Production of Hidden Photon Dark Matter: Recent Developments

Kazunori Nakayama(U.Tokyo)

2021/2/2, "Dark Matter as a Portal to New Physics", APCTP

Contents

1. Introduction

2. Vector coherent oscillation DM

3. Gravitational production of vector DM

1. Introduction

Light bosonic dark matter

Light boson is a candidate of dark matter

QCD axion, dilaton, string axion, hidden photon, …

Many light fields from string theory

"String Axiverse" [Arvanitaki et al (2009)]

Light mass can be ensured by symmetry

Axionic scalar: shift symmetry $\phi \rightarrow \phi + C$

Vector: gauge symmetry $A_\mu \to A_\mu + \partial_\mu \chi$

I will focus on vector boson (hidden photon)

[Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012)] \mathbf{L} . The exclusion regions labelled "Coulomb", "CAST" and "Solar Lifetime", "CAST" and "Solar Lifet

2

 $F_{\mu\nu}X^{\mu\nu}$

Production of Hidden Photon DM

- **Production mechanisms:**
	- Axionic coupling

[Agrawal, Kitajima, Reece, Sekiguchi, Takahashi (2018), Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018), Co, Pierce, Zhang, Zhao (2018)]

- Production from dark Higgs [Dror, Harigaya, Narayan (2018)]
- Production from cosmic string [Long, Wang (2019)]
- Coherent oscillation [Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012), KN (2019)]
- Gravitational production

[Graham, Mardon, Rajendran (2015), Ema, KN, Tang (2019)]

Production of Hidden Photon DM

- **Production mechanisms:**
	- Axionic coupling

[Agrawal, Kitajima, Reece, Sekiguchi, Takahashi (2018), Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018), Co, Pierce, Zhang, Zhao (2018)]

- Production from dark Higgs [Dror, Harigaya, Narayan (2018)]
- Production from cosmic string [Long, Wang (2019)]

Coherent oscillation [Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012), KN (2019)]

Gravitational production

[Graham, Mardon, Rajendran (2015), Ema, KN, Tang (2019)]

2. Vector coherent oscillation

Overview of history

<u>Nelson, Scholtz (2011)</u>

Vector coherent oscillation DM in minimal model

Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012)

Minimal model does not work.

Vector coherent oscillation DM in curvature coupling model

KN (2019)

Curvature coupling model does not work.

Vector coherent oscillation DM in kinetic function model

KN (2020)

Kinetic function model severely constrained by observation

2. Vector coherent oscillation

X 2-1. Minimal massive vector model

2-2. Curvature coupling model

2-3. Kinetic function model

2-4. Observational constraints

Scalar coherent oscillation

• Action
$$
S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right)
$$

\n• Eq. of.m $\ddot{\phi} + 3H\dot{\phi} + m^2 \phi = 0$
\n ϕ to ϕ to

Energy density

$$
\rho_{\phi} = T_{00} = \frac{1}{2} \left(\dot{\phi}^2 + m^2 \phi^2 \right) \simeq \frac{m^2 \phi_{\text{ini}}^2}{2} \left(\frac{a_0}{a(t)} \right)^3
$$

$$
\boxed{\frac{\Omega_\phi}{\Omega_{\rm DM}}\sim \left(\frac{m}{10^{-27}\,{\rm eV}}\right)^{1/2}\left(\frac{\phi_{\rm ini}}{M_P}\right)^2}
$$

(Light) scalar coherent oscillation is good DM candidate

Constraints

- **•** Halo structure $m \gtrsim 10^{-22}$ eV
- Isocurvature fluctuation $S_{\rm DM} \sim$ \bullet

$$
\rho_{\rm DM} \sim \frac{2 \delta \phi_{\rm ini}}{\phi_{\rm ini}} \sim \frac{H_{\rm inf}}{\pi \phi_{\rm ini}} \lesssim 9 \times 10^{-6}
$$

Vector coherent oscillation? **Vector**

• Action
$$
S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{MN} g^{KL} \mathcal{F}_{MK} \mathcal{F}_{NL} - \frac{1}{2} m^2 g^{MN} A_M \mathcal{A}_N \right)
$$

• Eq. of.m
$$
\ddot{A}_i + H \dot{A}_i + m^2 A_i = 0
$$

The action of the massive vector field *A^M* is given by

 $\overline{}$

but…

2. Vector coherent oscillation

2-1. Minimal massive vector model

X 2-2. Curvature coupling model

2-3. Kinetic function model

2-4.Observational constraints

Curvature coupling model **Curvature**

The observation in the previous subsection may lead us to interact a curvature coupling \sim

• Action
$$
\Delta \mathcal{L} = \frac{1}{2} \xi R g^{MN} A_M A_N
$$

\n
$$
R = 6(2H^2 + \dot{H})
$$

\n• Eq. of.m
$$
\ddot{A}_i + 3H\dot{A}_i + \left(m^2 + \left(\frac{1}{6} - \xi\right)R\right)\overline{A}_i = 0.
$$

[Turner, Widrow (1988) for Magnetogenesis]

 $c_{\rm max}$ the vector field was considered in the vector field was considered in the context of magnetogenesis ~ 1 • Taking $\xi = \frac{1}{6}$ \longrightarrow Same eq. of m as scalar field $L \rightarrow$ Cohoront oscillation of voctor field. the correlation of the community of the some finite α and α after the some finite α $[Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012)]$ 1 Taking $\xi = \frac{1}{6}$ \longrightarrow Same eq.of.m as scalar field Coherent oscillation of vector field

Taking account of not only zero mode but also fluctuation *^k*² ⁺ *^a*²*m*² *.* (37) Substituting the continues of the action, we find the the transverse model into continued model model model model model model model model with the second model model model model with the second model model model model mode *A*0(*k, t*) = *ⁱ k* + *a*₂*m*₂ • Taking account of not only zero mode but also fluctuation *Laking account of not only zero me*

$$
S = S_T + S_L \qquad \quad \vec{A} = \vec{A}_T + \hat{k} A_L
$$

$$
S_T = \int \frac{d^3 k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \vec{A}_T|^2 - (k^2 + a^2 \overline{m}^2) |\vec{A}_T|^2 \right) \qquad \text{: Transverse}
$$

$$
S_L=\int\frac{d^3kd\tau}{(2\pi)^3}\frac{1}{2}\left(\frac{a^2\overline{m}^2}{k^2+a^2\overline{m}^2}|\partial_\tau A_L|^2-a^2\overline{m}^2|A_L|^2\right)\text{ :Longitudinal}
$$

 $\sqrt{m^2}$ \overline{m}^2 $m^2 \equiv m^2 - \xi R \simeq -2H^2$

Wrong sign of longitudinal kinetic term: **Ghost instability !** [KN (2019); Himmetoglu, Contaldi, Peloso (2008) in the context of vector curvaton] $\frac{1}{2}$ *|*@⌧*A* f*L|* f*L| f f*¹
*f*² *Mrong sign of longitudinal kinetic term: Ghost i*² ⌘ *^k*² ⁺ *^m*² $\mathsf{KNI}\left(2019\right)$: Himmetr 'rong s *|*@⌧*A* f*L|* ² !² *^L|A* f*L|* $kinetic term$ *<u>Ghost</u> i A
<i>A* $\textsf{K}\textsf{N}$ (2019); Himmetc

Taking account of not only zero mode but also fluctuation *^k*² ⁺ *^a*²*m*² *.* (37) Substituting the continues of the action, we find the the transverse model into continued model model model model model model model model with the second model model model model with the second model model model model mode *A*0(*k, t*) = *ⁱ k* + *a*₂*m*₂ • Taking account of not only zero mode but also fluctuation *Laking account of not only zero me*

$$
S = S_T + S_L \qquad \quad \vec{A} = \vec{A}_T + \hat{k} A_L
$$

$$
S_T = \int \frac{d^3 k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \vec{A}_T|^2 - (k^2 + a^2 \overline{m}^2) |\vec{A}_T|^2 \right) \qquad \text{: Transverse}
$$

$$
S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(\frac{a^2 \overline{m}^2}{k^2 + a^2 \overline{m}^2} |\partial_\tau A_L|^2 - a^2 \overline{m}^2 |A_L|^2 \right) \text{ :} \text{Longitudinal}
$$

$$
\overline{m}^2 \equiv m^2 - \xi R \simeq -2H^2
$$

Wrong sign of longitudinal kinetic term: **Ghost instability !** [KN (2019); Himmetoglu, Contaldi, Peloso (2008) in the context of vector curvaton] $\frac{1}{2}$ *|*@⌧*A* f*L|* f*L| f f*¹
*f*² *Mrong sign of longitudinal kinetic term: Ghost i*² ⌘ *^k*² ⁺ *^m*² $\mathsf{KNI}\left(2019\right)$: Himmetr 'rong s *|*@⌧*A* f*L|* ² !² *^L|A* f*L|* $kinetic term$ *<u>Ghost</u> i A
<i>A* $\textsf{K}\textsf{N}$ (2019); Himmetc

• Curvature coupling means tachyonic mass of vector field

$$
\mathcal{L} = \frac{1}{2} \xi R g^{MN} A_M A_N \qquad \xi = \frac{1}{6}
$$

 \bullet Ref. in the Hubble mass term in the equation of motion and incorrectly derived coherently derived cohe **o** fractiyon Tachyonic mass <>>
Negative kinetic term in Higgs picture

$$
\mathcal{L} = +|D_M\Phi|^2 = +e^2|\Phi|^2 A_M A^M
$$

Higgs & NG mode (longitudinal vector boson) become ghost

2. Vector coherent oscillation

2-1. Minimal massive vector model

2-2. Curvature coupling model

X 2-3. Kinetic function model

2-4.Observational constraints

Vector with kinetic function $i + h$ *f* 2 *|A ^T* (*k*)*|* ² + (*f* ² *k*² + *a*² *m*²)*|A ^T* (*k*)*|* **VVILII NIIICLI**

[KN (2019)] *(kN*(2019)

$$
S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} f^2(\phi) g^{MN} g^{KL} \mathcal{F}_{MK} \mathcal{F}_{NL} - \frac{1}{2} m^2 g^{MN} A_M \mathcal{A}_N \right)
$$

Eq. of.m $\frac{\ddot{a}}{\ddot{A}_i} + 3H\dot{A}_i + \left(\frac{m^2}{f^2} + 2H^2 + \dot{H} - H\frac{\dot{f}}{f} - \frac{\ddot{f}}{f} \right) \overline{A}_i = 0$

[Ratra (1992) for magnetogenesis; Dimopoulos, Karciauskas, Wagstaff (2009) for vector curvaton] \overline{a}

• If $f^2 \propto a^{\alpha}(t)$: $\alpha = -4$ or 2 Hubble mass term is cancelled. *Aⁱ* + *f* ² \longrightarrow Vector coherent oscillation is nossible discharged the equation of motion becomes the equation becomes the equation becomes the equation becomes the e
The equation becomes the equation becomes the equation becomes the equation of the equation of the equation of **Vector coherent oscillation is possible!**

Concrete form of kinetic function ⁴ ⌘*^µ*⇢ **2** *f* 2

$$
f(\phi) = \exp\left(-\frac{\gamma}{2M_P^2} \int \frac{V}{V_\phi} d\phi\right) \qquad (\phi : \text{inflation})
$$

 \bullet Chaotic inflation **e** Chaotic inflation:

$$
f(\phi) = \exp\left(-\frac{\gamma}{4n} \frac{\phi^2}{M_P^2}\right), \qquad V(\phi) = \frac{\lambda \phi^n}{n}
$$

• Hilltop inflation

$$
f(\phi) = \exp\left(-\frac{\gamma}{4n(n-2)}\frac{v^n}{M_P^2 \phi^{n-2}}\right), \qquad V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{v}\right)^n\right]^2
$$

$$
\overline{A_i} = \text{const}
$$

$$
\overline{A_i} = \text{const} \qquad H < m \quad \overline{A_i} \sim a^{-\frac{3}{2}} \cos(mt)
$$

Vector coherent oscillation !

O During inflation: \bullet During in that ion:

˙ '

 \mathbf{r}

$$
\overline{A} \propto a^{(|1+\alpha|-3)/2} = \begin{cases} a^{\alpha/2-1} & \text{for } 1+\alpha > 0\\ a^{-\alpha/2-2} & \text{for } 1+\alpha \le 0 \end{cases}
$$

Vector energy density increases during inflation 4*HM*² *^P V* 4

to the kinetic energy in (10). Since the *C*¹ term is dominant for 1 + ↵ *>* 0, actually the → *→ Backreaction to the inflaton becomes important* \rightarrow reaction to the inflaton becomes important

scales as ⇢*^A* / *f* ²*a*⁴ / *a*↵⁴. Below we consider the case of *<* 4 and *>* 2 separately. **Anisotropic inflation** happens *^P V V* = ³*^H ^V.* (15)

[Watanabe, Kanno, Soda (2009)] \mathbf{L} in order for the solution to be consistent with slow-roll equation of (14), the energy density of \mathbf{L} Innocent promission to promission to be consistent with supply the energy density of $\lceil W_{\text{atanab}} \rceil$ (2009)

Vector energy density is saturated at

$$
\frac{\rho_A}{\rho_\phi} = -2\epsilon_V \frac{\gamma + 4}{\gamma^2} \equiv R_A
$$

→ **Vector coherent oscillation after inflation! Ference inflation energy density.** The summarized energy of the summarized in the summarized of the summarized o **Note on the case of** $\alpha > 2$

• During inflation: $\overline{A} \propto a^{\alpha/2-1}$ It is consistent with the observed DM abundance (' 4 ⇥ 10¹⁰ GeV in terms of the energy I is consistent with the observed DM abundance (' 4 \pm $\frac{1}{2}$ \pm $\frac{1}{2}$

Vector energy density does not increase: $\rho_A \propto a^{-\alpha-2}$

 \longrightarrow No backreaction to the inflaton *AA*²

● To maintain vector condensate during inflation: after inflation, the final vector boson abundance is evaluated as Θ after inflation, the final vector boson abundance is evaluated as \mathbf{c}

$$
\frac{m_A}{f} \ll H_{\text{inf}} \longrightarrow \frac{\rho_A}{s} \ll 10^{-13} \,\text{GeV} \left(\frac{10^{-22} \,\text{eV}}{m_A} \right)^{1/2} \left(\frac{H_{\text{inf}}}{10^{14} \,\text{GeV}} \right)
$$

• Impossible to explain observed DM abundance

Loophole : introduce mass function $\mathcal{L} \sim -$ 1 2 $\mathcal{L}\sim-\frac{1}{2}h^2(\phi)m_A^2A_\mu^2$ ● Numerical calculation *^A*⌘*^µ*⌫*AµA*⌫ \overline{C} $\frac{10}{100}$ $(\alpha = -5)$

2

 $m_\phi^2 \phi^2$

• chaotic inflation

 \overline{V}

hilltop inflation

$$
V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{v}\right)^n \right]^2
$$

 $10⁷$

10-16

10

10

10-10

 $1₀$

 $1₀$

10

10-10

 10

 $1₀$

 10

 $1₀$

Vector DM abundance refeating, the radiation-dominated universe begins. The radiation-dominated by the ratio is denominated by the r
Property of the ratio is denominated by the ratio is described by the ratio is described by the ratio is desc completed at *H* = where denotes the inflaton decay width. After the completion of

⇢*A s* $\overline{\mathbf{C}}$ ✓⇢*^A* ⇢ $\overline{\mathbf{m}}$ *s* .
f I 3*R^A* $\overline{}$ \overline{a} ⇡²*g*⇤ The equation of motion of the vector field *A* ⌘ *fA/a* is given by Vector dynamics after inflation

$$
\ddot{\overline{A}} + 3H\dot{\overline{A}} + \left(m_A^2 + \frac{1 - 3w}{2}H^2\right)\overline{A} = 0
$$

↑

▲ *Hinal yester soborant oscillation abundant A* = *d*1*a*¹ + *d*2*a*(3*w*1)*/*² Final vector coherent oscillation abundance

$$
\frac{\rho_A}{s} \simeq \begin{cases} 3.7 \times 10^{-10} \,\text{GeV} \left(\frac{R_A}{0.1}\right) \left(\frac{m_A}{10^{-8} \,\text{GeV}}\right)^{1/2} \left(\frac{10^{14} \,\text{GeV}}{H_{\text{inf}}}\right)^{2/3} \left(\frac{T_R}{10^6 \,\text{GeV}}\right)^{4/3} & \text{for } m_A < \Gamma_\phi \\ 3.5 \times 10^{-10} \,\text{GeV} \left(\frac{R_A}{0.1}\right) \left(\frac{m_A}{1 \,\text{GeV}}\right)^{2/3} \left(\frac{10^{14} \,\text{GeV}}{H_{\text{inf}}}\right)^{2/3} \left(\frac{T_R}{10 \,\text{GeV}}\right) & \text{for } m_A > \Gamma_\phi \end{cases}
$$

 \blacksquare DM abundance can be explained in kinetic function model

2. Vector coherent oscillation

2-1. Minimal massive vector model

2-2. Curvature coupling model

2-3. Kinetic function model

Observational constraints

Dark matter production mechanism is constrained by the cosmic microwave background observation

[Planck CMB map]

1. Isocurvature fluctuation **2**. Statistical anisotropy Vector coherent oscillation DM: $\{$

> [KN (2020)] **There are no parameter region that satisfy both constraints!**

Isocurvature fluctuation

Long-wave fluctuation of hidden photon during inflation

DM isocurvature perturbation

Isocurvature power spectrum is (nearly) scale-invariant $(\alpha < -4)$

Observational constraint on DM isocurvature fluctuation

$$
\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} \lesssim 0.1 \longleftrightarrow S_{\rm DM} = \frac{\delta \rho_A}{\rho_A} \sim \frac{|\delta \overline{A}_T|}{\overline{A}} \sim \frac{H_{\rm inf}}{\pi \overline{A}_i} \lesssim 10^{-5}
$$

Statistical anisotropy

Vector condensate indicates preferred direction

Anisotropic expansion Statistical anisotropy of the pertrubation Anisotropic expansion
This effect turns out to be small

Curvature perturbation $\langle \zeta(\vec{k})\zeta^*(\vec{k}')\rangle$ \setminus = $2\pi^2$ $\frac{2\pi}{k^3}$ P_{ζ} (\vec{k}) $(2\pi)^3 \delta(\vec{k}-\vec{k}')$

 $\mathcal{P}_{\zeta}(\vec{k}) = \mathcal{P}_{\zeta}(k)$ $\mathcal{P}_{\zeta}(\vec{k}) = \mathcal{P}_{\zeta}(k, \theta)$: statistically isotropic : statistically anisotropic

 $\bar{\bar{A}}$

/(

taken *A*

$$
S = \int d^4x \sqrt{-g} \left(-\frac{f^2(\phi)}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} \right) \qquad f(\phi) = \exp \left(-\frac{\gamma}{2M_P^2} \int \frac{V}{V_\phi} d\phi \right)
$$

 $A = A + \delta A$ $\zeta = H - \delta \phi$ Perturbed action: $A_i = A_{i0} + \delta A_i$ $\zeta = -H_{\text{inf}}$ $\delta \phi$ $\dot{\phi}$ $\overline{}$ *F*^{*x*}(*x*)</sub> *k*) *z*(*x*) *d*</sup> where we defined *^h* = 0 for *h* = 1 and *^h* = for *h* = *f*, *E* [~] *^f* ⁼ *fA* **Parturbed action:** $A_i = A_{i0} + \delta A_i$ $\zeta = -H_{i0} \frac{\delta \phi}{\phi}$ • Perturbed action: $A_i = A_{i0} + \delta A_i$ *A H*=*m^A* $\frac{1}{\sqrt{4}}$

$$
\delta S \simeq \int d\tau d^3k \left(-\gamma \vec{E}^f \cdot \delta \vec{E}^f(\vec{k}) \zeta(-\vec{k}) \right) \equiv -\int d\tau \, \mathcal{H}_{\text{int}},
$$

f(*)* = exp $f(x) = f(x) - f(x)$ $\ddot{}$ in the Hamiltonian *H*int. We use the in-in formalism to calculate the two point function • Power s $\overline{\text{}}$ • Power spectrum at second order [Bartolo, Matarrese, Peloso, Ricciadorn (2012)] **8.2 POWER SPECTEUM at Secc**

$$
\left\langle \zeta(\vec{k},\tau)\zeta(\vec{k}',\tau) \right\rangle_{\text{2nd}} = -\int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \left\langle \left[\left[\zeta_k^0(\tau) \zeta_{k'}^0(\tau), \mathcal{H}_{\text{int}}(\tau_1) \right], \mathcal{H}_{\text{int}}(\tau_2) \right] \right\rangle
$$

backreaction of the vector field to the inflaton dynamics is negligible. Note also that *f* ' 1 spower opeeer and or quadrupoide as finition / **EX and** *Anisotropic power spectrum of quadrupolar asymmetry* • Anisotropic power spectrum or quadrupolar asymmetry λ α is streaming position of spectrums of **• Anisotropic power spectrum of quadrupolar asymmetry**

while for 2π and π the first term is in the first term is in the first term is ince E the first term is ince E

$$
\mathcal{P}_{\zeta}(\vec{k}) = \mathcal{P}_{\zeta}^{0}(k) \left[1 + g_{*} \sin^{2} \theta_{k}\right]
$$

 2.1 dynamics during inflation 2.5 dynamics during 2.5 \bullet Observe First let us consider the case of 4 and *h* = 1. After some computation, using the • Observational constraint: $-0.010 < g_* < 0.019$ [Planck(2015)] ~ *^f* = 0 and the second term ⇣ (*k*) ' 2*.*1 ⇥ 10⁹ at the present horizon scale [27], ✓*^k* is the angle

/(

taken *A*

$$
S = \int d^4x \sqrt{-g} \left(-\frac{f^2(\phi)}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} \right) \qquad f(\phi) = \exp \left(-\frac{\gamma}{2M_P^2} \int \frac{V}{V_\phi} d\phi \right)
$$

 $A = A + \delta A$ $\zeta = H - \delta \phi$ Perturbed action: $A_i = A_{i0} + \delta A_i$ $\zeta = -H_{\text{inf}}$ $\delta \phi$ $\dot{\phi}$ $\overline{}$ *F*^{*x*}(*x*)</sub> *k*) *z*(*x*) *d*</sup> **Parturbed action:** $A_i = A_{i0} + \delta A_i$ $\zeta = -H_{i0} \frac{\delta \phi}{\phi}$ • Perturbed action: $A_i = A_{i0} + \delta A_i$ *A H*=*m^A* $\frac{1}{\sqrt{4}}$

$$
\delta S \simeq \int d\tau d^3k \left(-\gamma \vec{E}^f \cdot \delta \vec{E}^f(\vec{k}) \zeta(-\vec{k}) \right) \equiv -\int d\tau \, \mathcal{H}_{\text{int}},
$$

 $\ddot{}$ in the Hamiltonian *H*int. We use the in-in formalism to calculate the two point function \bullet **Power** s $\overline{\text{}}$ • Power spectrum at second order [Bartolo, Matarrese, Peloso, Ricciadorn (2012)] **8.2 POWER SPECTEUM at Secc**

$$
\left\langle \zeta(\vec{k},\tau)\zeta(\vec{k}',\tau) \right\rangle_{\text{2nd}} = -\int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \left\langle \left[\left[\zeta_k^0(\tau) \zeta_{k'}^0(\tau), \mathcal{H}_{\text{int}}(\tau_1) \right], \mathcal{H}_{\text{int}}(\tau_2) \right] \right\rangle
$$

backreaction of the vector field to the inflaton dynamics is negligible. Note also that *f* ' 1 spower opeeer and or quadrupoide as finition / **EX and** *Anisotropic power spectrum of quadrupolar asymmetry* • Anisotropic power spectrum or quadrupolar asymmetry *k*0 λ α is other is position of α , α **• Anisotropic power spectrum of quadrupolar asymmetry**

while for 2π and π the first term is in the first term is in the first term is ince E the first term is ince E

$$
\mathcal{P}_{\zeta}(\vec{k}) = \mathcal{P}_{\zeta}^{0}(k) \left[1 + g_{*} \sin^{2} \theta_{k}\right]
$$

 2.1 dynamics during inflation 2.5 dynamics during 2.5 \bullet Observe First let us consider the case of 4 and *h* = 1. After some computation, using the • Observational constraint: $-0.010 < g_* < 0.019$ [Planck(2015)] ~ *^f* = 0 and the second term

⇣ (*k*) ' 2*.*1 ⇥ 10⁹ at the present horizon scale [27], ✓*^k* is the angle

$$
g_* \simeq \frac{\gamma^2 (\alpha + 1)^4}{18\epsilon M_P^2} \left(\frac{\Gamma(\nu)}{\Gamma(3/2)}\right)^2 \left(H_{\text{inf}} A^f(\tau)\right)^2 \frac{k^{4+\alpha}}{(-2\tau)^{2+\alpha}} I(\tau)
$$

$$
I(\tau) \equiv \left[\int_{\tau_i}^{\tau} d\tau_1 \tau_1^{\alpha+3} \right]^2 = \begin{cases} \log^2(\tau/\tau_i) & \text{for } \alpha = -4\\ \left(\frac{\tau^{\alpha+4}}{\alpha+4} \right)^2 & \text{for } \alpha < -4 \end{cases}
$$

$$
|g_*| = \frac{\gamma^2(\alpha+1)^2}{3 \cdot 2^{\alpha+4}(\alpha+4)^2} \left(\frac{\Gamma(\nu)}{\Gamma(3/2)}\right)^2 \frac{\rho_A(\tau)}{\epsilon \rho_\phi(\tau)} (k\tau)^{\alpha+4} \geq C \frac{\rho_A(\tau)}{\epsilon \rho_\phi(\tau)} e^{-(\alpha+4)N(k)}
$$

described in the beginning of this section). In such a case *N*(*k*) should be regarded as an

$$
|g_*| \sim C \frac{\mathcal{P}_\zeta^0}{\mathcal{P}_S} e^{-2(\alpha+4)N_{\rm st}}
$$

|g⇤*|* = **Note:** \mathcal{P}_S $\sim \frac{H_{\rm inf}^2}{r^2} \sim \frac{\rho_\phi}{r^2} \frac{H_{\rm inf}^2}{r^2}$ $\frac{\varphi}{\Lambda} \frac{-\ln\Gamma}{\Lambda^2}$ \overline{f} $\bullet \quad \mathsf{Note}^\bullet \quad \mathcal{P}_\text{c} \sim \frac{H_\text{inf}^2}{\frac{M_\text{inf}^2}{\lambda}} \sim \frac{\rho_\phi}{\frac{H_\text{inf}^2}{\lambda}} \sim \frac{\rho_\phi}{\frac{M_\text{c}}{\lambda}}$ $P_S \sim$ $H_{\rm inf}^2$ \overline{A}_i^2 \sim ρ_{ϕ} ρ_A $H_{\rm inf}^2$ M^2_P \sim ρ_{ϕ} ρ_A $(\epsilon \mathcal{P}_{\zeta})$ H_{inf}^2 $\rho_\phi H_{\text{inf}}^2$ ρ_ϕ ρ_ϕ • Note: $\mathcal{P}_S \sim \frac{\text{Im}}{A^2} \sim \frac{I \times \mu}{\rho_A} \frac{\text{Im}}{M_P^2} \sim \frac{I \times \mu}{\rho_A} (\epsilon \mathcal{P}_\zeta)$ A_i *PA IIP PA* Note:

No-go theorem [KN (2020)]

 $-0.010 < g_* < 0.019$ **CMB constraint:** $-0.010 < g_* < 0.019$ $\frac{10}{\mathcal{D}_e} \lesssim 0.1$

There are no parameter region that satisfy both constraints!

Loophole & Summary

Curvaton scenario Now we are going to discuss constraint on the vector DM scenario from the vector DM scenario from the isocurva
In the vector DM scenario from the isocurvature from the isocurvature from the isocurvature from the isocurvatu

Scalar other than inflaton produces density perturbation

$$
\delta\phi \sim \frac{H_{\rm inf}}{2\pi} \neq \sqrt{\epsilon}M_P\zeta
$$

\n
$$
\longrightarrow |g_*| \sim C \frac{\mathcal{P}_{\zeta}^0}{\mathcal{P}_{S}^0} e^{-2(\alpha+4)N_{\rm st}} \qquad \mathcal{P}_{\zeta}^0 \ll \mathcal{P}_{\zeta}^{\text{(obs)}}
$$

- Non-standard thermal history after inflation
- $\bigcap_{t\in\mathbb{R}}\{f(t)\in\mathbb{R}^n\mid t\geq 0\}$ **e** Other way! Other way?

Vector coherent oscillation DM scenario is not excluded, rector corierent oscination Dr r scerial ions not excluded,
but severely constrained and need complicated model. the kinetic energy vanishes for the *C*¹ solution in (12). Therefore, the statistical anisotropy but severely constrained and need complicated model.

3. Gravitational productuion of vector DM

3. Gravitational Production

X 3-1. Gravitational production of scalar

3-2. Gravitational production of vector

Particle Production

[Dolgov, Kirilova (1990), Traschen, Brandenberger (1990)]

- Inflaton coherent oscillation: $\phi = \phi(t) \cos(m_{\phi} t)$
- Interaction with light particle χ

$$
\mathcal{L}_{int} = \frac{\phi}{M} (\partial_{\mu} \chi)^2 \longrightarrow \text{``decay''} \qquad \Gamma_{\phi \to \chi \chi} = \frac{m_{\phi}^3}{32\pi M^2}
$$
\n
$$
\mathcal{L}_{int} = g^2 \phi^2 \chi^2 \longrightarrow \text{``annihilation''} \quad \Gamma_{\phi \phi \to \chi \chi} \sim \frac{g^4 \phi^2}{8\pi m_{\phi}}
$$

Time dependence of effective mass/kinetic term leads to particle production

Gravitational Particle Production

[Parker (1969), Ford (1986)]

• Real scalar field interacting only through gravity

$$
S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_P^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 \right)
$$

FRW metric: $ds^2 = -dt^2 + a^2(t) d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$

$$
\blacktriangleright \left(S = \int d\tau d^3 x \frac{a^2(\tau)}{2} \left[\chi'^2 - (\nabla \chi)^2 - a^2 m_\chi^2 \chi^2 \right] \right)
$$

• Time dependent mass/kinetic term from scale factor **"Gravitational particle production"**

Gravitational Particle Production

[Parker (1969), Ford (1986)]

• Real scalar field interacting only through gravity

$$
S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_P^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 \right)
$$

FRW metric: $ds^2 = -dt^2 + a^2(t) d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$

$$
\blacktriangleright \left(S = \int d\tau d^3 x \frac{a^2(\tau)}{2} \left[\chi'^2 - (\nabla \chi)^2 - a^2 m_\chi^2 \chi^2 \right] \right)
$$

• Time dependent mass/kinetic term from scale factor **"Gravitational particle production"**

Gravitational Particle Production

[Parker (1969), Ford (1986)]

• Real scalar field interacting only through gravity

$$
S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_P^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 \right)
$$

FRW metric: $ds^2 = -dt^2 + a^2(t) d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$

$$
S = \int d\tau d^{3}x \frac{a^{2}(\tau)}{2} \left[\chi'^{2} - (\nabla \chi)^{2} - a^{2} m_{\chi}^{2} \chi^{2} \right]
$$

• Time dependent mass/kinetic term from scale factor **"Gravitational particle production"**

p*^g* \bullet *^R* ¹ $\frac{1}{2}$ \mathbf{r} \cdot \cdot \cdot \cdot where M is the reduced Planck scale and R the reduced Planck scale and R the canonical curvature, α Scalar production during inflation di proguction au C_{coll} (⌧)(*d*⌧ ² ⁺ *^d*[~]

$$
S = \int d\tau d^3x \frac{a^2(\tau)}{2} \left[\chi'^2 - (\nabla \chi)^2 - a^2 m_\chi^2 \chi^2 \right]
$$

$$
= \int d\tau d^3x \frac{1}{2} \left[\tilde{\chi}'^2 - (\partial_i \tilde{\chi})^2 - m_\chi^{(\text{eff})2} \tilde{\chi}^2 \right] \qquad \tilde{\chi} \equiv a\chi
$$

 $m_{\chi}^{\rm (eff)2}=a^2m_{\chi}^2$ $\frac{2}{\chi}-\frac{a^{\prime\prime}}{a}$ mass: $m_{\chi}^{(\text{eff})2} = a^2 m_{\chi}^2 - \frac{a}{a}$ $m_{\chi}^{\rm (eff)2}$ $= a$ $\overline{)}$ n_∞^2 $\frac{a'}{a}$ \overline{a} ◆ $=$ a^2 $\sqrt{ }$ m_χ^2 $\frac{2}{\chi}-2H^{2}+\frac{\dot{\phi}^{2}}{2M}$ $2M_P^2$! Effective mass: $m_\chi^{(\text{eff})2} = a^2 m_\chi^2 - \frac{a}{a} = a^2 \left(m_\chi^2 - 2H^2 + \frac{\varphi}{2M_P^2} \right)$ $\overline{}$ α *ⁱ* ^e ⁺ *^m*(e↵)2 ^e = 0*.* (4)

- θ and θ denotes the Hubble parameter and we have used the Friedmann equation in θ **• Tachyonic mass during inflation if** $m_\chi^2 < 2H_{\rm inf}^2$ if $m_{\chi}^2 < 2H_{\text{inf}}^2$ ^e = 0*.* (4) **Transformaries as contingular as a continuous motion of the following equation of motion, i.e.,** $m_\chi < 2 H_{\rm inf}$
	- **ass d** ✓*a*˙ *a* = 1
1
1 3*M*² $\overline{1}$ 2 **Illating mass during reheating and as the following motion, and the following equation** eneating **Oscillating mass during reheating**

(Nearly) scale invariant spectrum at the end of inflation

$$
\langle \chi^2(k) \rangle \simeq \left(\frac{H_{\rm inf}}{2\pi}\right)^2
$$

Note on gravitational production

Gravitational production also works during inflaton oscillation

[Ema, Jinno, Mukaida, KN (2015); Ema, KN, Tang (2018); Chung, Kolb, Long (2018)]

$$
m_{\chi}^{\text{(eff)}2} = \left\langle a^2 \right\rangle \left[m_{\chi}^2 - 2 \left\langle H \right\rangle^2 - \left(m_{\chi}^2 - 2 \left\langle H \right\rangle^2 \right) \frac{\varphi^2}{4 M_P^2} + \left\langle H \right\rangle \frac{\varphi \dot{\varphi}}{M_P^2} + \frac{\dot{\varphi}^2}{2 M_P^2} \right]
$$

- k~0 mode accumlates to form scalar condensate \rightarrow Scalar coherent oscillation
- (Nearly) scale invariant spectrum

Strong constraint from isocurvature fluctuation

Note on gravitational production

Gravitational production also works during inflaton oscillation

[Ema, Jinno, Mukaida, KN (2015); Ema, KN, Tang (2018); Chung, Kolb, Long (2018)]

$$
m_{\chi}^{(\text{eff})2} = \langle a^2 \rangle \left[m_{\chi}^2 - 2 \langle H \rangle^2 - \left(m_{\chi}^2 - 2 \langle H \rangle^2 \right) \frac{\varphi^2}{4M_P^2} + \langle H \rangle \frac{\varphi \dot{\varphi}}{M_P^2} + \frac{\dot{\varphi}^2}{2M_P^2} \right]
$$

"Slow" part

k~0 mode accumlates to form scalar condensate

 \rightarrow Scalar coherent oscillation

(Nearly) scale invariant spectrum

Strong constraint from isocurvature fluctuation

Note on gravitational production

Gravitational production also works during inflaton oscillation

[Ema, Jinno, Mukaida, KN (2015); Ema, KN, Tang (2018); Chung, Kolb, Long (2018)]

$$
m_{\chi}^{(\text{eff})2} = \langle a^2 \rangle \left[m_{\chi}^2 - 2 \langle H \rangle^2 - \left(m_{\chi}^2 - 2 \langle H \rangle^2 \right) \frac{\varphi^2}{4M_P^2} + \langle H \rangle \frac{\varphi \dot{\varphi}}{M_P^2} + \frac{\dot{\varphi}^2}{2M_P^2} \right]
$$

"Slow" part
"Fast" part

k~0 mode accumlates to form scalar condensate

 \rightarrow Scalar coherent oscillation

(Nearly) scale invariant spectrum

Strong constraint from isocurvature fluctuation

3. Gravitational Production

3-1. Gravitational production of scalar

X 3-2. Gravitational production of vector

Minimal massive vector **1 Viinimal massive vector**

Massive vector boson minimally coupled to gravity **8.** Massive vector boson minimally coupled to gravity

Z

$$
S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu \rho} g^{\nu \sigma} F_{\mu \nu} F_{\rho \sigma} - \frac{1}{2} m^2 g^{\mu \nu} A_{\mu} A_{\nu} \right]
$$

$$
= \int d\tau d^3x \left[-\frac{1}{4} \eta^{\mu \rho} \eta^{\nu \sigma} F_{\mu \nu} F_{\rho \sigma} - \frac{1}{2} a^2 m^2 \eta^{\mu \nu} A_{\mu} A_{\nu} \right]
$$

In the massless limit, vector does not "feel" gravity. Mass term induces gravitational production mixture mixtures of the standard model of a candidate photon. The state of \overline{A} $\frac{1}{3}$ for contribution $\frac{1}{3}$ mixing with the standard model hypercharge photon. Then *A^µ* is stable and a candidate of In the massiess limit, vector does Mass term induces gravitational production

Eraham, Mardon, Rajendran (2015)] The vector boson mass can be regarded as a regarded as σ regarded as σ results mechanism. In this mechanism. In this mechanism mechanism. In this mechanism. In this mechanism. In this mechanism. In this mechanism. In

Transverse-longitudinal decomposition Substituting the action, we find the action, we find the transverse model we find the transverse model model model model model we find the set of the transverse model we find the transverse model we find the set of the set *k*2 معه ا *^k*² ⁺ *^a*²*m*² *.* (37)

$$
S=S_T+S_L
$$

$$
S_T = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \vec{A}_T|^2 - (k^2 + a^2 m^2) |\vec{A}_T|^2 \right)
$$

$$
S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(\frac{a^2 m^2}{k^2 + a^2 m^2} |\partial_\tau A_L|^2 - a^2 m^2 |A_L|^2 \right)
$$

z *disverse mode is similar to stall with t f*(*b*) \overline{f} **f** \overline{f} $\$, Transverse mode is similar to scalar with conformal coupling

free prode production is subdoming *^L|A* compared with longitudinal mode \mathbf{r} *|*@⌧*A* f*L|* ² !² *^L|A* f*L| ,* !² *^L* = *<i>f* 2 *f* Transverse mode production is subdominant where the e \sim e \sim e \sim e \sim e \sim e \sim

f(a) $\frac{1}{2}$ **angitudinal mode** , For convenience, we also present the equation of motion in the original basis: $\mathcal{L}(\mathcal{L})$

T (

~

$$
S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \widetilde{A}_L|^2 - (k^2 + m_L^2) |\widetilde{A}_L|^2 \right)
$$

$$
m_L^2 = a^2 m^2 - \frac{k^2}{k^2 + a^2 m^2} \left(\frac{a''}{a} - \frac{a'^2}{a^2} \frac{3a^2 m^2}{k^2 + a^2 m^2} \right)
$$

From the transverse mode $\binom{n}{i}$ is $\binom{n}{i}$ in the limit is evident that it is conformal in the limit is conformal in the limit in the limit is conformal in the limit in the limit is conformal in the limit in the limi High momentum $(k/a > m)$ $\mu = u \left(m \overline{I}$ p \int *e*^{\int} *e^r* \int *e*^{\int} *e*<sup> $$ $m_L^2 \simeq a^2(m^2 - 2H_{\rm ir}^2)$ \int_{inf}^{2}

m \rightarrow **Superhorizon amplification during inflation Example Supernonizon amplincation during innation** Superhorizon amplification during inflation

• Low momentum
$$
(k/a < m)
$$
 $m_L^2 \simeq a^2 \left(m^2 + H_{\text{inf}}^2 \frac{k^2}{a^2 m^2} \right)$

 \longrightarrow No *A^T* ($\sqrt{2}$ → **No amplification** *k,h* + *A*⇤ No amplification

Again we find that it is peaked around *k* = *k*⇤ where *k*⇤ ⌘ *a*(*H* = *m*)*m*, which is now ● Note: Note:

- Gravitational production also works during inflaton oscillation \bullet Ema, KN, Tang (2019)]
y after inflation s depende on thormal his [Ema, KN, Tang (2019)]
- ⇡²*g*⇤ 32⇡²*M*³*/*² *P* Final abundance depends on thermal history after inflation [Ema, KN, Tang (2019), Ahmed, Grzadkowski, Socha (2020)]
- Thermal production from SM scattering with graviton exchange \bullet

[Garny, Sasndora, Sloth (2015), Tang, Wu (2016)]

C Final vector boson abundance

$$
\frac{\rho_L}{s} \simeq \begin{cases} \frac{\mathcal{C}_L T_{\rm R} H_{\rm inf} m}{4M_P^2} & \text{for } H_{\rm inf} < m \\ \frac{T_{\rm R} H_{\rm inf}^2}{32\pi^2 M_P^2} & \text{for } H_{\rm R} < m < H_{\rm inf} \\ \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \frac{m^{1/2} H_{\rm inf}^2}{32\pi^2 M_P^{3/2}} & \text{for } m < H_{\rm R} \end{cases}
$$

Vector DM is possible for wide range of mass & inflation scale Figure 2: Illustration of the with the large of the state vector boate $s_{10} = 6$ σ σ $m\gtrsim 10^{-6}\,\mathrm{eV}\qquad\quad H_{\mathrm{inf}}\lesssim 10^{14}\,\mathrm{GeV}$

Comment on XENONIT anomaly

- XENON1T found excess electronic recoil events
- Hidden photon DM with small kinetic mixing can explain it [Alonso-Alvarez et al (2020)] $m_V = 2.7 \,\text{keV} \qquad \epsilon = 7 \times 10^{-16}$
- Gravitational production works for this mass range blue dots are the events observed by XENON1T in the background, the background, the background, the background
The background, the background, the background, the background, the background, the background, the background blue dots are the events observed by XENON1T in the background, the background, the background, the background,

4. Summary

Summary

- Vector coherent oscillationDM scenario is (almost) excluded.
- Gravitational production is one of the simple scenario for vector DM for $m \gtrsim 10^{-6}\,\mathrm{eV}$
- Other production mechanims are also viable. Cosmic string, Higgs decay, axion coupling…

Appendix

Axion coupling

[Agrawal et al (2018), Bastero-Gil et al (2018), Co et al (2018)]

Axion couling to vector boson

$$
{\cal L} = C \frac{a}{f} F_{\mu\nu} \widetilde{F}^{\mu\nu}
$$

- Axion dynamics induces vector production One polarization mode even becomes tachyonic for $k \leq C\dot{a}/f$
- Backreaction stops the tachyonic growth
- Relic density:

$$
\Omega_A h^2 \sim 0.2 \left(\frac{40}{C}\right) \left(\frac{m_A}{10^{-9} \text{ eV}}\right) \left(\frac{10^{-8} \text{ eV}}{m_a}\right)^{1/2} \left(\frac{f}{10^{14} \text{ GeV}}\right)^2
$$

[Agrawal et al (2018)]

Cosmic string

Vector mass comes from hidden U(1) Higgs field ³*/*²*n^A* / *^t* ¹*/*². The relic abundance of cosmological dilution, *n^A* ⇠ *t* CCCU THE 33 CUTTLS IIV

> \longrightarrow Cosmic strings 0*M*² **P** Cosmic string

- Light vector boson (longitudinal component) is emitted by string dynamcis. where *H*⁰ = 100*h* km*/*sec*/*Mpc is the Hubble constant and **MPL SERVE ' 22.43**NUMBER IS THE REDUCED ON THE REDUCED SERVE IS THE REDUCED PLANCE MASS. HERE, we define the reduced Planck mass of the reduced Planck mass. Here, we define the reduced Planck mass. Here, we define the red *s* \sim $\frac{1}{2}$ is emitted sity at time *t* when the plasma temperature is *T*(*t*). Dark pho
	- **y** decay also exect to boson ¹⁶ ◆ ✓*E*¯*^A* \tilde{h} , Harigaya, INarayan Higgs decay also emits vector boson [Dror, Harigaya, Narayan (2018)]

[Long, Wang (2019)]

Isotropy of vector background

we have defined a \mathcal{L}_i $(0, 0, 1, 2)$ Vector background $A_i = (0, 0, A_z)$

Energy-momentum tensor A ^{*i*} A ^{*n*} A *n* A ^{*n*} A *n* A ^{*n*} A *n* $\mathsf{C}\mathsf{C}\mathsf{C}\mathsf{C}\mathsf{C}$

$$
T_{xx} = T_{yy} = \frac{f^2}{2} \dot{A}_z^2 - \frac{1}{2} m^2 A_z^2, \qquad T_{zz} = -\frac{f^2}{2} \dot{A}_z^2 + \frac{1}{2} m^2 A_z^2 \qquad T_{ij} = 0 \quad (i \neq j)
$$

Anisotropic during slow-roll (H>>m) field just behaves as non-relativistic matter and does not induce anisotropic expansion. On

 t_{best} of the universe, it induces anisotropic expansion. In our scenario studied below, the vector However, vector energy is negligible in this stage

Isotropic (pressure-less matter) during oscillation (H <<m)

 2 ˙*f* ! Background expansion is isotropic

Delta-N and isocurvature mode To be definite, we consider CDM isocurvature perturbation, but an extension to other types of the isocurvature perturbations is straightforward. types of the isocurvature perturbations is straightforward.

We we we we we write the perturbed space in the perturbed space in the perturbed space of the perturbed space in the space of the s **•** Perturbed metric

$$
ds^{2} = -\mathcal{N}^{2}dt^{2} + a^{2}(t)e^{2\psi}\gamma_{ij} (dx^{i} + \beta^{i}dt) (dx^{j} + \beta^{j}dt)
$$

C Spatial curvature on arbitrary time slice flyth Malik Sasaki (2004)] **group scale factor of the curvature of the curvature curvature successive** curvature spatial scales, $\frac{1}{2}$ ϵ or the curvature perturbation. On sufficiently large perturbation. On sufficiently large spacial scales, ϵ Compatible curvature on arbitrary thrie slice effective, main, based Spatial curvature on arbitrary time slice [Lyth, Malik, Sasaki (2004)]

$$
\psi(t_f, \vec{x}) = N(t_f, t_i; \vec{x}) - \log \frac{a(t_f)}{a(t_i)} = \delta N(t_f, t_i; \vec{x})
$$

- \sim δ Define $\zeta_i(\vec{x})$ as curvature on the slice $\delta \rho_i(\vec{x}) = 0$
- DM isocurvature perturbation: $S_{DM} \equiv 3(\zeta_{DM} \zeta_r)$

[Wands, Malik, Lyth, Liddle (2000)] [Kawasaki, KN, Sekiguchi, Suyama, Takahashi (2008)]

Isocurvature fluctuation

- Light field during inflation obtain long wave fluctuations
- Inflaton fluctuation $\,\delta\phi$
	- curvature perturbation

density perturbation

$$
\zeta = -H_{\rm inf} \frac{\delta \phi}{\dot{\phi}}
$$

$$
\frac{\delta \rho_r}{4\rho_r} = \zeta
$$

DM isocurvature perturbation

$\rho_{r}(\vec{x})$ $\rho_{\rm DM} (\vec{x})$

 \overline{x} \vec{x}

Inflationary quantum fluctuation which its sign, and hence its sign, and is stable and is stable and is a candidate of DM. We assume that α where M is the reduced Planck scale, R is the reduced Planck scale, R is the inflaton field R is the inflaton field R Inflationary quantum fluctuation does not have a direct coupling to the inflaton and other standard model fields. It interacts where the conformal Hubble parameter \mathcal{H} is a conform

- -Sitter background, zero-point fluctuation is enhanced at superhorizon regime (infla We use the Friedmann-Robertson-Walker metric: We use the Friedmann-Robertson-Walker metric: We use the Friedma
Robertson-Walker metric: We use the Friedmann-Robertson-Walker metric: We use the Friedmann-Robertson-Walker m with de-Sitter background zero-point fluctuati which is stated its state its state in the state of DM. We are not determined that it and is a canonical state of DM. We assume that it and it an de superflorizon regning (inflational y in r background, zo of background, zero-point fluctuation is enhanced at superhorizon regime (inflationary fluctuation). nhan
、 ✓*a*⁰ In de-Sitter background, zero-point fluctuation is enhanced
- only field during inflation ◆ ○Calar Ticle eding "Middler" where the conformal Hubble parameter *H* is Scalar field during inflation coupling corresponds to **a** \mathbf{f} = 1*/6.* \mathbf{f} = 1*/* We use the Friedmann-Robert manner metric of the Friedmann R

$$
S=\int d^4x\sqrt{-g}\left(\frac{1}{2}(M_P^2-\xi\chi^2)R-\frac{1}{2}\partial_\mu\chi\partial^\mu\chi-\frac{1}{2}m_\chi^2\chi^2\right)
$$

$$
S = \int d\tau d^3x \frac{1}{2} \left[\tilde{\chi}'^2 - (\partial_i \tilde{\chi})^2 - m_{\chi}^{(\text{eff})2} \tilde{\chi}^2 \right], \qquad \tilde{\chi} \equiv a\chi
$$

$$
m_{\chi}^{\text{(eff)}2} \equiv a^2 m_{\chi}^2 - (1 - 6\xi) \frac{a''}{a}
$$

Where the prime design respect to the derivative with respect to $\mathcal{L}_{\text{diff}}$ **Canonical quantization** and vacuum state. Let us define the Fourier mode as \blacksquare quantization with respect to denote the derivation \mathbf{q} $\frac{1}{\sqrt{2}}$

$$
\widetilde{\chi}(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left[a_{\vec{k}} \chi_k(\tau) + a_{-\vec{k}}^{\dagger} \chi_k^*(\tau) \right] e^{i\vec{k}\cdot\vec{x}} \qquad \left[a_{\vec{k}}, a_{\vec{k}'}^{\dagger} \right] = (2\pi)^3 \delta(\vec{k} - \vec{k}'),
$$

Inflationary quantum fluctuation <u>Innationary quantum inuctuation</u> Inflationary quantum fluctuation <u>ournational y quantum nuttiation</u> en provincial en la regional de la regional de la regional de la regional de la

- Mode equation: $\chi_k'' + \omega_k^2 \chi_k = 0$, $\omega_k^2 \equiv k^2 + m_{\chi}^{(\text{eff})2}$ $d = M$ ode equation: $\chi'' + \omega^2 \chi_i = 0$ $\omega^2 = k^2 + m^{\text{(eff)}}^2$ next section.
- \bullet Solution: **d** Solution:

$$
\chi_k(\tau) = e^{\frac{i(2\nu+1)\pi}{4}} \frac{1}{\sqrt{2k}} \sqrt{\frac{-\pi k\tau}{2}} H_{\nu}^{(1)}(-k\tau), \qquad \nu^2 \equiv \frac{9}{4} - 12\xi - \frac{m_{\chi}^2}{H^2}
$$

$$
\chi_k(\tau) \simeq \begin{cases} \frac{1}{\sqrt{2k}} e^{-ik\tau} & \text{for } k/a \gg H_{\rm inf} & \text{: subhorizon limit} \\ i \frac{aH_{\rm inf}}{\sqrt{2}k^{3/2}} & \text{for } k/a \ll H_{\rm inf} & \text{: superhorizon limit} \end{cases} \qquad H_1^{-1}
$$

3

 x^k or α

inflation \vdots MD or RD

 $(k/a)^{-1}$

t

Instead, the mode function develops with time, which may be interpreted as the particle may be interpreted as **•** Power spectrum in superhorizon limit

$$
\langle \chi_k \chi_{k'}^* \rangle = \frac{a^2 H_{\text{inf}}^2}{2k^3} (2\pi)^3 \delta(\vec{k} - \vec{k}')
$$

Scalar (inflaton) fluctuation on

$$
\left\langle \delta\phi(\vec{k})\delta\phi^*(\vec{k}') \right\rangle = \frac{2\pi^2 a^2}{k^3} \mathcal{P}_{\phi}(k)(2\pi)^3 \delta(\vec{k} - \vec{k}') \qquad \mathcal{P}_{\phi}(k) \simeq \left(\frac{H_{\rm inf}}{2\pi}\right)^2
$$

Transverse vector fluctuation homogeneous mode *A* (13), as expected. **The transverse vector independent inflation is defined as a spectrum of the spectrum intervalse as in the special special special special intervalse in the special special special special special special special special s**

$$
\left\langle \vec{A}_T(k) \cdot \vec{A}_T^*(k') \right\rangle = \frac{4\pi^2 a^2}{k^3} \mathcal{P}_T(k) (2\pi)^3 \delta(\vec{k} - \vec{k}')
$$

$$
\mathcal{P}_T(k) \sim \left(\frac{H_{\text{inf}}}{2\pi}\right)^2 \left(\frac{2aH_{\text{inf}}}{k}\right)^{-\alpha - 4} \sim |\delta \vec{A}_T|
$$

- 2 tor fluctuati $\overline{}$ on is independent $\overline{2}$ Vector fluctuation is independent of the inflaton Isocurvature fluctuation constrained by CMB.
	- Observational constraint on DM isocurvature fluctuation

$$
\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} \lesssim 0.1 \longleftrightarrow S_{\rm DM} = \frac{\delta \rho_A}{\rho_A} \sim \frac{|\delta \overline{A}_T|}{\overline{A}} \sim \frac{H_{\rm inf}}{\pi \overline{A}_i} \lesssim 10^{-5}
$$

•
$$
\rho_r(\vec{x})e^{-4\delta N_r(\vec{x})} = \overline{\rho_r}
$$
 $\longrightarrow \zeta_r(\vec{x}) = \delta N_r(\vec{x}) = \frac{\delta \rho_r(\vec{x})}{4\rho_r}$

•
$$
\rho_{\rm DM}(\vec{x})e^{-3\delta N_{\rm DM}(\vec{x})} = \overline{\rho_{\rm DM}} \longrightarrow \zeta_{\rm DM}(\vec{x}) = \delta N_{\rm DM}(\vec{x}) = \frac{\delta \rho_{\rm DM}(\vec{x})}{3\rho_{\rm DM}}
$$

$$
S_{\rm DM} = \frac{\delta \rho_{\rm DM}}{\rho_{\rm DM}} - \frac{3}{4} \frac{\delta \rho_r}{\rho_r}
$$

$\mathcal{L} = -\frac{\epsilon}{2}$ Kinetic mixing with standard model gauge $\mathcal{L} = -\frac{\epsilon}{2} F_{MN}^{(Y)} F^{MN}$ 2 z_{res}

String theory and light field

- Compactification of 6D through Calabi-Yau manifold
- Hodge-number of Calabi-Yau manifold

$$
h_{0,0} \t 1
$$
\n
$$
h_{1,0}, h_{0,1} \t 0,0
$$
\n
$$
h_{2,0}, h_{1,1}, h_{0,2} \t 0, h_{1,1}, 0
$$
\n
$$
h_{3,0}, h_{2,1}, h_{1,2}, h_{0,3} \t 1, h_{2,1}, h_{1,2}, 1
$$
\n
$$
h_{3,1}, h_{2,2}, h_{1,3} \t 0, h_{1,1}, 0
$$
\n
$$
h_{3,2}, h_{2,3} \t 0, 0
$$
\n
$$
h_{3,3} \t 1
$$

(p,q)-form is classified by the *hp,q* basis

→ 4D light field is related with Hodge number

Euler number: $\chi = h_{1,1} - h_{2,1}$

Light field in typeIIB theory

NS-NS

R-R