Advanced Nuclear Physics

Nuclear Theory Group,
Tohoku University
Kouichi Hagino

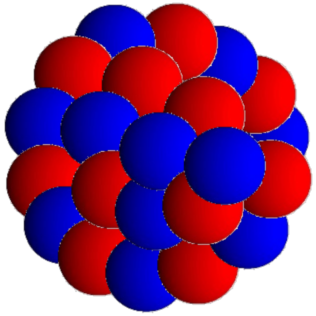
原子核理論特論

東北大学→ Kyoto U.
原子核理論研究室
萩野浩一

Contents

Nuclei: aggregate of nucleons (protons and neutrons)

→ ***Nuclear Many-Body Problems***



(Low-energy) Nuclear Physics:

to understand <u>rich nature</u> of atomic nuclei starting from nucleon-nucleon interactions

- size, mass, density, shape
- excitations
- decays
- nuclear reactions

two kinds of particle: protons and neutrons

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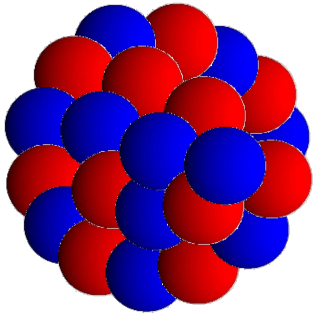
Nuclei: aggregate of nucleons (protons and neutrons)

→ ***Nuclear Many-Body Problems***

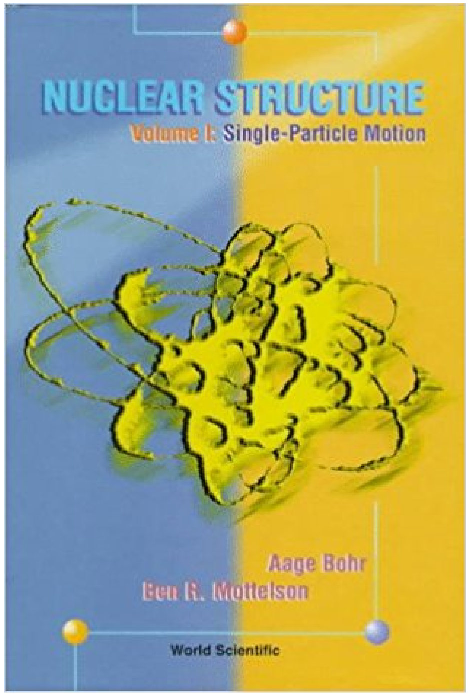


microscopic descriptions of atomic nuclei

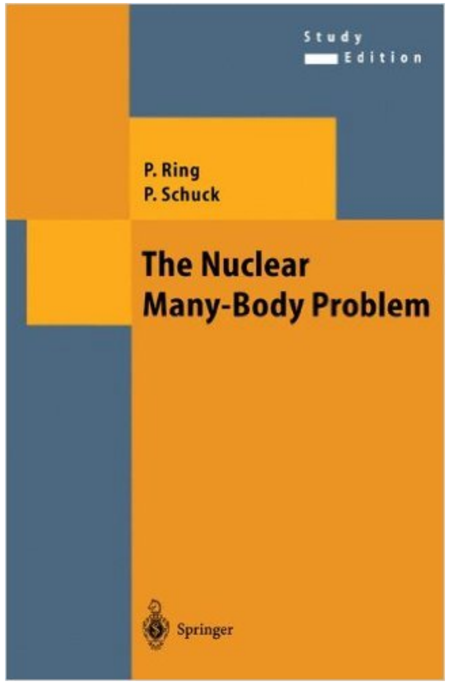
- Liquid drop model
- Single-particle motion and shell structure
- **Hartree-Fock approximation**
- Bruckner theory
- Pairing correlations and superfluid Nuclei
- Angular momentum and number projections
- 1-neutron and 2-neutron halo nuclei
- **Random phase approximation (RPA)**
- **Nuclear reactions**

Contents

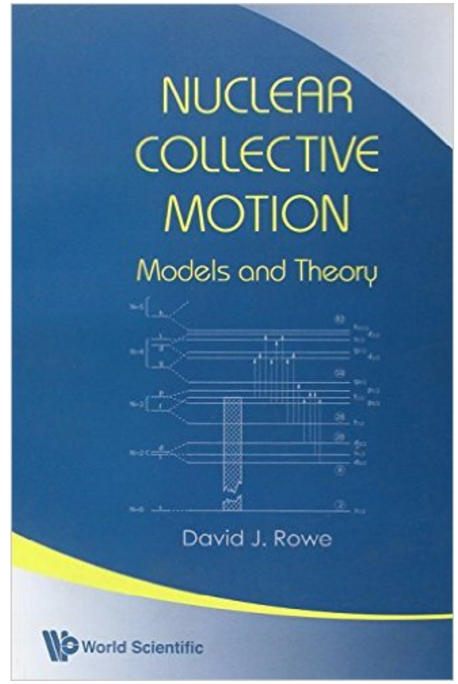
References



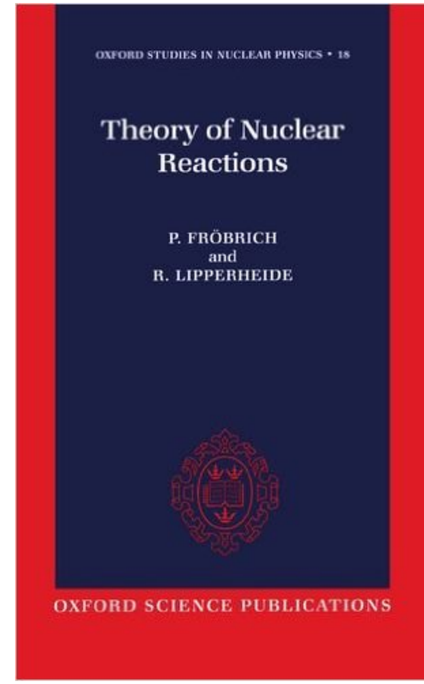
Bohr-Mottelson



Ring-Schuck



Rowe

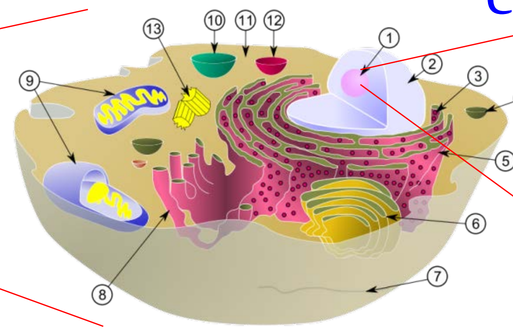


Frobrich -Lipperheide

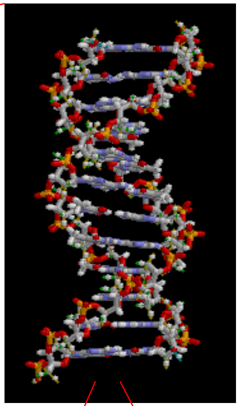
- From Nucleons to Nucleus : Jouni Suhonen
- Introductory nuclear physics : Samuel S. Wong

Introduction: atoms and atomic nuclei

cells



DNA

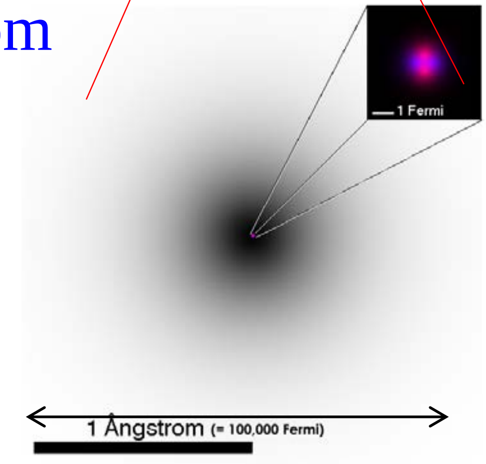


~ 50 cm

~ μm = 10^{-6} m

~ 10^{-8} m

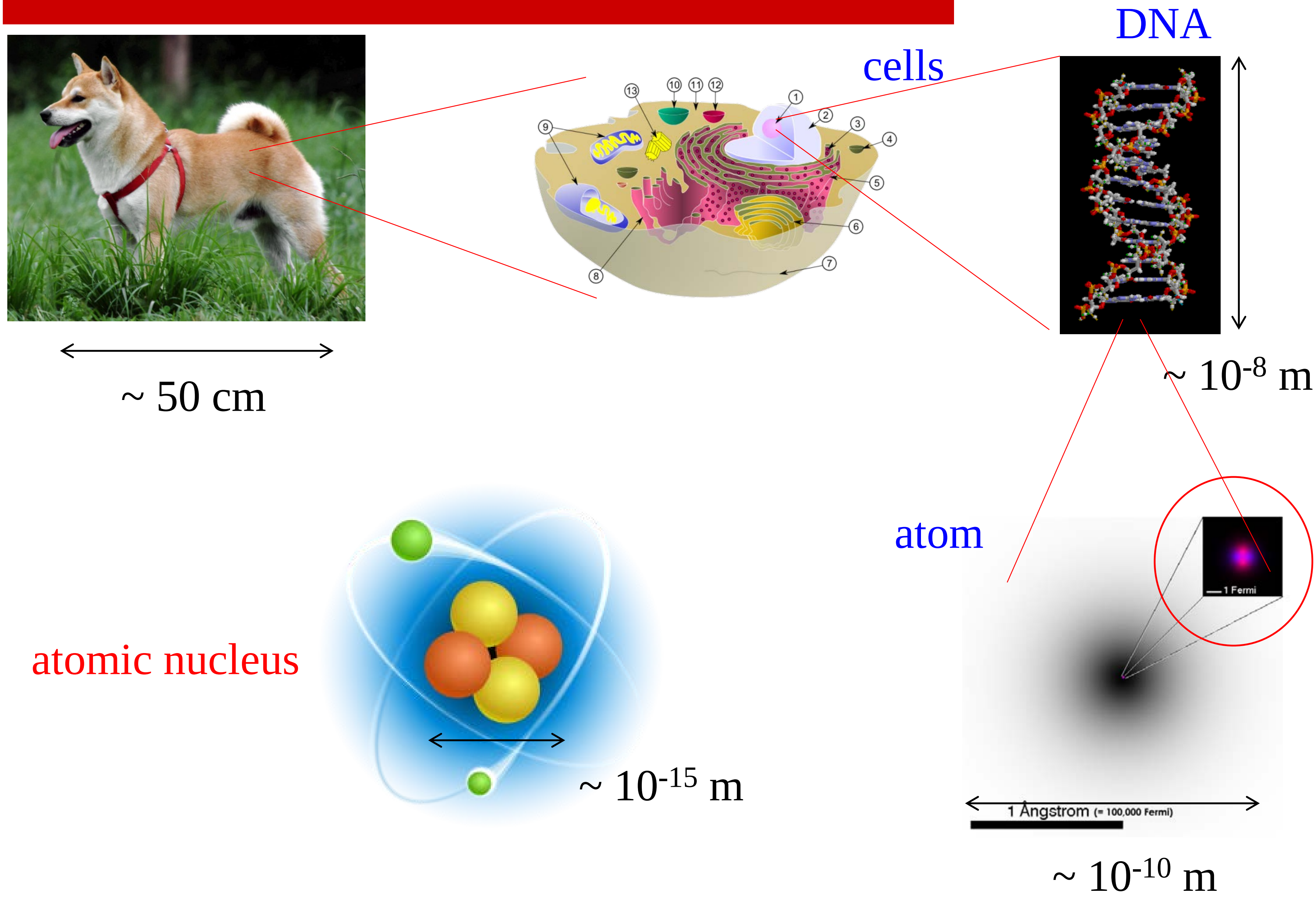
atom

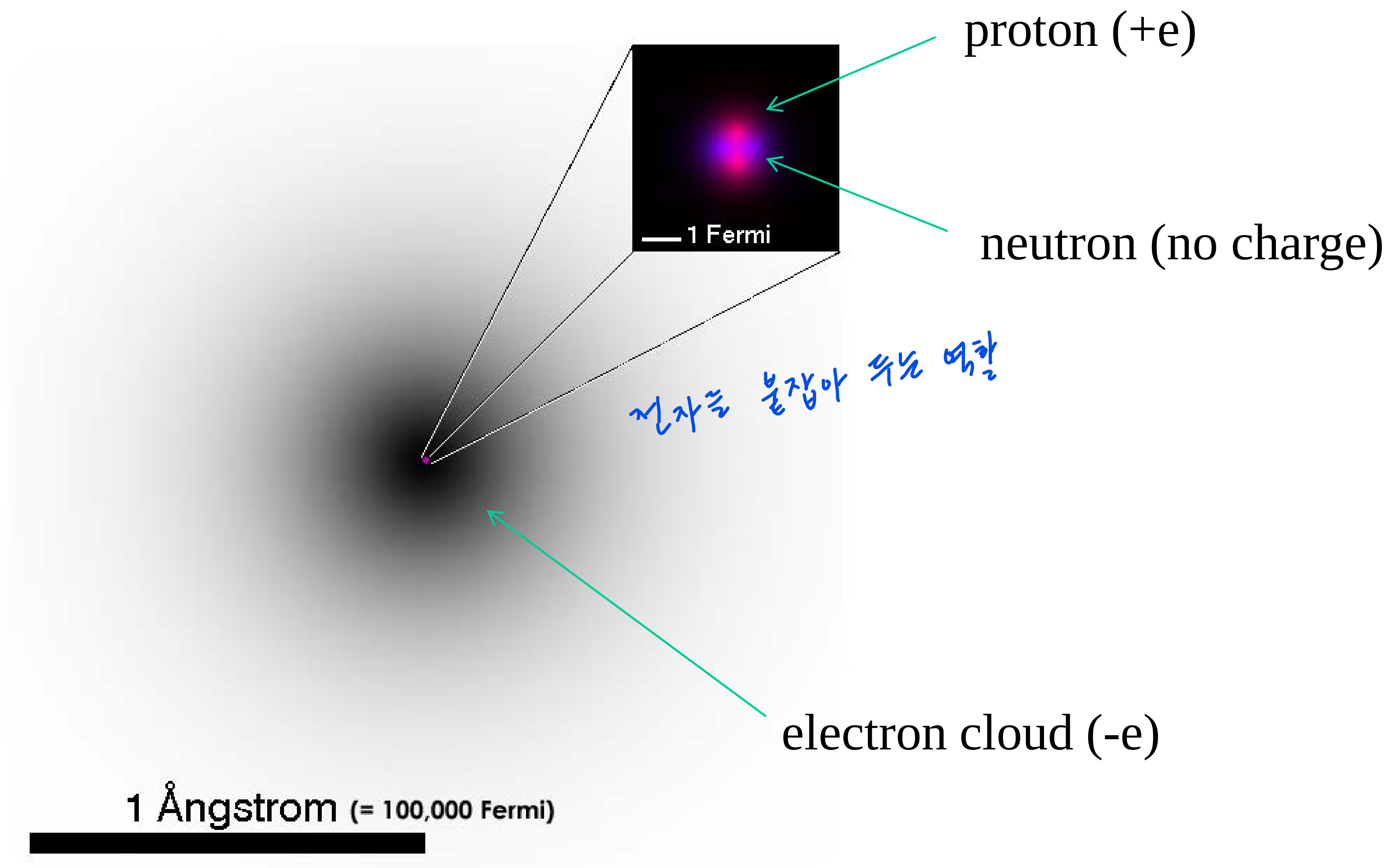


~ 10^{-10} m

Everything is made of atoms.

Introduction: atoms and atomic nuclei

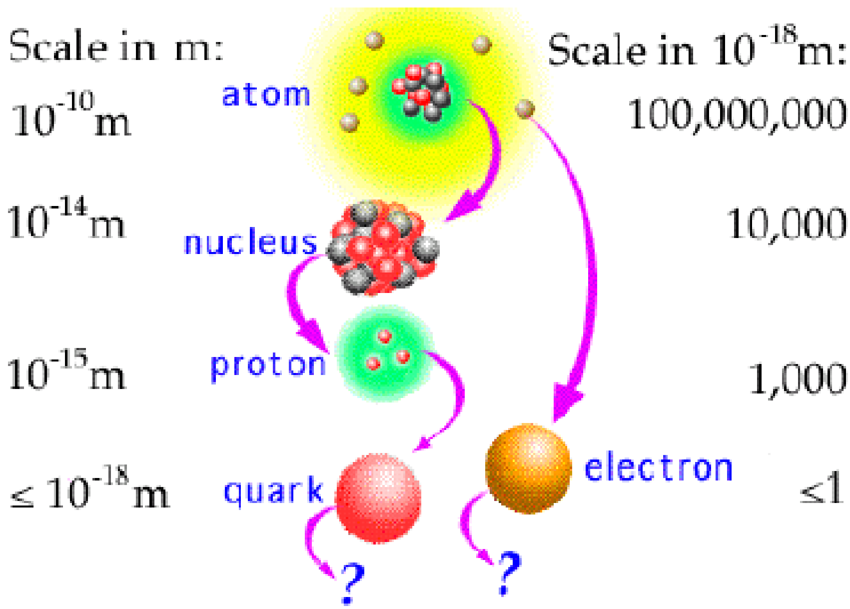




- Neutral atoms: # of protons = # of electrons
- Chemical properties of atoms ⟶ # of electrons
- $M_p \sim M_n \sim 2000\ M_e$ ⟶ the mass of atom ~ the mass of nucleus

99.99%   부피 $1/10^{12}$

Nuclear Physics



$1\text{ fm} = 10^{-15}\text{ m}$

Nucleus as a *quantum many body system*

microscopic system of many mutually interacting particles.

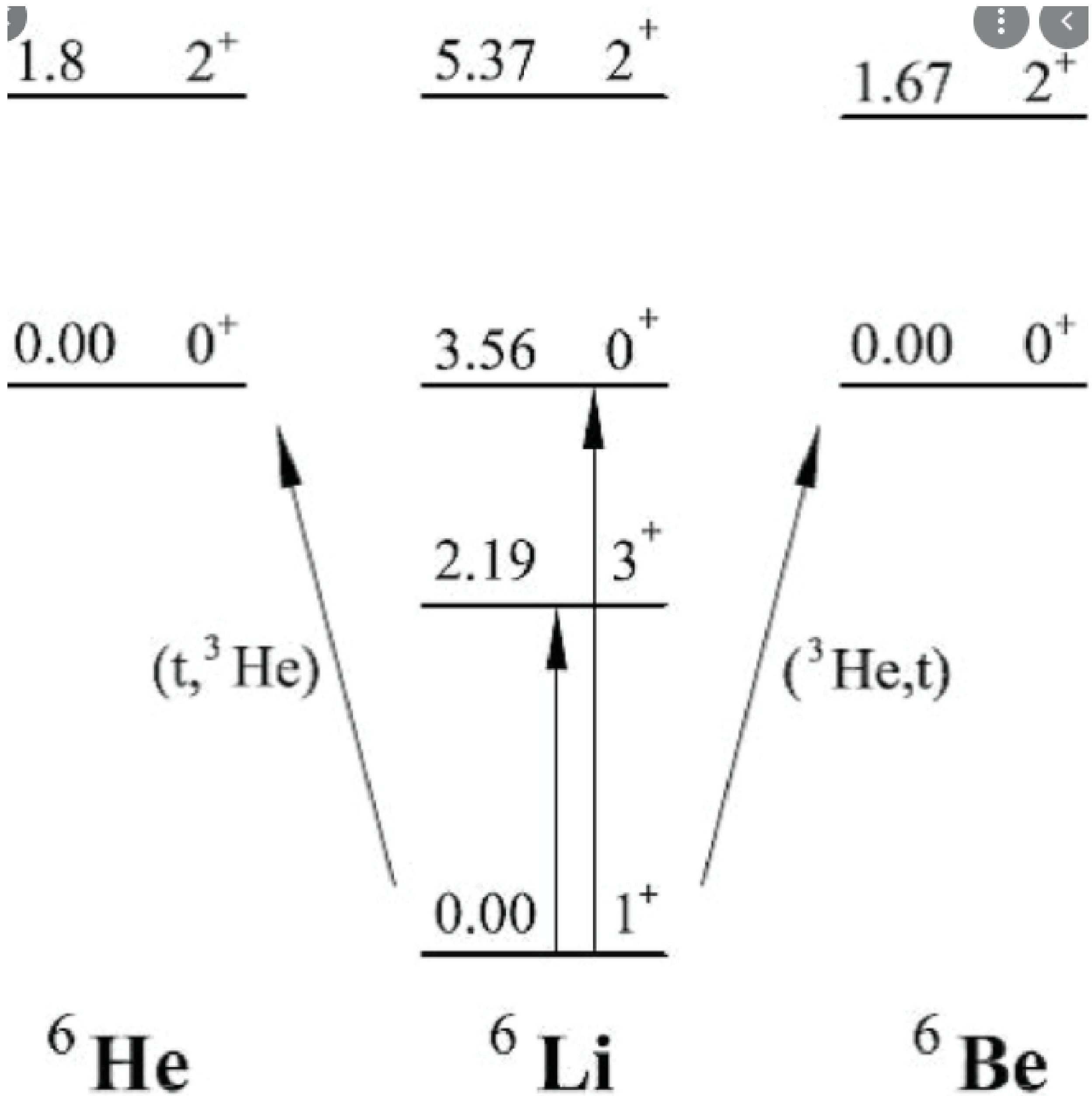
Basic ingredients:

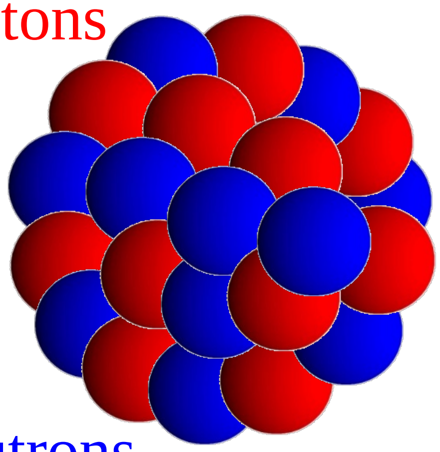
	charge	mass (MeV)	spin
Proton	+e	938.256	½+
Neutron	0	939.550	½+

if, w/o E.M Int. Isospin symm. n=p "Isobaric Analogue in same mass"

(note) $n \longrightarrow p + e^- + \bar{\nu}$ (10.4 min)

half-life

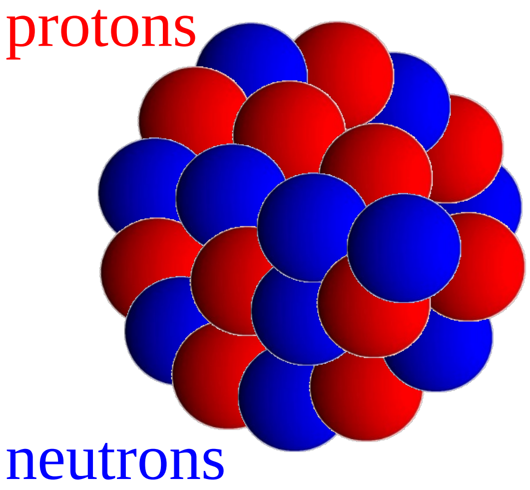

1.8 2+
0.00 0+
5.37 2+
3.56 0+
2.19 3+
0.00 1+
1.67 2+
0.00 0+
(t,3He)
(3He,t)
6He
6Li
6Be



- Nucleons are not stopping inside a nucleus.
  (they move relatively freely)

- Yet, they are not completely independent.
  A nucleus keeps its shape while nucleons influence among themselves so that a nucleon does not escape.

a self-bound system



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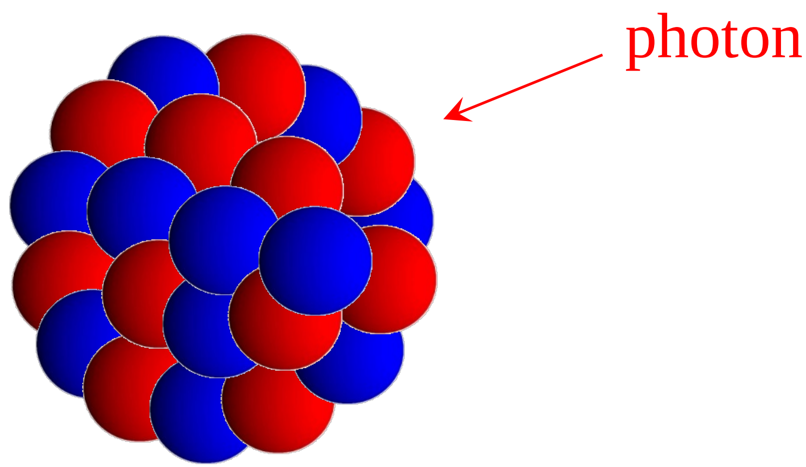
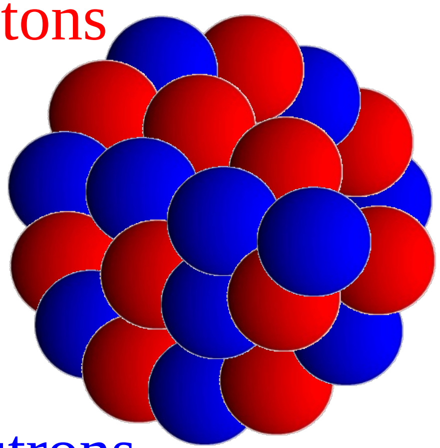
a self-bound system

What happens if a photon is absorbed into a nucleus?
- one nucleon simply starts moving faster?





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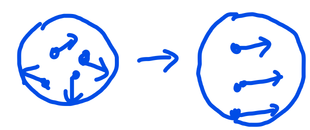
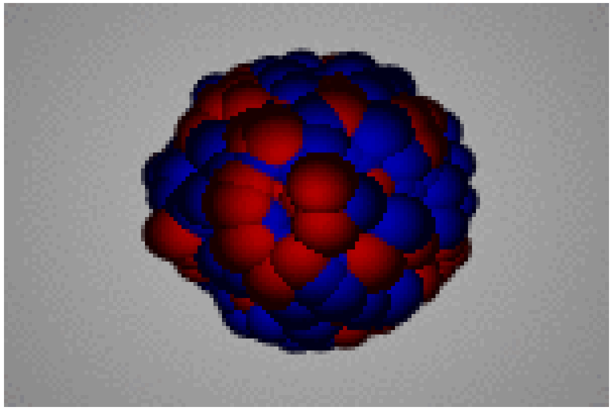
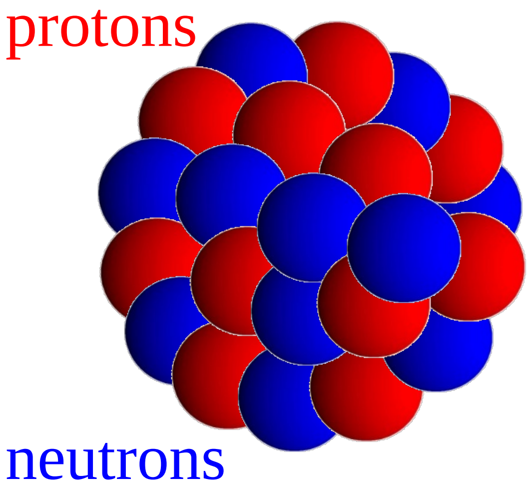




Very coherent motion can happen

Collective motions



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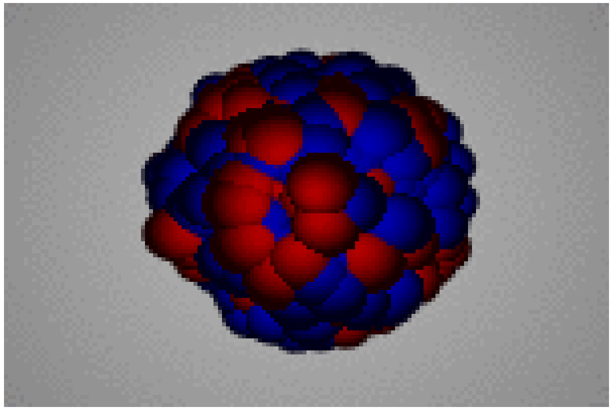

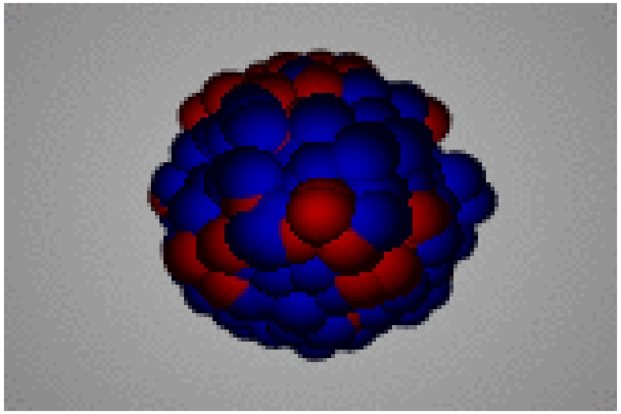

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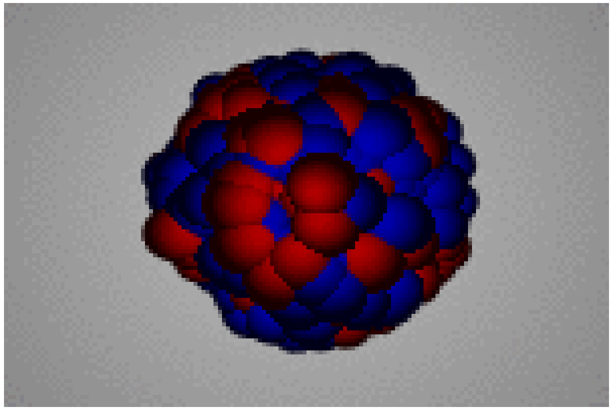

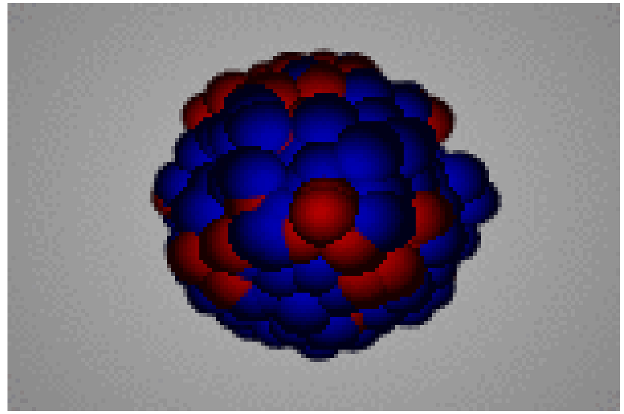

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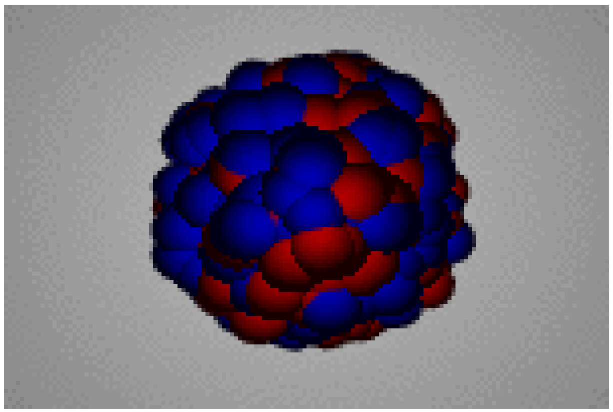
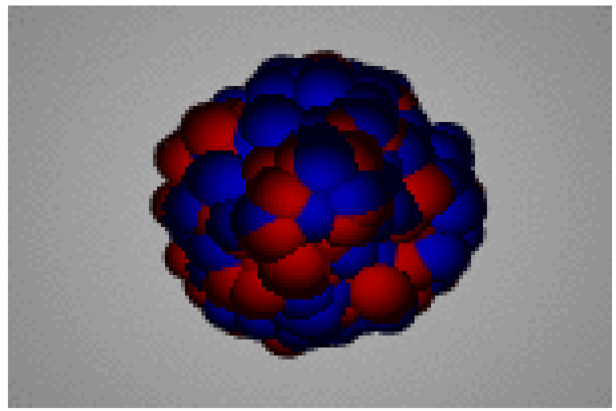
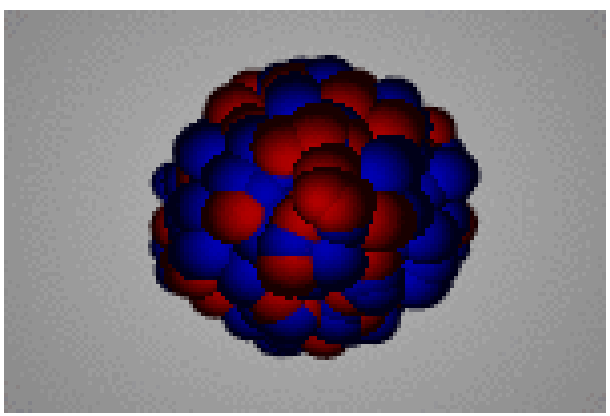
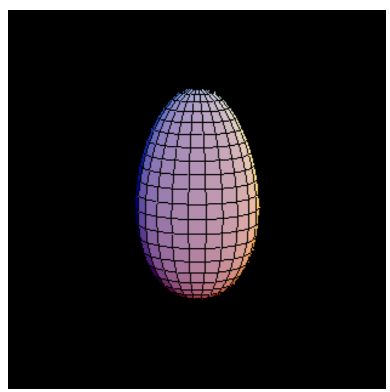
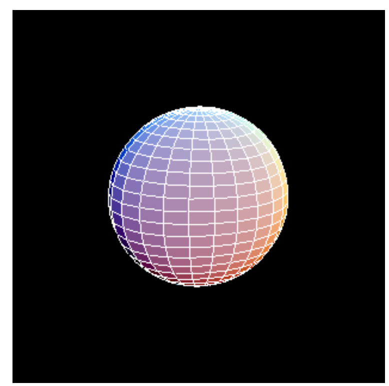


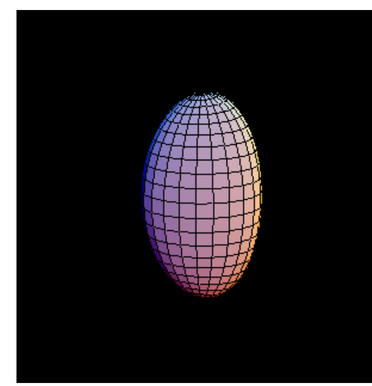


a variety of motions
→ very rich!

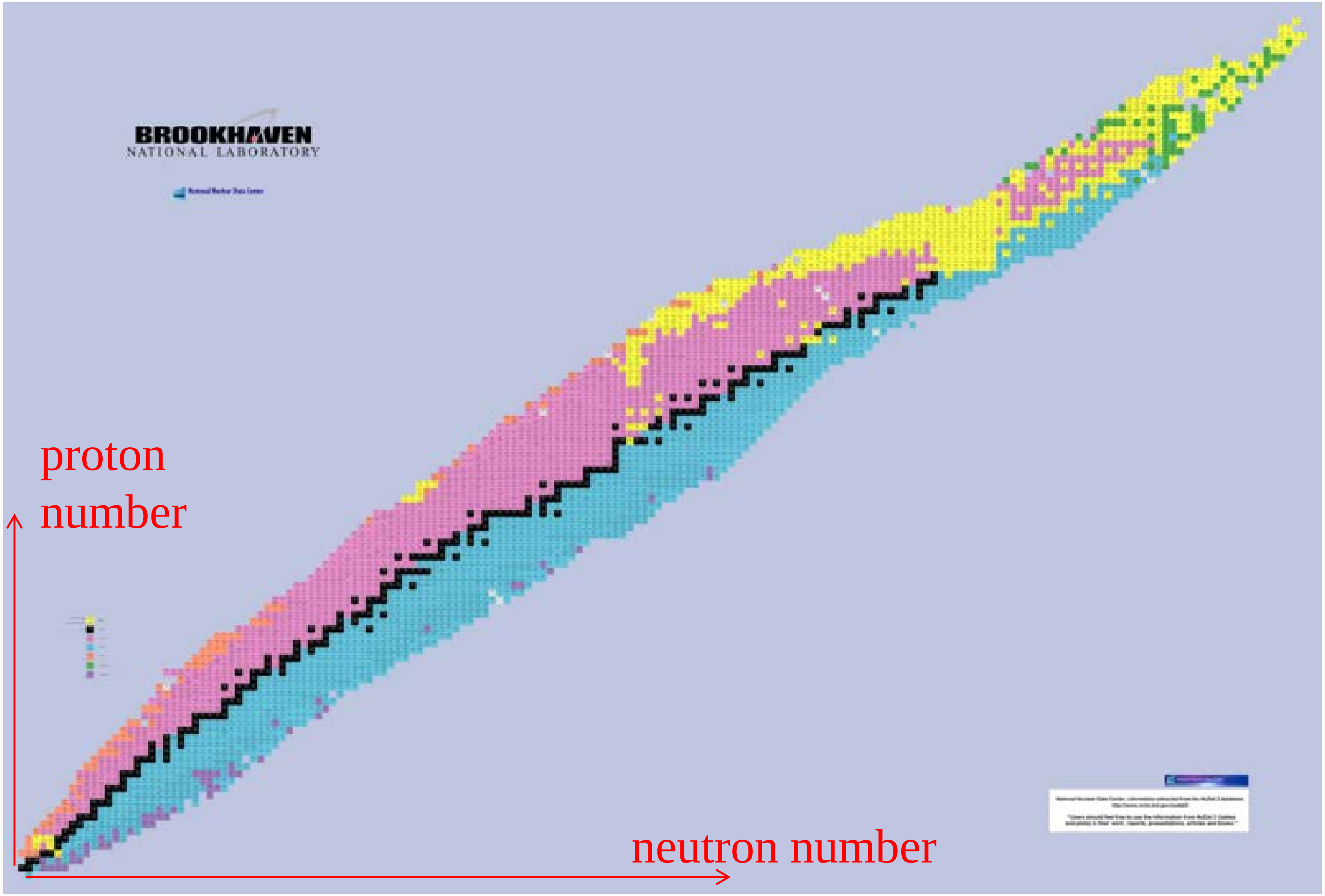








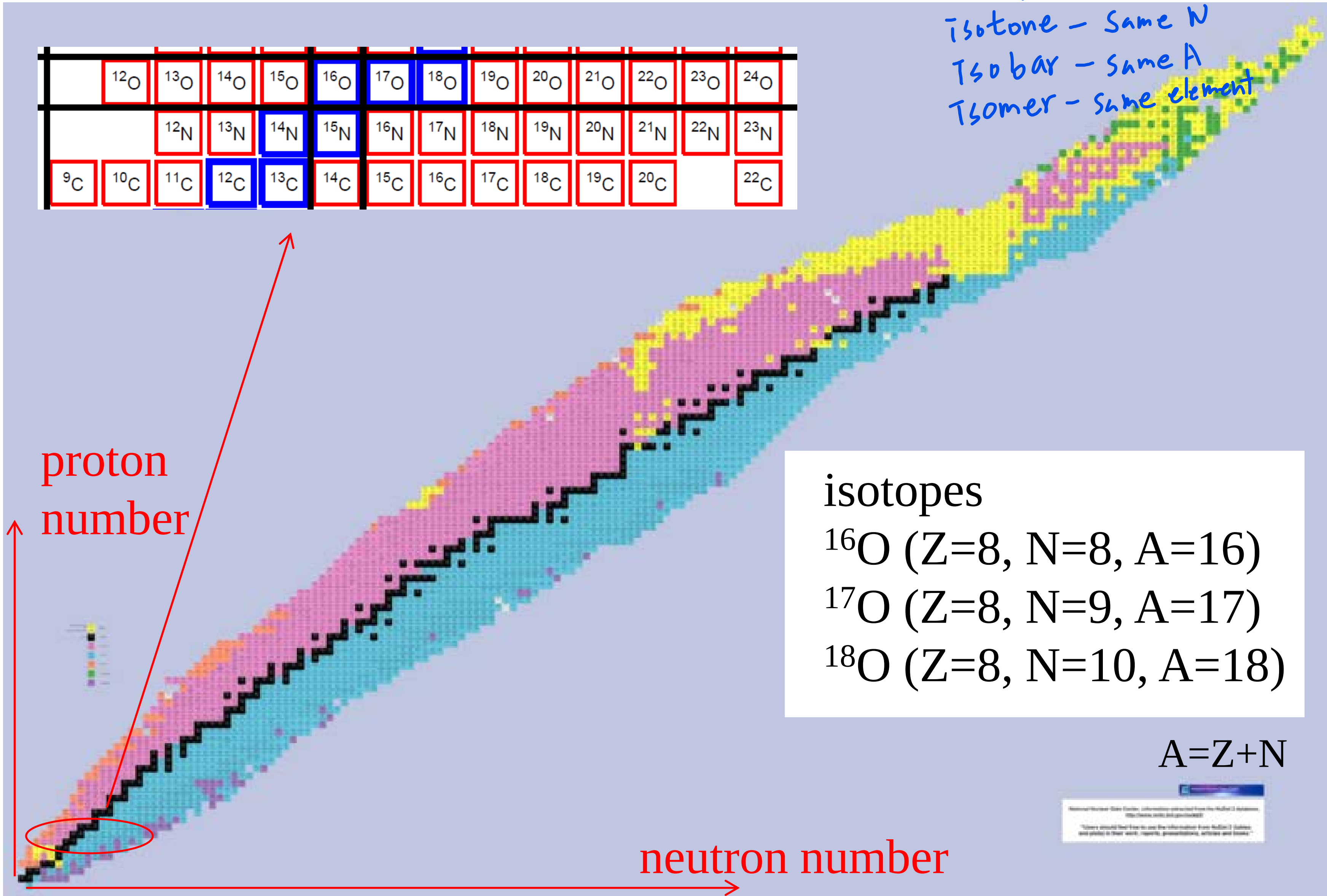
Nuclear Chart: 2D map of atomic nuclei



Periodic table: protons only, no neutrons

Nuclear Chart: 2D map of atomic nuclei

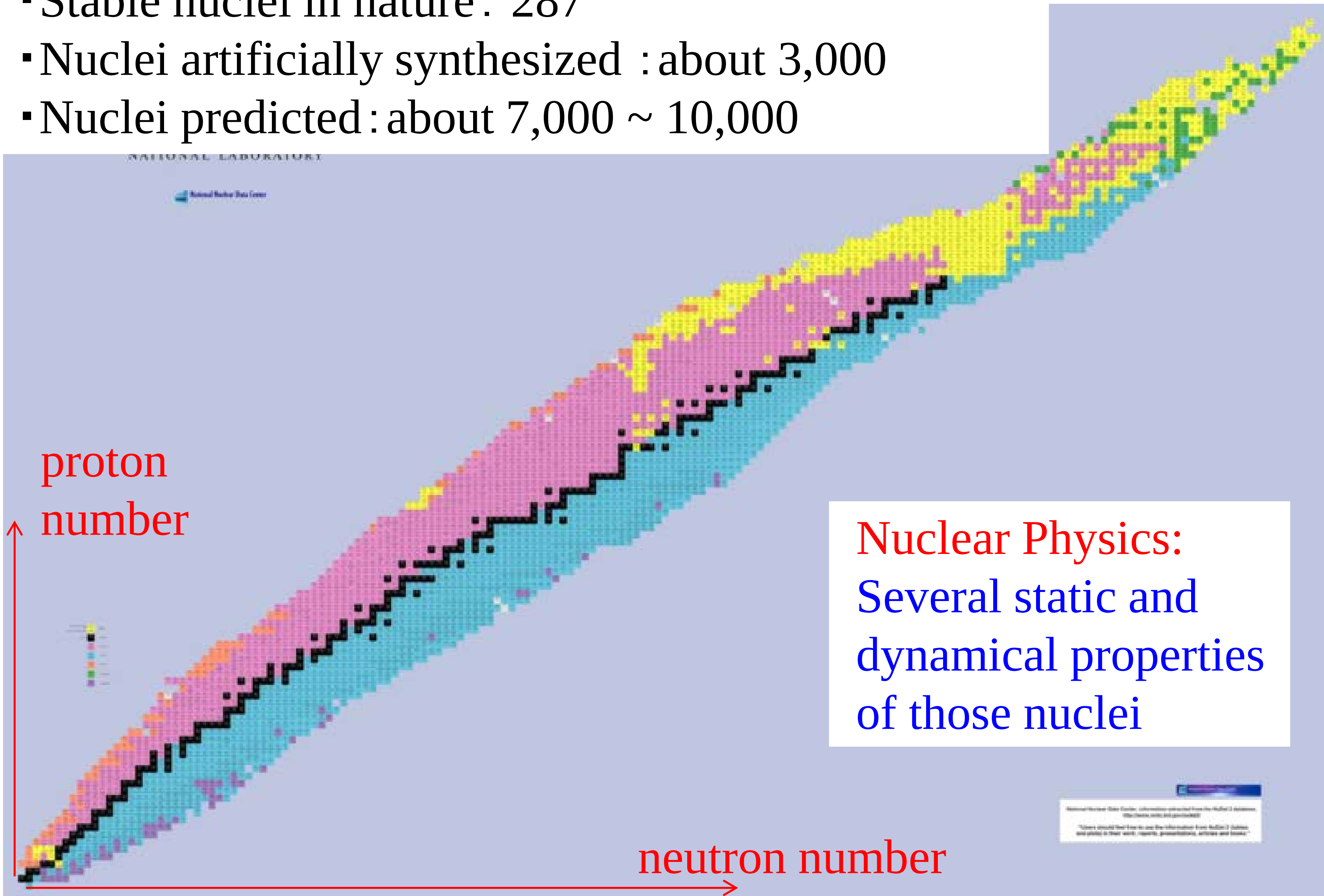
Isotope – same Z
Isotone – same N
Isobar – same A
Isomer – same element



isotopes
^{16}O (Z=8, N=8, A=16)
^{17}O (Z=8, N=9, A=17)
^{18}O (Z=8, N=10, A=18)

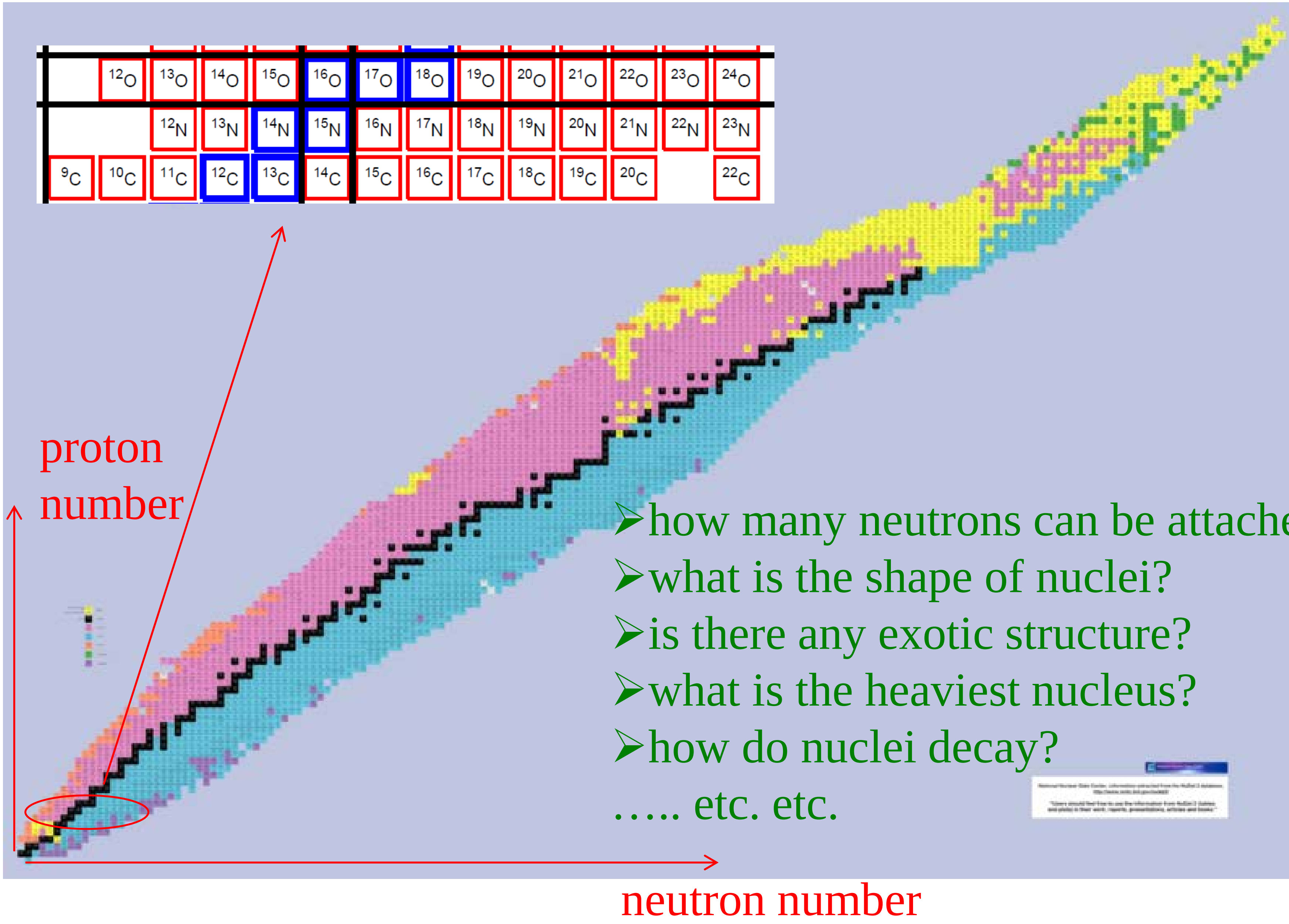
A=Z+N

- Stable nuclei in nature： 287
- Nuclei artificially synthesized ：about 3,000
- Nuclei predicted：about 7,000 ~ 10,000



Nuclear Physics:
Several static and dynamical properties of those nuclei

Nuclear Chart: 2D map of atomic nuclei

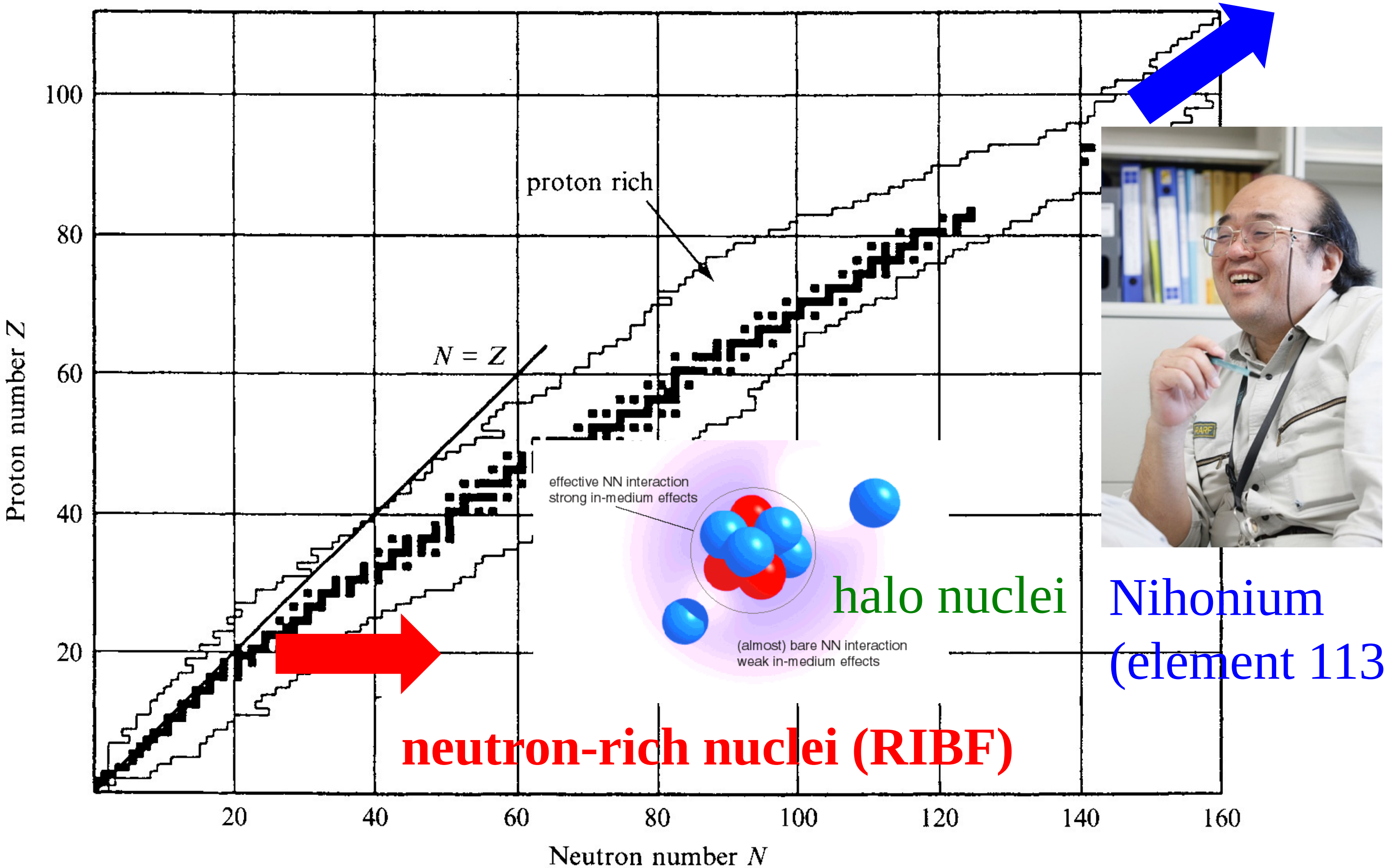


- how many neutrons can be attached?
- what is the shape of nuclei?
- is there any exotic structure?
- what is the heaviest nucleus?
- how do nuclei decay?

….. etc. etc.

Extension of nuclear chart: frontier of nuclear physics

superheavy elements

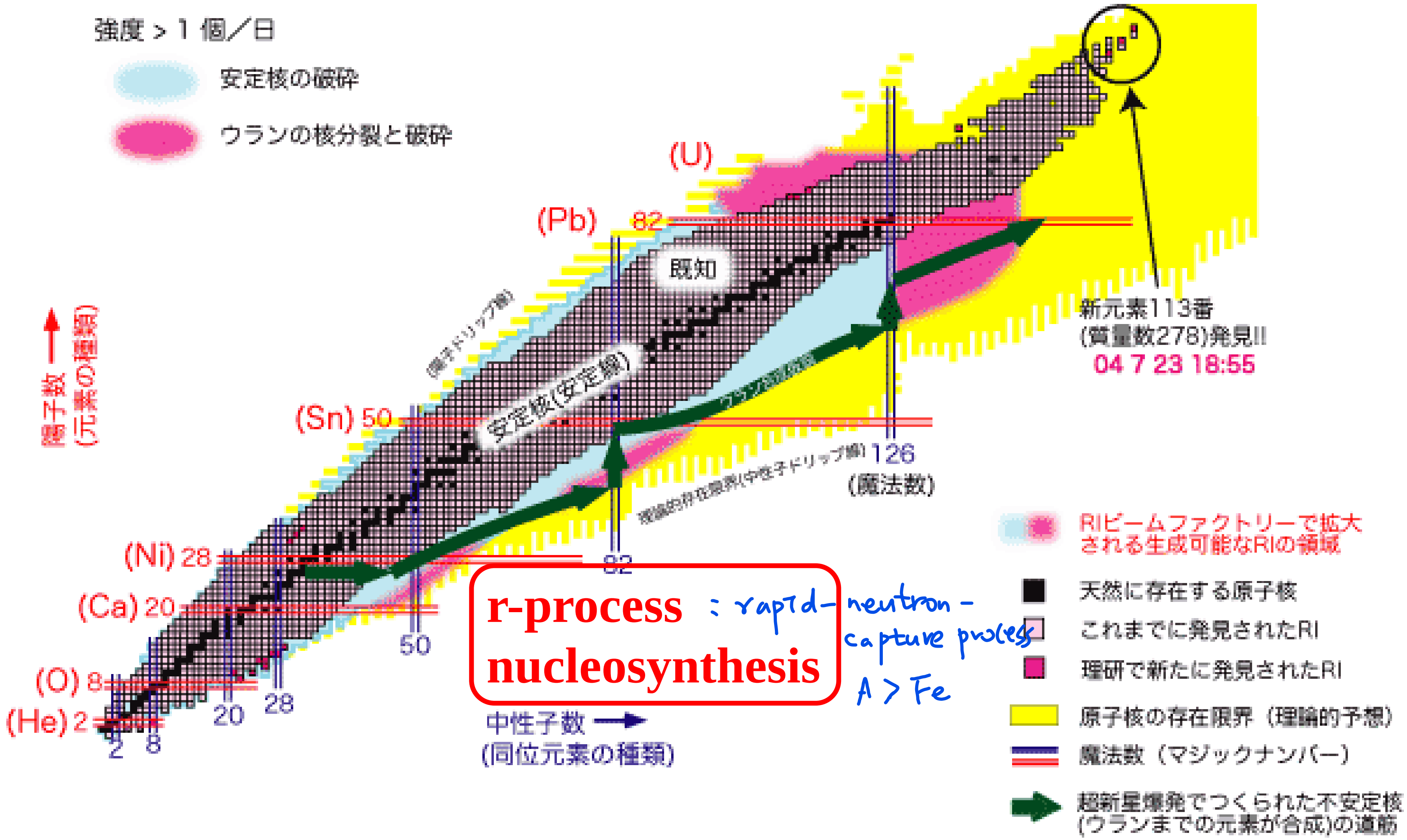


halo nuclei

Nihonium
(element 113)

neutron-rich nuclei (RIBF)

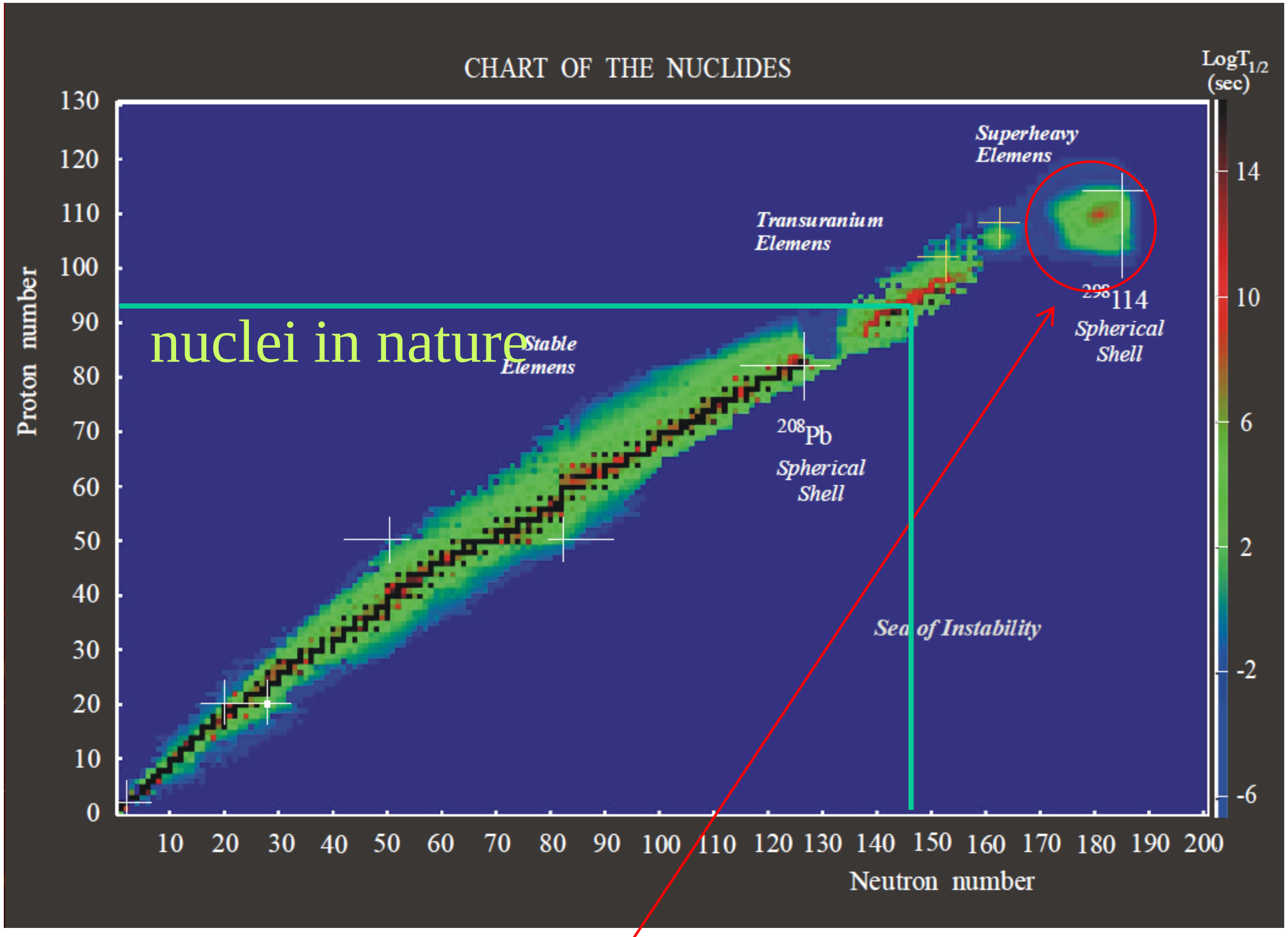
Neutron-rich nuclei (RIBF at RIKEN)



r-process nucleosynthesis : rapid-neutron-capture process

A > Fe

Prediction of island of stability: an important motivation of SHE study



Yuri Oganessian

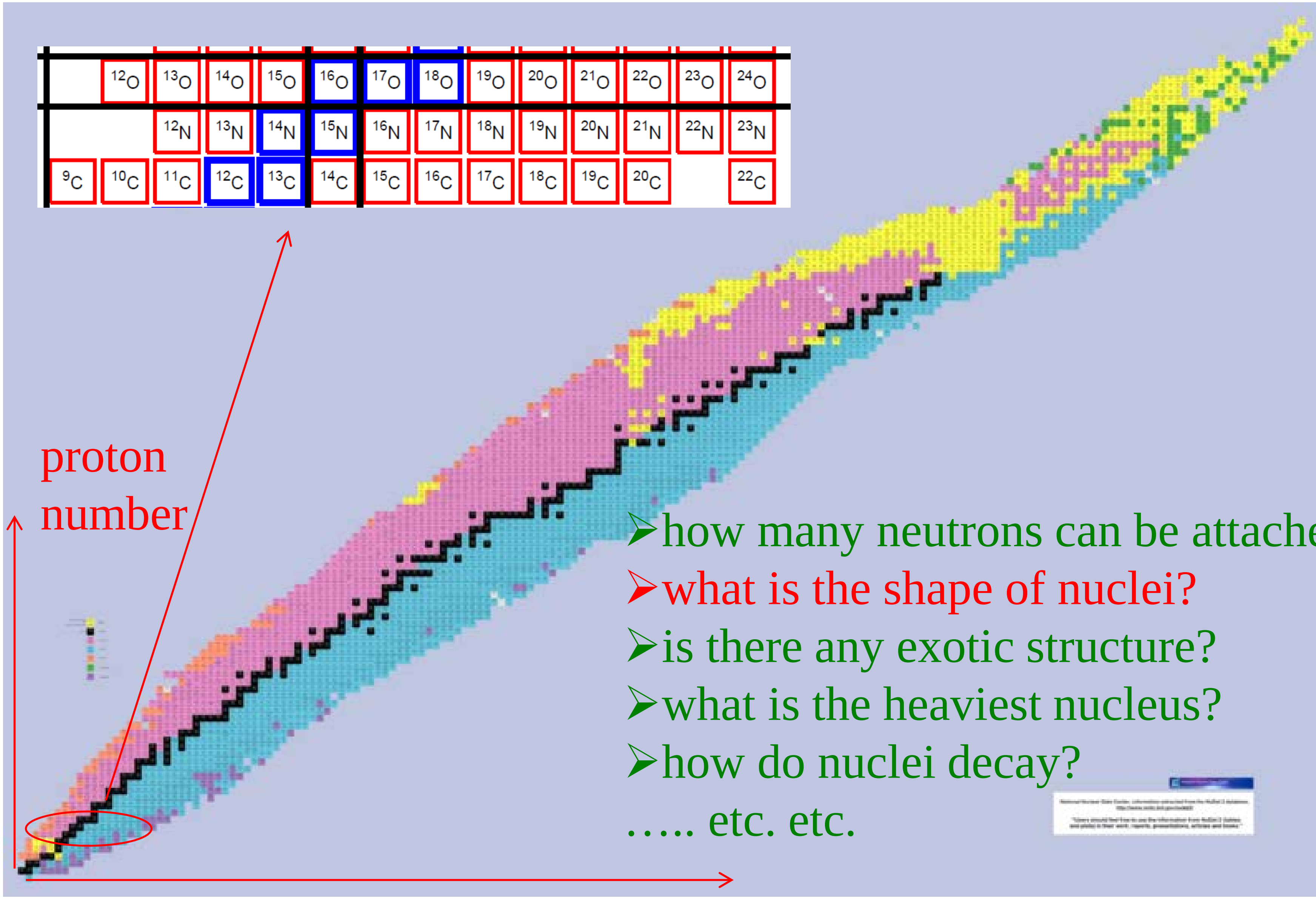
island of stability around Z=114, N=184

W.D. Myers and W.J. Swiatecki (1966), A. Sobiczewski et al. (1966)

⟶ modern calculations: Z=114,120, or 126, N=184

e.g., H. Koura et al. (2005)

Nuclear Chart: 2D map of atomic nuclei



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- what is the shape of nuclei?
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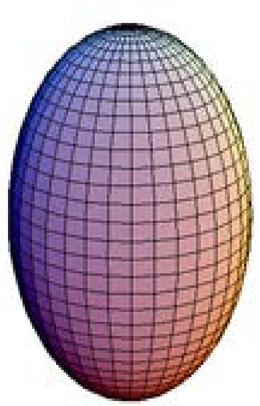
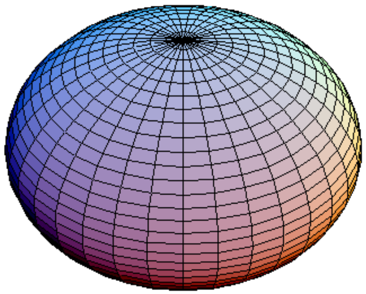
- how do nuclei decay?

….. etc. etc.

a nucleus is not always spherical

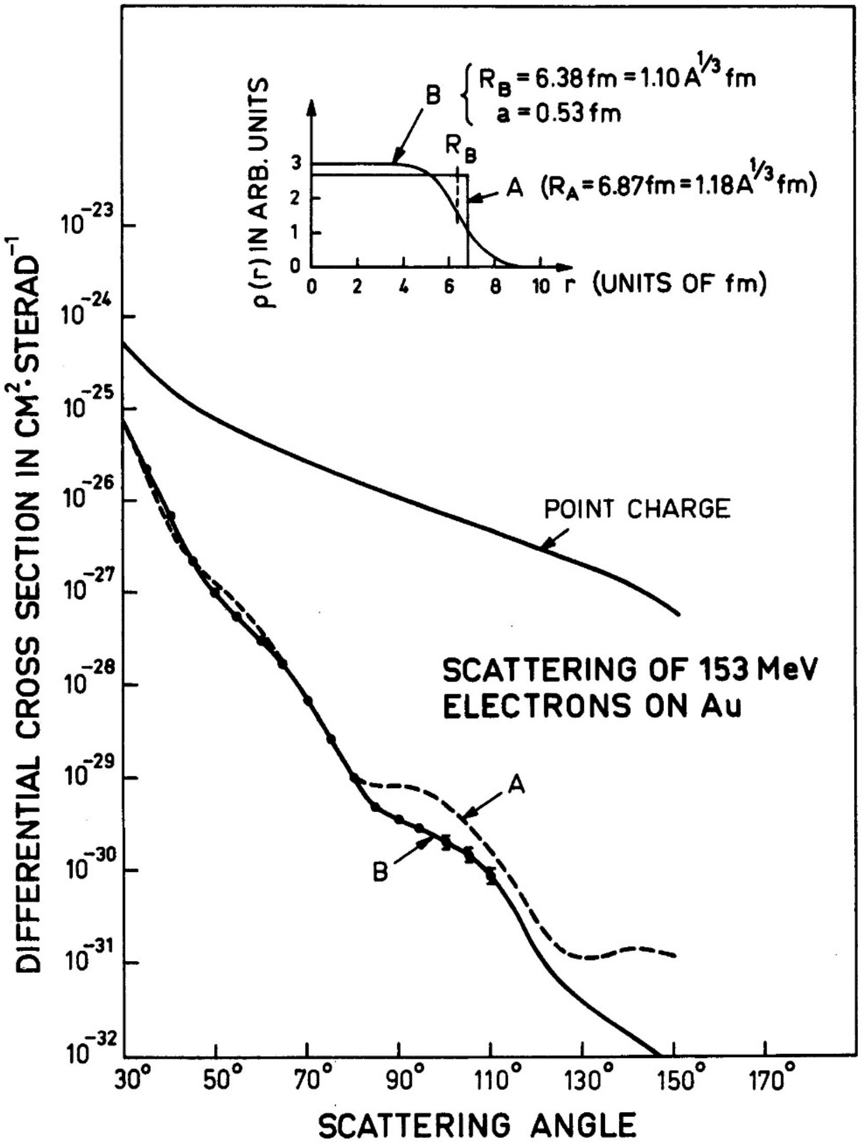




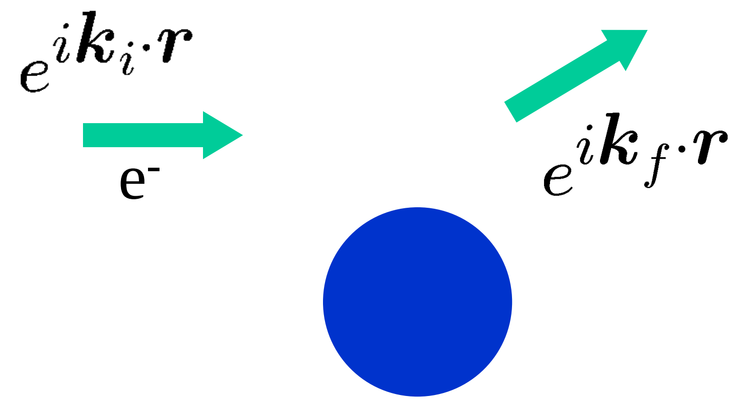
Quantum shape dynamics

Density Distribution

High energy electron scattering



Born approximation:



$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2}\,|F(\boldsymbol{q})|^2$$

Form factor

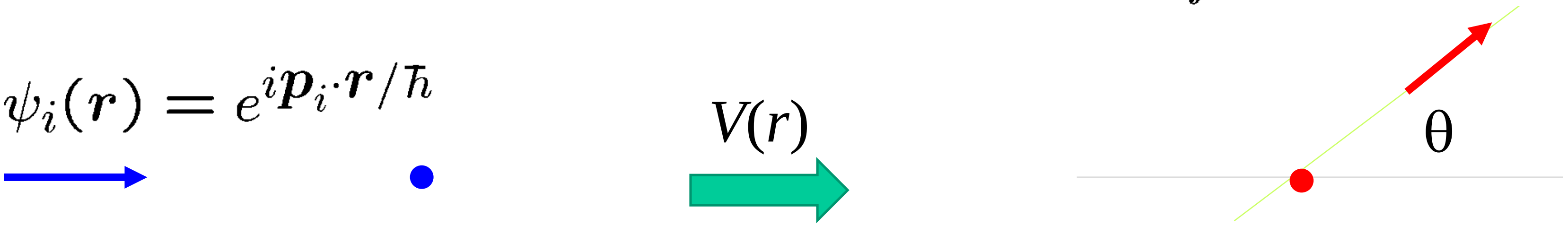
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho(\boldsymbol{r})\,d\boldsymbol{r}$$

(Fourier transform of the density)

Born approximation

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

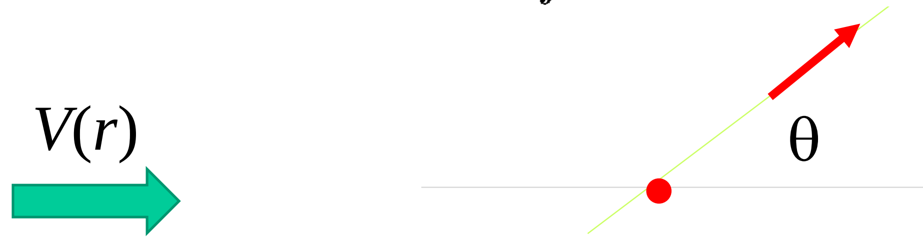
$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$



Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$



$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2$$

momentum transfer

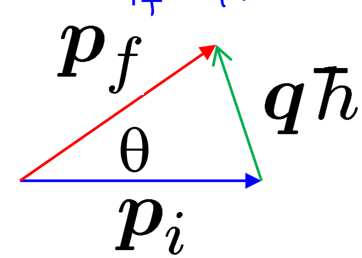
$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar} V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)$$

incident flux: $j_{\rm inc} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\rm inc}} = \int d\Omega \frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2$$

$$= \frac{d\sigma}{d\Omega}$$

If, $\vec{p}_f = \vec{p}_i$



$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_P^2 e^4}{(4E\sin^2\theta/2)^2} |F(\boldsymbol{q})|^2$$

$$= \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right) |F(\boldsymbol{q})|^2$$

Form factor

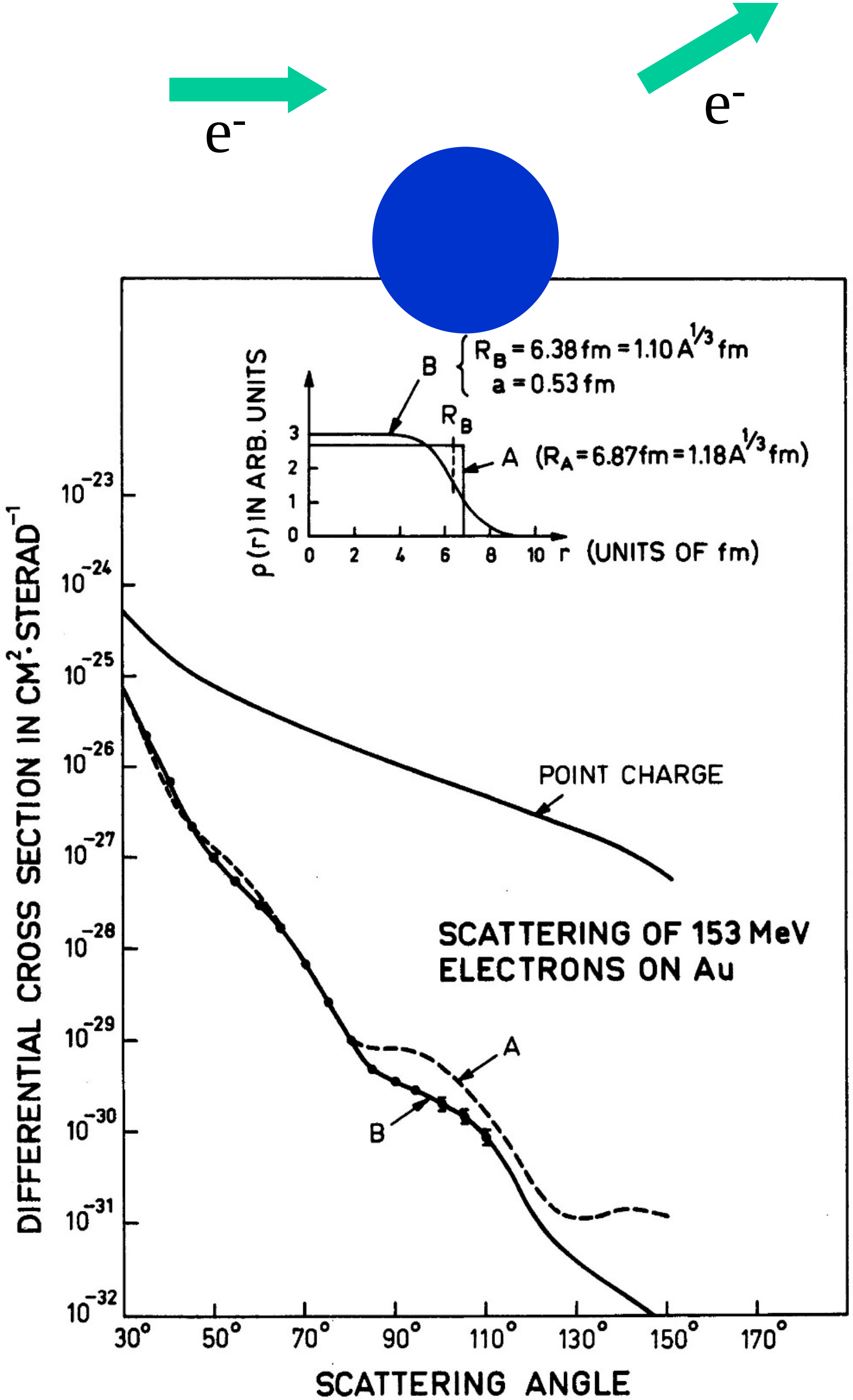
$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})\, d\boldsymbol{r}$$

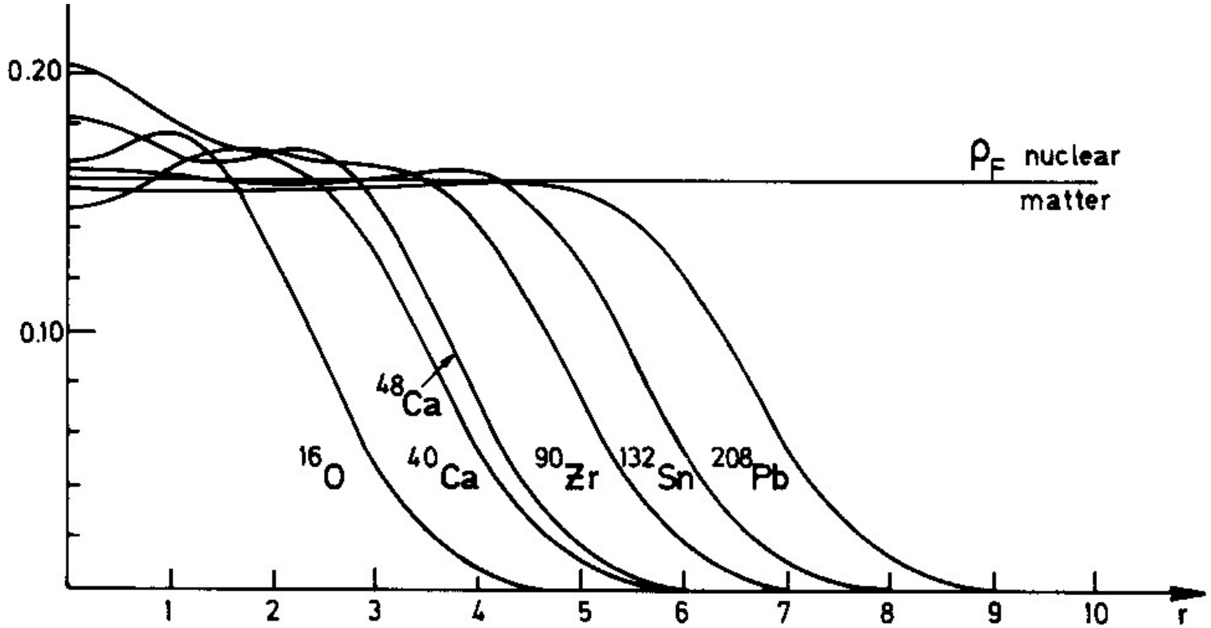
* relativistic correction:

$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega}$$

$$= \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right)$$

$$\sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2\frac{\theta}{2} \qquad (v \rightarrow c)$$





Fermi distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R_0)/a]}$$

$$\rho_0 \sim 0.17 \ (\mathrm{fm}^{-3})$$

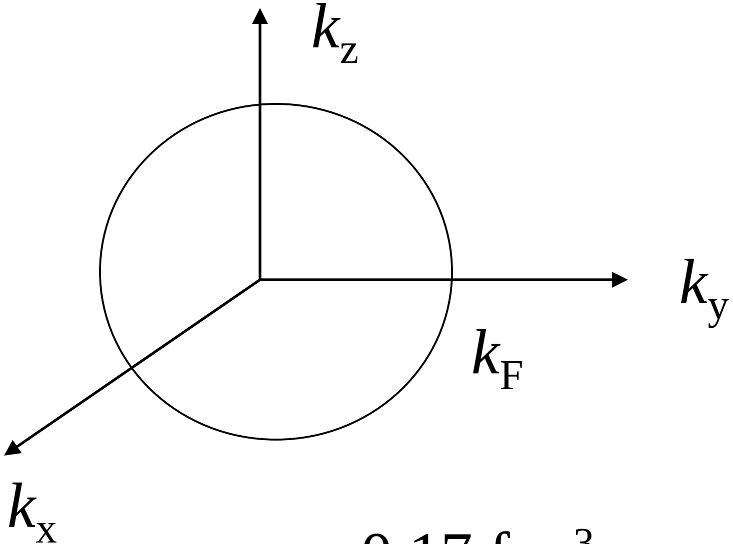
$$R_0 \sim 1.1 \times A^{1/3} \ (\mathrm{fm})$$

$$a \sim 0.57 \ (\mathrm{fm})$$

Saturation property

Momentum Distribution

Fermi gas approximation



$$\rho = 2 \times 2 \times 4\pi \int_0^{k_F} \frac{k^2 dk}{(2\pi)^3}$$

↑↓

$$= \frac{2}{3\pi^2} k_F^3$$

(note: spin-isospin degeneracy)

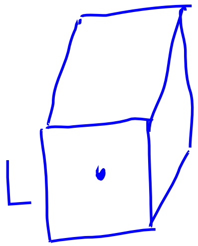
ρ = 0.17 fm^{-3} ⟶ $k_F \sim 1.36$ fm^{-1}

$p = \hbar k = mv$

$$\Longleftrightarrow \quad \frac{v_F}{c} = \frac{k_F \cdot \hbar c}{mc^2} = 0.285$$

$= p^2$

Fermi energy: $\epsilon_F = \frac{k_F^2 \hbar^2}{2m} \sim 37$ (MeV)



free particle $\psi(\vec{r}) = \frac{1}{\sqrt{L^3}} e^{i\vec{k}\cdot\vec{r}}$

by B. C $\quad k_x = \frac{2\pi}{L} n_x, \quad k_y = \frac{2\pi}{L} n_y, \quad k_z = \frac{2\pi}{L} n_z$

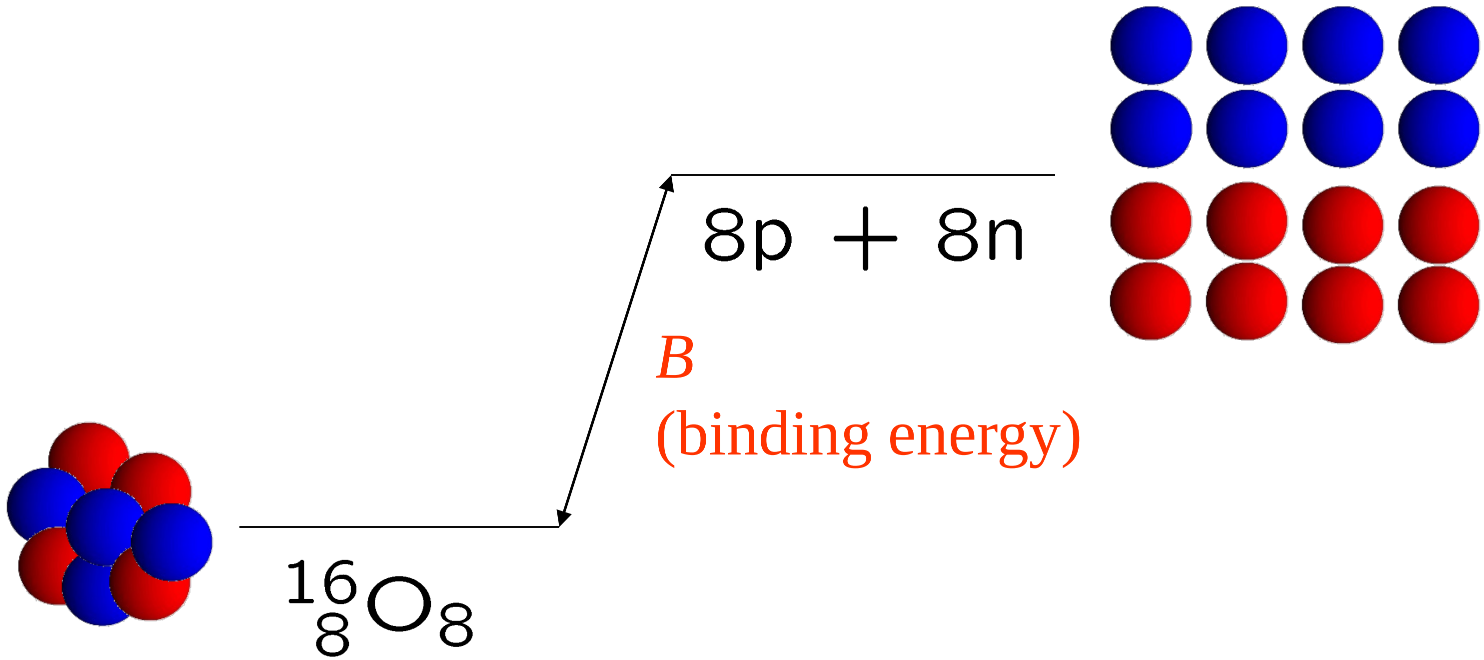
$$n_x = \frac{L}{2\pi} k_x$$

$$dn = \left(\frac{L}{2\pi}\right)^3 d^3k$$

$$A = \int_0^{\varepsilon_F} dn$$

\# of state : $\rho = \frac{A}{L^3} = \int_0^{k_F} \frac{d^3k}{(2\pi)^3} = 4\pi \int_0^{k_F} \frac{k^2\,dk}{(2\pi)^3}$

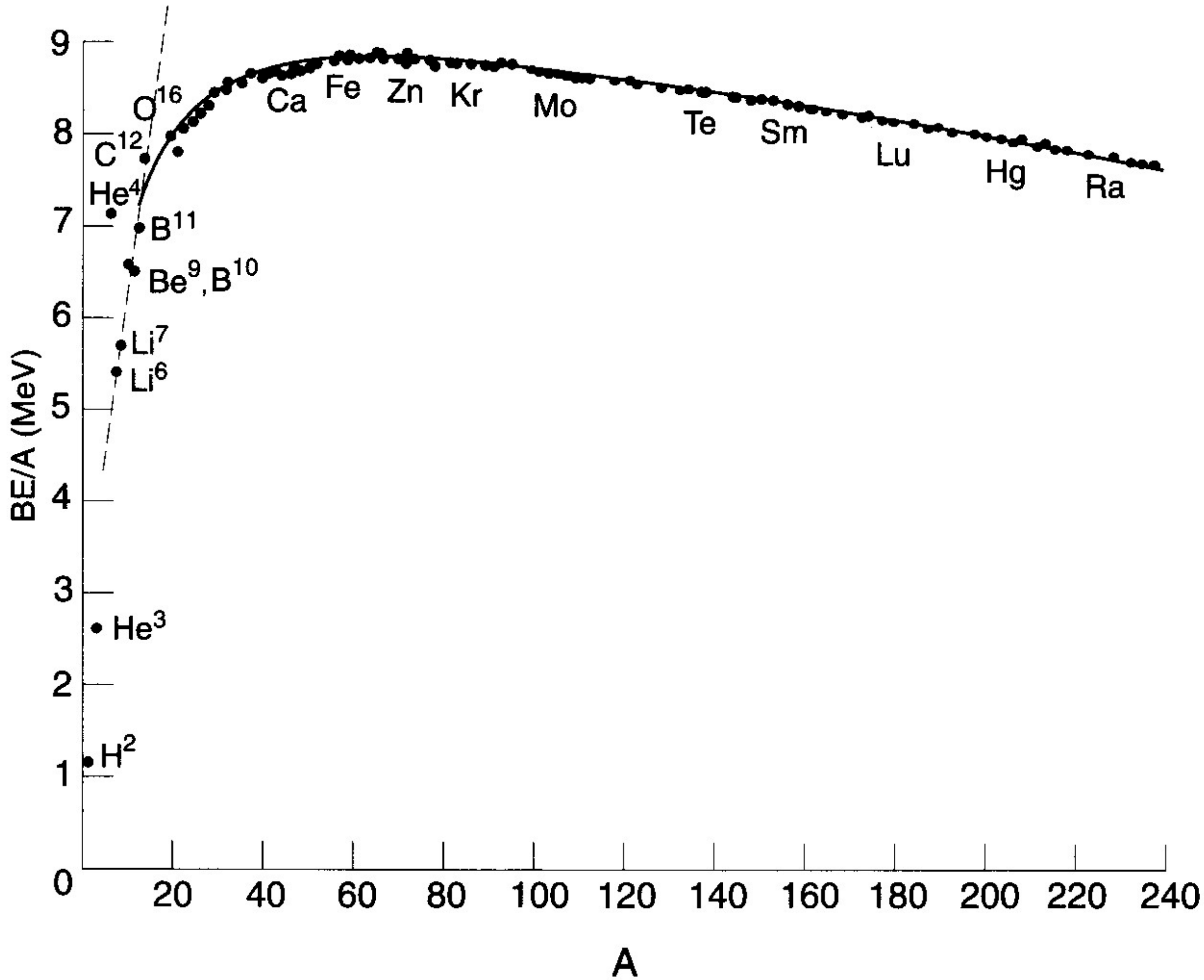
Nuclear Mass



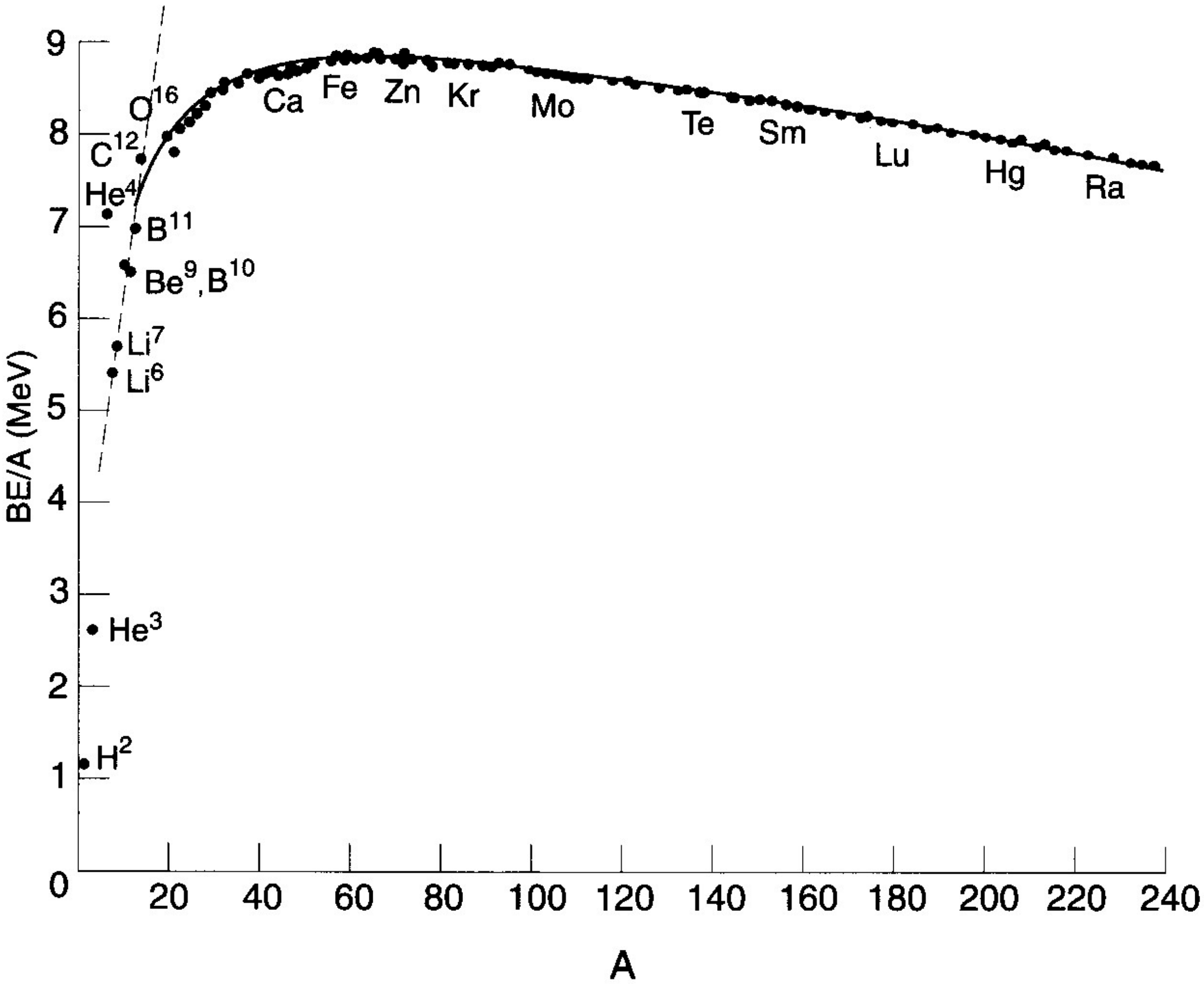
$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$

↑ binding E

$B > 0$



BE/A (MeV)
A
9
8
7
6
5
4
3
2
1
0
20
40
60
80
100
120
140
160
180
200
220
240
O^{16}
C^{12}
He^4
B^{11}
Be^9, B^{10}
Li^7
Li^6
He^3
H^2
Ca
Fe
Zn
Kr
Mo
Te
Sm
Lu
Hg
Ra



1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force

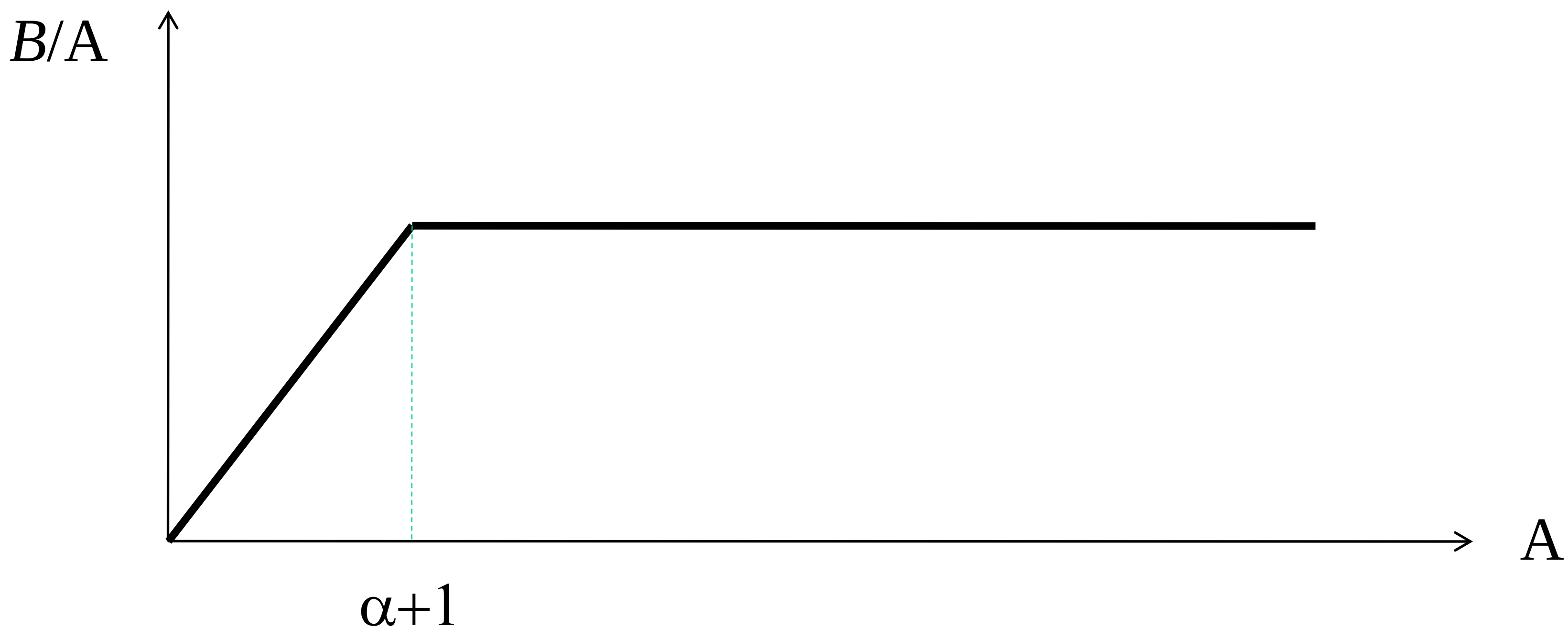
핵자는 주위의 몇몇 이웃하고만 상호작용함

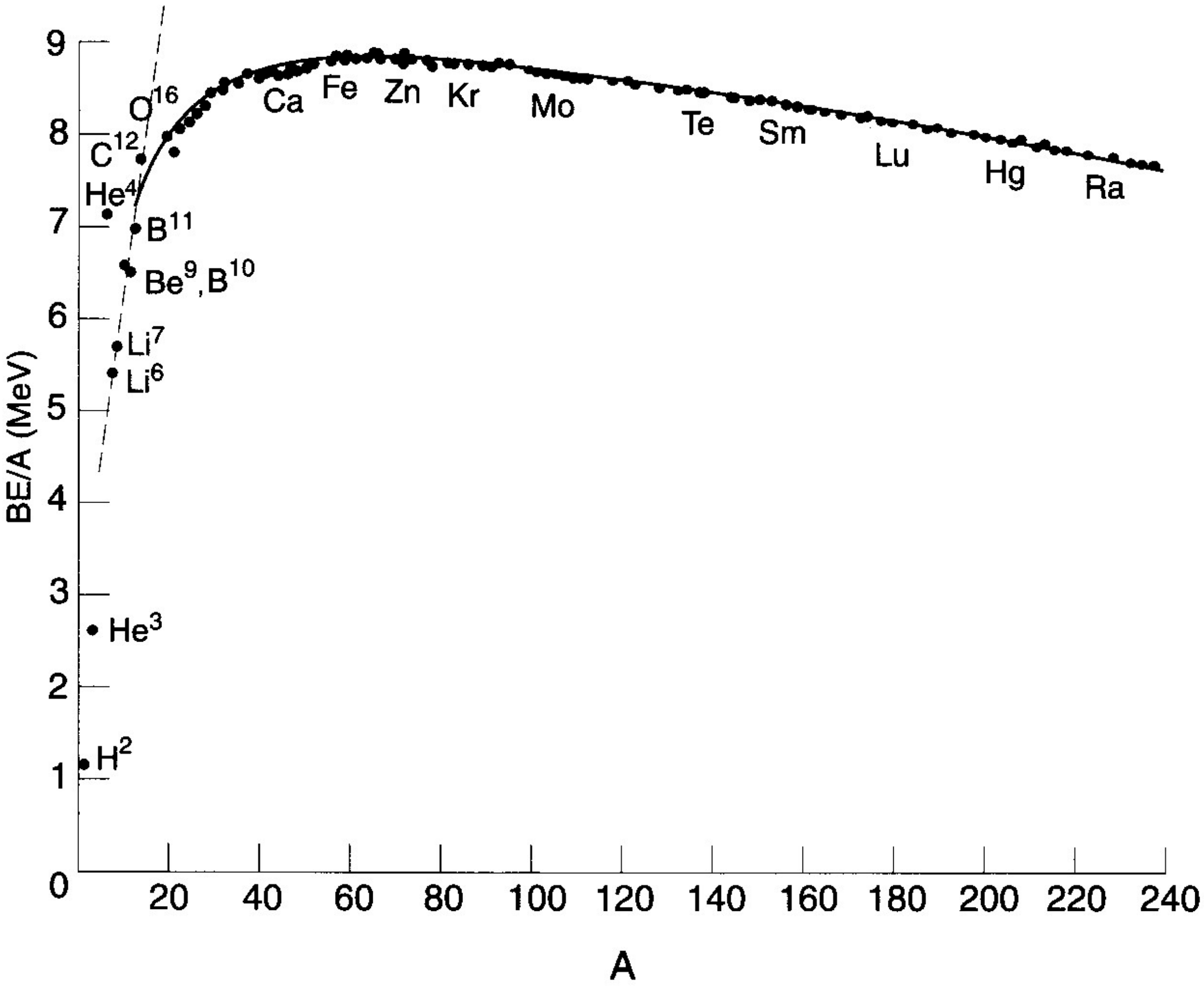
If one nucleon interacts only with surrounding α nucleons

$B \sim \alpha A/2 \longrightarrow B/A \sim \alpha/2$ (const.)

For $A < \alpha+1$, one nucleon interacts with all the other nucleons

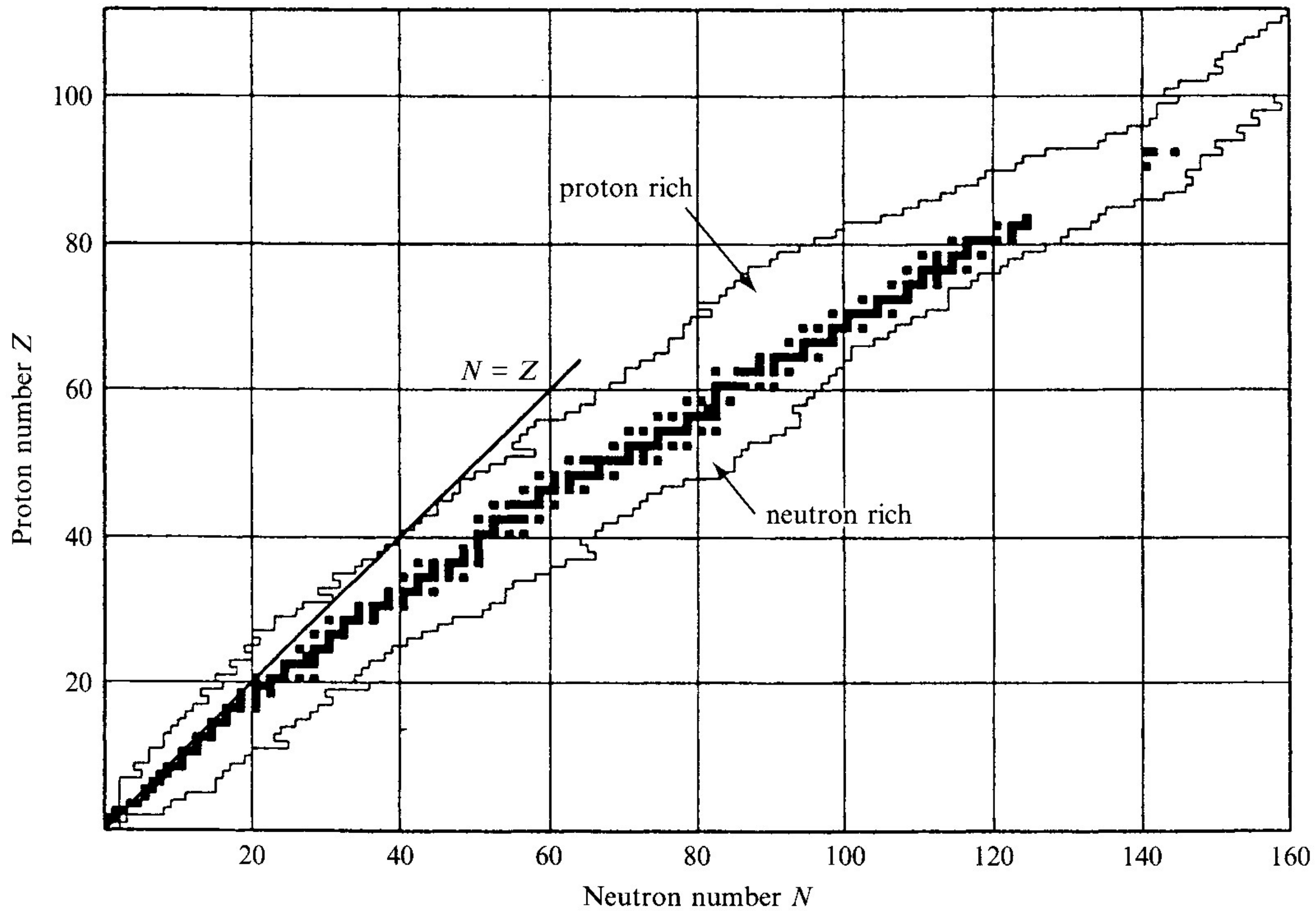
$$\longrightarrow B/A \propto A$$



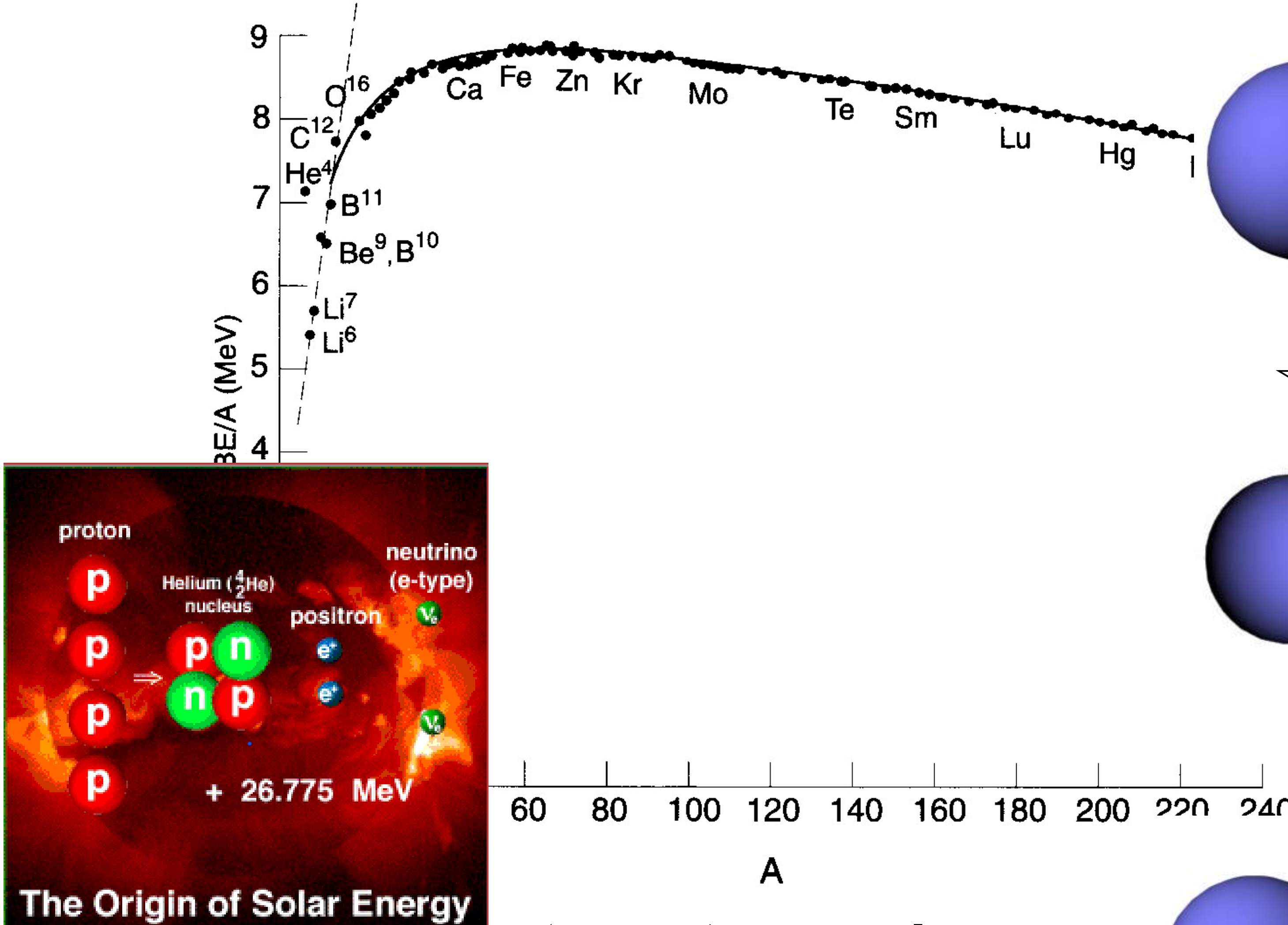


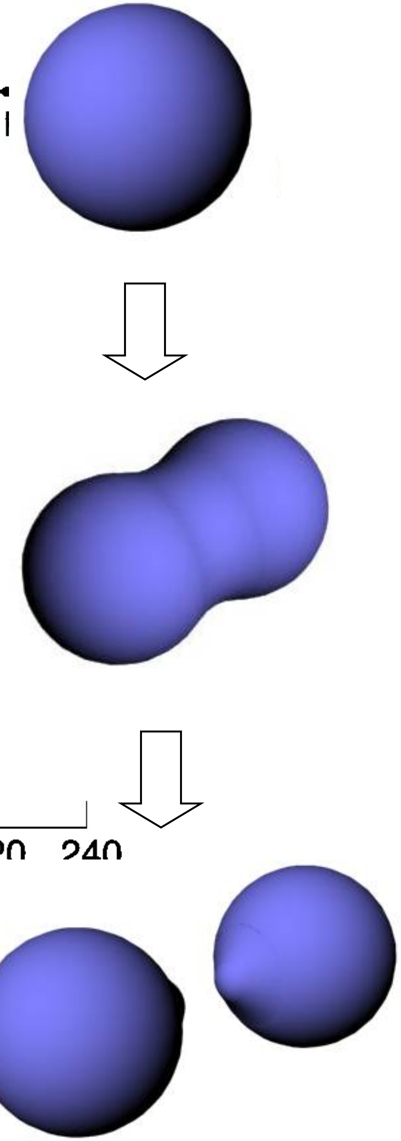
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$





1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

Semi-empirical mass formula

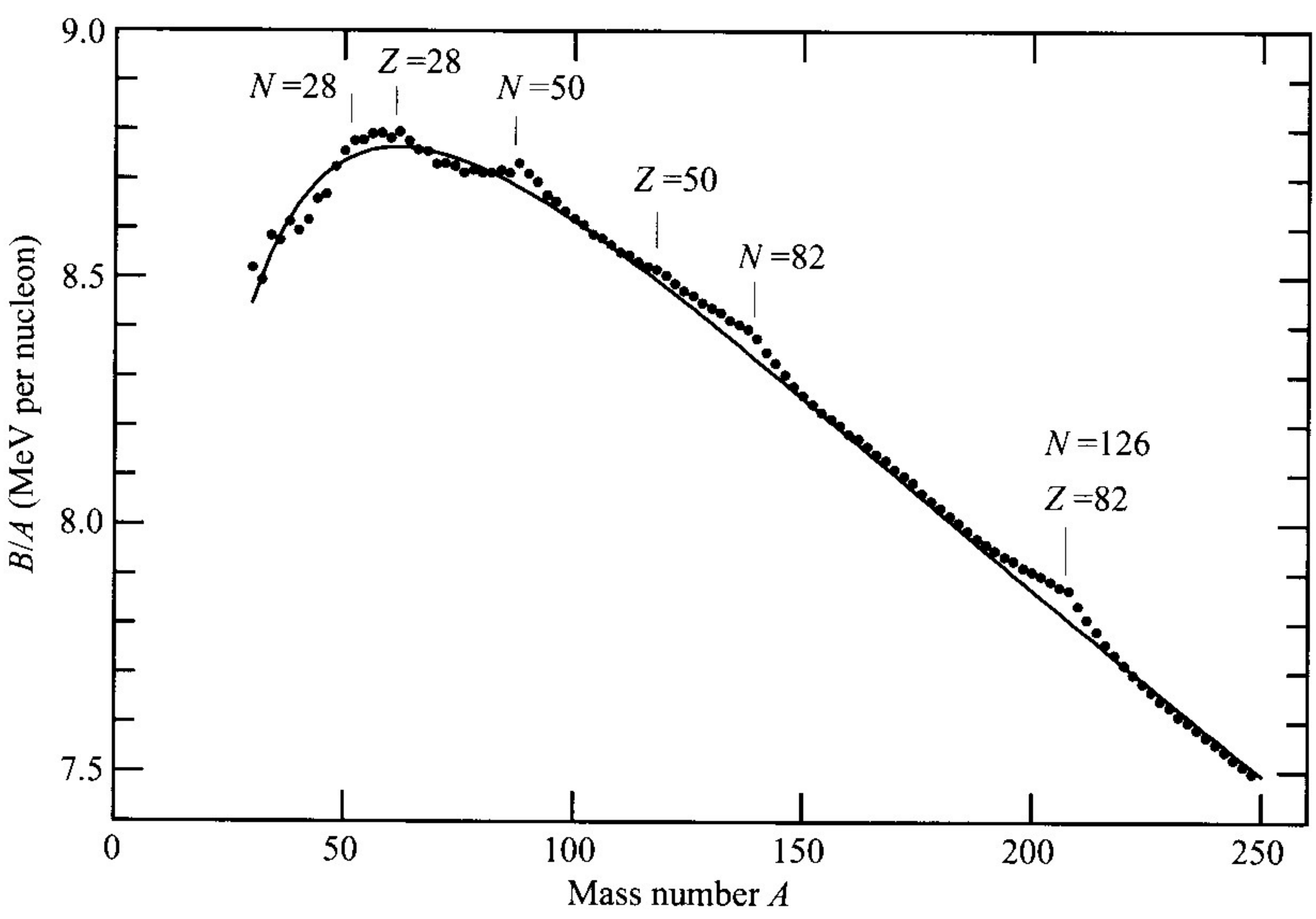
$R = r_0 A^{\frac{1}{3}}$

(Bethe-Weizacker formula: Liquid-drop model)

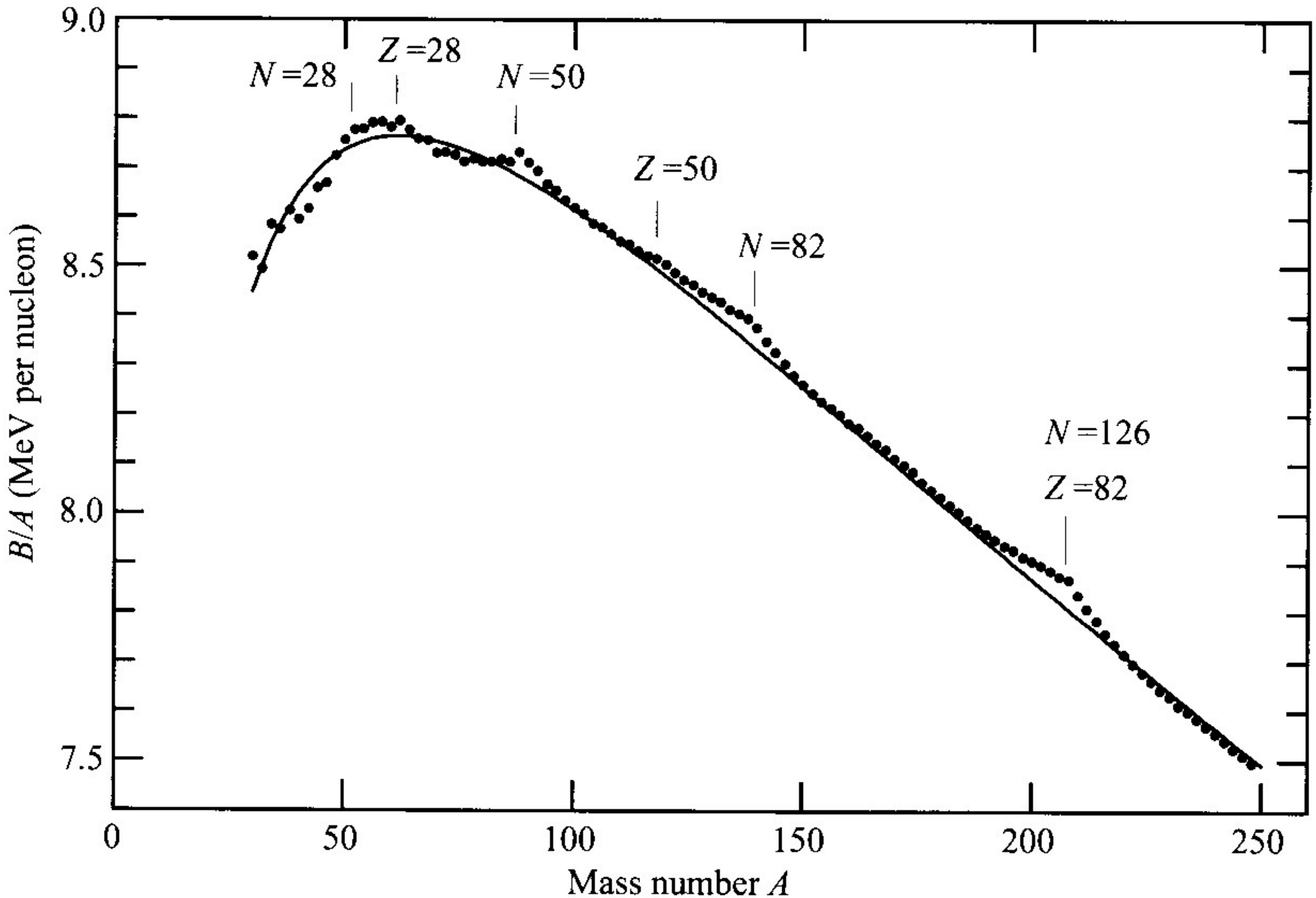
$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

- Volume energy: $a_v A$
- Surface energy: $-a_s A^{2/3}$
- Coulomb energy: $-a_C Z^2 / A^{1/3}$
- Symmetry energy: $-a_{\text{sym}} (N-Z)^2 / A$

How well does the Bethe-Weizacker formula reproduce the data?



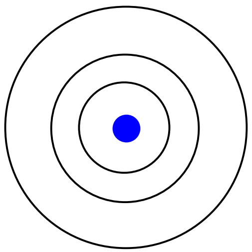
How well does the Bethe-Weizacker formula reproduce the data?



cf. N,Z = 2, 8, 20, 28, 50, 82, 126: large binding energy
“magic numbers”

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)



shell structure