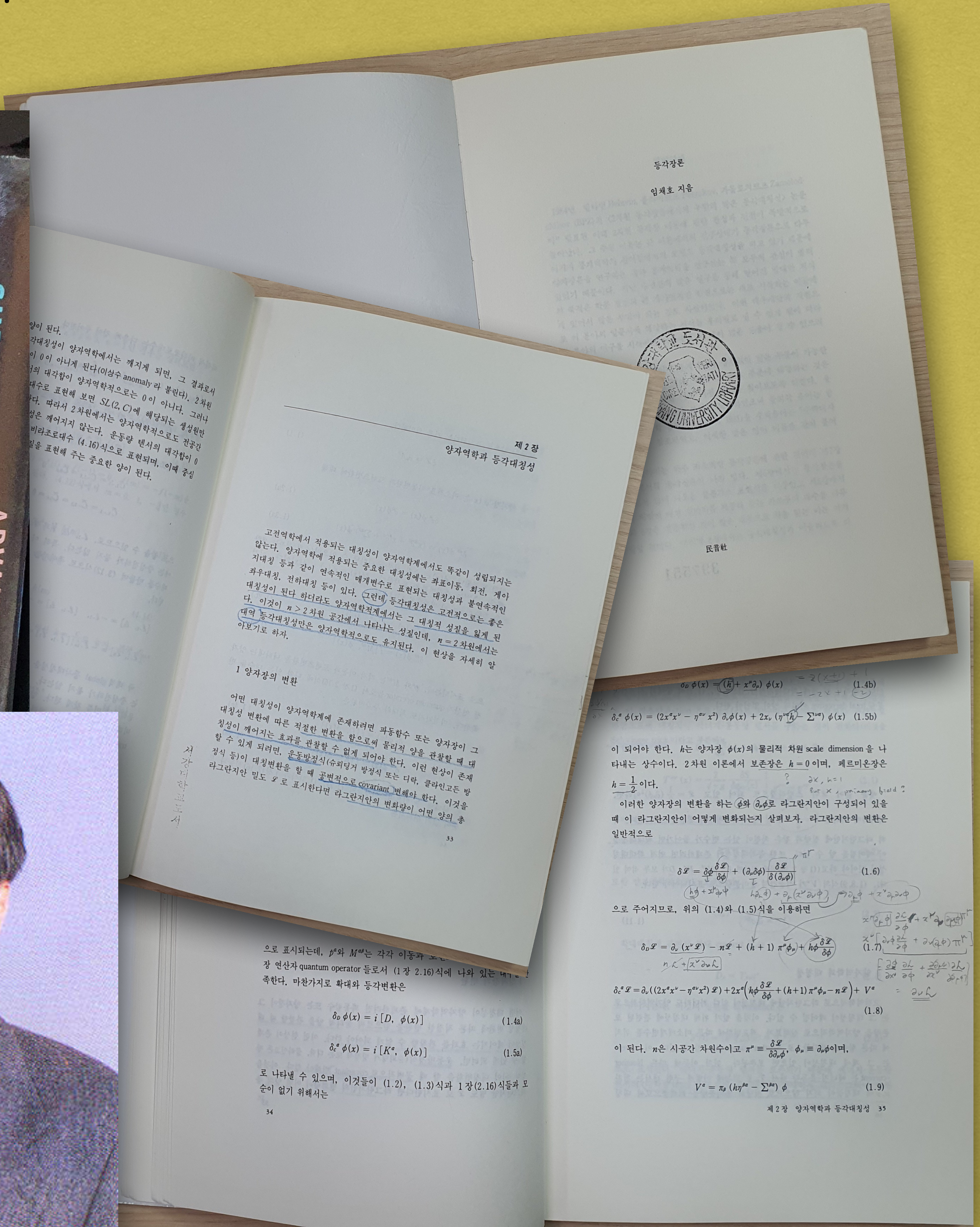
Research Directions in Quantum Field Theory and String Theory 2020

The workshop for Prof. *Chaiho Rim*

Applications of AdS/CFT
or Holographic duality

Keun-Young Kim
at GIST

1998년부터 저와 함께 세계 이곳 저곳을 돌아서
지금은 제 책꽂이에 자리 잡은 책이 하나 있습니다.
바로 임채호 교수님의 “등각장론” 입니다.



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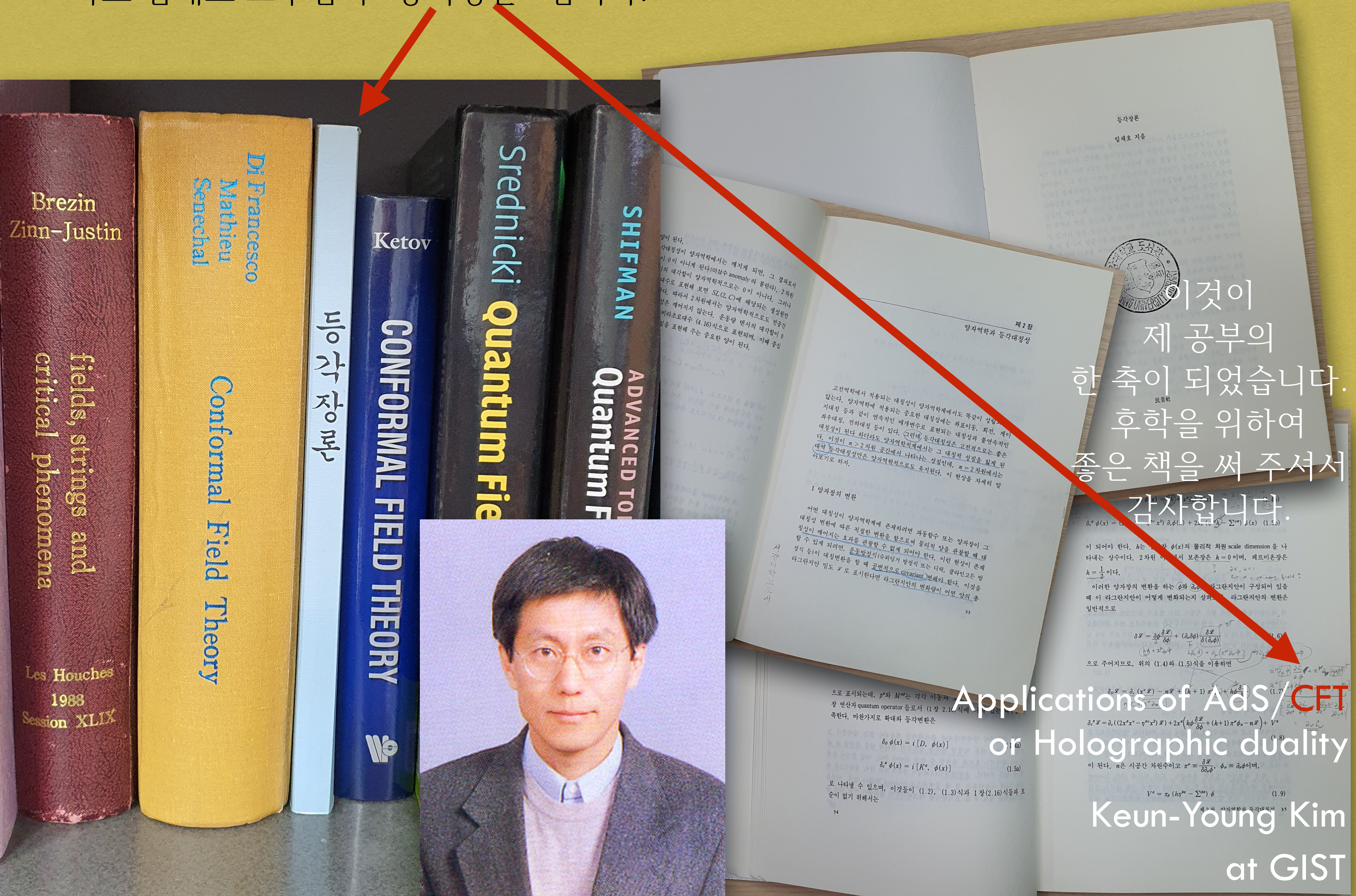

이것이
제 공부의
한 축이 되었습니다.
후학을 위하여
좋은 책을 써 주셔서
감사합니다.

Applications of AdS/CFT
or Holographic duality

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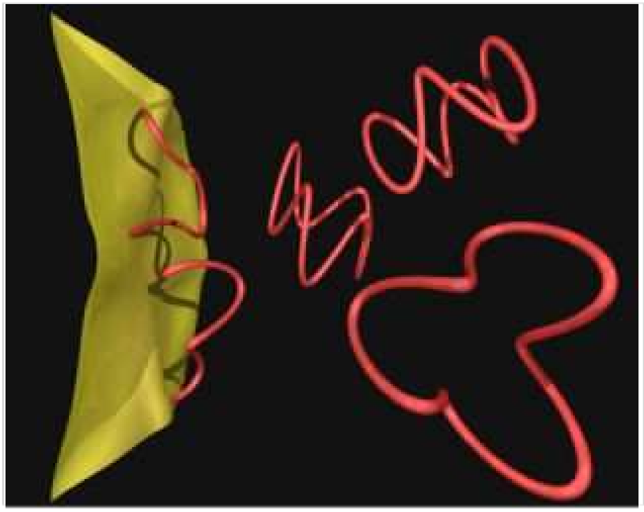
The beginning

N D3 Branes



Type IIB string theory on $AdS_5 \times S^5$		$\mathcal{N} = 4$ $U(N)$ Super Yang–Mills theory
$g_s\,, \quad R/l_s$	$\longleftrightarrow$	$g_{\rm YM}\,, \quad N$
	$g_{\rm YM}^2 = 4\pi g_s$ $(R/l_s)^4 = 4\pi g_s N$	
Classical strings $g_s \to 0\,, \quad R/l_s$ fixed.	$\longleftrightarrow$	't Hooft limit $N \to \infty\,, \quad \lambda = g_{\rm YM}^2 N$ fixed.
Classical supergravity $g_s \to 0\,, \quad R/l_s \to \infty\,.$	$\longleftrightarrow$	Large 't Hooft coupling limit $N \to \infty\,, \quad \lambda \to \infty\,.$

$$Z_{\rm String} = Z_{\rm Field\ \ Theory}$$

$$Z_{\rm Gravity}^{\rm On-Shell} = Z_{\rm Field\ \ Theory} \equiv e^{-W_{FT}}$$

$$W_{FT} = S_{\rm gravity}^{\rm on-shell} + \mathcal{O}\left(1/N^2\right) + \mathcal{O}\left(1/\sqrt{\lambda}\right)$$

AdS space

Anti de Sitter (AdS)

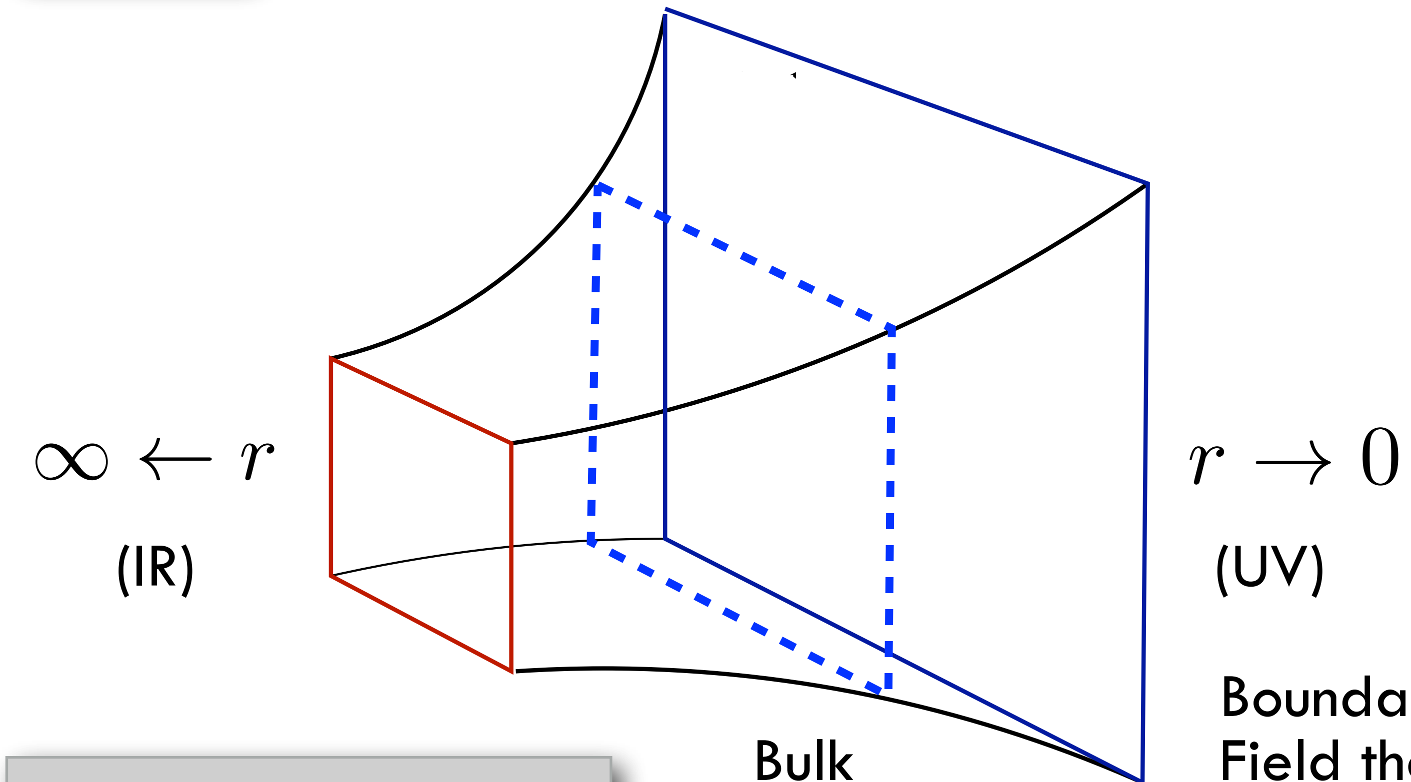
Action

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R + \frac{d(d-1)}{L^2}\right) \qquad (\kappa^2 = 8\pi G_N)$$

Negative cosmological constant: $R - 2\Lambda$

Solution

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx^i dx^i\right)$$

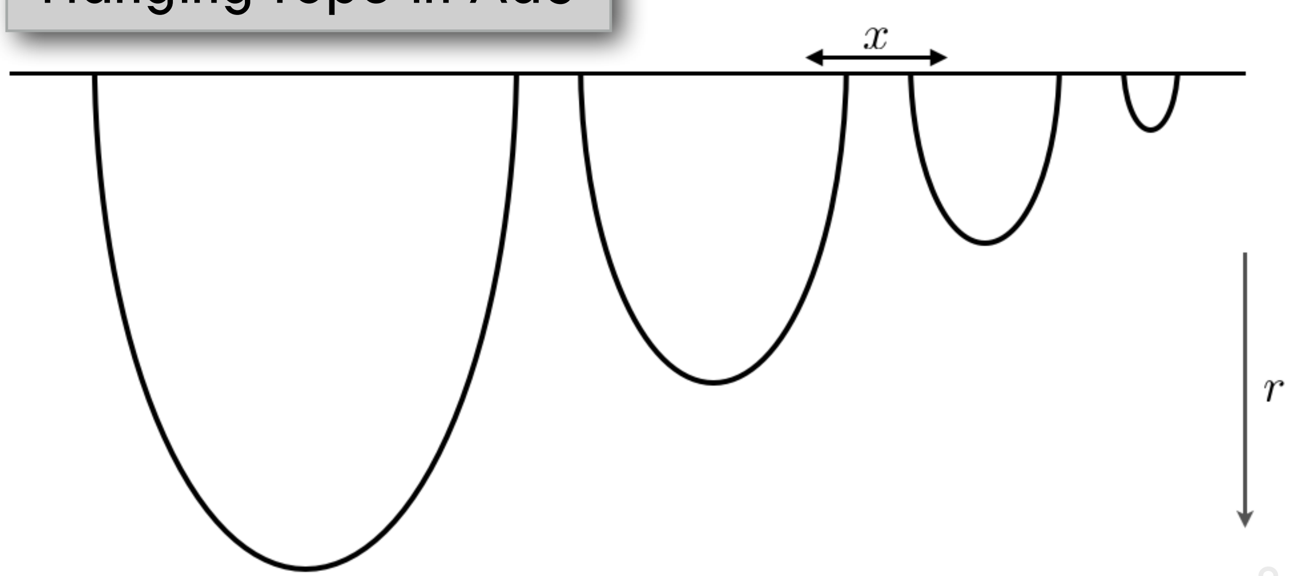


Boundary
Field theory lives here!

1. r (holographic direction): Energy scale
2. AdS is a Box
3. Conformal Invariance

$$x^\mu \rightarrow \lambda x^\mu\,, \quad r \rightarrow \lambda r$$

Hanging rope in AdS



Trap for gravity

A slowly moving massive particle feels a gravitational potential

$$\sim \sqrt{-g_{00}} = \frac{1}{r}$$

Thermal AdS (AAdS)

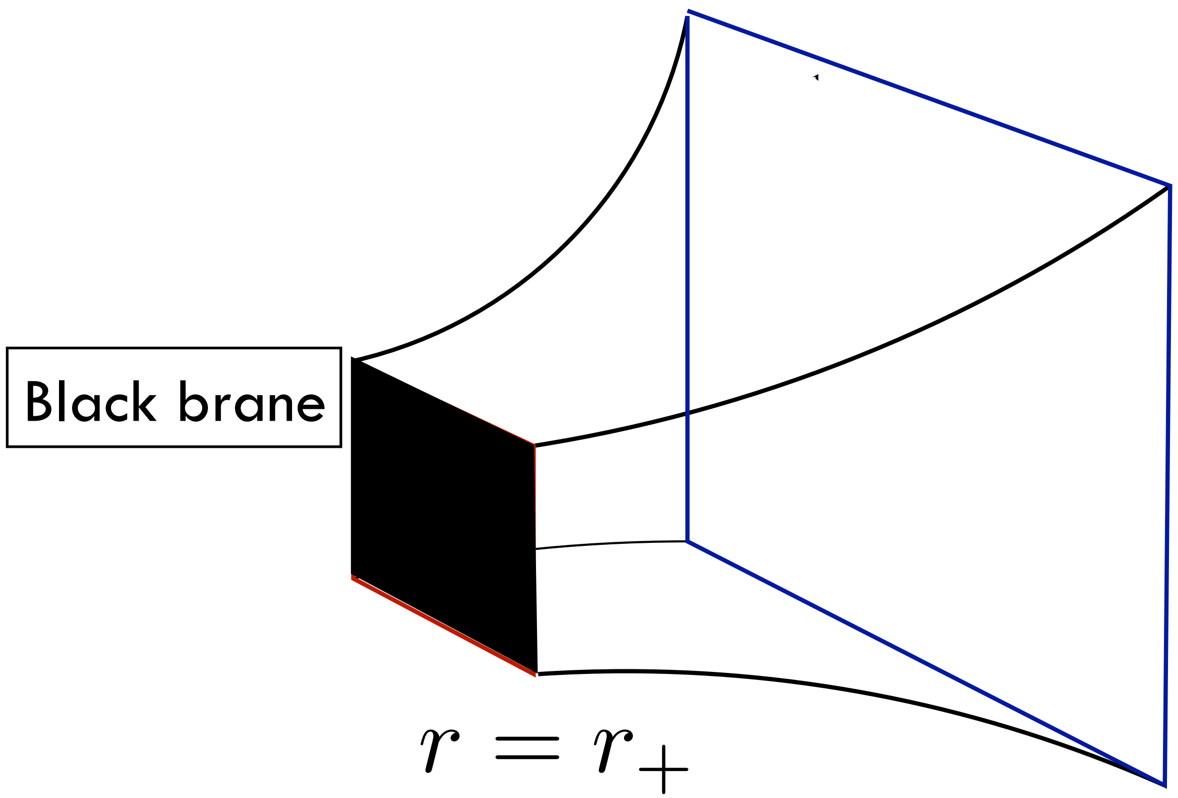
Action

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R + \frac{d(d-1)}{L^2}\right)$$

Solution

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right)$$

$$f(r) = 1 - \left(\frac{r}{r_+}\right)^d \qquad T = \frac{f'(r_+)}{4\pi} = \frac{d}{4\pi r_+}$$



Thermodynamics

$$F = -T\log Z = TS_E[g_\star] = -\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} V_{d-1} T^d$$

$$S = -\frac{\partial F}{\partial T} = \frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^{d-1}} V_{d-1} T^{d-1}$$ = area/4G

Bekenstein-Hawking

Charged AdS (AAdS)

Action

$$S = \int d^{d+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{d(d-1)}{L^2}\right) - \frac{1}{4g^2}F^2\right]$$

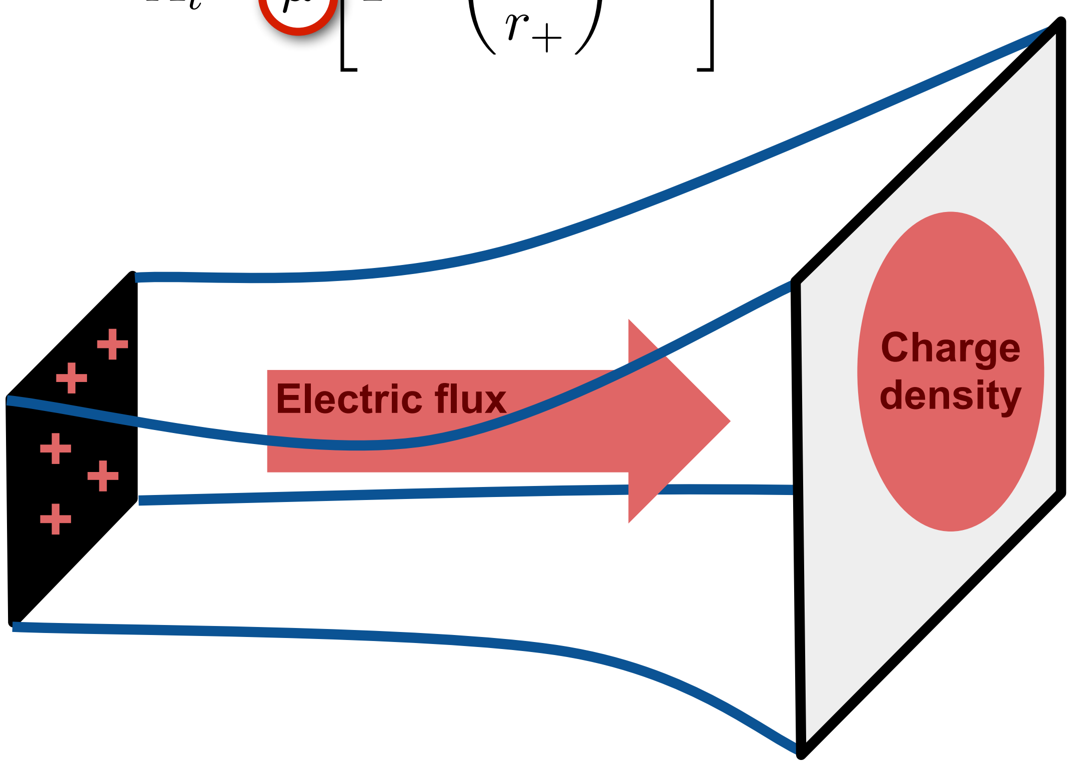
Solution

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right)$$

$$f(r) = 1 - \left(1 + \frac{r_+^2\mu^2}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^d + \frac{r_+^2\mu^2}{\gamma^2}\left(\frac{r}{r_+}\right)^{2(d-1)}$$

$$\gamma^2 = \frac{(d-1)g^2L^2}{(d-2)\kappa^2}$$

$$A_t = \mu\left[1 - \left(\frac{r}{r_+}\right)^{d-2}\right]$$



$$T = \frac{1}{4\pi r_+}\left(d - \frac{(d-2)r_+^2\mu^2}{\gamma^2}\right)$$

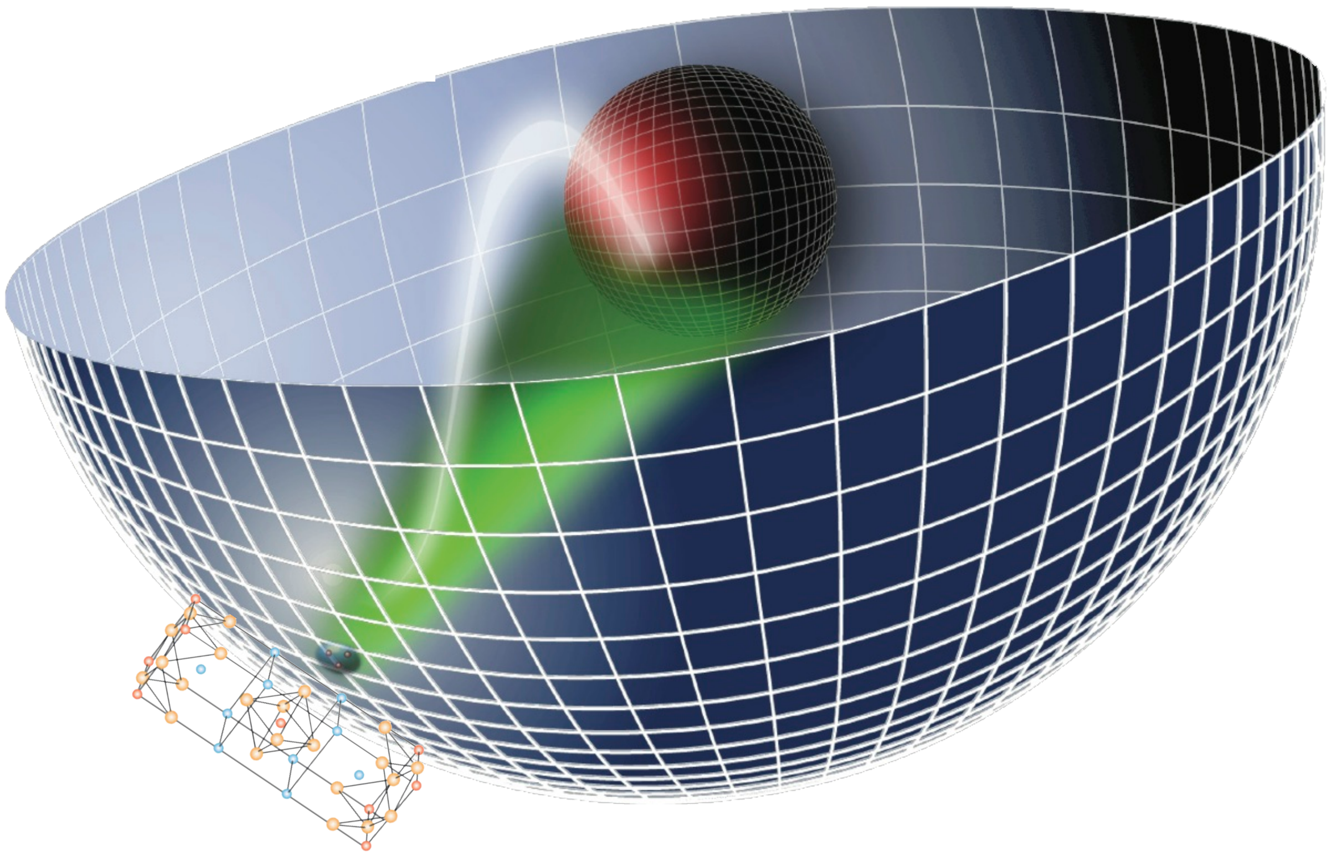
$$\Omega = -\frac{L^{d-1}}{2\kappa^2 r_+^d}\left(1 + \frac{r_+^2\mu^2}{\gamma^2}\right)V_{d-1} = \mathcal{F}\left(\frac{T}{\mu}\right)V_{d-1}T^d$$

Field ~ Operator

Holographic "duality"

gravity system: 5D

Classical fields



$$g_{\mu\nu} \sim T^{\mu\nu}$$

$$A_\mu \sim J^\mu$$

field theory: 4D

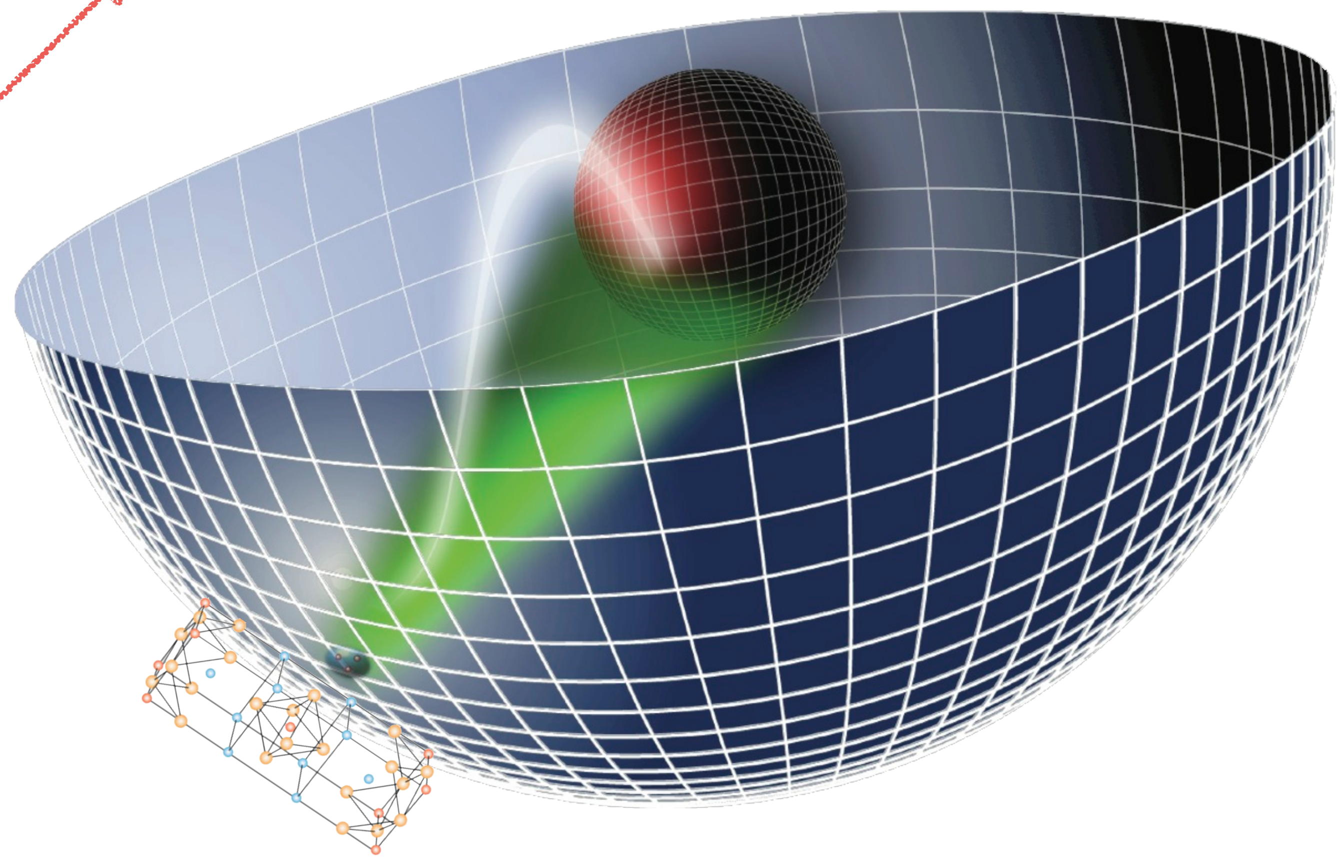
Quantum operators

Holographic "duality"

"Classical" gravity system: 5D

Classical fields

Narrow sense



$$g_{\mu\nu} \sim T^{\mu\nu}$$

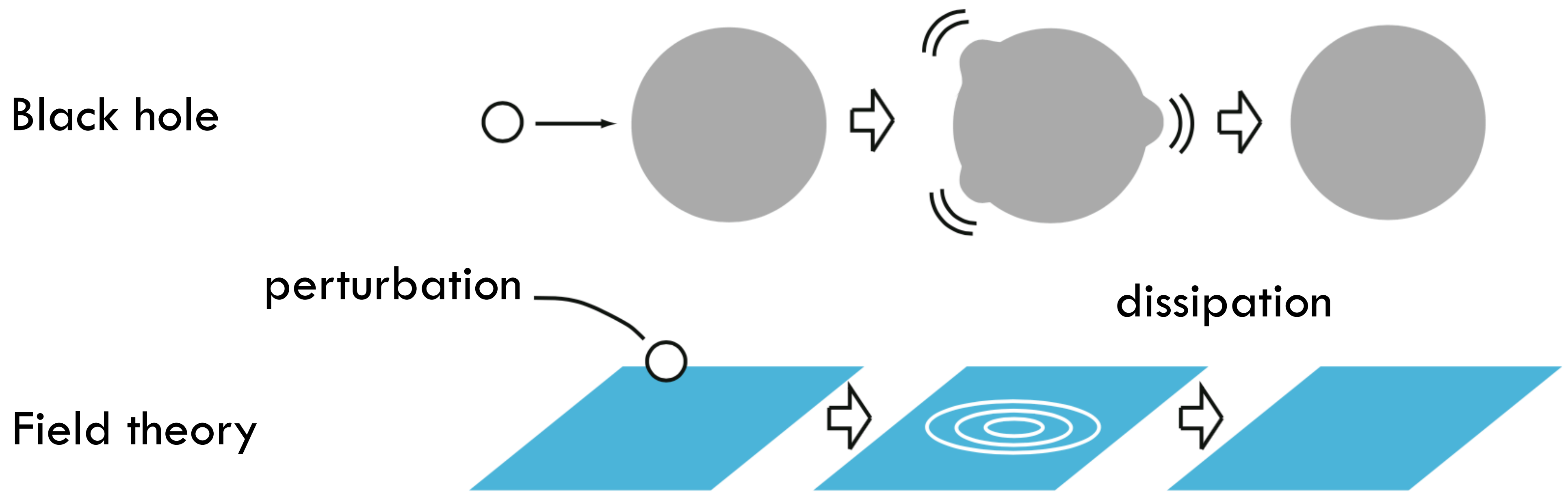
$$A_\mu \sim J^\mu$$

"Strongly" coupled field theory: 4D

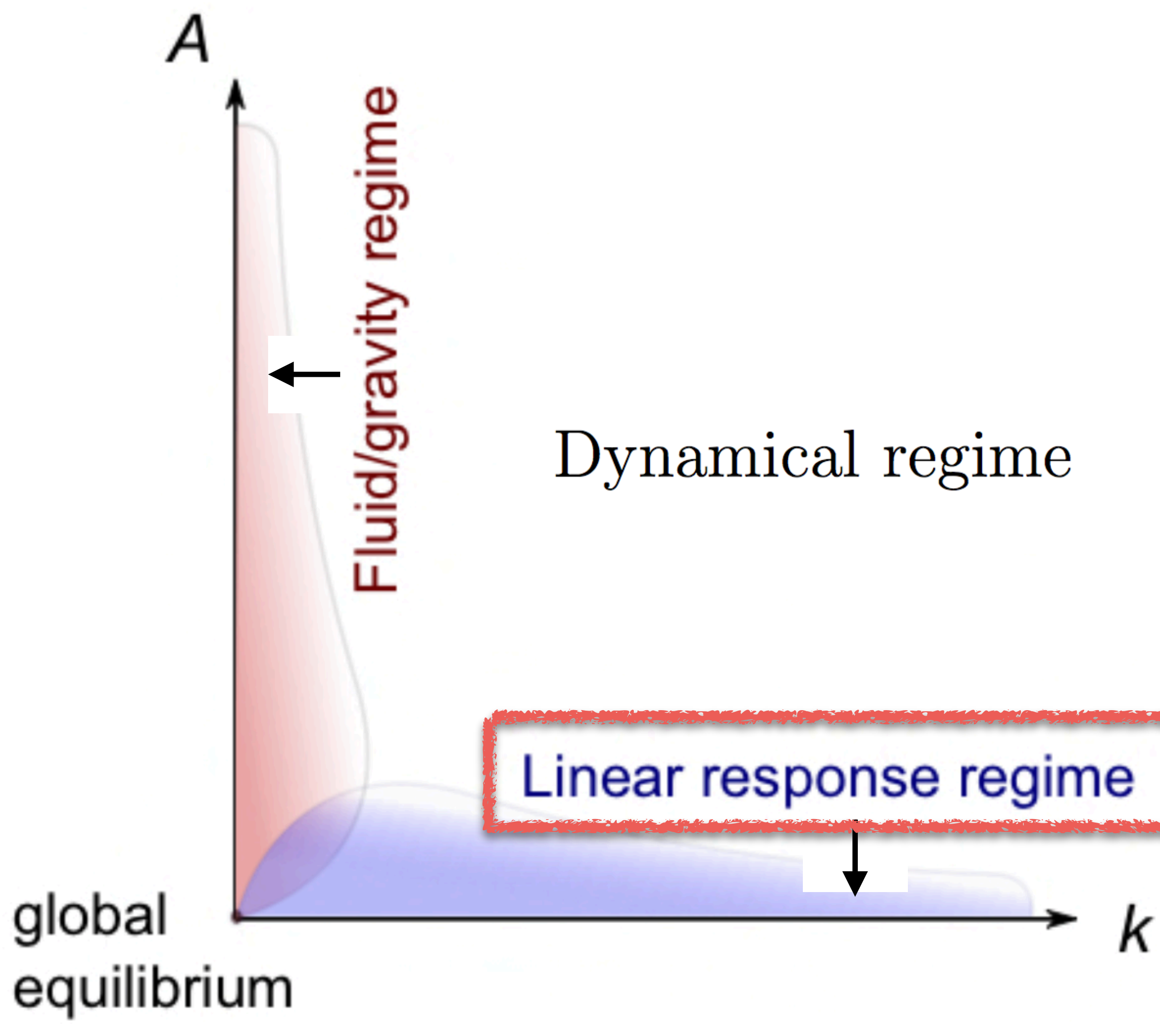
Quantum operators

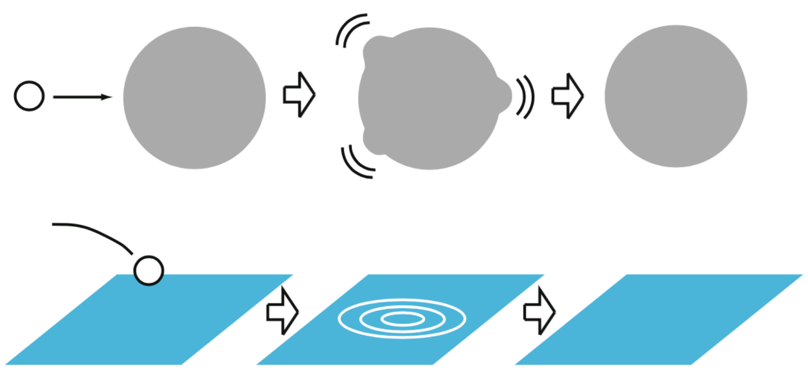
Non-equilibrium physics

Black hole hydrodynamics



Non-equilibrium process:
transport coefficients (viscosity, conductivity)


A
Fluid/gravity regime
Dynamical regime
Linear response regime
global
equilibrium
k



Linear response

(Classical Mechanics: Hamilton-Jacobi theory)

On-shell action as a function of the coordinates

$$\delta S = \left[\frac{\partial L}{\partial \dot q}\delta q\right]_{t_1}^{t_2} + \int_{t_1}^{t_2}\left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot q}\right)\delta q dt$$

$$\frac{\delta S}{\delta q} = p \qquad\qquad (\delta q(t_1) = 0)$$

One-point function

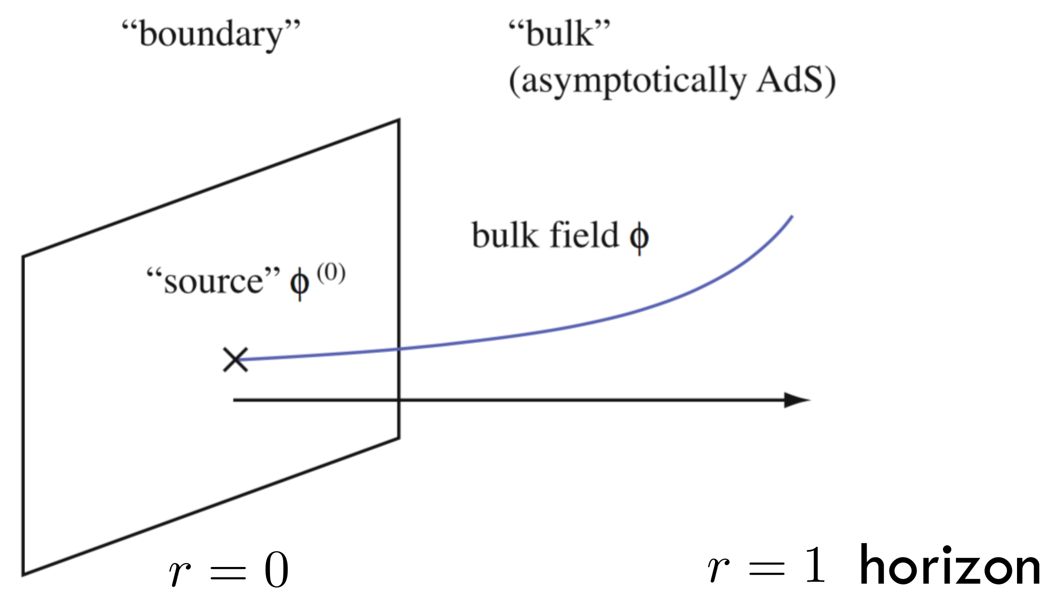
$$\langle O \rangle_s = \frac{\delta \underline{S}[\phi^{(0)}]}{\delta \phi^{(0)}}$$

GKP-Witten Relation

$$Z_{\text{gauge}} = Z_{\text{AdS}}$$

$$\left\langle \exp\left(i\int \phi^{(0)} O\right)\right\rangle = e^{i\underline{S}[\phi|_{u=0}=\phi^{(0)}]}.$$

Corollary



$$\delta A_x(r,\omega) = \frac{E}{i\omega} + J_x(\omega) r + \cdots$$

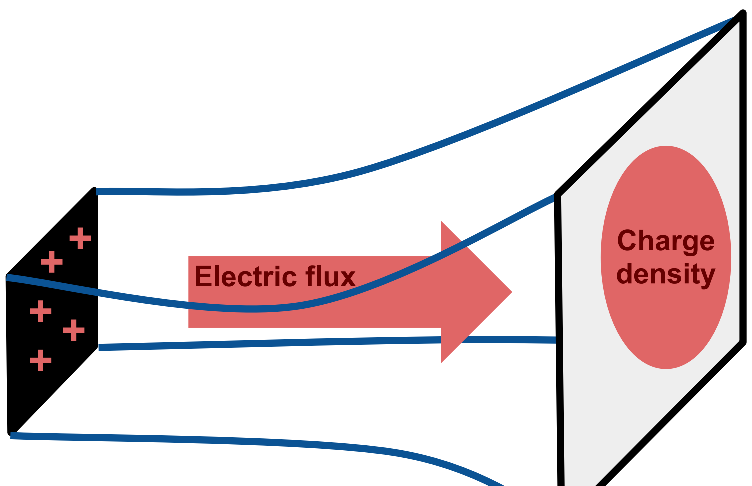
Source — Expectation value

Holographic conductivity

- Einstein-Maxwell system

$$S_{\rm EM} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right]$$

- Reissner-Nordstrom-AdS black hole
  ~ Boundary field theory at finite temperature and density



Hartnoll, 1106.4324

$$+ \quad (f\delta A_x')' + \frac{\omega^2}{f}\delta A_x - \frac{4\mu^2 r^2}{\gamma^2 r_+^2}\delta A_x = 0$$

$$E_j = -\partial_t A_j$$
$$A_j \sim e^{-i\omega t}$$

- Electric conductivity

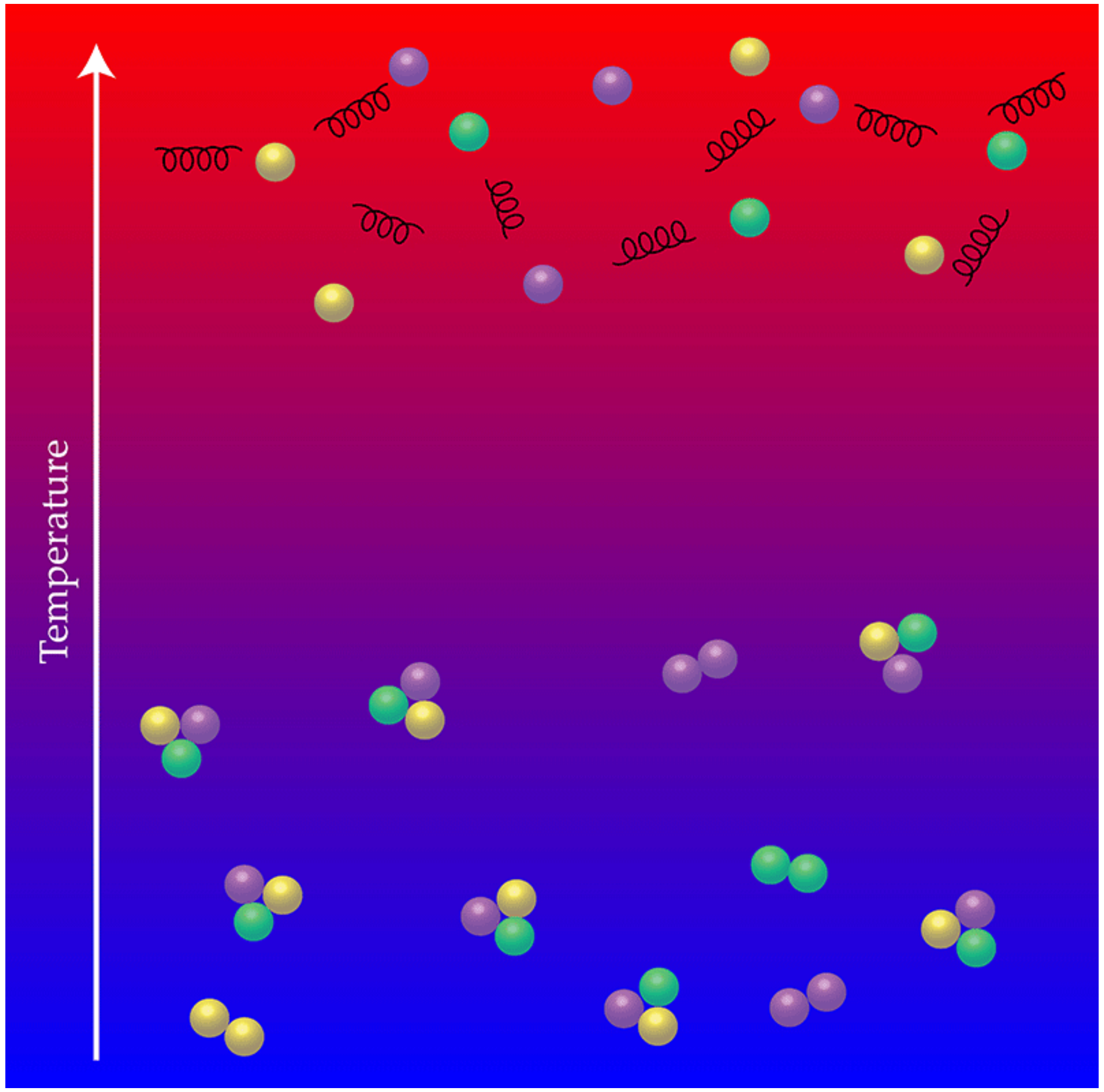
  Tail of the field~ $\delta A_x(r,\omega) = \frac{E}{i\omega} + J_x(\omega) r + \cdots \qquad J_x = \sigma E_x$

- on-shell action ~ $\delta A_x(\delta A_x') \sim (E/i\omega)J_x \sim (E/i\omega)\frac{J_x}{(E/i\omega)}(E/i\omega) \sim \delta A_x^{(0)} G_{11} \delta A_x^{(0)}$

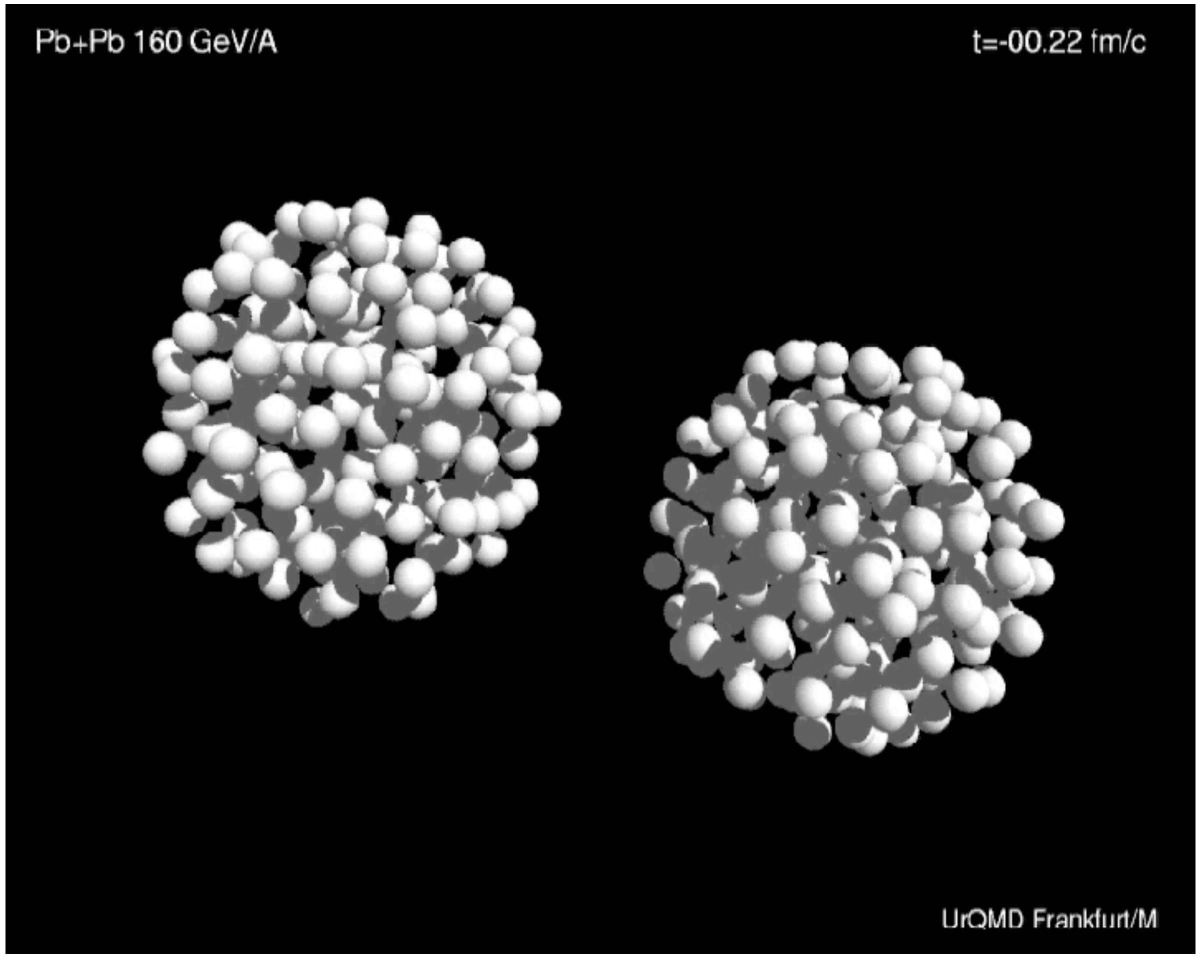
$$\frac{G_{11}}{i\omega} \sim \frac{J_x}{E} = \sigma$$

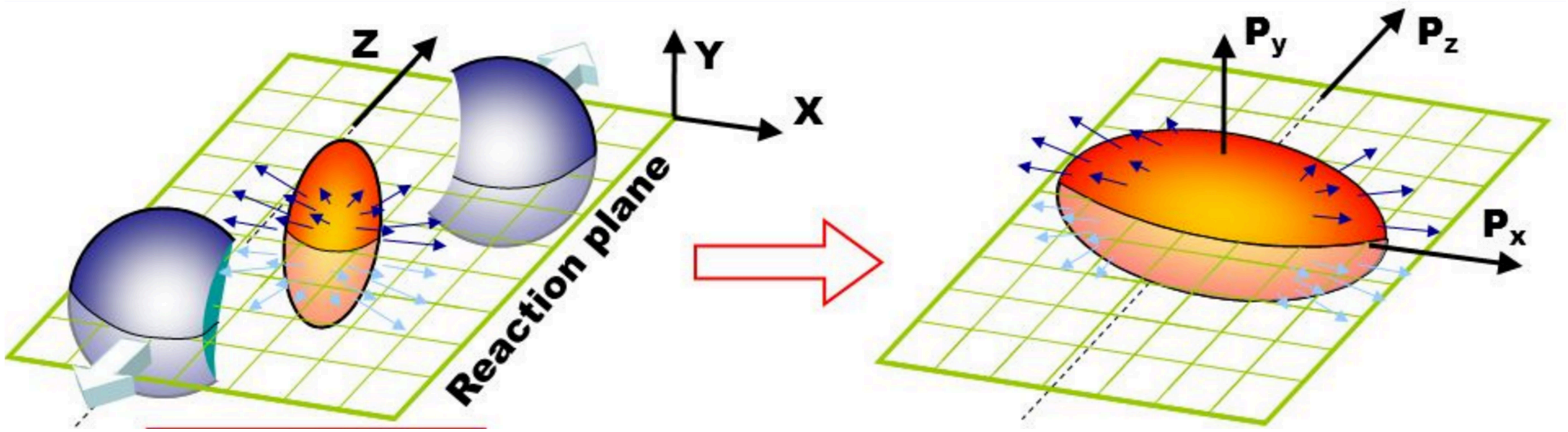
Strong coupling problems
- QCD/nuclear physics

Quark gluon plasma



Elliptic flow: shear viscosity



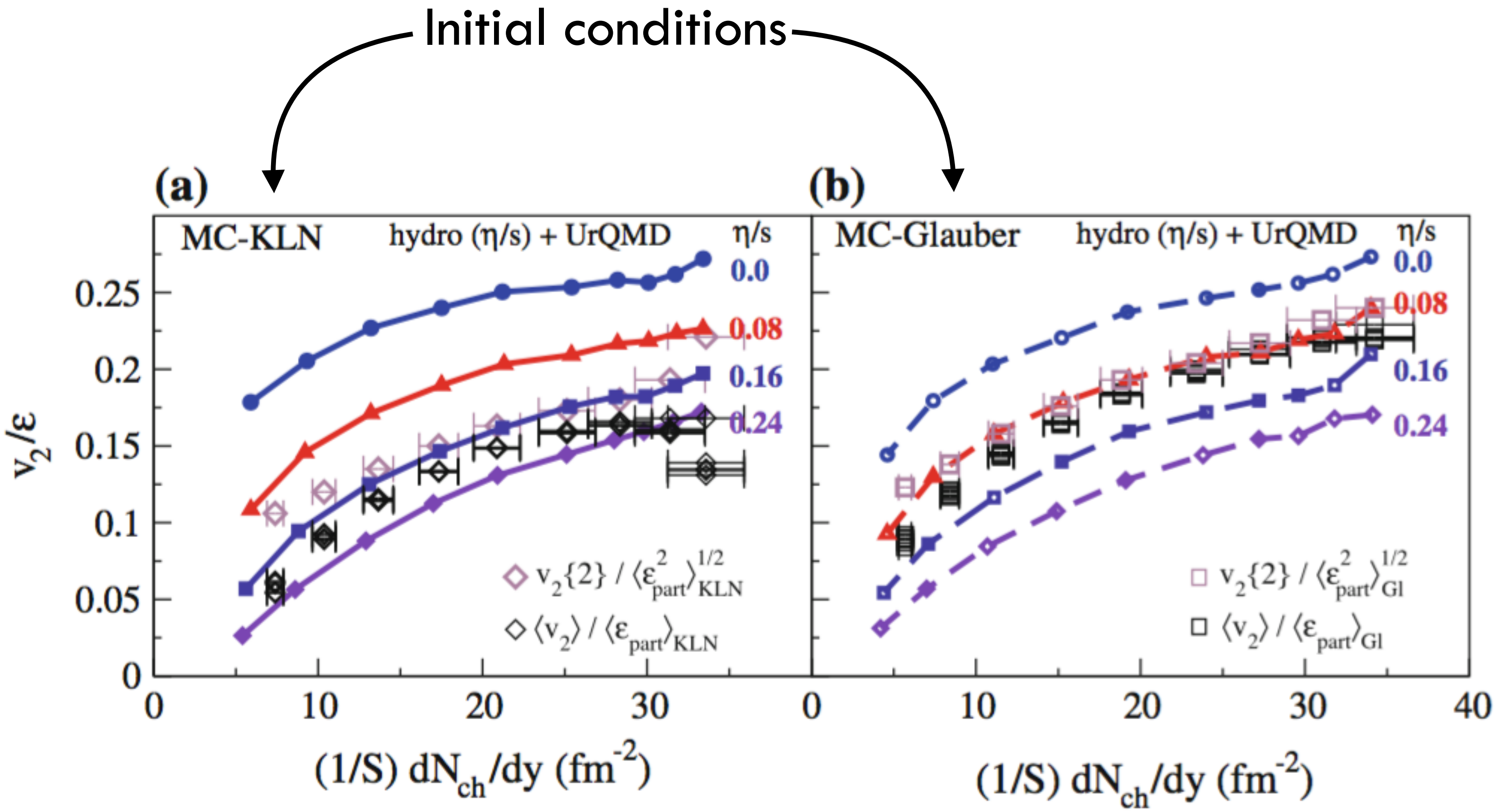


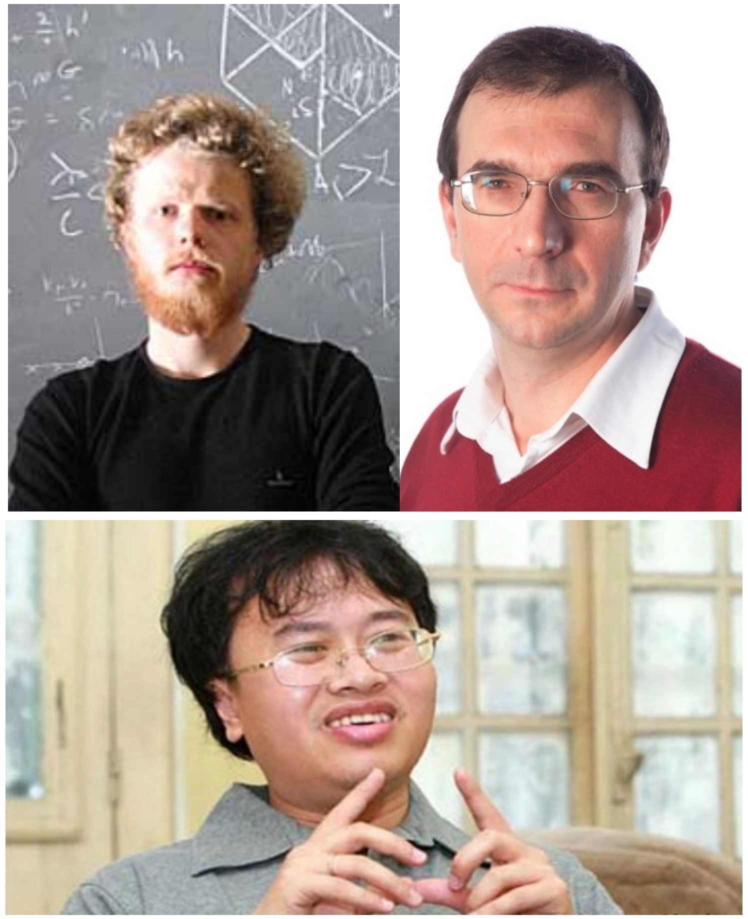
large elliptic flow

small viscosity/entropy

strong coupling

Experiment vs hydrodynamics simulation





Comparison between RHIC results and hydrodynamic simulations

Holographic (gravity) result

$$\eta = \frac{1}{16\pi G_5}\left(\frac{r_0}{L}\right)^3 = \frac{\pi}{8}N_c^2 T^3$$

$$s = \frac{1}{4G_5}\left(\frac{r_0}{L}\right)^3 = \frac{1}{2}\pi^2 N_c^2 T^3$$

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$$ (KSS bound)

A friend of mine in nuclear physics joked….

The first useful paper to come out of string theory

Strong coupling problems

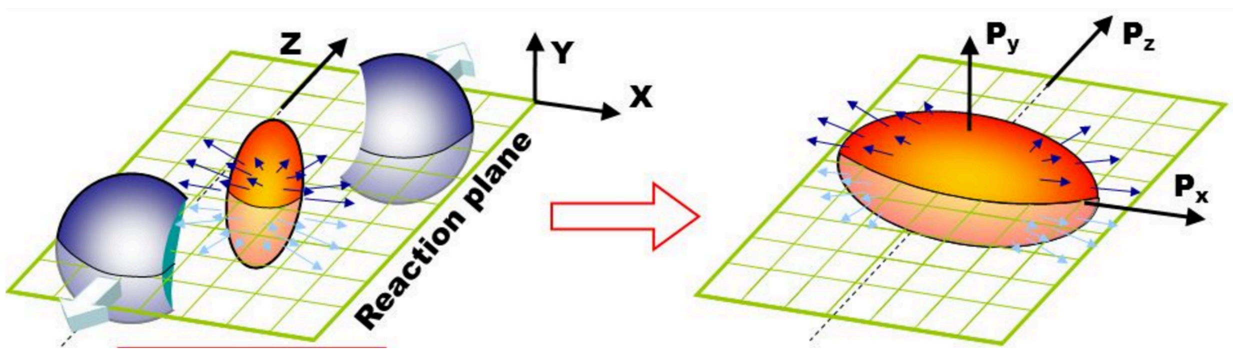
- More than QCD/nuclear physics

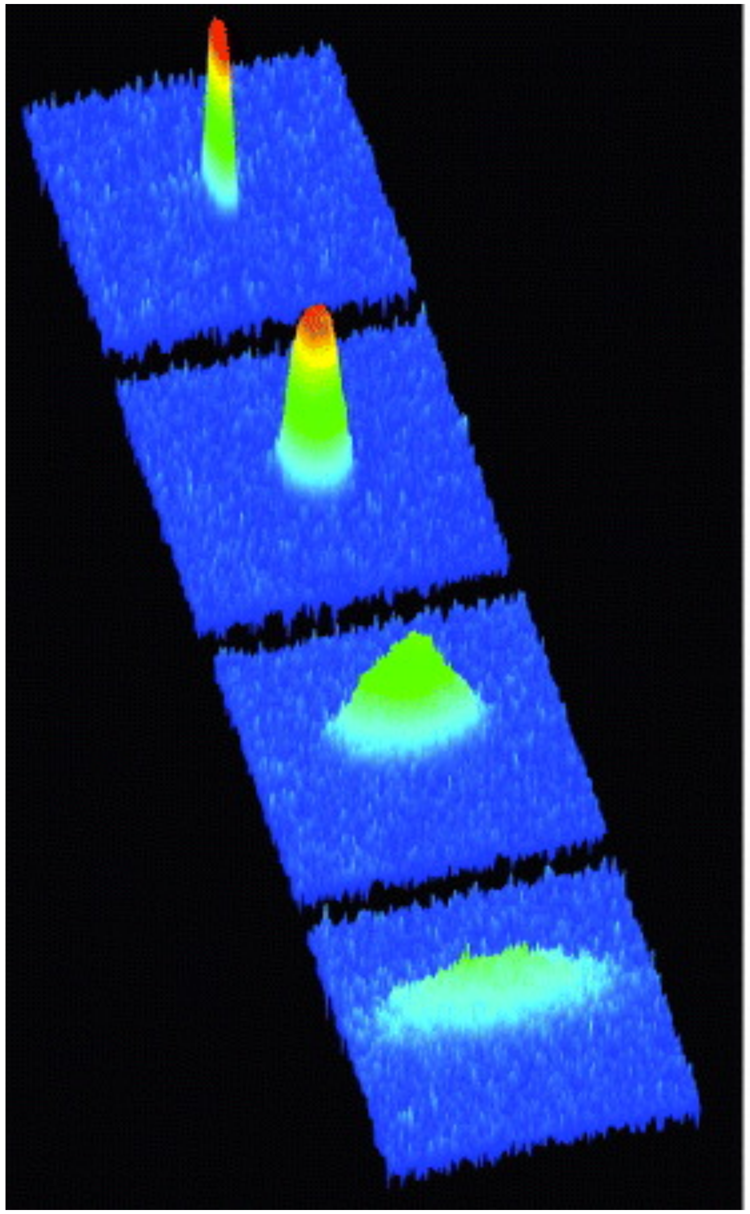
Observation of a Strongly Interacting Degenerate Fermi Gas of Atoms

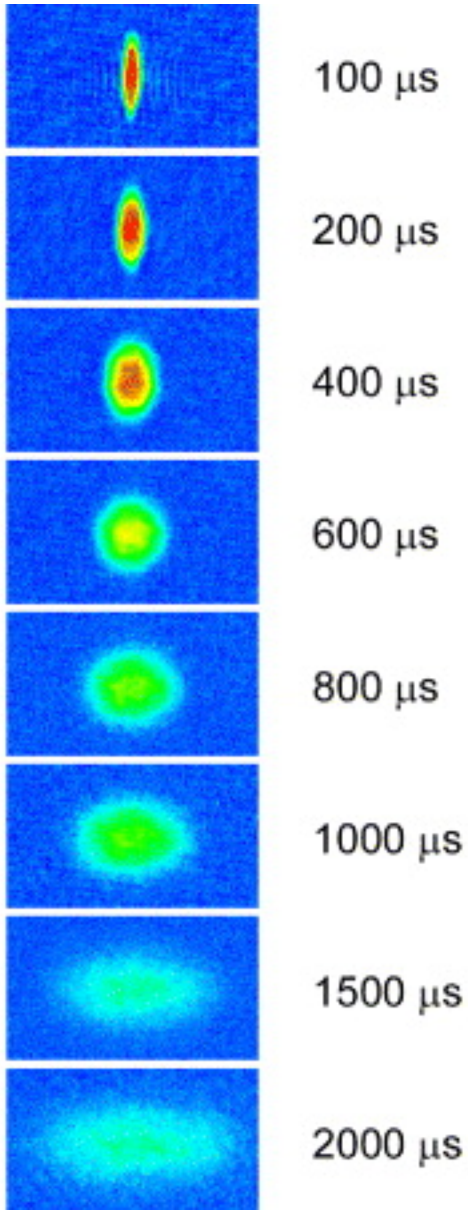
K. M. O'Hara, S. L. Hemmer, M. E. Gehm, S. R. Granade, J. E. Thomas*

+ Author Affiliations

Science 13 Dec 2002:
Vol. 298, Issue 5601, pp. 2179-2182
DOI: 10.1126/science.1079107

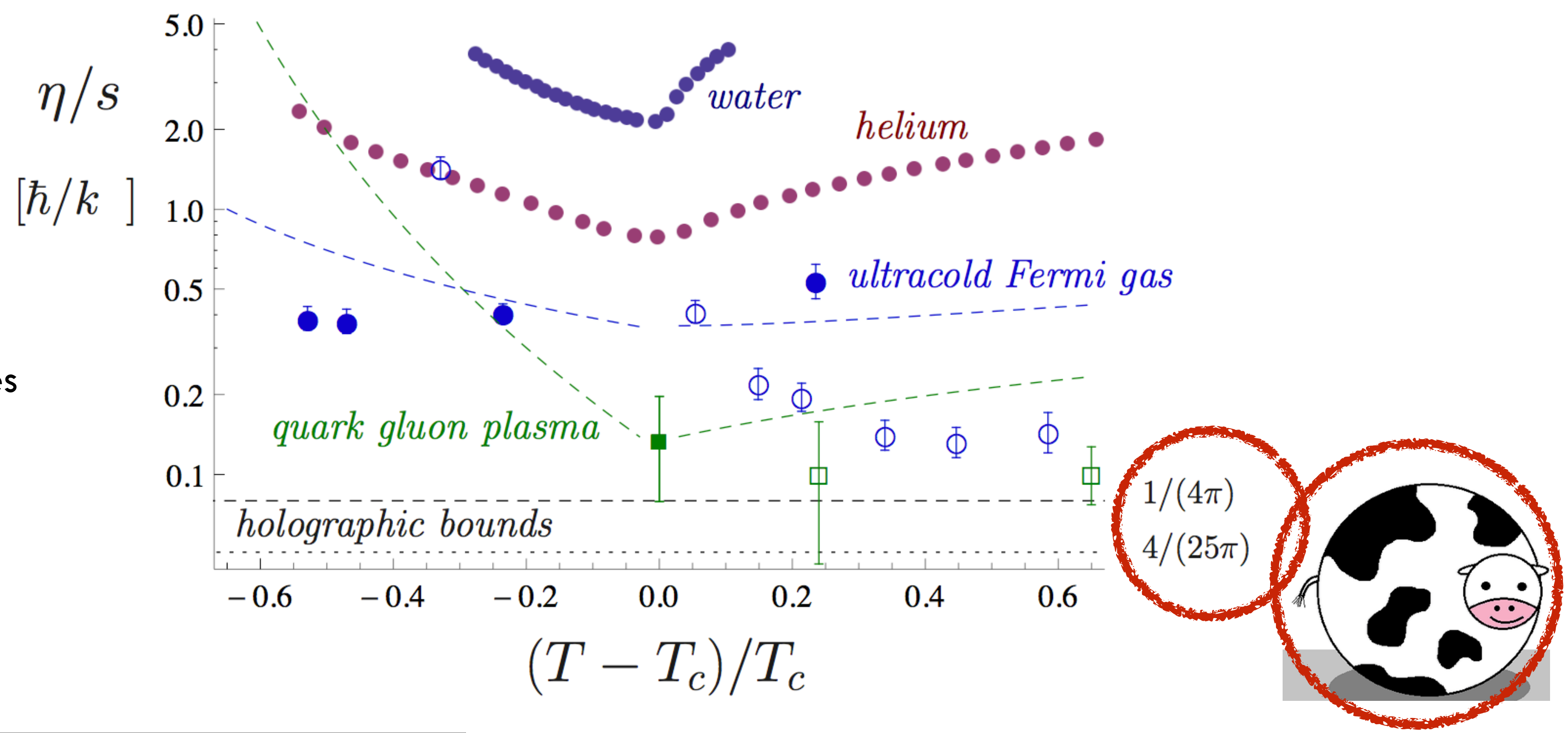




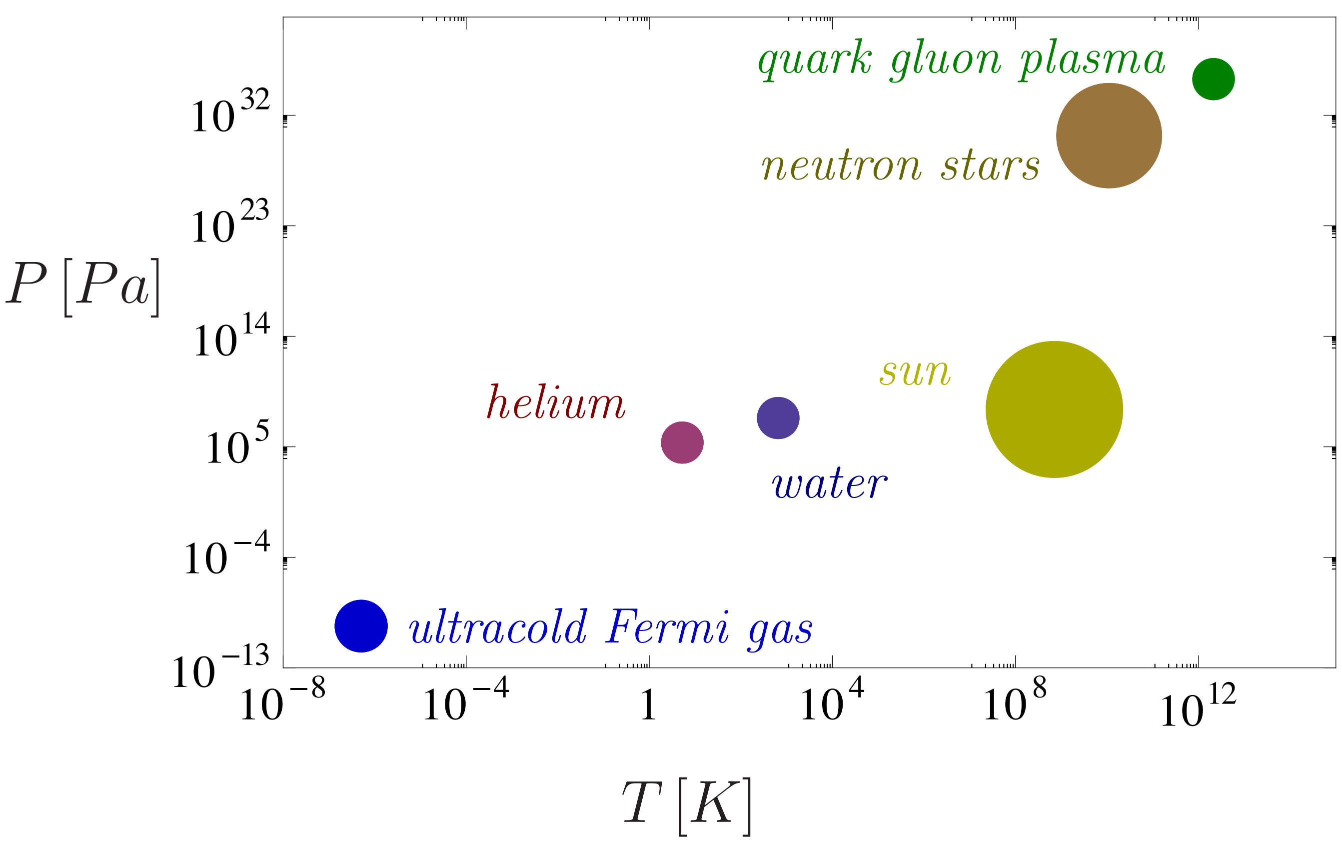


Elliptic flow of a strongly interacting Fermi gas as a function of time after release from a cigar-shaped optical trap

Universal bound



Experiment: colored circles
Lattice: open squares and circles
Theory: dashed curves



- Gas-liquid transition of water, helium
- Superfluid transition of Fermi gas
- Deconfinement transition of QCD

Strong coupling problems
AdS/CMT

AdS/CMT

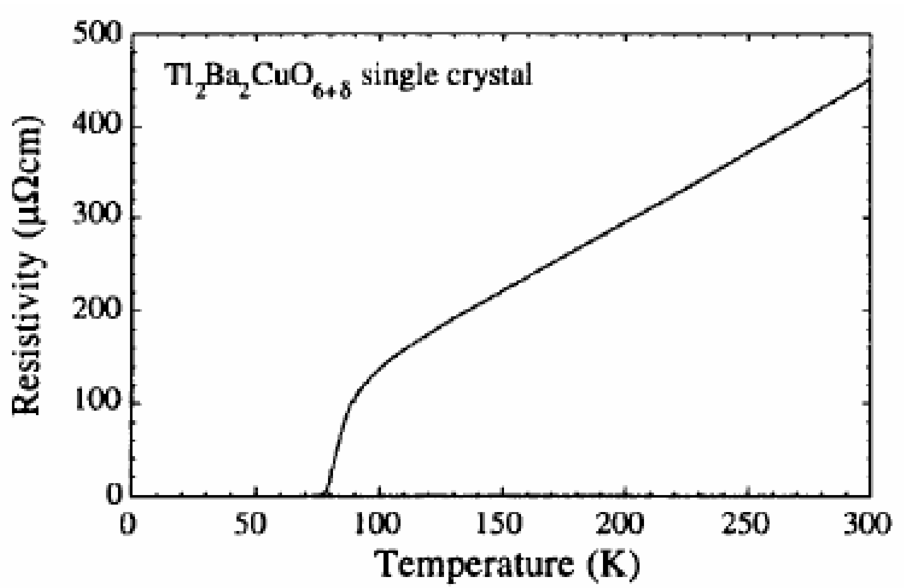
It is often said
the conductivity
is
the first quantity to be measured
and
the last to be understood

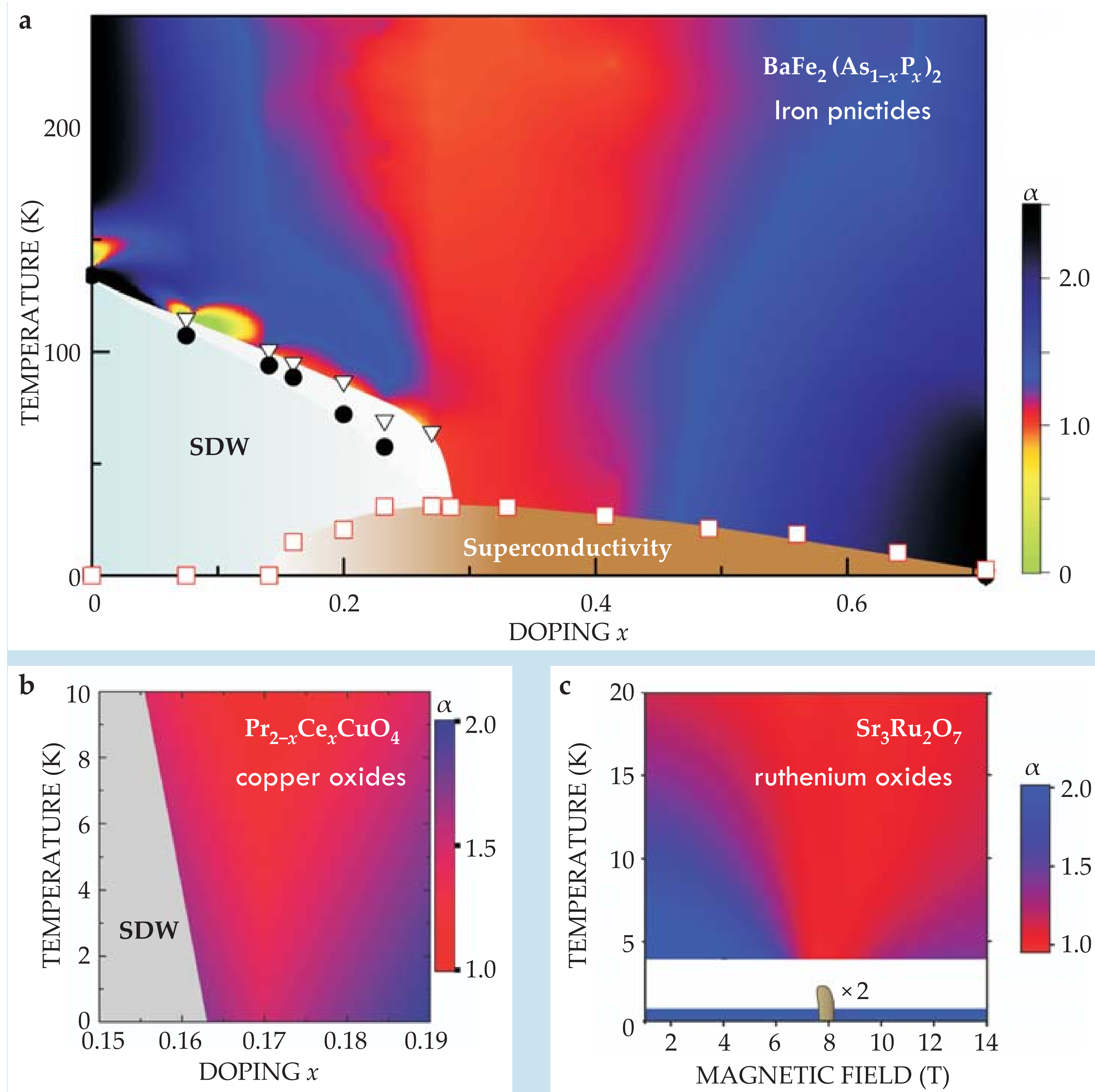
Strong interaction
~ No quasi particle picture & Fast thermalization
~ No Fermi-liquid theory
~ "Strange metal" or "Non-Fermi-liquid"

Linear-T-resistivity

[Sachdev, Keimer: 2011, Physics today]

Resistivity: $\rho \sim \rho_0 + AT^{\alpha}$





Fermi liquid theory $\rho \sim T^2$

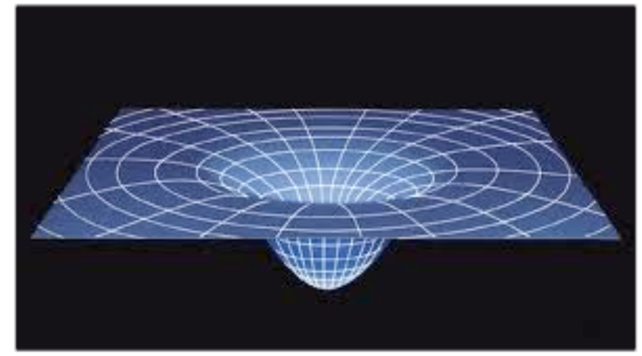
Anomalous and universal

Effective classical gravity for strong correlation

String theory

Good for microscopic understanding

Top-down models



Classical gravity in higher dimension

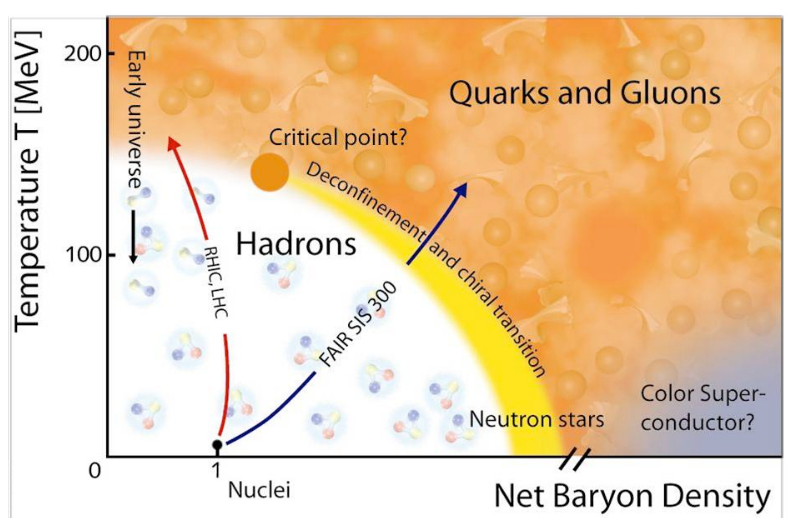
Bottom-up models

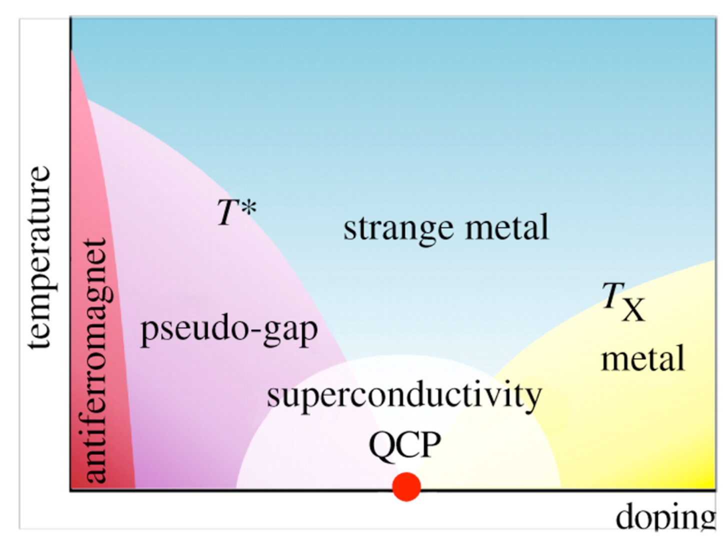
Good for universality

Guide:

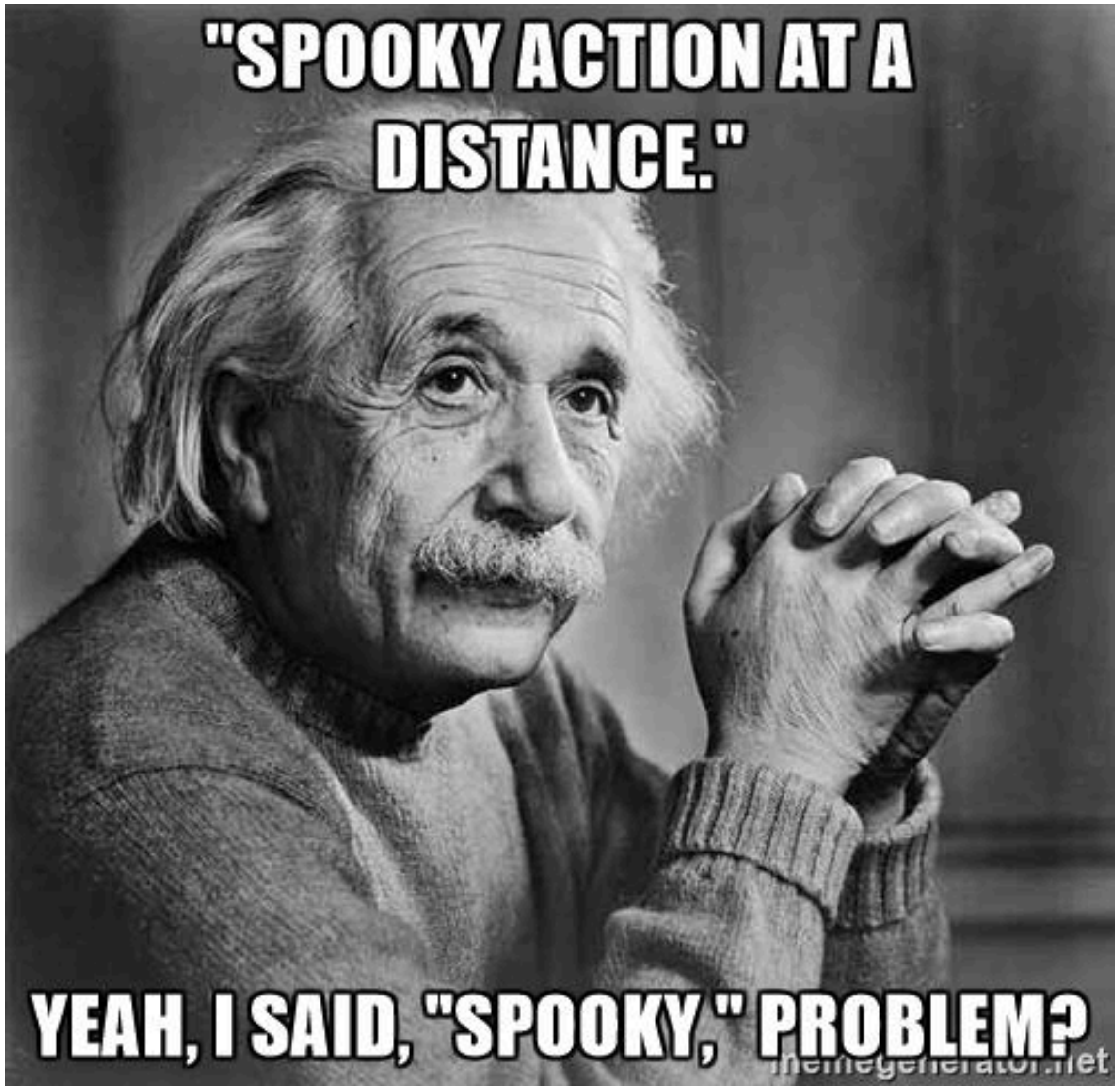
- Self-consistency
- Compare with the first principle computation
- Intuition from CMT
- Experimental results (Proposal for Exp.)

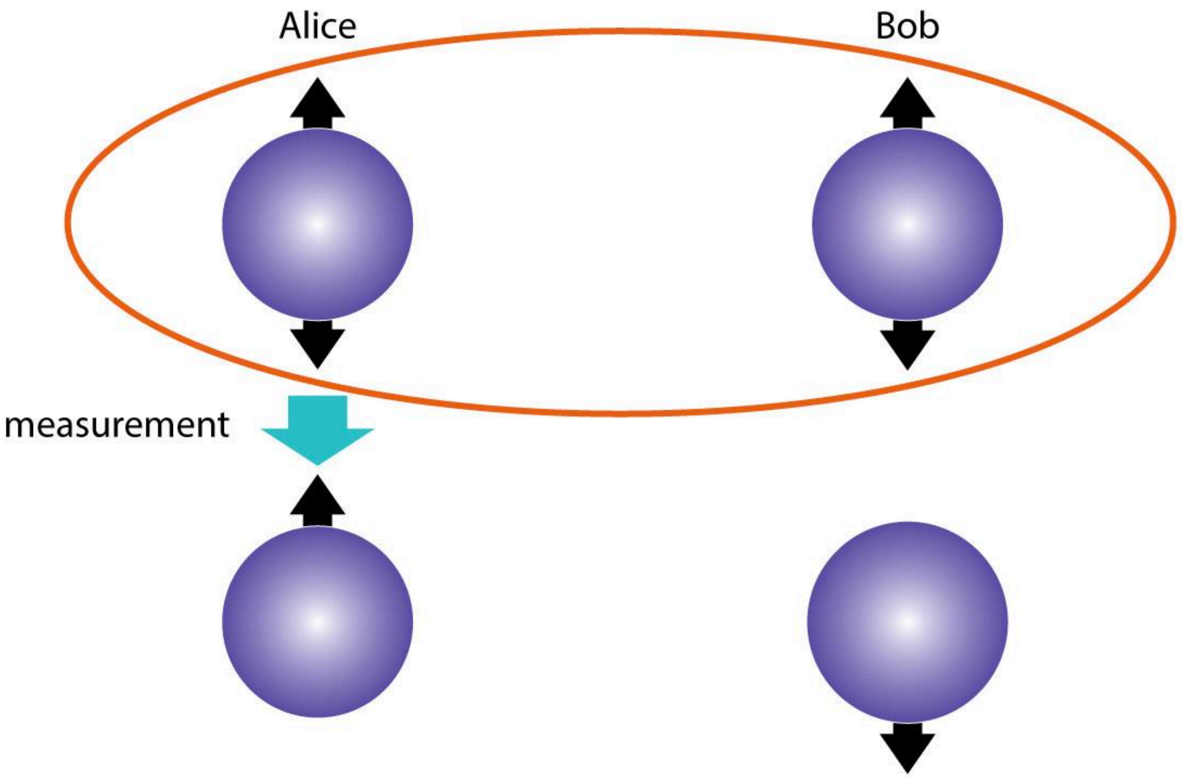
Strong correlation

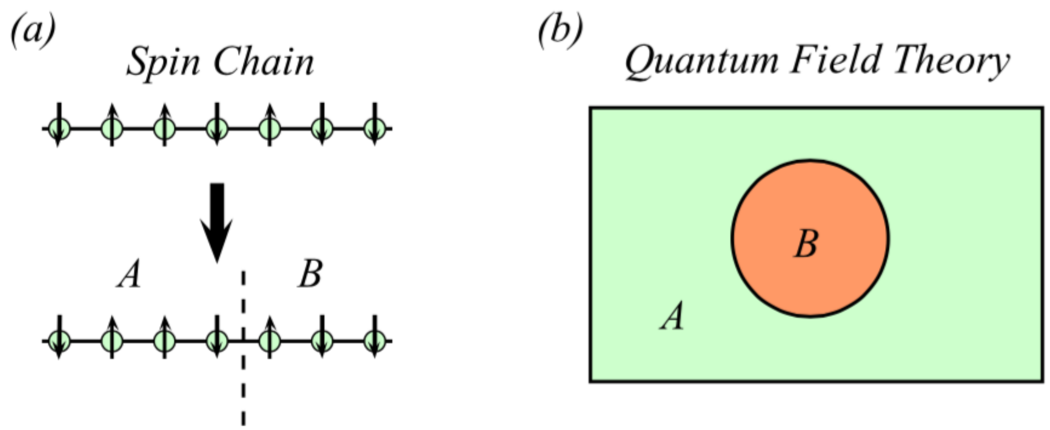




AdS/Quantum Information
-Entanglement entropy


"SPOOKY ACTION AT A DISTANCE."
YEAH, I SAID, "SPOOKY," PROBLEM?
memegenerator.net


Alice
Bob
measurement



$$S_A = -\mathrm{tr}_A\, \rho_A \log \rho_A$$

$$\rho_A = \mathrm{tr}_B\ \rho_{tot}$$

The Simplest Example: two spins (2 qubits)

(i) $|\Psi\rangle = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \otimes \left[|\uparrow\rangle_B + |\downarrow\rangle_B\right]$

$\Rightarrow \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \cdot \left[\langle\uparrow|_A + \langle\downarrow|_A\right].$

Not Entangled

$$S_A = 0$$

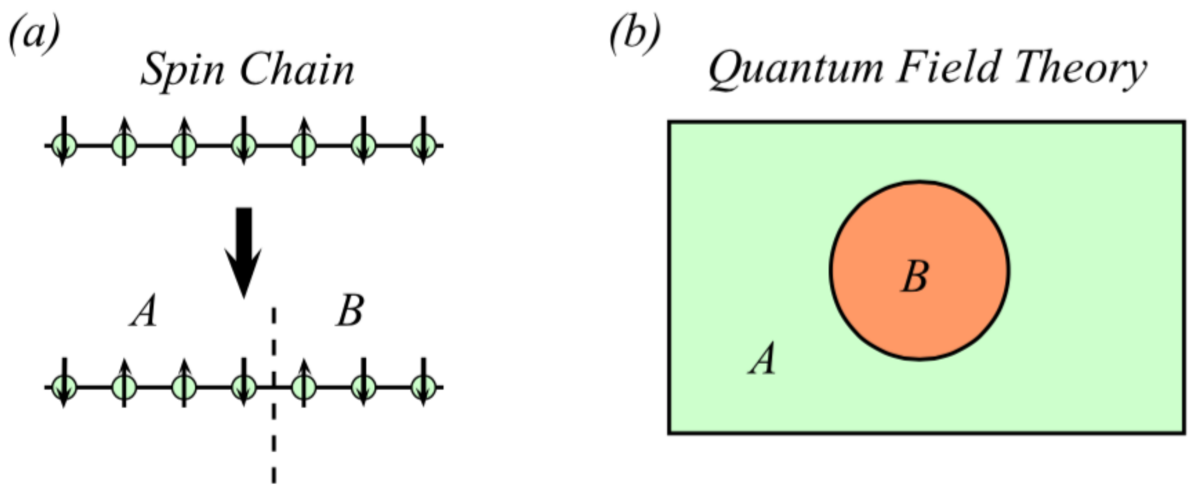
(ii) $|\Psi\rangle = \left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right] / \sqrt{2}$

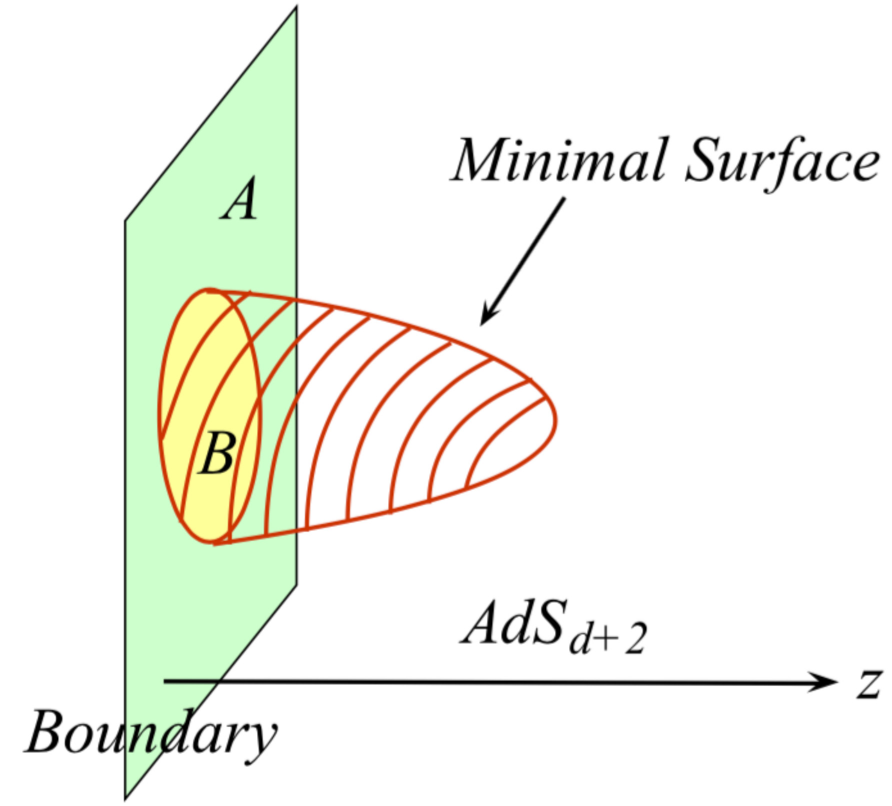
$\Rightarrow \rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|\uparrow\rangle_A\langle\uparrow|_A + |\downarrow\rangle_A\langle\downarrow|_A\right]$

Entangled

$$S_A = \log 2$$

Entanglement Entropy

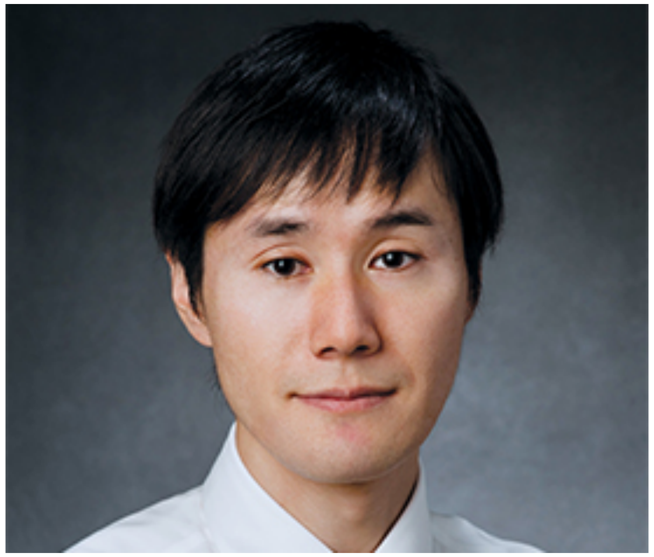




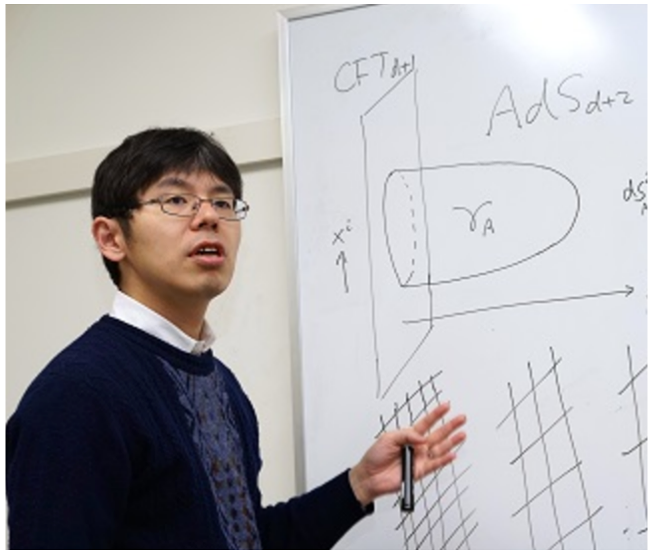
$$S_A = -\mathrm{tr}_A\, \rho_A \log \rho_A$$

$$\rho_A = \mathrm{tr}_B\ \rho_{tot}$$

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N^{(d+2)}}$$



Shines Ryu

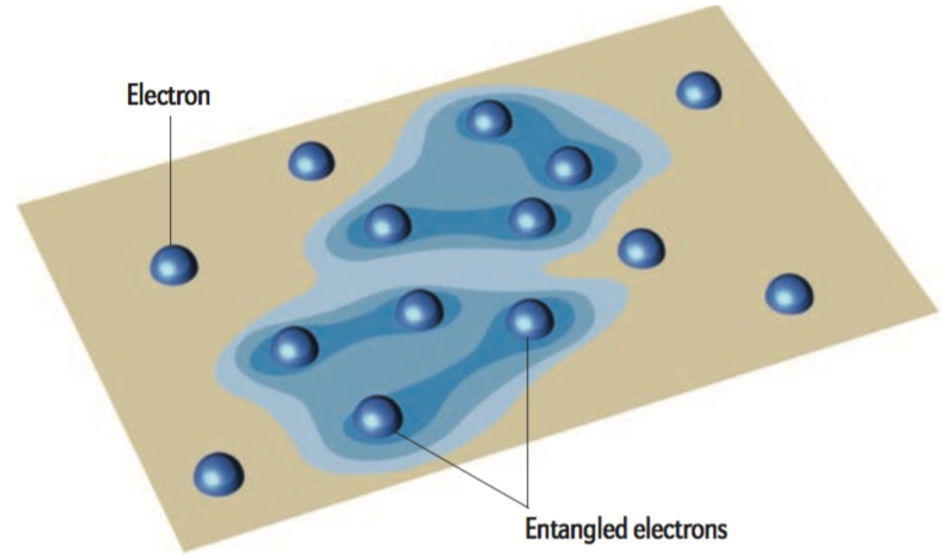


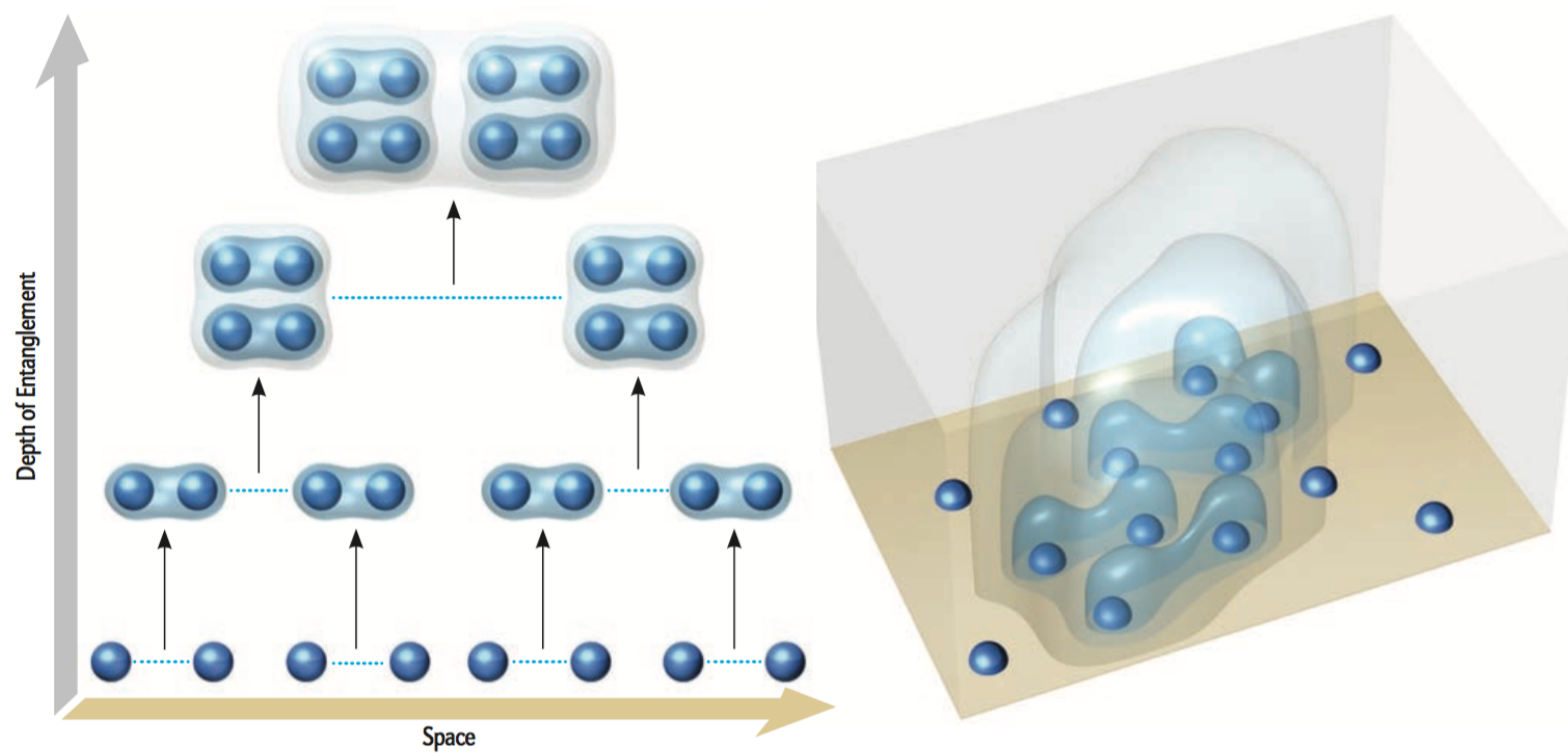
Tadashi Takayanagi

Successful agreements
with field theory computation

Emerging AdS?

Entanglement Entropy and Quantum information





AdS/Quantum Information
-Entanglement is not enough?

Complexity

[Susskind: 1402.5674
Stanford and Susskind: 1406.2678]

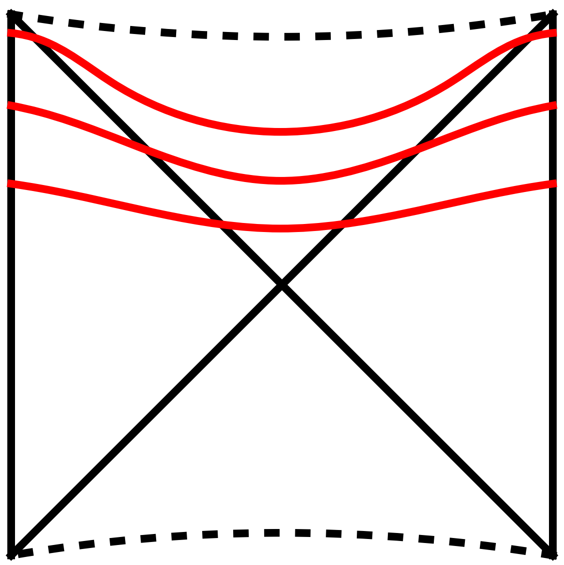
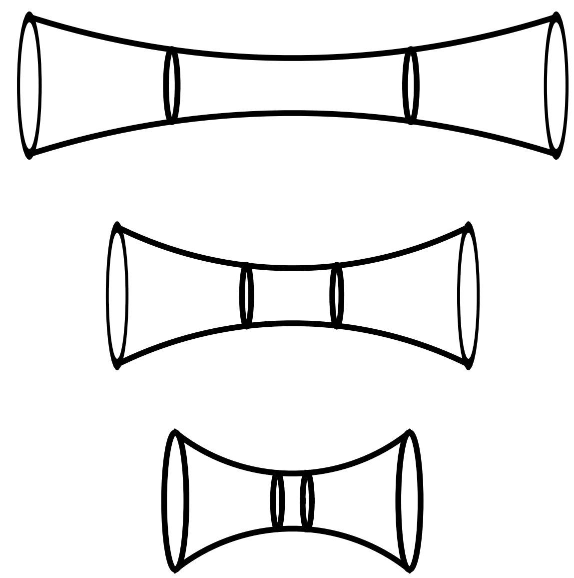


Fig. from [Koji, Norihiro, Sotaro: 1707.03840]



$$\mathcal{C}_V = \max_{\partial\Sigma = t_L \cup t_R} \left[\frac{V(\Sigma)}{G_N \ell}\right]$$

$$|\mathrm{TFD}\rangle := Z^{-1/2} \sum_\alpha \exp[-E_\alpha/(2T)] |E_\alpha\rangle_L |E_\alpha\rangle_R$$

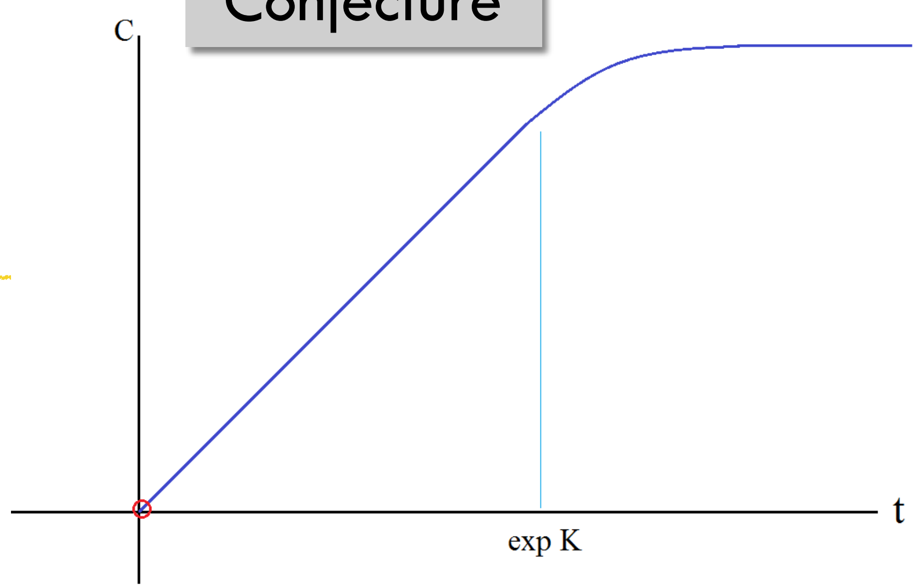
$$|\mathrm{TFD}(t_L, t_R)\rangle := e^{-i(t_L H_L + t_R H_R)} |\mathrm{TFD}\rangle$$

[Maldacena: hep-th/0106112]

Entanglement is not enough?

1. Einstein-Rosen bridge increases even after thermalization
2. The field theory meaning of this?
3. Physics inside black hole?

Conjecture



Complexity $|R\rangle \longrightarrow |T\rangle$

Minimum number of operation

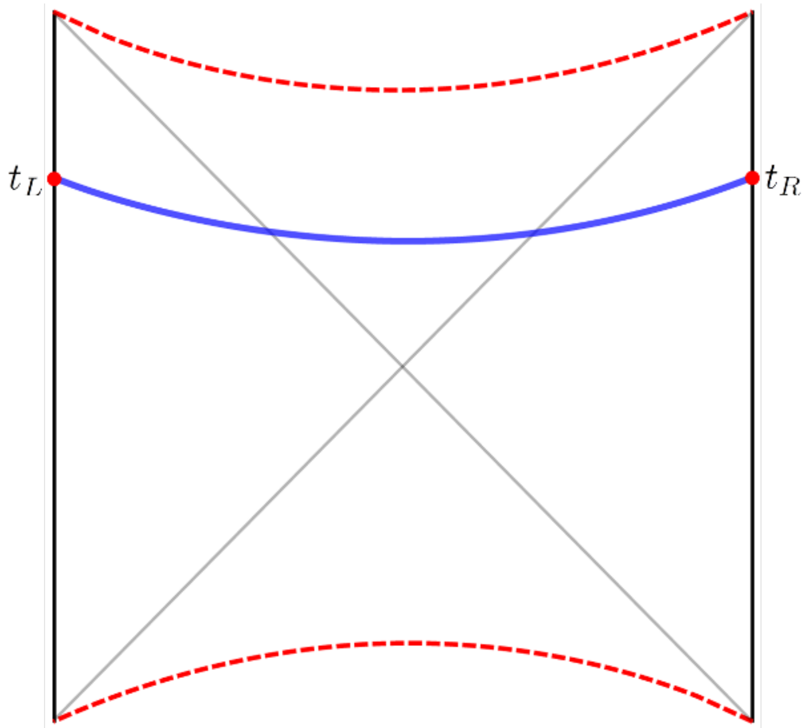
K-coins $(00110110001....)$ $\mathcal{C}_{max} \sim K \sim t_{max.comp}$

K-qubits $|\psi\rangle = \sum_1^{2^K} \alpha_i |i\rangle$ $\mathcal{C}_{max} \sim 2^K \sim t_{max.comp}$

Holographic conjecture for complexity

CV (complexity-volume)

[Susskind: 1402.5674
Stanford and Susskind: 1406.2678]

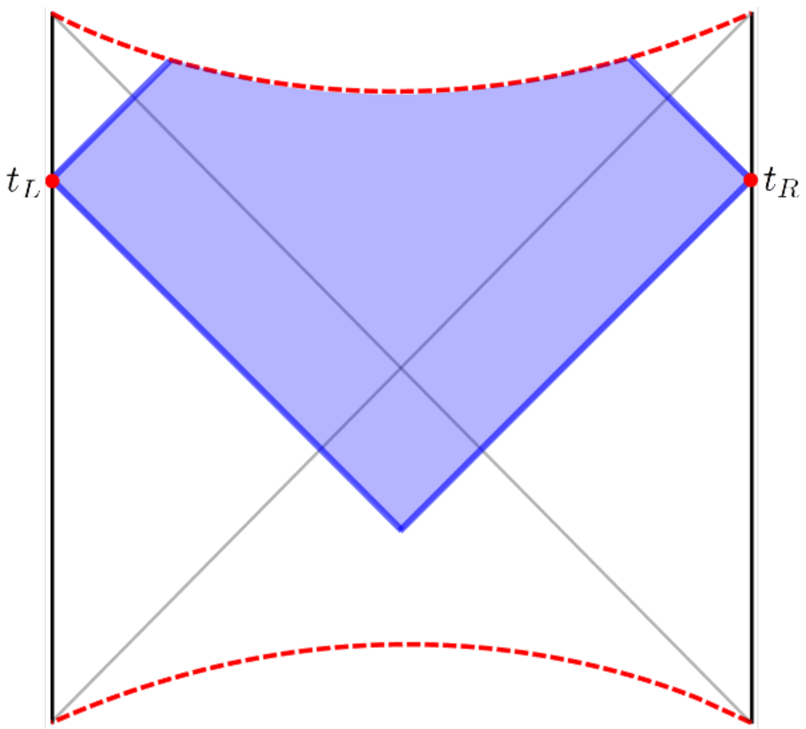


$$\mathcal{C}_V = \max_{\partial\Sigma = t_L \cup t_R} \left[\frac{V(\Sigma)}{G_N \ell} \right]$$

- Equation of motion
- Free scale: ambiguity

CA (complexity-action)

[Brown, Roberts, Susskind Swingle and Zhao:
1509.07876, 1512.04993]



$$\mathcal{C}_A = \frac{I_{\mathrm{WDW}}}{\pi \hbar}$$

- Boundary terms
- Singularity

Fig. from [Jefferson, Myers: 1707.08570]

Field theory conjecture for complexity

- Complexity geometry is a Finsler geometry.

- Puzzle: for a given operator, the right(-invariant) complexity and Left(-invariant) complexity are different.

$$\tilde{F}(H_r = i\dot{c}c^{-1}) \neq \tilde{F}(H_l = ic^{-1}\dot{c})$$

- By considering physical conditions, the puzzle is resolved: Left/right equivalence and bi-invariace. $\tilde{F}(H_\alpha) = \tilde{F}(\hat{U}H_\alpha\hat{U}^\dagger)$

$$\tilde{F}(H_r = i\dot{c}c^{-1}) = \tilde{F}(H_l = ic^{-1}\dot{c})$$

- SU(n) operator complexity is uniquely determined.

$$\mathcal{C}(\hat{O}) = \min\left\{\mathrm{Tr}\sqrt{\bar{H}\bar{H}^\dagger} \mid \forall\, \bar{H}, s.t., \exp(-i\bar{H}) = \hat{O}\right\} \quad \text{(Finsler)}$$

$$\mathcal{C}(\hat{O}) = \min\left\{\left[\mathrm{Tr}\left(\bar{H}\bar{H}^\dagger\right)^{\frac{p}{2}}\right]^{\frac{1}{p}} \mid \forall\, \bar{H}, s.t., \exp(-i\bar{H}) = \hat{O}\right\}$$

Applications?

- SYK model: complexity growth in a chaotic model

Complexity growth

SYK model

$$H(\mathcal{J},N) = \sum_{i<j<k<l}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l$$

(N Majorana Fermions)

mean = 0 variance $\sigma^2 = \frac{6\mathcal{J}^2}{N^3}$

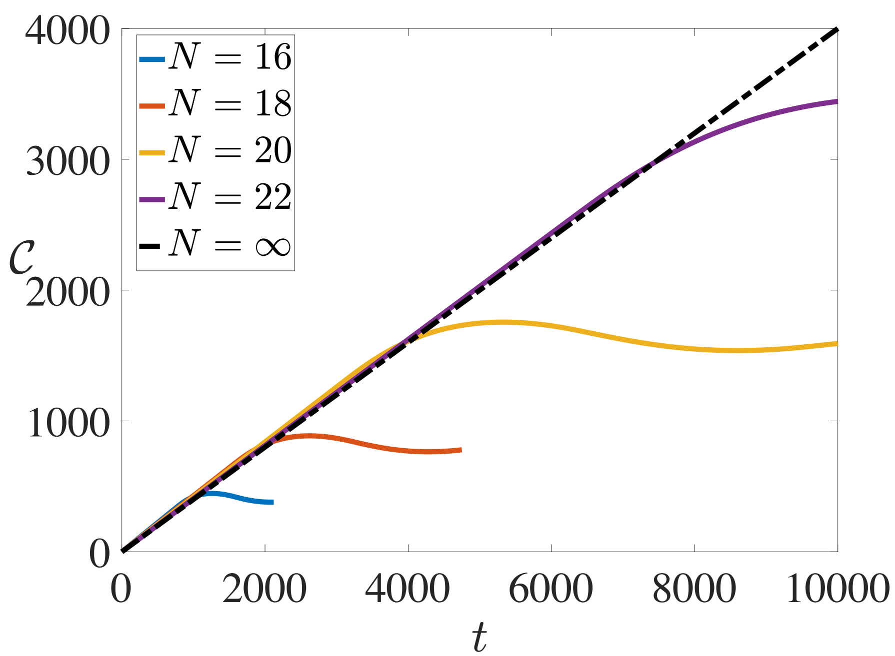
Complexity

$$\mathcal{C}(t) = \min\left\{\lambda\ \mathrm{Tr}\sqrt{VV^\dagger}\ \middle|\ \forall V,\quad s.t.\ \exp(-iV) = \hat{U}(t) = \exp(-iH(\mathcal{J},N)t)\right\}$$

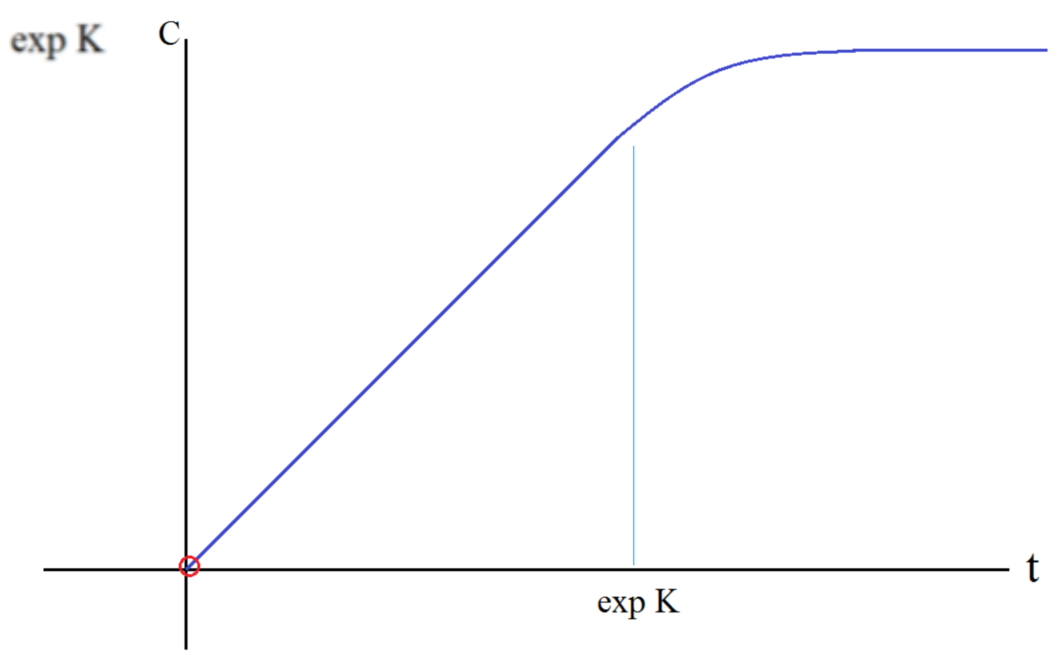
$$V = \sum_{n=1}^{2^{N/2}}(E_n t + 2\pi k_n)|n\rangle\langle n| \qquad k_n \in \mathbb{N} \quad \sum_{n=1}^{2^{N/2}} k_n = 0$$

$$\hat{U}(t) = \sum_{n=1}^{2^{N/2}} e^{-iE_n t}|n\rangle\langle n|$$

$$\mathcal{C}(t) = \min\left\{\sum_{n=1}^{2^{N/2}}|E_n t + 2\pi k_n|\ \middle|\ \forall k_n \in \mathbb{N},\quad s.\ t.\ \sum_{n=1}^{2^{N/2}} k_n = 0\right\}$$



$$C_c \sim 2^{N/2}$$
$$t_c \sim 2^{N/2}$$



Complexity growth

$$\mathcal{C}(t) = \min\left\{ \sum_{n=1}^{2^{N/2}} |E_n t + 2\pi k_n| \;\middle|\; \forall k_n \in \mathbb{N},\quad s.\ t.\ \sum_{n=1}^{2^{N/2}} k_n = 0 \right\}$$

$$\mathcal{C}(t) \approx \sum_{n=1}^{2^{N/2}} |E_n t - 2\pi [[E_n t/2\pi]]|$$

$$[[1.2]] = 1, [[1.7]] = 2 \text{ and } [[-2.7]] = -3$$

$$\mathcal{C}(t) \approx \sum_{n=1}^{2^{N/2}} |E_n| t \qquad t < \frac{\pi}{E_{\max}}$$

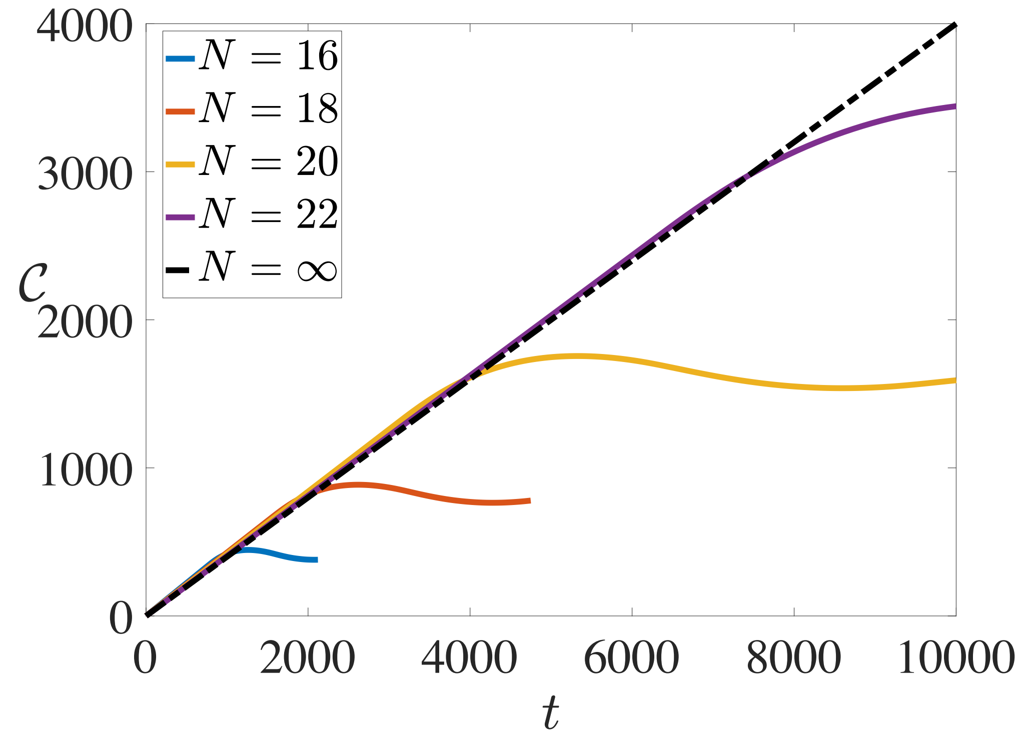
$$t_c \sim \frac{\pi}{E_{\max}} \sim \frac{2^{N/2}}{\langle E \rangle}$$

"total energy" $\langle E \rangle \approx \sum_{n=1}^{2^{N/2}} |E_n|$

$$\mathcal{C}_c \approx \langle E \rangle t_c$$

$$\frac{dC}{dt} \sim \frac{C_c}{t_c} \sim \langle E \rangle$$

$$\sim 2^{N/2}$$



More general analysis:
Statistical properties of eigenvalues
work in progress

SYK information

$$\langle E \rangle = 2^{N/2} \langle E_{\max}(\mathcal{J}, N) \rangle$$

$$\langle E_{\max}(\mathcal{J}, N) \rangle \approx (0.055 + 0.029N)\mathcal{J}$$

For fixing J (consequently fixing E_max), see

"Quantum complexity of time evolution with chaotic Hamiltonian" [1905.05765] ,Balasubramanian, DeCross, Kar, Parrikar

Summary and outlook

History of gauge/gravity duality

97 98 99 00 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19

AdS/QCD
Quark-Gluon Plasma:
Small $\frac{\eta}{s}$ bound

Nuclear Physics:
Sakai-Sugimoto model

AdS/QCD
bottom-up model

Quantum information:
Entanglement entropy

AdS/CMT

Holographic
superconductor

AdS/tensor-network

AdS/CMT: Strange metal

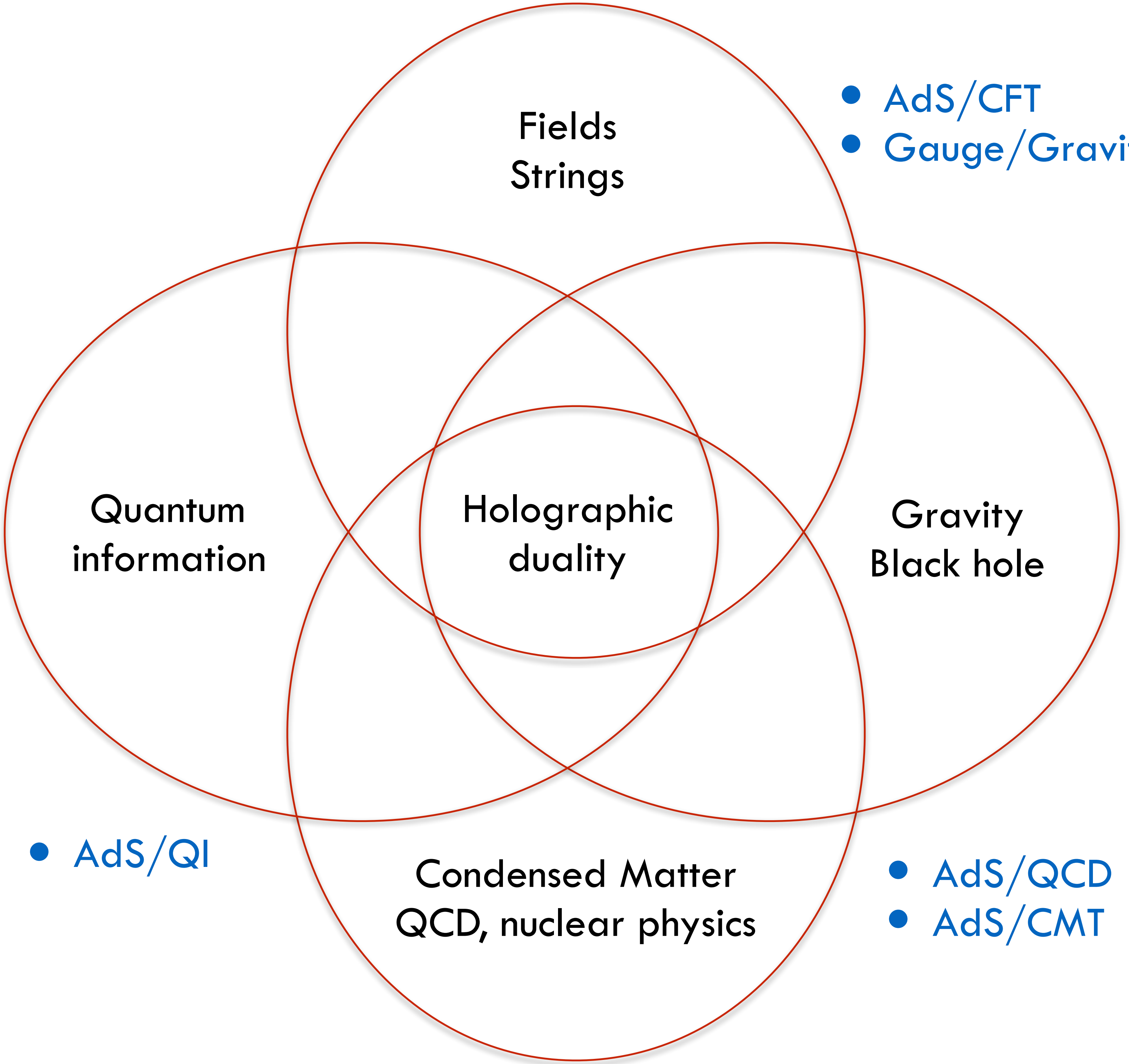
Many many QCD,
CMT, gravity
applications,
Quantum information

AdS/CFT

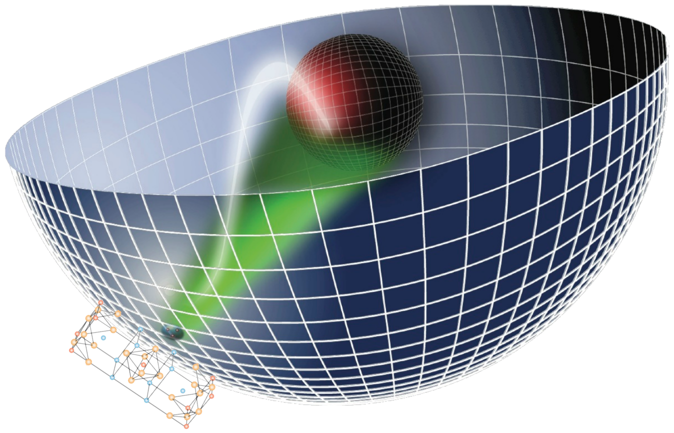
1. Finding more supporting evidences.
2. Towards less symmetric cases.
3. Adding flavours.

The principle is
not proven yet,
But many supporting evidences

In a bigger context: overview on holographic duality



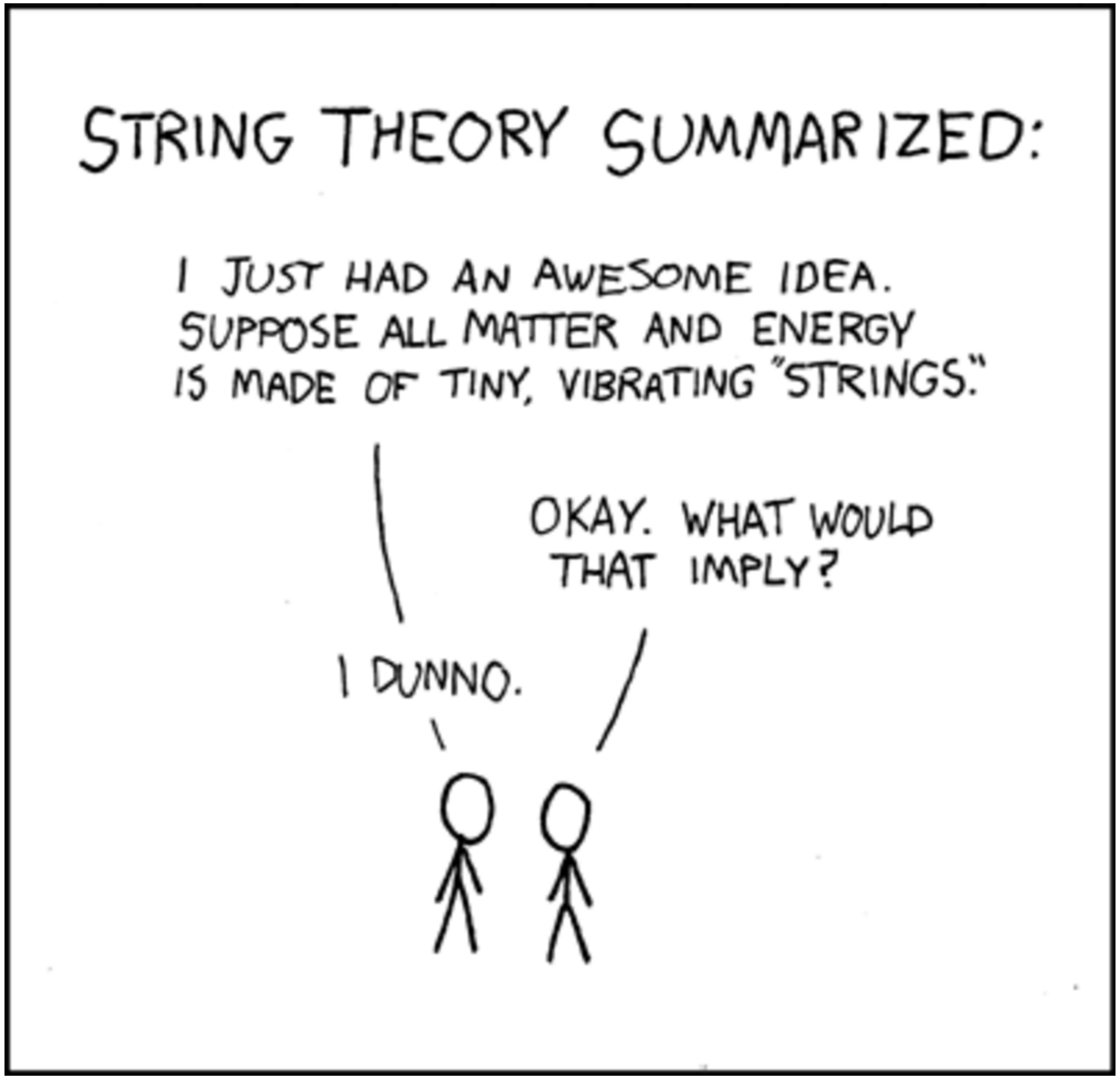
- AdS/CFT
- Gauge/Gravity

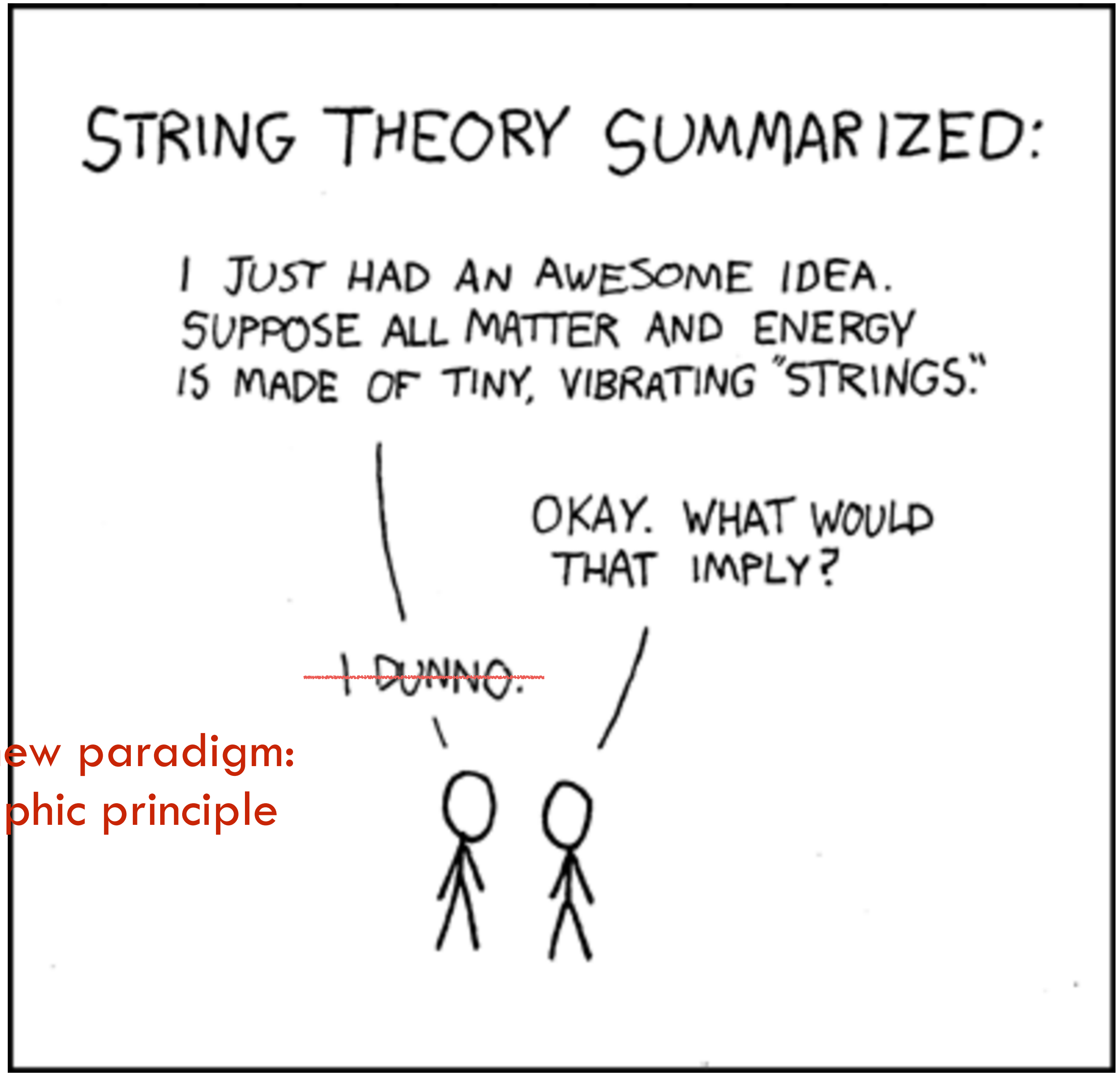


- AdS/QI

- AdS/QCD
- AdS/CMT

Guide:
- Self-consistency
- Compare with the first principle simulation
- Intuition from CMT/QI
- Proposal for CMT/QI
- Experimental results
- Proposal for Exp.


STRING THEORY SUMMARIZED:
I JUST HAD AN AWESOME IDEA. SUPPOSE ALL MATTER AND ENERGY IS MADE OF TINY, VIBRATING "STRINGS."
OKAY. WHAT WOULD THAT IMPLY?
I DUNNO.



It led a new paradigm:
Holographic principle

Research Directions in Quantum Field Theory and String Theory 2020

The workshop for Prof. *Chaiho Rim*

축하드립니다!!

김근영 드림

Thank you