Remarks on non-singular black hole models

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Based on:

"Information loss problem and a 'black hole` model with a closed apparent horizon", V.F., JHEP 1405 (2014) 049;

"Notes on non-singular models of black holes", V. F., Phys.Rev. D94 (2016) no.10, 104056;

"Quantum radiation from a sandwich black hole", V. F. and A. Zelnikov, Phys.Rev. D95 (2017) no.4, 044042;

"Quantum radiation from an evaporating non-singular black hole", V. F. and A. Zelnikov, Phys.Rev. D95 (2017) no.12, 124028.

According to GR: Singularity exists inside a black hole.

Theorems on singularities: Penrose and Hawking.

There exists a curvature singularity inside
a stationary BH in the Einstein gravity $\Rightarrow$
This theory is UV incomplete.

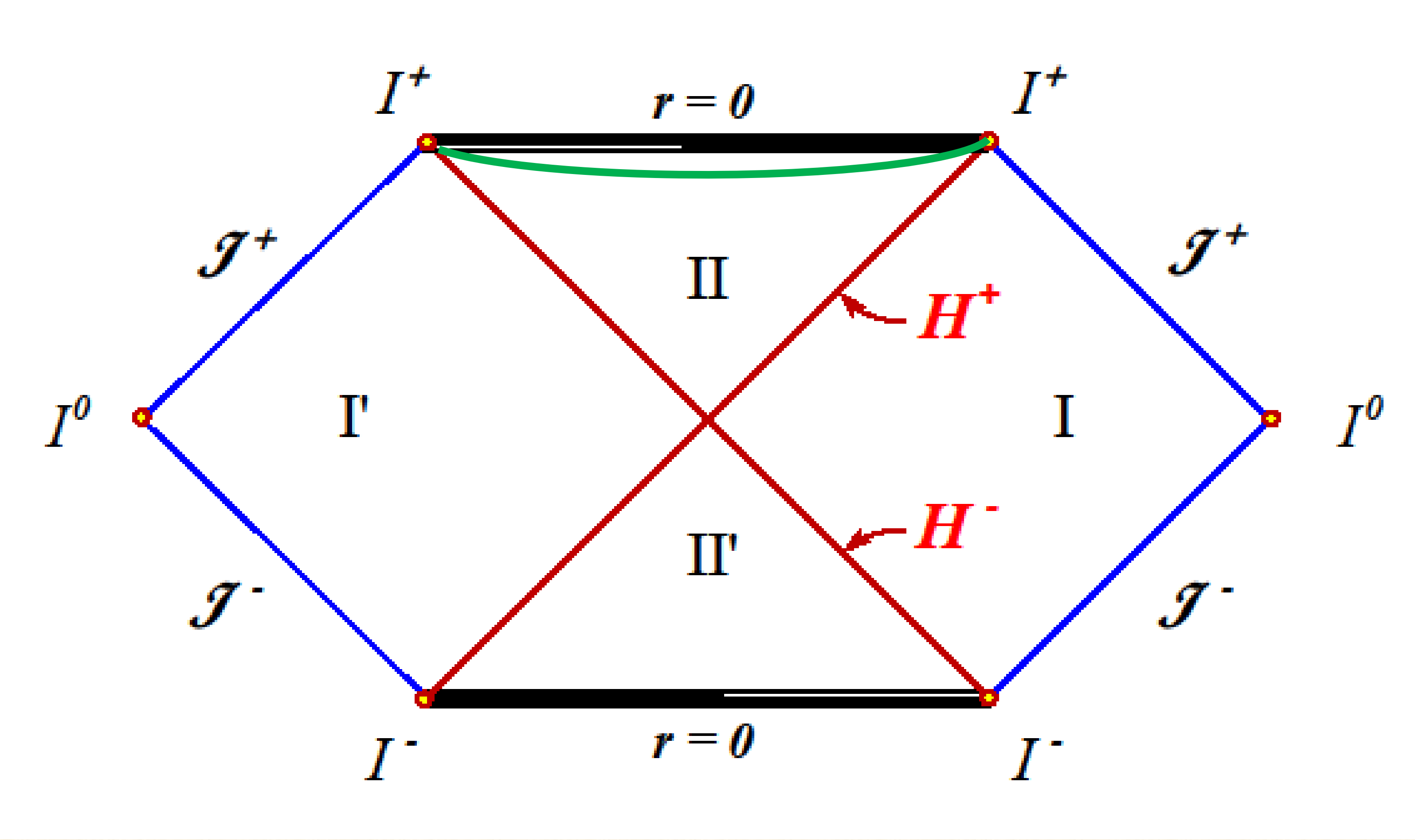
Expectations: When curvature becomes high (e.g. reaches the Planckian value) the classical GR should be modified. Singularities of GR would be resolved.

$$\mathbf{R}^2 = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \frac{48M^2}{r^6},$$

$$\mathbf{R}^2 = \ell^{-4} \Rightarrow r_* = \ell \left(4\sqrt{3} \frac{M}{\ell} \right)^{1/3}$$

$$M \sim 4 \times 10^6 M_\odot, \quad \ell \sim 10^{-33} \mathrm{cm},$$

$$r_* \sim 10^{-7} \mathrm{cm} \gg \ell$$



I^+
$r = 0$
I^+
$\mathcal{J}^+$
II
H^+
$\mathcal{J}^+$
I^0
I'
I
I^0
H^-
II'
$\mathcal{J}^-$
$\mathcal{J}^-$
I^-
$r = 0$
I^-

$$S^2 \times R$$

$$ds^2 = -\frac{dr^2}{\frac{2M}{r} - 1} + \left(\frac{2M}{r} - 1\right)dt^2 + r^2 d\omega^2,$$

$$\tau = \int dr \sqrt{\frac{r}{2M - r}} \approx \frac{2}{3\sqrt{2M}} r^{3/2},$$

$$ds^2 \sim -d\tau^2 + a\,\tau^{-2/3} dt^2 + b\,\tau^{4/3} d\omega^2$$

$$T_{evap} \sim t_{Pl}(M / m_{Pl})^3, \quad M = 10^{15} g, \quad T_{evap} \sim 10^{17} \sec,$$

$$L_{evap} \sim 10^{28} cm, \quad r_* = l_{Pl}(M / m_{Pl})^{1/3},$$

$$L \sim l_{Pl}(M / m_{Pl})^{11/3} \sim L_{evap}(M / m_{Pl})^{2/3} \sim 10^{13} L_{evap}$$

Modified gravity: Options:

(i) Vacuum polarization and particle creation $\rightarrow$
Effective action (higher derivatives and non-locality);

(ii) Modified fundamental gravity (higher derivatives, $f(R)$ theory, etc.);

(iii) Non-local modification (Ghost-free gravity);

(iv) Gravity as an emergent phenomenon (strings, loops, etc.)

Phenomenological description

(i) There exists the critical energy scale parameter μ . The corresponding fundamental length is $\ell = \frac{\hbar}{\mu c}$;

(ii) In the domain where $\Re \ll \ell^{-2}$ the metric obeys the Einstein equations with small corrections;

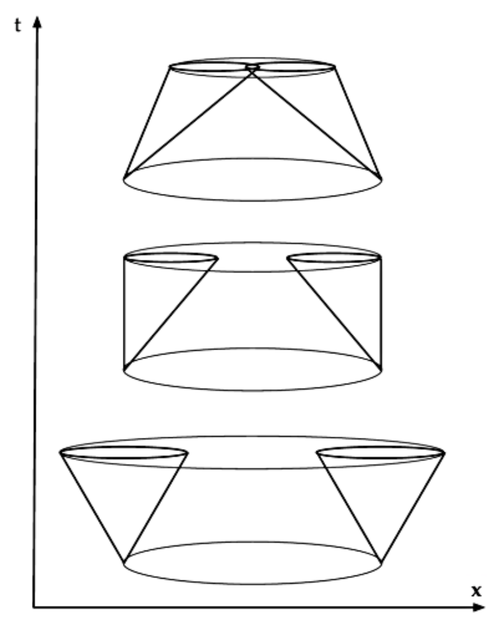
(iii) In the domain where $\Re \sim \ell^{-2}$ the Einstein equations should be modified;

(iv) Limiting curvature condition: $|\Re| \leq \frac{C}{\ell^2}$. C is a universal constant, defined by the theory and independent of the parameters of the solution.

[Markov, JETP Lett. 36, 265 (1982)]

Remark on inflation theory.

`Quasi-local definition’ of BH: Apparent horizon



A compact smooth surface *B* is called a trapped surface if both, in- and out-going null surfaces, orthogonal to *B*, are non-expanding .

A trapped region is a region inside *B*.

A boundary of all trapped regions is called an apparent horizon.

Null energy condition: $T_{\mu\nu}l^{\mu}l^{\mu} \geq 0$

Trapped surface + NEC

=Event horizon existence

In a ST obeying the null-energy condition the apparent horizon lies inside (or coincides with) the true event horizon.

In classical physics in order to prove the existence of a BH (in an exact mathematical sense) it is not necessary to wait infinite time, but it is sufficient to check the existence of the trapped surface `now'.

In quantum physics the energy conditions could be violated. An example is an evaporating black hole (negative energy flux through the horizon reduces its mass).

It is possible, in principle, that the apparent horizon exists, but there is no event horizon.

We shall focus on this option.

General form of SS metric in advanced time coordinates

$$ds^2 = -\alpha^2 f\, dv^2 + 2\alpha\, dv\, dr + r^2 d\Omega^2, \quad f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}\ .$$

$f(v,r)|_{r\to\infty} = 1$. Apparent horizon: $f = 0$.

Red-shift function: α(v,r). In a static ST: $\xi_t^2 = -\alpha^2 f$,

$$\Re^2 \sim 4\left(\frac{[(\nabla r)^2 - 1]}{r^2}\right)^2 \Rightarrow$$ If ST is regular at $r = 0$, the apparent horizon cannot cross this line.

ST is regular at $r=0$, if curvature invariants are finite there:

$$f = 1 + \tfrac{1}{2} f_2(v) r^2 + \dots \quad ,$$

$$\alpha = \alpha_0(v)[1 + \tfrac{1}{2}\alpha_2(v) r^2 + \dots].$$

We use normalization: $\alpha(v,r)|_{r\to\infty} = 1$, then the rate of the proper time at the center, τ, and the rate of the Killing time at infinity, v, are connected as: $d\tau = \alpha_0(v) dv$.

(i) An apparent horizon in a regular metric cannot cross $r = 0$.

(ii) It has two branches: outer- and inner-horizons.

(iii) Non-singular BH model with a closed apparent horizon

[V.F. and G.Vilkovisky, Phys. Lett., 106B, 307 (1981)]

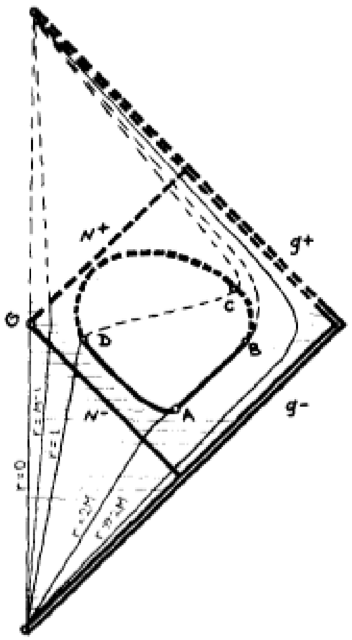


Fig. 1. Penrose diagram for the collapse of the null shell ($M \gg 1$). Solid (dashed) lines are used for the known (hypothetical) details of the picture. The shaded region is the region of validity of the obtained asymptotic solution. The line $N^- \cup N^+$ is the world line of the null shell. The closed and dashed bold line $ABCD$ is the apparent horizon. The light lines are the level lines r = const.

Other publications on regular BH models with closed apparent horizons

T. A. Roman and P.G. Bergmann (1983);
P. Bolashenko and V.F. (1986)
S. N. Solodukhin, (1999);
S. A. Hayward (2006);
S. Ansoldi (2008);
C. Bambi, D. Malafarina, L. Modesto (2013);
V. N. Lukas and V. N. Strokov (2013);
V. Frolov (2014);
J.M. Bardeen (2014);
C. Rovelli and F. Vidotto (2014);
T. De Lorenzo, C. Pacilio , C. Rovelli, S. Speziale (2015);

D.I. Kazakov, S.N. Solodukhin (1993)
P. Hajicek (2002);
D. Grumiller (2003, 2004;
J. Ziprick and G. Kunstatter (2010)

Static SS non-singular black-hole metrics

Remark: All stationary BH solutions in General Relativity can be written in the form, where the metric coefficients are rational functions of the coordinates.

We assume that $f=\dfrac{P_n(r)}{Q_n(r)}$. It has the form $=\dfrac{r^n+\dots}{r^n+\dots}\to 1$ at $r\to\infty$.

Example 1: Bardeen regular black hole [1968]

$$f=1-\frac{2Mr^2}{(r^2+q^2)^{3/2}}\to 1-\frac{2M}{|q|^3}r^2+\dots\,.\quad \Re\sim\frac{M}{|q|^3}.$$

Neither this metric nor its modification ($q\to\ell$) satisfies the limiting curvature condition.

Example 2: Metric with $f = 1 - \frac{2Mr}{r^2 + \ell^2} \sim 1 - \frac{2M}{\ell^2} r + \ldots$

is non-regular.

Metrics with $n \leq 2$ cannot be consistent metrics of a non-singular black hole. [V.F. PR D94,104056 (2016)].

Example 3 (n=3): Hayward metric [2007]:

$$f = 1 - \frac{2M r^2}{r^3 + 2M\ell^2 + \ell^3}, \quad (\alpha = 1).$$

Non-singular evaporating black hole: $M = M(v)$.

Non-singular model of black hole

$$dS^2 = -\alpha^2 f dv^2 + 2\alpha dvdr + r^2 d\omega^2 ,$$

$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2} ,$$

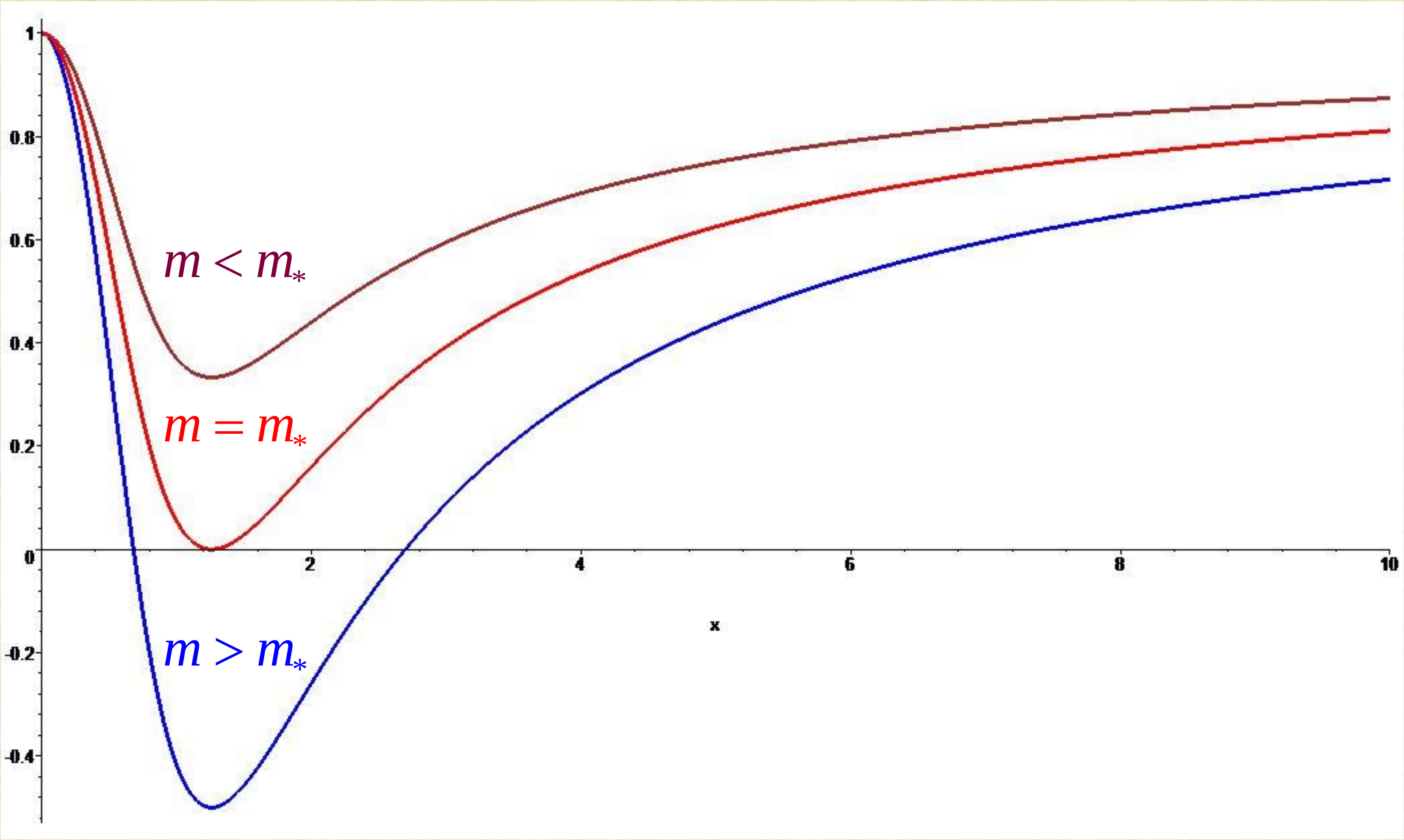
Standard: α=1;

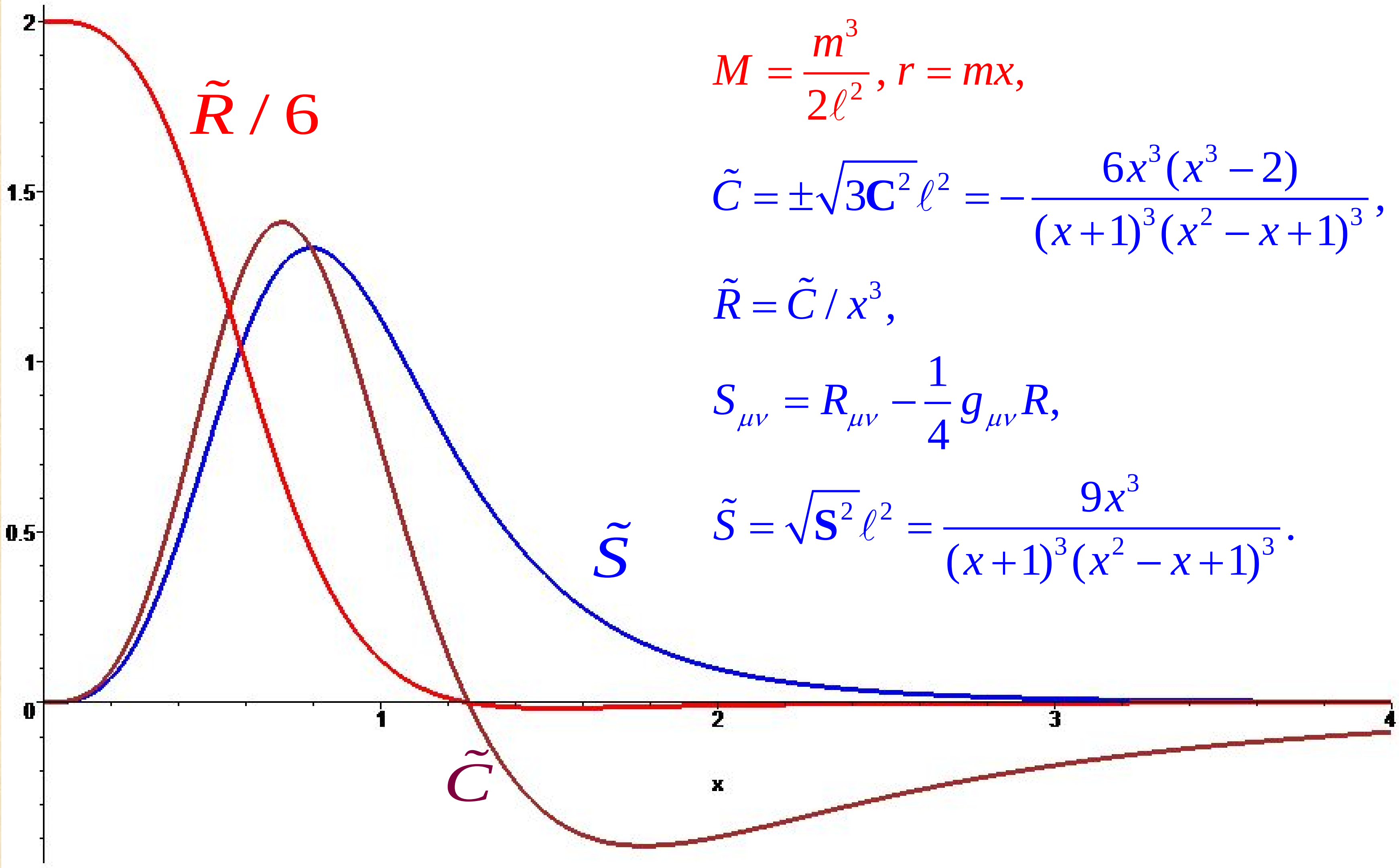
Modified: $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell^{n-k}}$.

For α=1 $\Rightarrow M = \frac{m^3}{2\ell^2}, \quad r = mx,$

$$f = 1 - \frac{m^2}{\ell^2} \frac{x^2}{1 + x^3}$$

$$m_* = (27/4)^{1/6}\,\ell \Leftrightarrow M_* = \frac{3\sqrt{3}}{4}\,\ell$$





$$M = \frac{m^3}{2\ell^2}, \; r = mx,$$

$$\tilde{C} = \pm\sqrt{3\mathbf{C}^2}\,\ell^2 = -\frac{6x^3(x^3-2)}{(x+1)^3(x^2-x+1)^3},$$

$$\tilde{R} = \tilde{C}/x^3,$$

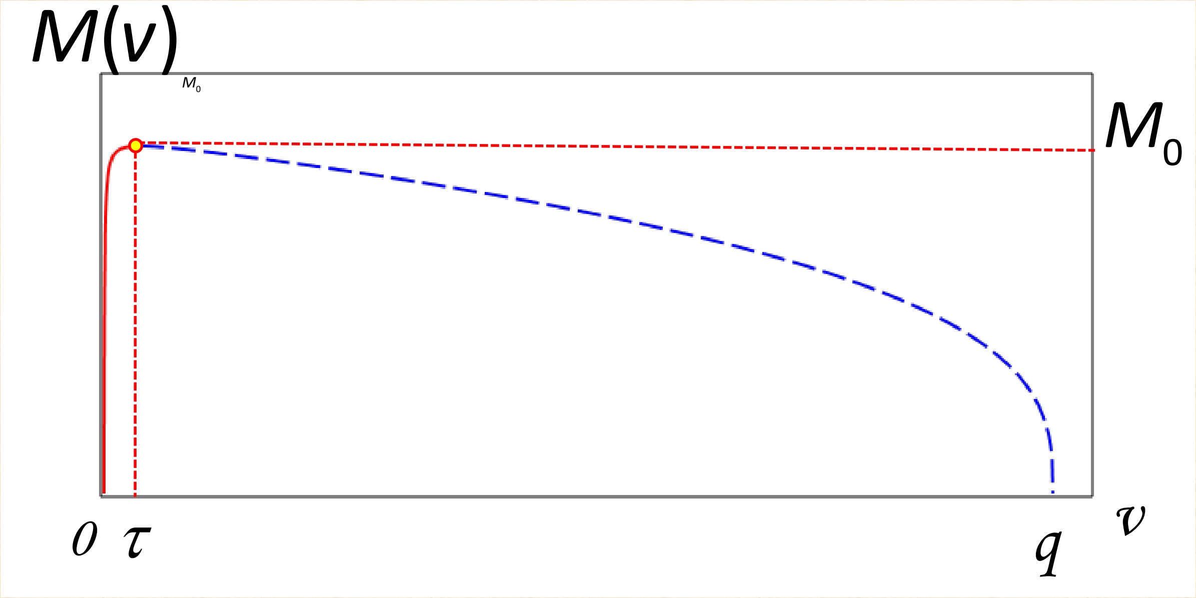
$$S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R,$$

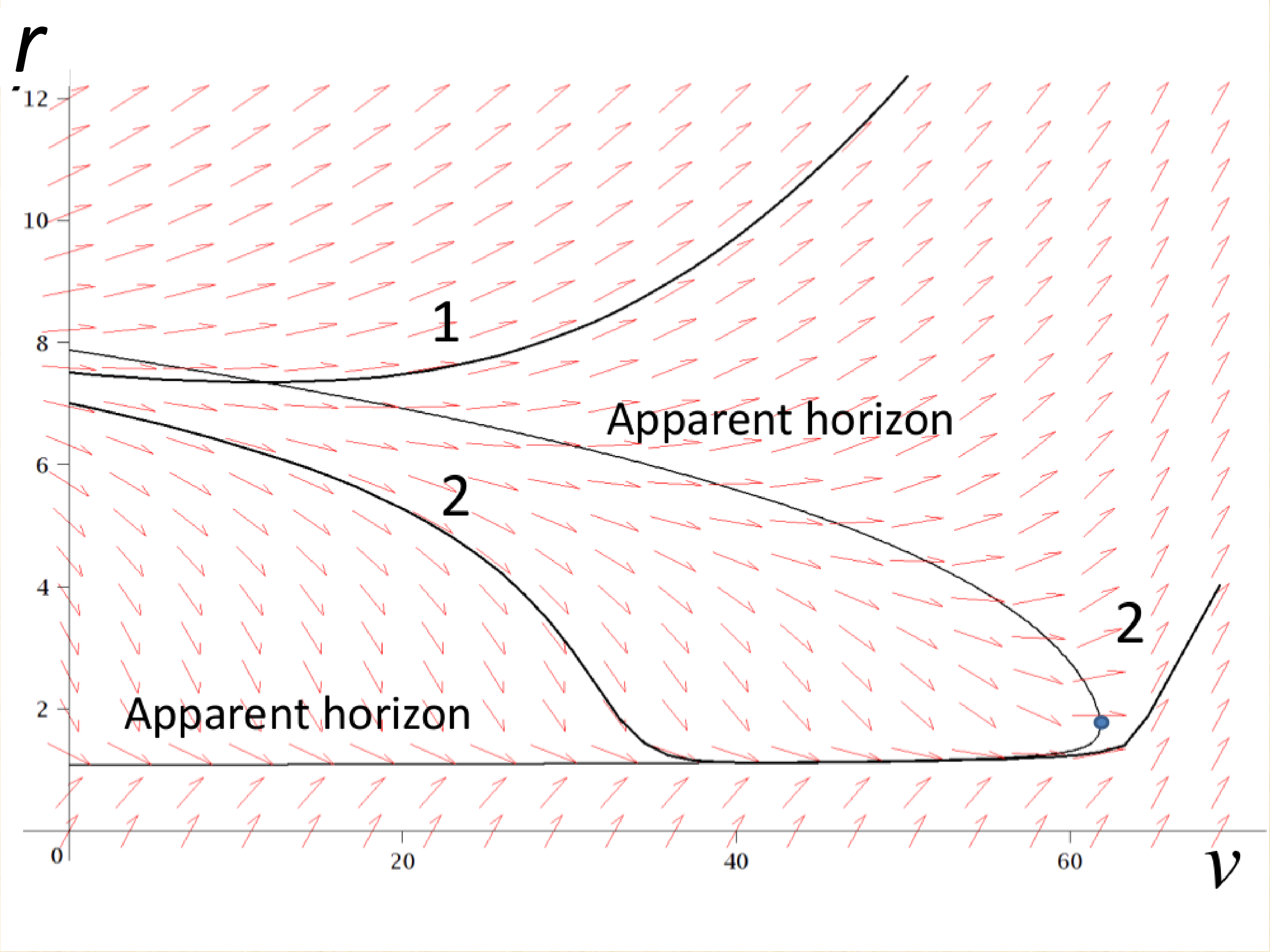
$$\tilde{S} = \sqrt{\mathbf{S}^2}\,\ell^2 = \frac{9x^3}{(x+1)^3(x^2-x+1)^3}.$$

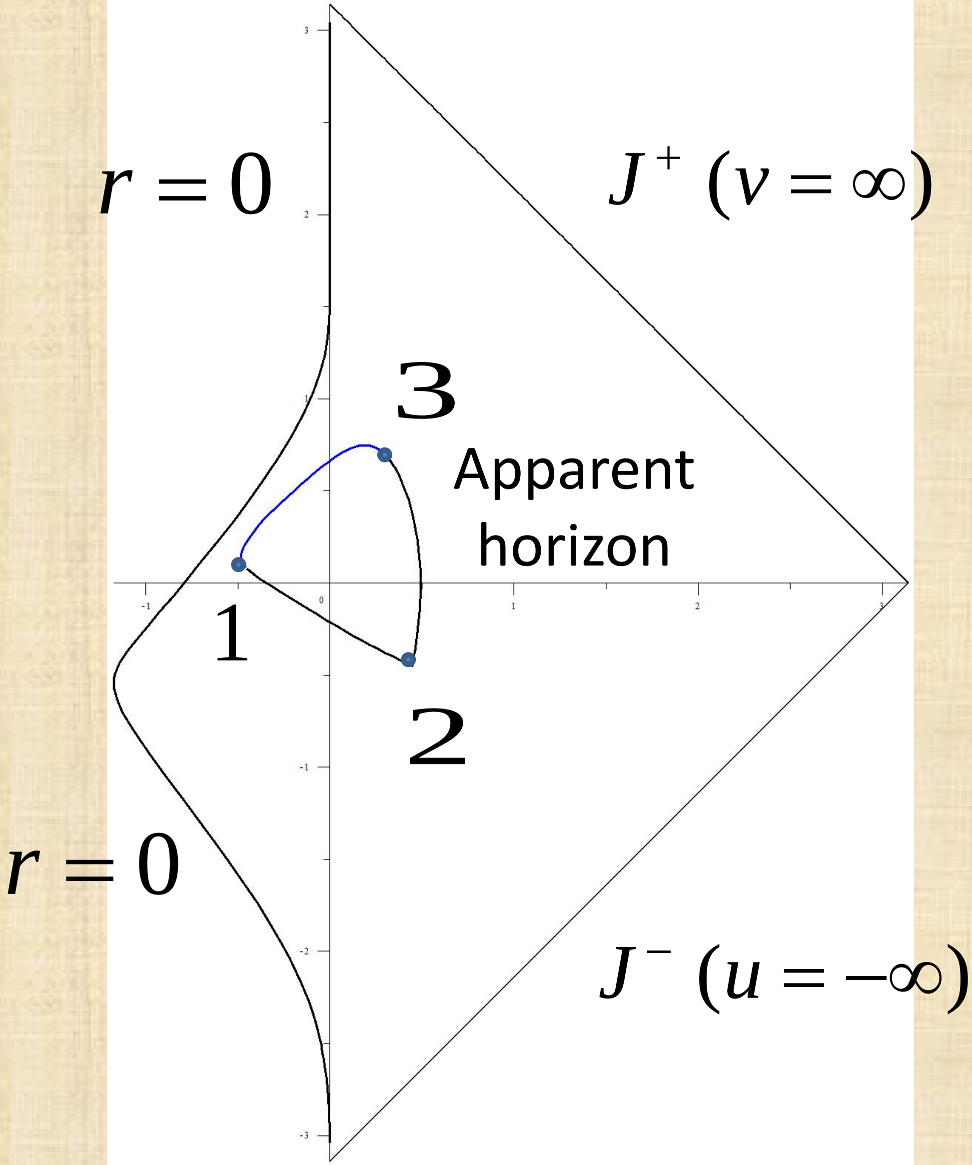
Non-singular model of an evaporating black hole

$$dS^2 = -f dv^2 + 2dvdr + r^2 d\omega^2 \,,$$

$$f = 1 - \frac{2M(v)r^2}{r^3 + 2M(v)\ell^2} \quad (\alpha = 1)$$




r
12
10
8
6
4
2
0
20
40
60
v
1
2
2
Apparent horizon
Apparent horizon



$r = 0$
$J^{+}\ (v = \infty)$
3
Apparent
horizon
1
2
$r = 0$
$J^{-}\ (u = -\infty)$

Information Loss Puzzle

$$\begin{pmatrix} \text{Pure} \\ \text{state} \end{pmatrix} \overset{\text{Collapse}}{\Rightarrow} \begin{pmatrix} \text{Black} \\ \text{hole} \end{pmatrix} \overset{\text{Evaporation}}{\Rightarrow} \begin{pmatrix} \text{Thermal} \\ \text{radiation} \end{pmatrix}$$

Information Loss Puzzle

$$\begin{pmatrix}\text{Pure} \\ \text{state}\end{pmatrix} \overset{\text{Collapse}}{\Rightarrow} \begin{pmatrix}\text{Black} \\ \text{hole}\end{pmatrix} \overset{\text{Evaporation}}{\Rightarrow} \begin{pmatrix}\text{Thermal} \\ \text{radiation}\end{pmatrix}$$

A model of a black hole with a closed apparent horizon is one of the options that were discussed in the connection with the information loss paradox.

Aharonov, Casher, Nussinov [1987];Carlitz, Willey [1987]; Preskill [1992]

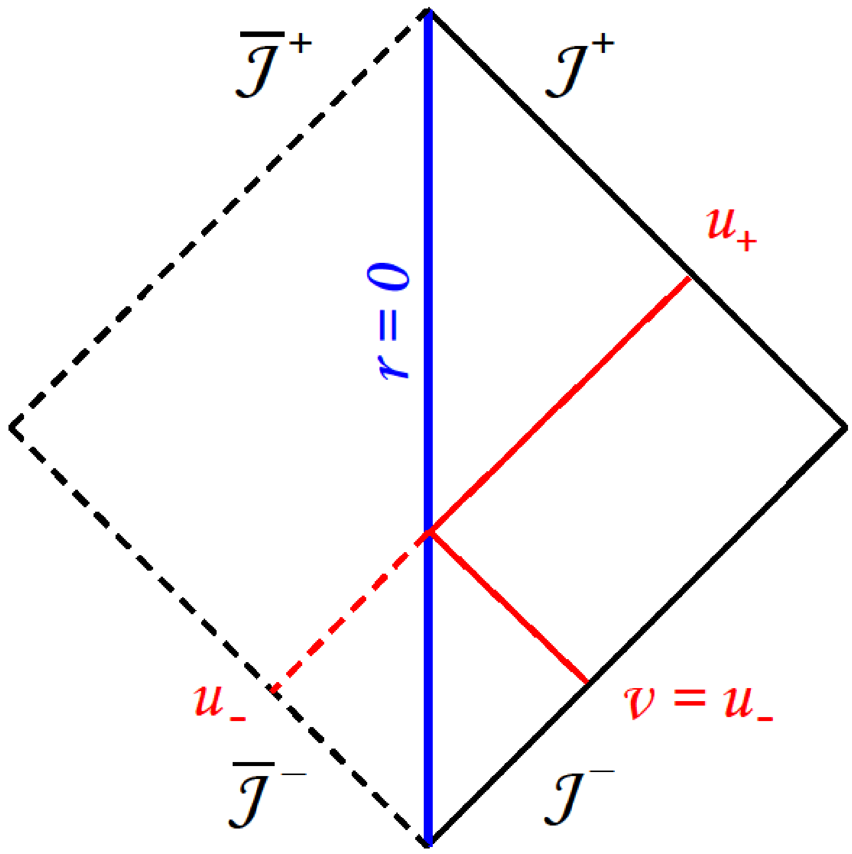
"The final stage of the evaporation process must take a very long time," $T_{ev} \geq M^4$.

Self-consistency problem

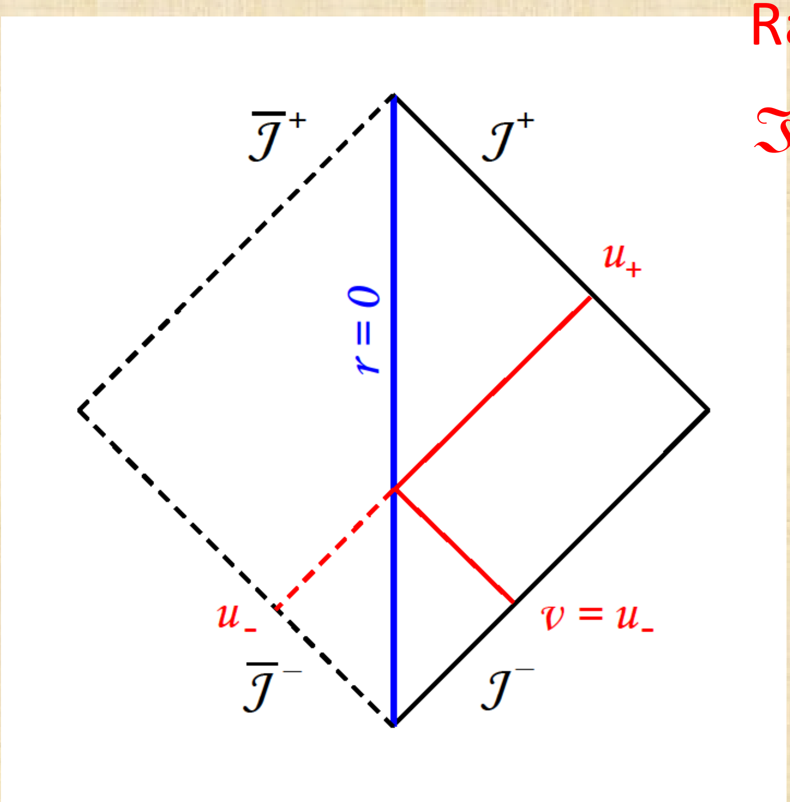
[Bolashenko and V.F. (1986)]

Quantum effects

We consider a quantum massless scalar fields, propagating in the background of a non-singular black hole. We use 2D approximation. The corresponding expectation value of the stress-energy tensor can be easily obtained from the known conformal anomaly.
[Christensen and Fulling, PD, D15, 2088 (1977)].
It can also be derived from Polyakov effective action.
[V.F. and Vilkovisky, in Quantum Gravity, p.267 (1984)]



$\bar{\mathcal{J}}^+$
$\mathcal{J}^+$
$r = 0$
u_+
u_-
$v = u_-$
$\bar{\mathcal{J}}^-$
$\mathcal{J}^-$



Radial null rays provide us with maps:

$\mathfrak{I}^- \leftrightarrow \mathfrak{I}^+ : \quad u_- = u_-(u_+)$ and $u_+ = u_+(u_-)$

$$\dot{E} = \frac{1}{48\pi}\left[-2\frac{d^2P}{du_+^2} + \left(\frac{dP}{\mathrm{du}_+}\right)^2\right],$$

$$P = \ln\left(\frac{du_-}{du_+}\right).$$

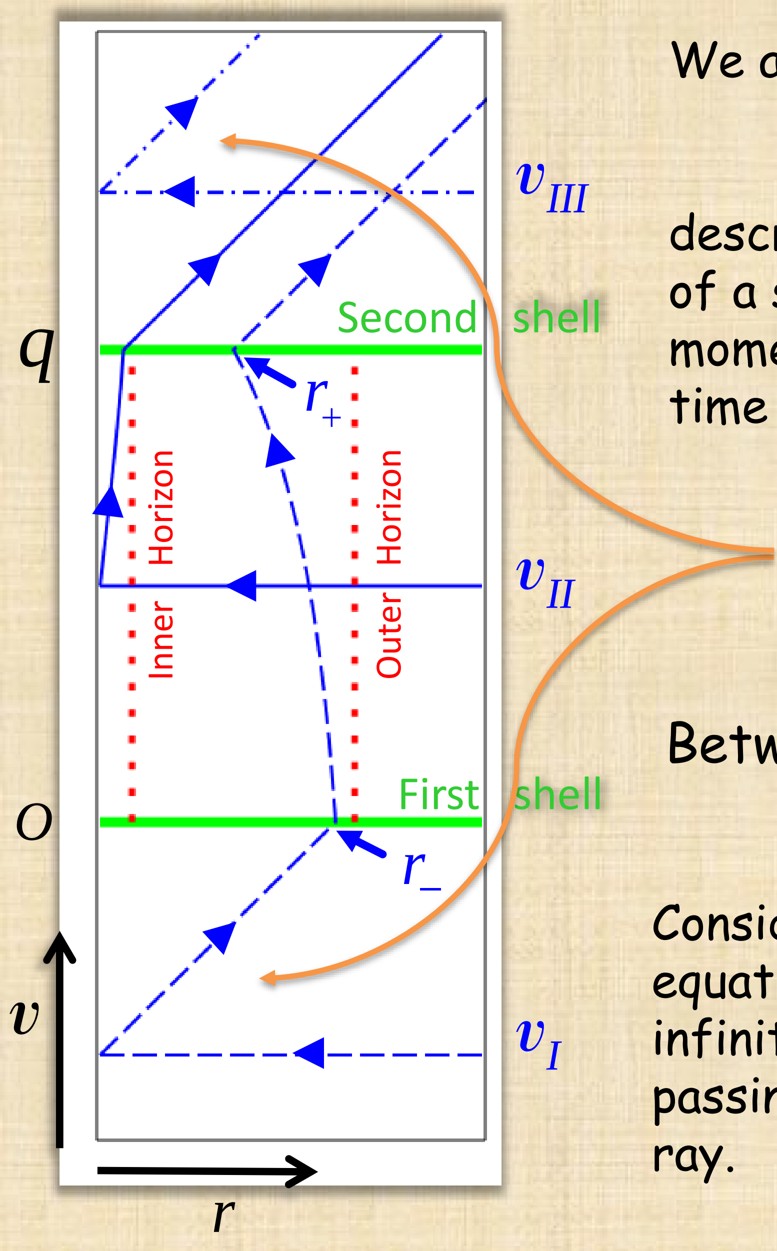
Gain function

The gain function β describes amplification of the particles energy, i.e., the ratio of the final energy of a photon to its initial energy.

To compute the energy fluxes $\mathcal{E}$ and the gain function $\beta = \frac{du_-}{du_+} = e^P$, one needs to know the map $u_-(u_+)$.

$$\beta = \frac{E_+}{E_-} = \frac{1}{\alpha_0}\exp\left[-\int \kappa dv\right], \quad \kappa = \frac{1}{2}\partial_r(\alpha f).$$

Sandwich black hole



We assume that a regular metric

$$ds^2 = -\alpha^2 f\, dv^2 + 2\alpha\; dvdr + r^2\; d\omega^2$$

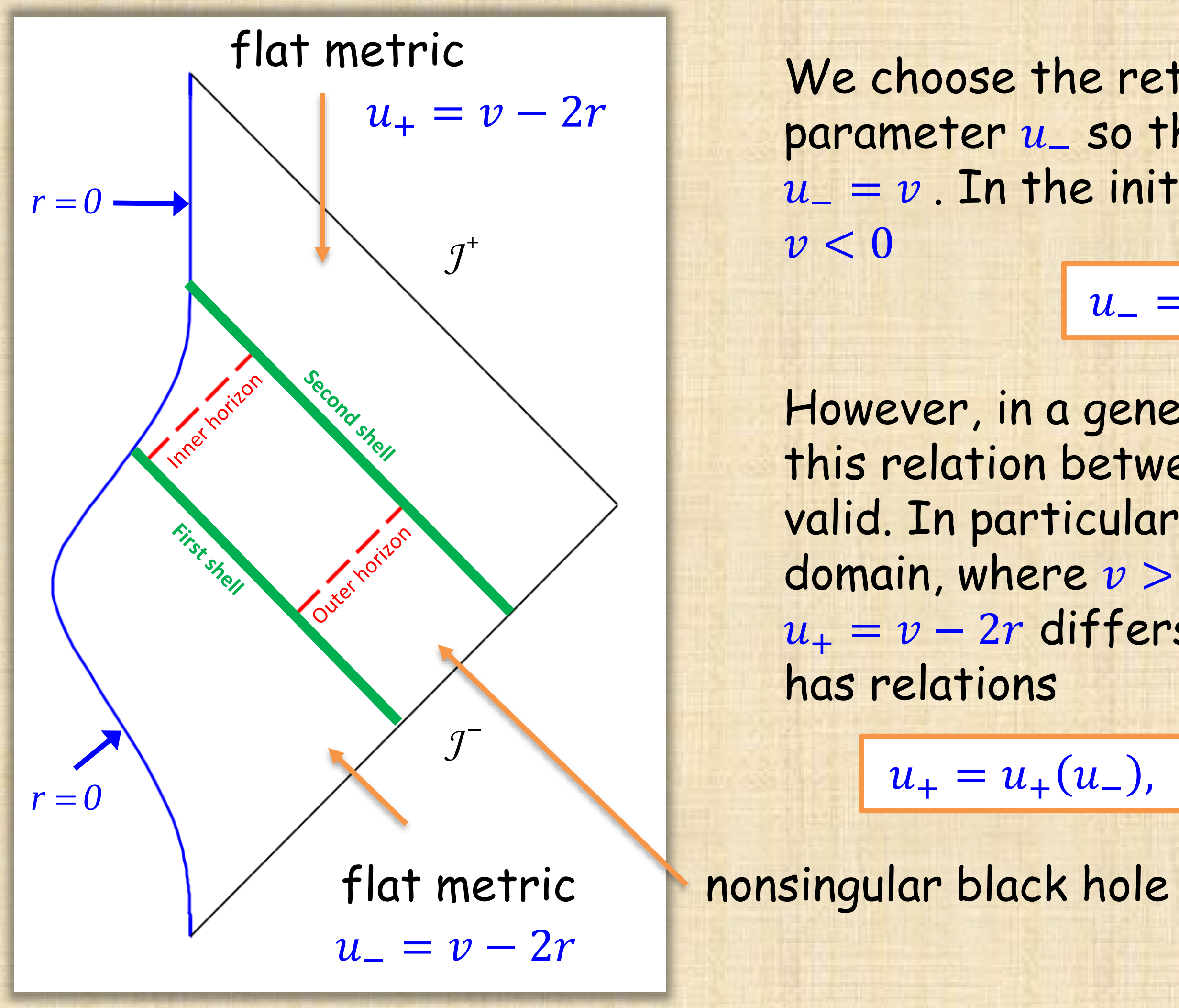
describes the black hole, which is created as a result of a spherical collapse of null shell of mass M at the moment $v = 0$, and which disappears after some finite time q after the collapse of the second shell -M.

$$f = \alpha = 1 \qquad \text{for } v < 0 \quad \text{and} \quad v > q$$

Between the shells – (modified) Hayward metric.

Consider an incoming radial null ray described by the equation $v = const$. It propagates from the past null infinity I^- and reaches the center $r = 0$. After passing the center, it becomes an outgoing radial null ray.

Penrose diagram of a sandwich black hole



We choose the retarded null time parameter u_- so that at $r=0$ one has $u_- = v$. In the initial flat domain, where $v < 0$

$$u_- = v - 2r$$

However, in a general case, for $v > 0$ this relation between u_- and v is not valid. In particular, in the final flat domain, where $v > q$, the null coordinate $u_+ = v - 2r$ differs from u_-, and one has relations

$$u_+ = u_+(u_-), \qquad u_- = u_-(u_+)$$

Scheme of calculations:

$$u_- = -2r_-, u_+ = q - 2r_+,$$

$$q = 2\int_{r_-}^{r_+} \frac{dr}{\alpha f} \Rightarrow r_- = r_-(r_+)$$

Useful parametrization of the Hayward metric: $(M,\ell) \rightarrow (r_{in}, r_{out})$.

We denote $p = \dfrac{r_{out}}{r_{in}}$, and use r_{in} as a scale parameter:

$$f = \frac{(r-p)(r-1)[r+p/(p+1)]}{r^3 + p^2/(p+1)}.$$

For $M \gg \ell$, $p \approx 2M$, $r_{in} \approx \ell$, $\kappa_- \approx -1$ $(\approx -\ell^{-1})$, $\kappa_+ \approx \dfrac{1}{2p}$ $(\approx \dfrac{1}{4M})$.

$$\kappa_1 = \frac{(p-1)(p+2)}{2p(p^2+p+1)}, \kappa_2 = -\frac{(p-1)(2p+1)}{2(p^2+p+1)},$$

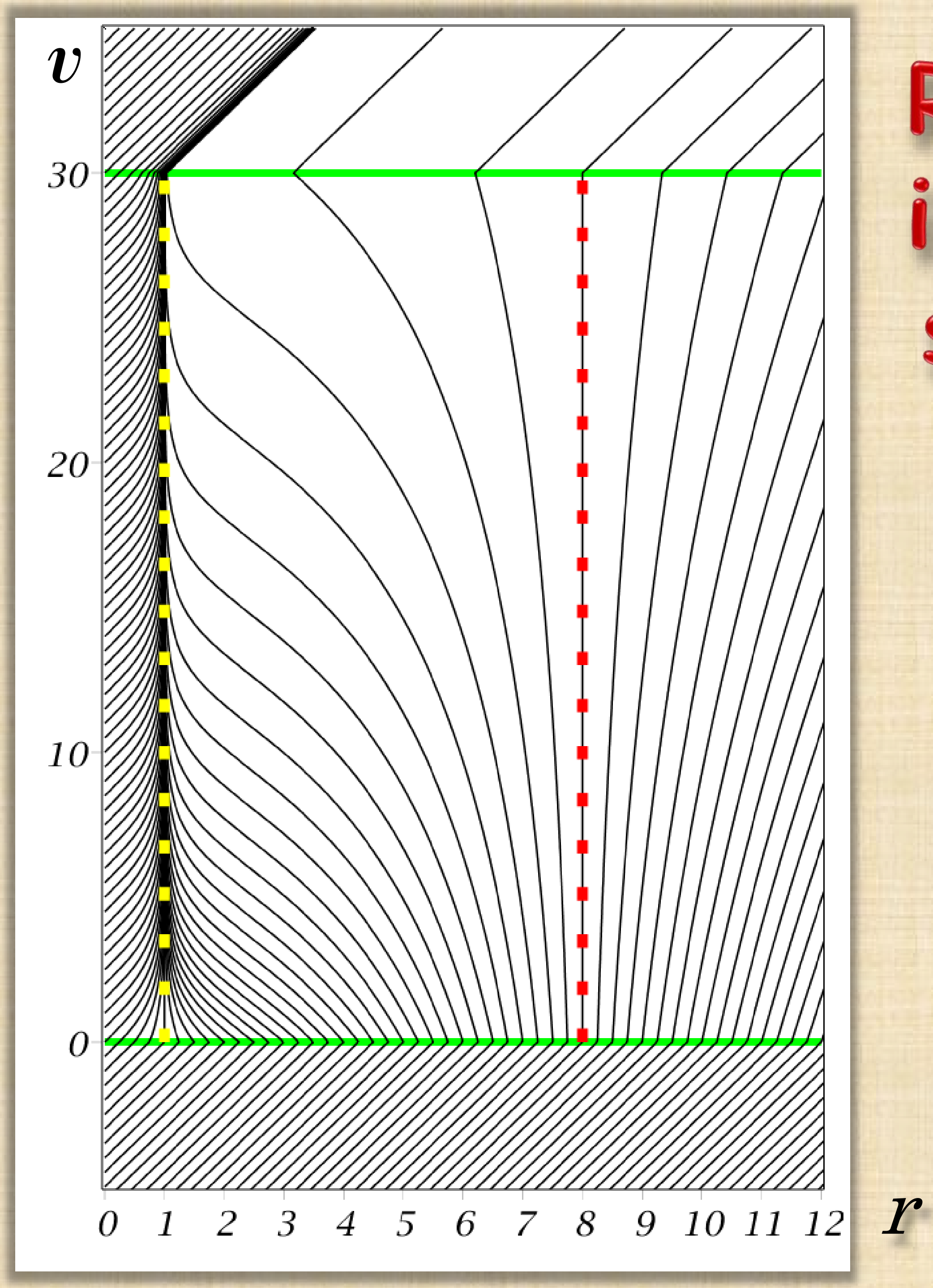
$$\kappa_0 = \frac{(p+1)(p+2)(2p+1)}{2p(p^2+p+1)}.$$

$$\kappa_1 \approx \frac{1}{2p} \approx \frac{1}{4M}, \quad \kappa_2 \approx -1;$$

$$Q = \int_0^r \frac{dr}{f}, \quad f^{-1} = 1 + \frac{1}{2\kappa_1(r-r_1)} + \frac{1}{2\kappa_2(r-r_2)} + \frac{1}{2\kappa_0(r-r_0)}.$$

$$Q(r) = r + \frac{1}{2\kappa_2}\ln|r-1| + \frac{1}{2\kappa_1}\ln\left(\frac{|r-p|}{p}\right) + \frac{1}{2\kappa_0}\ln\left(\frac{|r-r_0|}{|r_0|}\right).$$

Outgoing rays: $u = v - 2Q(r) = const$



Radial null rays in a nonsingular sandwich black hole

$$ds^2 = -\alpha^2 f dv^2 + 2\alpha\, dv dr$$

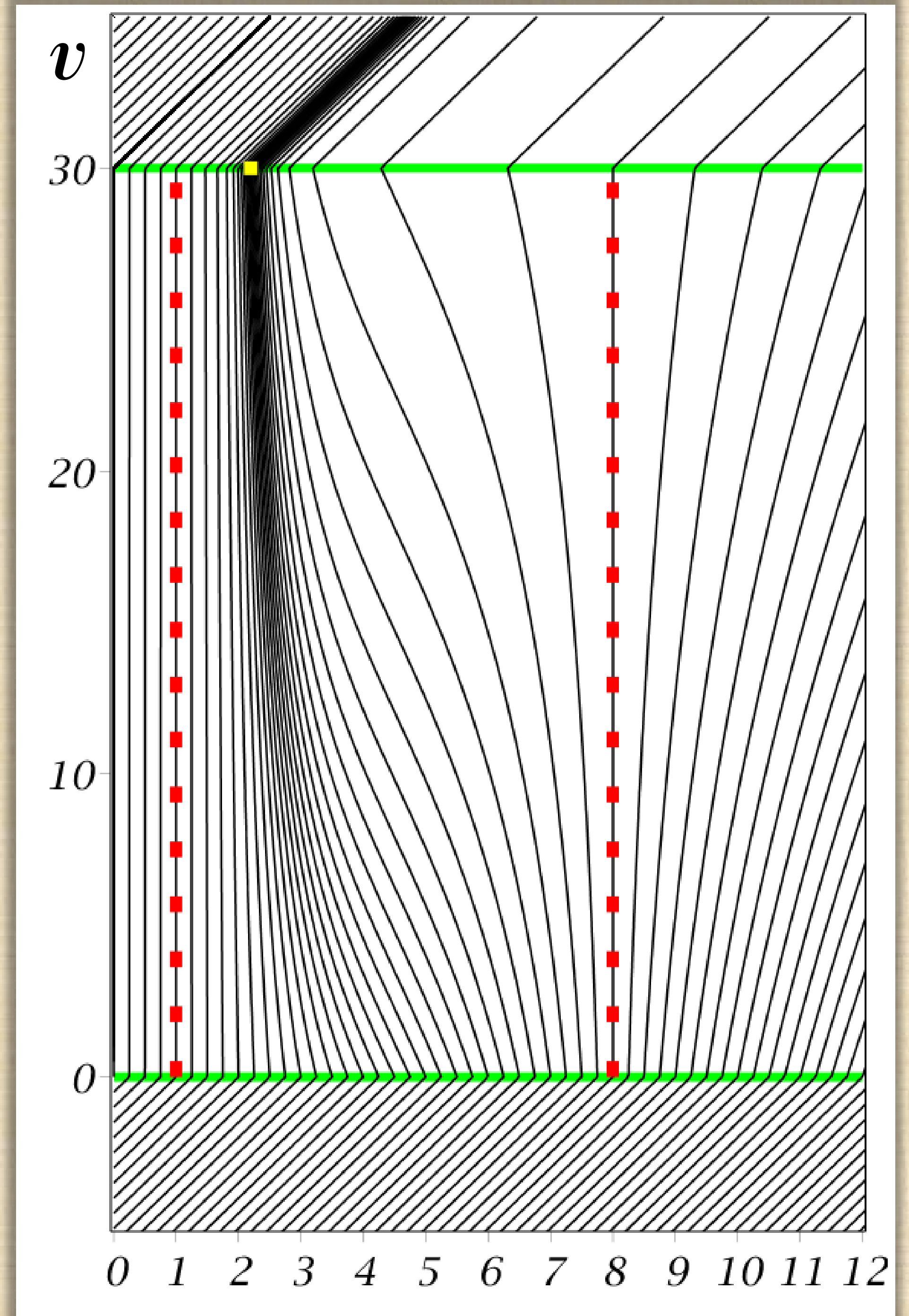
$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}$$

$$f = \frac{(r-r_1)(r-r_2)(r-r_0)}{r^3 - r_1 r_2 r_0}, \qquad p = r_1/r_2$$

Standard model $\alpha = 1$

$$p = 8, \qquad q = 30$$

Radial null rays in a nonsingular sandwich black hole



$$ds^2 = -\alpha^2 f dv^2 + 2\alpha\, dv dr$$

$$f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}$$

$$f = \frac{(r-r_1)(r-r_2)(r-r_0)}{r^3 - r_1 r_2 r_0}, \qquad p = r_1/r_2$$

Modified model $\alpha = \dfrac{r^n + 1}{r^n + 1 + p^k}$

$p = 8, \qquad q = 30,$

$n = 6, \qquad k = 4$

Sequence of events (as seen by an external observer):

(i) black hole formation;

(ii) Hawking radiation;

(iii) signal from the second shell;

(iv) radiation from the black hole interior;

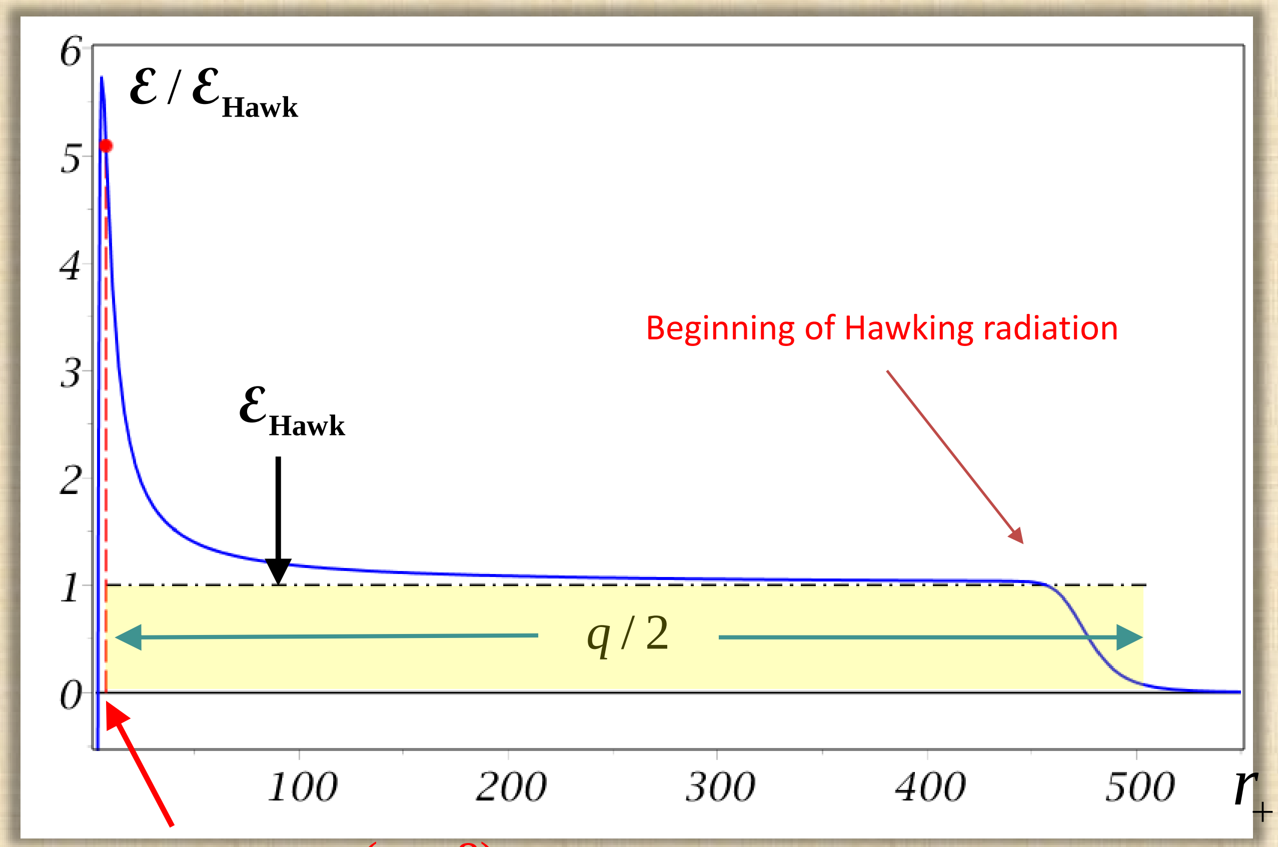
(v) ourburst of radiation from the inner domain (near inner horizon);

(vi) Mass inflation; $\kappa \sim -\alpha_0$;

(vii) Total emitted energy is always positive, its density can be negative during short time.

Hawking radiation

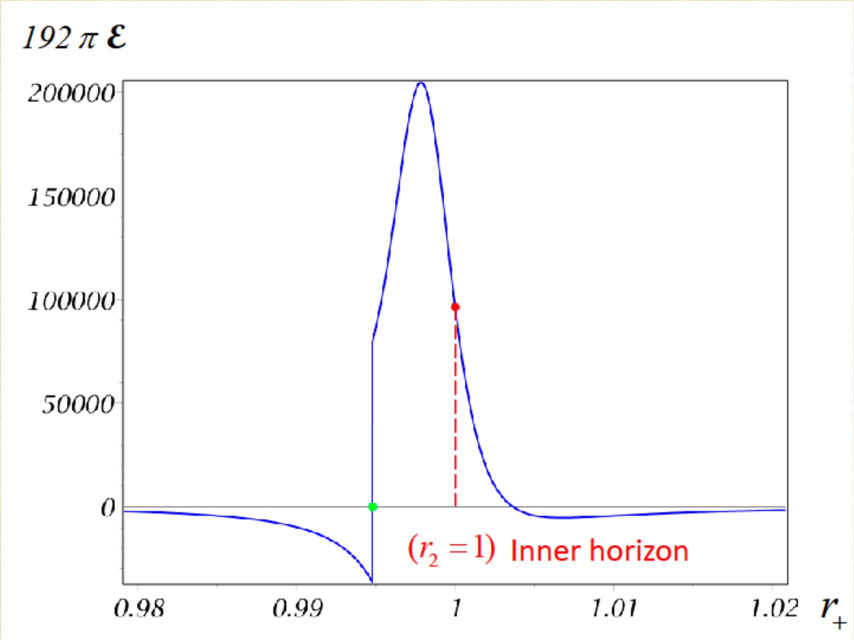
The Hawking result for the quantum energy flux from a black hole is correctly reproduced, when the mass parameter p and the lifetime of the blackhole q are large. The shape of the curve is almost the same for both standard and modified models. Duration of the almost constant tail of quantum radiation is approximately equal to q (lifetime of the black hole).



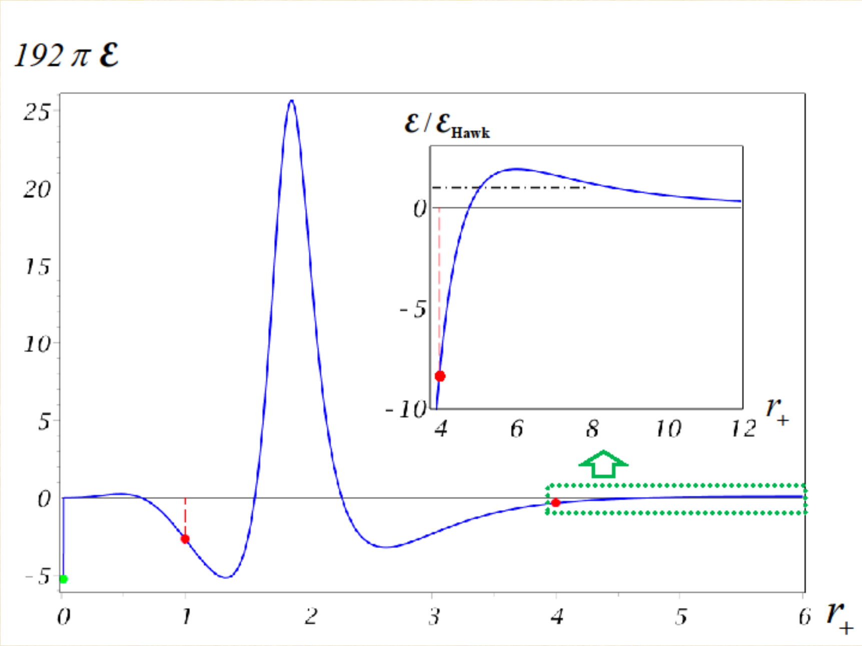
$$\mathcal{E}_{Hawk} = \frac{\kappa_1{}^2}{48\pi} = \frac{1}{192\,\pi\, p^2}$$

$q = 1000$

$p = 4, q = 30, \alpha_0 = 1$



$p = 4, q = 30, \alpha_0 \sim p^{-4}$



$q \sim p^3, \quad \Delta E \sim \exp(p^3)$

$q \sim p^3, \quad \Delta E \sim p^3$

"Mass inflation" is cured, however "gravity acceleration" mechanism still works. Self-consistency problem.

Main results:

(i) Properly reproduced Hawking radiation from the outer horizon (for $q \gg M$);

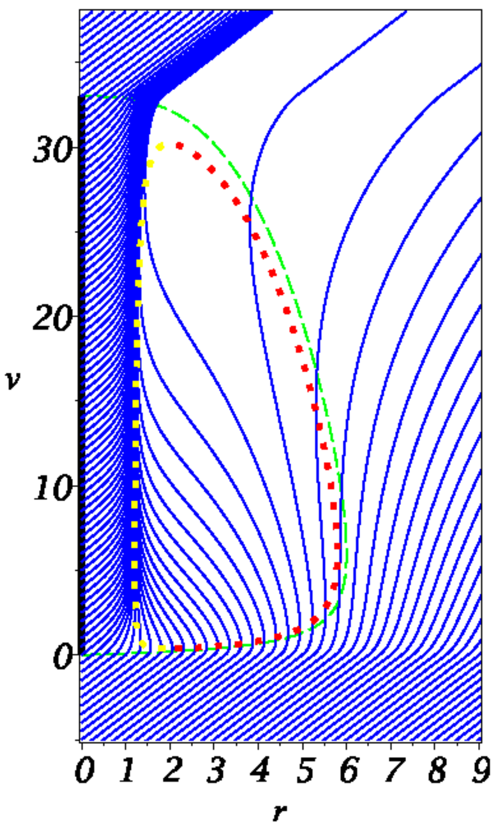
(ii) For α=1 -- huge outburst of the quantum radiation from the inner horizon: $\Delta E \sim \exp(q) \sim \exp(\Delta v/\ell)$. This radiation comes from the inner horizon during time interval $\Delta u_+ \sim \exp(-q) \sim \exp(-\Delta v/\ell)$;

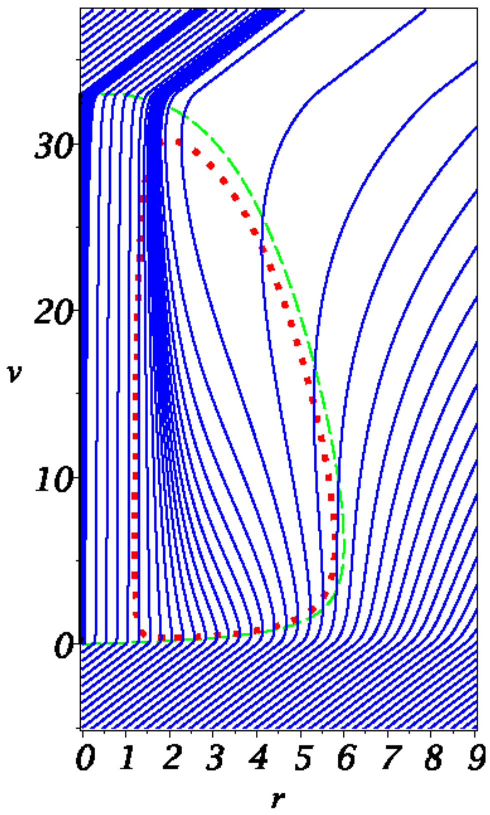
(iii) Mass inflation mechanism (Israel, Poisson [1990]);

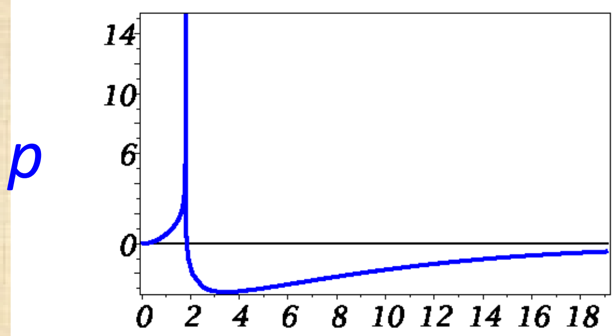
(iv) For a special choice of α ($\alpha_0 \ll 1$) outburst of the energy can be reduced to the power law;

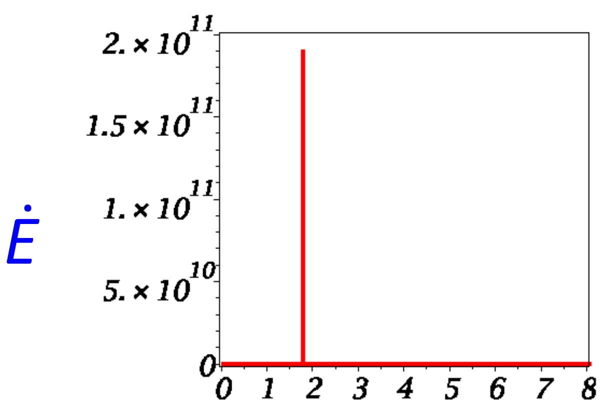
(v) Self-consistency problem remains [Bolashenko and V.F. (1986)].

“Realistic” non-singular model of an evaporating black hole

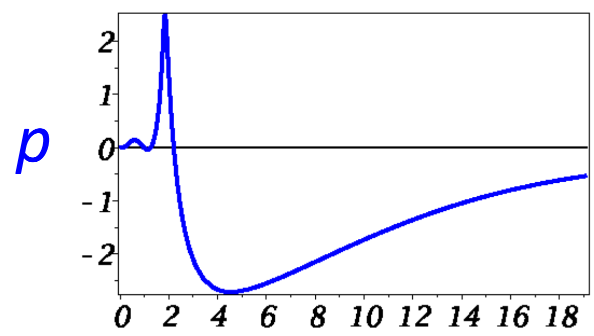


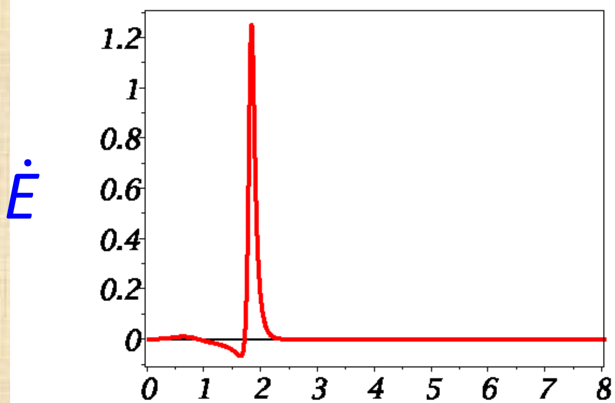






$M = 3, \quad \alpha = 1$





$M = 3, \quad \alpha_0 \sim M^{-3}$

Summary and Discussion

(i) Non-singular models of evaporating BHs

(ii) Quantum radiation from BH interior

2D approximation (its validity?);

(iii) "Bracket" formalism;

(iv) Sandwich model [2 shells, 2 parameres]

vs. a "realistic" non-singular models

(v) Two mechanisms of energy amplification:

"Gravity accelerator" vs "Mass inflation";

(vi) Properly chosen red-shift factor helps to cure

the mass inflation problem;

(vii) Information loss and self-consistency problems;

(viii) Back-reaction of created radiation?!

Polyakov action:

$$W[g] = -\frac{b}{2}\int d^2x\,\sqrt{g}\,R\frac{1}{\Box}R, \quad \Box\Phi = R,$$

$$\left\langle T^{\mu\nu}\right\rangle = -\frac{1}{2\sqrt{g}}\frac{\delta W[g]}{\delta g_{\mu\nu}} =$$

$$b\{-2\nabla^{\mu}\nabla^{\nu}\Phi - \nabla^{\mu}\Phi\nabla^{\nu}\Phi + g^{\mu\nu}[2R + \frac{1}{2}(\nabla\Phi)^2]\},$$

$$\left\langle T^{\mu\nu}\right\rangle g_{\mu\nu} = 2bR.$$

"Bracket" formalism

Let $y(x)$ be a function, and $x(y)$ is its inverse.

$$[y,x]=\ln|y'|,\quad <y,x>=\frac{y''}{y'},\quad \{y,x\}=\frac{y'''}{y'}-\frac{3}{2}\left(\frac{y''}{y'}\right)^2.$$

Schwarz derivative

$$[y,x]=-[x,y],\quad <y,x>=-y'<x,y>,\quad \{y,x\}=-(y')^2\{x,y\}.$$

Chain rule: $f\circ g(z)=f(g(z))\Rightarrow$

$$[f\circ g,z]=[f,g]\big|_{g=g(z)}+[g,z],$$

$$<f\circ g,z>=<f,g>\big|_{g=g(z)}g'(z)+<g,z>,$$

$$\{f\circ g,z\}=\{f,g\}\big|_{g=g(z)}(g'(z))^2+\{g,z\}.$$

Observables on I^+ in terms of "brackets".

Gain function: $\beta = \dfrac{du_-}{du_+}, \quad P = \ln\beta = [u_-, u_+],$

"Radiation entropy" $S(v) = -\dfrac{1}{12}P,$

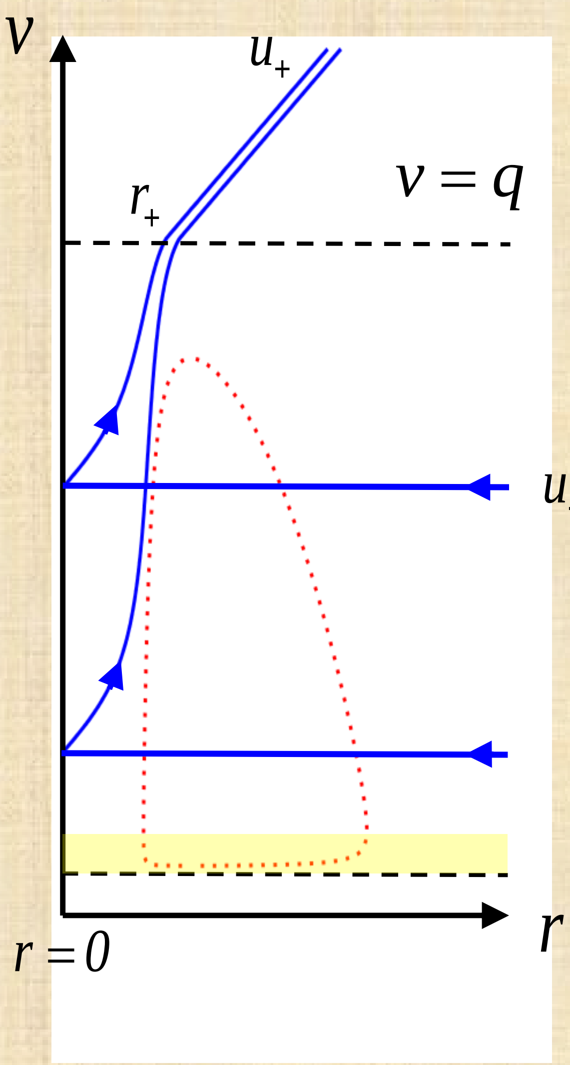
[Bianchi, DeLorenzo, Smerlak, JHEP, 06, 1280 (2015)]

Density of out-going trajectories: $W = \langle u_-, u_+ \rangle.$

Energy flux: $\dot{E} = -\dfrac{1}{24\pi}\{u_-, u_+\}, \{u_-, u_+\}$ - Schwarz derivative,

[M.Reuter, CQG, 6, 1149 (1989)]

Basic equations in the bracket formalism



$$\frac{dr}{dv}=Z(v,r),\quad Z=\frac{1}{2}\alpha f,\quad Z_m(v)=\left.\frac{\partial^m Z(v,r)}{\partial r^m}\right|_{r=r_0(v)}$$

$$p(v)=\ln r_1(v),\quad w(v)=\frac{r_2(v)}{r_1(v)},$$

$$\varepsilon(v)=\frac{r_3(v)}{r_1(v)}-\frac{3}{2}\left(\frac{r_2(v)}{r_1(v)}\right)^2.$$

$$\frac{dr_0}{dv}=Z_0,\quad \frac{dp}{dv}=Z_1,$$

$$\frac{dw}{dv}=Z_2e^p,\quad \frac{d\varepsilon}{dv}=Z_3e^{2p}.$$

$$p(u\_)=w(u\_)=\varepsilon(u\_)=0$$

$$P = \ln\beta = \ln\left(\frac{du_-}{du_+}\right) = [u_-, u_+] = -p(q) - \ln\alpha_0\,,$$

$$W = < u_-, u_+ > = e^{-p(q)}\left[\frac{1}{2}w(q) - \frac{\alpha_0}{\alpha_0^2}\right],$$

$$-24\pi\dot{E} = \{u_-, u_+\} = -e^{-2p(q)}\left[\frac{1}{4}\varepsilon(q) + \frac{1}{\alpha_0^2}\{x, u_-\}\right],$$

$$\{x, u_-\} = \frac{1}{2}\alpha_0^2 a_2 + \frac{\alpha_0}{\alpha_0} - \frac{3}{2}\left(\frac{\alpha_0}{\alpha_0}\right)^2.$$