Remarks on non-singular black hole models

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Based on:

"Information loss problem and a 'black hole` model with a closed apparent horizon", V.F., JHEP 1405 (2014) 049;

Phys.Rev. D94 (2016) no.10, 104056;

V. F. and A. Zelnikov, Phys.Rev. D95 (2017) no.4, 044042;

Based on:

"Information loss problem and a 'black hole' model with a

closed apparent horizon", V.F., JHEP 1405 (2014) 049;

"Notes on non-singular models of black holes", V.F.,

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"Notes on non-singular models of black holes", V.F.,

Phys.Rev. D94 (2016) no.10, 104056;

"Quan **Example 18 Sand A. Solution From a sandwich black hole's model with a same absolute apparent horizon".** V.F., JHEP 1405 (2014) 049; "Notes on non-singular models of black holes", V. F., Phys.Rev. D94 (2016) no.10, 104056; "Quantum radiation from an evaporating non-singular
black hole", V. F. and A. Zelnikov, Phys.Rev. D95 (2017) no.12, 124028. **Based on:**

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"Notes on non-singular models of black

Phys.Rev. D94 (2016) no.10, 104056;

"Quantum radiation from a sandwich bl

V. F.

According to GR: Singularity exists inside a black hole. Theorems on singularities: Penrose and Hawking.

There exists a curvature singularity inside a stationary BH in the Einstein gravity \Rightarrow
This theory is UV incomplete. s UV incomplete.

Expectations: When curvature becomes high (e.g. reaches the Planckian value) the classical GR should be modified. Singularities of GR would be resolved.

$$
\mathbf{R}^{2} = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \frac{48M^{2}}{r^{6}},
$$

$$
\mathbf{R}^{2} = \ell^{-4} \Rightarrow r_{*} = \ell \left(4\sqrt{3} \frac{M}{\ell} \right)^{1/3}
$$

$$
M \sim 4 \times 10^{6} M_{\odot}, \quad \ell \sim 10^{-33} \text{cm},
$$

$$
r_{*} \sim 10^{-7} \text{cm} \gg \ell
$$

 $-d\tau^2 + a \tau^{-2/3} dt^2 + b \tau^{4/3} d\omega^2$

$$
T_{evap} \sim t_{Pl} (M/m_{Pl})^3, \quad M = 10^{15} g, \quad T_{evap} \sim 10^{17} \text{ sec},
$$

$$
L_{evap} \sim 10^{28} cm, \quad r_* = l_{Pl} (M/m_{Pl})^{1/3},
$$

$$
L \sim l_{Pl} (M/m_{Pl})^{11/3} \sim L_{V} (M/m_{Pl})^{2/3} \sim 10^{13} L_{V}
$$

$$
L_{evap} \sim 10^{28} \, \text{cm}, \quad r_* = l_{Pl} \left(M / m_{Pl} \right)^{1/3},
$$

 $(M/m_{Pl})^3, M=10^{15} g, T_{evc}$ ²⁸ cm, $r_* = l_{Pl} (M/m_{Pl})^{1/3},$ $\left(\begin{array}{ccc} \rho_{Pl} \end{array} \right)^3, & M = 10^{15} g, & T_{evap} \sim 10^{17} s$
 $r_* = l_{Pl} (M/m_{Pl})^{1/3},$
 $\left(\begin{array}{ccc} 11/3 & \sim L_{evap} (M/m_{Pl})^{2/3} \sim 10^{13} L_{evap} \end{array} \right)$ $(M / m_{Pl})^3$, $M = 10^{15} g$, $T_{evap} \sim 10^{17} \text{ sec}$,
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 $10^{28}\,cm, & r_*= l_{Pl} (M\,/\,m_{Pl})^{1/3}, \ (M\,/\,m_{Pl})^{11/3}\sim L_{evap} (M\,/\,m_{Pl})^{2/3}\sim$ $t_{Pl}(M/m_{Pl})^3$, $M = 10^{15} g$, $T_{evap} \sim 10^{17} \text{ sec}$
 $10^{28} cm$, $r_* = l_{Pl}(M/m_{Pl})^{1/3}$,
 $(M/m_{Pl})^{11/3} \sim L_{evap}(M/m_{Pl})^{2/3} \sim 10^{13} L_{evap}$ $\begin{array}{l} \displaystyle \mu_{\rm vap} \sim t_{Pl} (M\,/\,m_{Pl})^3, \quad M = \! 10^{15}\,g \, , \[2mm] \displaystyle \mu_{\rm evap} \sim \! 10^{28}\,cm, \quad r_{\rm s} = \!l_{Pl} (M\,/\,m_{Pl})^{1/2} \[2mm] \displaystyle \rho_{\rm vap} \sim l_{Pl} (M\,/\,m_{Pl})^{11/3} \sim L_{\rm evap} (M\,/\,m_{Pl}) \end{array}$ $\sim t_{Pl} (M/m_{Pl})^3$, $M = 10^{15} g$, $T_{evap} \sim 10^{17} \text{ sec}$,
 $\sim 10^{28} cm$, $r_* = l_{Pl} (M/m_{Pl})^{1/3}$,
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 $L_{evap} \sim 10^{28} cm$, $r_* = l_{Pl} (M / m_{Pl})^{1/3}$,
 $L \sim l_{Pl} (M / m_{Pl})^{11/3} \sim L_{evap} (M / m_{Pl})^{2/3} \sim 10^{13} L$ $T_{evap} \sim t_{Pl} (M/m_{Pl})^3, \quad M = 10^{15} g, \quad T_{evap}$
 $L_{evap} \sim 10^{28} cm, \quad r_* = l_{Pl} (M/m_{Pl})^{1/3},$
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 $L_{evap} \sim 10^{28}$ cm, $r_* = l_{Pl} (M / m_{Pl})^{1/3}$,
 $L \sim l_{Pl} (M / m_{Pl})^{11/3} \sim L_{evap} (M / m_{Pl})^{2/3} \sim 10^{13} L$

Modified gravity: Options:

(i) Vacuum polarization and particle creation \rightarrow Effective action (higher derivatives and non-locality); (ii) Modified fundamental gravity (higher derivatives, Modified gravity: Options:
(i) Vacuum polarization and particle creation \rightarrow
Effective action (higher derivatives and
non-locality);
(ii) Modified fundamental gravity (higher derivative
 $f(R)$ theory, etc.);
(iii) Non-loc (strings, loops, etc.) *f*
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Phenomenological description

(ii) In the domain where $\Re \ll \ell^{-2}$ the metric obeys the Einstein equations (i) There exists the critical energy scale parameter $\,\mu$. The corresponding **Phenomenologica**

(i) There exists the critical energy scale production in the domain where $\Re \ll \ell^2$ the merginal corrections;

(ii) In the domain where $\Re \ll \ell^2$ the merginal corrections;

(iii) In the domain where fundamental length is $\ell = \frac{\hbar}{\mu c}$; (iii) In the domain where $\Re \sim \ell^{-2}$ the Einstein equations should be modified; $\mathfrak{R} \sim \ell^{-2}$ the Einstein equations should be modified;
ndition: $|\mathfrak{R}| \!\leq\! \frac{C}{\ell^2}.$ C is a universal constant, defined with small corrections; **(i)** There exists the critical energy scale parameter μ . The corresponding
fundamental length is $\ell = \frac{\hbar}{\mu c}$;
(ii) In the domain where $\Re \ll \ell^2$ the metric obeys the Einstein equations
with small corrections;
(ii **gical description**

cale parameter μ . The corresponding

ne metric obeys the Einstein equations

ne Einstein equations should be modified;
 $\Re \leq \frac{C}{\ell^2}$. *C* is a universal constant, defined

c of the parameters [Markov, JETP Lett. 36, 265 (1982)]
Remark on inflation theory. by the theory and independent of the parameters of the solution.
[Markov, JETP Lett. 36, 265 (1982)] Remark on inflation theory.]

`Quasi-local definition' of BH: Apparent horizon

A compact smooth surface *B* is called a trapped surface if both, in- and out-going null surfaces, **local definition' of BH:**
 pparent horizon

A compact smooth surface B is called a tra

surface if both, in- and out-going null sur

orthogonal to B , are non-expanding .

A trapped region is a region inside B .

A orthogonal to B , are non-expanding.

A trapped region is a region inside B.

A boundary of all trapped r egions is called an apparent horizon.

 t \prime

Null energy condition: $T_{\mu\nu}l^{\mu}l^{\mu} \ge 0$

Trapped surface + NEC
=Event horizon existence

In a ST obeying the null-energy condition the apparent horizon lies inside (or coincides with) the true event horizon.

In classical physics in order to prove the existence of a BH (in an exact mathematical sense) it is not necessary to wait infinite time, but it is sufficient to check the existence of the trapped surface `now'. In quantum physics the energy conditions could be violated. An example is an evaporating black hole (negative energy flux through the horizon reduces its mass).

It is possible, in principle, that the apparent horizon exists, but there is no event horizon.

We shall focus on this option.

 General form of SS metric in advanced time coordi

General form of SS metric in

advanced time coordinates
 $\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\Omega^2$, $f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$.
 $\rightarrow \infty = 1$. Apparent horizon: $f = 0$.

ift function: $\alpha(v,r)$. In a static ST: $\xi_t^2 = -\alpha^2 f$,
 $\left(\frac{\Gamma(\n$ α dv dr + r²d Ω^2 , $f = (\nabla r)^2 = g^{\mu\nu} r_{\mu} r_{\nu}$. General form of SS metric in

advanced time coordinates
 $=-\alpha^2 f d v^2 + 2\alpha dv dr + r^2 d \Omega^2$, $f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$.
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 meral form of SS metric in

lvanced time coordinates
 $f dv^2 + 2\alpha dv dr + r^2 d\Omega^2$, $f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$.

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iunction: $\alpha(v,r)$. In a static ST: $\xi_t^2 = -\alpha^2 f$,
 $\frac{\nabla r}{r^2}$ $\left(\frac{1}{r^2}\$ General form of SS metric in

advanced time coordinates
 $\begin{aligned}\n^2 &= -\alpha^2 f \, d\nu^2 + 2\alpha \, d\nu dr + r^2 d\Omega^2, \quad f = (\nabla r)^2 = g^\mu\nu, r) \big|_{r \to \infty} = 1. \end{aligned}$ Apparent horizon: $f = 0$.

Ad-shift function: $\alpha(\nu, r)$. In a static ST: $\xi_t^$ General form of SS metric in

advanced time coordinates
 $ds^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\Omega^2$, $f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$.
 $f(v,r)|_{r \to \infty} = 1$. Apparent horizon: $f = 0$.

Red-shift function: $\alpha(v,r)$. In a static ST: $\xi_t^2 =$ tic ST: $\xi_t^2 = -\alpha^2 f$,)

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= 0, the General form of SS metric in

advanced time coordinates
 $ds^2 = -\alpha^2 f \, dv^2 + 2\alpha \, dv dr + r^2 d\Omega^2$, $f = (\nabla r)^2 = g^{\mu\nu} r_{,f}$
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Red-shift function: $\alpha(v,r)$. In a static ST: $\xi_t^2 = -\alpha$ *General form of SS metric in*
 *radvanced time coordinates***
** $ds^2 = -\alpha^2 f \, dv^2 + 2\alpha \, dv \, dr + r^2 d\Omega^2$ **,** $f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$ **.
** $f(v,r)|_{v \to \infty} = 1$ **. Apparent horizon:** $f = 0$ **.

Red-shift function:** $\alpha(v,r)$ **. In a stati** : in

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advanced time coordinates
 $ds^2 = -\alpha^2 f \, dv^2 + 2\alpha \, dv \, dr + r^2 \, d\Omega^2$, $f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$,
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Red-shift function: $\alpha(v,r)$. In a static ST: **ST:** $\xi_t^2 = -\alpha^2 f$, General form of SS metric in

advanced time coordinates
 $-\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\Omega^2$, $f = (\nabla r)^2 = g^{\mu\nu} r_{,\mu} r_{,\nu}$.
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 $f = 1 + \frac{1}{2} f_2(v) r^2 + ...$,
 $\alpha = \alpha_0(v) [1 + \frac{1}{2} \alpha_2(v) r^2 + ...].$
We use normalization: $\alpha(v,r)|_{r \to \infty} = 1$,
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center, τ then the rate of the proper time at the center, τ , and the rate of the Killing time $r \rightarrow \infty$ regular at $r = 0$, if curvature
riants are finite there:
 $f = 1 + \frac{1}{2}f_2(v)r^2 + ...$,
 $\alpha = \alpha_0(v)[1 + \frac{1}{2}\alpha_2(v)r^2 + ...]$.
use normalization: $\alpha(v,r)|_{r\to\infty} = 1$,
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 $\begin{aligned} \text{re} \ -\text{Re} \ \text{at the} \ \text{is} \ d\tau & = \alpha_0(\nu) d\nu. \end{aligned}$ $r = 0$, if curvature

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 $(v)r^2 + ...$,
 $+\frac{1}{2}\alpha_2(v)r^2 + ...$].

lization: $\alpha(v,r)|_{r\to\infty} = 1$,

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ants are finite there:
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e normalization: $\alpha(v,r)|_{r \to \infty} = 1$,

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invariants are finite there:
 $f = 1 + \frac{1}{2}f_2(v)r^2 + ...$,
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We use normalization: $\alpha(v,r)|_{r \to \infty} = 1$,
then the rate of the proper time at the
center, τ , a

(i) An apparent horizon in a regular metric cannot cross $r = 0$. (ii) It has two branches: outer- and inner-horizons. (iii) Non-singular BH model with a closed apparent horizon
[V.F. and G.Vilkovisky, Phys. Lett., 106B, 307 (1981)] [V.F. and G. Vilkovisky, Phys. Lett., 106B, 307 (1981)]

Fig. 1. Penrose diagram for the collapse of the null shell $(M \ge 1)$. Solid (dashed) lines are used for the known (hypothetical) details of the picture. The shaded region is the region of validity of the obtained asymptotic solution. The line $N^{-} \cup N^{+}$ is the world line of the null shell. The closed and dashed bold line $ABCD$ is the apparent horizon. The light lines are the level lines $r = const$.

Other publications on regular BH models with closed apparent horizons

T. A. Roman and P.G. Bergmann (1983); P. Bolashenko and V.F. (1986) S. N. Solodukhin, (1999); S. A. Hayward (2006); S. Ansoldi (2008); C. Bambi, D. Malafarina, L. Modesto (2013); V. N. Lukas and V. N. Strokov (2013); V. Frolov (2014); J.M. Bardeen (2014); C. Rovelli and F. Vidotto (2014); T. De Lorenzo, C. Pacilio , C. Rovelli, S. Speziale (2015);

D.I. Kazakov, S.N. Solodukhin (1993) P. Hajicek (2002); D. Grumiller (2003, 2004; J. Ziprick and G. Kunstatter (2010)

Static SS non-singular black-hole metrics

Remark: All stationary BH solutions in General Relativity can be written in the form, where the metric coefficients are rational functions of the coordinates.

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Example 1: Bardeen regular black hole [1968]

atic SS non-singular black-hole met

nark: All stationary BH solutions in General Relativity

be written in the form, where the metric coefficients

rational functions of the coordinates.

assume that $f = \frac{P_o(r)}{Q_o(r)}$. singular black-hole metrics

BH solutions in General Relativity

orm, where the metric coefficients

of the coordinates.

(r). It has the form $=\frac{r^n + ...}{r^n + ...} \rightarrow 1$ at $r \rightarrow \infty$.

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 $\frac{1}{r^2} \rightarrow$ **- Singular black-hole**
 n BH solutions in General Relation

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 $f = \frac{P_n(r)}{Q_n(r)}$. It has the form $= \frac{r^n + ...}{r^n + ...} \rightarrow 1$ at $r \rightarrow \infty$.

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all stationary BH solutions in General Relativity

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be written in the form, where the metric coefficients

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ssume that $f = \frac{P_n(r)}{Q$ satisfies the limiting curvature condition. *Mriary BH solutions in General Relativity*
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 Mrsume that $f = \frac{P_n(r)}{Q_n(r)}$. It has the form $= \frac{r^n + ...}{r^n + ...$ **5 non-singular black-hole metrics**

ationary BH solutions in General Relativity

in the form, where the metric coefficients

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at $f = \frac{P_a(r)}{Q_a(r)}$. It has the form $= \frac{r^n + ...}{r^n + ...} \rightarrow 1$ at r $q \rightarrow \ell$ ic SS non-singular black-hole metrics
 $x:$ All stationary BH solutions in General Relativity

written in the form, where the metric coefficients

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Example 2: Metric with $f = 1 - \frac{2Mr}{r^2 + \ell^2} \sim 1 - \frac{2M}{\ell^2}r +$.

is non-regular.

Metrics with $n \le 2$ cannot be consistent metrics of

a non-singular black hole. [V.F. PR D94,104056 (201 Example 2: Metric with $f = 1 - \frac{2Mr}{r^2 + \ell^2} \sim 1 - \frac{2M}{\ell^2}r + ...$

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Example 3 (n=3): Hayward metr xample 2: Metric with $f = 1 - \frac{2Mr}{r^2 + \ell^2} \sim 1 - \frac{2M}{\ell^2}r + ...$

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g black hole: $M = M(v)$. $=1-\frac{2r}{r^2+\ell^2}-1-\frac{2r}{r^2}$ $-\frac{2Mr}{r^2 + \ell^2} \sim 1 - \frac{2M}{\ell^2}r + ...$
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 $\frac{2Mr^2}{r^3 + 2Me^2 + \ell^3}$ Example 2: Metric with $f = 1 - \frac{2Mr}{r^2 + \ell^2} \sim 1 - \frac{2M}{\ell^2}r +$.

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Example 3 (n=3): Hayward metric mple 2: Metric with $f = 1 -$
is non-regular.
trics with $n \le 2$ cannot be com-singular black hole. [V.F.]
mple 3 (n=3): Hayward metr
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}$, ($\alpha =$
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D7]:
 M = *M*(*v*). que 2: Metric with $f = 1 - \frac{2Mr}{r^2 + \ell^2} \sim 1 - \frac{2M}{\ell^2}r + ...$

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n ≤ 2 cannot be consistent metrics of a non-singular black hole. [V.F. PR D94,104056 (201 6)].

E xample 3 (n=3): Hayward metric [2007]:

 $3'$ $2Mr^2$ $+2M\ell^2+\ell$

Non-singular evaporating black hole: $M = M(v)$.

Non-singular model of black hole ar model of black hole
 $\alpha^2 f d v^2 + 2 \alpha d v d r + r^2 d \omega^2$,
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,
 $d: \alpha = 1$;
 $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n}$.

 $dS^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\omega^2$,

gular model of black
 $e^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\omega^2$,
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,

ndard: α =1; $2Mr²$ $x^2 f dv^2 + 2$
 $x^2 f dv^2 + 2$
 $\vdots \alpha = 1;$
 $\therefore \alpha = \frac{m^2}{r^n + 1}$
 $\Rightarrow M = \frac{m^2}{2\ell}$
 $\frac{x^2}{1 + x^3}$ ar mode
 $\alpha^2 f d v^2 + 2 \alpha$
 $f = 1 - \frac{2N}{r^3 + 2}$
 $f: \alpha = 1;$
 $f: \alpha = \frac{m^3}{r^n + \ell}$
 $\Rightarrow M = \frac{m^3}{2\ell^2}$
 $\frac{r^2}{1 + x^3}$ $+2M\ell^2$

Standard: α =1;

del of t
 $+2\alpha dvdr+r$
 $\frac{2Mr^{2}}{3}+2M\ell^{2}$
 $\frac{r^{n}+\ell^{n}}{1+\ell^{n}+(2M)^{n}}$ Modified: α = singular model of bl
 $dS^2 = -\alpha^2 f d v^2 + 2\alpha dv dr + r^2 c$
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,

Standard: $\alpha = 1$;

Modified: $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell}$

For $\alpha = 1 \Rightarrow M = \frac{m^3}{2\ell^2}$, $r = mx$,
 $f = 1 - \frac{m^2}{\ell^2} \frac{x^2}{1 + x^3}$ of black
 $dr + r^2 d\omega^2$,
 $\frac{1}{r^2}$,
 $+\ell^n$
 $(2M)^k \ell^{n-k}$.
 $=mx$, **of bl**

vdr + r²d
 $\frac{1}{2}$,
 $\frac{1}{2}$,
 $\frac{n + \ell^n}{n}$

+ (2M)^k ℓ^n

r = mx, $\begin{aligned} \textbf{p} & \textbf{else} \textbf{le} \textbf{le} \textbf{le} \textbf{le} \textbf{$ **ingular model of black** $dS^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\omega^2$,
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,

tandard: $\alpha = 1$;

Addified: $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell^{n-k}}$.

or $\alpha = 1 \Rightarrow M = \frac{m^3}{2\ell^2}$, $r = mx$,
 $f = 1 - \frac{m^2}{\ell^2} \frac{x^2}{1 + x^$ $r^n+\ell$ **odel of black h**
 $r^2 + 2\alpha dvdr + r^2d\omega^2$,
 $\frac{2Mr^2}{r^3 + 2M\ell^2}$,
 \vdots
 $\frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell^{n-k}}$.
 $=\frac{m^3}{2\ell^2}$, $r = mx$,
 $\frac{m^3}{\ell^3}$ **singular model of black**
 $dS^2 = -\alpha^2 f d v^2 + 2\alpha d v d r + r^2 d \omega^2$,
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,

Standard: $\alpha = 1$;

Modified: $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell^{n-k}}$.

For $\alpha = 1 \Rightarrow M = \frac{m^3}{2\ell^2}$, $r = mx$,
 $f = 1 - \frac{m^2}{\ell^2} \frac{x^$ $\alpha =$ ar model of black hole
 $\alpha^2 f d v^2 + 2 \alpha d v d r + r^2 d \omega^2$,
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,
 $d: \alpha = 1$;
 $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell^{n-k}}$.
 $\Rightarrow M = \frac{m^3}{2\ell^2}$, $r = mx$,
 $\frac{r^2}{r^2} \frac{x^2}{1 + x^3}$ ular model of black hole
 $=-\alpha^2 f d v^2 + 2\alpha d v d r + r^2 d \omega^2$,
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,

dard: $\alpha = 1$;

fied: $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k \ell^{n-k}}$. $+\ell^n$ $=\frac{1}{\sqrt{2\pi}}$ $+ \ell^{n} + (2l)$

3 $2'$ singular model of b
 $dS^2 = -\alpha^2 f d v^2 + 2\alpha dv dr + r^2$
 $f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2}$,

Standard: $\alpha = 1$;

Modified: $\alpha = \frac{r^n + \ell^n}{r^n + \ell^n + (2M)^k}$

For $\alpha = 1 \Rightarrow M = \frac{m^3}{2\ell^2}$, $r = mx$, α =1 \Rightarrow M

$$
f=1-\frac{m^2}{\ell^2}\frac{x^2}{1+x^3}
$$

 $m_* = (27/4)^{1/6} \ell \Leftrightarrow M_* = \frac{3\sqrt{3}}{4} \ell$

Non-singular model of an evaporating black hole

singular model of a
porating black hole
 $\alpha^2 = -fdv^2 + 2dvdr + r^2d\omega^2,$
=1- $\frac{2M(v)r^2}{r^3 + 2M(v)\ell^2}$ ($\alpha = 1$) 2 ular mode

ting black
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{2M(v)r^2}{r^3+2M(v)\ell^2}$ $2dvdr + r^2d\omega^2$, $\alpha = 1$ $+2M(v)\ell$

Information Loss Puzzle

Information Loss Puzzle

A model of a black hole with a closed apparent horizon is one of the options that were discussed in the connection with the information loss paradox. **Aharonov, Casher, Nussinov [1987];Carlitz, Willey [1987]; Preskill [1992]**

must take a very long time," $T_{av} \geq M^4$. "The final stage of the evaporation process

Aharonov, Casher, Nussinov [1987];Carlitz, Willey [19]
Preskill [1992]
"The final stage of the evaporation proce
must take a very long time," $T_{ev} \geq M^4$.
Self-Consistency problem
[Bolashenko and V.F. (1986)] Self-consistency problem [Bolashenko and V.F. (1986)]

Quantum effects

We consider a quantum massless scalar fields,
propagating in the background of a non-singular black hole. We use 2D approximation. The corresponding expectation value o f the stress-energy tensor can be [Christensen and Fulling, PD, D15, 2088 (1977)]. easily obtained from the known conformal anomaly. It can also be derived from Polyakov effective action. [V.F. and Vilkovisky, in Quantum Gravity, p.267 (1984)]

Radial null rays provide us with maps: $\mathfrak{T} \leftrightarrow \mathfrak{T}^+$: $u_{-} = u_{-}(u_{+})$ and $u_{+} = u_{+}(u_{-})$

$$
\dot{E} = \frac{1}{48\pi} \left[-2\frac{d^2P}{du_+^2} + \left(\frac{dP}{du_+}\right)^2 \right],
$$

$$
P = \ln\left(\frac{du_-}{du_+}\right).
$$

The gain function β describes amplification of the particles energy, i.e., the ratio of the final energy of a photon to its initial energy.

To compute the energy fluxes $\mathcal E$ and the gain function $\beta =$ $du_-\$ du_+ $= e^{P}$, one needs to know the map $u_-(u_+).$ **Gain function**

gain function β describes amplification

the particles energy, i.e., the ratio of the

l energy of a photon to its initial energy.

compute the energy fluxes $\mathcal E$ and the gain

ction $\beta = \frac{du}{du_+} = e^$ **Gain function**
 ain function β describes amplification
 e particles energy, i.e., the ratio of the

energy of a photon to its initial energy.
 mpute the energy fluxes ε and the gain

ion $\beta = \frac{du}{du_{+}} = e^{P}$, *Gain function*
 d function β describes amplification
 darticles energy, i.e., the ratio of the
 dergy of a photon to its initial energy.
 dute the energy fluxes $\mathcal E$ *and the gain*
 d $\beta = \frac{du_{-}}{du_{+}} = e^{P}$

 α_{0} \blacksquare $+$ α $E_{_+}$ 1

Sandwich black hole

We assume that a regular metric

 $ds^2 = -\alpha^2 f dv^2 + 2\alpha dv dr + r^2 d\omega^2$

describes the black hole, which is created as a result of a spherical collapse of null shell of mass M at the moment $v = 0$, and which disappears after some finite time *q* after the collapse of the second shell -M.

 $f = \alpha = 1$ for $v < \theta$ and $v > q$

Between the shells – (modified) Hayward metric.

Consider an incoming radial null ray described by the equation $v = const$. It propagates from the past null v_I infinity I^- and reaches the center $r = 0$. After passing the center, it becomes an outgoing radial null ray.

Penrose diagram of a sandwich black hole

We choose the retarded null time parameter $u_$ so that at $r = 0$ one has $u_-=v$. In the initial flat domain, where $\nu < 0$

 $u_-=v-2r$

However, in a general case, for $v > 0$ this relation between $u_-\,$ and v is not valid. In particular, in the final flat domain, where $v > q$, the null coordinate $u_+ = v - 2r$ differs from u_- , and one has relations

 $u_+ = u_+(u_-), \qquad u_- = u_-(u_+)$

nonsingular black hole

Scheme of calculations:

Scheme of calcula:
 $u_{-} = -2r_{-}, u_{+} = q - 2r_{+},$
 $q = 2 \int_{r_{-}}^{r_{+}} \frac{dr}{\alpha f} \Rightarrow r_{-} = r_{-}(r_{+})$ heme of calculat
=-2r_, u₊ = q - 2r₊,
2 $\int_{r_-}^{r_+} \frac{dr}{\alpha f}$ \Rightarrow r_ = r_(r₊) *r r* π of calcula
 $r_1, u_+ = q - 2r_+$
 $\frac{dr}{xf} \Rightarrow r_- = r_-(r_+)$ *dr* Scheme of calcula
 $a_{-} = -2r_{-}$, $a_{+} = q - 2r_{+}$,
 $q = 2 \int_{r_{-}}^{r_{+}} \frac{dr}{\alpha f} \Rightarrow r_{-} = r_{-}(r_{+})$ - 11 Scheme of calculations:
 $\begin{aligned} \n\begin{aligned}\n&= -2r_-, u_+ = q - 2r_+, \\
&= 2 \int\limits_{r_-}^{r_+} \frac{dr}{\alpha f} \Rightarrow r_- = r_-(r_+) \n\end{aligned}\n\end{aligned}$ $\begin{aligned} &\frac{1}{2}\text{calculations:}\ &\frac{1}{2} = q - 2r_{_+},\ &\frac{1}{2} = r_{_-}(r_{_+}) \end{aligned}$: heme of calculations:
=-2r_, u₊ =q-2r₊,
=2 $\int_{r_1}^{r_1} \frac{dr}{\alpha f}$ \Rightarrow r_ = r_(r₊) cheme of calculations:
=-2r_, u₊ =q-2r₊,
=2 $\int_{r_-}^{r_+} \frac{dr}{\alpha f}$ \Rightarrow r_ = r_(r₊)

 $f = \frac{r^3 + p^2/(p+1)}{r^3 + p^2/(p+1)}$.
For $M \gg l$, $p \approx 2M$, $r_{in} \approx l$, $K_{-} \approx -1$ $(\approx -l^{-1})$, K_{+} Useful parametrization of the Hayward metric: $(M, \ell) \rightarrow (r_{in}, r_{out})$. We denote $p = \frac{I_{out}}{I}$, and use r_{in} as a scale parameter: trization of the Hayward metric: $(M,\ell) \rightarrow (r_n, r_{out})$.
 $\frac{r_{out}}{r_n}$, and use r_n as a scale parameter:
 $\frac{-\rho)(r-1)[r + p/(p+1)]}{r^3 + p^2/(p+1)}$.
 $2M, r_n \approx \ell, \kappa \approx -1 \approx -\ell^{-1}, \quad \kappa_+ \approx \frac{1}{2p} (\approx \frac{1}{4M})$. $=\frac{V}{r}$ $\frac{\rho}{r}$ e Hayward metric: $(M,\ell) \rightarrow (r_m, r_{out})$.
 $\frac{r_m}{r_m}$ as a scale parameter:
 $\frac{1}{(p+1)!}$.
 ≈ -1 ($\approx -\ell^{-1}$), $K_+ \approx \frac{1}{2p}$ ($\approx \frac{1}{4M}$). tion of the Hayward metric: $(M,\ell) \rightarrow (r_m, r_{out})$,

, and use r_m as a scale parameter:
 $\frac{(r-1)[r + p/(p+1)]}{(p+1)}$.
 $r_m \approx \ell, \kappa_- \approx -1 \approx -\ell^{-1}$, $\kappa_+ \approx \frac{1}{2p} (\approx \frac{1}{4M})$. Useful parametrization of the Hayward metric: $(M, \ell) \rightarrow (r_n, r_{out})$.

We denote $p = \frac{r_{out}}{r_n}$, and use r_n as a scale parameter:
 $f = \frac{(r-p)(r-1)(r+p/(p+1))}{r^2+p^2/(p+1)}$.

For $M \gg \ell$, $p \approx 2M$, $r_n \approx \ell$, $\kappa_- \approx -1$ $(\approx -\ell^{-1})$, metrization of the Hayward me
 $p = \frac{r_{out}}{r_{in}}$, and use r_{in} as a scale pa
 $\frac{(r - p)(r - 1)[r + p/(p + 1)]}{r^3 + p^2/(p + 1)}$.
 $p \approx 2M$, $r_{in} \approx \ell$, $K_{-} \approx -1$ ($\approx -\ell^{-1}$), parametrization of the Hayward me

note $p = \frac{r_{out}}{r_{in}}$, and use r_{in} as a scale p:
 $f = \frac{(r-p)(r-1)[r+p/(p+1)]}{r^3+p^2/(p+1)}$.
 $\gg \ell$, $p \approx 2M$, $r_{in} \approx \ell$, $\kappa_{-} \approx -1$ $(\approx -\ell^{-1})$, eful parametrization

e denote $p = \frac{r_{out}}{r_{in}}$, and
 $f = \frac{(r - p)(r - 1)}{r^3 + p^2}$

r $M \gg \ell$, $p \approx 2M$, $r_{in} \approx$) is the Hayward me as r_{in} as a scale point $\frac{1+p/(p+1)!}{(p+1)!}$.
 $\kappa_{-} \approx -1 \approx -e^{-1}$, *ization of the Hayward metric:* $(M, \ell) \rightarrow (r_{in}, r_{out})$.
 $\frac{f_{out}}{r_{in}}$, and use r_{in} as a scale parameter:
 $\frac{p}{r_{in}}$
 $\frac{p(r-1)[r + p/(p+1)]}{r^3 + p^2/(p+1)}$.
 $M, r_{in} \approx \ell, \kappa_{-} \approx -1 \ (\approx -\ell^{-1}), \ \ \kappa_{+} \approx \frac{1}{2p} \ (\approx \frac{1}{4M})$. ization of the Hayward metric: $(M,\ell) \rightarrow (r_m, r_{out})$.
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 M, $r_m \approx \ell, \kappa \approx -1$ ($\approx -\ell^{-1}$), $\kappa_+ \approx \frac{1}{2p}$ ($\approx \frac{1}{4M}$). metrization of the Hayw
 $p = \frac{r_{out}}{r_{in}}$, and use r_{in} as a
 $\frac{(r-p)(r-1)[r+p/(p+1)}{r^3+p^2/(p+1)}$
 $p \approx 2M$, $r_{in} \approx l$, $K_{-} \approx -1$ (and *r rametrization of the Hayward me*
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 f = $\frac{(r - p)(r - 1)[r + p/(p + 1)]}{r^3 + p^2/(p + 1)}$.
 f, $p \approx 2M$, $r_{in} \approx l$, $\kappa_{-} \approx -1$ ($\approx -\ell^{-1}$) ation of the Haywar
 r, and use r_{in} as a so
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 $r_{$ *ful parametrization of the Hayward metric:* $(M, \ell) \rightarrow$
denote $p = \frac{r_{out}}{r_n}$, and use r_n as a scale parameter:
 $f = \frac{(r - p)(r - 1)[r + p/(p + 1)]}{r^3 + p^2/(p + 1)}$.
 $M \gg \ell$, $p \approx 2M$, $r_n \approx \ell$, $\kappa_- \approx -1$ ($\approx -\ell^{-1}$), $\kappa_+ \approx \frac{1$ rd metric: $(M,\ell) \rightarrow (r_m, r_{out})$.

cale parameter:

..
 $-\ell^{-1}), \quad \kappa_+ \approx \frac{1}{2p} \left(\approx \frac{1}{4M} \right).$ etric: $(M, \ell) \rightarrow (r_{in}, r_{out})$.
parameter:
), $\kappa_{+} \approx \frac{1}{2p}$ ($\approx \frac{1}{4M}$).

 $1\frac{1}{1}$ ≈ 1 ≈ 1

$$
\kappa_1 = \frac{(p-1)(p+2)}{2p(p^2+p+1)}, \kappa_2 = -\frac{(p-1)(2p+1)}{2(p^2+p+1)},
$$

\n
$$
\kappa_0 = \frac{(p+1)(p+2)(2p+1)}{2p(p^2+p+1)}.
$$

\n
$$
\kappa_1 \approx \frac{1}{2p} \approx \frac{1}{4M}, \quad \kappa_2 \approx -1;
$$

\n
$$
Q = \int_0^r \frac{dr}{f}, \quad f^{-1} = 1 + \frac{1}{2\kappa_1(r-r_1)} + \frac{1}{2\kappa_2(r-r_2)} + \frac{1}{2\kappa_0(r-r_0)}.
$$

\n
$$
Q(r) = r + \frac{1}{2r} \ln|r-1| + \frac{1}{2r} \ln\left(\frac{|r-p|}{p}\right) + \frac{1}{2r} \ln\left(\frac{|r-r_0|}{r}\right).
$$

$$
\kappa_1 \approx \frac{1}{2p} \approx \frac{1}{4M}, \quad \kappa_2 \approx -1;
$$

$$
\kappa_1 = \frac{(p-1)(p+2)}{2p(p^2+p+1)}, \kappa_2 = -\frac{(p-1)(2p+1)}{2(p^2+p+1)},
$$
\n
$$
\kappa_0 = \frac{(p+1)(p+2)(2p+1)}{2p(p^2+p+1)}.
$$
\n
$$
\kappa_1 \approx \frac{1}{2p} \approx \frac{1}{4M}, \quad \kappa_2 \approx -1;
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\n
$$
Q = \int_0^r \frac{dr}{f}, \quad f^{-1} = 1 + \frac{1}{2\kappa_1(r-r_1)} + \frac{1}{2\kappa_2(r-r_2)} + \frac{1}{2\kappa_0(r-r_0)}.
$$
\n
$$
Q(r) = r + \frac{1}{2\kappa_2} \ln|r - 1| + \frac{1}{2\kappa_1} \ln\left(\frac{|r-p|}{p}\right) + \frac{1}{2\kappa_0} \ln\left(\frac{|r-r_0|}{|r_0|}\right).
$$
\nOutgoing rays: $u = v - 2Q(r) = const$

Outgoing rays: $u = v - 2Q(r) = const$

in a nonsingular
sandwich black
hole $\begin{array}{l} \n\text{1} & \text{1} & \$ dial null ra

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ndwich blad
 $\frac{hole}{1-\frac{2Mr^2}{r^3+2M\ell^2+\ell^3}}$
 $\frac{(r-r_1)(r-r_2)(r-r_0)}{r^3-r_1r_2r_0}$, p

ndard model $\alpha = 1$ andwich blass (andwich blass (andwich blass (as 2 = -a²fdv² + 2a dvdr

f = 1 - $\frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}$

f = $\frac{(r-r_1)(r-r_2)(r-r_0)}{r^3 - r_1 r_2 r_0}$, p

standard model $\boxed{a = 1}$ **ial null rays

nonsingular**
 dwich black
 hole
 $\frac{2Mr^2}{r^3+2M\ell^2+\ell^3}$
 $\frac{1-\frac{2Mr^2}{r^3+2M\ell^2+\ell^3}}{r^3-r_1r_2r_0}$, $p=r_1/r_2$
 dard model $\alpha = 1$

 $ds^2 = -\alpha^2 f dv^2 + 2\alpha dv dr$

$$
f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}
$$

$$
f = \frac{(r - r_1)(r - r_2)(r - r_0)}{r^3 - r_1 r_2 r_0}, \qquad p = r_1/r_2
$$

Standard model

 $\alpha = 1$

 $p = 8$, $q = 30$

102
 102

<u> $\frac{2Mr^2}{4v^2 + 2\alpha \, dvdr}$
 $\frac{2Mr^2}{1 + 2M\ell^2 + \ell^3}$
 $\frac{r_2(r - r_0)}{1 + r_2 r_0}$, $p = r_1/r_2 r_0$
 p = $\frac{r^n + 1}{r^n + 1 + p^k}$ </u> $ds^2 = -\alpha^2 f dv^2 + 2\alpha dv dr$ sandwich black
hole dial null ra

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ndwich blad
 $\frac{hole}{1-\frac{2Mr^2}{r^3+2M\ell^2+\ell^3}}$
 $\frac{(r-r_1)(r-r_2)(r-r_0)}{r^3-r_1r_2r_0}$, p

dified model $\alpha = \frac{r^n-r_0}{r^n+1}$ **andwich blass (andwich blass)**
 respectively andwich blass (andwich blass)
 r **= 1 -** $\frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}$ **
** $f = \frac{(r-r_1)(r-r_2)(r-r_0)}{r^3 - r_1 r_2 r_0}$ **, p

Modified model** $\alpha = \frac{r^n}{r^n + 1}$ **ial null rays

nonsingular**
 dwich black
 hole
 $\frac{2Mr^2}{r^3+2M\ell^2+\ell^3}$
 $\frac{r}{r^3-r_1r_2r_0}$
 $\frac{r}{r^3-r_1r_2r_0}$
 ied model $\alpha = \frac{r^n+1}{r^n+1+p^k}$

$$
f = 1 - \frac{2Mr^2}{r^3 + 2M\ell^2 + \ell^3}
$$

$$
f = \frac{(r - r_1)(r - r_2)(r - r_0)}{r^3 - r_1 r_2 r_0}, \qquad p = r_1/r_2
$$

Modified model $\alpha =$ $p = 8,$ $q = 30,$ $r^n + 1$ $n = 6, \quad k = 4$

(i) black hole formation; (ii) Hawking radiation; (iii) signal from the second shell;
(iv) radiation from the black hole interior; Sequence of events (as seen by an external observer) (vi) Mass inflation; $\kappa \sim -\alpha_0$; (v) ourburst of radiation from the inner domain (near inner horizon); (vii) Total emitted energy is always positive, its density can be negative during short time. vents (as seen by

 ll observer):

ttion;

on;

second shell;

he black hole interior;

ation from the inner

ner horizon);
 $\kappa \sim -\alpha_0$;

nergy is always positive,

be negative during short time.

Hawking radiation

The Hawking result for the quantum energy flux from a black hole is correctly reproduced, when the mass parameter p and the lifetime of the blackhole q are large. The shape of the curve is almost the same for both standard and modified models. Duration of the almost constant tail of quantum radiation is approximately equal to q (lifetime of the black hole).

 $p = 4$, $q = 30$, $\alpha_{0} = 1$

 $p = 4$, $q = 30$, $\alpha_{\rm o} \sim p^{-4}$

"Mass inflation" is cured, however "gravity acceleration" mechanism still works. Self-consistency problem.

Main results:

(i) Properly reproduced Hawking radiation from the outer horizon (for $q \gg M$); (ii) For α =1 -- huge outburst of the quantum radiation from the inner horizon: $\Delta E \sim \exp(q) \sim \exp(\Delta v/\ell)$. This radiation **Main results:**

roduced Hawking radi
 $q \gg M$);

uge outburst of the quorizon: $\Delta E \sim \exp(q) \sim e$:

the inner horizon dur

q) ~ $\exp(-\Delta v/\ell)$;

on mechanism (Israel

ll choice of α ($\alpha_0 \ll 1$)

uced to the power law

ency p **in results:**

ed Hawking radiation from the outer

1);

utburst of the quantum radiation from

: $\Delta E \sim \exp(q) \sim \exp(\Delta v/\ell)$. This radiation

nner horizon during time interval
 $\exp(-\Delta v/\ell)$;

echanism (Israel, Poisson [1990]);
 comes from the inner horizon during time interval **Main results:**

(i) Properly reproduced Hawking radiation from the outer

horizon (for $q \gg M$);

(ii) For α =1 -- huge outburst of the quantum radiation from

the inner horizon: $\Delta E \sim \exp(q) \sim \exp(\Delta v/\ell)$. This radiation

c $\Delta u_+ \sim \exp(-q) \sim \exp(-\Delta v/\ell);$ **n results:**

d Hawking radiation from the outer

);

tburst of the quantum radiation from
 $\Delta E \sim \exp(q) \sim \exp(\Delta v/\ell)$. This radiation

ner horizon during time interval

p(- $\Delta v/\ell$);

chanism (Israel, Poisson [1990]);

e of $\$ **Main results:**

roperly reproduced Hawking radiation from the outer

orizon (for $q \gg M$);

Sor α =1 -- huge outburst of the quantum radiation from

the inner horizon: $\Delta E \sim \exp(q) \sim \exp(\Delta v/\ell)$. This radiation

comes from t **Aain results:**

bluced Hawking radiation from the outer
 $\gg M$);

e outburst of the quantum radiation from

ron: $\Delta E \sim \exp(q) \sim \exp(\Delta v/\ell)$. This radiation

e inner horizon during time interval
 $\sim \exp(-\Delta v/\ell)$;

mechanism (Isr can be reduced to the power law; (v) Self-consistency problem remains [Bolashenko and V.F. (1986)]. **esults:**
wking radiation from the outer
st of the quantum radiation from
exp(q) ~ exp($\Delta v/\ell$). This radiation
norizon during time interval
 v/ℓ);
ism (Israel, Poisson [1990]);
 α ($\alpha_0 \ll 1$) outburst of the energy
p

"Realistic" non-singular model of an evaporating black hole

(i) Non-singular models of evaporating BHs (ii) Quantum radiation from BH interior 2D approximation (its validity?); (iii) "Bracket" formalism; (iv) Sandwich model [2 shells, 2 parameres] Summary and Discussion vs. a "realistic" non-singular models (v) Two mechanisms of energy amplification: "Gravity accelerator" vs "Mass inflation"; (vi) Properly chosen red-shift factor helps to cure the mass inflation problem; (vii) Information loss and self-consistency problems; (viii) Back-reaction of created radiation?!

Polyakov action: $W[g] = -\frac{b}{2}\int d^2x \sqrt{g} R \frac{1}{n}R$, $\Box \Phi = R$, $\left\langle T^{\mu\nu}\right\rangle =-\frac{1}{2\sqrt{g}}\frac{\delta W[g]}{\delta g_{\mu\nu}}=$

b{-2V^uV^v Φ - V^u Φ V^v Φ + g^{uv}[2R + $\frac{1}{2}$ (V Φ)²]},

 $\langle T^{\mu\nu}\rangle g_{\mu\nu} = 2bR.$

"Bracket" formalism

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"Bracket" formalism
\nLet
$$
y(x)
$$
 be a function, and $x(y)$ is its inverse.
\n
$$
[y,x] = \ln |y'|, \quad \langle y, x \rangle = \frac{y''}{y'}, \quad \{y, x\} = \frac{y''}{y'} - \frac{3}{2} \left(\frac{y''}{y'}\right)^2.
$$
\nSchwarz derivative
\n
$$
[y,x] = -[x,y], \quad \langle y, x \rangle = -y' \langle x, y \rangle, \quad \{y, x\} = -(y')^2 \{x, y\}.
$$
\nChain rule: $f \circ g(z) = f(g(z)) \Rightarrow$
\n
$$
[f \circ g, z] = [f, g]_{g=g(z)} + [g, z],
$$
\n
$$
\langle f \circ g, z \rangle = \langle f, g \rangle_{g=g(z)} g'(z) + \langle g, z \rangle,
$$
\n
$$
\langle f \circ g, z \rangle = \{f, g\} |_{g=g(z)} (g'(z))^2 + \{g, z\}.
$$

Schwarz derivative Schwarz derivative

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y, x >=
 $y, x > 0$
 $\Rightarrow f(x) + 1$
 $> |g| = g(x)$
 $\Rightarrow g(x)$ **"Bracket" fort**

Let $y(x)$ be a function, and $x(y)$ is its

[y, x] = ln | y'|, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle$

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[y, x] = -[x, y], $\langle y, x \rangle = -y' \langle x, y \rangle$

Chain rule: $f \circ g(z) = f(g(z)) \Rightarrow$

[$f \circ g, z$] = [f, g]_{$|_{g=g(z)}$} + [g, **"Bracket" for**

Let $y(x)$ be a function, and $x(y)$ is it
 $[y, x] = \ln |y'|$, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, y \rangle$

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Chain rule: $f \circ g(z) = f(g(z)) \Rightarrow$
 $[f \circ g, z] = [f, g]_{g=g(z)} + [g, z],$
 $\langle f \circ g, z \rangle = \langle f, g \rangle_{g=g(z$ () acket"
 $x > = \frac{y''}{y'}$,
 $x > = -y' <$
 $= f(g(z)) =$
 $\bigg|_{g=g(z)} g'(z) -$
 $(g'(z))^2 + g(z)$ **"Bracket" formalism**

Let $y(x)$ be a function, and $x(y)$ is its inverse.
 $[y, x] = \ln |y'|$, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle = \frac{y'''}{y'} - \frac{3}{2} \left(\frac{y''}{y'} \right)^2$.

Schwarz derivative
 $[y, x] = -[x, y]$, $\langle y, x \rangle = -y' \langle x, y \rangle$, $\langle y, x \rangle = (g'(z))^2 + (g, z).$ Schwarz derivative
 $-y' < x, y>, \{y, x\} = -(y,$
 $(x)) \Rightarrow$
 $(x, z),$
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 $(y, z) + (z, z),$
 $(y, z) + (z, z),$ $[f \circ g, z] = [f, g]_{g = g(z)} + [g, z],$ $\langle f \circ g, z \rangle = \langle f, g \rangle \Big|_{g=g(z)} g'(z) + \langle g, z \rangle,$ acket" formalism

n, and $x(y)$ is its inverse.
 $x \geq \frac{y''}{y'}$, $\{y, x\} = \frac{y'''}{y'} - \frac{3}{2} \left(\frac{y''}{y'}\right)^2$.

Schwarz derivative
 $x \geq -y' < x, y >$, $\{y, x\} = -(y')^2 \{x, y\}$.
 $= f(g(z)) \Rightarrow$
 $\bigcup_{x \in x(x)} g'(z) + \langle g, z \rangle$,
 $\bigg|_{x = g(x)} g'(z)$ "Bracket" formalism

x) be a function, and $x(y)$ is its inverse.
 $= \ln |y'|$, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle = \frac{y'''}{y'} - \frac{3}{2} \left(\frac{y''}{y'} \right)^2$.

Schwarz derivative
 $= -[x, y]$, $\langle y, x \rangle = -y' \langle x, y \rangle$, $\langle y, x \rangle = -(y')^2 \langle x, y \rangle$.

ru "Bracket" formalism

(a) be a function, and $x(y)$ is its inverse.
 $\ln |y'|$, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle = \frac{y'''}{y'} - \frac{3}{2} \left(\frac{y''}{y'} \right)^2$.

Schwarz derivative
 $\langle -[x, y], \langle y, x \rangle = -y' \langle x, y \rangle, \quad \langle y, x \rangle = -\langle y' \rangle^2 \langle x, y \rangle$.

Th **"Bracket" formalism**

(x) be a function, and $x(y)$ is its inverse.
 $= \ln |y'|$, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle = \frac{y'''}{y'} - \frac{3}{2} \left(\frac{y''}{y'} \right)^2$.

Schwarz derivative
 $= -[x, y]$, $\langle y, x \rangle = -y' \langle x, y \rangle$, $\langle y, x \rangle = -\langle y' \rangle^2 / x, y$. **"Bracket" formalism**

t y(x) be a function, and x(y) is its inverse.
 $x = \ln |y'|$, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle = \frac{y'''}{y'} - \frac{3}{2} \left(\frac{y''}{y'} \right)^2$.

Schwarz derivative
 $x = \{x, y\}$, $\langle y, x \rangle = -y' \langle x, y \rangle$, $\langle y, x \rangle = -\langle y' \rangle^2 /$ *g g z g g zg g zy x x y < y x > y < x y > {y x} function, and x(y) is its*
function, and x(y) is its
j, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle$
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z], $\langle y, x \rangle = -y' \langle x, y \rangle$
f $\circ g(z) = f(g(z)) \Rightarrow$
f, $g|_{g=g(z)} + [g, z],$
 $\langle f, g \rangle_{g=g(z)} g'(z) + \langle g, g \rangle$
 $g|_{g=g(z)} (g'(z))^2 + \langle g, z \rangle$ *f f f f g <i>f g <i>f f f f g <i>f g <i>f <i>g l* **Bracket" forms**

Let $y(x)$ be a function, and $x(y)$ is its inv
 $[y, x] = \ln |y'|$, $\langle y, x \rangle = \frac{y''}{y'}$, $\langle y, x \rangle =$

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 $[y, x] = -[x, y]$, $\langle y, x \rangle = -y' \langle x, y \rangle$,

Chain rule: $f \circ g(z) = f(g(z)) \Rightarrow$
 $[f \circ g, z] = [f, g]_{g = g(z)} + [g, z]$,
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s of "brackets". $\,$., u_{+}], $\,$ (2015)]

Gain function: $\beta = \frac{du}{du_+}$, P
"Radiation entropy" $S(v) =$ du_{+} ⁷

 $\frac{1}{R}$ 2^{\sim}

[Bianchi, DeLorenzo, Smerlak, JHEP, 06, 1280 (2015)]

Density of out-going trajectories: $W =$.

bservables on:

ain function: $\beta = \frac{du}{du_+}$,

Radiation entropy" $S(v)$

ianchi, DeLorenzo, Smerlal

ensity of out-going traj

nergy flux: $\dot{E} = -\frac{1}{24\pi}$.

[M.Reuter, CQG, 6, 1149] **Son I⁺ in term**:
 $\frac{du_{-}}{du_{+}}, \quad P = \ln \beta = [u$

" $S(v) = -\frac{1}{12}P$,

Smerlak, JHEP, 06, 1280

ng trajectories: $W = < -\frac{1}{24\pi} \{u_{-}, u_{+}\}$, $\{u_{-}, u_{+}\}$, $\{1149(1989)\}$ t **in terms of "brath"**
 $P = \ln \beta = [u_-, u_+]$,
 $\frac{1}{12}P$,

JHEP, 06, 1280 (2015)]

stories: $W = < u_-, u_+ >$.
 $\frac{1}{12}u_+, u_+ = \frac{1}{12}u_+v_+ + \frac{1}{12}u_+v_+ + \frac{1}{12}u_+v_+ + \frac{1}{12}u_+v_+ + \frac{1}{12}u_+v_+ + \frac{1}{12}u_+v_+ + \frac{1}{12}u$ les on I⁺ in terms of "brackets".
 $\beta = \frac{du_-}{du_+}, \quad P = \ln \beta = [u_-, u_+],$

opy" $S(v) = -\frac{1}{12}P$,

zo, Smerlak, JHEP, 06, 1280 (2015)]

going trajectories: $W = < u_-, u_+ >$.
 $\frac{1}{24\pi} = -\frac{1}{24\pi}u_-, u_+ f, \{u_-, u_+ f\}$ - Schwarz d **S on I⁺ in terms of "brackets".**
 $=\frac{du_{-}}{du_{+}}, \quad P = \ln \beta = [u_{-}, u_{+}],$

y" $S(v) = -\frac{1}{12}P$,

Smerlak, JHEP, 06, 1280 (2015)]

ing trajectories: $W = \langle u_{-}, u_{+} \rangle$.
 $-\frac{1}{24\pi} \{u_{-}, u_{+}\}$, $\{u_{-}, u_{+}\}$ - Schwarz derivativ **Observables on I⁺ in terms of "brackets".**

Gain function: $\beta = \frac{du_-}{du_+}$, $P = \ln \beta = [u_-, u_+]$,

"Radiation entropy" $S(v) = -\frac{1}{12}P$,

[Bianchi, DeLorenzo, Smerlak, JHEP, 06, 1280 (2015)]

Density of out-going trajector **on I⁺ in terms of**
 $\frac{du_-}{du_+}$, $P = \ln \beta = [u_-, u_+]$
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 z trajectories: $W = < u_-, u_+$
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149 (1989)] **on I⁺ in terms of "bra**
 $\frac{du_{-}}{du_{+}}, \quad P = \ln \beta = [u_{-}, u_{+}],$
 $S(v) = -\frac{1}{12}P,$

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g trajectories: $W = < u_{-}, u_{+}>$.
 $\frac{1}{24\pi}\{u_{-}, u_{+}\}$, $\{u_{-}, u_{+}\}$ - Schwarz

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[M.Reuter, CQG, 6, 1149 (1989)] **Observables on I⁺ in terms of "brackets".**

Gain function: $\beta = \frac{du}{du_+}$, $P = \ln \beta = [u_-, u_+]$,

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Density of out-going trajectorie ' in terms of "brackets".
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JHEP, 06, 1280 (2015)]

tories: $W =$.

, $u_+, \int, \{u_-, u_+ \}$ - Schwarz derivative,

989)] Energy flux: $\dot{E} = -\frac{1}{24\pi} \{u_-, u_+\}$, $\{u_-, u_+\}$ - Schwarz derivative, **es on I⁺ in terms of "brackets".**
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ppy" $S(v) = -\frac{1}{12}P$,

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(oing trajectories: $W = \langle u_-, u_+ \rangle$.
 $= -\frac{1}{24\pi} \{u_-, u_+ \}$, $\{u_-, u_+ \}$ - Schwarz de **Dies on I⁺ in terms of "brackets".**
 $E = \frac{du_{-}}{du_{+}}, \quad P = \ln \beta = [u_{-}, u_{+}],$

tropy" $S(v) = -\frac{1}{12}P$,

enzo, Smerlak, JHEP, 06, 1280 (2015)]
 E-going trajectories: $W = \langle u_{-}, u_{+} \rangle$.
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 xs: W = < *u*₋, *u*₊ >.
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Schwarz derivati **in term**
 $u = \ln \beta = [u$
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Basic equations in the bracket formalism

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\begin{aligned}\n\text{in the bracket formula} \text{for } \mathbf{m} \text{ is } \mathbf{m} \text{ is } \mathbf{m} \text{ and } \mathbf{
$$

$$
P = \ln \beta = \ln \left(\frac{du_-}{du_+}\right) = [u_-, u_+] = -p(q) - \ln \alpha_0,
$$

$$
W = = e^{-p(q)} \left[\frac{1}{2}w(q) - \frac{\alpha_0}{\alpha_0^2}\right],
$$

$$
P = \ln \beta = \ln \left(\frac{du_{-}}{du_{+}}\right) = [u_{-}, u_{+}] = -p(q) - \ln \alpha_{0},
$$

\n
$$
W = \langle u_{-}, u_{+} \rangle = e^{-p(q)} \left[\frac{1}{2}w(q) - \frac{\alpha_{0}}{\alpha_{0}^{2}}\right],
$$

\n
$$
-24\pi \dot{E} = \{u_{-}, u_{+}\} = -e^{-2p(q)} \left[\frac{1}{4}\varepsilon(q) + \frac{1}{\alpha_{0}^{2}}\{x, u_{-}\}\right],
$$

\n
$$
\{x, u_{-}\} = \frac{1}{2}\alpha_{0}^{2}a_{2} + \frac{\alpha_{0}}{\alpha_{0}} - \frac{3}{2}\left(\frac{\alpha_{0}}{\alpha_{0}}\right)^{2}.
$$

$$
P = \ln \beta = \ln \left(\frac{du_{-}}{du_{+}}\right) = [u_{-}, u_{+}] = -p(q) - \ln \alpha_{0},
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\n
$$
W = \langle u_{-}, u_{+} \rangle = e^{-p(q)} \left[\frac{1}{2}w(q) - \frac{\alpha_{0}}{\alpha_{0}^{2}}\right],
$$

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$$
-24\pi \dot{E} = \{u_{-}, u_{+}\} = -e^{-2p(q)} \left[\frac{1}{4}\varepsilon(q) + \frac{1}{\alpha_{0}^{2}}\{x, u_{-}\}\right],
$$

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\{x, u_{-}\} = \frac{1}{2}\alpha_{0}^{2}a_{2} + \frac{\alpha_{0}}{\alpha_{0}} - \frac{3}{2}\left(\frac{\alpha_{0}}{\alpha_{0}}\right)^{2}.
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$$
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