Eternal traversable wormholes and Experimental probes

Research Directions in Quantum Field Theory and String Theory 2020
서강대. Feb 26 (Wed) 2020

Chanju Kim (Ewha Womans Univ.)

Based on
Dongsu Bak, CK, Sang-Heon Yi, JHEP1808 (2018) 140 [arXiv:1805.12349];
JHEP1903 (2019) 155 [arXiv:1901.07679];
JHEP1912 (2019) 005 [arXiv:1907.13465]

AdS_2 Dilaton Gravity [Jackiw-Teitelboim]

- Action

$$I = I_{\rm top} + \frac{1}{16\pi G}\int_M d^2x\sqrt{-g}\,\phi\left(R+\frac{2}{\ell^2}\right) + I_{\rm surf} + I_M(g,\chi)$$

where

$$I_{\rm top} = \frac{\phi_0}{16\pi G}\int_M d^2x\sqrt{-g}\,R$$

$$I_{\rm surf} = -\frac{1}{8\pi G}\int_{\partial M}\sqrt{-\gamma}\,(\phi_0+\phi)\,K$$

$$I_M = -\frac{1}{2}\int_M d^2x\sqrt{-g}(\nabla\chi\cdot\nabla\chi + m^2\chi^2)$$

ℓ : AdS radius
γ_{ij} : Induced metric
K : Extrinsic curvature
ϕ : Dilaton
χ : Matter

- Equations of Motion

$$R + \frac{2}{\ell^2} = 0\,, \quad \rightarrow AdS_2$$

$$\nabla^2\chi - m^2\chi = 0\,,$$

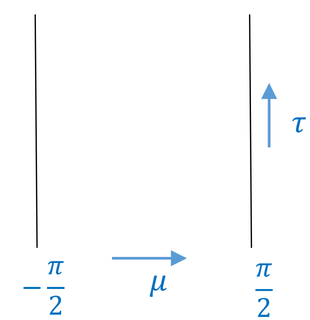
$$\nabla_a\nabla_b\phi - g_{ab}\nabla^2\phi + g_{ab}\phi = -8\pi G T_{ab}$$

where

$$T_{ab} = \nabla_a\chi\nabla_b\chi - \frac{1}{2}g_{ab}\left(\nabla\chi\cdot\nabla\chi + m^2\chi^2\right)$$

AdS_2 in the global coordinates

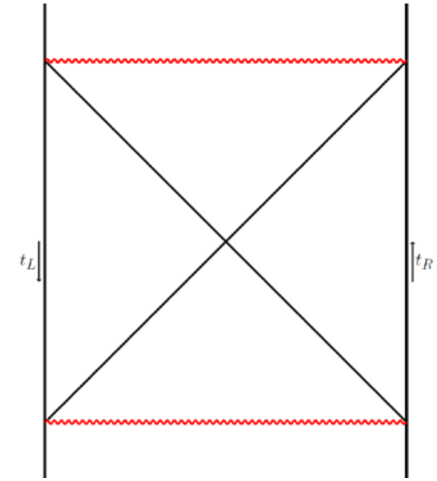
$$ds^2 = \frac{\ell^2}{\cos^2\mu}\left(-d\tau^2 + d\mu^2\right)$$



AdS_2 Black Hole

- The most general vacuum solution ($T_{ab} = 0$)

$$\phi = \bar{\phi}\, L\, \frac{(b+b^{-1})\cos(\tau-\tau_B) - (b-b^{-1})\sin\mu}{2\cos\mu}$$



Penrose diagram with $b = 1$

- Coordinate transformation

$$\frac{r}{L} = \frac{(b+b^{-1})\cos(\tau-\tau_B) - (b-b^{-1})\sin\mu}{2\cos\mu}$$

$$\tanh\frac{tL}{\ell^2} = \frac{2\sin(\tau-\tau_B)}{(b+b^{-1})\sin\mu - (b-b^{-1})\cos(\tau-\tau_B)}$$

➔ AdS_2 black hole metric (indep. of b and τ_B)

$$ds^2 = -\frac{r^2-L^2}{\ell^2}dt^2 + \frac{\ell^2}{r^2-L^2}dr^2$$

$$\phi = \bar{\phi}\, r$$

- Two-sided black hole
- Location of singularity: determined by $\phi_0 + \phi = 0$

  $\phi_0 + \phi =$ radius squared in the transverse space from the viewpoint of the dimensional reduction from higher dimensions

- Gibbons-Hawking temperature

$$T = \frac{1}{2\pi}\frac{L}{\ell^2} \qquad \Rightarrow \; \beta = \frac{2\pi\ell^2}{L}$$

- Entropy and Energy

$$S = S_0 + \mathcal{C}T = \frac{\phi_0 + \bar{\phi}\,L}{4G} \qquad \left(\mathcal{C} = \frac{\pi\bar{\phi}\ell^2}{2G} \right)$$

$$E = \frac{1}{2}\mathcal{C}T^2$$

- This two-sided AdS black hole is dual to the thermofield double of $\mathrm{CFT1}_L \times \mathrm{CFT1}_R$

$$|\Psi(0)\rangle = \frac{1}{\sqrt{Z}}\sum_{n,n'} \langle n|\mathcal{U}|n'\rangle\, |n'\rangle \otimes |n\rangle = \frac{1}{\sqrt{Z}}\sum_{n} e^{-\frac{\beta}{2}E_n}\, |n\rangle \otimes |n\rangle$$

- Degrees of freedom reside on the boundaries
    - Their dynamics is described by the **Schwarzian dynamics**

- Boundary values of the metric and dilaton ($\epsilon \to 0$: cutoff)

$$ds^2|_{\partial M} = -\frac{1}{\epsilon^2} d\tilde{u}^2 \,, \qquad \phi|_{\partial M} = \frac{\ell\,\bar{\phi}}{\epsilon}$$

  - $\tilde{u}$: boundary time. $\tilde{u} \in (-\infty, +\infty)$
  - The boundary dynamical degree is given by $\tau(\tilde{u})$

- Inserting these into the on-shell action, we get

$$S = \int d\tilde{u}\,[-\phi_L\,\mathrm{Sch}(\tilde{u}) - \phi_R\,\mathrm{Sch}(\tilde{u})]$$

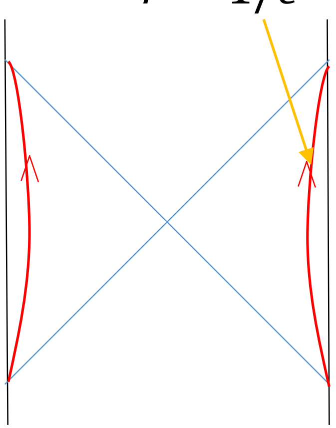
where $\phi_L = \phi_R = \bar{\phi}$ and

$$\{f, \tilde{u}\} = -\frac{1}{2}\frac{f''^2}{f'^2} + \left(\frac{f''}{f'}\right)'$$

$$\mathrm{Sch}(\tilde{u}) \equiv \left\{ \tan\frac{\tau(\tilde{u})}{2}, \tilde{u} \right\}$$

- The Schwarzian equation of motion is solved by

$$\sin \tau_{R/L} = \tanh\frac{2\pi\tilde{u}}{\beta}, \qquad or \quad t_{R/L} = \pm\tilde{u}$$



Double Trace Deformation

- Turn on a scalar χ with mass m $(-\frac{1}{4} < m^2 < 0)$
  - Dual to a scalar primary operators of dimension $\Delta_\pm = \frac{1}{2}\left(1 \pm \sqrt{1+4m^2}\right)$
  - For the double trace deformation, we are interested in $\Delta = \Delta_- \in \left(0, \frac{1}{2}\right)$
- Solve the scalar equation for χ with the boundary conditions

$$\chi(t,r)|_{R/L} = \frac{\alpha_{R/L}}{r^\Delta} + \cdots + \frac{\beta_{R/L}}{r^{1-\Delta}} + \cdots$$

- The mixed boundary condition

$$\beta_L(t) = h(-t)\alpha_R(-t)\,, \qquad \beta_R(t) = h(t)\alpha_L(-t)$$

corresponds to the double trace deformation of the Hamiltonian in the boundary theory (relevant def. of dimension 2Δ)

$$\delta H(\tilde{u}) = -h(\tilde{u})\mathcal{O}_R(\tilde{u})\mathcal{O}_L(-\tilde{u})$$

- 1-loop stress tensor

$$T_{ab} = \lim_{x' \to x} \left[\partial_a \partial'_b G(x,x') - \frac{1}{2} g_{ab} g^{\rho\sigma} \partial_\rho \partial'_\sigma G(x,x') - \frac{1}{2} g_{ab} m^2 G(x,x') \right]$$

Where $G(x,x') = \langle \chi(t)\chi(t') \rangle$ is the two point function which can be evaluated by

$$G = i \int_{t_0}^{t} d\tilde{t} \, \langle [\delta H(\tilde{t}), \chi_R(t)], \chi_R(t') \rangle + i \int_{t_0}^{t'} d\tilde{t} \, \langle \chi_R(t) [\delta H(\tilde{t}), \chi_R(t')] \rangle$$

- 1-loop stress tensor is linear in h ➔ the average null energy condition can be violated

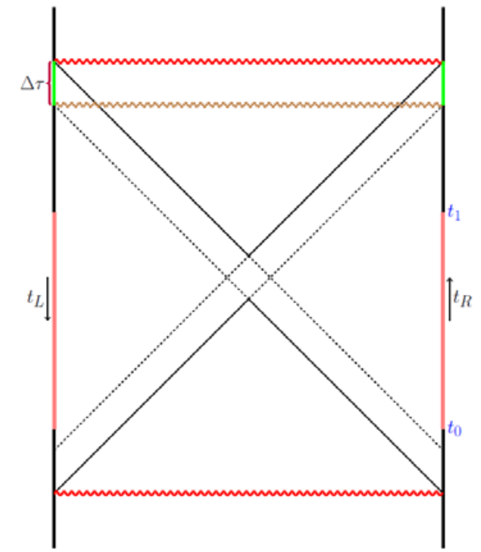
$$\int_{-\infty}^{\infty} T_{UU} \, dU < 0$$

➔ The wormhole becomes traversable. [Gao,Jafferis,Wall]

- Can calculate the change in the dilaton ϕ

  ➔ identify the change of the location of the singularity. [Bak,CK,Yi]

  - Delayed by $\Delta\tau$



- The general solution of the dilaton equation of motion can be written as

$$\phi = \phi_{hom} + \varphi$$

where ϕ_{hom} is a vacuum solution and

$$\varphi(u,v) = \int_{u_0}^{u} dp \frac{\sin(p-u)\cos(p-v)}{\cos(u-v)} T_{uu}(p,v)$$

$$u = \frac{1}{2}(\tau+\mu),\quad v = \frac{1}{2}(\tau-\mu)$$

- The system may equivalently be described by the boundary action

[Maldacena,Qi]

$$S = \int d\tilde{u}\,[-\phi_L\,\mathrm{Sch}(\tilde{u}) - \phi_R\,\mathrm{Sch}(\tilde{u})] + S_{\rm int}$$

where $\phi_L = \phi_R = \bar{\phi}$ and

$$S_{\rm int} \equiv \frac{g}{2^{2\Delta}} \int d\tilde{u} \left[\frac{\tau'_L(\tilde{u})\tau'_R(\tilde{u})}{\cos^2 \frac{\tau_L(\tilde{u})-\tau_R(\tilde{u})}{2}} \right]$$

with [Bak,CK,Yi]

$$g = \frac{h}{2\pi} \frac{2^{2\Delta-1}\Gamma^2(\Delta)}{\Gamma(2\Delta)}$$

Eternal Traversable Wormhole (ETW)

- Can be realized by turning on the double trace deformation from the infinite past with $\phi_{hom} = 0$ and

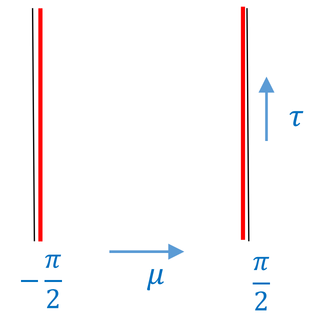
$$\phi_{\rm ETW}(u,v) = \frac{1}{2}\frac{4\ell^2\bar{h}\Delta N_\Delta}{2^{2\Delta}}\frac{B(2-\Delta,2-\Delta)}{1-\Delta}\frac{\sin^{3-2\Delta}|u-v|}{\cos|u-v|} \times F\left(1-\Delta,1-\Delta;\frac{5}{2}-\Delta\,|\,\sin^2(u-v)\right).$$

- Comparing this with the boundary conditions, one finds

$$\tau'(\tilde{u}) = \left(\frac{g\Delta}{2^{2\Delta}\bar{\phi}}\right)^{\frac{1}{2(1-\Delta)}} = \text{constant}$$

➔ Eternal Traversable Wormhole

- Exactly the same result can also be obtained from the Schwarzian theory.

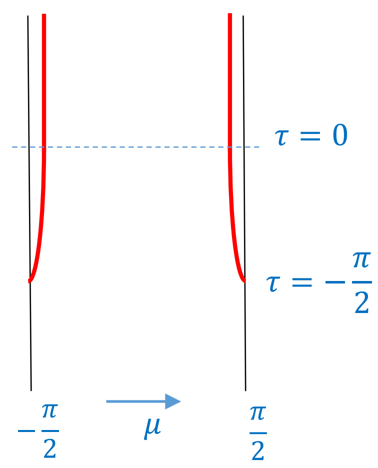


Transition from BH to ETW

- Begin with BH spacetime
- Turn on the double trace deformation at $\tau = 0$ with a suitable interaction strength so that the boundary trajectory is smoothly connected, leading to

$$\tilde{u} = \theta(-\tau)\frac{\ell^2}{L}\text{arctanh}\,\sin\tau + \theta(\tau)\frac{\ell^2}{L}\tau \qquad (\tau > -\pi/2)$$

➔ This solution describes a transition from BH to ETW state.



Experimental Probes of Wormholes

- From the boundary viewpoint, the existence of the bulk is nontrivial.
- Can we probe the bulk experimentally? How?
- Two channels of signal propagation from one boundary to the other
  - Boundary channel
  - Bulk channel
  - Two signals are mixed but have distinct physical characteristics
    - Boundary channel: direct LR interaction through δH (No time delay)
    - Bulk channel: respect bulk causality (Nonzero time delay)

- Send a signal through the bulk from one side at time $\tilde{u}_s$ to the other.
- The signal is observed at the other side at time $\tilde{u}_o$,

$$\tilde{u}_o = \frac{\beta}{2} + \theta(\tilde{u}_s)\tilde{u}_s + \frac{\beta}{2\pi}\theta(-\tilde{u}_s)\,\mathrm{arcsin}\,\mathrm{tanh}\,\frac{2\pi\tilde{u}_s}{\beta}$$

- Signals sent before turning on the boundary interaction ($-\infty < \tilde{u}_s < 0$) come out at the other side during the time $\frac{\beta}{4} < \tilde{u}_o < \frac{\beta}{2}$.
- Signals through the bulk channel are blue-shifted
  - Initial frequency ω_s
  - Observed frequency ω_o

$$\omega_o = \omega_s \cosh\frac{2\pi\tilde{u}_s}{\beta}$$

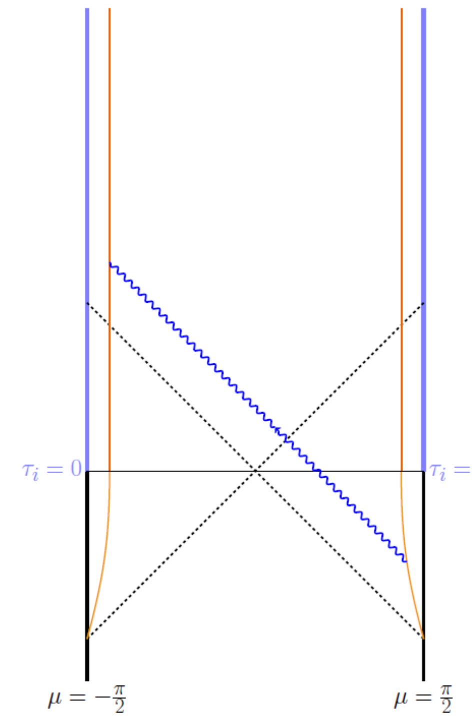
  - Earlier signals are more blue-shifted

$$\tilde{u}_s = -\beta \rightarrow \frac{\omega_o}{\omega_s} \simeq 268$$

$$\tilde{u}_s = -2\beta \rightarrow \frac{\omega_o}{\omega_s} \simeq 143376$$

- Boundary channel: no blue-shift factor, instantaneous

➔ can clearly distinguish two channels

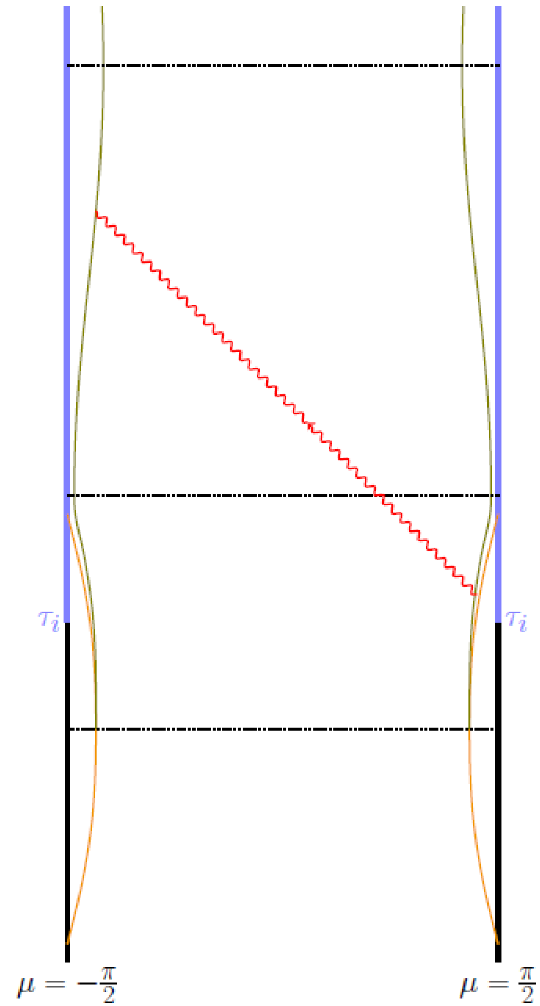


- More general solution: BH to an excited ETW state
- Signals sent before turning on the LR interaction through bulk channel emerges to the other side after some amount of time with **modulated frequencies**, either blue-shifted or red-shifted

$$\omega_0 = \omega_s \frac{\tau'(\tilde{u}_o)}{\tau'(\tilde{u}_s)}$$

➔ Clear evidence of

- the exisitence of the bulk channel
- traversability of the wormhole



- Experimental realization of SYK model with
  - Ultracold gases
  - Graphene flakes
  - Quantum wires
  - 3D topological insulators

➔ Holographic dual of traversable wormhole by entangling two SYK model systems