}{g_{ag}} \sim \frac{f_a^2}{\Lambda^2} \ll 1$$

ii) String-theoretic axion

$$\epsilon \sim \left(\frac{f_b}{f_a}\right)^{1/3} \gg \frac{f_b}{f_a} \quad \Rightarrow \quad \frac{g_{a\gamma}}{g_{ag}} \sim \left(\frac{f_a}{f_b}\right)^{2/3} \sim \sqrt{\frac{\tau_L}{\tau_S}} \gg 1$$

in a Large Volume Scenario

Conclusions

- Axions have quantized couplings due to its periodicity, and it sets a natural scale for the axion couplings as the periodic field range.
- Many well-motivated cosmological scenarios involving axions however require various hierarchies in axion couplings.
- Those hierarchies may originate from the axion landscape through certain patterns of mixing among multiple axions.