Neutrino Oscillations in Dark Matter

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Based on arXiv: 1909.10478 & 2012.09474
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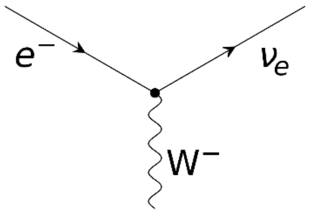
Outline

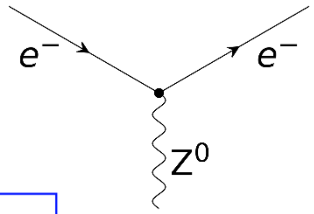
- Neutrino oscillations in vacuum
  - Flavors and masses
- Neutrino oscillations in hot/dense matter
  - Wolfenstein potential **Wolfenstein, 1978**
  - Adiabatic conversion of solar neutrinos **Mikheyev & Smirnov, 1985**
- Dispersion relation @ a finite temperature
  - Thermal-induced mass **Weldon, 1982**
    Mannheim 1988, Pal &Pham 1989, Nieves 1989, …
- Neutrino-DM interactions
  - High-momentum limit **Ge & Murayama 2019, Choi, Chun, & JKK 2019**
  - General solutions in various limits **Choi, Chun, & JKK 2020**

Neutrino oscillation in vacuum

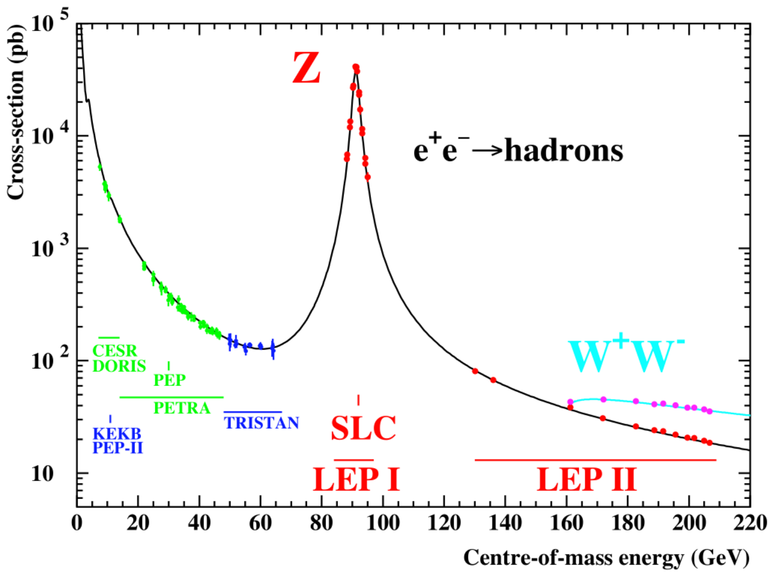
- Flavored neutrinos: Weak interaction eigenstates
  - Production & Detection

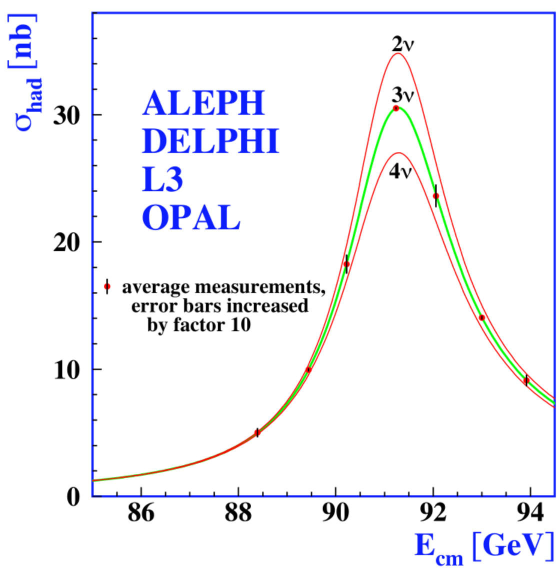
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$





- The number of neutrinos

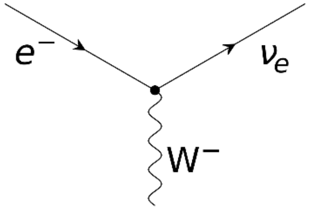


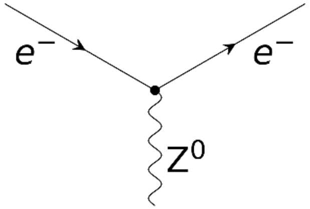


Neutrino oscillation in vacuum

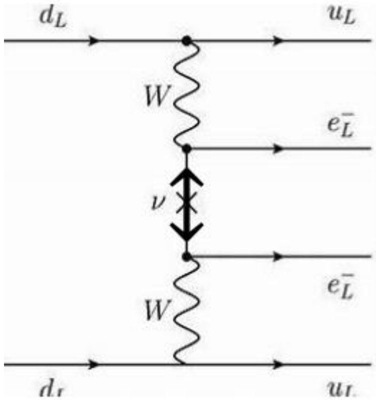
- Flavored neutrinos: Weak interaction eigenstates
  - Production & Detection

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$





- Massive neutrinos: Majorana VS Dirac
  - Dirac: $\nu \neq \nu^c \;\; (\nu^c \sim N)$
  - Majorana: $\nu = \nu^c \;\; (\nu_R \sim \nu_L^*)$
  - neutrinoless-double beta decay



Neutrino oscillation in vacuum

- Flavor eigenstates ≠ Mass eigenstates
  - $\nu_\alpha = U_{\alpha i}\nu_i$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot P_M$$

$$P_M = \mathrm{Diag}[1, e^{i\varphi_2}, e^{i\varphi_2}]$$

- Two-flavor neutrino propagation in vacuum

$$\nu_e \rightarrow \nu_\mu$$

$$|\nu_e(0)\rangle = c_\theta|\nu_1\rangle + s_\theta|\nu_2\rangle$$

$$U = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix}$$

$$|\nu_e(t)\rangle = c_\theta\, e^{i\phi_1}|\nu_1\rangle + s_\theta e^{i\phi_2}|\nu_2\rangle$$

$$\phi_i = E_i t - \boldsymbol{p}_i L$$

Neutrino oscillation in vacuum

- Ultra-relativistic limit ($t \cong L$)

$$E_i \approx \boldsymbol{p} + \frac{m_i^2}{2\boldsymbol{p}} \approx E + \frac{m_i^2}{2E}$$

$$\Delta\phi = \phi_2 - \phi_1 \approx \frac{\Delta m^2 L}{2E}$$

- Conversion probability

$$P_{e\mu} = \left|\langle \nu_\mu | \nu_e(t) \rangle\right|^2 = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Neutrino oscillation in vacuum

- Neutrino propagation Hamiltonian

$$i\frac{d}{dt}\psi = H\,\psi$$

$$\psi = \begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix};\ \begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

- Hamiltonian

$$H = \frac{m^2}{2E} \quad \Rightarrow \quad \begin{aligned} H_\nu &= \frac{M^+M}{2E} = \frac{U^+m^2U}{2E}, \\ H_{\bar{\nu}} &= \frac{MM^+}{2E} = \frac{Um^2U^+}{2E}\ \left(\frac{Vm^2V^+}{2E}\right) \end{aligned}$$

- Two-flavor evolution

$$H = \frac{\Delta m^2}{4E}\begin{bmatrix}-c_{2\theta} & s_{2\theta}\\ s_{2\theta} & c_{2\theta}\end{bmatrix} \rightarrow \ P_{e\mu} = \left|\langle\nu_\mu|e^{iHt}|\nu_e\rangle\right|^2$$

Neutrino oscillation in matter

- Wolfenstein Potential

  L. Wolfenstein, 1978

  - Coherent forward scattering of neutrinos leaving the medium unchanged must be taken into account

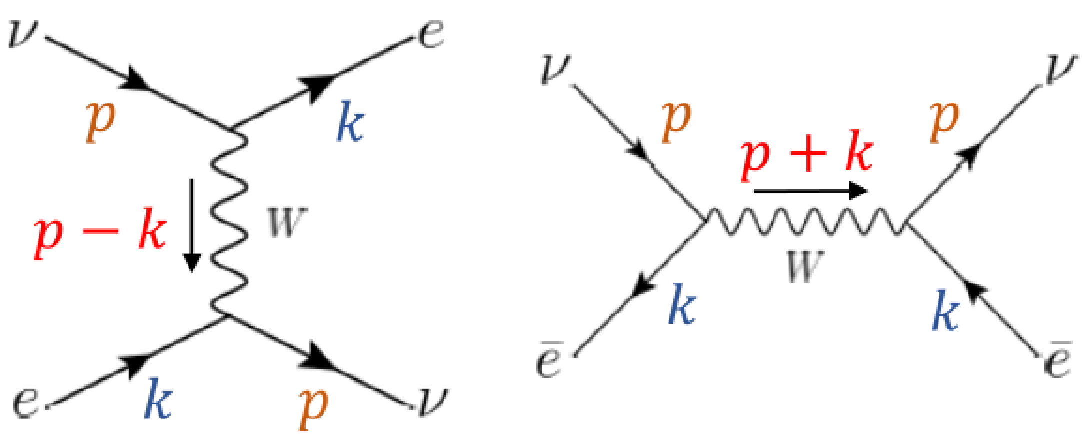
- Consider neutrino/anti-neutrino propagation in a general background
  - electron, positron

- Effective Hamiltonian

$$\mathcal{H}_{eff} = 2\sqrt{2}\, G_F m_W^2 \frac{\overline{\nu_{eL}} \gamma^\mu e_L\, \overline{e_L} \gamma_\mu \nu_{eL}}{m_W^2 - q^2}$$

Neutrino oscillation in matter

- **Coherent forward scattering**

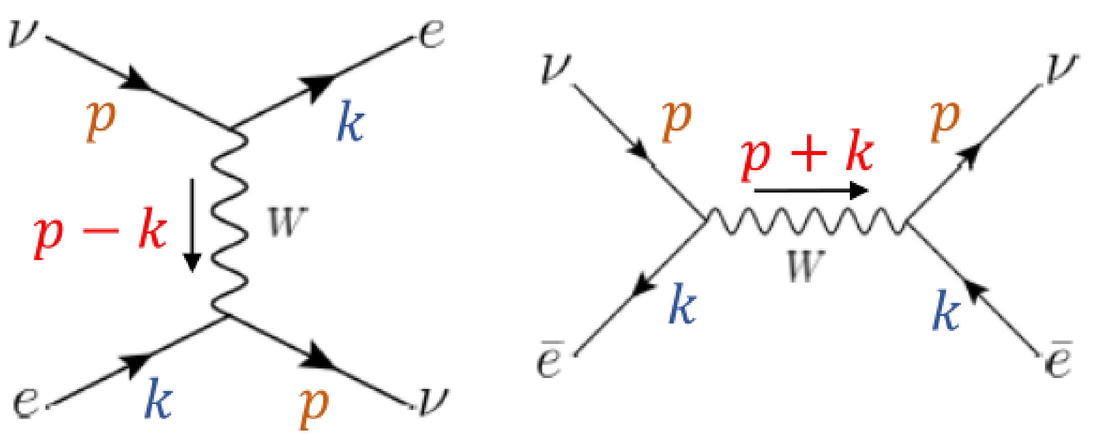


- $$\langle \mathcal{H}_\nu \rangle = \not{k}\,\sqrt{2} G_F m_W^2 \left[\frac{N_e/m_e}{m_W^2 - (p-k)^2} - \frac{N_{\bar{e}}/m_e}{m_W^2 - (p+k)^2} \right]$$

$$\langle \mathcal{H}_{\bar{\nu}} \rangle = \not{k}\,\sqrt{2} G_F m_W^2 \left[\frac{N_{\bar{e}}/m_e}{m_W^2 - (p-k)^2} - \frac{N_e/m_e}{m_W^2 - (p+k)^2} \right]$$

Neutrino oscillation in matter

- **Coherent forward scattering**



- Generalized matter potential

$$\epsilon \equiv \frac{N_e - N_{\bar{e}}}{N_e + N_{\bar{e}}}$$

$$V^{SM}_{\nu,\bar{\nu}} = \sqrt{2} G_F (N_e + N_{\bar{e}}) \frac{\pm\epsilon\, m_W^4 - 2 m_W^2 m_e E_\nu}{m_W^4 - 4 m_e^2 E_\nu^2}$$

Standard MSW effect

- Standard matter potential

L. Wolfenstein, 1978

  - $\epsilon = 1 \ (N_{\bar{e}} = 0)$
  - $m_W^2 \gg 2 m_e E_\nu$

$$\pm\sqrt{2} G_F N_e$$

- Matter potential @ high energy
  - $$V_{\nu,\bar{\nu}}^{SM} \approx \frac{\sqrt{2} G_F m_W^2 (N_e + N_{\bar{e}})}{2 m_e E_\nu}$$

Neutrino Oscillations without mass?

- Probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_M \sin^2\left(\frac{\Delta m_M^2 x}{4E}\right)$$

DM models

- A lot of models of DM and mediator

$$\mathcal{L}' = g_{\alpha i}\, \overline{f_{iL}} \gamma^{\mu} \nu_{\alpha L}\, X_{\mu} + h.c.$$

$$g_{\alpha i}\, \bar{f}_R \nu_{\alpha L}\, \phi_i + h.c.$$

$$g_{\alpha i}\, \overline{f_{iR}} \nu_{\alpha L}\, \phi + h.c.$$

$$g_{\alpha \beta}\, \overline{\nu^c_{\beta R}} \nu_{\alpha L}\, \phi + h.c.$$

$$g_{\alpha \beta}\, \overline{\nu^c_{\beta R}} \nu_{\alpha L}\, \phi + y\, \phi\, \bar{f}_R f_L + h.c.$$

General formulation

- Equation of motion in the momentum space

$$(\not{p} - \not{\Sigma})u_L = (M^\dagger + \bar{\Sigma}_0)u_R,$$

$$(\not{p} - \bar{\not{\Sigma}})u_R = (M + \Sigma_0)u_L,$$

  - $\not{\Sigma} \equiv \Sigma_\mu \gamma^\mu,\ \bar{\not{\Sigma}} \equiv \bar{\Sigma}_\mu \gamma^\mu,\ \Sigma_0$: corrections

R. F. Sawyer, 1999
P. Q. Hung, 2000
A. Berlin, 2016
S. F. Ge, S. Parke, 2019
H. Davoudiasl, G. Mohlabeng, M. Sulliovan, 2019
G. D'Amico, T. Hamill, N. Kaloper, 2018
F. Capozzi, I. Shoemaker, L. Vecchi 2018

- In a Lorenz invariant medium:
  - $\not{\Sigma} = \not{p}\,\Sigma_1 + \not{k}\,\Sigma_2;\ \bar{\not{\Sigma}} = \not{p}\,\bar{\Sigma}_1 + \not{k}\,\bar{\Sigma}_2,$

- Canonical basis of the kinetic term:

$$u_L \simeq \left(1 + \frac{\Sigma_1}{2}\right)\tilde{u}_L,$$

$$u_R \simeq \left(1 + \frac{\bar{\Sigma}_1}{2}\right)\tilde{u}_R,$$

General formulation

- **The Equation of Motion**

$$(\not{p} - \not{k}\Sigma_2)\tilde{u}_L = \tilde{M}^\dagger \tilde{u}_R,$$
$$(\not{p} - \not{k}\bar{\Sigma}_2)\tilde{u}_R = \tilde{M}\tilde{u}_L.$$

- Correction to the neutrino mass matrix

$$\tilde{M} \simeq \left(1 + \frac{\bar{\Sigma}_1}{2}\right) M \left(1 + \frac{\Sigma_1}{2}\right)$$

  - Original mass term is modified
  - For large parameter space, the mass correction is subdominant

General formulation

- **The Equation of Motion**

$$(\not{p} - \not{k}\Sigma_2)\tilde{u}_L = \tilde{M}^\dagger \tilde{u}_R,$$
$$(\not{p} - \not{k}\bar{\Sigma}_2)\tilde{u}_R = \tilde{M}\tilde{u}_L.$$

- Neutrino/ anti-neutrino Hamiltonian

$$H_\nu = E_\nu + \frac{\tilde{M}^\dagger \tilde{M}}{2E_\nu} + k^0\Sigma_2,$$
$$H_{\bar{\nu}} = E_\nu + \frac{\tilde{M}\tilde{M}^\dagger}{2E_\nu} + k^0\bar{\Sigma}_2,$$

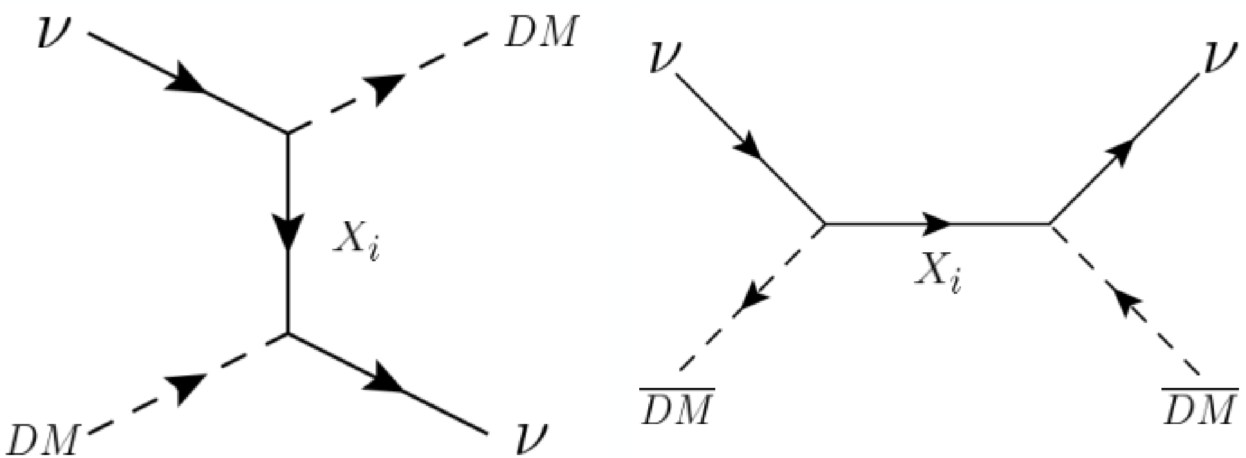
DM model

- Bosonic DM (ϕ) and fermionic messenger (X_i)

- Lagrangian

$$\mathcal{L}_{int} = g_{\alpha i}\bar{X}_i P_L \nu_\alpha \phi^* + h.c.$$

- **Coherent forward scattering**



General formulation

Ki-Young Choi, Eung Jin Chun, JKK

- Corrections

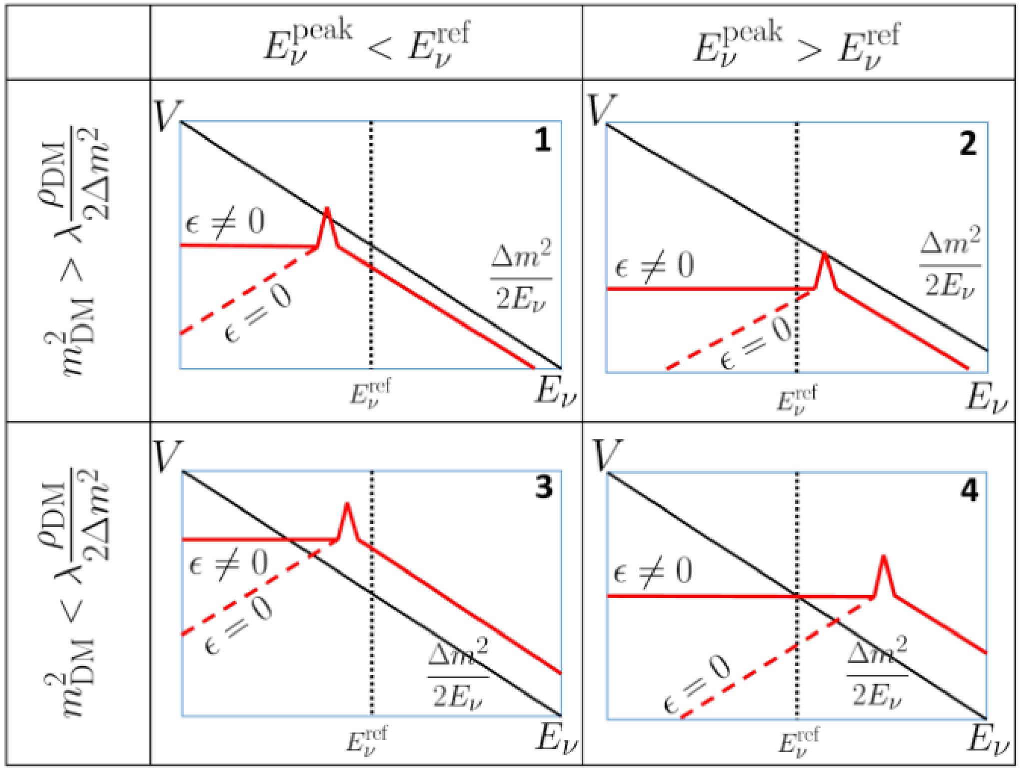
$$\Sigma_1\,(\text{or }\bar{\Sigma}_1) \simeq \frac{\lambda^{(T)}}{2}\frac{\rho_{DM}}{m_{DM}^2}\frac{\pm\epsilon\, 2m_{DM}E_\nu - m_X^2}{m_X^4 - 4m_{DM}^2E_\nu^2},$$

$$\Sigma_2\,(\text{or }\bar{\Sigma}_2) \simeq \frac{\lambda^{(T)}}{2}\frac{\rho_{DM}}{m_{DM}^2}\frac{\pm\epsilon\, m_X^2 - 2m_{DM}E_\nu}{m_X^4 - 4m_{DM}^2E_\nu^2},$$

- $\lambda_{\alpha\beta} \equiv g_{\alpha i}^{*}\, g_{\beta i}\ \ (\lambda^T = \lambda^*)$
- $\epsilon \equiv (\rho_{DM} - \rho_{\overline{DM}})/(\rho_{DM} + \rho_{\overline{DM}})$
- $\varepsilon = 0, m_X \to 0$: S-F Ge, Murayama 1904.02518

Neutrino potential

Ki-Young Choi, Eung Jin Chun, JKK



- Change of shape:

$$E_\nu^{peak} = \frac{m_X^2}{2m_{DM}}$$

- **Low Energy Limit:**

$$V_{\nu,\bar{\nu}}^{DM} \simeq \pm\epsilon\frac{\lambda^{(T)}}{4}\frac{\rho_{DM}}{m_{DM}^2 E_\nu^{peak}}.$$

- **High Energy limit:**

$$V_{\nu,\bar{\nu}}^{DM} \simeq \frac{\lambda^{(T)}}{2}\frac{\rho_{DM}/m_{DM}^2}{2E_\nu}$$

Two-flavor oscillation

- The effective Hamiltonian

$$\mathcal{H}_M = \frac{\Delta m^2}{4E} \begin{pmatrix} -(\cos 2\theta - x) & \sin 2\theta + y \\ \sin 2\theta + y & \cos 2\theta - x \end{pmatrix}$$

  - $x \equiv \frac{(V_{\mu\mu} - V_{\tau\tau})/2}{\Delta m^2/4E}$, and $y \equiv \frac{V_{\mu\tau}}{\Delta m^2/4E}$

- The mixing angle & mass squared difference in the medium

$$\sin^2 2\theta_M = \frac{(\sin 2\theta + y)^2}{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2},$$

$$\Delta m_M^2 = \Delta m^2 \sqrt{(\cos 2\theta - x)^2 + (\sin 2\theta + y)^2}.$$

Mass difference between ν&ν̄

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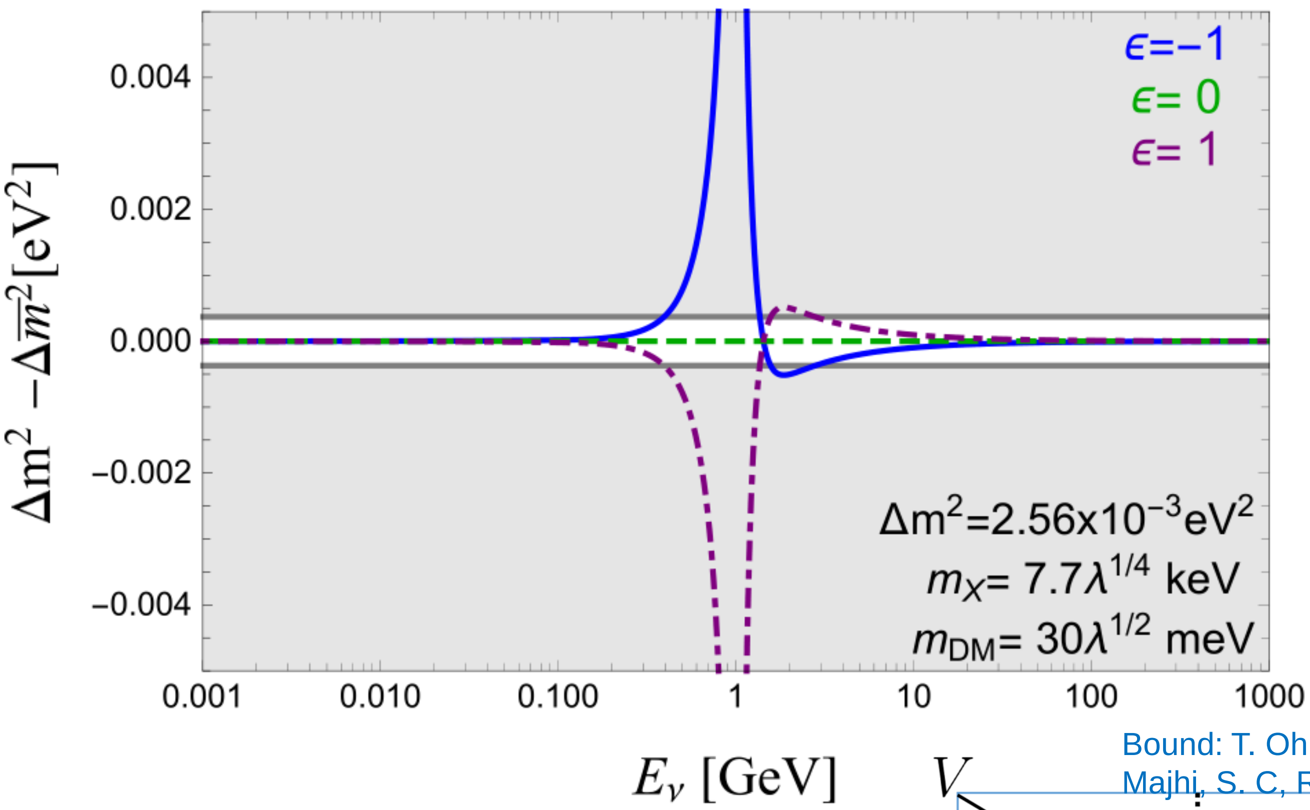
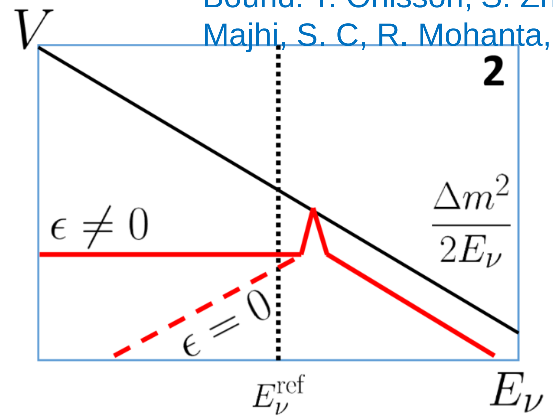
- 

Bound: T. Ohlsson, S. Zhou, 2015, R. Majhi, S. C, R. Mohanta, 2019



- $E_\nu^{Peak} = 1$GeV
- $x \to 0.75$ @ High Energy limit

Modified mixing angle

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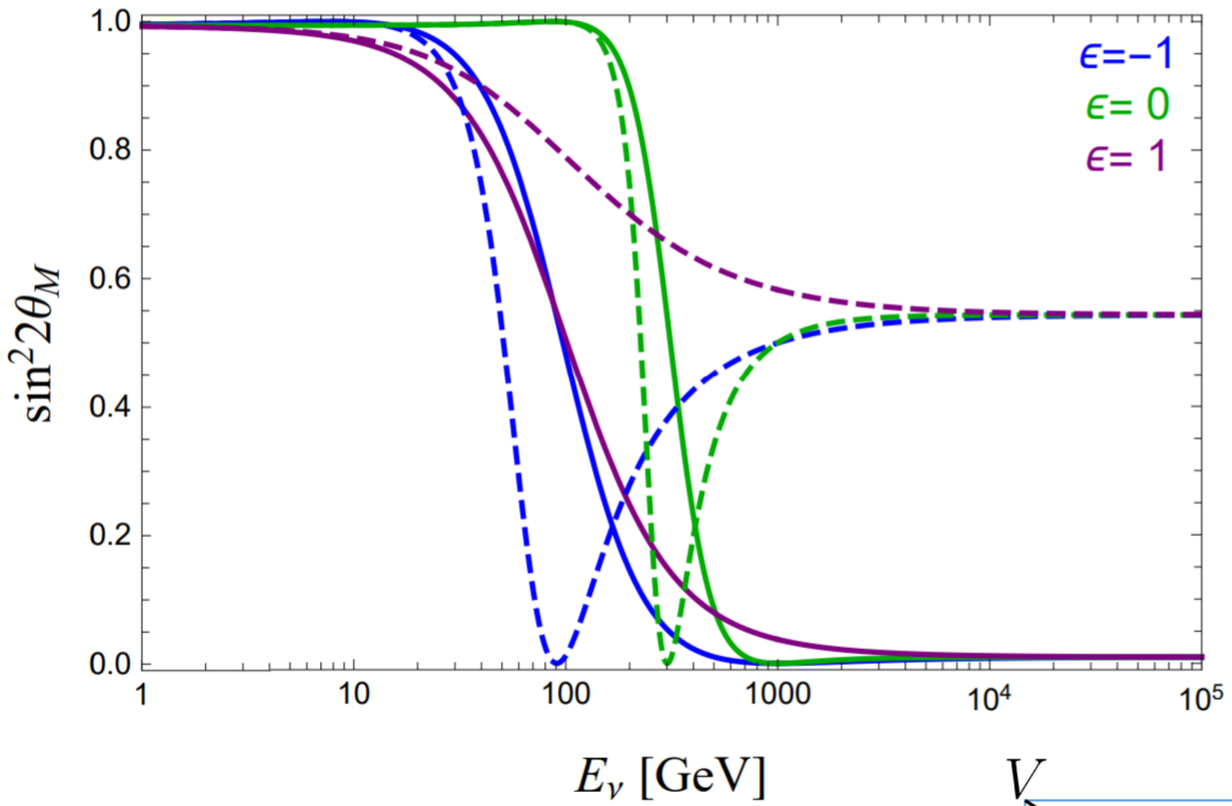
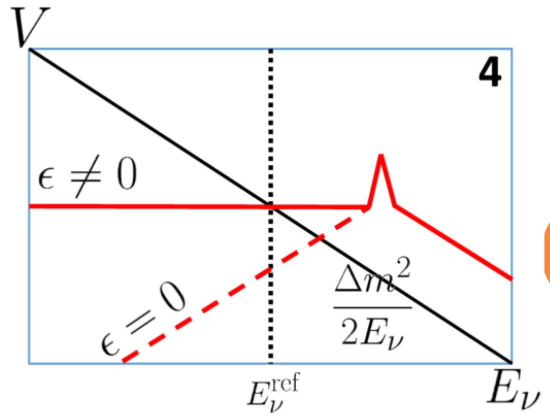
- 

- $E_\nu^{Peak} = 1\text{TeV}$
- Solid line: $x \to 10, y \to 0$
- Dashed line: $x \to 10, y \to 10$



DM assisted neutrino oscillation

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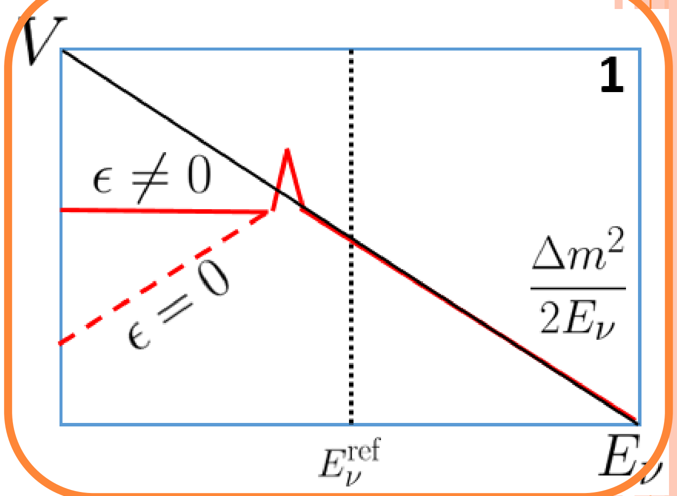
- In the case of $m_X^2 \ll 2m_{DM}E_\nu$ **(Peak energy << 1MeV)**

$$V_{\nu,\bar{\nu}}^{DM} \simeq \frac{\lambda^{(T)}}{2}\frac{\rho_{DM}/m_{DM}^2}{2E_\nu}$$

$$\approx \frac{3\times 10^{-3}\text{eV}^2}{2E_\nu}\lambda^{(T)}\left(\frac{20\text{meV}}{m_{DM}}\right)^2$$



- $$\lambda = \frac{2m_{DM}^2}{\rho_{DM}}U^*\text{diag}(\Delta m^2)U^T,$$

$$\simeq \begin{pmatrix} 0.026 & 0.091 & 0.085 \\ 0.091 & 0.381 & 0.408 \\ 0.085 & 0.408 & 0.477 \end{pmatrix}\left(\frac{20\text{meV}}{m_{DM}}\right)^2\left(\frac{0.3\,\text{GeV cm}^{-3}}{\rho_{DM}}\right)$$

- Standard neutrino oscillation can occur from the symmetric DM effect even for **massless neutrino**.

General Eqs for dispersion

- For $(m_\phi, \vec{0})$& $p = (E, p\hat{z})$, Equation of motion for ν_L, ν_R is solved by

$$L \cdot R - m_\nu^2 \pm H = 0$$

$$L \equiv p - \Sigma_L \equiv (L_0, \hat{z}L_z)$$
$$R \equiv p - \Sigma_R \equiv (R_0, \hat{z}R_z)$$

$$H \equiv L_z R_0 - R_z L_0$$

$$0 = (E^2 - \mathrm{p}^2)(1 - \Sigma_{1L})(1 - \Sigma_{1R}) - m_\nu^2 + m_\phi^2 \Sigma_{2L}\Sigma_{2R}$$
$$- m_\phi (E \pm \mathrm{p})\Sigma_{2L}(1 - \Sigma_{1R}) - m_\phi (E \mp \mathrm{p})\Sigma_{2R}(1 - \Sigma_{1L}) \xrightarrow{E \to \mathrm{p}} 2\mathrm{p}\, m_\phi \begin{cases} \Sigma_{2L}(1 - \Sigma_{1R}) \\ \Sigma_{2R}(1 - \Sigma_{1L}) \end{cases}$$

- Approximate solutions for

$$E^2_{\nu_1, \nu_2} \approx \mathbf{p}^2 + m_\nu^2 + m_\nu^2 (\Sigma_{1L}^{(0)} + \Sigma_{1R}^{(0)})$$
$$+ m_\phi \left(E_0 (\Sigma_{2L}^{(0)} + \Sigma_{2R}^{(0)}) \pm \mathbf{p} (\Sigma_{2L}^{(0)} - \Sigma_{2R}^{(0)}) \right)$$

Application to different types

- Wely ($m_\nu = 0$):

$$\left(E_{\nu,\bar{\nu}} - \mathrm{p}\right)\left(1 - \Sigma_{1L,R}\right) - m_\phi \Sigma_{2L,R} = 0 \;\Rightarrow\; E_{\nu,\bar{\nu}} \approx \mathrm{p} + m_\phi \Sigma_{2L,R}^{(0)}$$

- Majorana ($\nu = \nu^c$):

$$(u_R = v_L^c, v_R = u_L^c)$$

$$\Sigma_R^u(p) = [\Sigma_L^v(p)]^* = -[\Sigma_L^u(-p)]^* = [\Sigma_L^u(p)]^*_{\epsilon \to -\epsilon}$$

$$E_{\nu,\bar{\nu}} \approx E_0 + \frac{m_\nu^2}{2E_0}\left(\Sigma_{1L}^{(0)} + \Sigma_{1R}^{(0)}\right) + \frac{m_\phi}{2}\left(\left(\Sigma_{2L}^{(0)} + \Sigma_{2R}^{(0)}\right) \pm \frac{\mathrm{p}}{E_0}\left(\Sigma_{2L}^{(0)} - \Sigma_{2R}^{(0)}\right)\right)$$

- Dirac ($\nu \neq \nu^c, m_\nu \neq 0$) with $\Sigma_R = 0$:

$$(E^2 - \mathrm{p}^2)(1 - \Sigma_{1L}) - m_\nu^2 - m_\phi \Sigma_{2L}(E \pm \mathrm{p}) = 0$$

$$E_{\nu_L,\nu_R} \approx E_0 + \frac{m_\nu^2}{2E_0}\Sigma_{1L}^{(0)} + \frac{m_\phi}{2}\Sigma_{2L}^{(0)}\left(1 \pm \frac{\mathrm{p}}{E_0}\right)$$

Neutrino propagator

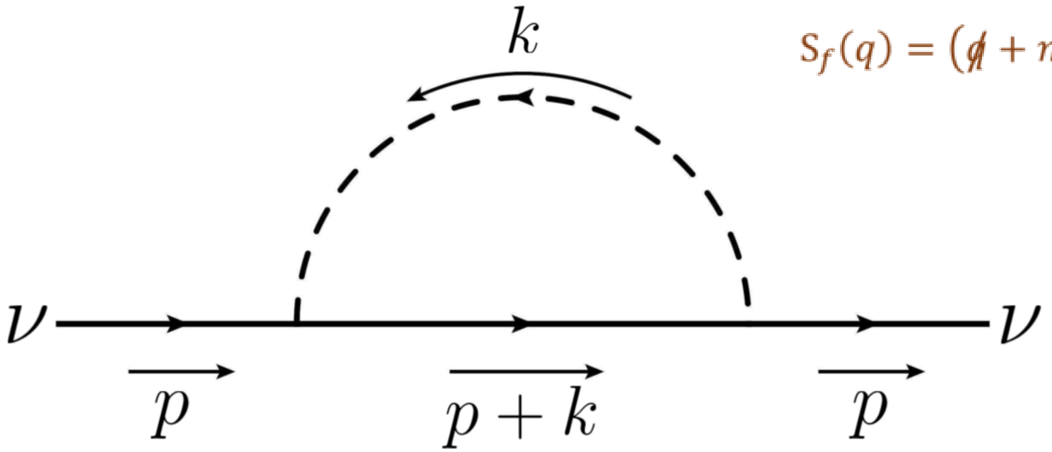
- Finite temperature/ density calculation

$$S_\nu^{-1}(p) = (\not{p} - \not{\Sigma}) = (\not{p} - \not{p}\Sigma_1 - \not{k}\Sigma_2)$$

$$\not{\Sigma} = i\, g g^\dagger \int \frac{d^4k}{(2\pi)^4} \Delta_\phi(k)\, S_f(p+k)$$

$$\Delta_\phi(k) = \frac{1}{k^2} - 2\pi i\, \delta\left(k^2 - m_\phi^2\right) f_\phi(k)$$

$$S_f(q) = (\not{q} + m_f)\left(\frac{1}{q^2 - m_f^2} + 2\pi i\, \delta\left(q^2 - m_f^2\right) f_f(q)\right)$$



Self-energy corrections

$$\Sigma_{1L}^{u,v}(p), \Sigma_{1L,R} = S(p) \pm \epsilon A(p)$$
$$\Sigma_{2L}^{u,v}(p), \Sigma_{2L,R} = A(p) \pm \epsilon S(p)$$

$$S(p) = \delta m^2 \frac{p^2 + m_\phi^2 - m_f^2}{\left(p^2 + m_\phi^2 - m_f^2\right)^2 - 4m_\phi^2 E^2}$$

$$A(p) = \delta m^2 \frac{-2m_\phi E}{\left(p^2 + m_\phi^2 - m_f^2\right)^2 - 4m_\phi^2 E^2}$$

$$\epsilon \equiv \frac{N_\phi - N_{\bar{\phi}}}{N_\phi + N_{\bar{\phi}}} \qquad \delta m^2 \equiv |g|^2 \frac{N_\phi + N_{\bar{\phi}}}{2\, m_\phi}$$

- Decoupling limit: $m_f^2 \gg \delta m^2, m_\nu^2, 2m_\phi \mathrm{p} \gg m_\phi^2$
- Heavy neutrino limit: $m_\nu^2 \gg \delta m^2, m_f^2, 2m_\phi \mathrm{p}$
- High momentum limit: $2m_\phi \mathrm{p} \gg m_f^2, \delta m^2, m_\nu^2$
- High density limit: $\delta m^2 \gg m_f^2, m_\nu^2, 2m_\phi \mathrm{p}$

Dispersion of Weyl/Majorana ν

- Decoupling limit: $m_f^2 \gg \delta m^2, m_\nu^2, 2m_\phi \mathrm{p} \gg m_\phi^2$
- Weyl: $m_\nu = 0$

  - High momentum limit:

$$E_{\nu,\bar{\nu}} \simeq \mathbf{p} + \frac{m_\nu^2}{2\mathbf{p}}\left(1 - 2\frac{\delta m_\nu^2}{m_f^2}\right) \mp \frac{\delta m_\nu^2}{m_f^2}\epsilon m_\phi$$

  - Zero momentum limit: $E_{\nu,\bar{\nu}} \approx m_\nu\left(1 - \frac{\delta m^2}{m_f^2}\right)$

Dispersion of Weyl/Majorana ν

- Heavy Neutrino limit: $m_\nu^2 \gg \delta m^2, m_f^2, 2m_\phi \mathrm{p}$

  - High momentum limit:

$$E_{\nu,\bar{\nu}} \simeq \mathbf{p} + \frac{m_\nu^2 + 2\delta m_\nu^2}{2\mathbf{p}} \pm \frac{\delta m_\nu^2}{m_\nu^2}\epsilon m_\phi$$

  - Zero momentum limit: $E_{\nu,\bar{\nu}} \approx m_\nu \left(1 + \frac{\delta m^2}{m_\nu^2}\right)$

Dispersion of Weyl/Majorana ν

- High momentum limit: $2m_\phi \mathrm{p} \gg m_f^2, \delta m^2, m_\nu^2$

$$E_{\nu,\bar{\nu}} \simeq \mathbf{p} + \frac{m_\nu^2 + \delta m_\nu^2}{2\mathbf{p}} \mp \epsilon \frac{\delta m_\nu^2 (m_\nu^2 - m_f^2)}{4 m_\phi \mathbf{p}^2}$$

- High density limit: $\delta m^2 \gg m_f^2, m_\nu^2, 2m_\phi \mathrm{p}$

$$E_{\nu,\bar{\nu}}^2 \approx \mathbf{p}^2 + m_M^2 + 2\delta m_\nu^2 \frac{m_f^2 \pm \epsilon m_\phi \mathbf{p}}{m_M^2 + \delta m_\nu^2}$$

$$m_M \equiv \frac{1}{2}\left(m_\nu + \sqrt{m_\nu^2 + 4\delta m^2}\right)$$

Dispersion of Dirac ν

- Decoupling limit: $m_f^2 \gg \delta m^2, m_\nu^2, 2m_\phi \mathrm{p} \gg m_\phi^2$

$$E_{\nu_1,\bar{\nu}_1} \approx \mathbf{p} + \frac{m_\nu^2}{2\mathbf{p}}\left(1 - \frac{\delta m_\nu^2}{m_f^2}\right) \mp \frac{\delta m_\nu^2}{m_f^2}\epsilon m_\phi$$

$$E_{\nu_2,\bar{\nu}_2} \approx \mathbf{p} + \frac{m_\nu^2}{2\mathbf{p}}\left(1 - \frac{\delta m_\nu^2}{m_f^2}\right),$$

- Heavy Neutrino limit: $m_\nu^2 \gg \delta m^2, m_f^2, 2m_\phi \mathrm{p}$

$$E_{\nu_1,\bar{\nu}_1} \approx \mathbf{p} + \frac{m_\nu^2 + \delta m_\nu^2}{2\mathbf{p}},$$

$$E_{\nu_2,\bar{\nu}_2} \approx \mathbf{p} + \frac{m_\nu^2 + \delta m_\nu^2}{2\mathbf{p}} \mp \frac{\delta m_\nu^2}{m_\nu^2}\epsilon m_\phi,$$

Dispersion of Dirac ν

- High momentum limit: $2m_\phi \mathrm{p} \gg m_f^2, \delta m^2, m_\nu^2$

$$E_{\nu_1,\bar{\nu}_1} \simeq \mathbf{p} + \frac{m_\nu^2 + \delta m_\nu^2}{2\mathbf{p}} \pm \epsilon \frac{\delta m_\nu^2}{2\mathbf{p}} \frac{m_f^2}{2m_\phi \mathbf{p}},$$

$$E_{\nu_2,\bar{\nu}_2} \simeq \mathbf{p} + \frac{m_\nu^2}{2\mathbf{p}} \left(1 \pm \epsilon \frac{\delta m_\nu^2}{2m_\phi \mathbf{p}}\right),$$

- High density limit: $\delta m^2 \gg m_f^2, m_\nu^2, 2m_\phi \mathrm{p}$

$$E_{\nu_1,\nu_2}^2 \approx \mathbf{p}^2 + m_D^2 + \delta m_\nu^2 \frac{m_f^2 - \epsilon m_\phi (E_0 \mp \mathbf{p})}{m_D^2},$$

$$\boxed{m_D \equiv \sqrt{m_\nu^2 + \delta m^2}} \neq m_M!!$$

Effective mass in dark medium

- Mass-squared:

$$\delta m^2 = \frac{gg^+}{2}\frac{\rho_{\phi+\bar{\phi}}}{2m_\phi}$$

$$E \xrightarrow{\mathrm{p}\to 0} m_{M,D}$$

$$m_M = \frac{1}{2}\left(m_\nu + \sqrt{m_\nu^2 + 4\delta m^2}\right)$$

$$m_D = \sqrt{m_\nu^2 + \delta m^2}$$

- Having an effect on neutrino oscillations:

$$\mathrm{p}^2 \gg 2m_\phi \mathrm{p} \gg \delta m^2, m_\nu^2 \Rightarrow E \sim \frac{m_\nu^2 + \delta m^2}{2\mathrm{p}}$$

$$\mathrm{p}^2 \gg \delta m^2, m_\nu^2 \gg 2m_\phi \mathrm{p} \Rightarrow E \sim \frac{m_{M,D}^2}{2\mathrm{p}}$$

Effective mass in dark medium

- Wely neutrinos can oscillates due to local DMs

$$\delta m_{loc}^2 = g^2 \frac{\rho_{DM}^{loc}}{2m_{DM}^2} \sim \Delta m_{\text{sol, atm}}^2 \quad \Longrightarrow m_\phi \approx 0.03 g \text{ eV} \left(\frac{2.5 \times 10^{-3} \text{ eV}^2}{\delta m_{loc}^2}\right)^{\frac{1}{2}}$$

- Cosmological limitation
  - Neutrino were heavier at earlier time

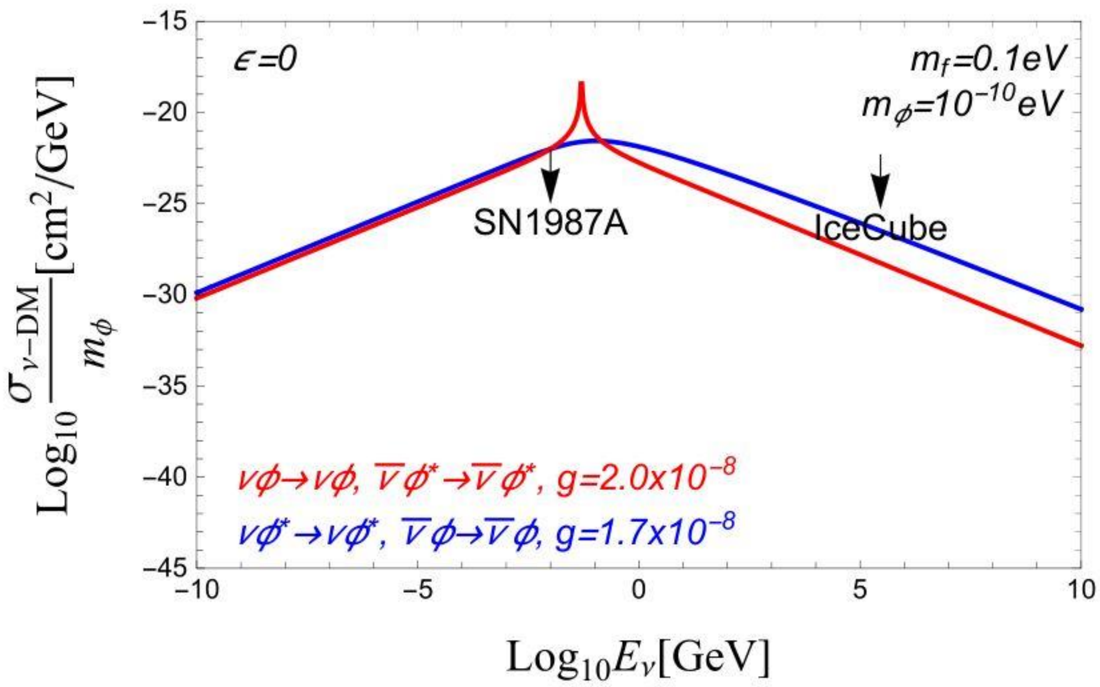
$$\delta m^2(z) = \delta m_{loc}^2 \frac{\rho_{DM}^0}{\rho_{DM}^{loc}} (1+z)^3 \approx 6650 \left(\frac{z}{1100}\right)^3 \delta m_{loc}^2$$

  - Around the CMB decoupling, neutrinos cannot be too heavy:

$$\delta m^2(z) \lesssim (0.1\text{eV})^2 \Rightarrow \delta m_{loc}^2 < 10^{-6}\text{eV}^2$$

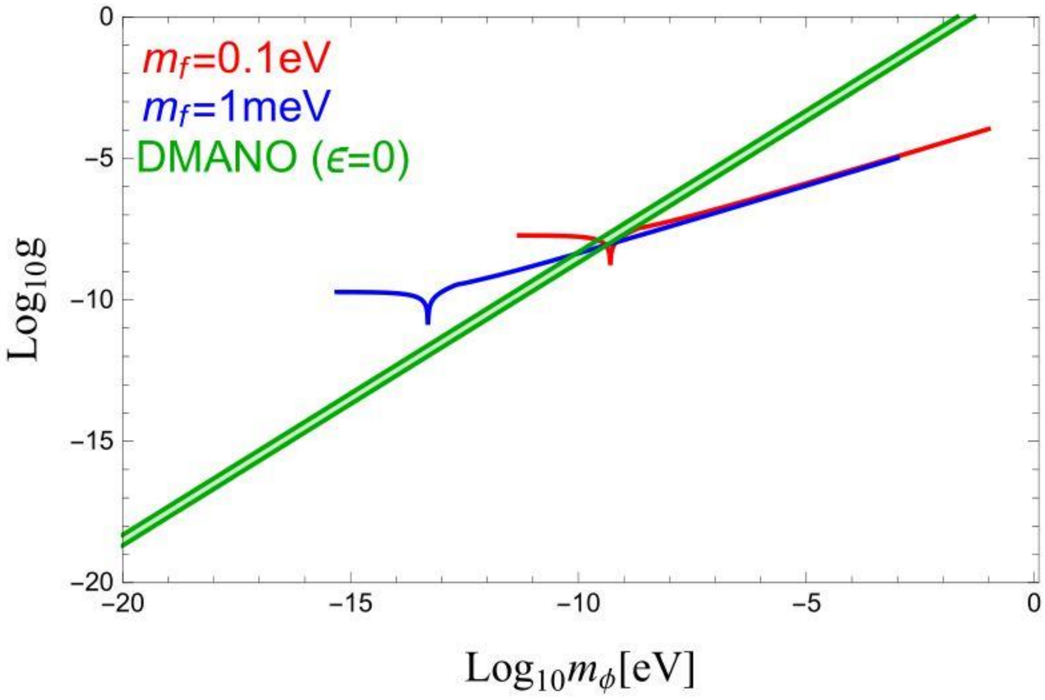
DM assisted neutrino oscillation

- Elastic scattering cross-sections of neutrinos with DM



DM assisted neutrino oscillation

- The allowed region for the DM-assisted neutrino oscillation



Conclusions

- General dispersion relations of neutrinos propagating in a dark medium are found up to 1st order in perturbation considering the 4 limiting cases
- Effective mass is generated and the neutrino oscillations may be due to the matter effect
- The scenario is limited by the cosmological mass generation and the neutrino-DM interaction is constrained by the astrophysical neutrino observations
- UV-completion for the origin of scalar DM?

Conclusions

- General dispersion relations of neutrinos propagating in a dark medium are found up to 1st order in perturbation considering the 4 limiting cases

Thank you very much.