Production of Hidden Photon Dark Matter: Recent Developments

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Contents

1. Introduction

Light bosonic dark matter

- Light boson is a candidate of dark matter

  QCD axion, dilaton, string axion, hidden photon, …

- Many light fields from string theory

  “String Axiverse” [Arvanitaki et al (2009)]

- Light mass can be ensured by symmetry

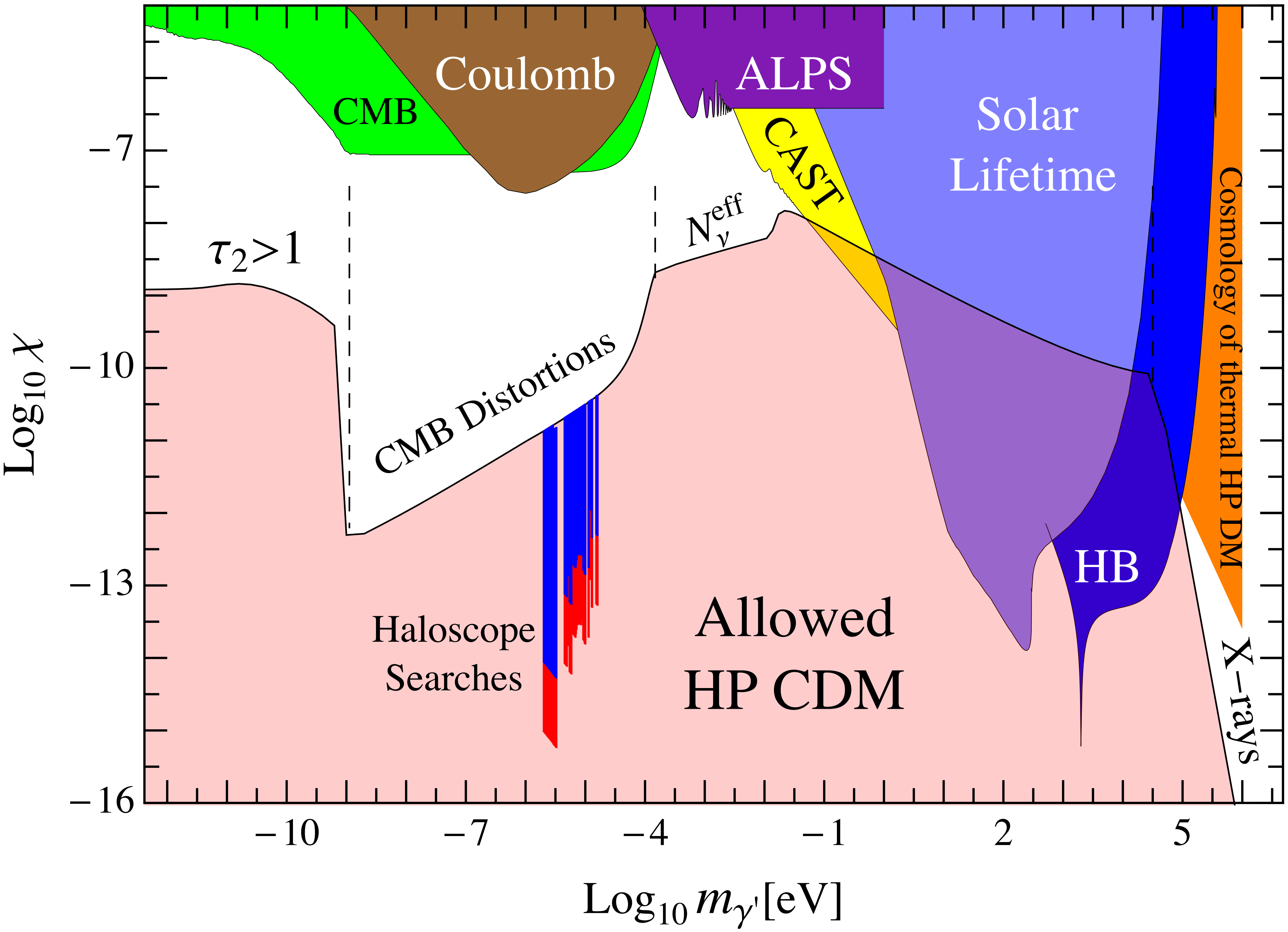
  Axionic scalar : shift symmetry $\phi \to \phi + C$

  Vector : gauge symmetry $A_\mu \to A_\mu + \partial_\mu \chi$

- I will focus on vector boson (hidden photon)

Constraint on kinetic mixing of hidden photon

$$\mathcal{L} = -\frac{\chi}{2} F_{\mu\nu} X^{\mu\nu}$$



[Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012)]

Production of Hidden Photon DM

- Production mechanisms:
  - Axionic coupling [Agrawal, Kitajima, Reece, Sekiguchi, Takahashi (2018), Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018), Co, Pierce, Zhang, Zhao (2018)]
  - Production from dark Higgs [Dror, Harigaya, Narayan (2018)]
  - Production from cosmic string [Long, Wang (2019)]
  - Coherent oscillation [Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012), KN (2019)]
  - Gravitational production [Graham, Mardon, Rajendran (2015), Ema, KN, Tang (2019)]

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2. Vector coherent oscillation

Overview of history

- Nelson, Scholtz (2011)

  Vector coherent oscillation DM in minimal model

- Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012)

  Minimal model does not work.

  Vector coherent oscillation DM in curvature coupling model

- KN (2019)

  Curvature coupling model does not work.

  Vector coherent oscillation DM in kinetic function model

- KN (2020)

  Kinetic function model severely constrained by observation

2. Vector coherent oscillation

★ 2-1. Minimal massive vector model

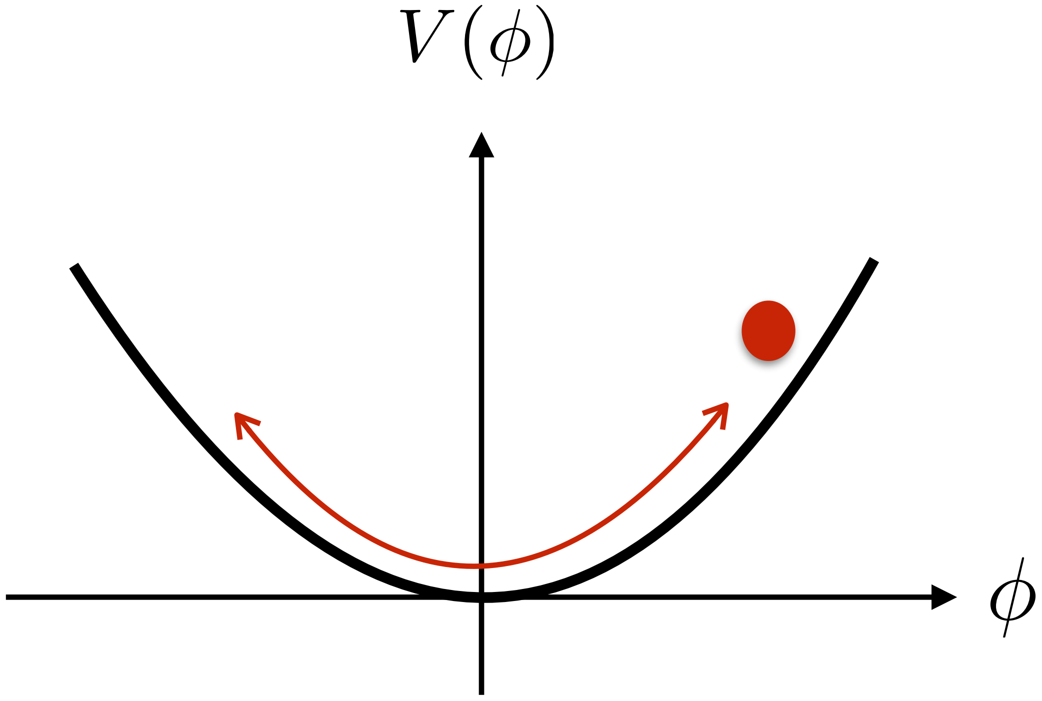
2-2. Curvature coupling model

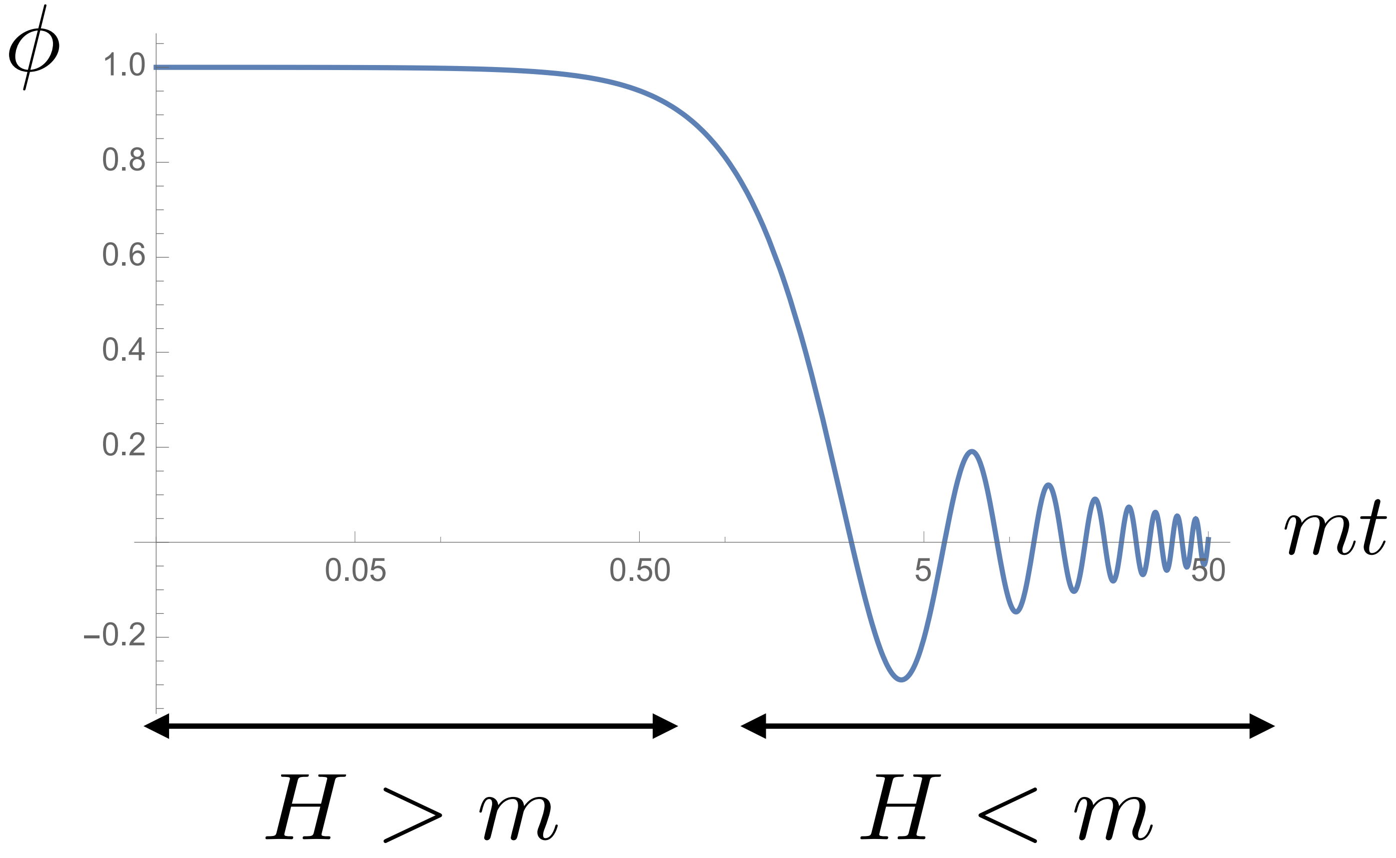
2-3. Kinetic function model

2-4. Observational constraints

Scalar coherent oscillation

- **Action** $S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right)$
- **Eq.of.m** $\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$





- Energy density

$$\rho_\phi = T_{00} = \frac{1}{2}\left(\dot{\phi}^2 + m^2\phi^2\right) \simeq \frac{m^2\phi_{\rm ini}^2}{2}\left(\frac{a_0}{a(t)}\right)^3$$

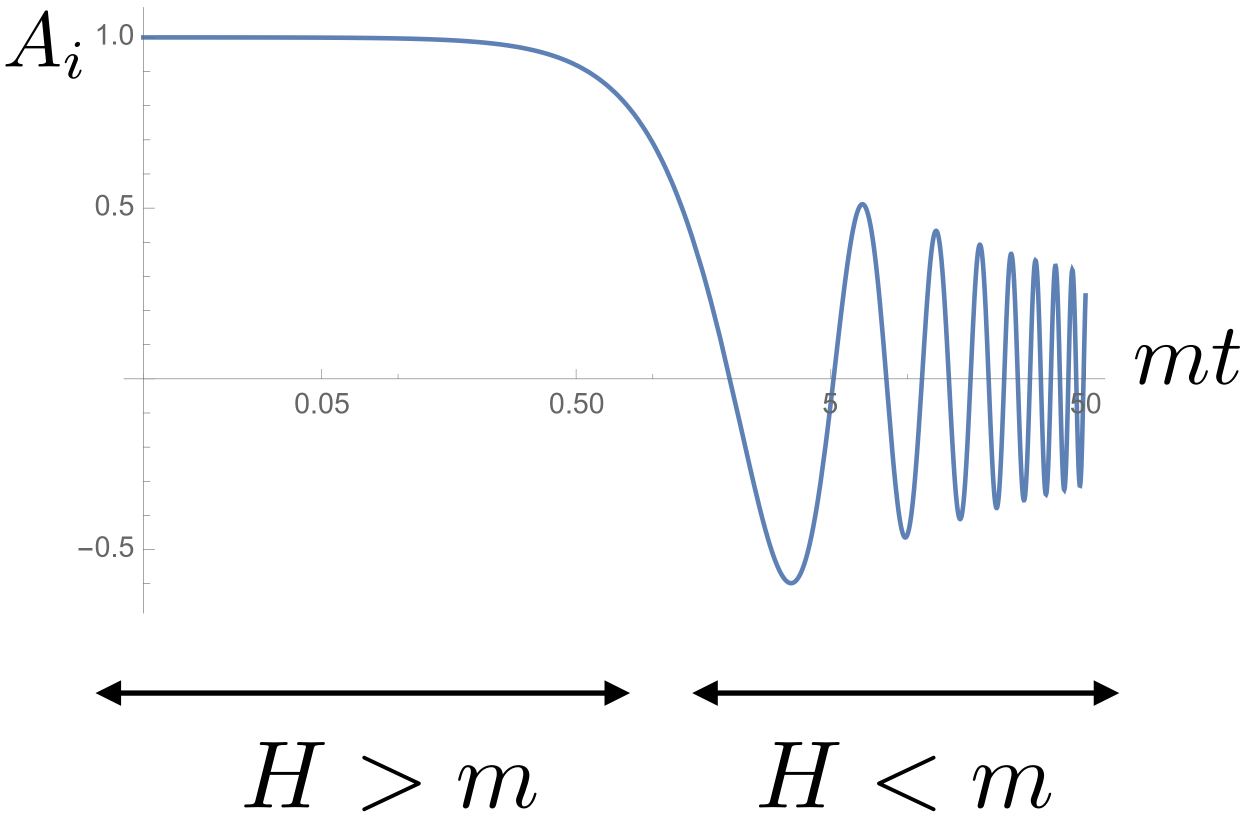
$$\boxed{\frac{\Omega_\phi}{\Omega_{\rm DM}} \sim \left(\frac{m}{10^{-27}\,{\rm eV}}\right)^{1/2}\left(\frac{\phi_{\rm ini}}{M_P}\right)^2}$$

(Light) scalar coherent oscillation is good DM candidate

- Constraints
  - Halo structure $m \gtrsim 10^{-22}\,{\rm eV}$
  - Isocurvature fluctuation $S_{\rm DM} \sim \frac{2\delta\phi_{\rm ini}}{\phi_{\rm ini}} \sim \frac{H_{\rm inf}}{\pi\phi_{\rm ini}} \lesssim 9 \times 10^{-6}$

Vector coherent oscillation?

- Action $S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{MN} g^{KL} \mathcal{F}_{MK} \mathcal{F}_{NL} - \frac{1}{2} m^2 g^{MN} \mathcal{A}_M \mathcal{A}_N \right)$
- Eq.of.m $\ddot{A}_i + H\dot{A}_i + m^2 A_i = 0$



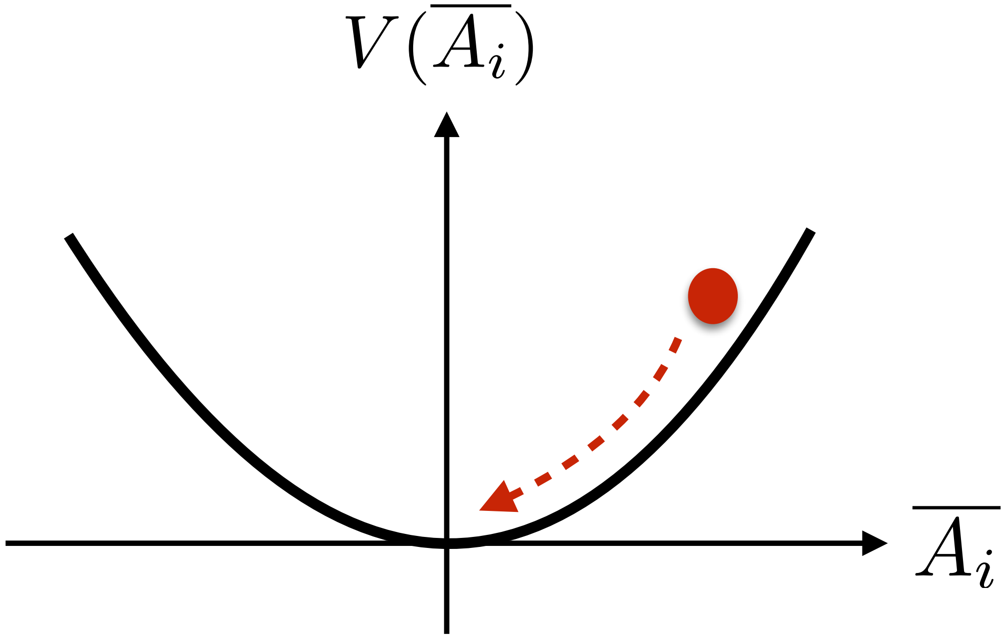
Coherent oscillation
similar to scalar?
but…

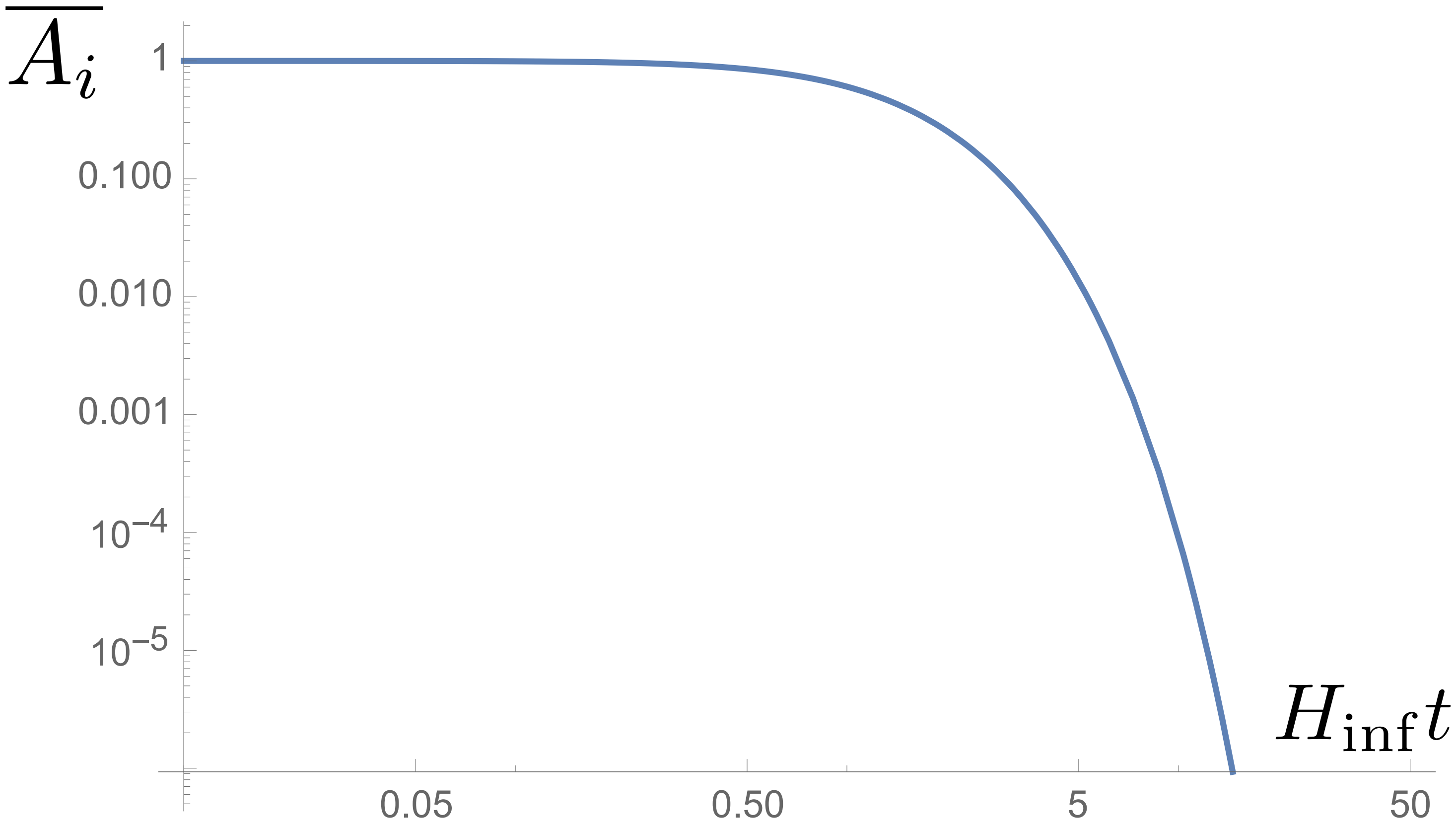
- Energy density $\rho_A = T_{00} = \frac{1}{2a^2}\left(\dot{A}_i^2 + m^2 A_i^2\right)$

→ Energy density exponentially decreases during inflation

- Eq.of.m of "Physical" field: $\overline{A_i} \equiv A_i/a$

$$\ddot{\overline{A_i}} + 3H\dot{\overline{A_i}} + (m^2 + 2H^2 + \dot{H})\overline{A_i} = 0$$





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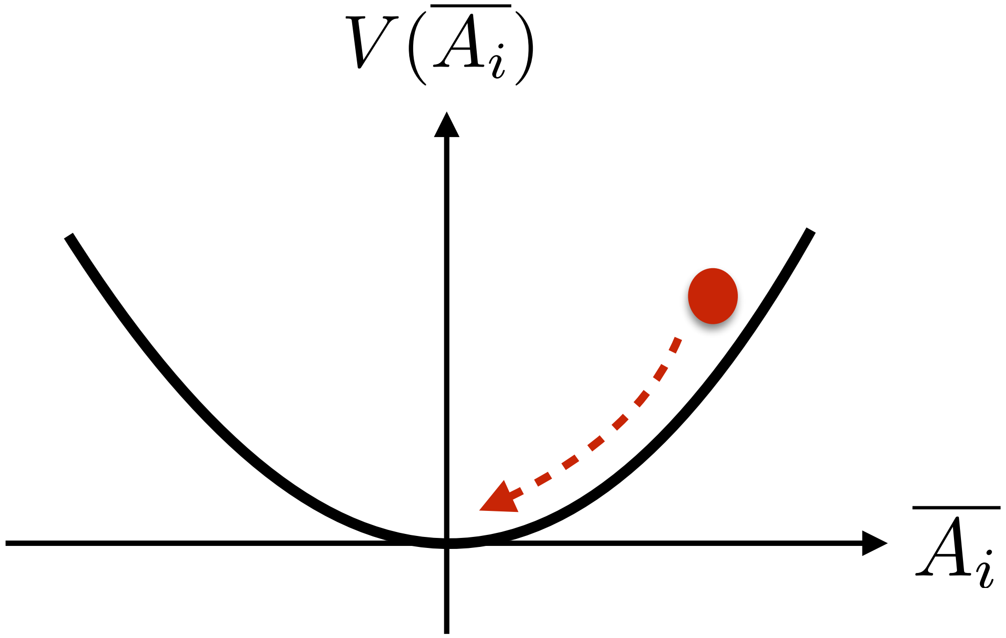

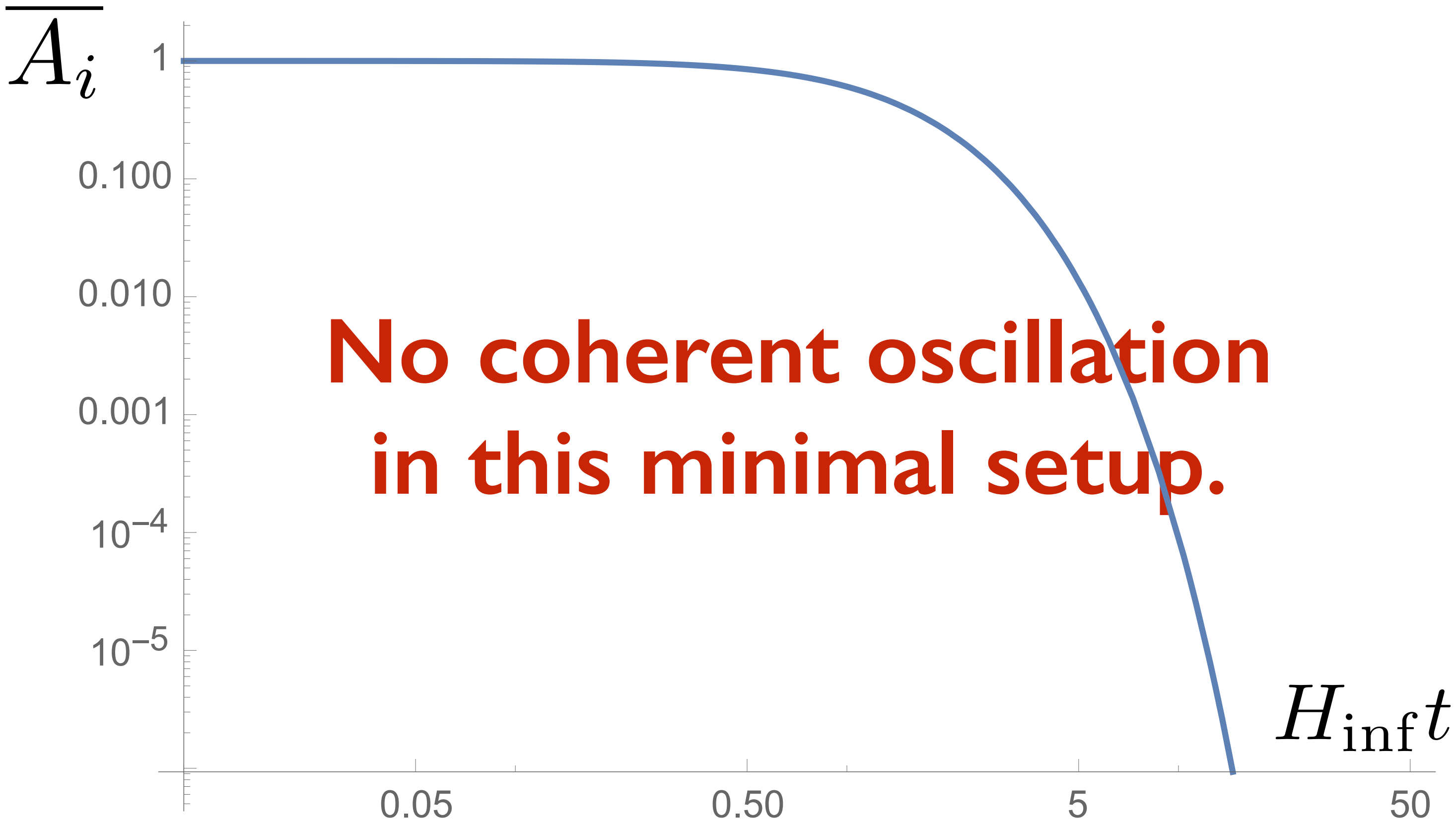

No coherent oscillation in this minimal setup.

2. Vector coherent oscillation

2-1. Minimal massive vector model

★ 2-2. Curvature coupling model

2-3. Kinetic function model

2-4. Observational constraints

Curvature coupling model

- Action $\Delta\mathcal{L} = \frac{1}{2}\xi R\, g^{MN} A_M A_N$

R : Ricci curvature

$R = 6(2H^2 + \dot{H})$

- Eq.of.m $\ddot{\overline{A}}_i + 3H\dot{\overline{A}}_i + \left(m^2 + \left(\frac{1}{6} - \xi\right) R\right) \overline{A}_i = 0.$

[Turner, Widrow (1988) for Magnetogenesis]

- Taking $\xi = \frac{1}{6}$ → **Same eq.of.m as scalar field**

→ Coherent oscillation of vector field

[Arias, Cadamuro, Goodsell, Jaeckel, Redondo, Ringwald (2012)]

- Taking account of not only zero mode but also fluctuation

$$S = S_T + S_L \qquad \vec{A} = \vec{A}_T + \hat{k} A_L$$

$$S_T = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \vec{A}_T|^2 - (k^2 + a^2 \overline{m}^2) |\vec{A}_T|^2 \right)$$:Transverse

$$S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(\frac{a^2 \overline{m}^2}{k^2 + a^2 \overline{m}^2} |\partial_\tau A_L|^2 - a^2 \overline{m}^2 |A_L|^2 \right)$$:Longitudinal

$$\overline{m}^2 \equiv m^2 - \xi R \simeq -2H^2$$

- Wrong sign of longitudinal kinetic term: **Ghost instability !**

[KN (2019); Himmetoglu, Contaldi, Peloso (2008) in the context of vector curvaton]

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Origin of ghost instability

- Curvature coupling means tachyonic mass of vector field

$$\mathcal{L} = \frac{1}{2}\xi R\, g^{MN} A_M A_N \qquad \xi = \frac{1}{6}$$

- Tachyonic mass ⟷ Negative kinetic term in Higgs picture

$$\mathcal{L} = +|D_M \Phi|^2 = +e^2 |\Phi|^2 A_M A^M$$

- Higgs & NG mode (longitudinal vector boson) become ghost

2. Vector coherent oscillation

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Vector with kinetic function

[KN (2019)]

- Action

$$S=\int d^4x\sqrt{-g}\left(-\frac{1}{4}f^2(\phi)g^{MN}g^{KL}\mathcal{F}_{MK}\mathcal{F}_{NL}-\frac{1}{2}m^2g^{MN}\mathcal{A}_M\mathcal{A}_N\right)$$

- Eq.of.m $\ddot{\overline{A}}_i+3H\dot{\overline{A}}_i+\left(\frac{m^2}{f^2}+2H^2+\dot{H}-H\frac{\dot{f}}{f}-\frac{\ddot{f}}{f}\right)\overline{A}_i=0$

[Ratra (1992) for magnetogenesis; Dimopoulos, Karciauskas, Wagstaff (2009) for vector curvaton]

- If $f^2\propto a^\alpha(t)\ :\ \alpha=-4\ \mathrm{or}\ 2$ Hubble mass term is cancelled.

⟶ Vector coherent oscillation is possible!

- **Concrete form of kinetic function**

$$f(\phi) = \exp\left(-\frac{\gamma}{2M_P^2}\int \frac{V}{V_\phi}d\phi\right) \qquad (\phi : \text{inflaton})$$

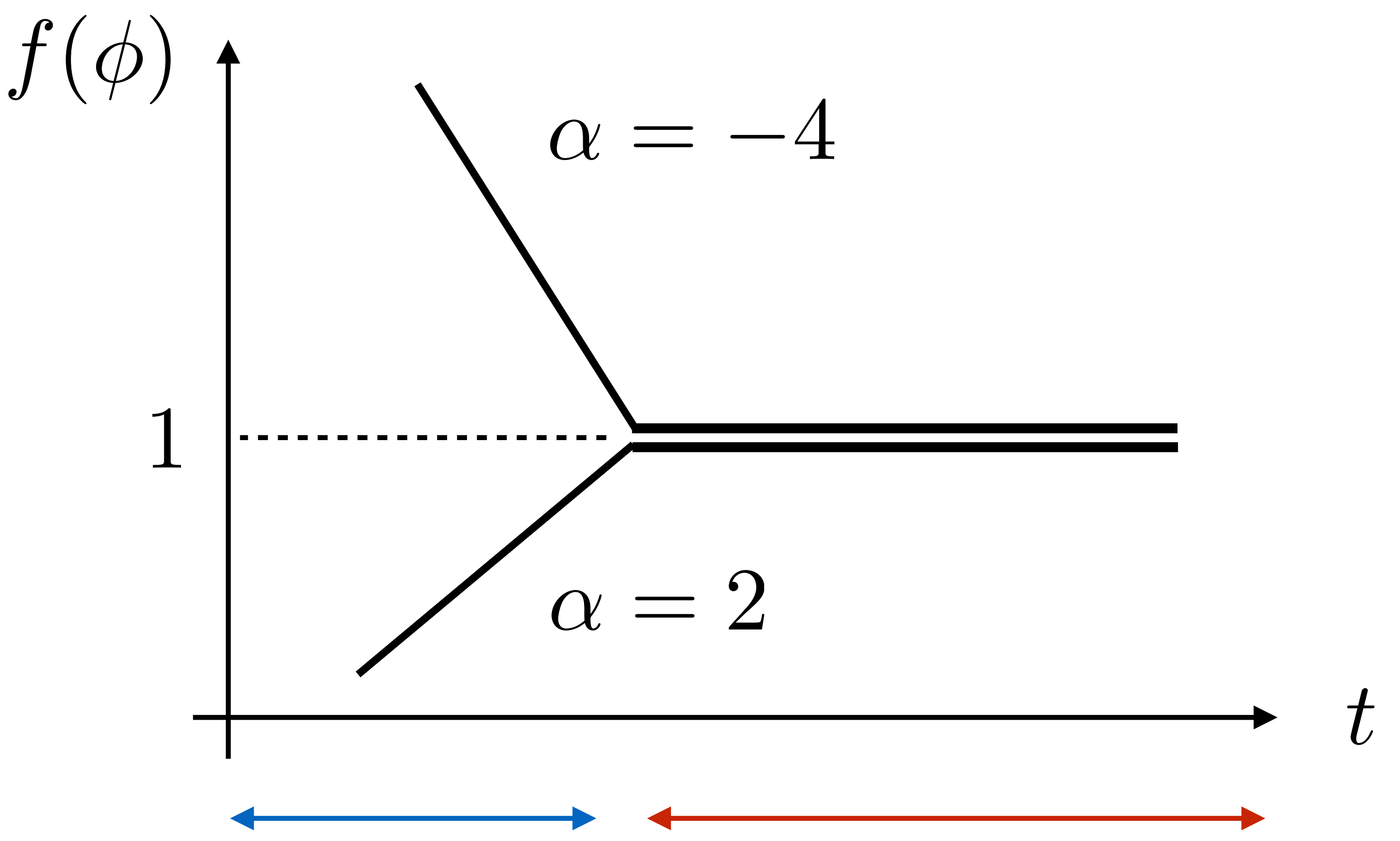
  - Chaotic inflation

$$f(\phi) = \exp\left(-\frac{\gamma}{4n}\frac{\phi^2}{M_P^2}\right), \qquad V(\phi) = \frac{\lambda\phi^n}{n}.$$

  - Hilltop inflation

$$f(\phi) = \exp\left(-\frac{\gamma}{4n(n-2)}\frac{v^n}{M_P^2\phi^{n-2}}\right), \qquad V(\phi) = \Lambda^4\left[1-\left(\frac{\phi}{v}\right)^n\right]^2$$

$$\longrightarrow \begin{cases} f^2 \propto a(t)^\gamma & \text{during inflation} \\ f^2 \simeq 1 & \text{after inflation} \end{cases}$$



inflation

after inflation

$$\ddot{\overline{A_i}} + 3H\dot{\overline{A_i}} + \frac{m^2}{f^2}\overline{A_i} = 0$$

$$\ddot{\overline{A_i}} + 3H\dot{\overline{A_i}} + \left(m^2 + \frac{1-3w}{2}H^2\right)\overline{A_i} = 0$$

$$\overline{A_i} = \mathrm{const}$$

$$H < m \qquad \overline{A_i} \sim a^{-\frac{3}{2}}\cos(mt)$$

Vector coherent oscillation !

Note on the case of $\alpha < -4$

- During inflation:

$$\overline{A} \propto a^{(|1+\alpha|-3)/2} = \begin{cases} a^{\alpha/2-1} & \text{for} \quad 1+\alpha > 0 \\ a^{-\alpha/2-2} & \text{for} \quad 1+\alpha \leq 0 \end{cases}$$

Vector energy density increases during inflation

$\longrightarrow$ Backreaction to the inflaton becomes important

$\longrightarrow$ **Anisotropic inflation** happens

[Watanabe, Kanno, Soda (2009)]

- Vector energy density is saturated at

$$\frac{\rho_A}{\rho_\phi} = -2\epsilon_V \frac{\gamma+4}{\gamma^2} \equiv R_A$$

$\longrightarrow$ Vector coherent oscillation after inflation!

Note on the case of $\alpha > 2$

- During inflation: $\overline{A} \propto a^{\alpha/2-1}$

  Vector energy density does not increase: $\rho_A \propto a^{-\alpha-2}$

  $\longrightarrow$ No backreaction to the inflaton

- To maintain vector condensate during inflation:

$$\frac{m_A}{f} \ll H_{\rm inf} \longrightarrow \frac{\rho_A}{s} \ll 10^{-13}\,{\rm GeV}\left(\frac{10^{-22}\,{\rm eV}}{m_A}\right)^{1/2}\left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)$$

- Impossible to explain observed DM abundance

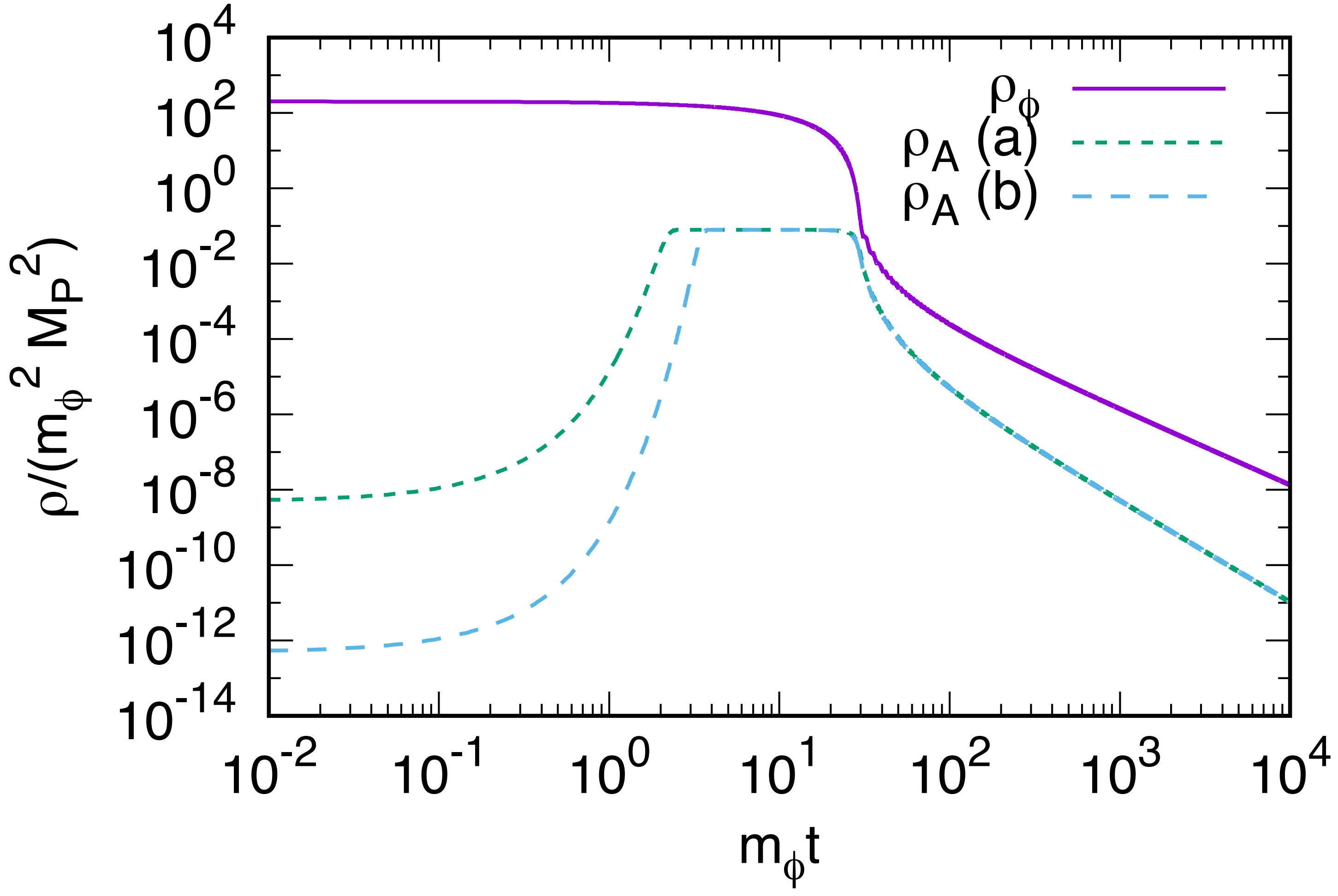
  Loophole : introduce mass function $\mathcal{L} \sim -\frac{1}{2}h^2(\phi)m_A^2 A_\mu^2$

- Numerical calculation

  $(\alpha = -5)$

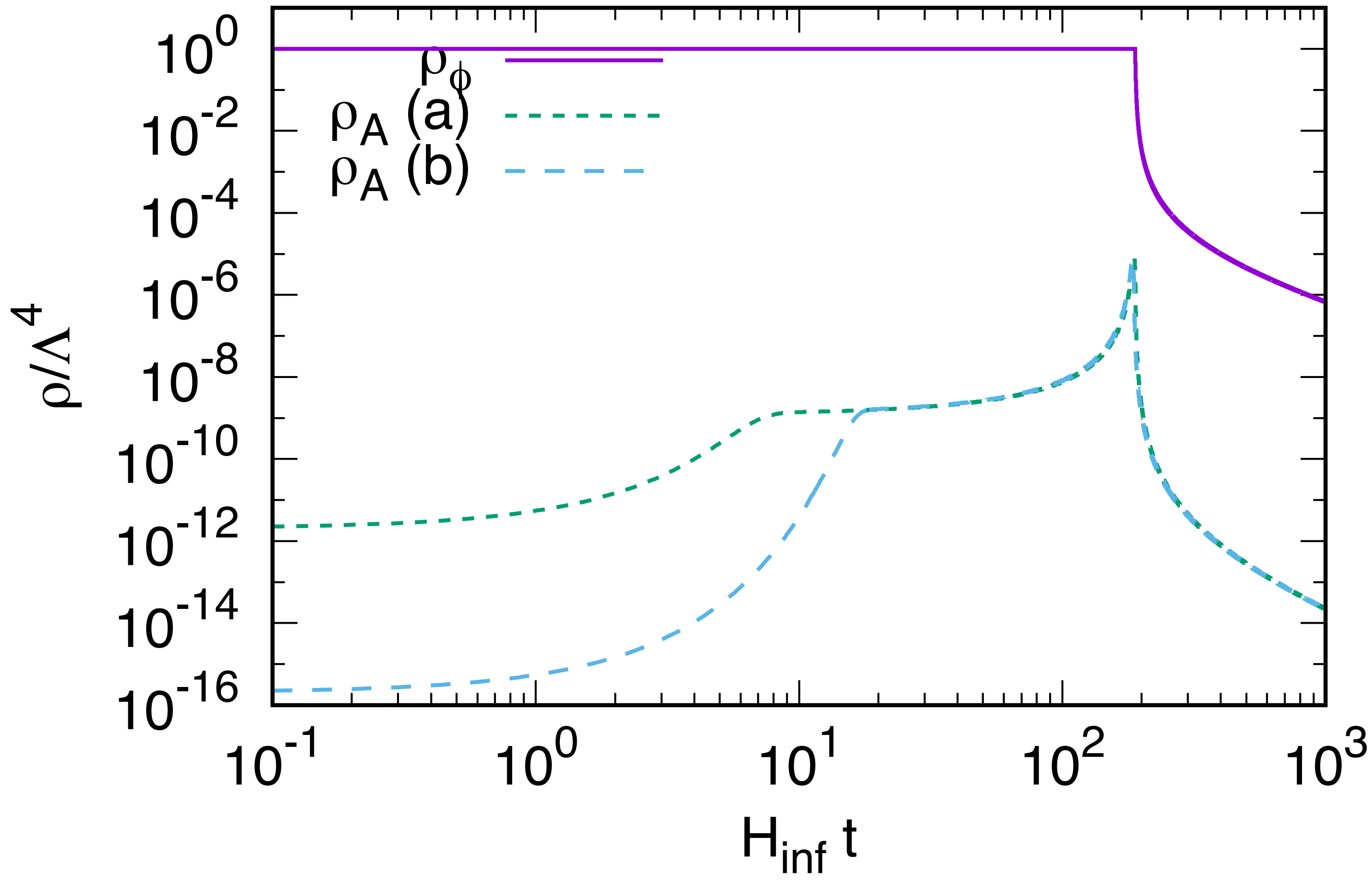
  - chaotic inflation

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2$$



  - hilltop inflation

$$V(\phi) = \Lambda^4\left[1-\left(\frac{\phi}{v}\right)^n\right]^2$$



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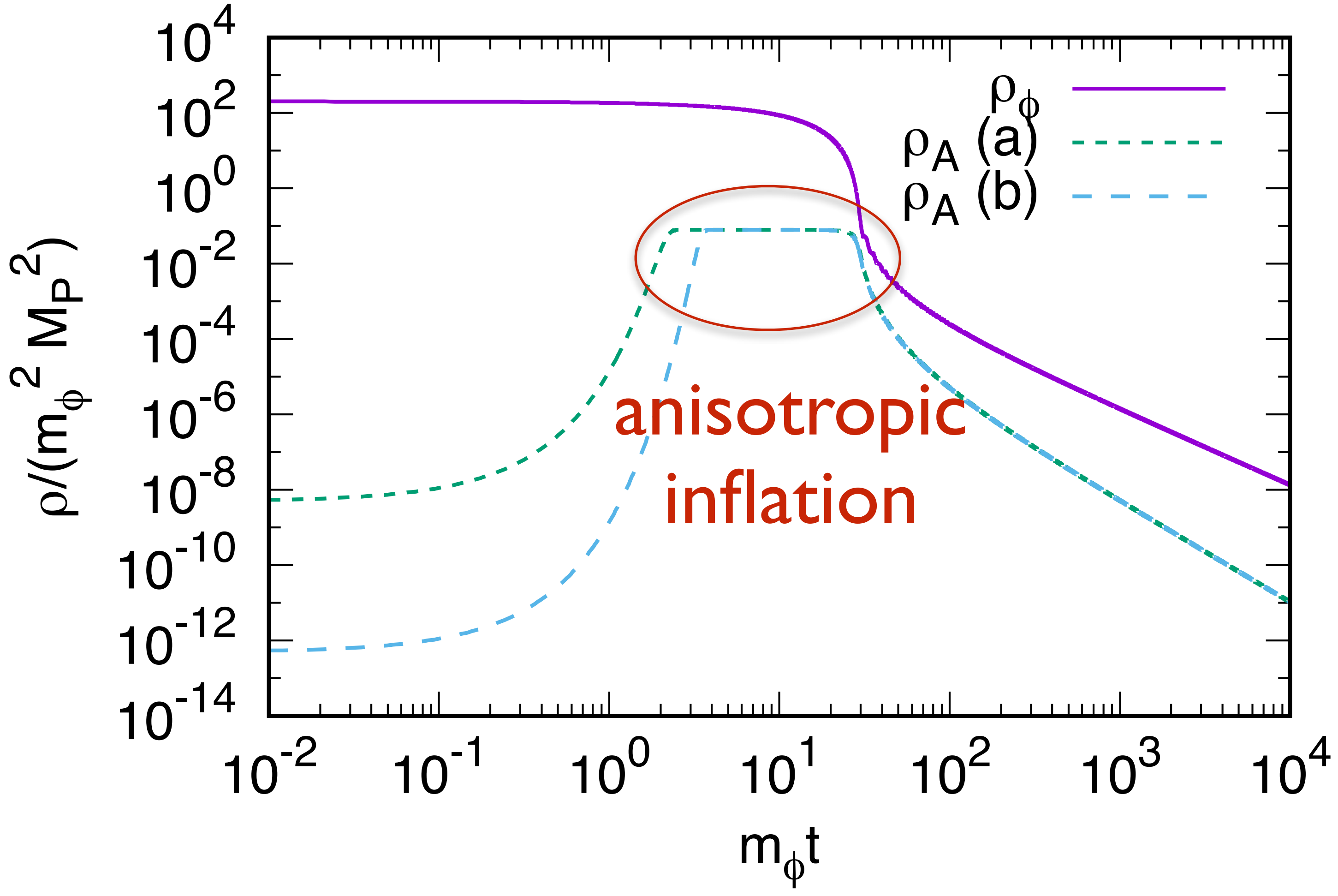

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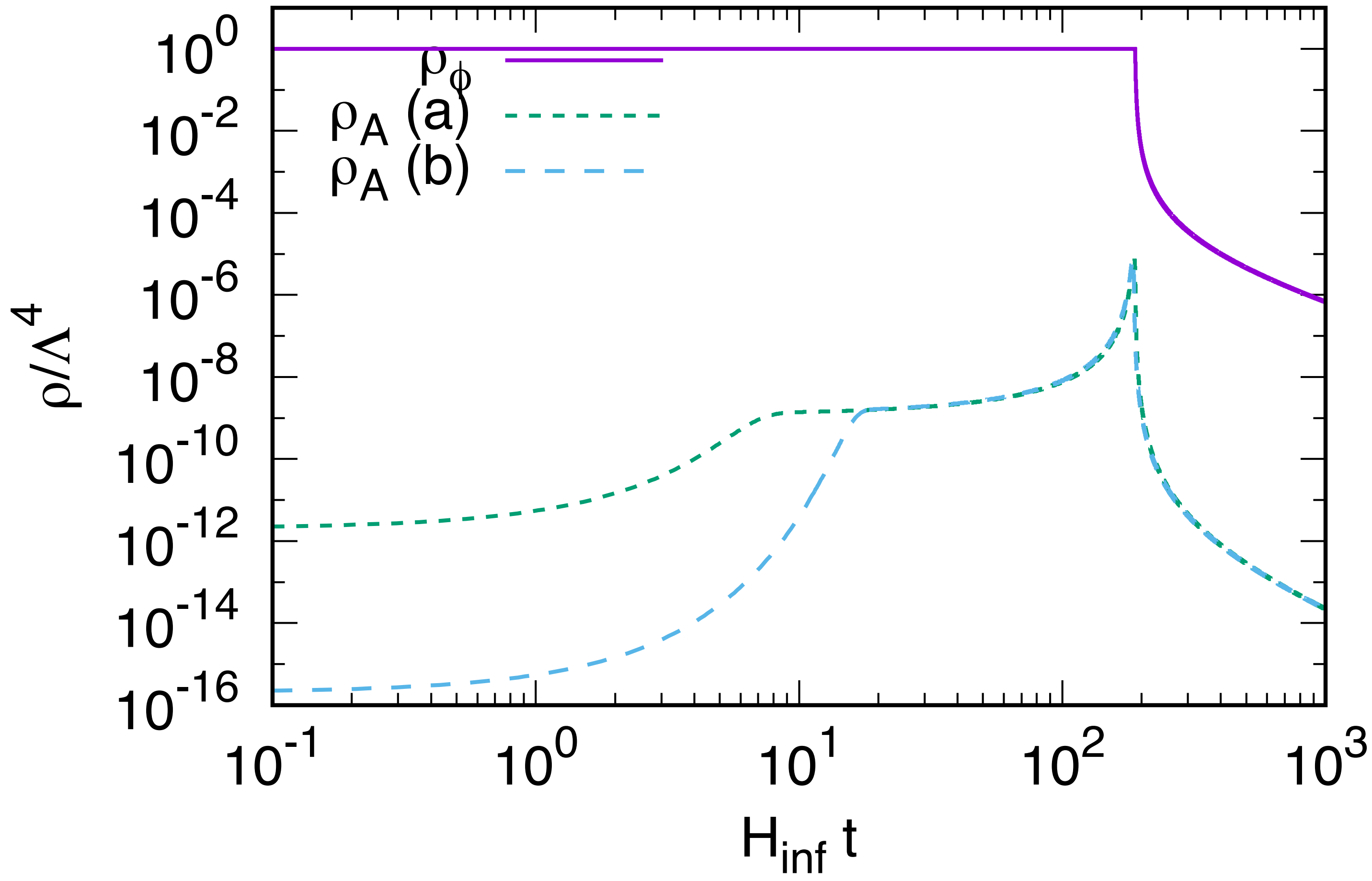

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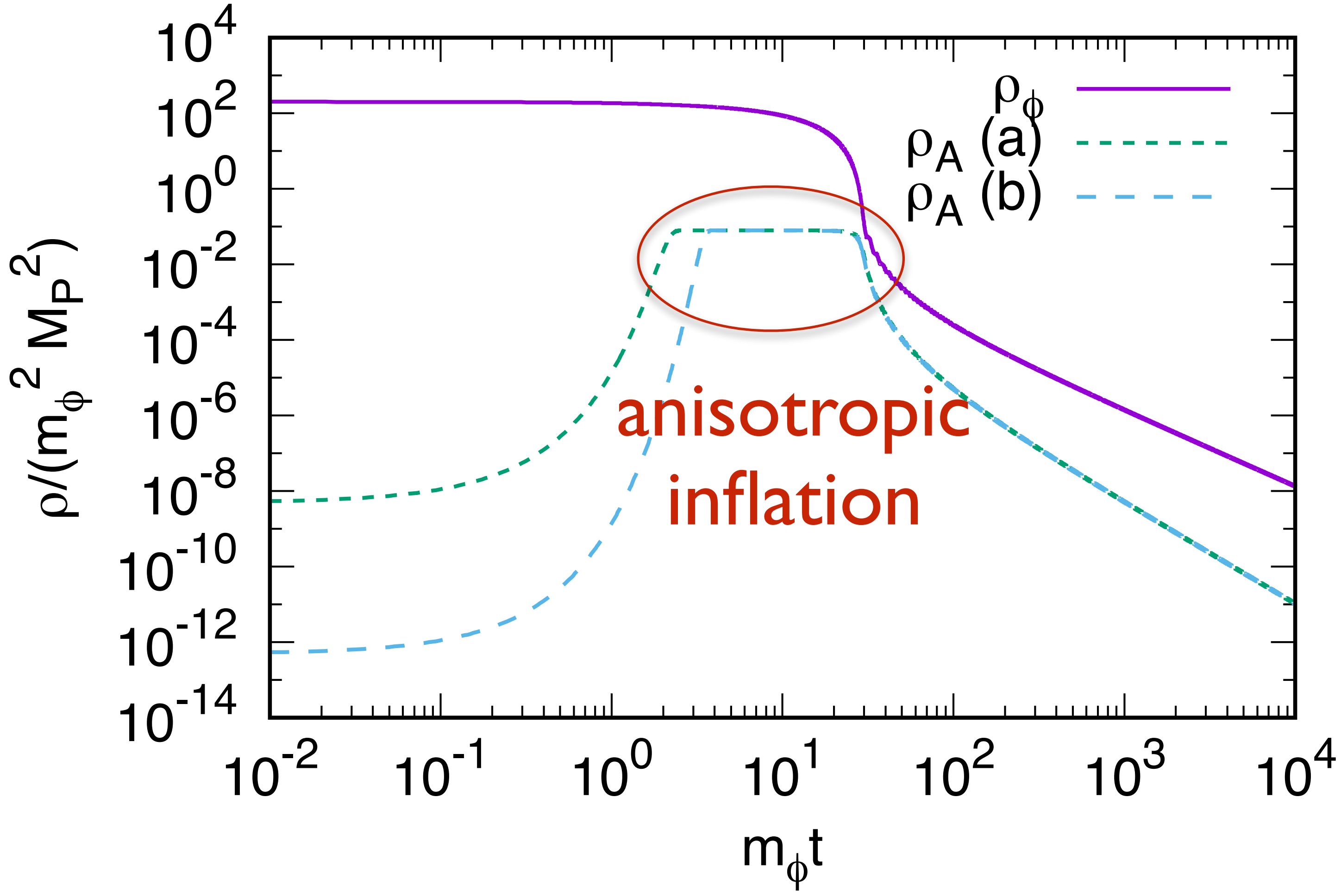

anisotropic inflation

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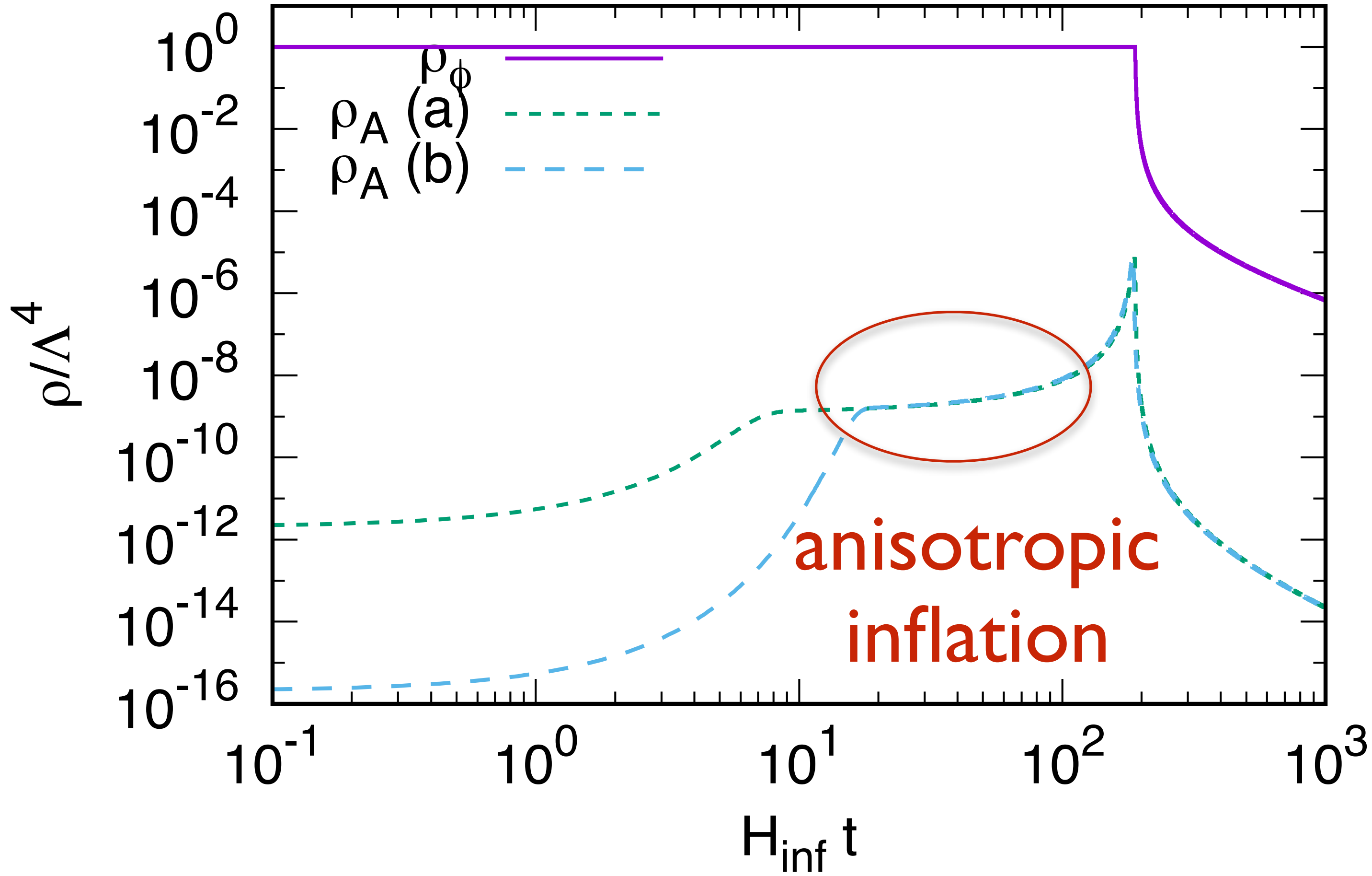

anisotropic inflation

Vector DM abundance

- Vector dynamics after inflation

$$\ddot{\overline{A}} + 3H\dot{\overline{A}} + \left(m_A^2 + \frac{1-3w}{2}H^2\right)\overline{A} = 0$$

- Final vector coherent oscillation abundance

$$\frac{\rho_A}{s} \simeq \begin{cases} 3.7\times10^{-10}\,\mathrm{GeV}\left(\frac{R_A}{0.1}\right)\left(\frac{m_A}{10^{-8}\,\mathrm{GeV}}\right)^{1/2}\left(\frac{10^{14}\,\mathrm{GeV}}{H_{\mathrm{inf}}}\right)^{2/3}\left(\frac{T_{\mathrm{R}}}{10^{6}\,\mathrm{GeV}}\right)^{4/3} & \text{for} \quad m_A < \Gamma_\phi \\ 3.5\times10^{-10}\,\mathrm{GeV}\left(\frac{R_A}{0.1}\right)\left(\frac{m_A}{1\,\mathrm{GeV}}\right)^{2/3}\left(\frac{10^{14}\,\mathrm{GeV}}{H_{\mathrm{inf}}}\right)^{2/3}\left(\frac{T_{\mathrm{R}}}{10\,\mathrm{GeV}}\right) & \text{for} \quad m_A > \Gamma_\phi \end{cases}$$

DM abundance can be explained in kinetic function model

2. Vector coherent oscillation

2-1. Minimal massive vector model

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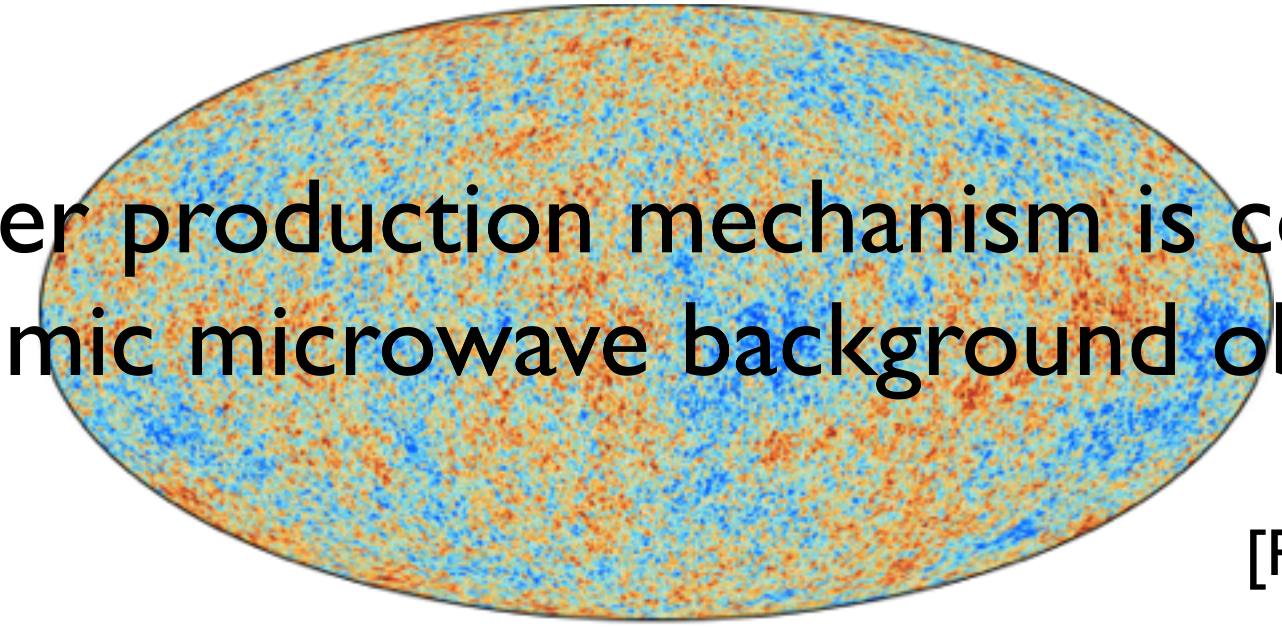
★ 2-4. Observational constraints

Observational constraints



Dark matter production mechanism is constrained by the cosmic microwave background observation

[Planck CMB map]

Vector coherent oscillation DM: $\begin{cases} \text{1. Isocurvature fluctuation} \\ \text{2. Statistical anisotropy} \end{cases}$

There are no parameter region that satisfy both constraints!

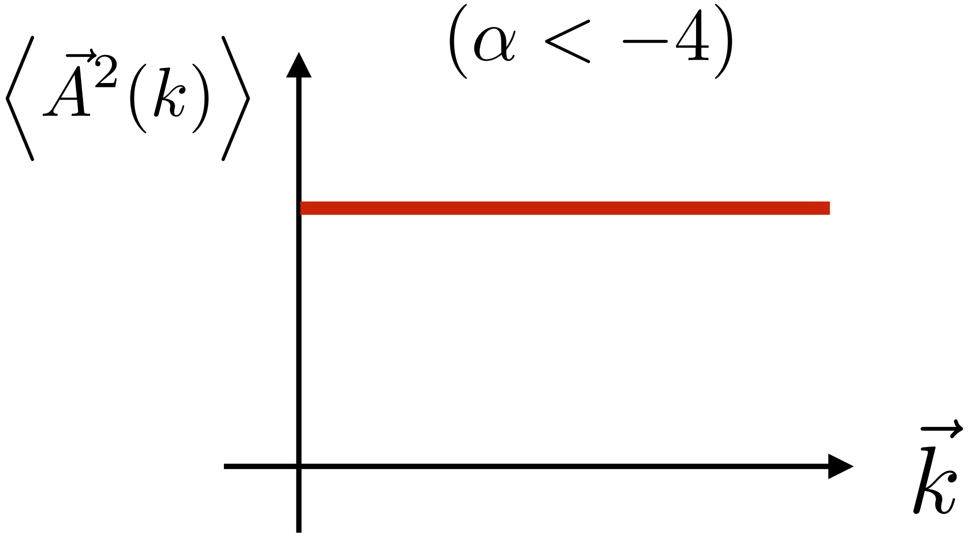
[KN (2020)]

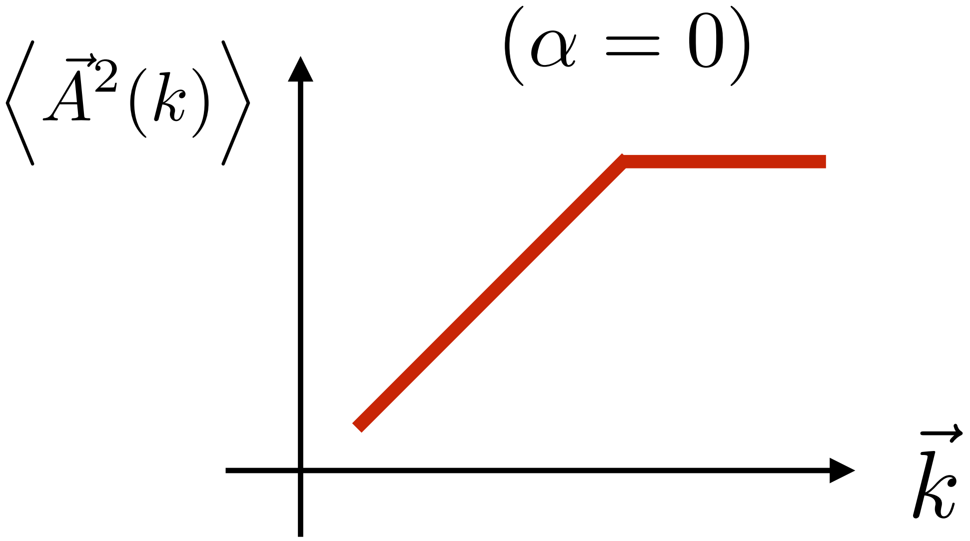
Isocurvature fluctuation

- Long-wave fluctuation of hidden photon during inflation

  ⟶ DM isocurvature perturbation

- Isocurvature power spectrum is (nearly) scale-invariant $(\alpha < -4)$



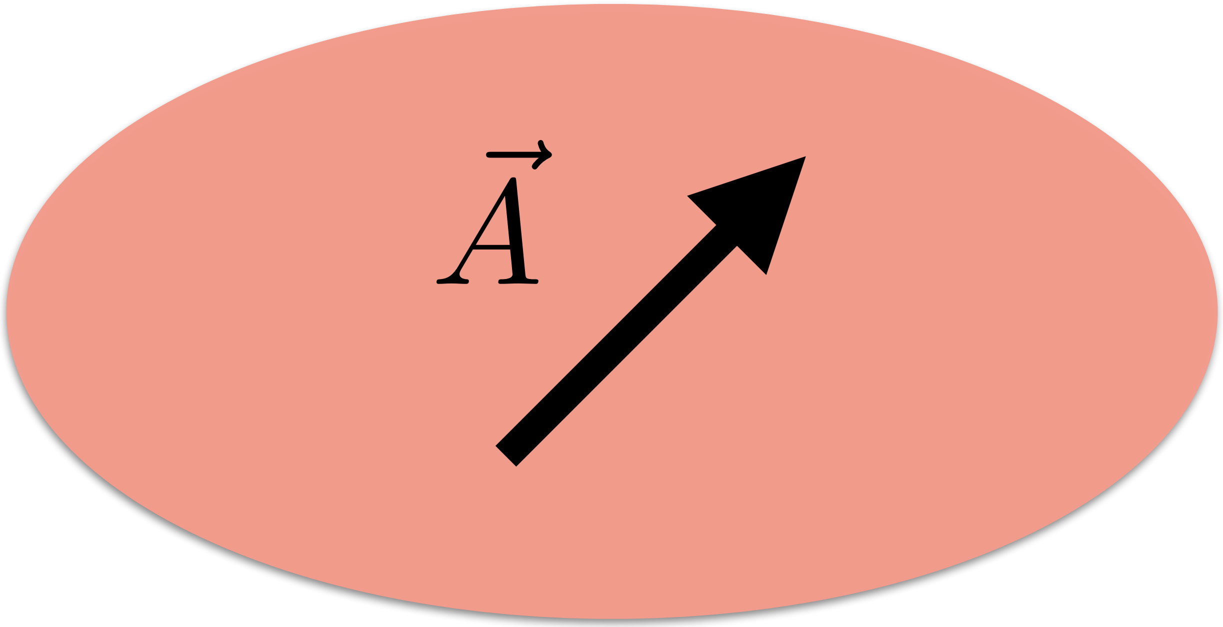


- Observational constraint on DM isocurvature fluctuation

$$\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} \lesssim 0.1 \longleftrightarrow S_{\rm DM} = \frac{\delta\rho_A}{\rho_A} \sim \frac{|\delta \overline{A}_T|}{\overline{A}} \sim \frac{H_{\rm inf}}{\pi \overline{A}_i} \lesssim 10^{-5}$$

Statistical anisotropy

- Vector condensate indicates
  preferred direction

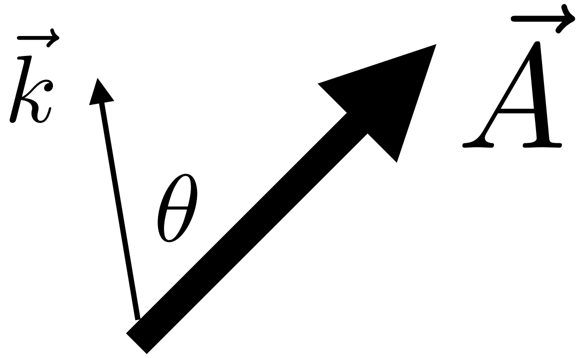


$\longrightarrow \left\{ \begin{array}{l} \textbf{Anisotropic expansion} \\ \quad \text{This effect turns out to be small} \\ \textbf{Statistical anisotropy of the pertrubation} \end{array} \right.$

- Curvature perturbation $\left\langle \zeta(\vec{k})\zeta^*(\vec{k}')\right\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(\vec{k})(2\pi)^3\delta(\vec{k}-\vec{k}')$

$\mathcal{P}_\zeta(\vec{k}) = \mathcal{P}_\zeta(k)$: statistically isotropic

$\mathcal{P}_\zeta(\vec{k}) = \mathcal{P}_\zeta(k,\theta)$: statistically anisotropic



Rough sketch

$$S = \int d^4x \sqrt{-g} \left(-\frac{f^2(\phi)}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} \right) \qquad f(\phi) = \exp\left(-\frac{\gamma}{2M_P^2} \int \frac{V}{V_\phi} d\phi \right)$$

- Perturbed action: $A_i = A_{i0} + \delta A_i \qquad \zeta = -H_{\rm inf} \frac{\delta\phi}{\dot{\phi}}$

$$\longrightarrow \qquad \delta S \simeq \int d\tau d^3k \left(-\gamma \vec{E}^f \cdot \delta \vec{E}^f(\vec{k}) \zeta(-\vec{k}) \right) \equiv -\int d\tau\, \mathcal{H}_{\rm int}$$

- Power spectrum at second order [Bartolo, Matarrese, Peloso, Ricciadorn (2012)]

$$\left\langle \zeta(\vec{k},\tau) \zeta(\vec{k}',\tau) \right\rangle_{\rm 2nd} = -\int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \left\langle \left[\left[\zeta_k^0(\tau) \zeta_{k'}^0(\tau), \mathcal{H}_{\rm int}(\tau_1) \right], \mathcal{H}_{\rm int}(\tau_2) \right] \right\rangle$$

- Anisotropic power spectrum of quadrupolar asymmetry

$$\mathcal{P}_\zeta(\vec{k}) = \mathcal{P}_\zeta^0(k) \left[1 + g_* \sin^2 \theta_k \right]$$

- Observational constraint: $\underline{-0.010 < g_* < 0.019}$ [Planck(2015)]

- <u>Rough sketch</u>

$$S = \int d^4x\sqrt{-g}\left(-\frac{f^2(\phi)}{4}\mathcal{F}_{MN}\mathcal{F}^{MN}\right) \qquad f(\phi) = \exp\left(-\frac{\gamma}{2M_P^2}\int\frac{V}{V_\phi}d\phi\right)$$

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- Power spectrum at second order [Bartolo, Matarrese, Peloso, Ricciadorn (2012)]

$$\left\langle\zeta(\vec{k},\tau)\zeta(\vec{k}',\tau)\right\rangle_{\rm 2nd} = -\int_{\tau_i}^{\tau}d\tau_1\int_{\tau_i}^{\tau}d\tau_2\left\langle\left[\left[\zeta_k^0(\tau)\zeta_{k'}^0(\tau),\mathcal{H}_{\rm int}(\tau_1)\right],\mathcal{H}_{\rm int}(\tau_2)\right]\right\rangle$$

- Anisotropic power spectrum of quadrupolar asymmetry

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- Observational constraint: $-0.010 < g_* < 0.019$ [Planck(2015)]

- **Concrete form**

$$g_* \simeq \frac{\gamma^2(\alpha+1)^4}{18\epsilon M_P^2}\left(\frac{\Gamma(\nu)}{\Gamma(3/2)}\right)^2 \left(H_{\rm inf}A^f(\tau)\right)^2 \frac{k^{4+\alpha}}{(-2\tau)^{2+\alpha}} I(\tau)$$

$$I(\tau) \equiv \left[\int_{\tau_i}^{\tau} d\tau_1\, \tau_1^{\alpha+3}\right]^2 = \begin{cases} \log^2(\tau/\tau_i) & \text{for} \quad \alpha = -4 \\ \left(\dfrac{\tau^{\alpha+4}}{\alpha+4}\right)^2 & \text{for} \quad \alpha < -4 \end{cases}$$

$$|g_*| = \frac{\gamma^2(\alpha+1)^2}{3\cdot 2^{\alpha+4}(\alpha+4)^2}\left(\frac{\Gamma(\nu)}{\Gamma(3/2)}\right)^2 \frac{\rho_A(\tau)}{\epsilon\rho_\phi(\tau)}(k\tau)^{\alpha+4} \quad \gtrsim \quad \mathcal{C}\frac{\rho_A(\tau)}{\epsilon\rho_\phi(\tau)} e^{-(\alpha+4)N(k)}$$

$$\longrightarrow \quad |g_*| \sim \mathcal{C}\frac{\mathcal{P}_\zeta^0}{\mathcal{P}_S} e^{-2(\alpha+4)N_{\rm st}}$$

- **Note:** $\mathcal{P}_S \sim \dfrac{H_{\rm inf}^2}{\overline{A}_i^2} \sim \dfrac{\rho_\phi}{\rho_A}\dfrac{H_{\rm inf}^2}{M_P^2} \sim \dfrac{\rho_\phi}{\rho_A}(\epsilon\mathcal{P}_\zeta)$

No-go theorem

[KN (2020)]

$$|g_*| \gtrsim \mathcal{C}\frac{\mathcal{P}_\zeta^0}{\mathcal{P}_S} \gg 1 \quad \text{for} \quad \frac{\mathcal{P}_S}{\mathcal{P}_\zeta^0} < 1$$

CMB constraint: $-0.010 < g_* < 0.019$ $\frac{\mathcal{P}_S}{\mathcal{P}_\zeta} \lesssim 0.1$

There are no parameter region that satisfy both constraints!

Loophole & Summary

- Curvaton scenario

Scalar other than inflaton produces density perturbation

$$\delta\phi \sim \frac{H_{\rm inf}}{2\pi} \neq \sqrt{\epsilon}M_P\zeta$$

$$\longrightarrow \quad |g_*| \sim \mathcal{C}\frac{\mathcal{P}_\zeta^0}{\mathcal{P}_S}e^{-2(\alpha+4)N_{\rm st}} \qquad \mathcal{P}_\zeta^0 \ll \mathcal{P}_\zeta^{(\rm obs)}$$

- Non-standard thermal history after inflation
- Other way?

Vector coherent oscillation DM scenario is not excluded, but severely constrained and need complicated model.

3. Gravitational productuion of vector DM

3. Gravitational Production

- ★ 3-1. Gravitational production of scalar
- 3-2. Gravitational production of vector

Particle Production

[Dolgov, Kirilova (1990), Traschen, Brandenberger (1990)]

- Inflaton coherent oscillation: $\phi = \widetilde{\phi}(t)\cos(m_\phi t)$
- Interaction with light particle χ

$$\mathcal{L}_{\rm int} = \frac{\phi}{M}(\partial_\mu\chi)^2 \longrightarrow \text{“decay”} \quad \Gamma_{\phi\to\chi\chi} = \frac{m_\phi^3}{32\pi M^2}$$

$$\mathcal{L}_{\rm int} = g^2\phi^2\chi^2 \longrightarrow \text{“annihilation”} \quad \Gamma_{\phi\phi\to\chi\chi} \sim \frac{g^4\phi^2}{8\pi m_\phi}$$

Time dependence of effective mass/kinetic term leads to particle production

Gravitational Particle Production

[Parker (1969), Ford (1986)]

- Real scalar field interacting only through gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}m_\chi^2\chi^2\right)$$

FRW metric: $ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$

$$\longrightarrow \quad S = \int d\tau d^3x \frac{a^2(\tau)}{2}\left[\chi'^2 - (\nabla\chi)^2 - a^2 m_\chi^2\chi^2\right]$$

- Time dependent mass/kinetic term from scale factor

“Gravitational particle production”

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“Gravitational particle production”

Scalar production during inflation

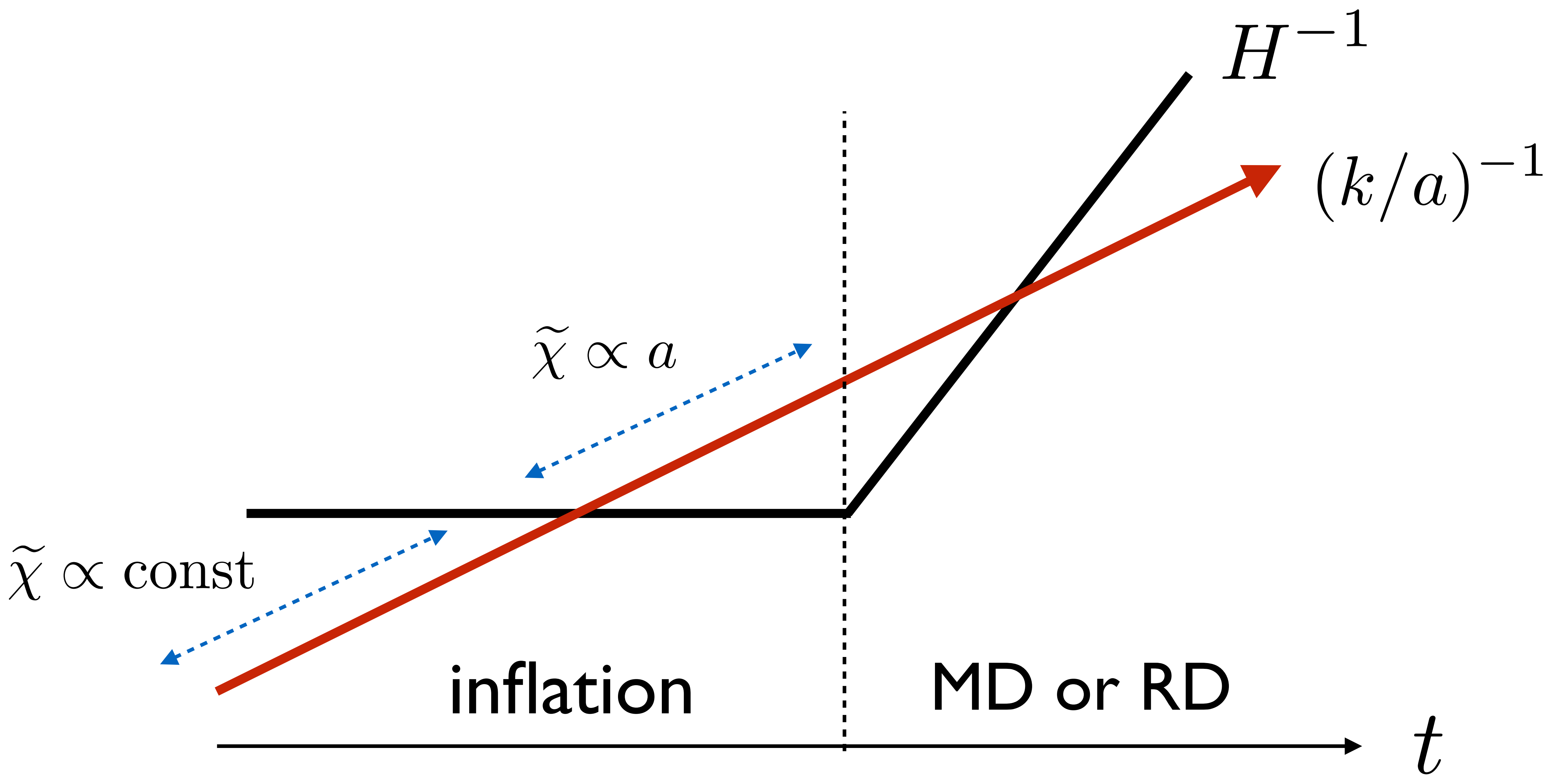
$$S = \int d\tau d^3x \frac{a^2(\tau)}{2}\left[\chi'^2 - (\nabla\chi)^2 - a^2 m_\chi^2 \chi^2\right]$$

$$= \int d\tau d^3x \frac{1}{2}\left[\widetilde{\chi}'^2 - (\partial_i\widetilde{\chi})^2 - m_\chi^{(\mathrm{eff})2}\widetilde{\chi}^2\right] \qquad \widetilde{\chi} \equiv a\chi$$

Effective mass: $m_\chi^{(\mathrm{eff})2} = a^2 m_\chi^2 - \frac{a''}{a} \quad = a^2\left(m_\chi^2 - 2H^2 + \frac{\dot{\phi}^2}{2M_P^2}\right)$

- Tachyonic mass during inflation if $m_\chi^2 < 2H_{\mathrm{inf}}^2$
- Oscillating mass during reheating

- Evolution of Fourier mode



- (Nearly) scale invariant spectrum at the end of inflation

$$\langle \chi^2(k) \rangle \simeq \left(\frac{H_{\rm inf}}{2\pi} \right)^2$$

Note on gravitational production

- Gravitational production also works during inflaton oscillation

[Ema, Jinno, Mukaida, KN (2015); Ema, KN, Tang (2018); Chung, Kolb, Long (2018)]

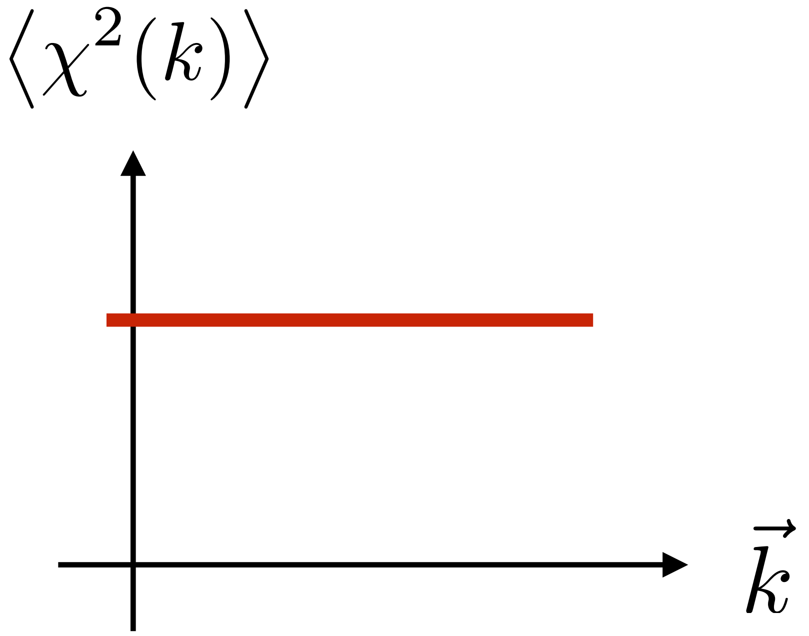
$$m_\chi^{(\mathrm{eff})2} = \langle a^2 \rangle \left[m_\chi^2 - 2\langle H \rangle^2 - \left(m_\chi^2 - 2\langle H \rangle^2 \right) \frac{\varphi^2}{4M_P^2} + \langle H \rangle \frac{\varphi\dot{\varphi}}{M_P^2} + \frac{\dot{\varphi}^2}{2M_P^2} \right]$$

- k~0 mode accumlates to form scalar condensate

$\longrightarrow$ Scalar coherent oscillation

- (Nearly) scale invariant spectrum

$\longrightarrow$ Strong constraint from isocurvature fluctuation



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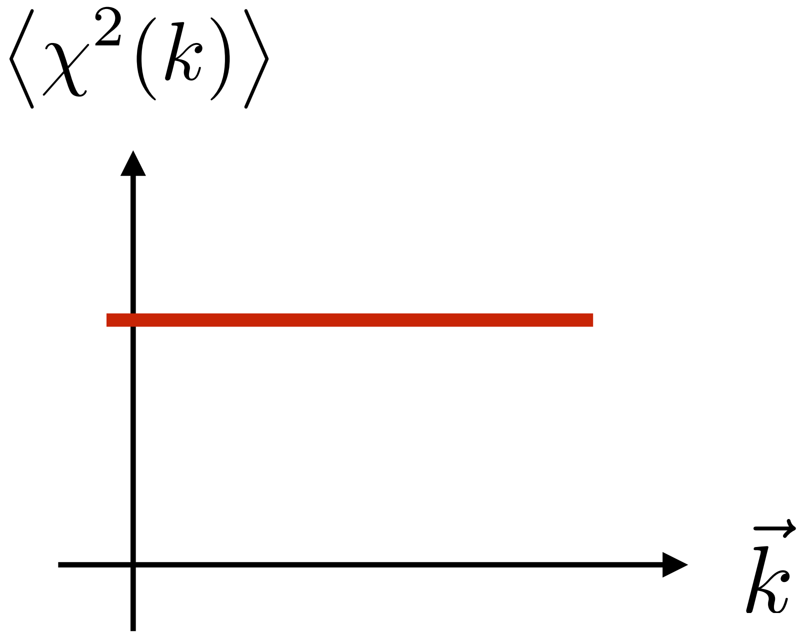
"Slow" part

- k~0 mode accumlates to form scalar condensate

→ Scalar coherent oscillation

- (Nearly) scale invariant spectrum

→ Strong constraint from isocurvature fluctuation



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"Slow" part

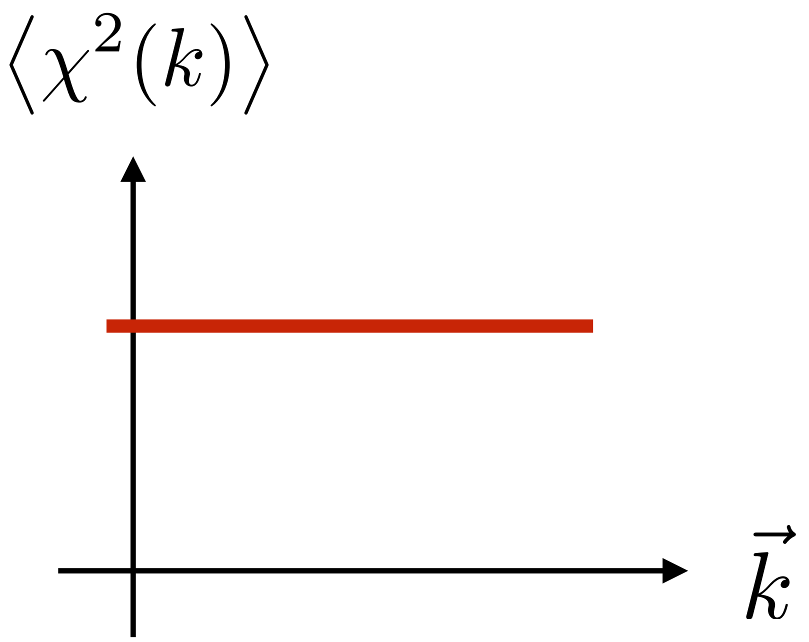
"Fast" part

- k~0 mode accumlates to form scalar condensate

→ Scalar coherent oscillation

- (Nearly) scale invariant spectrum

→ Strong constraint from isocurvature fluctuation



3. Gravitational Production

3-1. Gravitational production of scalar

★ 3-2. Gravitational production of vector

Minimal massive vector

- Massive vector boson minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}m^2 g^{\mu\nu}A_\mu A_\nu\right]$$

$$= \int d\tau d^3x\left[-\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}a^2 m^2\eta^{\mu\nu}A_\mu A_\nu\right]$$

In the massless limit, vector does not “feel” gravity.

Mass term induces gravitational production

[Graham, Mardon, Rajendran (2015)]

- Transverse-longitudinal decomposition

$$S = S_T + S_L$$

$$S_T = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \vec{A}_T|^2 - (k^2 + a^2 m^2) |\vec{A}_T|^2 \right)$$

$$S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(\frac{a^2 m^2}{k^2 + a^2 m^2} |\partial_\tau A_L|^2 - a^2 m^2 |A_L|^2 \right)$$

- Transverse mode is similar to scalar with conformal coupling

Transverse mode production is subdominant
compared with longitudinal mode

Longitudinal mode

$$S_L = \int \frac{d^3k d\tau}{(2\pi)^3} \frac{1}{2} \left(|\partial_\tau \widetilde{A}_L|^2 - (k^2 + m_L^2)|\widetilde{A}_L|^2 \right)$$

$$m_L^2 = a^2 m^2 - \frac{k^2}{k^2 + a^2 m^2} \left(\frac{a''}{a} - \frac{a'^2}{a^2} \frac{3a^2 m^2}{k^2 + a^2 m^2} \right)$$

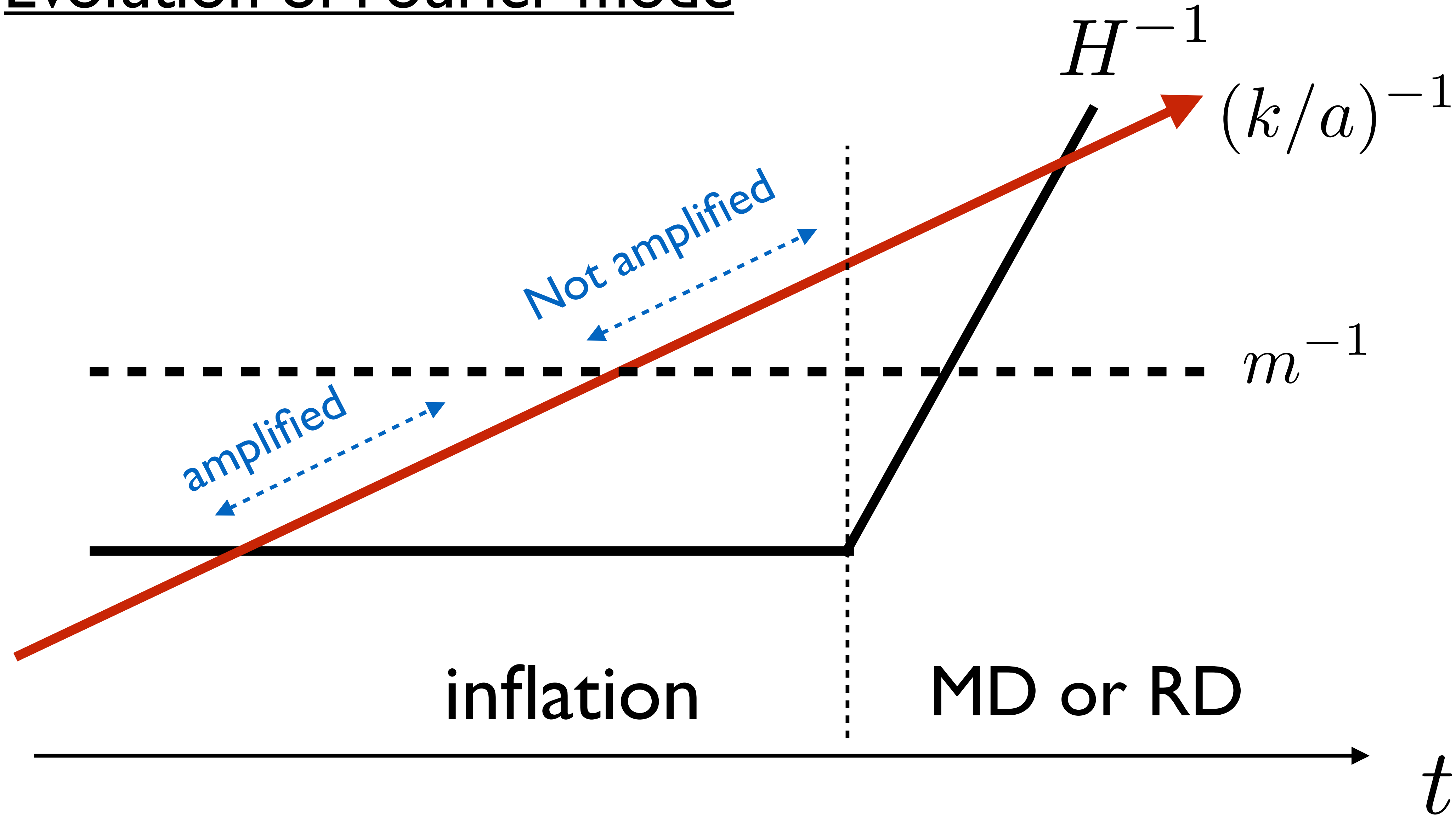
- High momentum $(k/a > m)$ $\quad m_L^2 \simeq a^2(m^2 - 2H_{\rm inf}^2)$

⟶ Superhorizon amplification during inflation

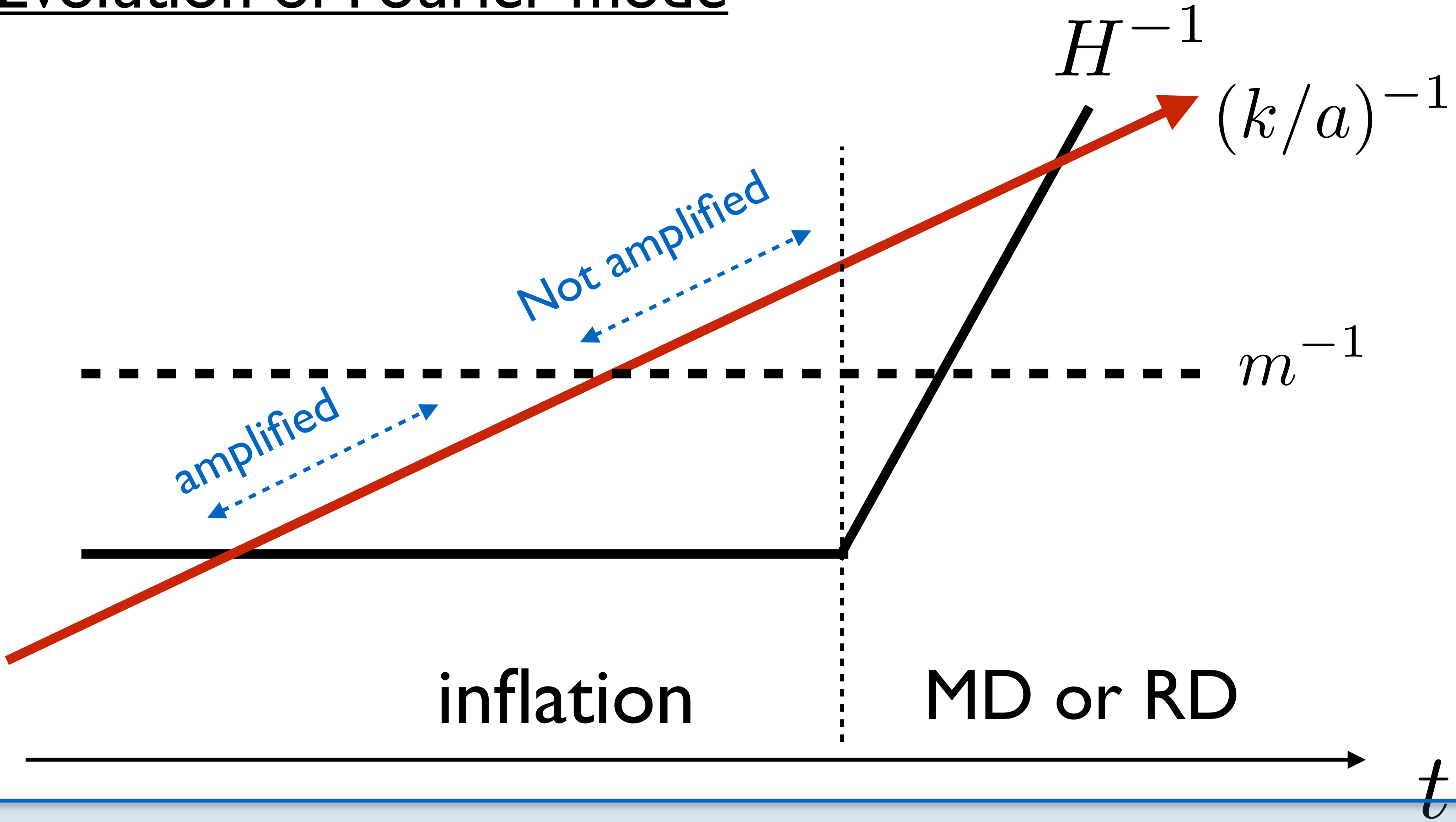
- Low momentum $(k/a < m)$ $\quad m_L^2 \simeq a^2 \left(m^2 + H_{\rm inf}^2 \frac{k^2}{a^2 m^2} \right)$

⟶ No amplification

- Evolution of Fourier mode



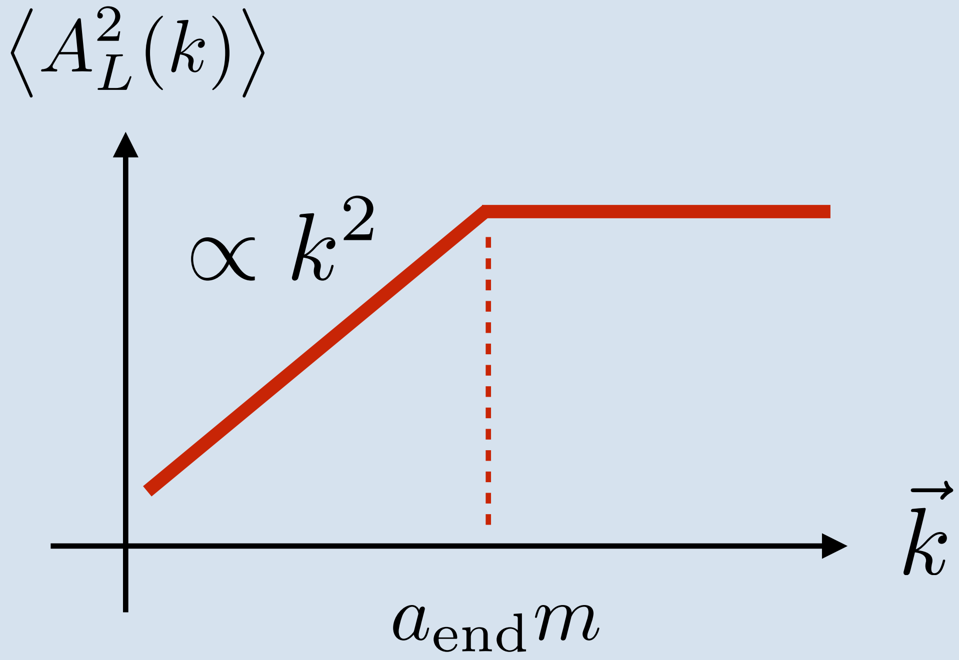
- Evolution of Fourier mode



- **Strongly blue spectrum**

→ No constraint from isocurvature perturbation!

[Graham, Mardon, Rajendran (2015)]



- Note:
    - Gravitational production also works during inflaton oscillation

      [Ema, KN, Tang (2019)]
    - Final abundance depends on thermal history after inflation

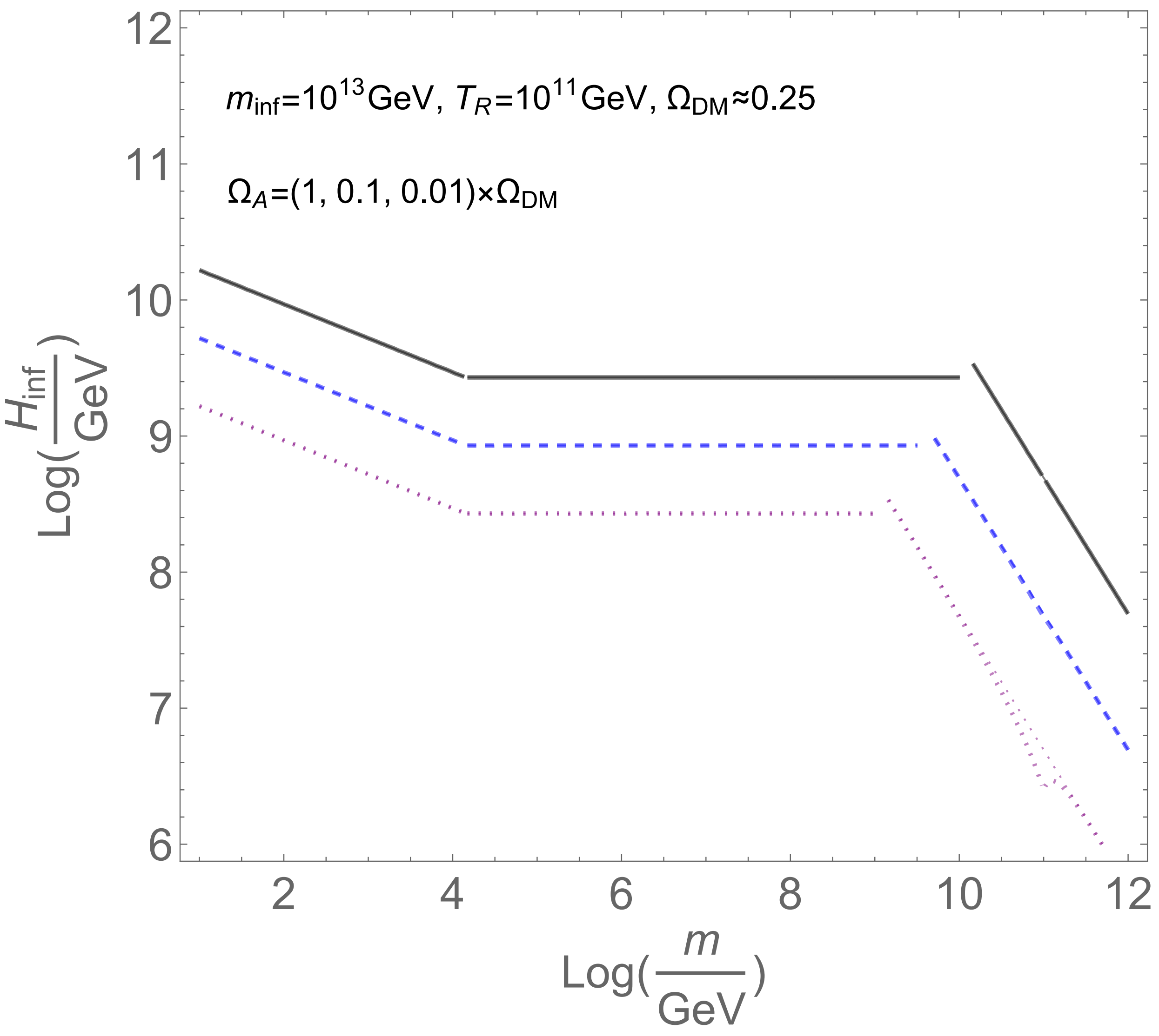
      [Ema, KN, Tang (2019), Ahmed, Grzadkowski, Socha (2020)]
    - Thermal production from SM scattering with graviton exchange

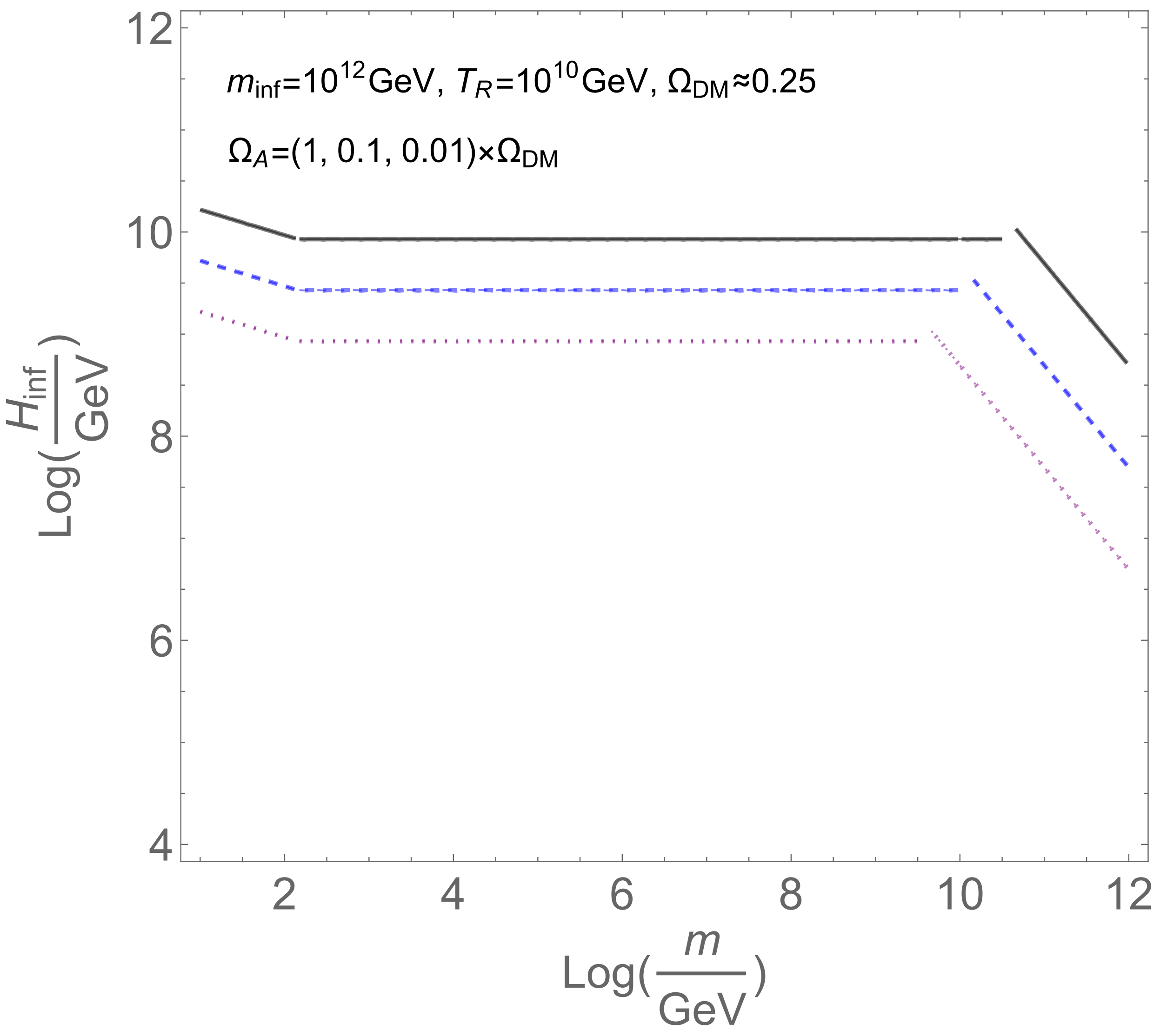
      [Garny, Sasndora, Sloth (2015), Tang, Wu (2016)]
- Final vector boson abundance

$$\frac{\rho_L}{s} \simeq \begin{cases} \dfrac{\mathcal{C}_L T_{\rm R} H_{\rm inf} m}{4M_P^2} & \text{for} \quad H_{\rm inf} < m \\ \dfrac{T_{\rm R} H_{\rm inf}^2}{32\pi^2 M_P^2} & \text{for} \quad H_{\rm R} < m < H_{\rm inf} \\ \left(\dfrac{90}{\pi^2 g_*}\right)^{1/4} \dfrac{m^{1/2} H_{\rm inf}^2}{32\pi^2 M_P^{3/2}} & \text{for} \quad m < H_{\rm R} \end{cases}$$

Contour of vector boson abundance

[Ema, KN, Tang (2019)]





Vector DM is possible for wide range of mass & inflation scale

$$m \gtrsim 10^{-6}\,\mathrm{eV} \qquad H_{\rm inf} \lesssim 10^{14}\,\mathrm{GeV}$$

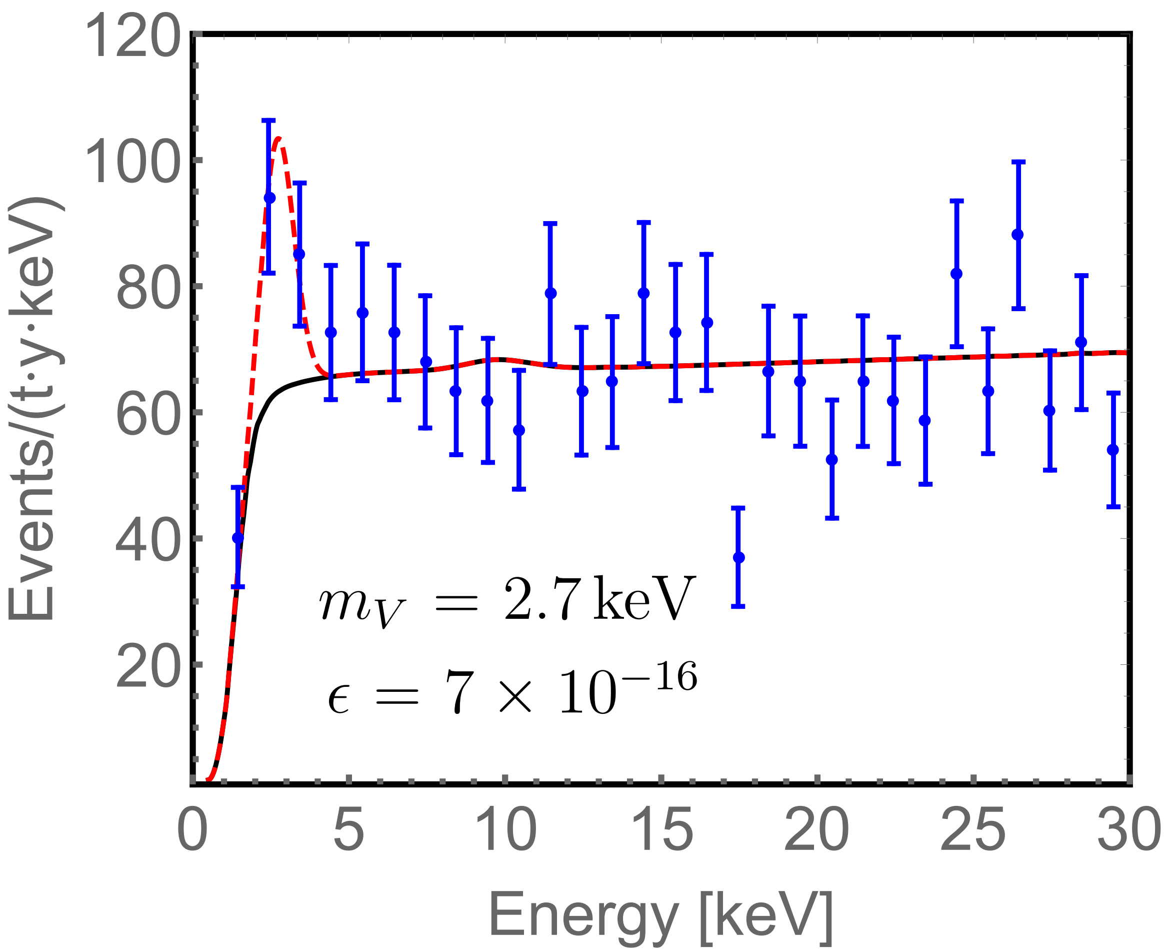
Comment on XENON1T anomaly

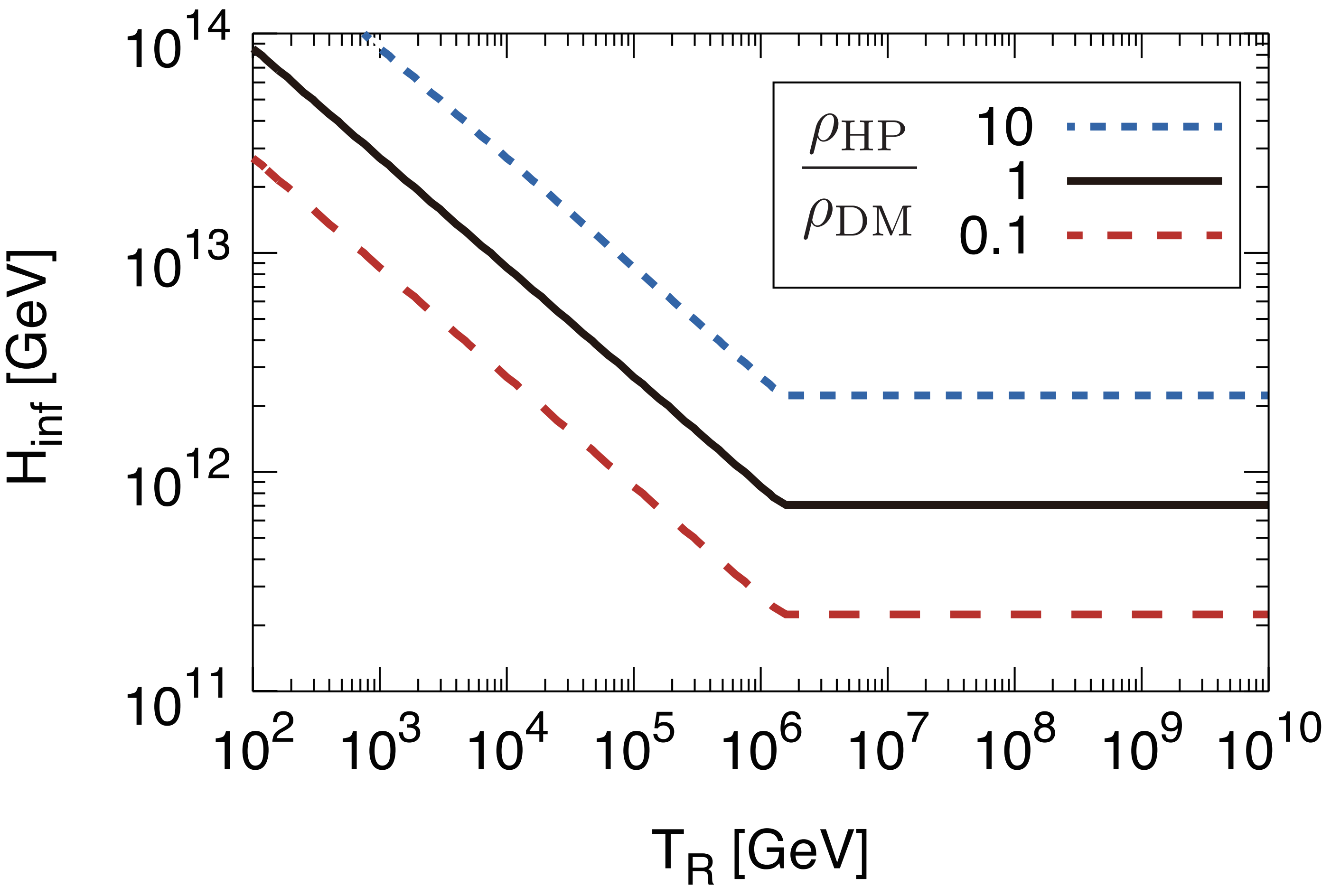
- XENON1T found excess electronic recoil events
- Hidden photon DM with small kinetic mixing can explain it

$m_V = 2.7\,\mathrm{keV} \qquad \epsilon = 7\times 10^{-16}$ [Alonso-Alvarez et al (2020)]

- Gravitational production works for this mass range

[KN, Tang (2020)]





4. Summary

Summary

- Vector coherent oscillationDM scenario is (almost) excluded.

- Gravitational production is one of the simple scenario for vector DM for $m \gtrsim 10^{-6}\,\mathrm{eV}$

- Other production mechanims are also viable.

Cosmic string, Higgs decay, axion coupling…

Appendix

Axion coupling

[Agrawal et al (2018), Bastero-Gil et al (2018), Co et al (2018)]

- Axion couling to vector boson $\mathcal{L} = C\frac{a}{f}F_{\mu\nu}\widetilde{F}^{\mu\nu}$

- Axion dynamics induces vector production

  One polarization mode even becomes tachyonic for $k \lesssim C\dot{a}/f$

- Backreaction stops the tachyonic growth

- Relic density:

$$\Omega_A h^2 \sim 0.2 \left(\frac{40}{C}\right)\left(\frac{m_A}{10^{-9}\,\mathrm{eV}}\right)\left(\frac{10^{-8}\,\mathrm{eV}}{m_a}\right)^{1/2}\left(\frac{f}{10^{14}\,\mathrm{GeV}}\right)^2$$

[Agrawal et al (2018)]

Cosmic string

- Vector mass comes from hidden U(1) Higgs field

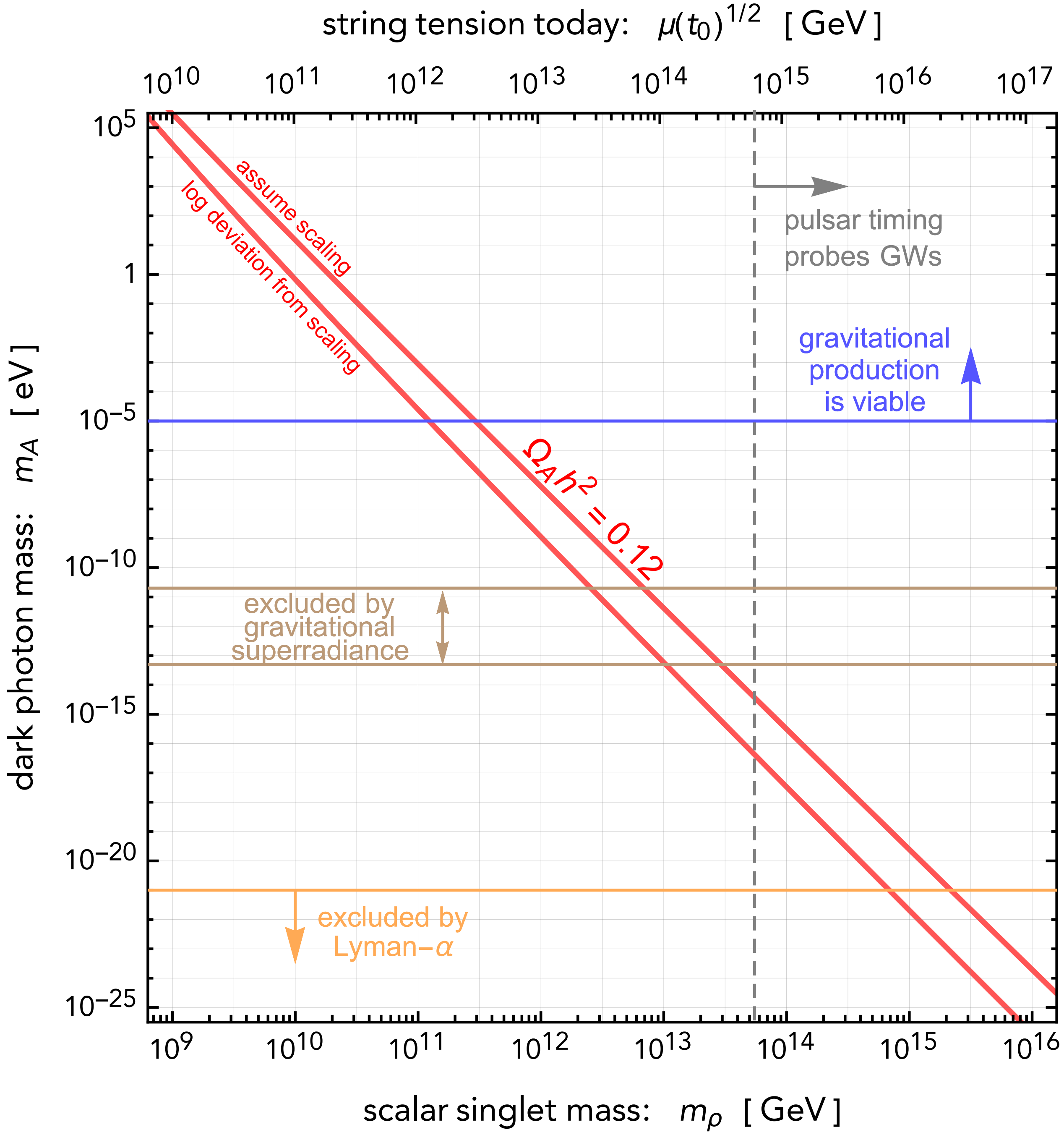
⟶ **Cosmic strings**

- Light vector boson (longitudinal component) is emitted by string dynamcis.

- Higgs decay also emits vector boson

[Dror, Harigaya, Narayan (2018)]

[Long, Wang (2019)]



Isotropy of vector background

- Vector background $A_i = (0, 0, A_z)$

  Energy-momentum tensor

$$T_{xx} = T_{yy} = \frac{f^2}{2}\dot{A}_z^2 - \frac{1}{2}m^2 A_z^2, \qquad T_{zz} = -\frac{f^2}{2}\dot{A}_z^2 + \frac{1}{2}m^2 A_z^2 \qquad T_{ij} = 0 \quad (i \neq j)$$

- **Anisotropic** during slow-roll (H>>m)

  However, vector energy is negligible in this stage

- **Isotropic** (pressure-less matter) during oscillation (H <<m)

  ⟶ Background expansion is isotropic

Delta-N and isocurvature mode

- Perturbed metric

$$ds^2 = -\mathcal{N}^2 dt^2 + a^2(t) e^{2\psi} \gamma_{ij} \left(dx^i + \beta^i dt\right) \left(dx^j + \beta^j dt\right)$$

- Spatial curvature on arbitrary time slice [Lyth, Malik, Sasaki (2004)]

$$\psi(t_f, \vec{x}) = N(t_f, t_i; \vec{x}) - \log \frac{a(t_f)}{a(t_i)} \quad = \delta N(t_f, t_i; \vec{x})$$

- Define $\zeta_i(\vec{x})$ as curvature on the slice $\delta\rho_i(\vec{x}) = 0$

- DM isocurvature perturbation: $S_{\rm DM} \equiv 3(\zeta_{\rm DM} - \zeta_r)$

[Wands, Malik, Lyth, Liddle (2000)]
[Kawasaki, KN, Sekiguchi, Suyama, Takahashi (2008)]

Isocurvature fluctuation

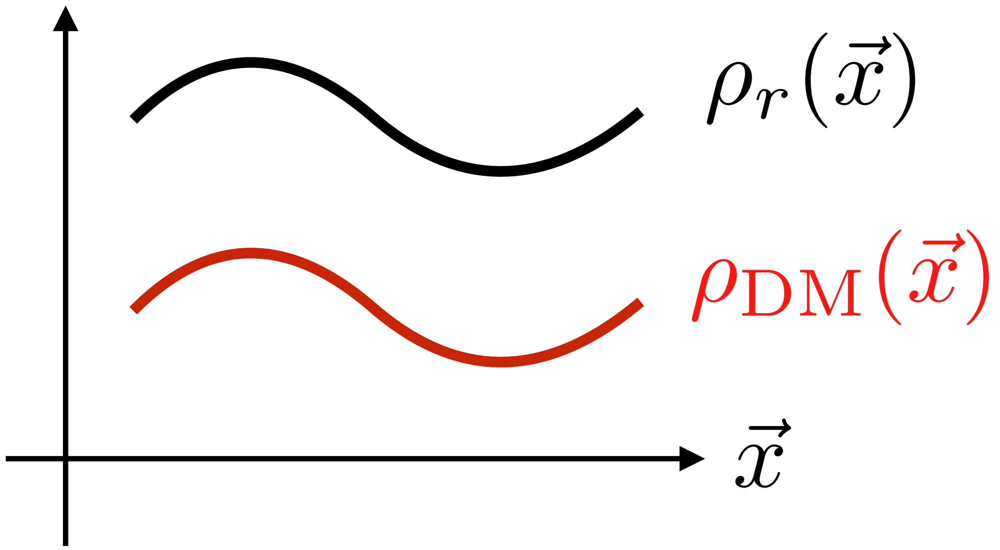
- Light field during inflation obtain long wave fluctuations
- Inflaton fluctuation $\delta\phi$

→ curvature perturbation $$\zeta = -H_{\rm inf}\frac{\delta\phi}{\dot\phi}$$

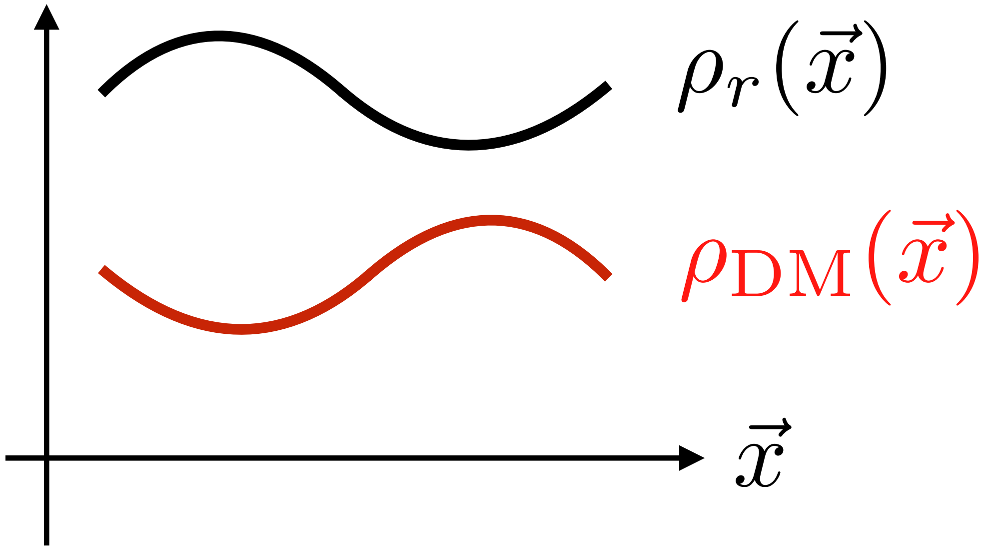
→ density perturbation $$\frac{\delta\rho_r}{4\rho_r} = \zeta$$

- DM isocurvature perturbation $$S_{\rm DM} = \frac{\delta\rho_{\rm DM}}{\rho_{\rm DM}} - \frac{3}{4}\frac{\delta\rho_r}{\rho_r} \neq 0$$

Adiabatic mode



Isocurvature mode



Inflationary quantum fluctuation

- In de-Sitter background, zero-point fluctuation is enhanced at superhorizon regime (inflationary fluctuation).

- Scalar field during inflation

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}(M_P^2 - \xi\chi^2)R - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2\right)$$

$$\longrightarrow \boxed{S = \int d\tau d^3x\frac{1}{2}\left[\widetilde{\chi}'^2 - (\partial_i\widetilde{\chi})^2 - m_\chi^{(\mathrm{eff})2}\widetilde{\chi}^2\right],} \qquad \widetilde{\chi} \equiv a\chi$$

$$m_\chi^{(\mathrm{eff})2} \equiv a^2 m_\chi^2 - (1-6\xi)\frac{a''}{a}$$

- Canonical quantization

$$\widetilde{\chi}(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[a_{\vec{k}}\chi_k(\tau) + a^\dagger_{-\vec{k}}\chi_k^*(\tau)\right]e^{i\vec{k}\cdot\vec{x}} \qquad \left[a_{\vec{k}}, a^\dagger_{\vec{k}'}\right] = (2\pi)^3\delta(\vec{k} - \vec{k}'),$$

Inflationary quantum fluctuation

- Mode equation: $\chi_k'' + \omega_k^2 \chi_k = 0, \qquad \omega_k^2 \equiv k^2 + m_\chi^{(\text{eff})2}$
- Solution:

$$\chi_k(\tau) = e^{\frac{i(2\nu+1)\pi}{4}} \frac{1}{\sqrt{2k}} \sqrt{\frac{-\pi k\tau}{2}} H_\nu^{(1)}(-k\tau), \qquad \nu^2 \equiv \frac{9}{4} - 12\xi - \frac{m_\chi^2}{H^2}$$

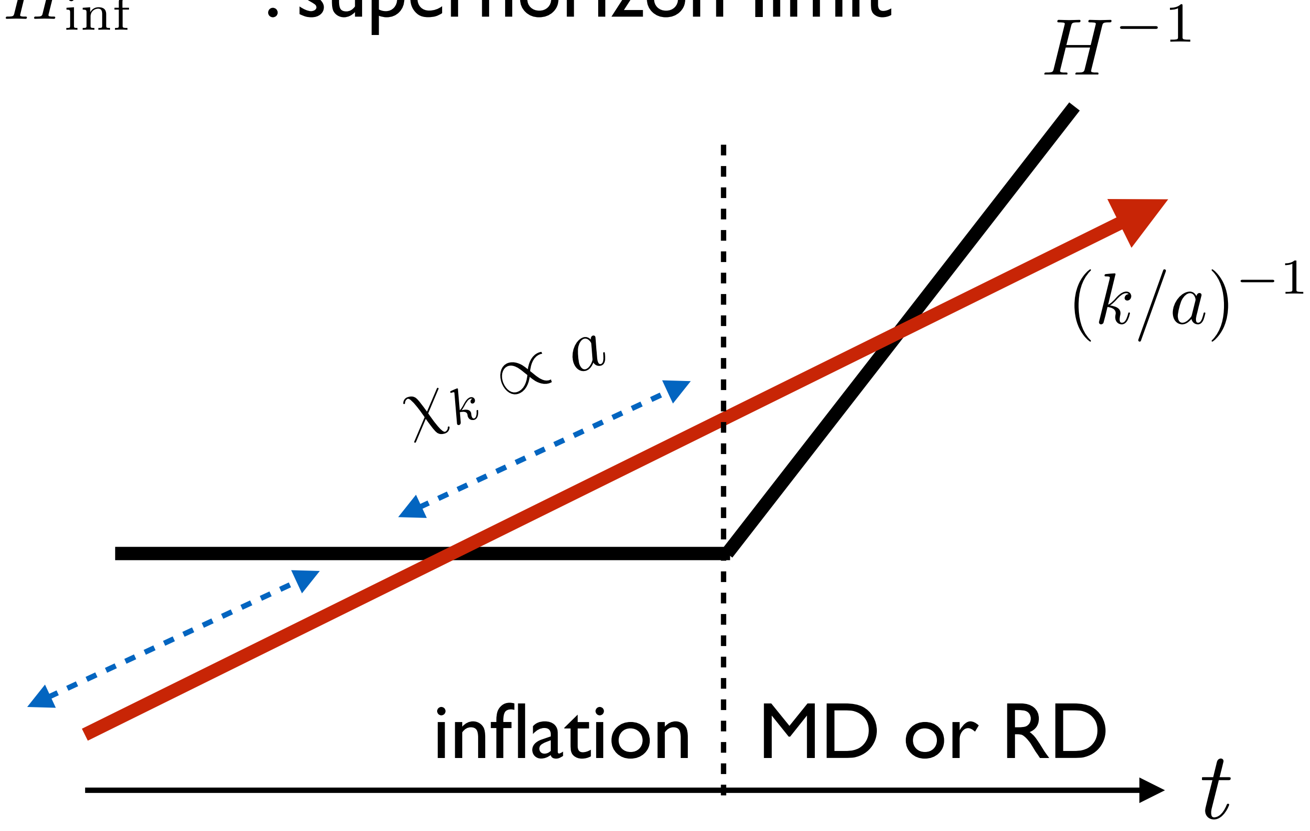
$$\chi_k(\tau) \simeq \begin{cases} \frac{1}{\sqrt{2k}} e^{-ik\tau} & \text{for } k/a \gg H_{\text{inf}} \\ i\frac{aH_{\text{inf}}}{\sqrt{2}k^{3/2}} & \text{for } k/a \ll H_{\text{inf}} \end{cases}$$

: subhorizon limit

: superhorizon limit

- Power spectrum in superhorizon limit

$$\langle \chi_k \chi_{k'}^* \rangle = \frac{a^2 H_{\text{inf}}^2}{2k^3} (2\pi)^3 \delta(\vec{k} - \vec{k}')$$



- Scalar (inflaton) fluctuation

$$\left\langle \delta\phi(\vec{k})\delta\phi^*(\vec{k}')\right\rangle = \frac{2\pi^2 a^2}{k^3}\mathcal{P}_\phi(k)(2\pi)^3\delta(\vec{k}-\vec{k}') \qquad \mathcal{P}_\phi(k) \simeq \left(\frac{H_{\rm inf}}{2\pi}\right)^2$$

- Transverse vector fluctuation

$$\left\langle \vec{A}_T(k)\cdot\vec{A}_T^*(k')\right\rangle = \frac{4\pi^2 a^2}{k^3}\mathcal{P}_T(k)(2\pi)^3\delta(\vec{k}-\vec{k}')$$

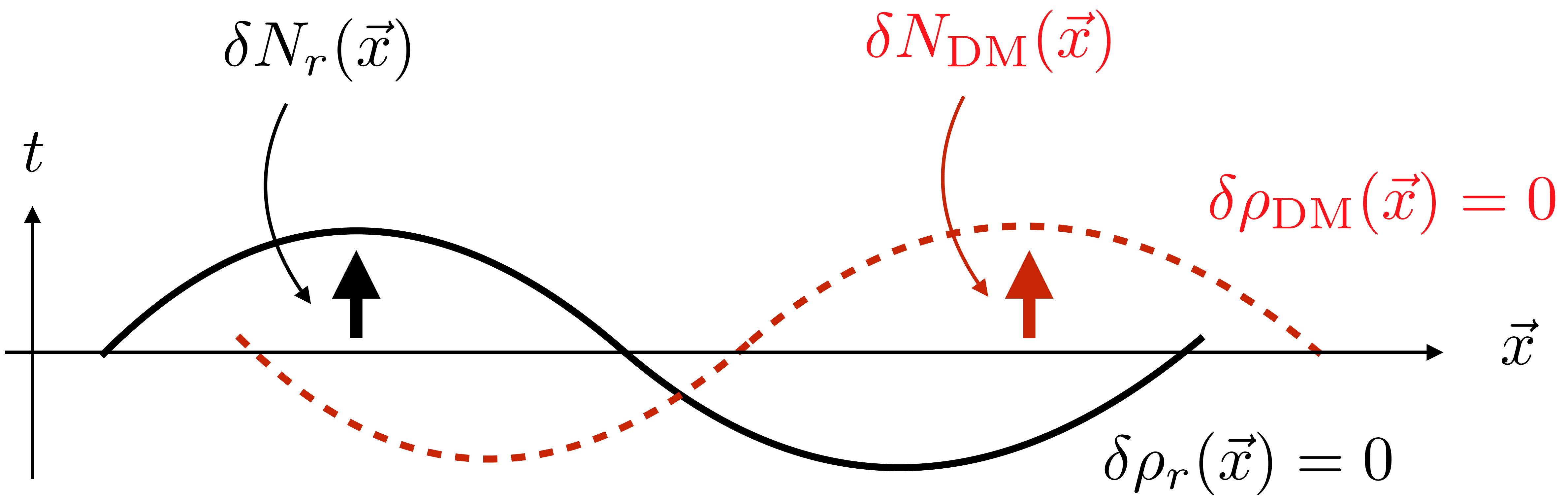
$$\mathcal{P}_T(k)\sim\left(\frac{H_{\rm inf}}{2\pi}\right)^2\left(\frac{2aH_{\rm inf}}{k}\right)^{-\alpha-4} \qquad \sim |\delta\vec{A}_T|$$

- Vector fluctuation is independent of the inflaton

$\longrightarrow$ Isocurvature fluctuation constrained by CMB.

- Observational constraint on DM isocurvature fluctuation

$$\frac{\mathcal{P}_S}{\mathcal{P}_\zeta}\lesssim 0.1 \quad\longleftrightarrow\quad S_{\rm DM} = \frac{\delta\rho_A}{\rho_A}\sim\frac{|\delta\overline{A}_T|}{\overline{A}}\sim\frac{H_{\rm inf}}{\pi\overline{A}_i}\lesssim 10^{-5}$$

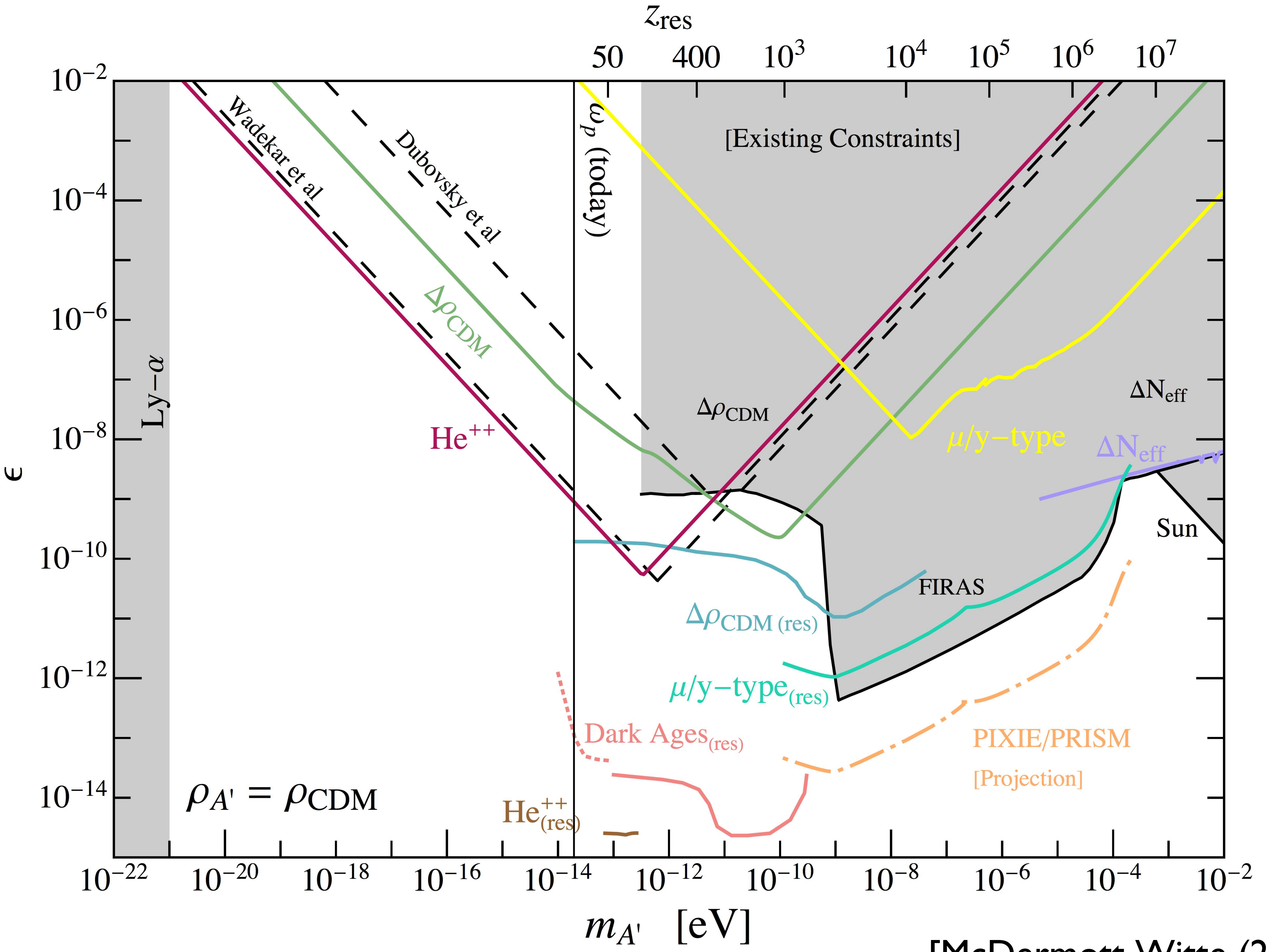


- $\rho_r(\vec{x})e^{-4\delta N_r(\vec{x})} = \overline{\rho_r} \qquad \longrightarrow \qquad \zeta_r(\vec{x}) = \delta N_r(\vec{x}) = \dfrac{\delta\rho_r(\vec{x})}{4\rho_r}$
- $\rho_{\rm DM}(\vec{x})e^{-3\delta N_{\rm DM}(\vec{x})} = \overline{\rho_{\rm DM}} \qquad \longrightarrow \qquad \zeta_{\rm DM}(\vec{x}) = \delta N_{\rm DM}(\vec{x}) = \dfrac{\delta\rho_{\rm DM}(\vec{x})}{3\rho_{\rm DM}}$

$$S_{\rm DM} = \frac{\delta\rho_{\rm DM}}{\rho_{\rm DM}} - \frac{3}{4}\frac{\delta\rho_r}{\rho_r}$$

Kinetic mixing with standard model gauge

$$\mathcal{L} = -\frac{\epsilon}{2} F^{(Y)}_{MN} F^{MN}$$



[McDermott, Witte (2019)]

String theory and light field

- Compactification of 6D through Calabi-Yau manifold
- Hodge-number of Calabi-Yau manifold

$$\begin{array}{c} h_{0,0} \\ h_{1,0}, h_{0,1} \\ h_{2,0}, h_{1,1}, h_{0,2} \\ h_{3,0}, h_{2,1}, h_{1,2}, h_{0,3} \\ h_{3,1}, h_{2,2}, h_{1,3} \\ h_{3,2}, h_{2,3} \\ h_{3,3} \end{array} = \begin{array}{c} 1 \\ 0, 0 \\ 0, h_{1,1}, 0 \\ 1, h_{2,1}, h_{1,2}, 1 \\ 0, h_{1,1}, 0 \\ 0, 0 \\ 1 \end{array}$$

- (p,q)-form is classified by the $h_{p,q}$ basis

$\longrightarrow$ 4D light field is related with Hodge number

- Euler number: $\chi = h_{1,1} - h_{2,1}$

Light field in typeIIB theory

	4d 10d	1 gravity	$h^+_{2,1}$ vector	$h^-_{2,1}$ chiral	$h^+_{1,1}$ chiral	$h^-_{1,1}$ chiral	1 chiral
NS-NS	G	$g_{\mu\nu}$		U_A	v_α		
	B_2					b_a	
	ϕ						ϕ
R-R	C_0						c_0
	C_2					c_a	
	C_4		A^i_μ		c_α		

Graviton

Hidden U(1)

Complex structure moduli

Kahler moduli

Dilaton