(Bethe-Weizacker formula: Liquid-drop model)

Q1

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

Fermi gas model : (N-Z)에 대하여 전개

$$E_K = \frac{C}{2^{2/3}}\left(A^{5/3} + \frac{5}{9}\frac{(N-Z)^2}{A^{1/3}}\right) + \mathcal{O}((N-Z)^4) + \cdots$$

* $(N-Z)^1, (N-Z)^3, \cdots$ 홀수 지수항은 상쇄

$N-Z \Leftrightarrow Z-N$

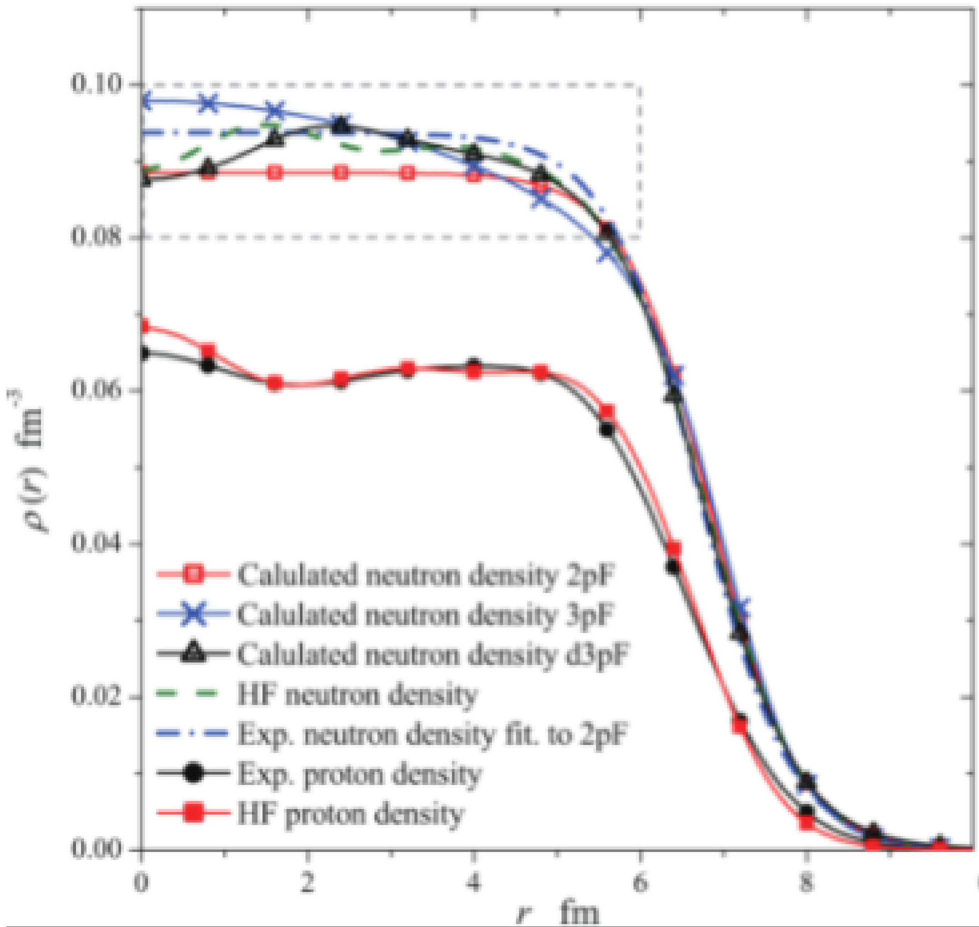
$$E_K \simeq \frac{C A^{2/3}}{2^{2/3}}\left(A^{-2/3} A^{5/3} + \frac{5}{9} A^{-2/3} \frac{(N-Z)^2}{A^{1/3}}\right)$$

$$= \left(\frac{C A^{2/3}}{2^{2/3}}\right) A + \left(\frac{5}{9}\frac{C A^{2/3}}{2^{2/3}}\right)\frac{(N-Z)^2}{A^{2/3} \cdot A^{1/3}}$$

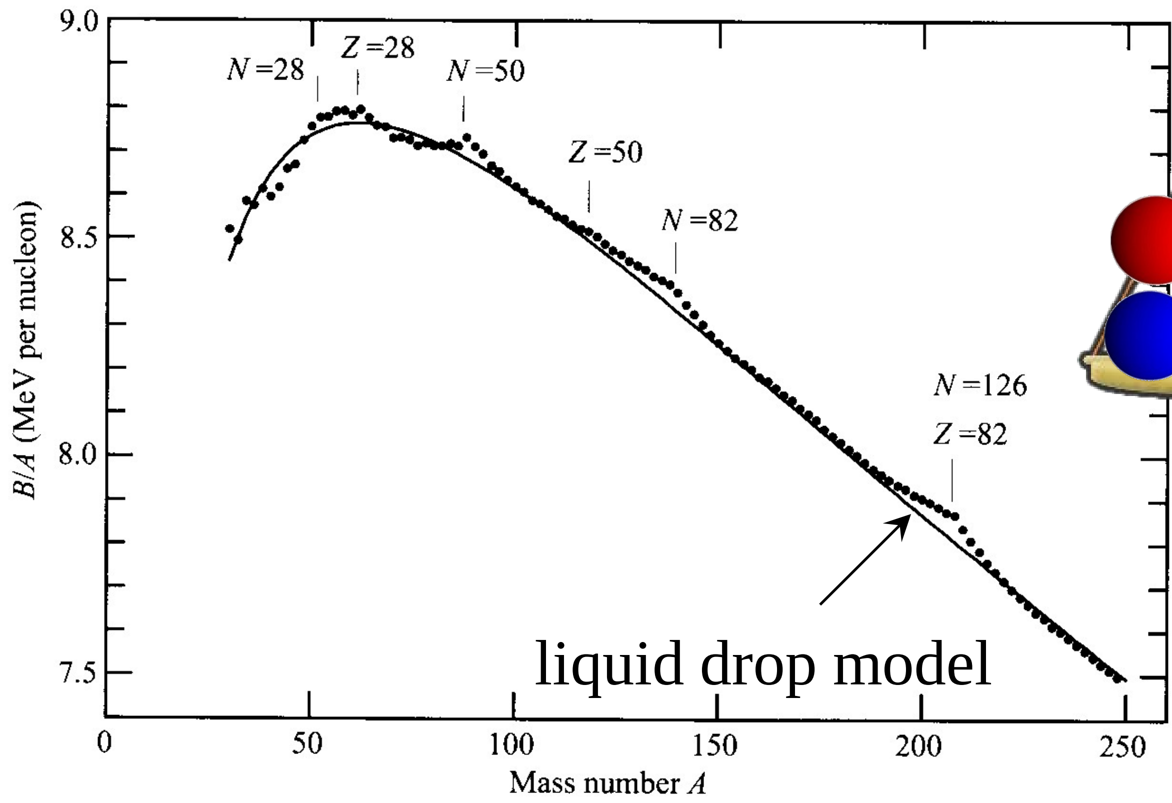
$$= a_v A + a_{sym} \frac{(N-Z)^2}{A}$$

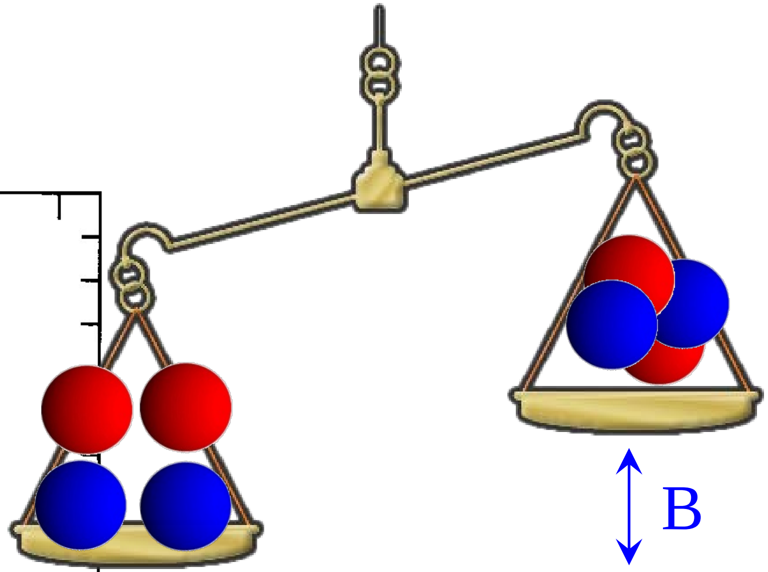
Q2

^{208}Pb


0.10
0.08
0.06
0.04
0.02
0.00
0
2
4
6
8
ρ(r) fm⁻³
r fm
Calulated neutron density 2pF
Calulated neutron density 3pF
Calulated neutron density d3pF
HF neutron density
Exp. neutron density fit. to 2pF
Exp. proton density
HF proton density

Shell Energy

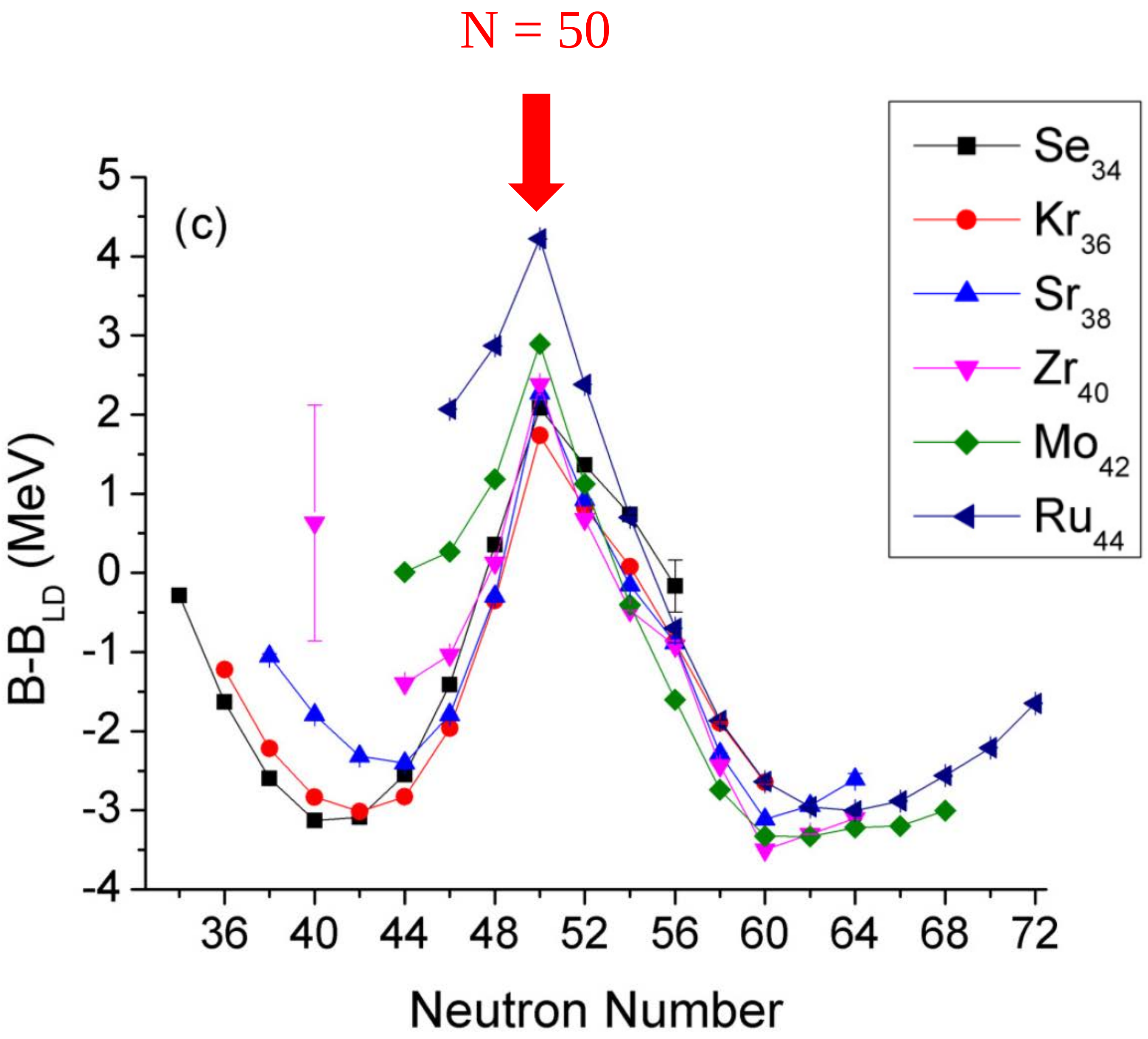




Extra binding for N or Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

⇨ Very stable

${}^{4}_{2}He_{2}, {}^{16}_{8}O_{8}, {}^{40}_{20}Ca_{20}, {}^{48}_{20}Ca_{28}, {}^{208}_{82}Pb_{126}$

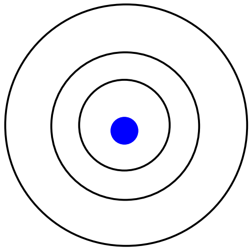


I. Bentley et al., PRC93 ('16) 044337

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

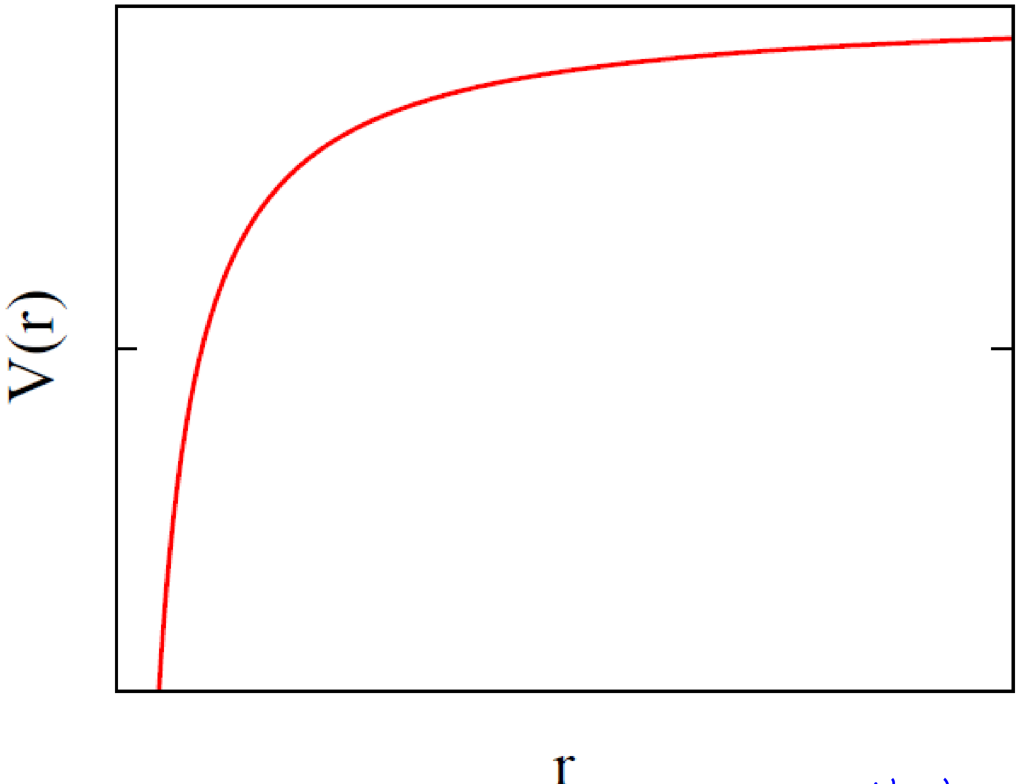
interpretation:



shell structure

[magic numbers for electrons]

Hydrogen-like potential: $V(r) = -\frac{Ze^2}{r}$



$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$n = n_r + l + 1$$

pricipal q.n — radial q.n — obital q.n

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}u(r) + \left[\frac{l(l+1)\hbar^2}{2mr^2} - \frac{Ze^2}{r}\right]u(r) = E u(r)$$

$$R_{nl} = \frac{u_{nl}(r)}{r}$$

magic numbers for electrons

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

3S 3P 3D

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

2S 2P

$$n = n_r + l + 1$$

1S

magic numbers for electrons

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)

(spin x l_z)

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

3S [2]   3P [6]   3D [10]

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

2S [2]   2P [6]

$$n = n_r + l + 1$$

1S [2]

magic numbers for electrons

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)

(spin x l_z)

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

3S [2] 3P [6] 3D [10]

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

2S [2] 2P [6]

$$n = n_r + l + 1$$

He

1S [2]

magic numbers for electrons

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)

(spin x l_z)

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$

3S [2] 3P [6] 3D [10]

2S [2] 2P [6] ⇒ $_{16}$Ne

$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$n = n_r + l + 1$$

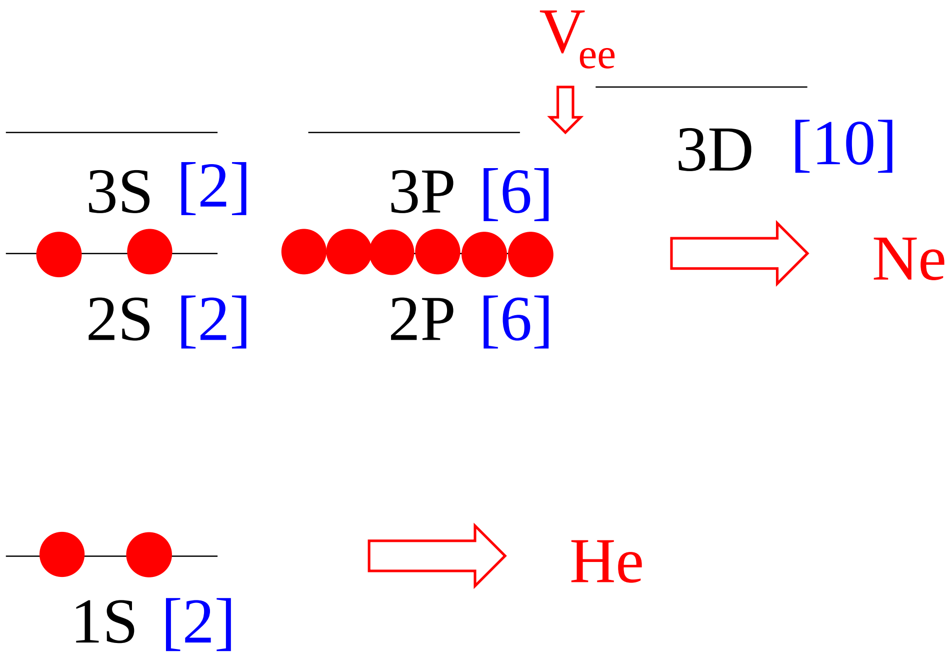
1S [2] ⇒ $_2$He

magic numbers for electrons

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 l +1)

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$



$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

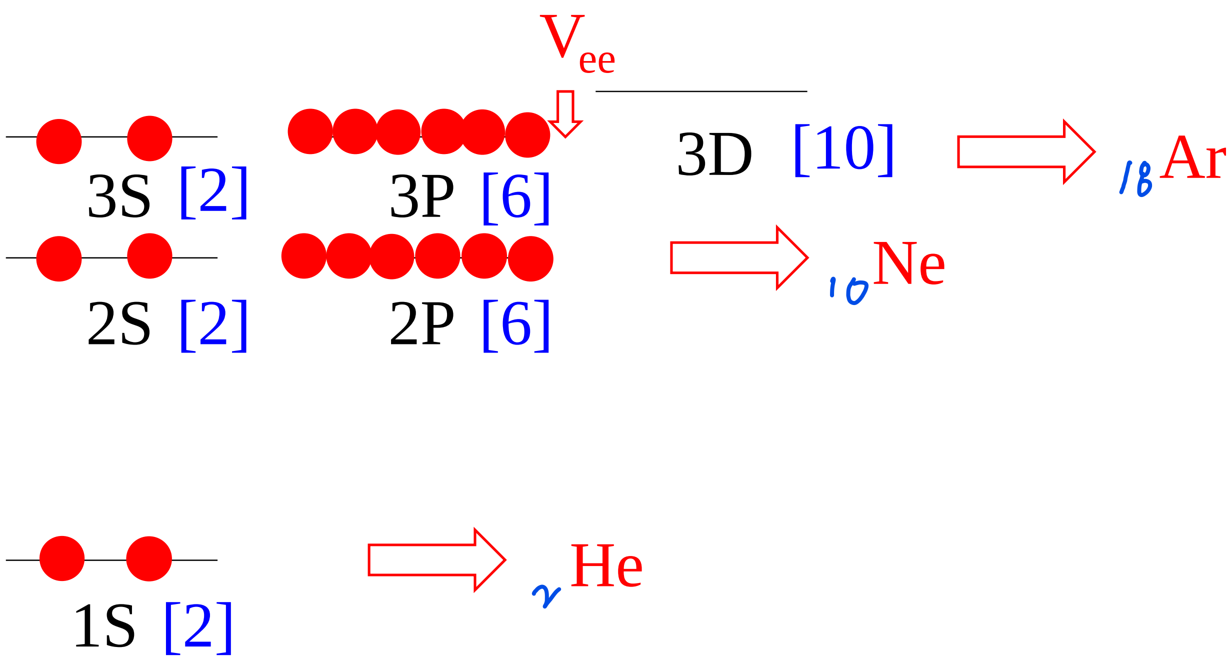
$$n = n_r + l + 1$$

magic numbers for electrons

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 l +1)

$$E_n = -\frac{(Z\alpha)^2}{2n^2} mc^2$$



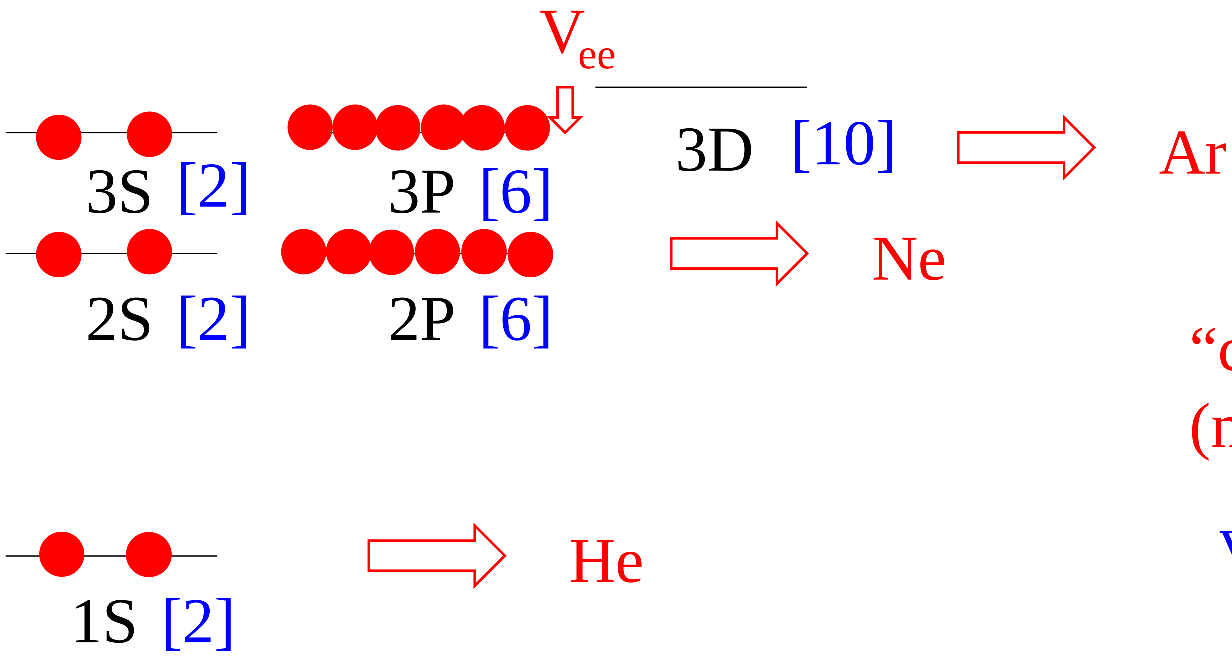
$$\alpha = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$n = n_r + l + 1$$

magic numbers for electrons

Hydrogen-like potential: $$V(r) = -\frac{Ze^2}{r}$$

degeneracy = 2 * (2 *l* +1)

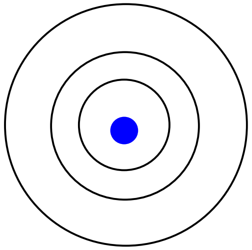


"closed shell"
(magic numbers)

very stable

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

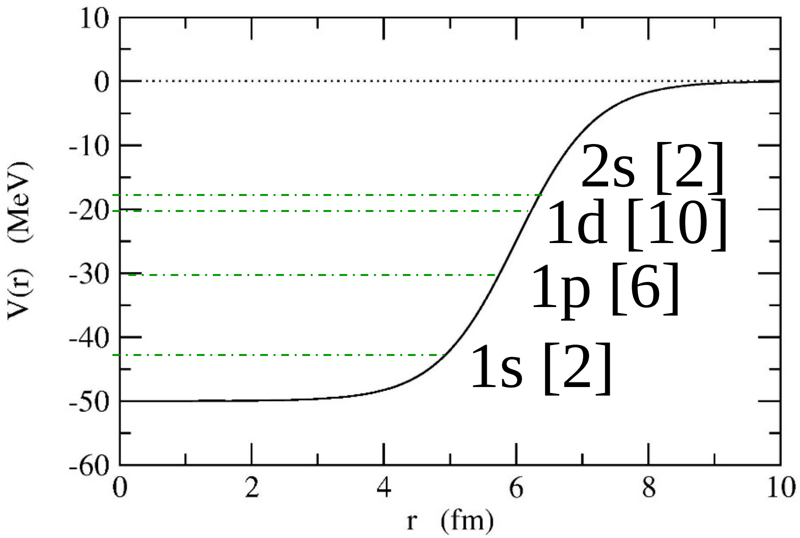


Shell structure

A similar attempt in nuclear physics: independent particle motion in a potential well

Woods-Saxon potential

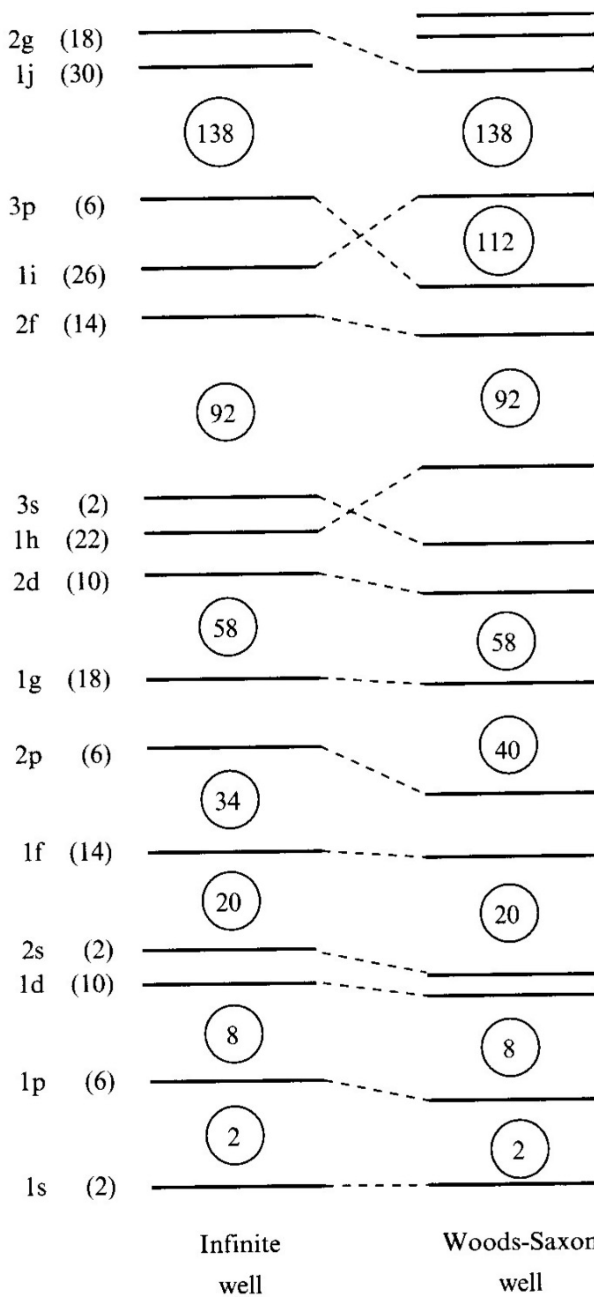
$$V(r) = \frac{-V_0}{1 + \exp[(r - R_0)/a]}$$



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

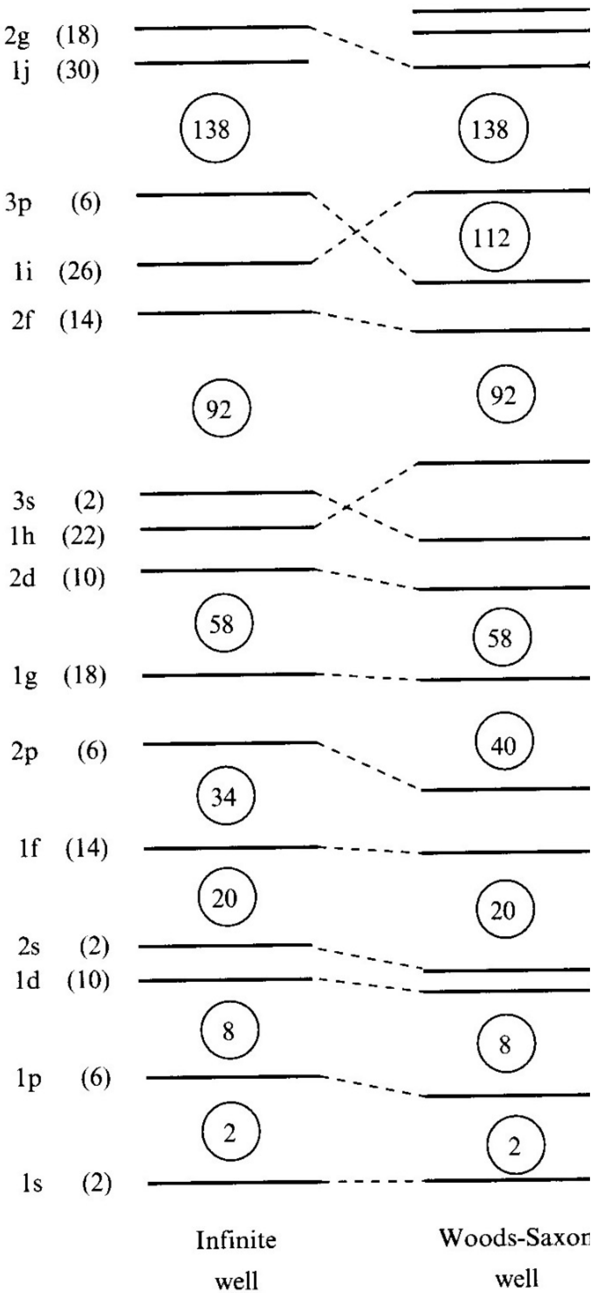
$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

degeneracy: 2*(2*l*+1)



Nuclear magic numbers:
2, 8, 20, 28, 50, 82, 126

?



Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).

Mayer and Jensen (1949): Nobel prize in 1963

Strong spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$V_{ls}(r) \sim -\lambda \frac{1}{r}\frac{dV}{dr} \qquad (\lambda > 0)$$

jj coupling shell model

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0 \Longrightarrow \psi_{lmm_s}(\boldsymbol{r}) = \frac{u_l(r)}{r}Y_{lm}(\hat{\boldsymbol{r}})\cdot\chi_{m_s}$$

Spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \Longrightarrow \boldsymbol{l}\cdot\boldsymbol{s} = \frac{1}{2}(\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)$

$$\begin{aligned} \psi_{jlm}(\boldsymbol{r}) &= \frac{u_{jl}(r)}{r}\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) \\ \mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) &= \sum_{m_l,m_s}\langle l\ m_l\ 1/2\ m_s|j\ m\rangle Y_{lm_l}(\hat{\boldsymbol{r}})\chi_{m_s} \end{aligned}$$

$$\begin{aligned} \boldsymbol{j}^2\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) &= j(j+1)\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) \\ j_z\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) &= m\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) \\ \boldsymbol{l}^2\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) &= l(l+1)\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) \\ \boldsymbol{s}^2\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) &= \frac{1}{2}\left(\frac{1}{2}+1\right)\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}}) \end{aligned}$$

jj coupling shell model

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0 \Longrightarrow \psi_{lmm_s}(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

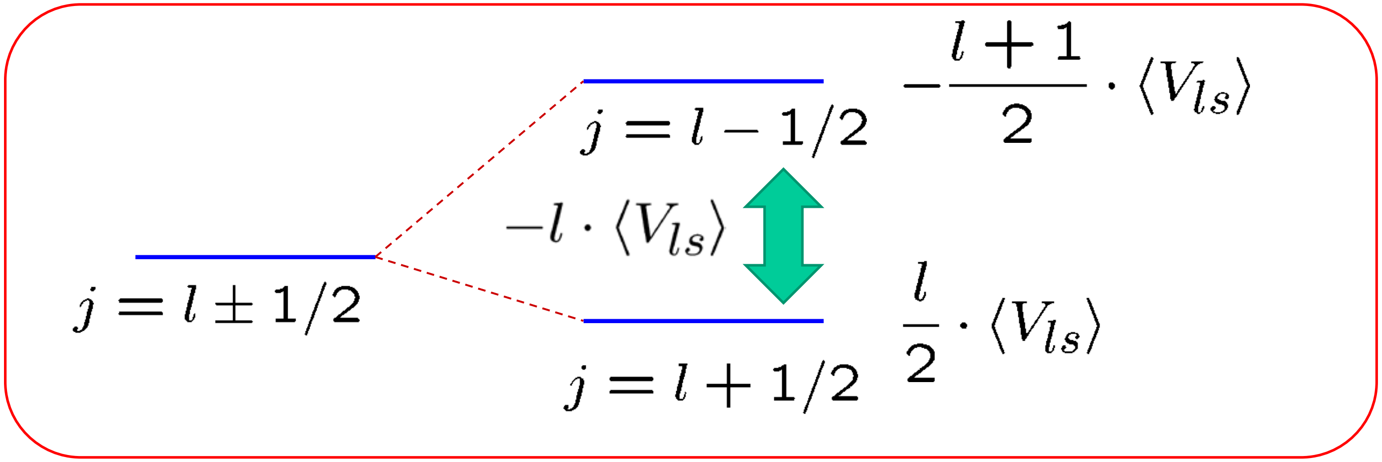
Spin-orbit interaction

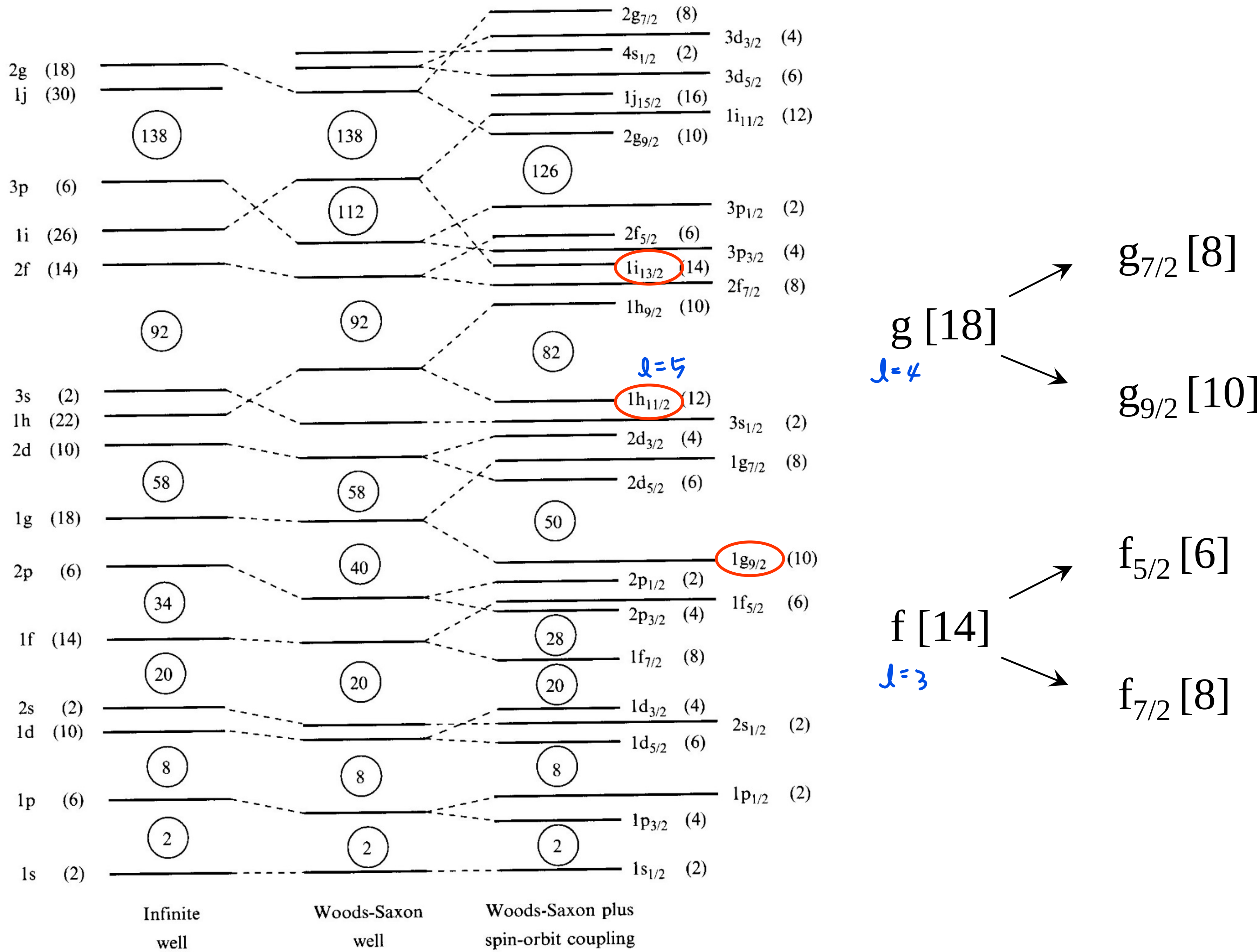
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

(note) $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \Longrightarrow \boldsymbol{l}\cdot\boldsymbol{s} = (\boldsymbol{j}^2 - \boldsymbol{l}^2 - \boldsymbol{s}^2)/2$

$= j(j+1) - l(l+1) - \frac{1}{2}\frac{3}{2}$

$$\boldsymbol{l}\cdot\boldsymbol{s} = \frac{l}{2}\ (j = l + 1/2), \quad -\frac{l+1}{2}\ (j = l - 1/2)$$





2g (18)
1j (30)
138
3p (6)
1i (26)
2f (14)
92
3s (2)
1h (22)
2d (10)
58
1g (18)
2p (6)
34
1f (14)
20
2s (2)
1d (10)
8
1p (6)
2
1s (2)
Infinite well
138
112
92
58
40
20
8
2
Woods-Saxon well
2g7/2 (8)
3d3/2 (4)
4s1/2 (2)
3d5/2 (6)
1j15/2 (16)
1i11/2 (12)
2g9/2 (10)
126
3p1/2 (2)
2f5/2 (6)
3p3/2 (4)
1i13/2 (14)
2f7/2 (8)
1h9/2 (10)
82
l=5
1h11/2 (12)
3s1/2 (2)
2d3/2 (4)
1g7/2 (8)
2d5/2 (6)
50
1g9/2 (10)
2p1/2 (2)
1f5/2 (6)
2p3/2 (4)
28
1f7/2 (8)
20
1d3/2 (4)
2s1/2 (2)
1d5/2 (6)
8
1p1/2 (2)
1p3/2 (4)
2
1s1/2 (2)
Woods-Saxon plus spin-orbit coupling
g [18]
l=4
g7/2 [8]
g9/2 [10]
f [14]
l=3
f5/2 [6]
f7/2 [8]

Single particle spectra

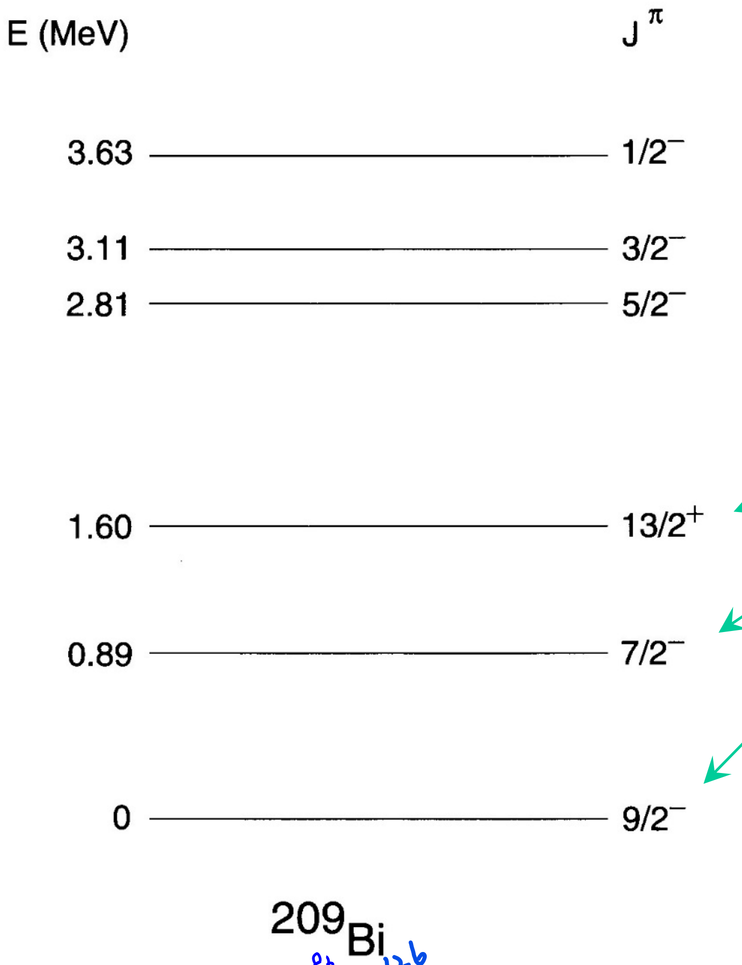
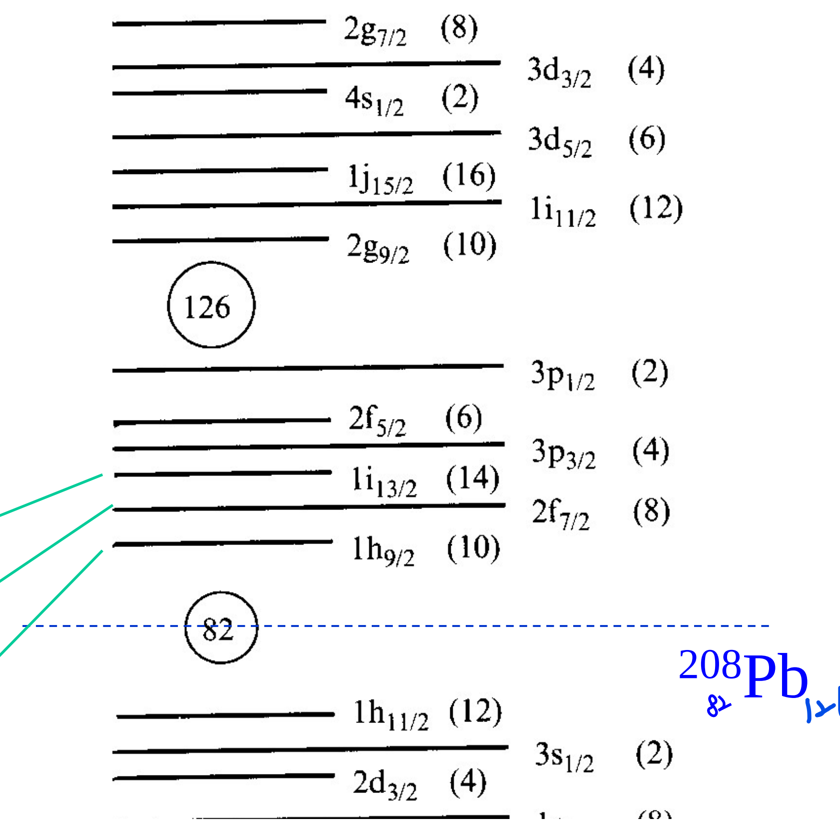


FIG. 3.6. Low-lying single-particle levels of ^{209}Bi.



$\ell = 0, 1, 2, 3, 4, 5, 6$
s p d f g h i

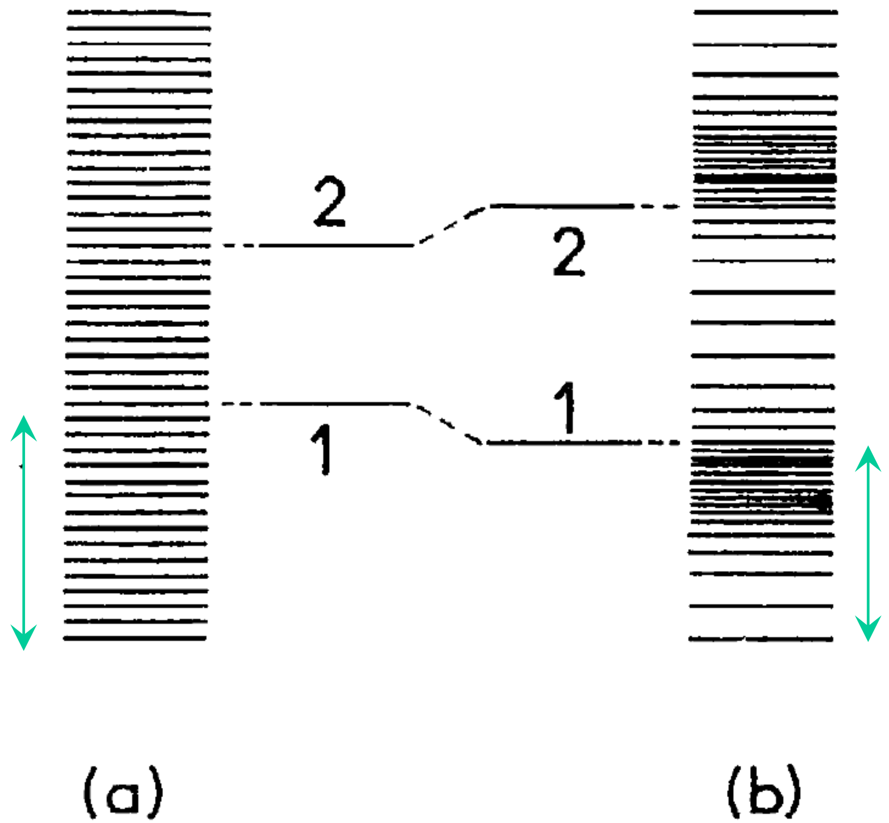
$\pi = (-)^{\ell}$

- How to construct V(r) microscopically?
- Does the independent particle picture really hold?

⟹ Later in this course

Why do closed-shell-nuclei become stable?

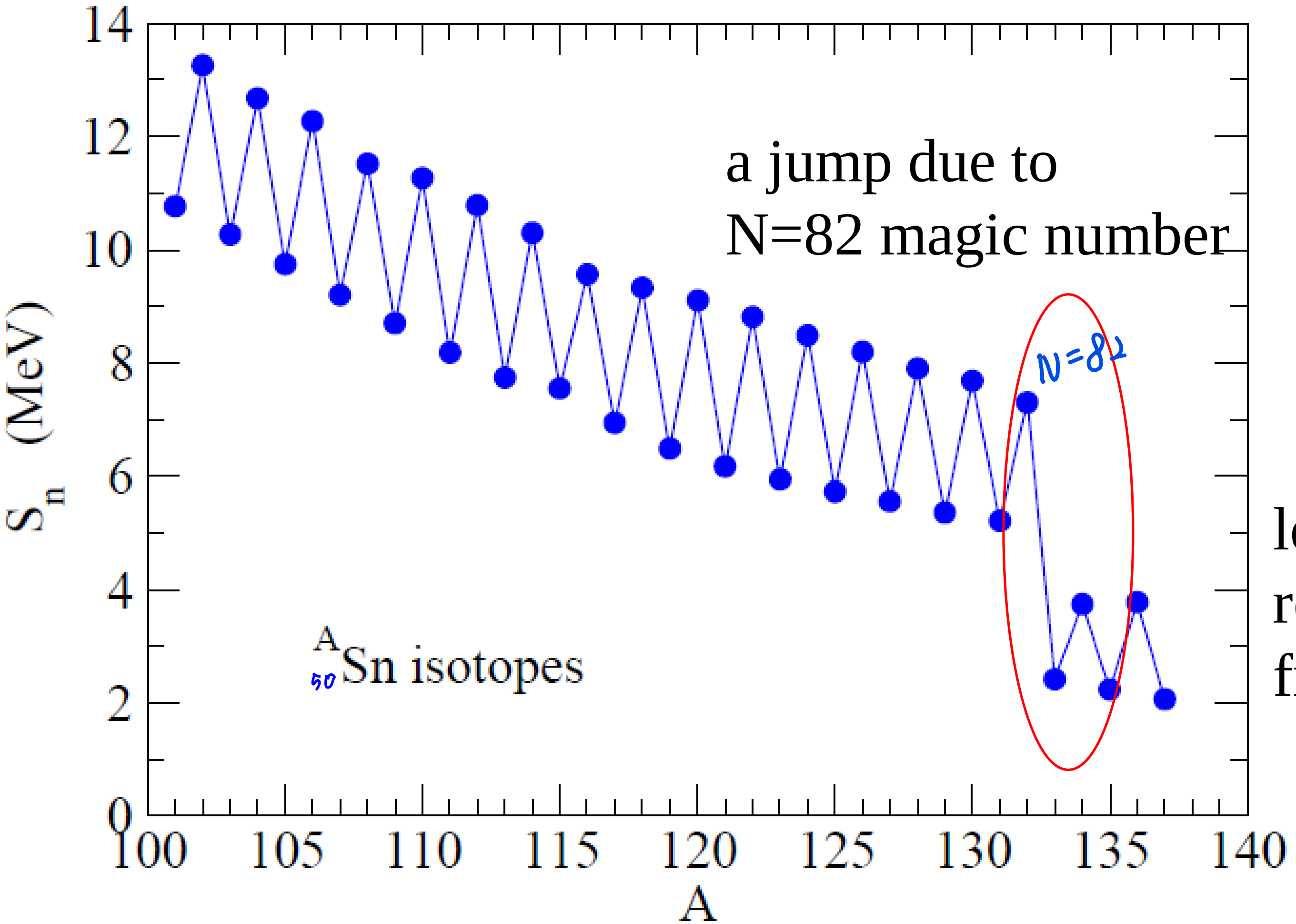
level density



smaller total energy (more stable)

uniform

non-uniform



less energy to remove a neutron from a higher orbit

1n separation energy: $S_n(A,Z) = B(A,Z) - B(A-1,Z)$

A lucky accident for the origin of life

Atomic magic numbers

electron #: 2, 10, 18, 36, 54, 86

inert gas：He, Ne, Ar, Kr, Xe, Rn

Nuclear magic numbers

proton # or neutron #

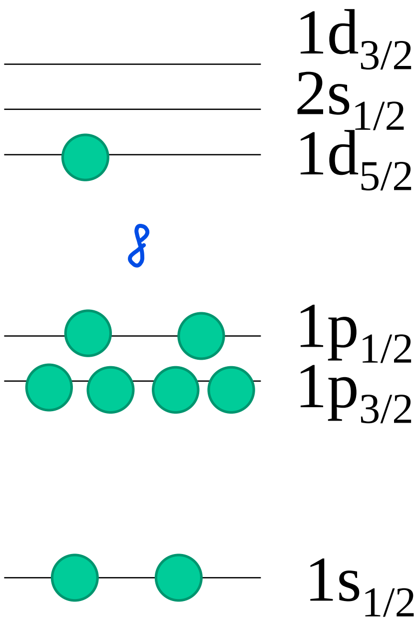
2, 8, 20, 28, 50, 82, 126

- e.g., ${}^{16}_{8}O_8$ (double magic)
- many oxygen nuclei: produced during nucleosynthesis
- oxygen: chemically active
- several complex chemical reactions, leading to the birth of life

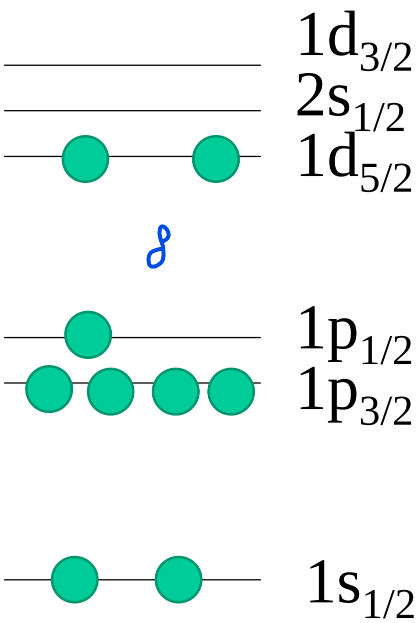
参考：望月優子　ビデオ「元素誕生の謎にせまる」　http://rarfaxp.riken.go.jp/~motizuki/contents/genso.html

single-j model

shell model



configuration 1



configuration 2

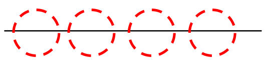
…… several others

angular momentum (spin) and parity for each configuration?

→ let us first investigate a single-j case single-j level: one level with an angular momentum j

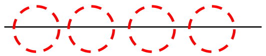
j

example: $j = p_{3/2}$



$p_{3/2}$

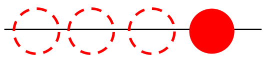
can accommodate 4 nucleons
(j_z= +3/2, +1/2, -1/2, -3/2)



$p_{3/2}$ can accommodate 4 nucleons (2j+1)

(j_z= +3/2, +1/2, -1/2, -3/2)

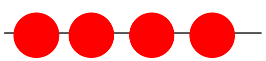
i) 1 nucleon



$p_{3/2}$ → $I^\pi = 3/2^-$ (2I+1)

(there are 4 ways to occupy this level)

ii) 4 nucleons



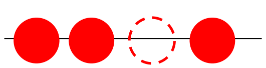
$p_{3/2}$ → $I^\pi = 0^+$

(there is only 1 way to occupy this level)

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

parity: (-1) x (-1) x (-1) x (-1) = +1

iii) 3 nucleons



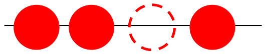
$p_{3/2}$ → $I^\pi = 3/2^-$

(there are 4 ways to make a hole)

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3$$

parity: (-1) x (-1) x (-1) = -1

iii) 3 nucleons



$p_{3/2}$

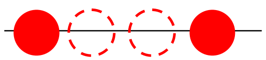
$$I = j_1 + j_2 + j_3$$

$\Rightarrow$ $I^{\pi} = 3/2^{-}$

(there are 4 ways to make a hole)

parity: (-1) x (-1) x (-1) = -1

iv) 2 nucleons



$p_{3/2}$

$$I = j_1 + j_2$$

there are 4 x 3/2=6 ways to occupy this level with 2 nucleons.

$\Rightarrow$ $I^{\pi} = 0^{+}$ [1] or 2^{+} [5]

3/2 + 3/2 → I = 0, ~~1~~, 2, ~~3~~

anti-symmetrization

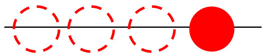
nn & pp : T=1
∴ J=I=even

In Isospin representation

two-coupled w.f. $|ab;JM;TM_T\rangle = \mathcal{N}_{ab}(JT)\left[c_a^{+} c_b^{+}\right]_{JM}^{TM_T}|\text{core}\rangle$

$$\mathcal{N}_{ab} = \frac{\sqrt{1-\delta_{ab}(-)^{J+T}}}{1+\delta_{ab}}$$ ∴ J+T=odd

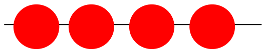
i) 1 nucleon



$p_{3/2}$ → $I^{\pi} = 3/2^{-}$

(there are 4 ways to occupy this level)

ii) 4 nucleons

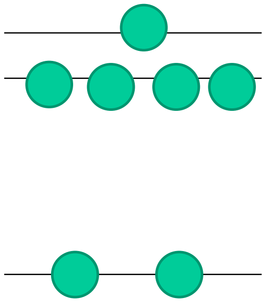


$p_{3/2}$ → $I^{\pi} = 0^{+}$

$$\boldsymbol{I} = \boldsymbol{j}_1 + \boldsymbol{j}_2 + \boldsymbol{j}_3 + \boldsymbol{j}_4$$

(there is only 1 way to occupy this level)

parity: (-1) x (-1) x (-1) x (-1) = +1



$1p_{1/2}$ → $I^{\pi} = 1/2^{-}$

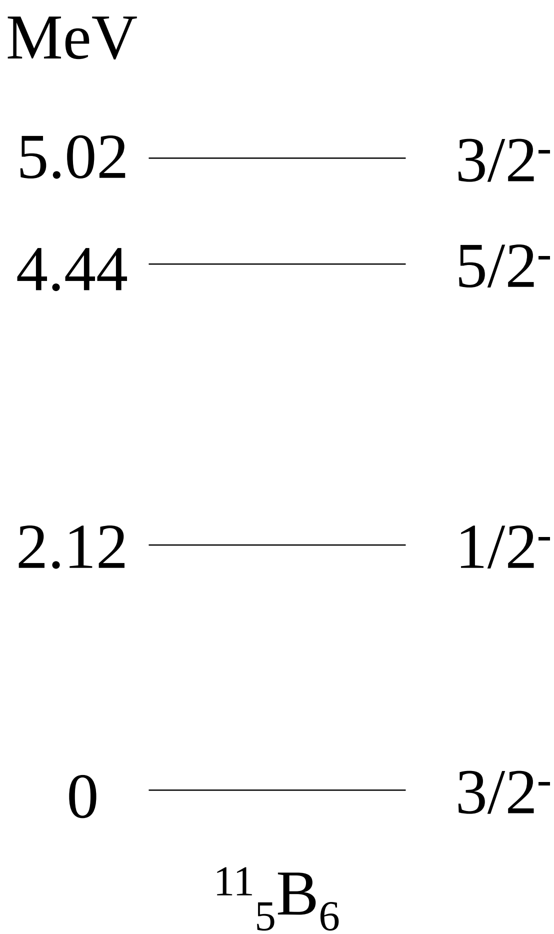
$1p_{3/2}$ → $I^{\pi} = 0^{+}$

$1s_{1/2}$ → $I^{\pi} = 0^{+}$

in total,
$I^{\pi} = 1/2^{-}$

example: (main) shell model configurations for $^{11}_{5}B_{6}$

cf. $^{12}C(e,e'K^{+})^{12}_{\Lambda}B$ (=$^{11}B+\Lambda$)



cf. $^{12}C(e,e'K^+)^{12}_{\Lambda}B$ (=$^{11}B+\Lambda$)

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Experiments with the High Resolution Kaon Spectrometer at JLab Hall C and the new spectroscopy of $^{12}_{\Lambda}B$ hypernuclei

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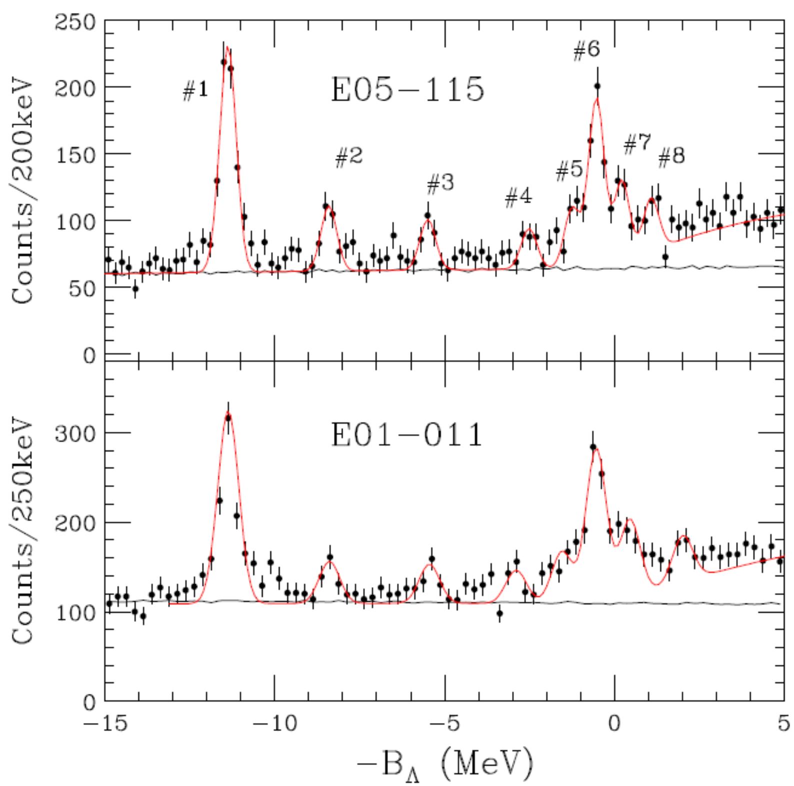
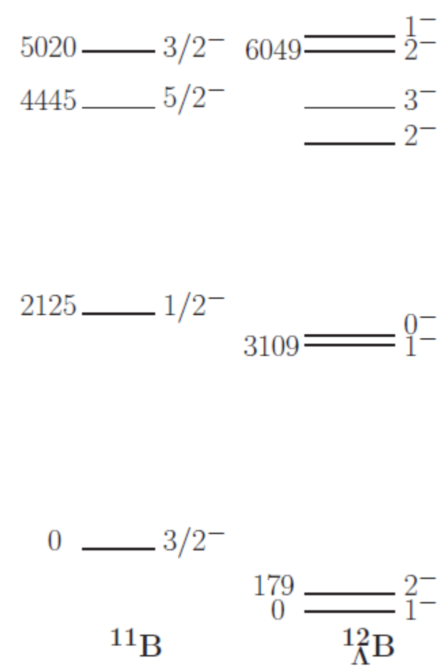
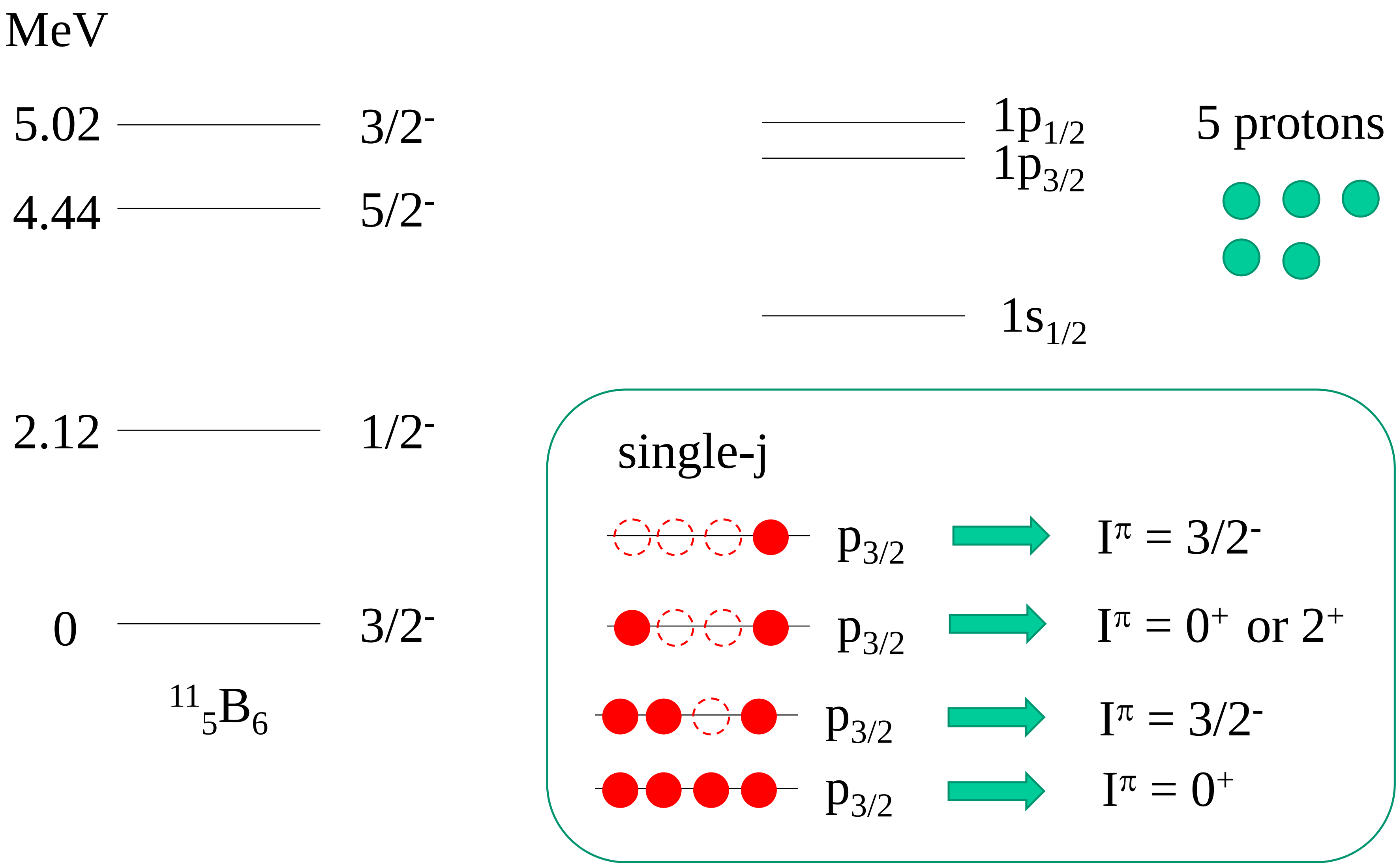




(b)

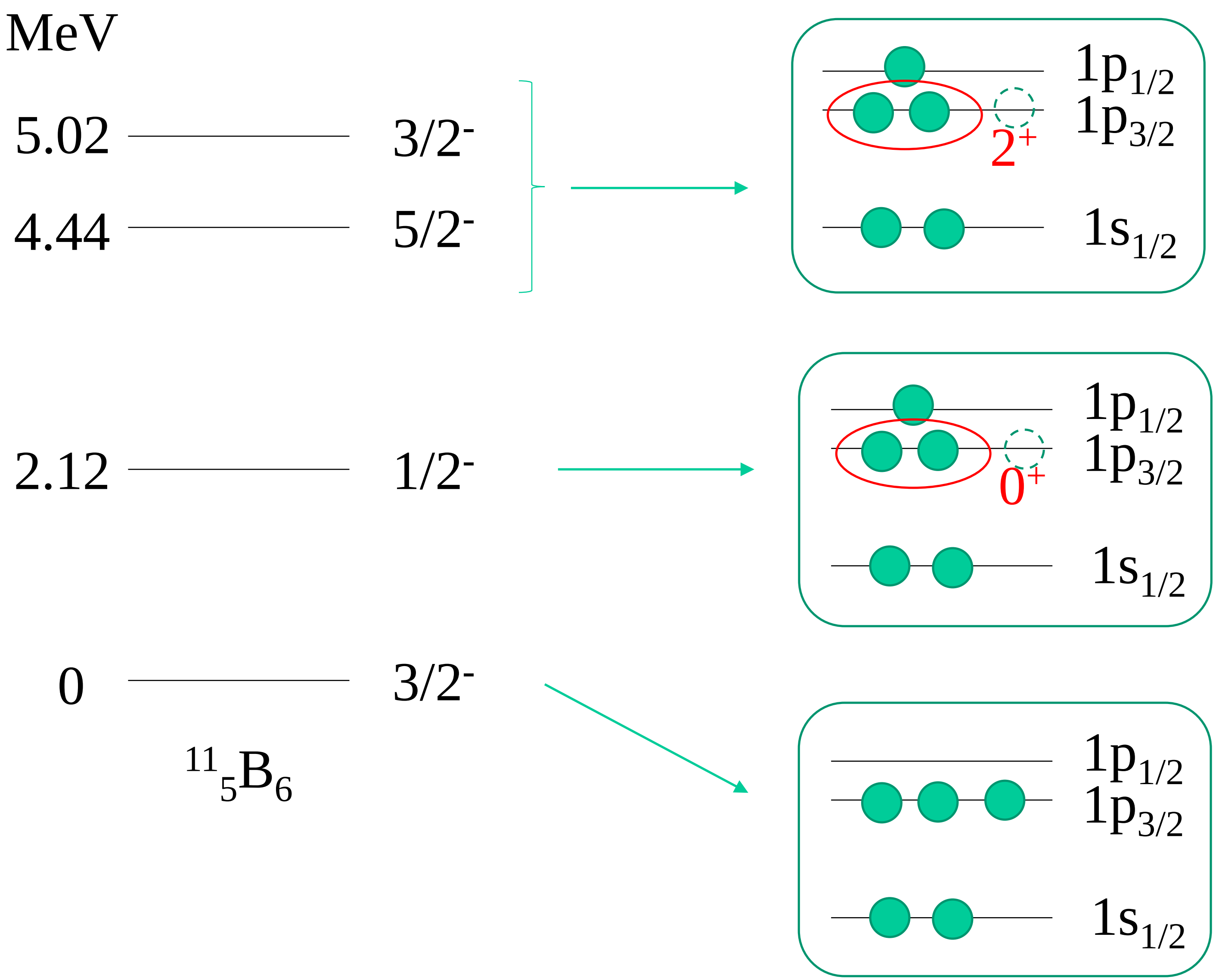
example: (main) shell model configurations for ${}^{11}_{5}B_6$

cf. ${}^{12}C(e,e'K^+){}^{12}_{\Lambda}B$ (=${}^{11}B+\Lambda$)

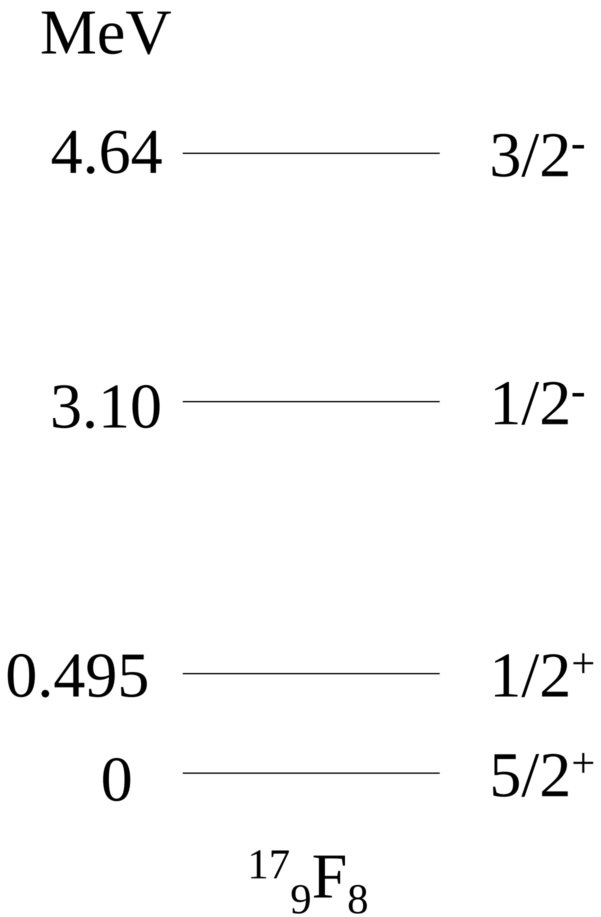


example: (main) shell model configurations for ^{11}B

cf. $^{12}C(e,e'K^+)^{12}{}_{\Lambda}B$ (=$^{11}B+\Lambda$)



another example: (main) shell model configurations for ^{17}F



another example: (main) shell model configurations for ^{17}F