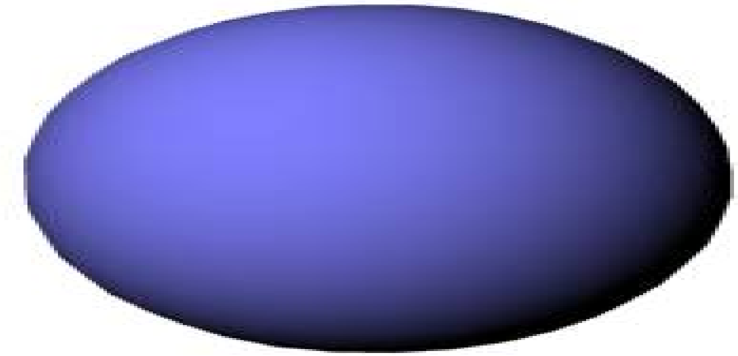
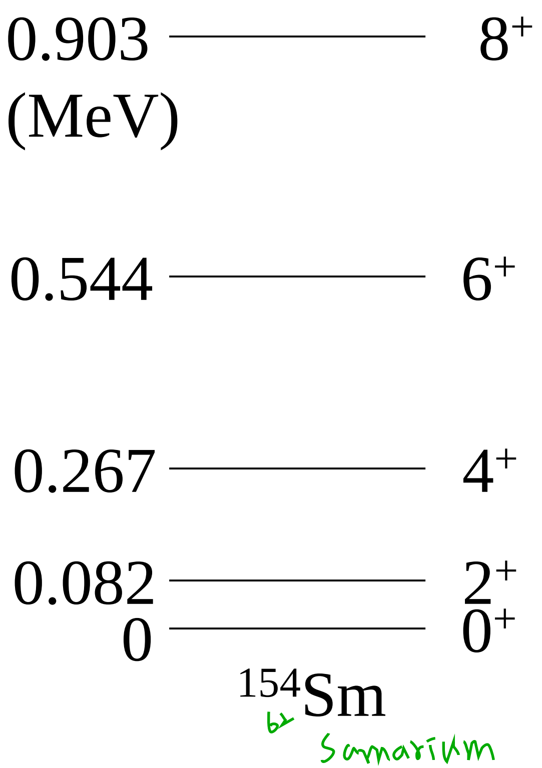
Nuclear Deformation



Nuclear Deformation

Excitation spectra of ^{154}Sm



$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

cf. Rotational energy of a rigid body (Classical mechanics)

(momentum of inertia (mr^2))

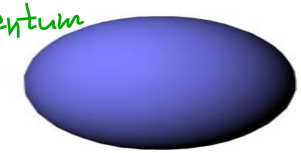
$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

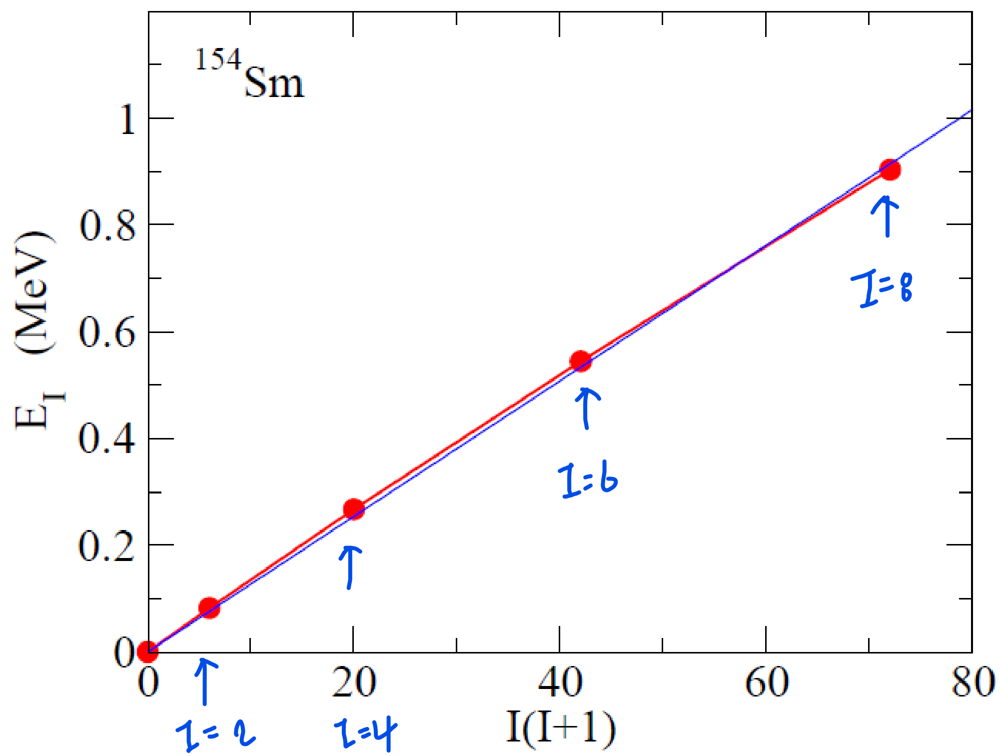
$$(I = \mathcal{J}\omega,\ \ \omega = \dot{\theta})$$

(angular momentum)

^{154}Sm is deformed

$\beta_2 = 0.341$ (B(E2))



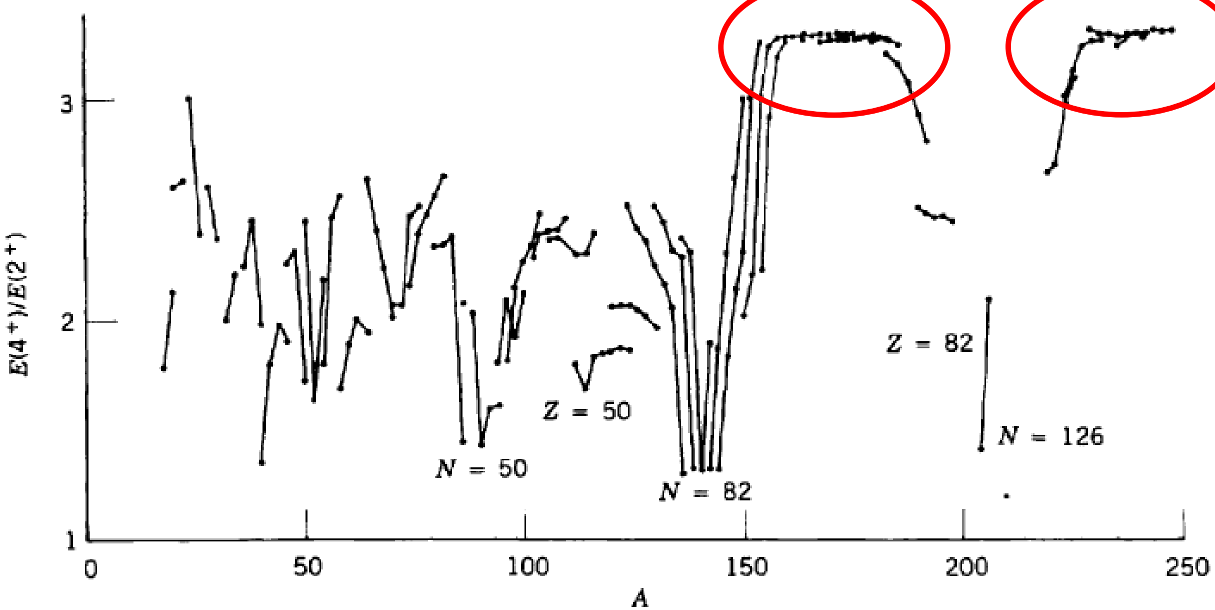


$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

$$\Rightarrow E_2 \propto 2 \times 3 = 6, \quad E_4 \propto 4 \times 5 = 20$$

$$\Rightarrow E_4/E_2 = 20/6 = 3.3333\cdots$$

E(4$^+$)/E(2$^+$)

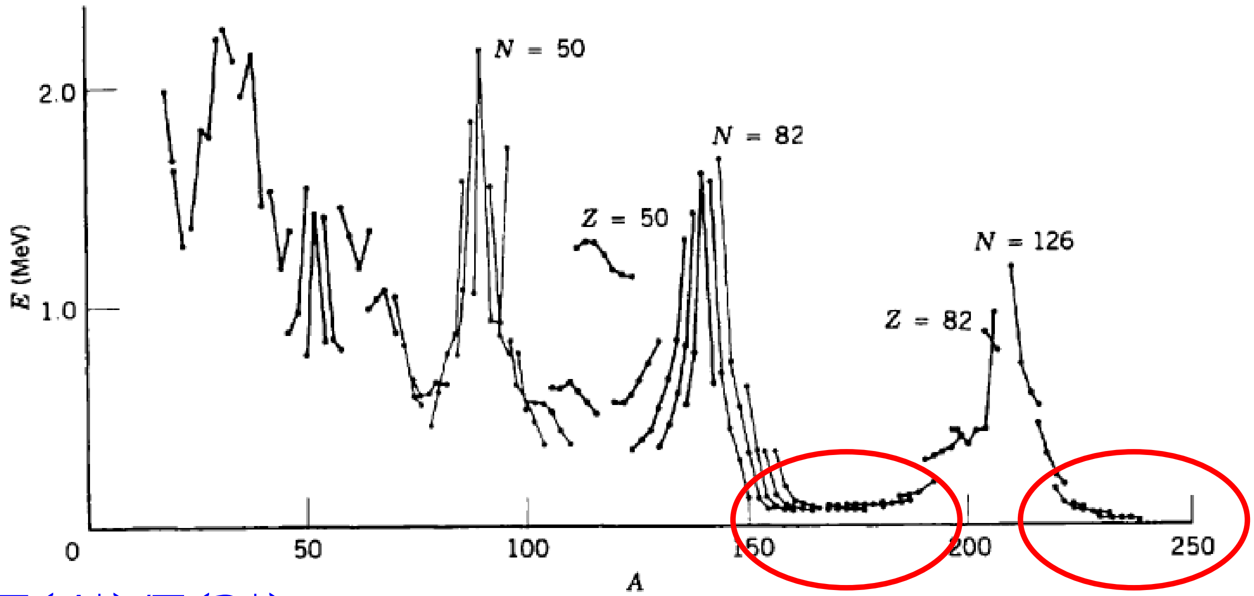


deformed nuclei:
E(4$^+$)/E(2$^+$) ~ 3.3

spherical nuclei:
E(4$^+$)/E(2$^+$) ~ 2

K.S. Krane, “Introductory Nuclear Physics”

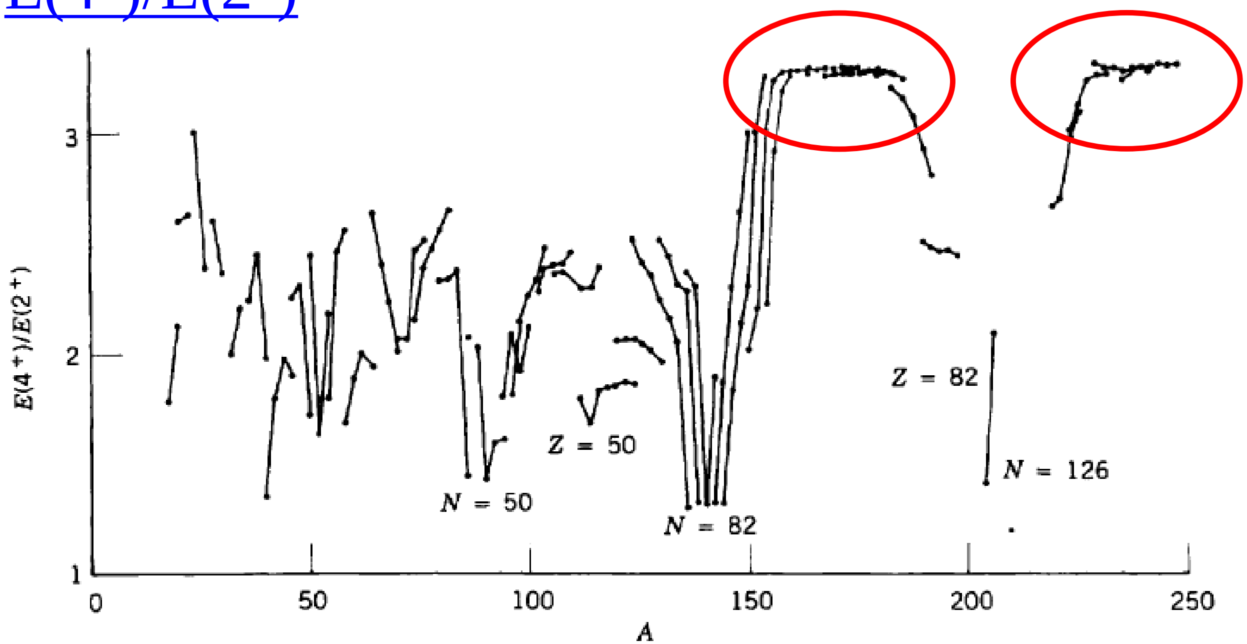
The energy of the first 2^+ state in even-even nuclei



a small energy
→ spontaneously
symm. breaking

deformed nuclei

$E(4^+)/E(2^+)$



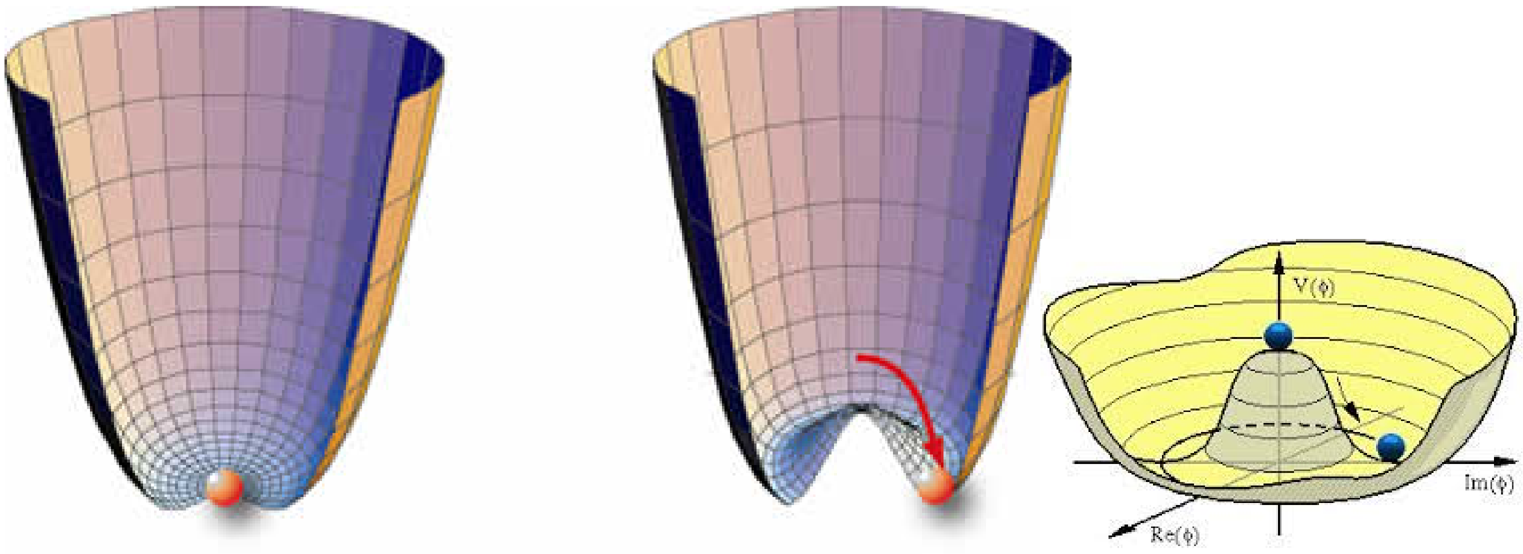
deformed nuclei:
$E(4^+)/E(2^+) \sim 3.3$

spherical nuclei:
$E(4^+)/E(2^+) \sim 2$

K.S. Krane, "Introductory Nuclear Physics"

Spontaneous symmetry breaking

The vacuum state does not have (i.e, the vacuum state violates) the symmetry which the Hamiltonian has.

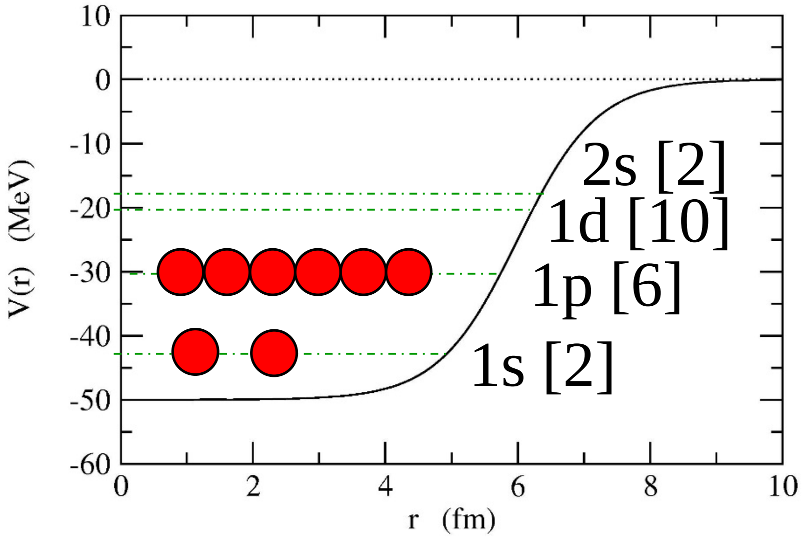


자발 대칭 깨짐이 일어나면 항상 파괴된 대칭의 수 만큼
Goldstone boson(mass=0)이 존재한다.

Nambu-Goldstone mode (zero energy mode) to restore the symmetry

Mean-field approximation and deformation

Mean-field approximation



$$H \sim \sum_i \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\rm MF}(\boldsymbol{r}_i) \right)$$

Slater determinant

$$\Psi_{\rm MF}(1,2,\cdots,A) = \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)]$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm MF}(\boldsymbol{r}) \right)\psi_k(\boldsymbol{r}) = \epsilon_k\psi_k(\boldsymbol{r})$$

the original many-body H:

$$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j)$$

$$= \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i^2 + V_{\rm MF}(\boldsymbol{r}_i) \right) + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i,\boldsymbol{r}_j) - \sum_i V_{\rm MF}(\boldsymbol{r}_i)$$

H_{res} = residual

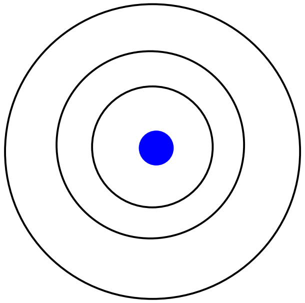
$\Psi_{\rm MF}$: does not necessarily possess the symmetries that H has.

Typical Examples

➢ Translational symmetry: always broken in nuclear systems

$$H = -\sum_{i=1}^{A}\frac{\hbar^2}{2m}\nabla_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i - \boldsymbol{r}_j) \rightarrow \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\nabla_i^2 + \underline{\underline{V_{\rm MF}(\boldsymbol{r}_i)}}\right)$$

(cf.) atoms

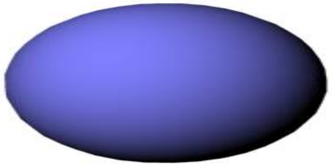


nucleus in the center

⟶ translational symmetry: broken from the begining

➢ Rotational symmetry

Deformed solution



Constrained Hartree-Fock method

$Q = \sqrt{16\pi B(E2)/5e^2}$, $\sqrt{16\pi/5}\,(3/4\pi)\,A R_0^2 \beta_2$

minimize $H' = H - \lambda \hat{Q}_{20}$ with a Slater determinant w.f.

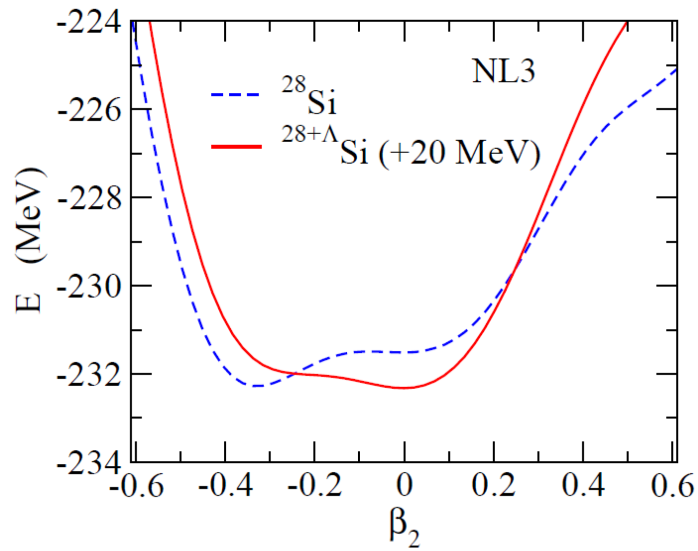
Constrained Variation

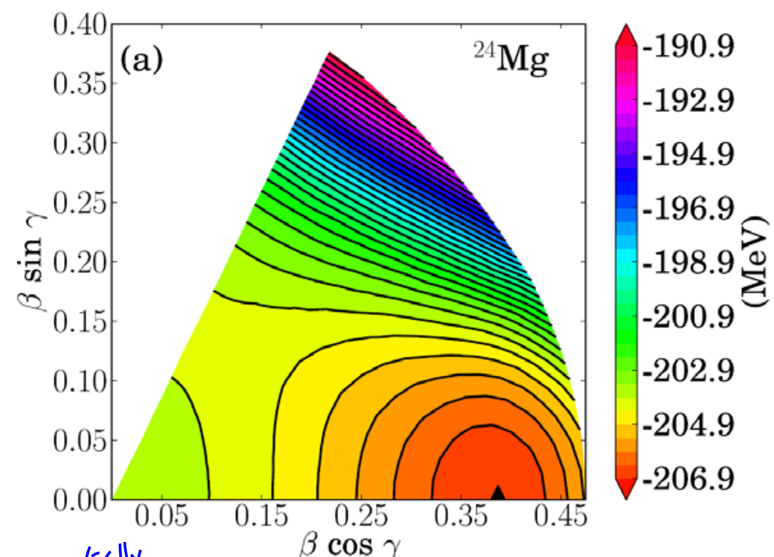
$\hat{Q}_{20} = \sum_i r_i^2 Y_{20}(\hat{\boldsymbol{r}}_i)$: quadrupole operator

λ : Lagrange multiplier, to be determined so that $\langle \hat{Q}_{20} \rangle = Q \propto R^2 \beta$

deformation parm.

⟶ $E(\beta)$: potential energy curve





axially triaxial

$E(\beta,\gamma)$: potential energy surface

One-particle motion in a deformed potential

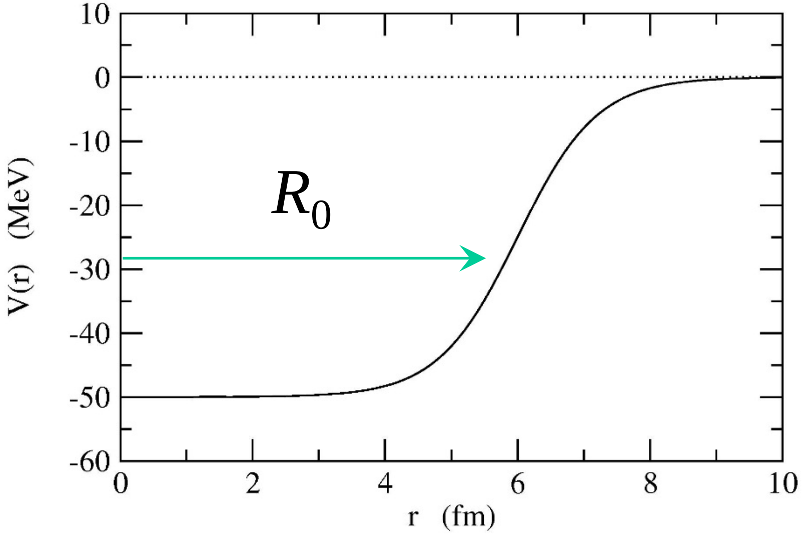
$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')$$

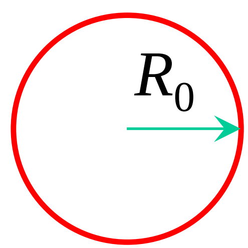
if the density is deformed, so is the mean-field potential

(example) a deformed Woods-Saxon potential

a spherical WS potential:

$$V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R_0}{a}\right)}$$





One-particle motion in a deformed potential

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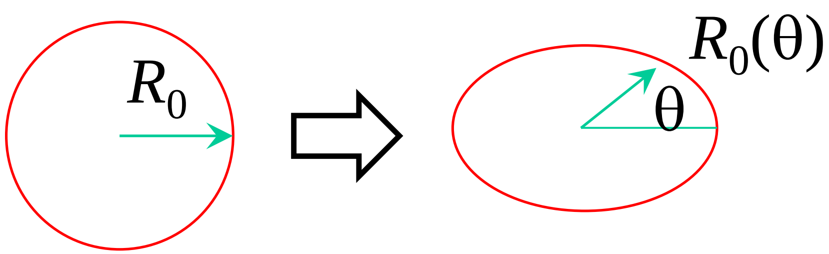
if the density is deformed, so is the mean-field potential

(example) a deformed Woods-Saxon potential

a WS potential:

$$V(r) = -\frac{V_0}{1+\exp\left(\frac{r-R_0}{a}\right)}$$

$$V(r) \rightarrow V(r,\theta) = -\frac{V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)}$$



$$R_0 \rightarrow R_0(1+\beta_2 Y_{20}(\theta))$$

$R_0(\theta)$

One-particle motion in a deformed potential

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r}, \boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r}, \boldsymbol{r}') = -g\delta(\boldsymbol{r} - \boldsymbol{r}')$$

if the density is deformed, so is the mean-field potential

(example) a deformed Woods-Saxon potential

$$V(r,\theta) = -\frac{V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)} = -\frac{V_0}{1+\exp\left(\frac{r-R_0-R_0\beta_2 Y_{20}(\theta)}{a}\right)}$$

$$f(x+h) = f(x) + h f'(x) + \frac{h^2 f''(x)}{2!} + \cdots$$

$$\sim -\frac{V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)} - R_0\beta_2 Y_{20}(\theta)\frac{d}{dr}\left[\frac{-V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)}\right]$$

$$\equiv V_0(r) + V_2(r)Y_{20}(\theta) \qquad V_2(r) = -R_0\beta_2 V_0'(r)$$

One-particle motion in a deformed potential

$$V(\boldsymbol{r}) \sim \int v(\boldsymbol{r},\boldsymbol{r}')\rho(\boldsymbol{r}')d\boldsymbol{r}' \sim -g\rho(\boldsymbol{r}) \qquad \text{if} \quad v(\boldsymbol{r},\boldsymbol{r}') = -g\delta(\boldsymbol{r}-\boldsymbol{r}')$$

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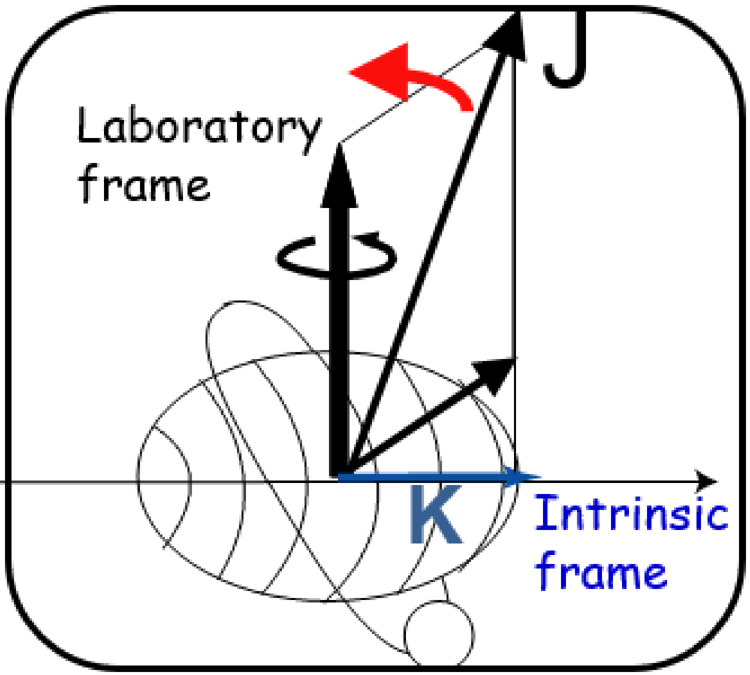
(example) a deformed Woods-Saxon potential

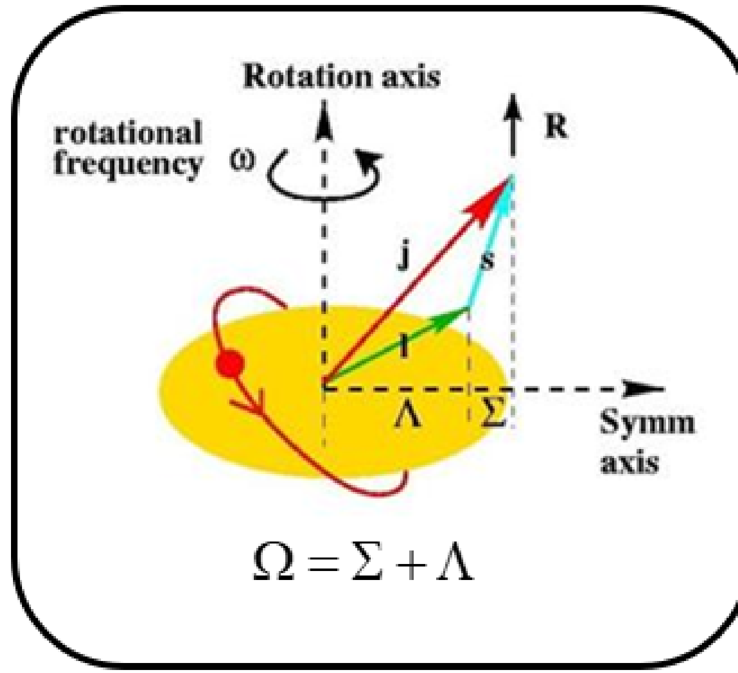
$$V(r,\theta) = -\frac{V_0}{1+\exp\left(\frac{r-R_0(\theta)}{a}\right)} = -\frac{V_0}{1+\exp\left(\frac{r-R_0-R_0\beta_2Y_{20}(\theta)}{a}\right)}$$

$$\sim -\frac{V_0}{1+\exp\left(\frac{r-R_0(\cancel{\theta})}{a}\right)} - R_0\beta_2Y_{20}(\theta)\frac{d}{dr}\left[\frac{-V_0}{1+\exp\left(\frac{r-R_0(\cancel{\theta})}{a}\right)}\right]$$

* non-spherical potential → angular momentum: not conserved

(Coupled system)





One-particle motion in a deformed potential

$$V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots$$

■the effect of Y_{20} term

Eigen-functions for β_2=0 (spherical pot.) :

$$\psi_{nll_z}(\boldsymbol{r}) = R_{nl}(r) Y_{ll_z}(\hat{\boldsymbol{r}})$$

eigen-values: E_{nl} (no dependence on l_z)

The change of energy due to the Y_{20} term (1st order perturbation theory):

$$E_{nl} \rightarrow E_{nl} + \underline{\langle \psi_{nll_z} | \Delta V | \psi_{nll_z} \rangle}$$

$$\Delta V(r) = -\beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta)$$

One-particle motion in a deformed potential

$$V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots$$

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$$E_{nl} \rightarrow E_{nl} + \langle \psi_{nll_z} | \Delta V | \psi_{nll_z} \rangle$$

$$\Delta V(r) = -\beta_2 R_0 \frac{dV_0(r)}{dr} Y_{20}(\theta)$$

$$\psi_{nll_z}(\boldsymbol{r}) = R_{nl}(r) Y_{ll_z}(\hat{\boldsymbol{r}})$$

$$\Delta E = -\beta_2 R_0 \int_0^\infty r^2 dr |R_{nl}(r)|^2 V_0'(r) \times \int d\hat{\boldsymbol{r}} Y^*_{ll_z}(\theta) Y_{20}(\theta) Y_{ll_z}(\theta)$$

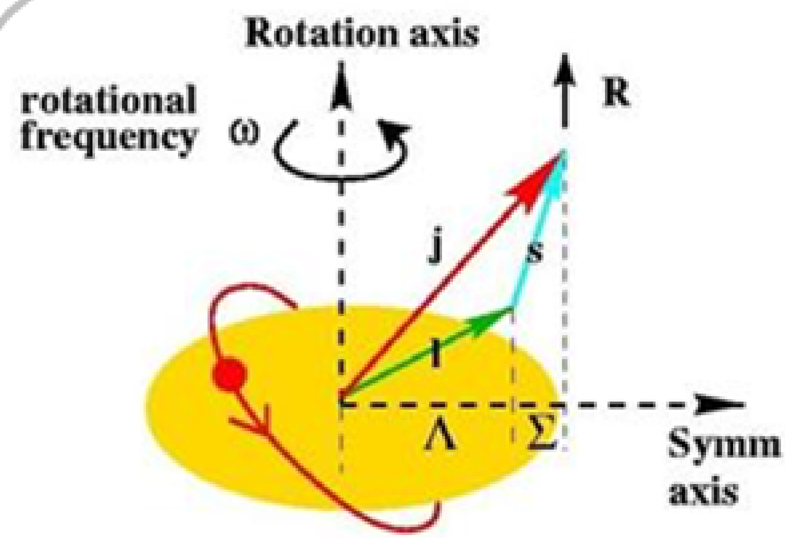
+ +

$$\propto -(3l_z^2 - l(l+1))$$

$$\int d\vec{r}\, Y^*_{\ell\ell_z}(\theta)\, Y_{20}\, Y_{\ell\ell_z}(\theta) = \int r^2 dr\, \underbrace{\sin\theta\, d\theta}_{-d\cos\theta}\, d\phi\, Y^*_{\ell\ell_z}(\theta)\, Y_{20}\, Y_{\ell\ell_z}(\theta)$$

$$= -(3\ell_z - \ell(\ell+1))$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta - 1) = \sqrt{\frac{5}{16\pi}}\,\frac{1}{r^2}(3z^2 - r^2)$$



$$\Omega = \Sigma + \Lambda$$

One-particle motion in a deformed potential

$$V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots$$

■the effect of Y_{20} term

$$E_{nl} \rightarrow E_{nl} + \langle \psi_{nll_z} | \Delta V | \psi_{nll_z} \rangle$$

$$\Delta E = -\beta_2 R_0 \int_0^\infty r^2 dr |R_{nl}(r)|^2 V_0'(r) \times \int d\hat{\boldsymbol{r}} Y^*_{ll_z}(\theta) Y_{20}(\theta) Y_{ll_z}(\theta)$$

+ +

$$\propto -(3l_z^2 - l(l+1))$$

$$\equiv \beta_2 \times \alpha_{nl} (3l_z^2 - l(l+1)) \quad (\alpha_{nl} > 0)$$

One-particle motion in a deformed potential

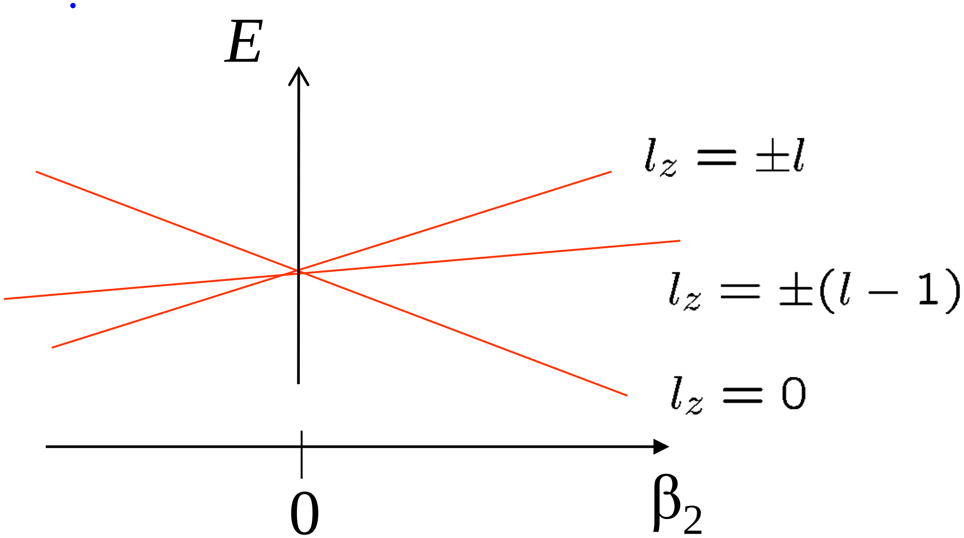
$$V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots$$

$$\Delta E = \beta_2 \times \alpha_{nl}\,(3l_z^2 - l(l+1)) \quad (\alpha_{nl} > 0)$$

for $\beta_2 > 0$

$\Delta E < 0$ ($l_z = 0$)

$\Delta E > 0$ ($l_z = 1$ → l) $\Delta E = 3l^2 - l^2 - l = 2l^2 - l > 0$



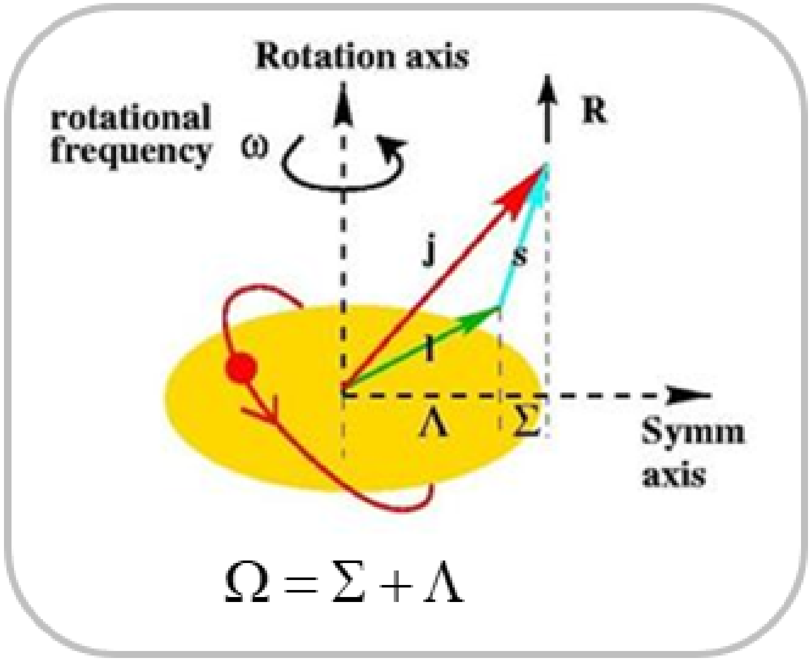
✓ degeneracy: resolved
(E: now depends on l_z)

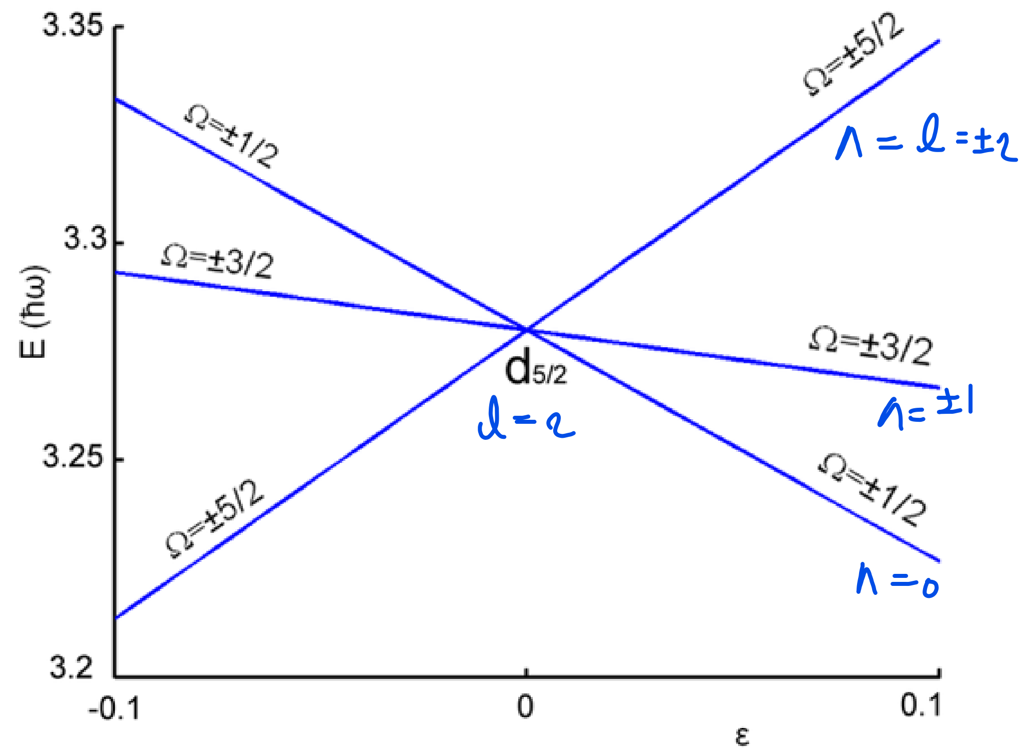
✓ degeneracy: $+l_z$ and $-l_z$

j projection on z → Ω
l → Λ
J → K
$\Omega = \Lambda \pm \frac{1}{2}$

In spherical basis, **j** is a good quantum number.
But in deformed basis, a projection of J on the nuclear symmetric axis z, **Ω**, is a good quantum number.

Deformed states, ±5/2, ±3/2, and ±1/2, are separated from the spherical state $d_{5/2}$.



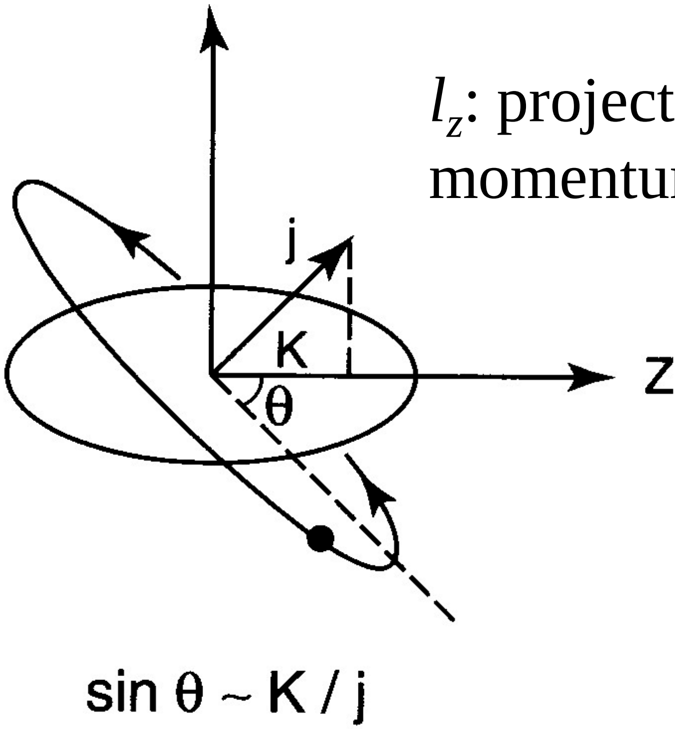


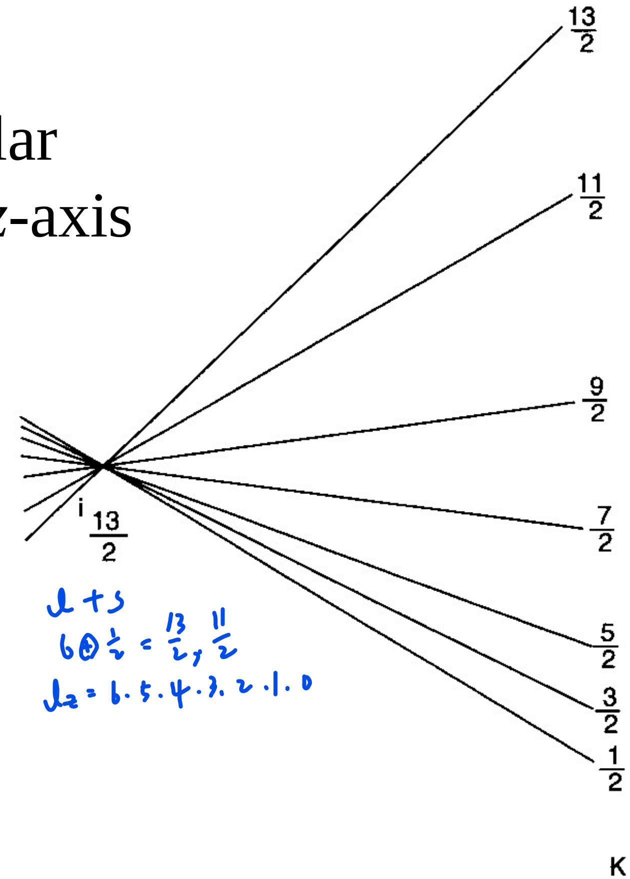
➢ Single particle states in deformed nucleus become more complex.

Geometrical interpretation

l = 0, 1, 2, 3, 4, 5, 6
s p d f g h i

l_z: projection of angular momentum onto the z-axis





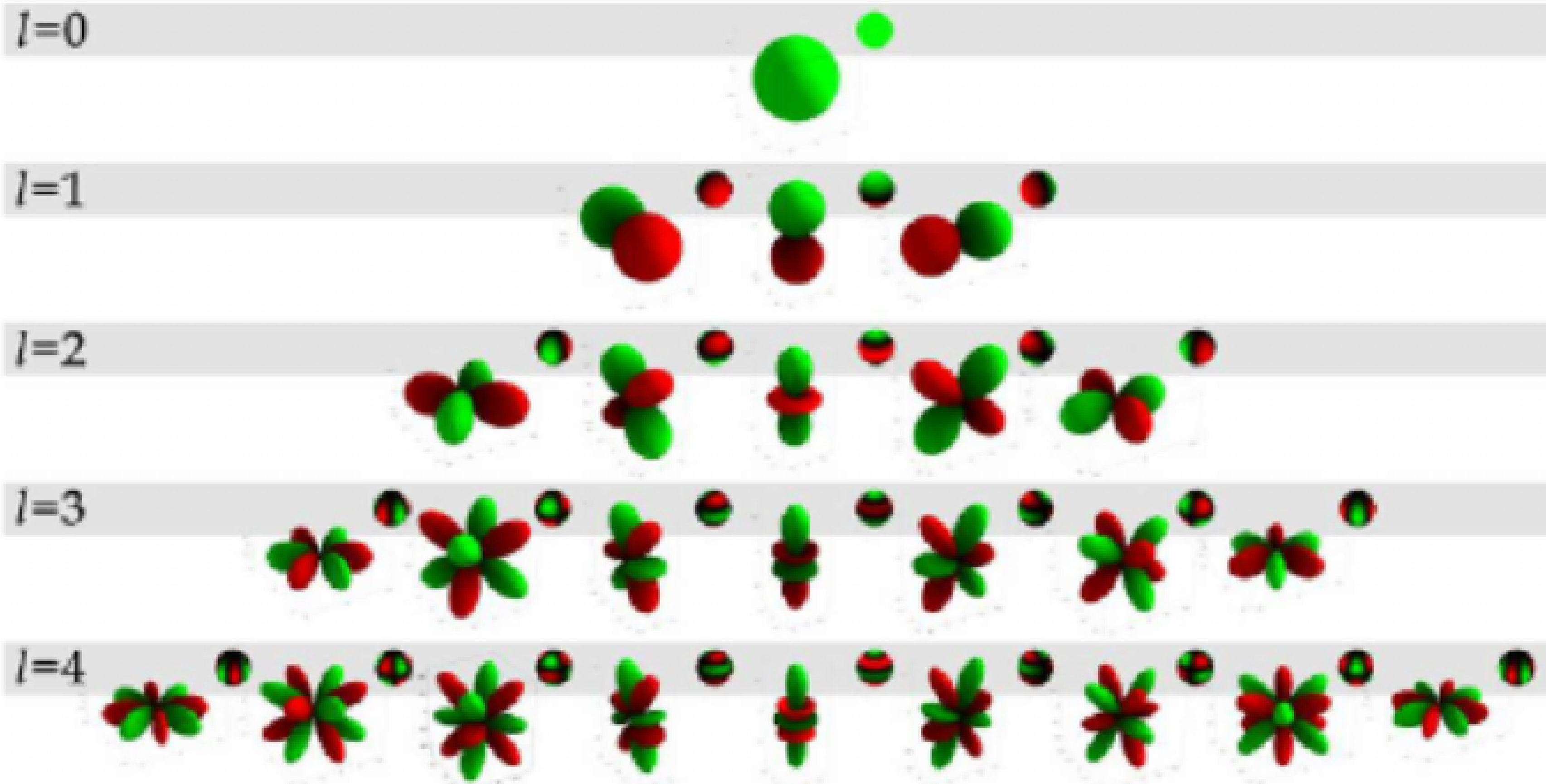
for $\beta_2 > 0$

small l_z ⟷ a motion along the longer axis $\theta = 90^\circ$, $l_z = 0$, E ↓

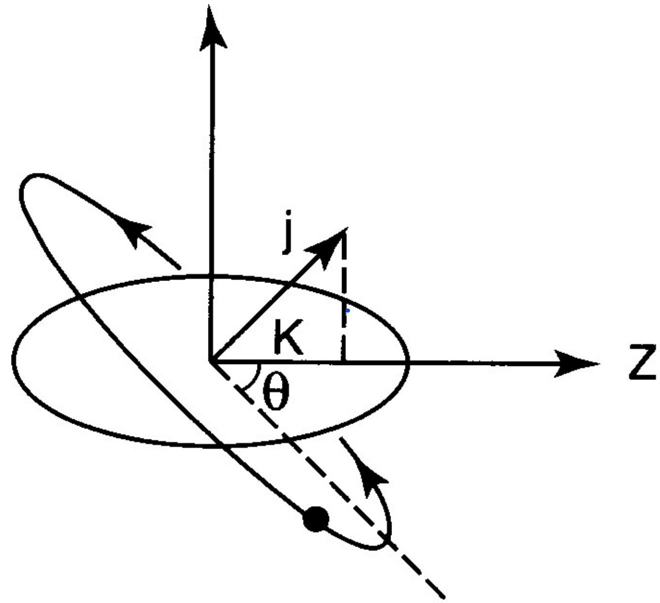
→ the energy is lowered

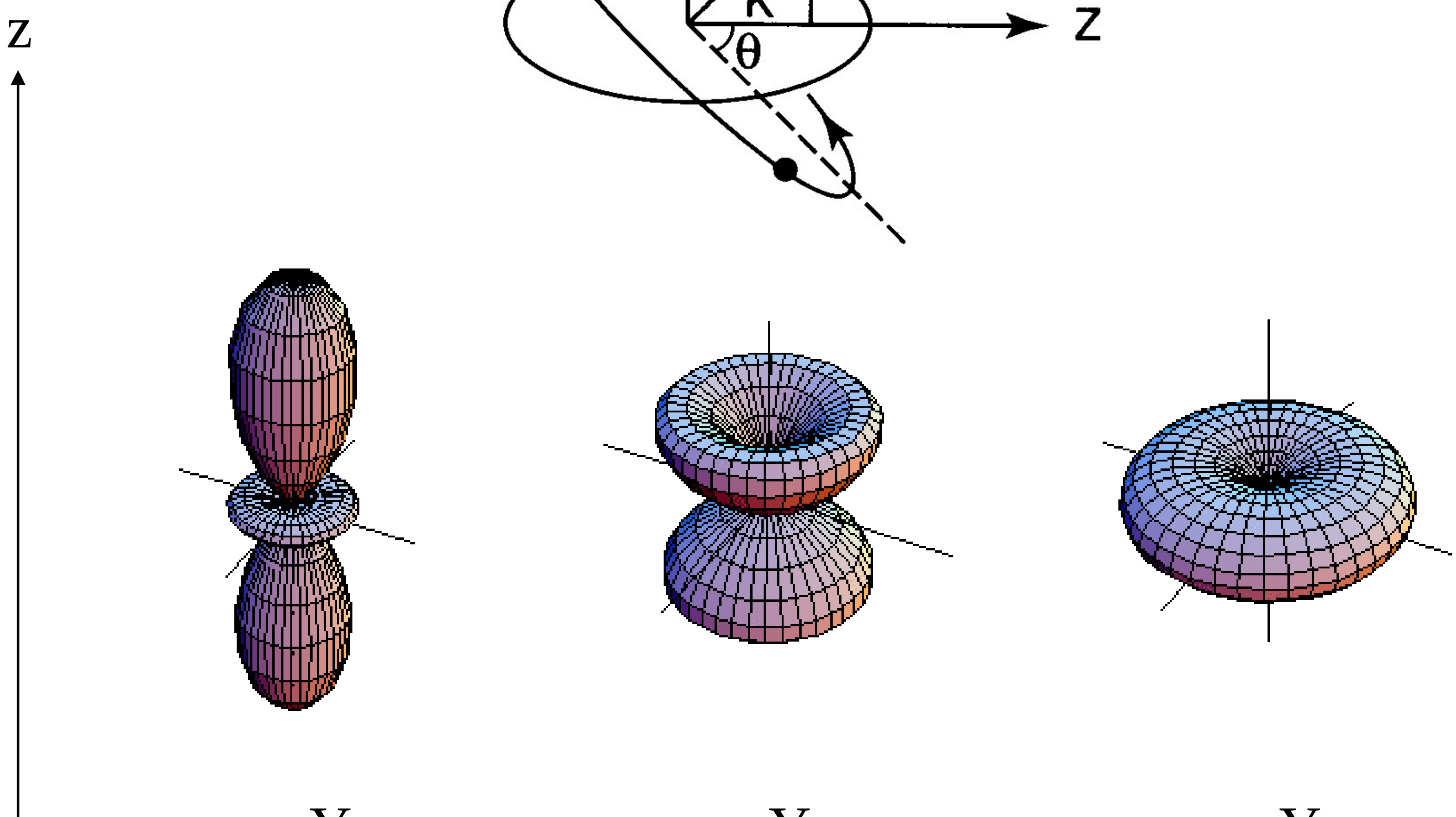
large l_z ⟷ a motion along the shorter axis $\theta = 0^\circ$, $l_z = l$, E ↑

→ the energy is increased



l=0
l=1
l=2
l=3
l=4





$r = Y_{20}$ (l_z=0) $r = Y_{21}$ (l_z=1) $r = Y_{22}$ (l_z=2)

$Y_{20} = 3z^2 - r^2$

$3z^2 - (x^2+y^2+z^2)$ if $x=y$

$= 2z^2 - 2x^2 > 0$

$z > x$ (prolate)

$z < x$ (oblate)

The nuclear shape in the deformed WS potential depends on ε, which is related to as the deformation parameter β_2.

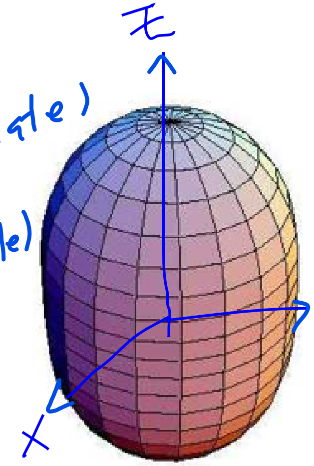
$$\varepsilon = 3(\omega_\perp - \omega_3)/(2\omega_\perp + \omega_3)$$

$$\beta_2 \approx \frac{2}{3}\sqrt{\frac{4\pi}{5}}\,\varepsilon$$

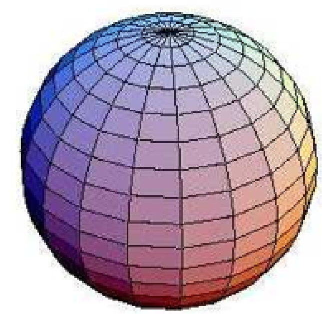
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta))$$

In experimental side, β_2 can be extracted from E2 transition probability.

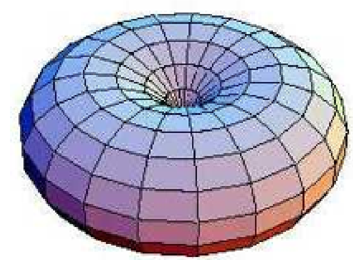
$$\beta_2 = \frac{4\pi}{3ZR_0^2}\left[\frac{B(E2\uparrow)}{e^2}\right]^{1/2} \qquad (R_0 = 1.2A^{1/3})$$



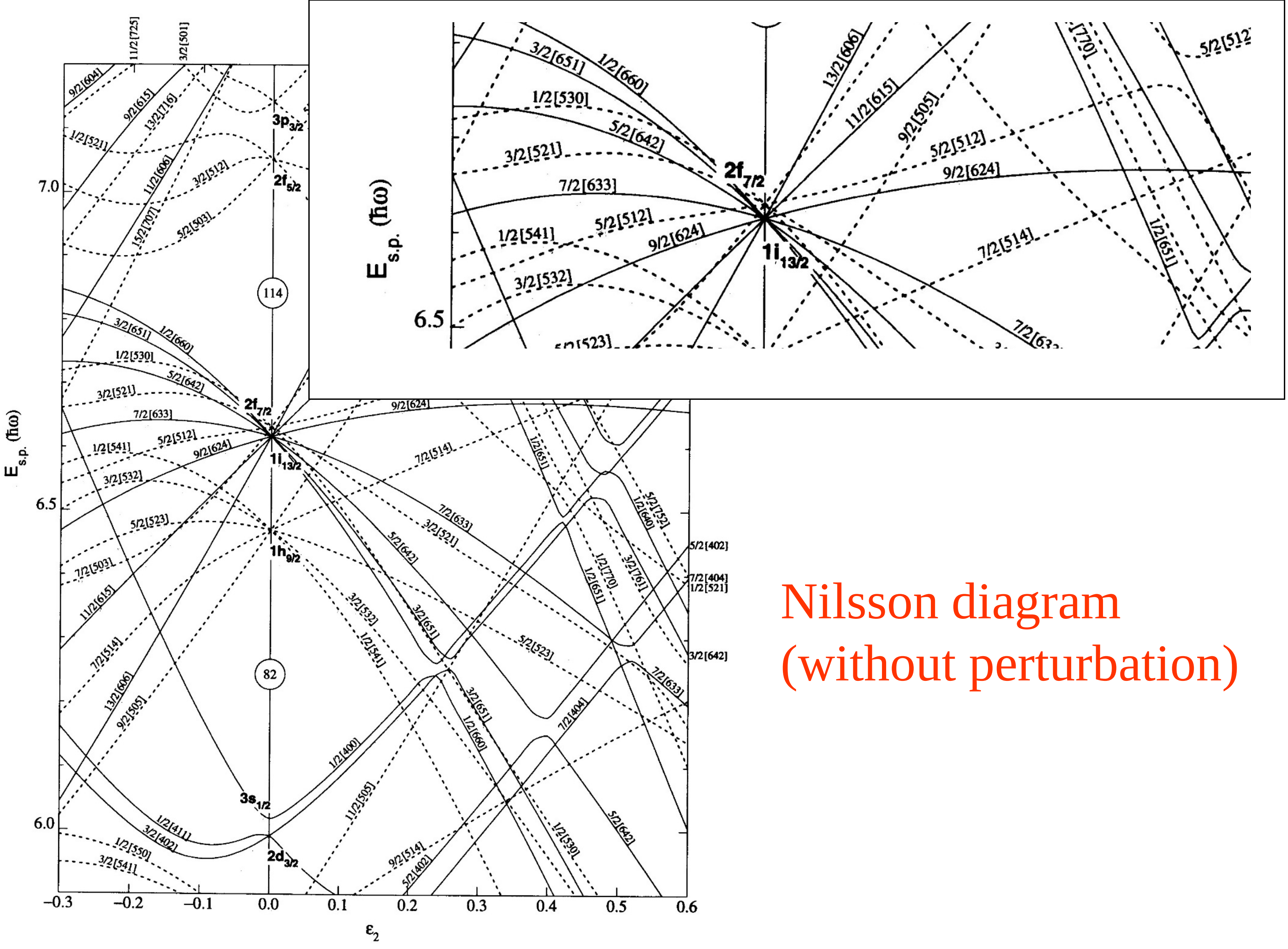
$\beta_2 > 0$, *prolate*



$\beta_2 = 0$, *spherical*



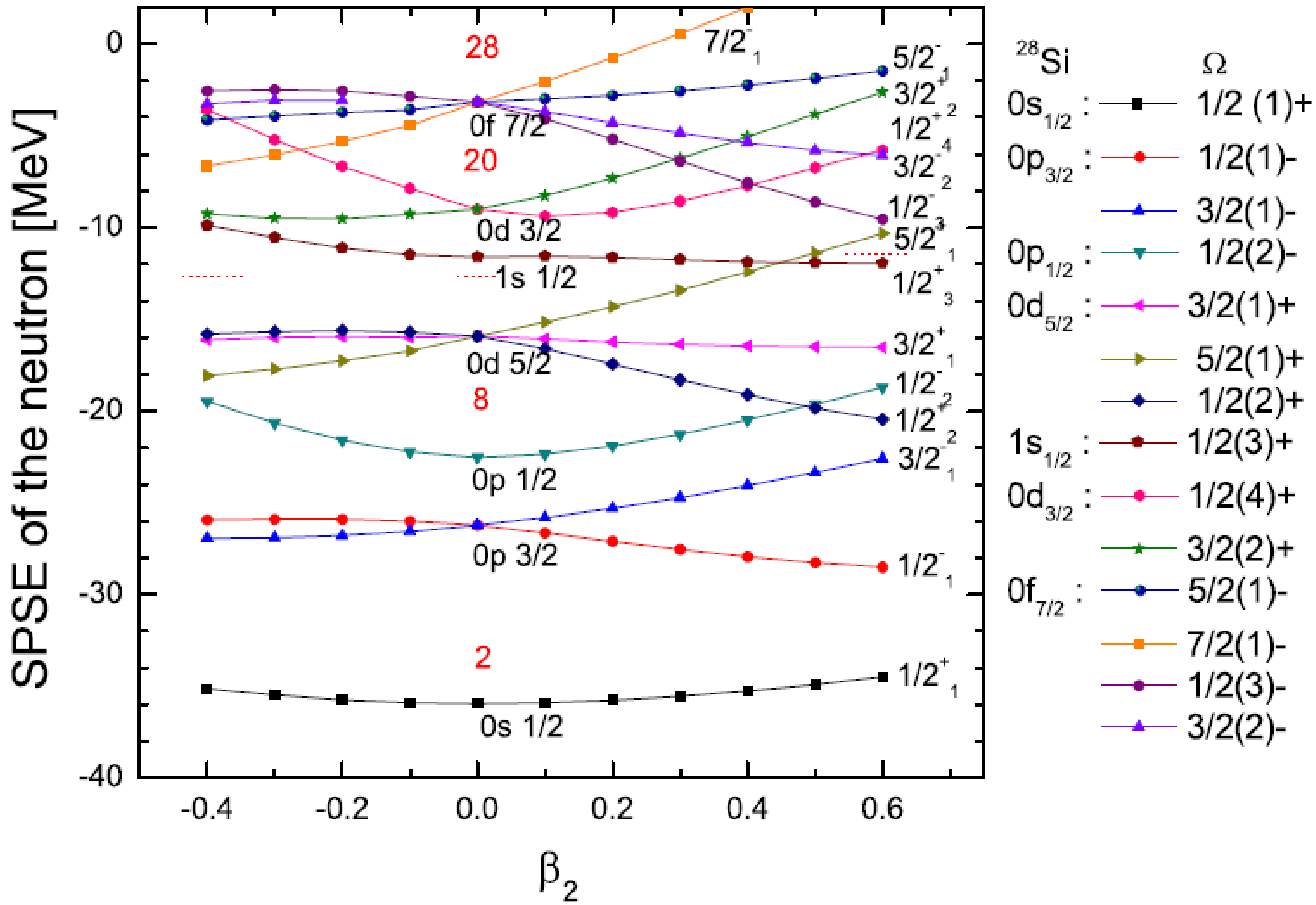
$\beta_2 < 0$, *oblate*



Nilsson diagram (without perturbation)

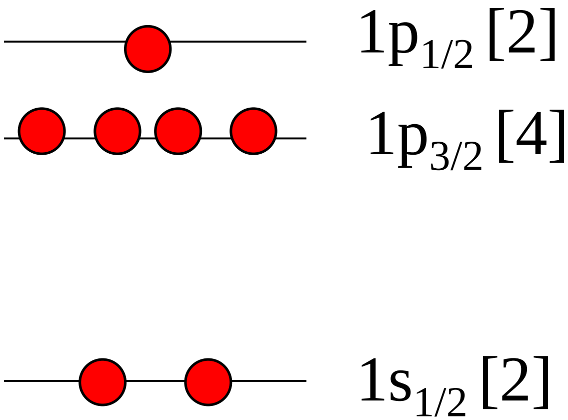
Figure 13. Nilsson diagram for protons, Z ≥ 82 ($\varepsilon_4 = \varepsilon_2^2/6$).

EPJA (2017)
Ha et al.



Level scheme of $^{11}_{4}Be_{7}$

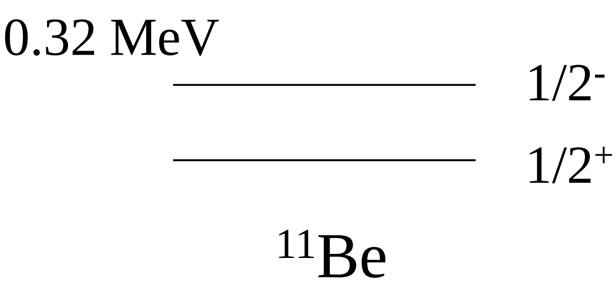
With a spherical potential :



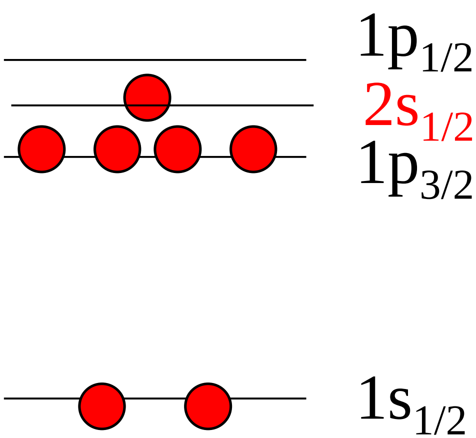
The g.s. of ^{11}Be : $I^{\pi} = 1/2^{-}$

very artificial

In reality…..

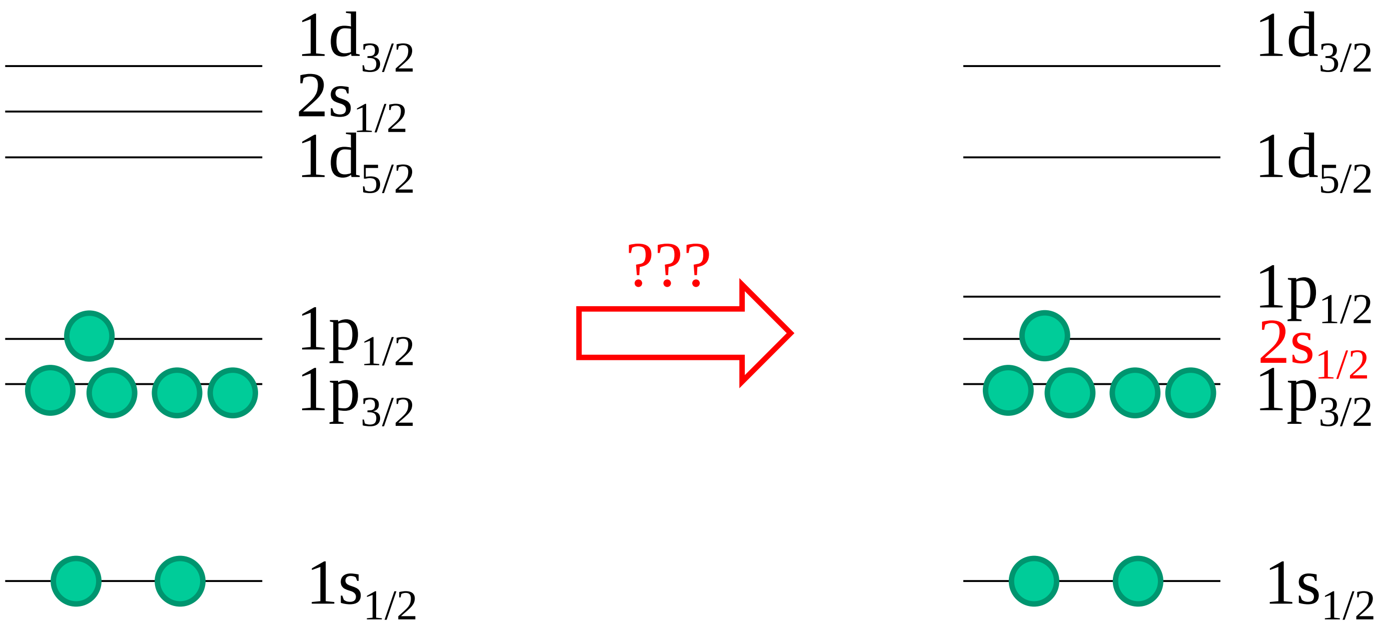


?



"parity inversion"

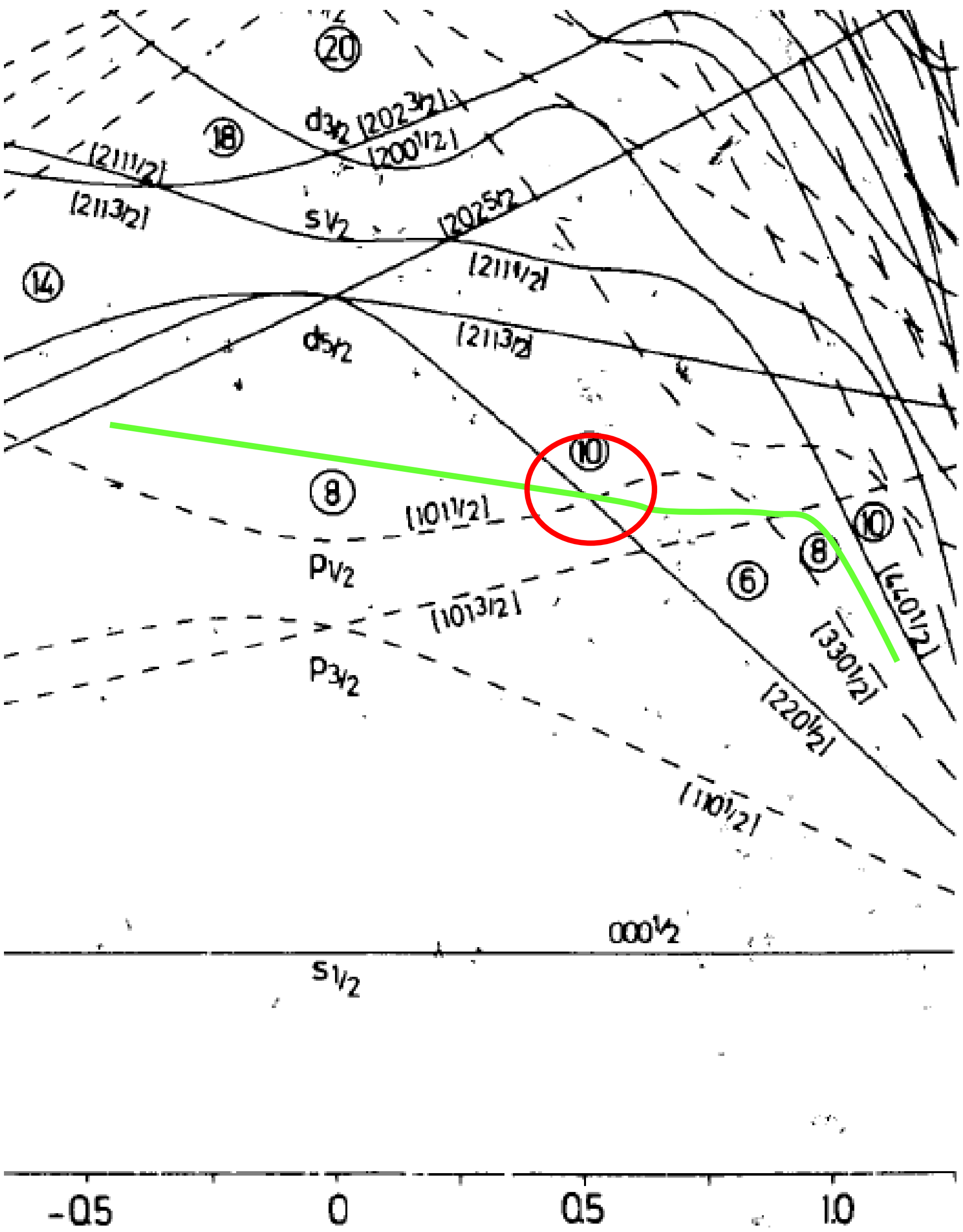
What happens if ^{11}Be is deformed?



Very unnatural.

The observed $1/2^+$ state can be more naturally explained if one considers a deformation of ^{11}Be.

$^{11}_{4}Be_{7}$



deformation parameter

0.32 MeV

$1/2^-$

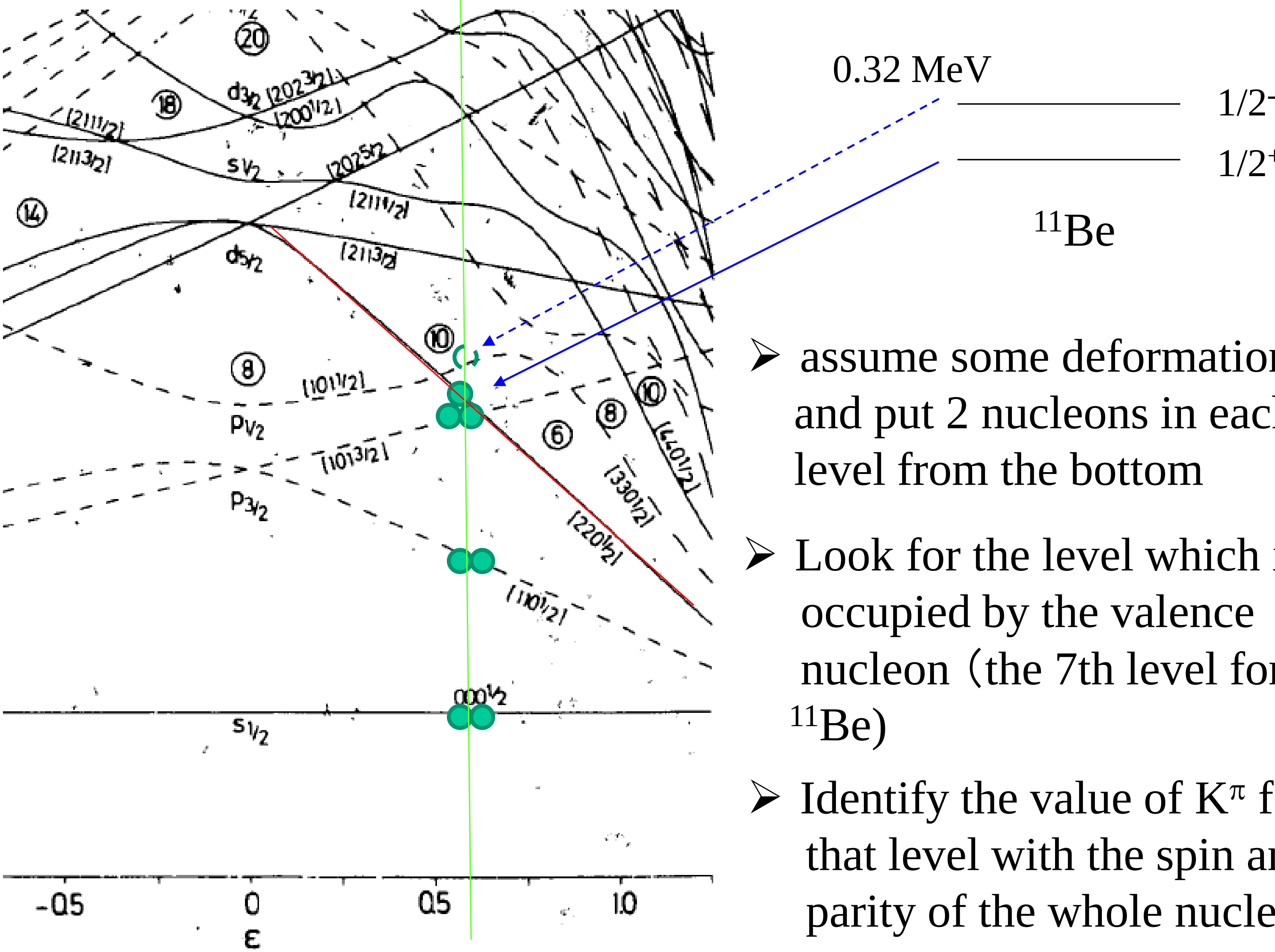
$1/2^+$

^{11}Be

- assume some deformation, and put 2 nucleons in each level from the bottom (degeneracy of +*K* and –*K*)
- Look for the level which is occupied by the valence nucleon（the 7th level for ^{11}Be)
- Identify the value of K^{π} for that level with the spin and parity of the whole nucleus.

cf. particle-rotor model

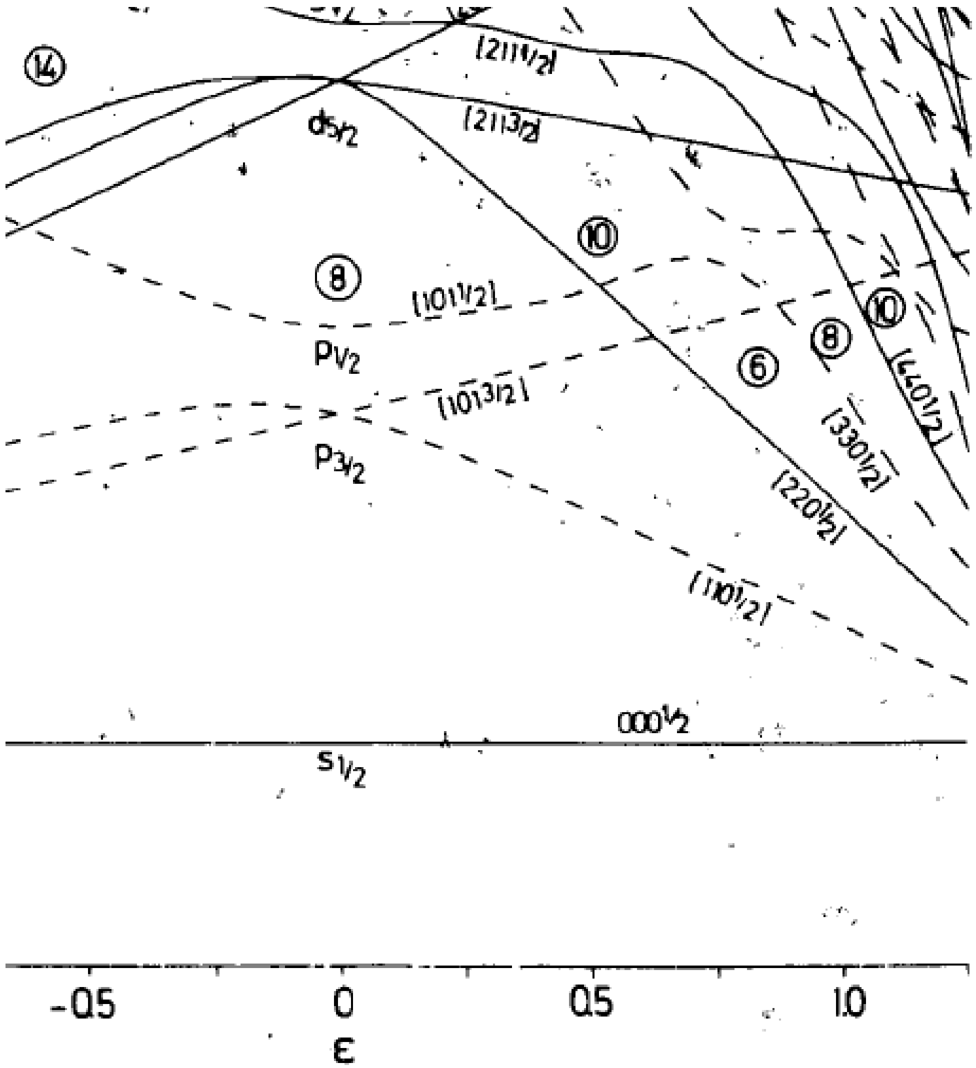
$^{11}{}_{4}Be_{7}$

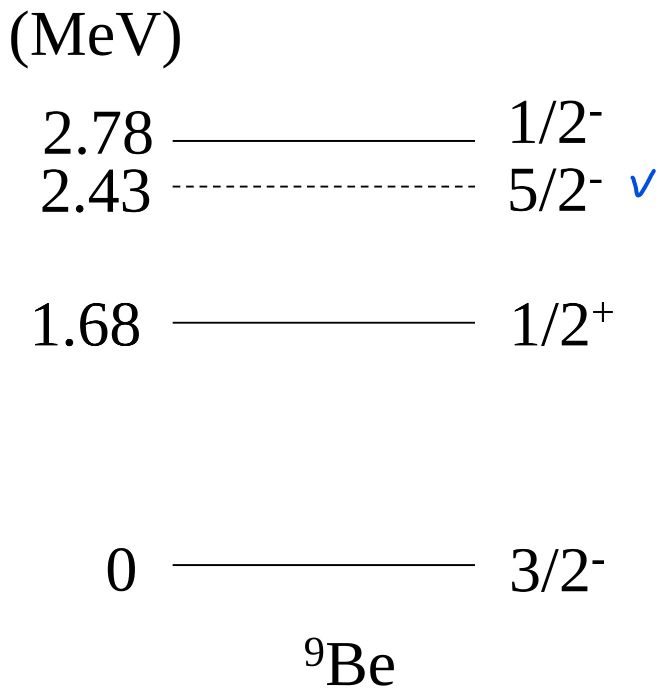


- assume some deformation, and put 2 nucleons in each level from the bottom
- Look for the level which is occupied by the valence nucleon（the 7th level for ^{11}Be)
- Identify the value of K^{π} for that level with the spin and parity of the whole nucleus.

Can the level scheme of $^{9}_{4}Be_5$ be explained in a similar way?

cf. $^{10}B(e,e'K^+)^{10}_{\Lambda}Be$ (= $^{9}Be+\Lambda$)

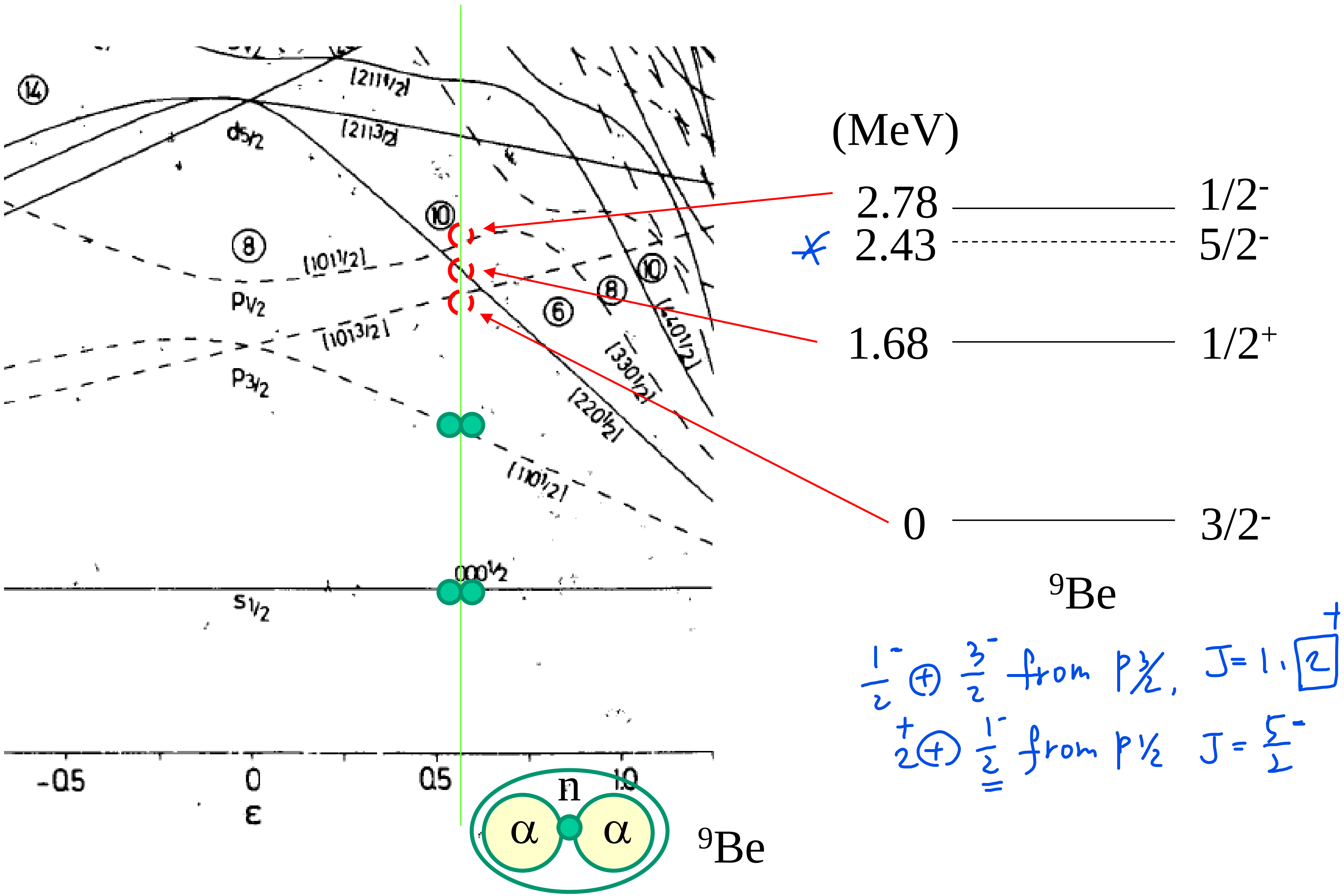




The $5/2^-$ state at 2.43 MeV: rotational state with the same configuration as the g.s. state (not considered here)

Can the level scheme of $^{9}_{4}Be_{5}$ be explained in a similar way?

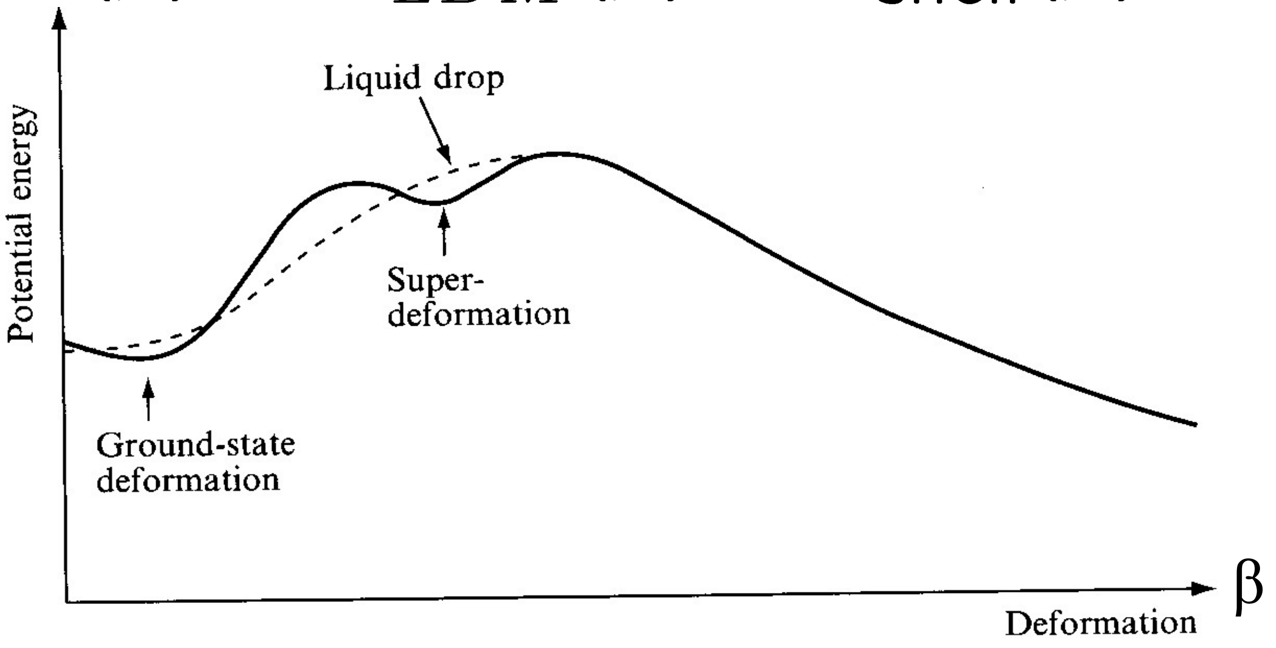
cf. $^{10}B(e,e'K^{+})^{10}_{\Lambda}Be$ (= $^{9}Be+\Lambda$)



nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\mathsf{shell}}(\beta)$$



LDM only → always spherical ground state energy gap if deformed

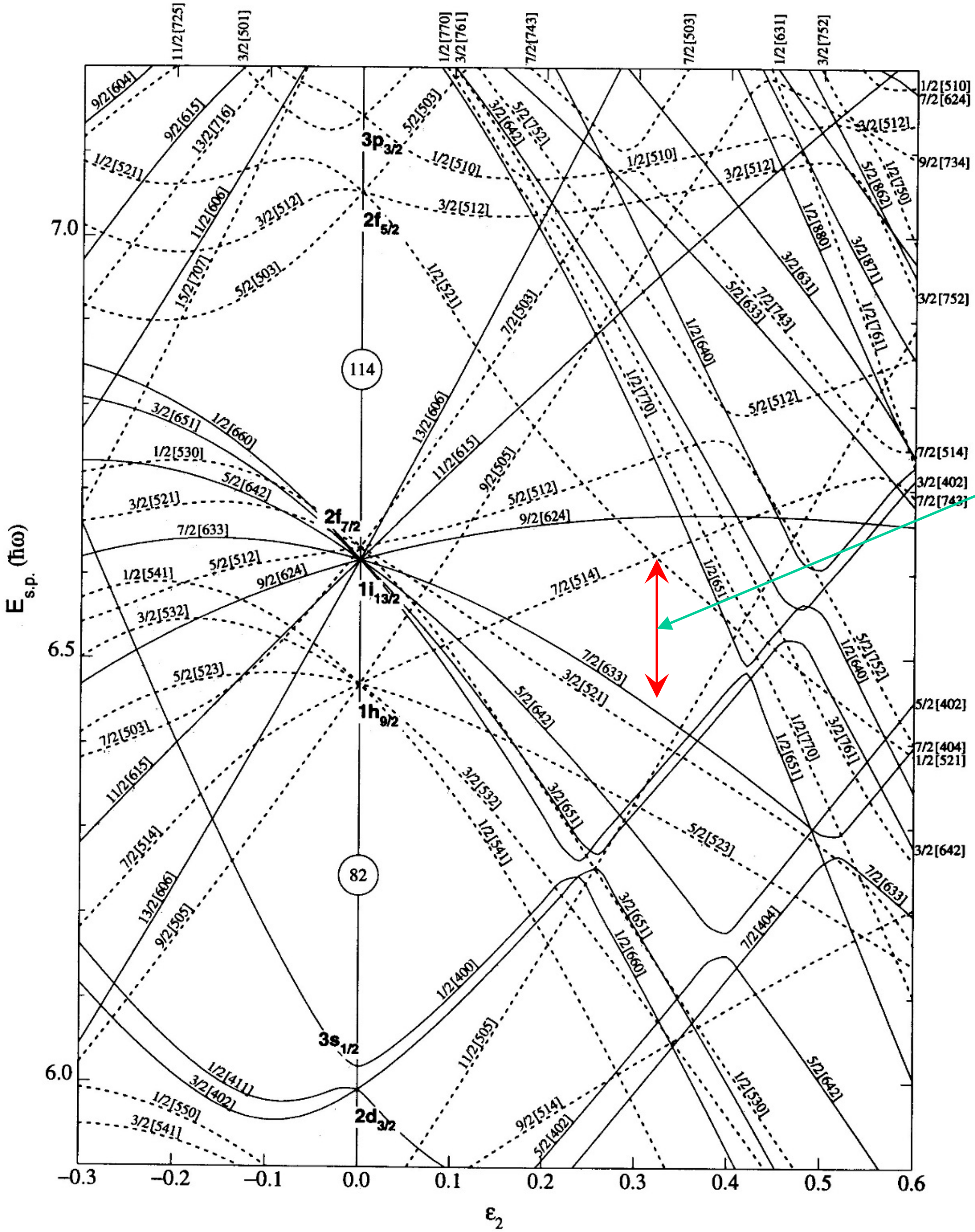


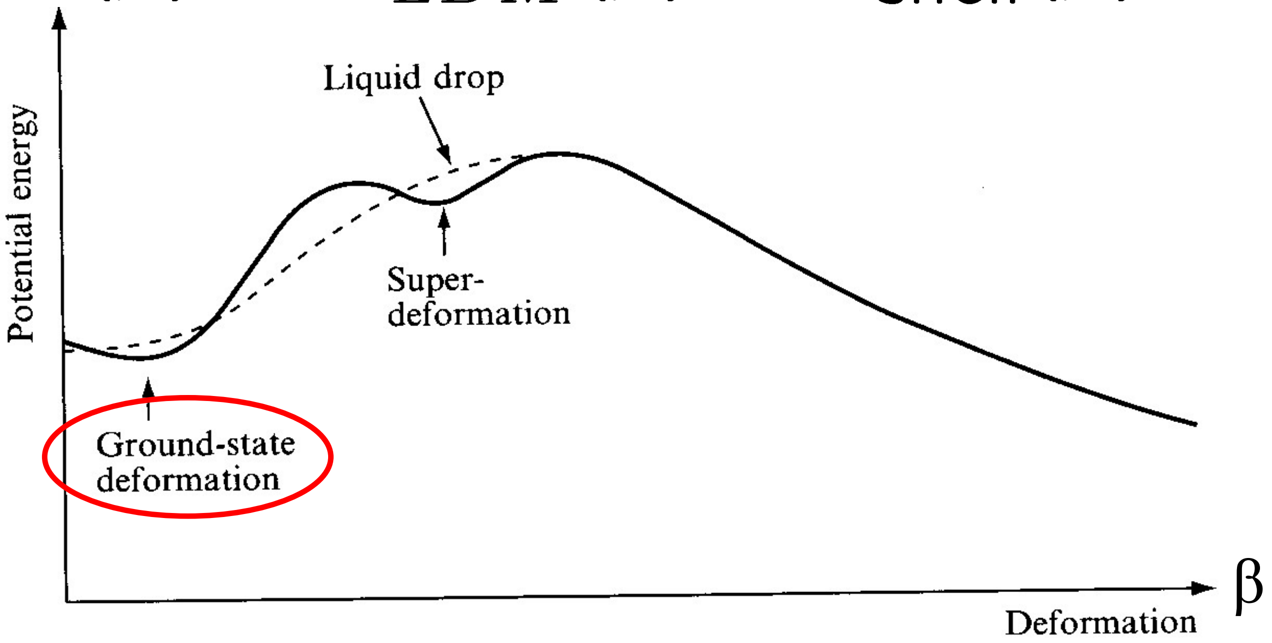
Figure 13. Nilsson diagram for protons, Z ≥ 82 ($\varepsilon_4 = \varepsilon_2^2/6$).

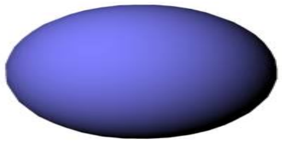
Nilsson diagram

nuclear deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\mathsf{shell}}(\beta)$$





LDM only → always spherical ground state

Shell correction → may lead to a deformed g.s.

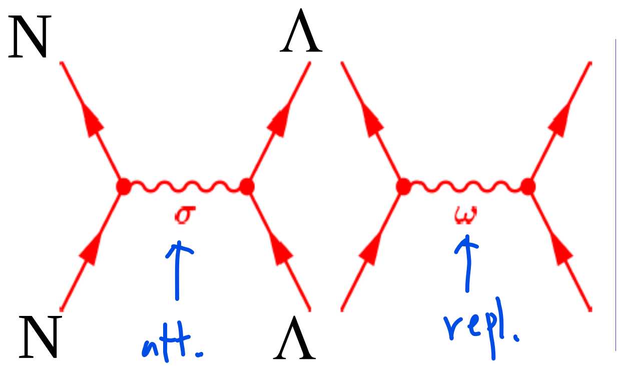
* Spontaneous Symmetry Breaking

RMF calculations for deformed hypernuclei

Hypernuclei: nucleus + Lambda particle

Effect of a Λ particle on nuclear shapes?

Relativistic Mean-field model

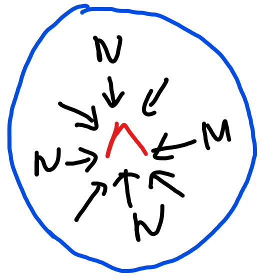


nucleon-nucleon interaction
via meson exchange

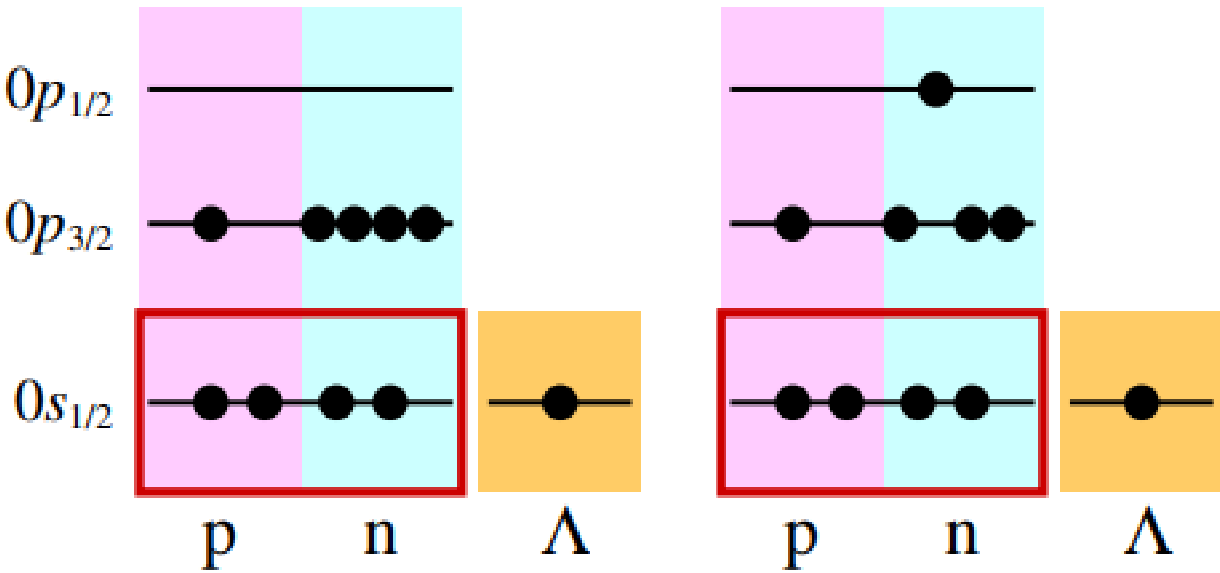
Λσ and Λω couplings

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

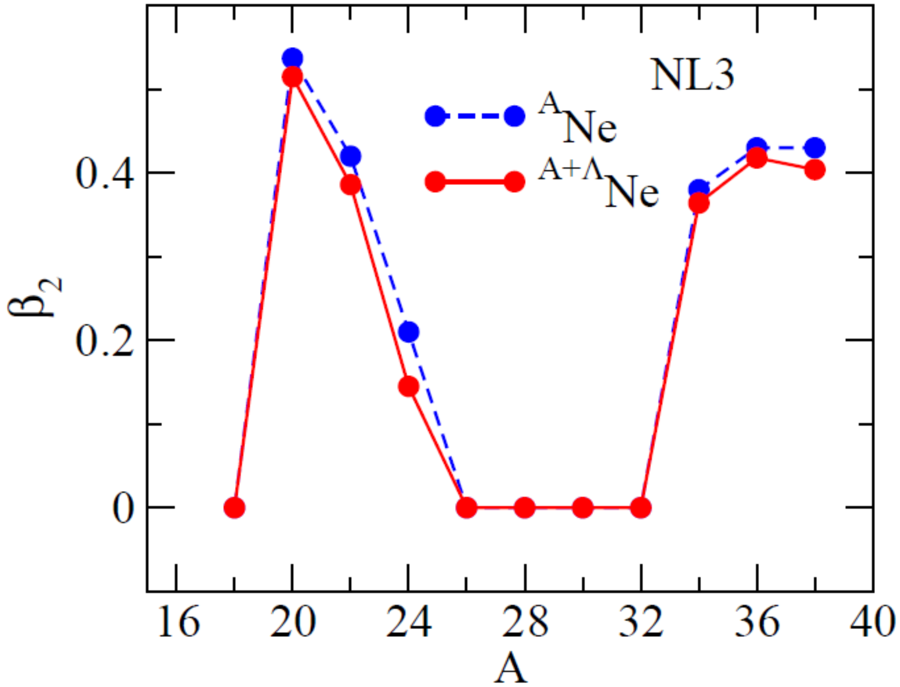
No Pauli principle Between N and Λ.



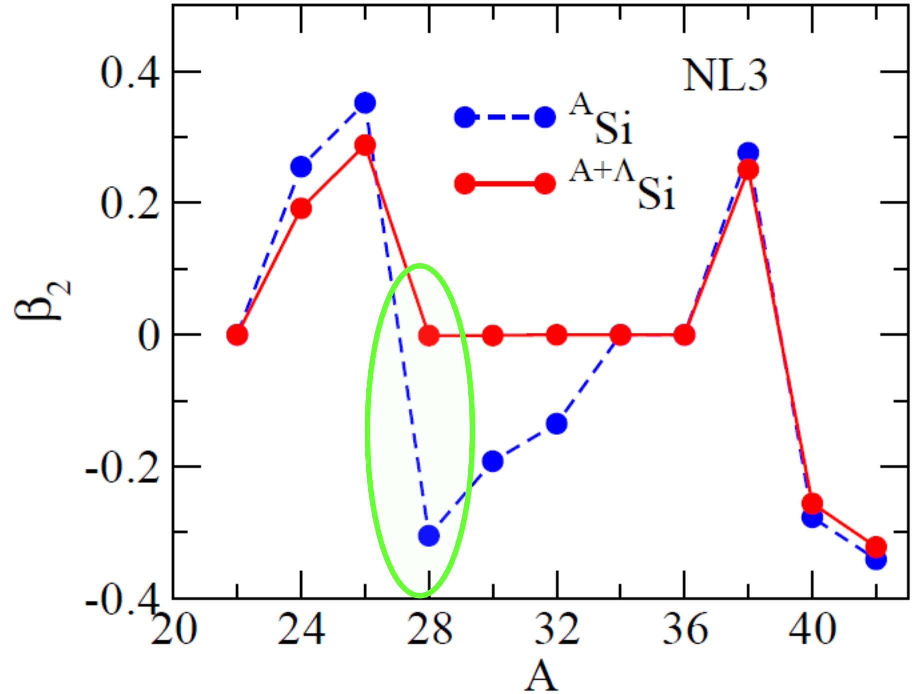
Examples of basis states



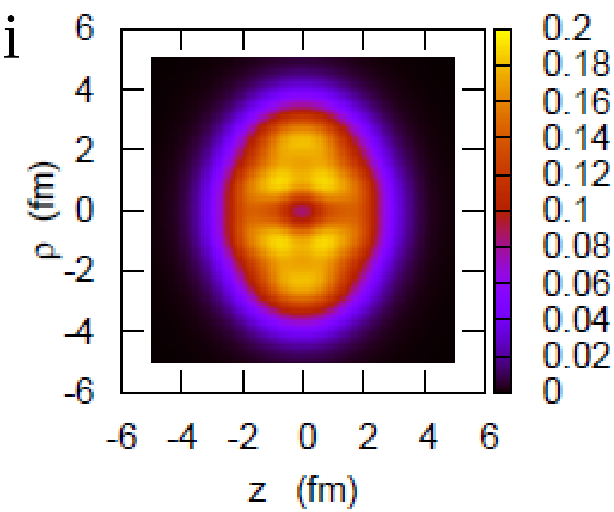
Ne isotopes



Si isotopes

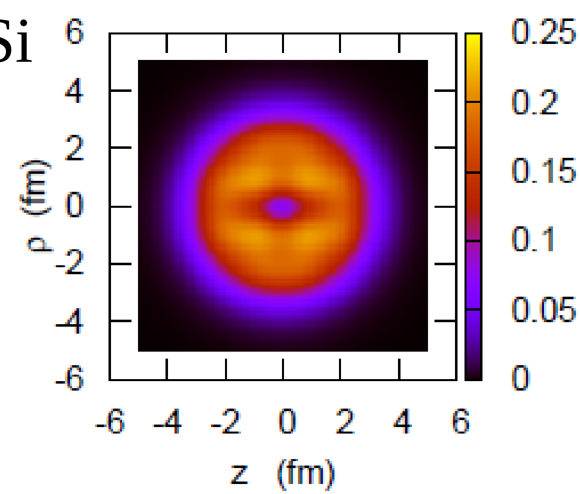


^{28}Si



Λ →

$^{29}{}_{\Lambda}$Si



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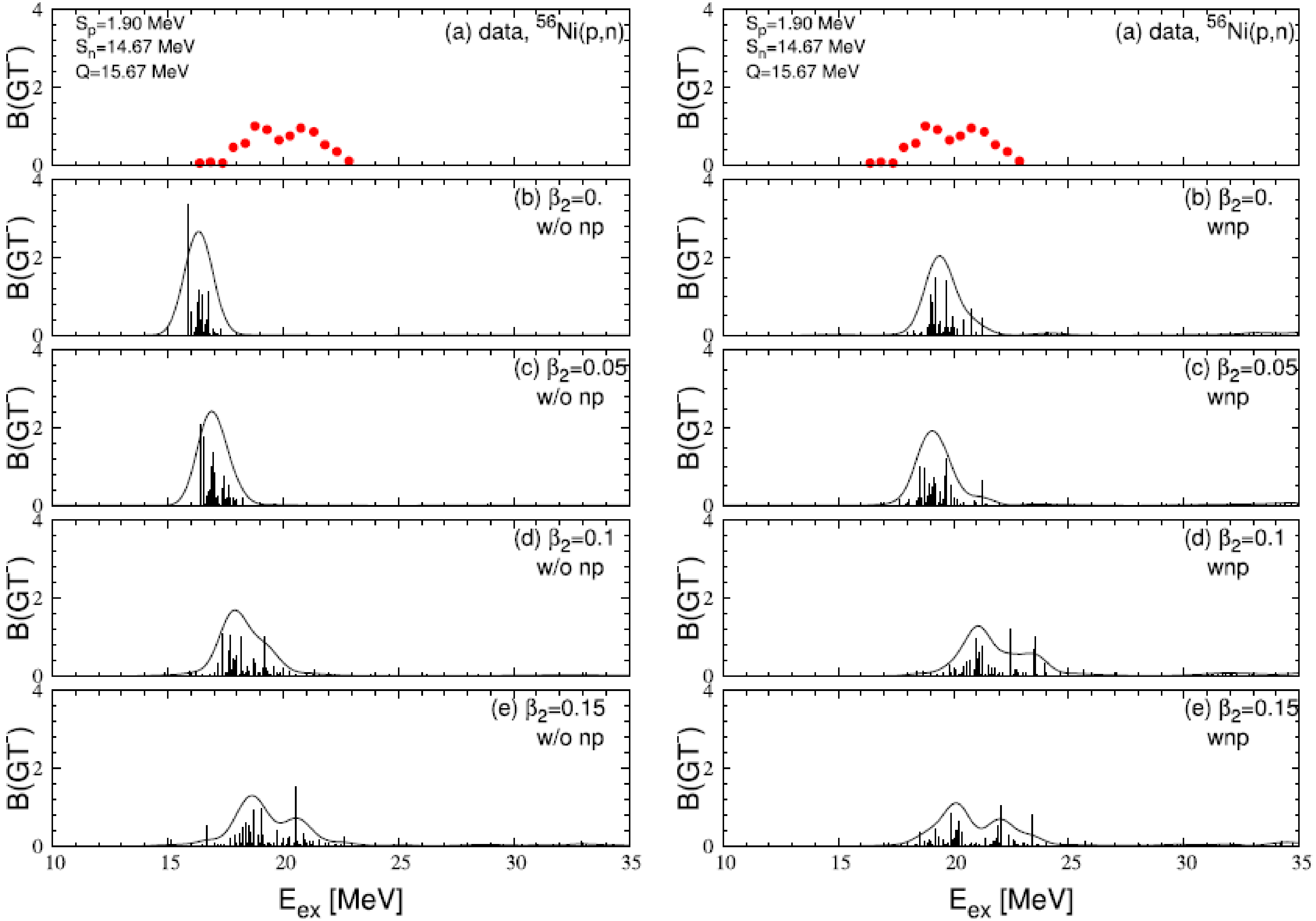

S_p=1.90 MeV
S_n=14.67 MeV
Q=15.67 MeV
(a) data, ^{56}Ni(p,n)
(b) β_2=0. w/o np
(c) β_2=0.05 w/o np
(d) β_2=0.1 w/o np
(e) β_2=0.15 w/o np
(b) β_2=0. wnp
(c) β_2=0.05 wnp
(d) β_2=0.1 wnp
(e) β_2=0.15 wnp
B(GT)
0
2
4
10
15
20
25
30
35
E_{ex} [MeV]