Inflation and Disformal Transformation

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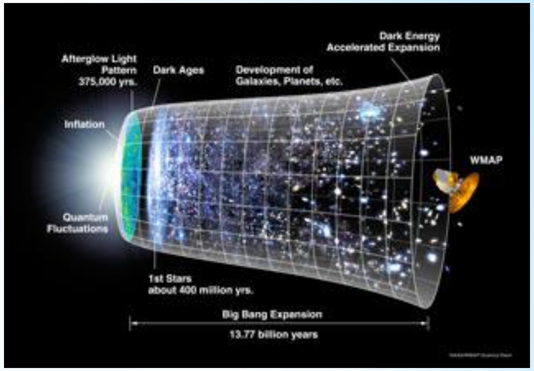
Big mystery in cosmology

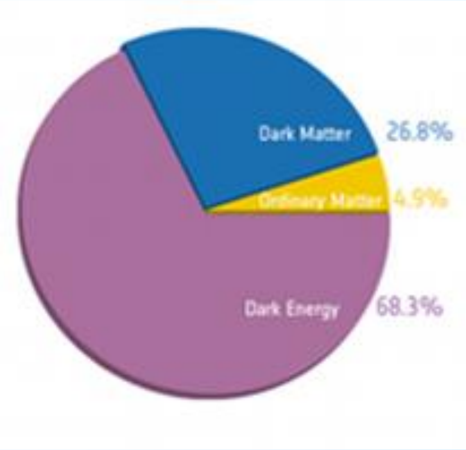
Acceleration of cosmic expansion

- Inflation: early stage of the Universe



What is an inflaton ?

- Present Acceleration



cosmological constant

$$\Lambda \sim 10^{-120} m_{PL}^2$$

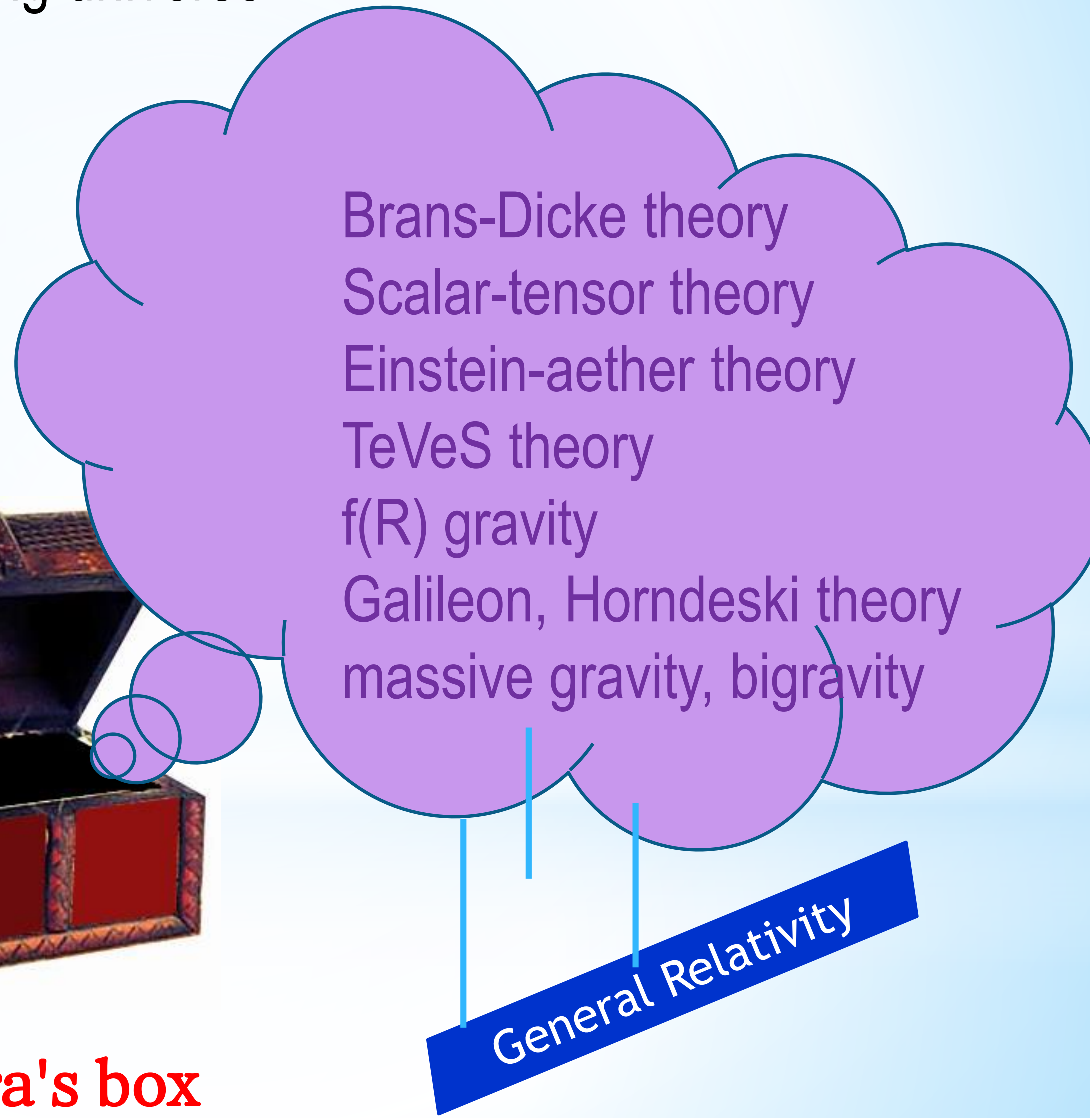
→
- Dark Energy
- Modified gravity

dark energy

google scholar

"modified gravity" & "accelerating universe"

-1995	3
1996-2000	11
2001-2005	118
2006-2010	516
2011-2015	994
2016,2017	377



Pandora's box

Inflation

So many models

The origin ?

Standard Model

Scalar field＝Higgs

Gravity is modified

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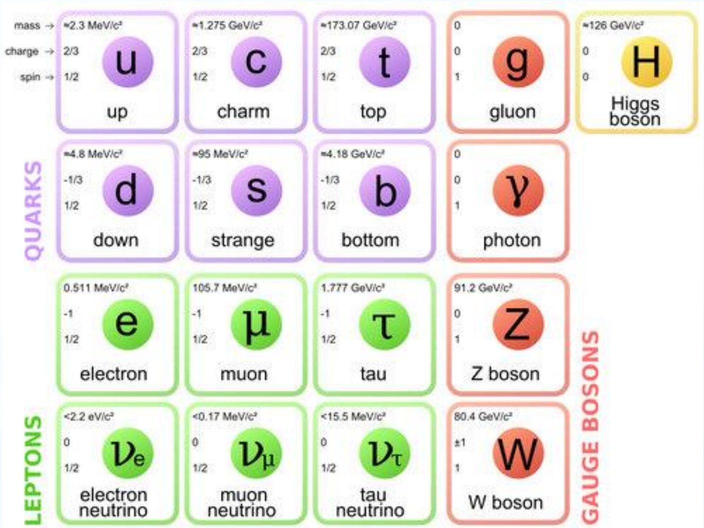
Higgs inflation

original (conventional)
new
hybrid
generalized

google scholar
“inflation" & “cosmology"

1970-1979	225
1980-1989	2510
1990-1999	8270
2000-2009	20100
2010-2017	18800



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How to analyze modified gravity theories

- Fundamental approach

  Unified theory/quantum correction/natural extension

  KKLT
  massive gravity

- Phenomenological approach

  Given back ground + possible extension

  PPN approach
  effective theory

- General relativistic approach

  description in the Einstein frame

  → ***present talk***

a gravitational theory with the action

$$S = \int d^D x \sqrt{-g} F(g^{\mu\nu}, R_{\mu\nu}, C^{\mu}{}_{\nu\rho,\sigma}, \psi)$$

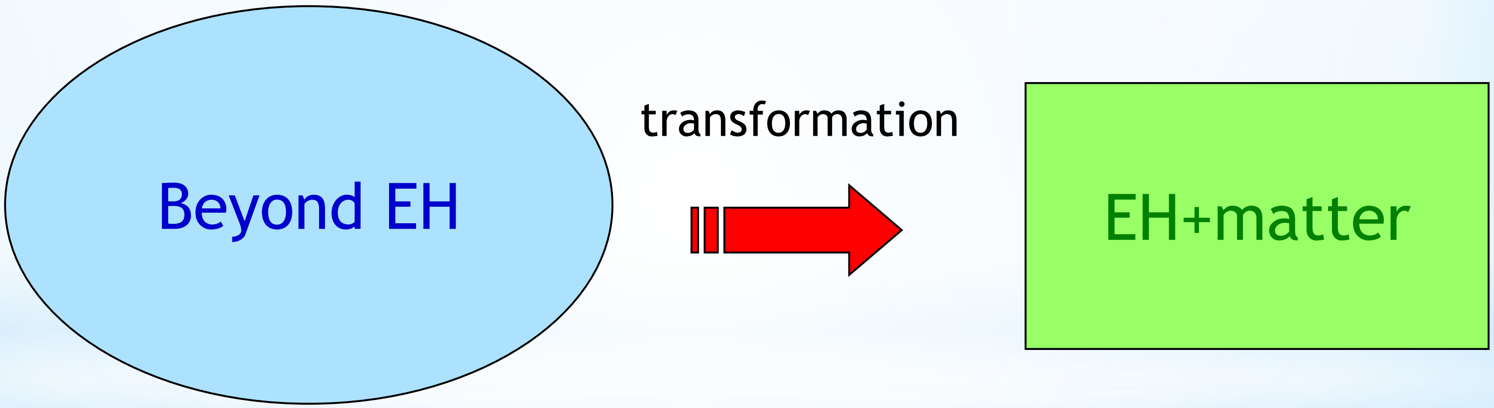
$F(\cdots)$ An arbitrary function

ψ matter fields including scalar fields

It may show some interesting properties of gravity
But it is too complicated to analyze it

Toward the EH action :

If we can find an equivalent gravitational theory only with the EH action by some transformation, it makes our discussion simpler.



Basic equations are very complicated

well known

1. A scalar-tensor type theory

KM (1989)

$$S = \int d^D x \sqrt{-g} \left[f(\phi) R - \frac{\epsilon_\phi}{2} (\nabla \phi)^2 - V(\phi) \right]$$

$$\hat{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu}$$

a conformal transformation

$$\omega = \frac{1}{D-2} \ln(2\kappa^2 |f(\phi)|)$$

$$S = \int d^D x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} R(\hat{g}) - \frac{1}{2} (\nabla \sigma)^2 - U(\sigma) \right]$$

$$\kappa\sigma = \int d\phi \left[\frac{\epsilon_\phi (D-2) f(\phi) + 2(D-1)(f'(\phi))^2}{2(D-2) f^2(\phi)} \right]^{1/2}$$

$$U(\sigma) = \epsilon_f [2\kappa^2 |f(\phi)|]^{-D/(D-2)} V(\phi)$$

(original) Higgs inflation

Bezrukov, Shaposhnikov (2008)

Spokoiny (1984); Salopek, Bond, Bardeen (1989); Futamase, KM (1989); Fakir, Unruh (1990)

Higgs field: +non-minimal coupling $(\xi < 0)$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2}\xi\phi^2 R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right]$$

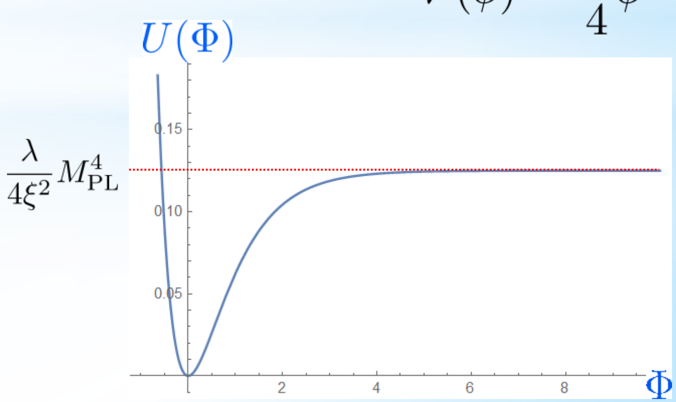
conformal transformation $\tilde{g}_{\mu\nu} = (1 - \xi\kappa^2\phi^2) g_{\mu\nu}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - U(\Phi) \right]$$

$$\frac{d\Phi}{d\phi} = \frac{1}{\sqrt{(1 - \xi\kappa^2\phi^2)}}$$

$$U(\Phi) = \frac{1}{(1 - \xi\kappa^2\phi^2)^2} V(\phi)$$

$$V(\phi) = \frac{\lambda}{4}\phi^4$$



2. F(R , ϕ) theory

$$S=\int d^Dx\sqrt{-g}\left[F(R,\phi)-\frac{\epsilon_\phi}{2}(\nabla\phi)^2\right]$$

KM (1989)

higher derivarives

Jakubiec, Kijowski (1987);
Magnano, Ferraris, Francaviglia, (1987);
Ferraris, Francaviglia, Magnano, (1988)

$$\hat{g}_{\mu\nu}=e^{2\omega}g_{\mu\nu}$$ a conformal transformation

$$\omega=\frac{1}{D-2}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]$$

$$\kappa\sigma=\sqrt{\frac{D-1}{D-2}}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]$$

"new degree of freedom"

$$S=\int d^Dx\sqrt{-\hat{g}}\left[\frac{1}{2\kappa^2}R(\hat{g})-\frac{1}{2}(\hat{\nabla}\sigma)^2\right.$$

$$\left.-\frac{\epsilon_\phi\epsilon_F}{2}e^{-\sqrt{\frac{D-1}{D-2}}\kappa\sigma}(\hat{\nabla}\phi)^2-U(\phi,\sigma)\right]$$

$$U(\phi,\sigma)=\epsilon_F\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]^{-D/(D-2)}\left(R\frac{\partial F}{\partial R}-F(R)\right)$$

A simple example

KM (1988)

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R+\alpha R^2\right]$$: Starobinski inflation

It contains higher derivatives

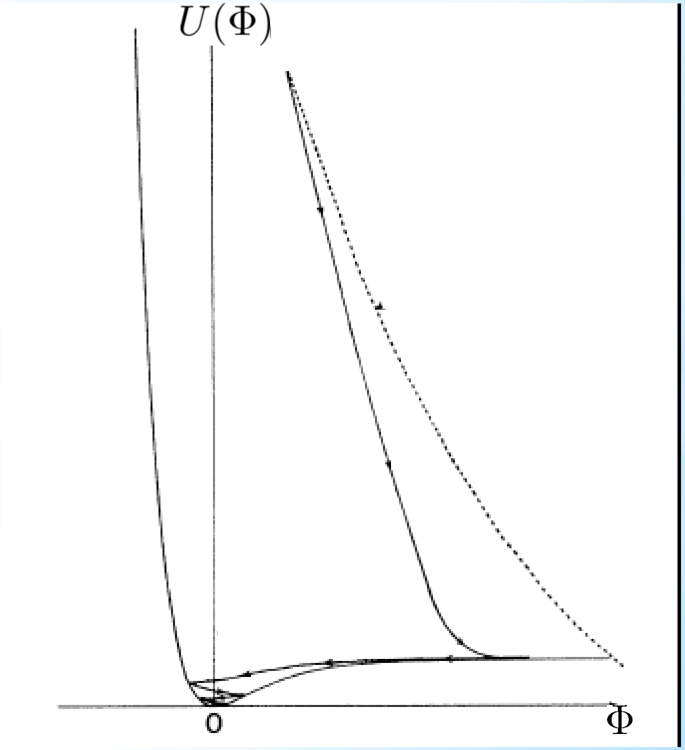
conformal transformation $\tilde{g}_{\mu\nu}=(1+2\alpha R)g_{\mu\nu}$

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2\kappa^2}\tilde{R}-\frac{1}{2}(\tilde{\nabla}\Phi)^2-U(\Phi)\right]$$

GR + a scalar field with a potential U(Φ)

$$\kappa\Phi=\sqrt{\frac{3}{2}}\ln\left[1+2\alpha R\right]$$

$$U(\Phi)=\frac{1}{8\alpha}\left(1-e^{-\sqrt{\frac{3}{2}}\kappa\Phi}\right)^2$$



It is easy to judge
whether inflation occurs or not

3. F($R_{\mu\nu}$) theory

Jakubiec, Kijowski , GRG 19 (1987) 719 ;
Magnano, Ferraris, Francaviglia, GRG 19 (1987) 465 ;
Ferraris, Francaviglia, Magnano, CQG. 5 (1988) L95

$$S = \int d^4x\sqrt{-g}F(g^{\mu\nu}, R_{\mu\nu})$$

$$\sqrt{-q}q^{\mu\nu} = 2\kappa^2 \sqrt{-g}\frac{\partial F}{\partial R_{\mu\nu}}$$

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-q}\left[R(q,\partial q,\partial q^2) + q^{\mu\nu}(C^\rho{}_{\rho\sigma}C^\sigma{}_{\mu\nu} - C^\rho{}_{\sigma\mu}C^\sigma{}_{\rho\nu}) - q^{\mu\nu}\mathcal{R}_{\mu\nu} + \frac{\sqrt{-g}}{\sqrt{-q}}F(\mathcal{R}_{\mu\nu}(g,q), g^{\alpha\beta})\right] + S_{\text{matter}}(g^{\alpha\beta},\psi)$$

$$C^\rho{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(\nabla^{(q)}_\mu g_{\nu\sigma} + \nabla^{(q)}_\nu g_{\mu\sigma} - \nabla^{(q)}_\sigma g_{\mu\nu}\right)$$

$$R_{\mu\nu} = \mathcal{R}_{\mu\nu}(g^{\alpha\beta}, q^{\gamma\delta})$$

The EH gravitational action + spin 2 field ($g^{\mu\nu}$) + other matter fields

new Higgs inflation   Germani, Kehagias (2010)

Higgs field: + derivative coupling

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\alpha G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right)-\frac{1}{2}(\nabla\phi)^2-V(\phi)\right]$$

The EH gravitational action ($q^{\mu\nu}$) + spin 2 field ($g^{\mu\nu}$) + other matter fields

Behavior ?

The previous method may not work

Instead, we may use a disformal transformation

disformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 \left(g_{\mu\nu} + u_\mu u_\nu \right)$$

u_μ: a timelike vector

$$u_\mu u^\mu = -\lambda^2$$

$$\sqrt{-\tilde{g}} = \Omega^4 (1-\lambda^2)^{\frac{1}{2}} \sqrt{-g}$$

$$\tilde{g}^{\mu\nu} = \Omega^{-2} \left(g^{\mu\nu} - \frac{1}{1-\lambda^2} u^\mu u^\nu \right)$$

Scale deformation in time direction

$$\tilde{\Gamma}^\mu_{\nu\rho} = \Gamma^\mu_{\nu\rho} + \gamma^\mu_{\nu\rho} \qquad \gamma^\mu_{\nu\rho} = f^\mu_{\nu\rho} + \omega^\mu_{\nu\rho}$$

$$f^\mu{}_{\rho\sigma} = \frac{1}{2} \left(g^{\mu\nu} - \frac{1}{1-\lambda^2} u^\mu u^\nu \right) \left[\nabla_\rho (u_\nu u_\sigma) + \nabla_\sigma (u_\nu u_\rho) - \nabla_\nu (u_\rho u_\sigma) \right]$$

$$\omega^\mu{}_{\rho\sigma} = \delta^\mu_\rho \nabla_\sigma \ln \Omega + \delta^\mu_\sigma \nabla_\rho \ln \Omega - \left(g^{\mu\nu} - \frac{1}{1-\lambda^2} u^\mu u^\nu \right) (g_{\rho\sigma} + u_\rho u_\sigma) \nabla_\nu \ln \Omega$$

$$\begin{aligned} \tilde{R} =& \Omega^{-2} \Big[\frac{2-\lambda^2}{2(1-\lambda^2)} R - \frac{1}{1-\lambda^2} \boxed{G_{\mu\nu} u^\mu u^\nu} \\ &+ \nabla_\mu (g^{\rho\sigma} \gamma^\mu_{\rho\sigma}) - \nabla^\rho \gamma^\mu{}_{\mu\rho} + g^{\rho\sigma} \gamma^\alpha_{\rho\sigma} \gamma^\mu{}_{\mu\alpha} - g^{\rho\sigma} \gamma^\beta{}_{\alpha\rho} \gamma^\alpha{}_{\beta\sigma} \\ &- \frac{1}{1-\lambda^2} u^\rho u^\sigma \left(\nabla_\mu \gamma^\mu_{\rho\sigma} - \nabla_\sigma \gamma^\mu{}_{\mu\rho} + \gamma^\mu{}_{\mu\alpha} \gamma^\alpha_{\rho\sigma} - \gamma^\beta{}_{\alpha\rho} \gamma^\alpha{}_{\beta\sigma} \right) \Big] \end{aligned}$$

$$
\begin{aligned}
\tilde{G}_{\mu\nu} &= \tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} \\
&= G_{\mu\nu} + \frac{1}{2(1-\lambda^2)}(g_{\mu\nu} + u_\mu u_\nu)u^\alpha u^\beta G_{\alpha\beta} \\
&\quad - \frac{\lambda^2}{4(1-\lambda^2)}(g_{\mu\nu} + u_\mu u_\nu)R - \frac{1}{2}u_\mu u_\nu R \\
&\quad + \nabla_\rho \gamma^\rho_{\mu\nu} - \nabla_\nu \gamma^\rho{}_{\rho\mu} + \gamma^\rho{}_{\rho\sigma}\gamma^\sigma_{\mu\nu} - \gamma^\sigma{}_{\rho\mu}\gamma^\rho{}_{\sigma\nu} \\
&\quad - \frac{1}{2}(g_{\mu\nu} + u_\mu u_\nu)\Big[\nabla_\alpha(g^{\rho\sigma}\gamma^\alpha_{\rho\sigma}) - \nabla^\rho\gamma^\sigma{}_{\sigma\rho} + g^{\rho\sigma}\gamma^\alpha_{\rho\sigma}\gamma^\beta{}_{\beta\alpha} - g^{\rho\sigma}\gamma^\beta{}_{\alpha\rho}\gamma^\alpha{}_{\beta\sigma} \\
&\quad - \frac{1}{1-\lambda^2}u^\rho u^\sigma\Big(\nabla_\alpha\gamma^\alpha_{\rho\sigma} - \nabla_\sigma\gamma^\alpha{}_{\alpha\rho} + \gamma^\beta{}_{\beta\alpha}\gamma^\alpha_{\rho\sigma} - \gamma^\beta{}_{\alpha\rho}\gamma^\alpha{}_{\beta\sigma}\Big)\Big]
\end{aligned}
$$

new Higgs inflation

$$u_\mu = \beta \nabla_\mu \phi \qquad \lambda^2 = 2\beta^2 X \qquad X = -\frac{1}{2}(\nabla\phi)^2$$

$$\tilde{G}_{\mu\nu}\tilde{\nabla}^\mu\phi\tilde{\nabla}^\nu\phi$$

$$= \frac{1}{\beta^2\Omega^4(1-\lambda^2)^2}\Big[\left(1-\frac{\lambda^2}{2}\right)\boxed{u^\mu u^\nu G_{\mu\nu}} - \frac{\lambda^4}{4}\boxed{R}$$

$$+ u^\mu u^\nu\left(\nabla_\rho\gamma^\rho_{\mu\nu} - \nabla_\nu\gamma^\rho_{\rho\mu} + \gamma^\rho_{\rho\sigma}\gamma^\sigma_{\mu\nu} - \gamma^\sigma_{\rho\mu}\gamma^\rho_{\sigma\nu}\right)$$

$$+ \frac{\lambda^2}{2}\Big\{(1-\lambda^2)\left(\nabla_\alpha(g^{\rho\sigma}\gamma^\alpha_{\rho\sigma}) - \nabla^\rho\gamma^\sigma_{\sigma\rho} + (g^{\rho\sigma}\gamma^\alpha_{\rho\sigma})\gamma^\beta_{\beta\alpha} - g^{\rho\sigma}\gamma^\beta_{\rho\alpha}\gamma^\alpha_{\sigma\beta}\right)$$

$$- u^\rho u^\sigma\left(\nabla_\alpha\gamma^\alpha_{\rho\sigma} - \nabla_\sigma\gamma^\alpha_{\alpha} + \gamma^\beta_{\beta\alpha}\gamma^\alpha_{\rho\sigma} - \gamma^\beta_{\alpha\rho}\gamma^\alpha_{\beta\sigma}\right)\Big\}\Big]$$

disformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2\left(g_{\mu\nu} + \beta^2\nabla_\mu\phi\nabla_\nu\phi\right)$$

$$\Omega^2 = \frac{(2-\lambda^2)}{2(1-\lambda^2)^{\frac{1}{2}}} \qquad \beta^2 = \alpha(1-\lambda^2)^{-\frac{1}{2}} \implies \lambda^4(1-\lambda^2) = 4\alpha^2 X^2$$

The EH gravitational action + Higgs field ϕ with higher-derivatives

The higher-derivative terms are too complicated

It may be better to analyze it in the original frame

However, if we can **ignore the higher-derivative terms**,
The analysis in the disformal frame becomes easy

Germani, Martucci, Moyassari (2012)

Slow-rolling inflationary phase

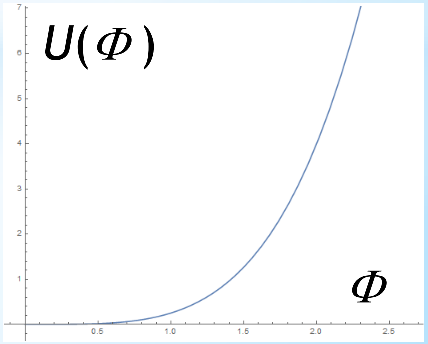
$$\mathcal{L}_{\rm Higgs} = -\sqrt{-\tilde{g}}\left[\left(\frac{1+\alpha V(\phi)}{2}\right)(\tilde{\nabla}\phi)^2 + V(\phi)\right] + \cdots$$

$$= -\sqrt{-\tilde{g}}\left[\frac{1}{2}(\tilde{\nabla}\Phi)^2 + U(\Phi)\right] + \cdots$$

$$\alpha = \frac{1}{M^2 M_{\rm PL}^2}$$

higher-derivative terms

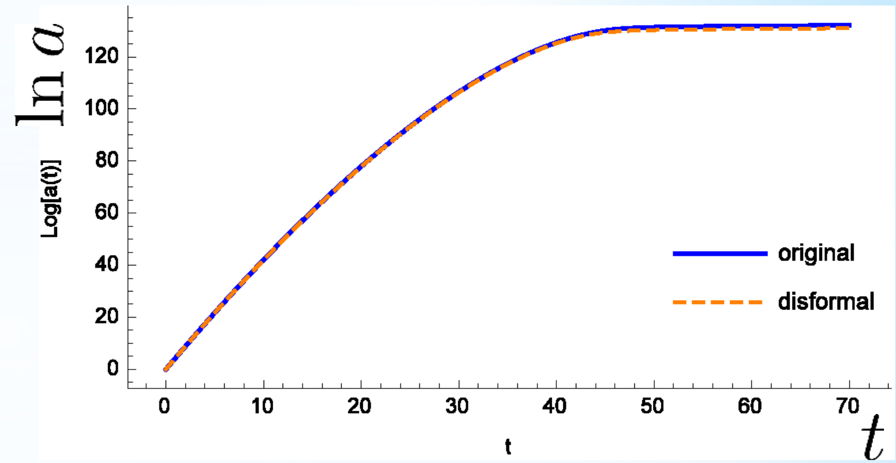
$$U(\Phi) = \begin{cases} \frac{\lambda}{4}\Phi^4 & \Phi \ll \Phi_{cr} \\ 3\sqrt[3]{\frac{3\lambda}{4}} M_{\rm PL}^4 \left(\frac{M}{M_{\rm PL}}\right)^{4/3}\left(\frac{\Phi}{M_{\rm PL}}\right)^{4/3} & \Phi \gg \Phi_{cr} \end{cases}$$



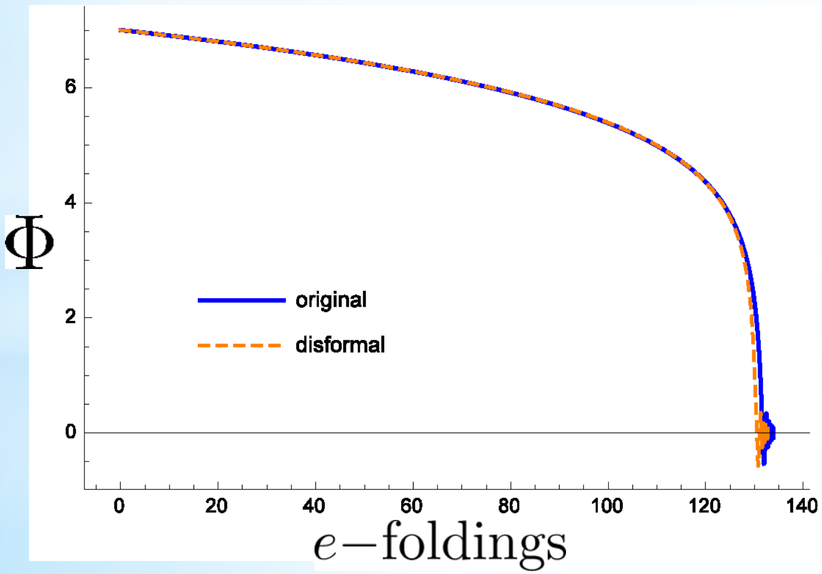
How well this truncation describes the original model ?

The original model
vs
the truncated one
after disformal transformation

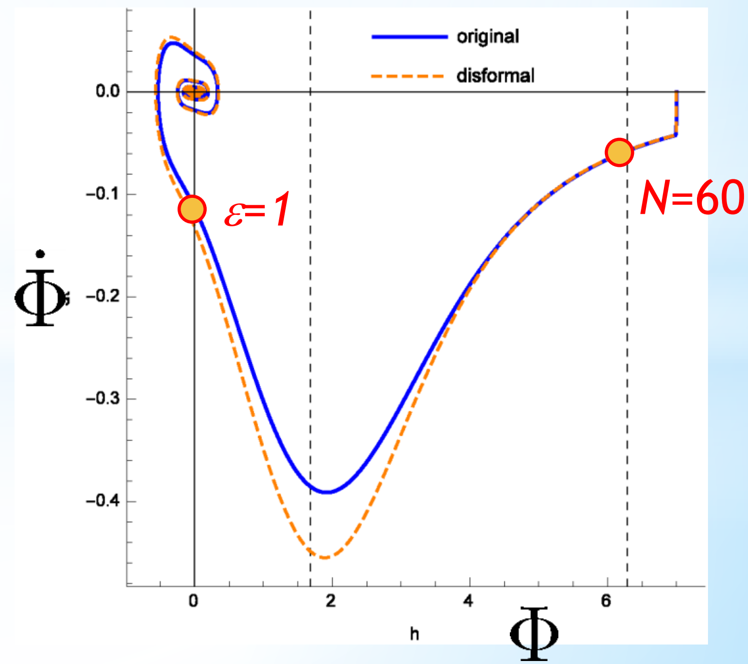
scale factor



Higgs field

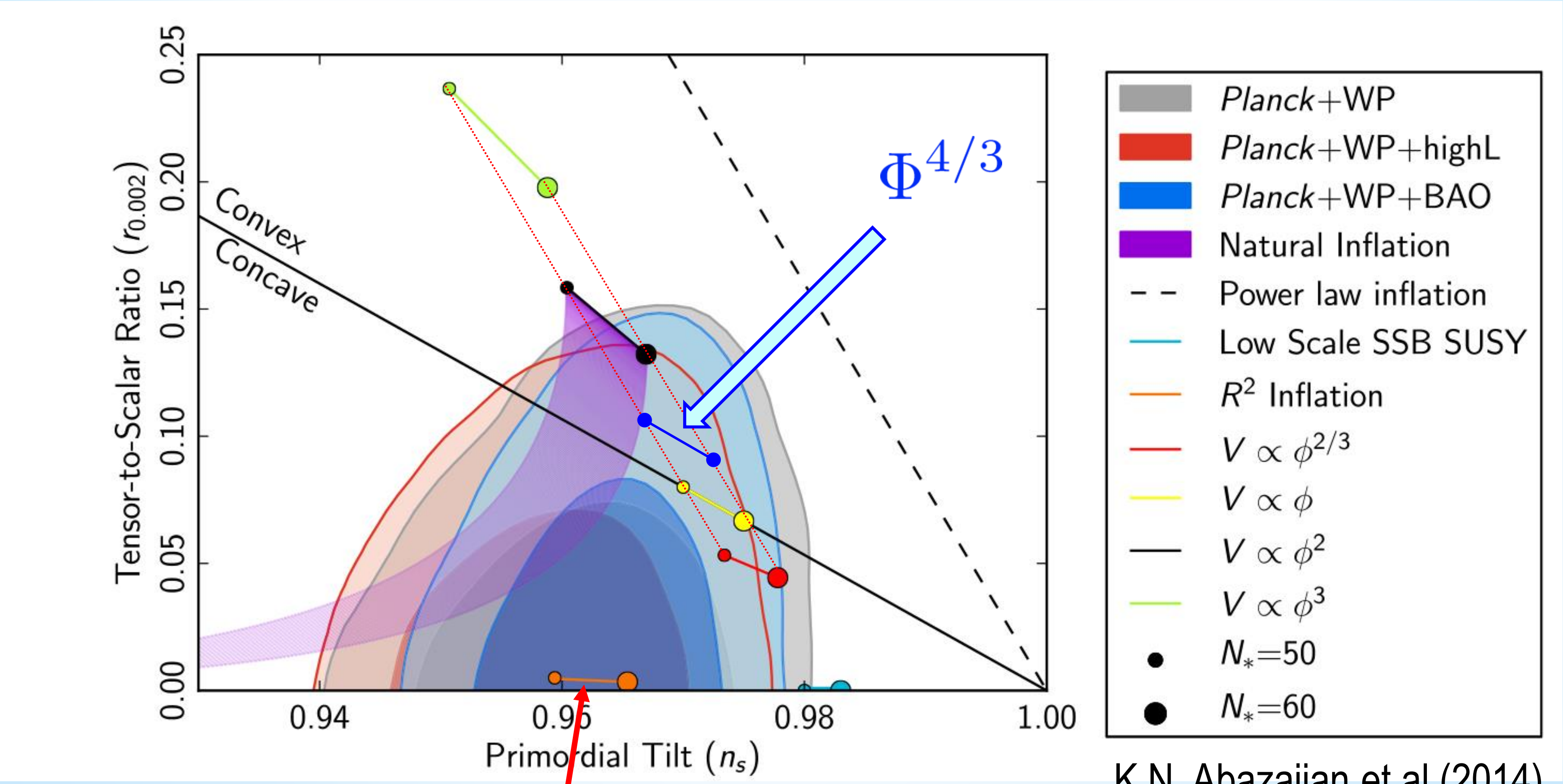


phase space of Higgs field



Observational constraint

perturbations



K.N. Abazajian et al (2014)

R^2(or original Higgs)

“original” Higgs Inflation

defect

$|\xi|$ is too large ($\xi \cong -10^5$)

r might be too small

new Higgs Inflation

defect

Observationally marginal

Hybrid ?

Hybrid Higgs Inflation (conventional+new)

Easther, KM, Musoke, Sato (2016)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\alpha G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi\right)-\frac{1}{2}\xi\phi^2R-\frac{1}{2}(\nabla\phi)^2-V(\phi)\right]$$

Kamada et al (2012): generalized Higgs inflation

↓ disformal transformation

EH action +

$$\mathcal{L}_{\rm Higgs}=-\sqrt{-g}\left[\left(\frac{(1-\xi(1-6\xi)\kappa^2\phi^2)+\alpha V(\phi)}{2(1-\xi\kappa^2\phi^2)^2}\right)(\nabla\phi)^2+\frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}\right]+\cdots$$

$$=-\sqrt{-g}\left[\frac{1}{2}(\nabla\Phi)^2+U(\Phi)\right]+\cdots$$

higher-derivative terms

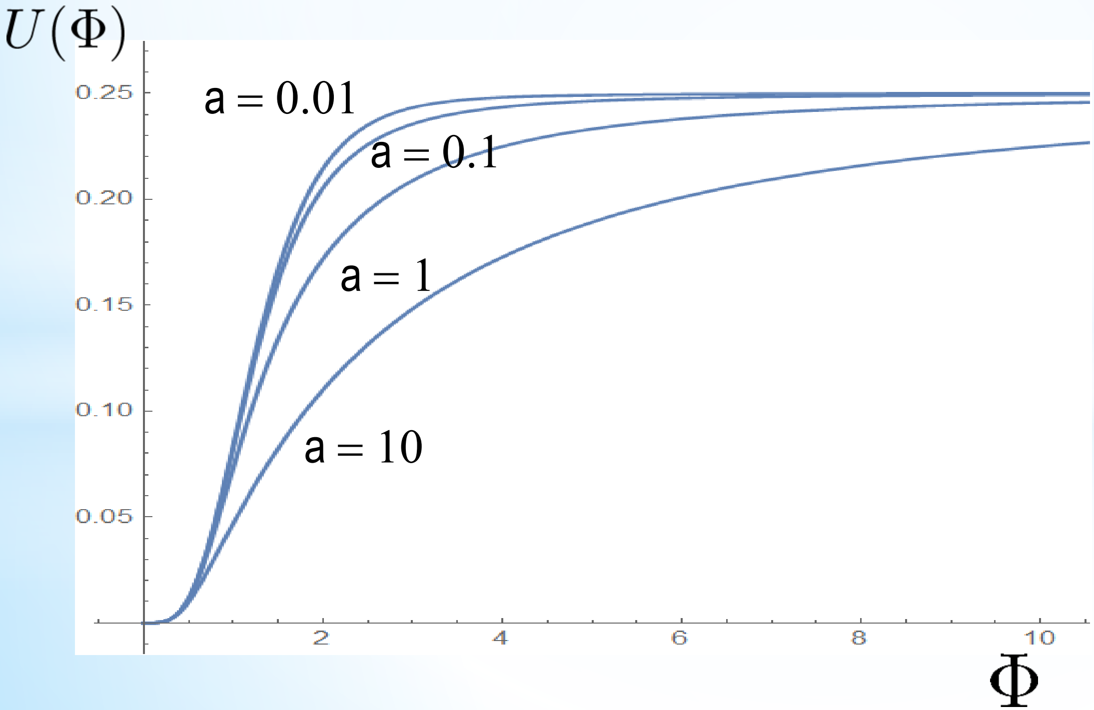
$$\Phi=\int\frac{\sqrt{(1-\xi(1-6\xi)\kappa^2\phi^2)+\alpha V(\phi)}}{(1-\xi\kappa^2\phi^2)}d\phi \qquad U(\Phi)=\frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}$$

$$V = \frac{\lambda}{4}\phi^4 \qquad \alpha = \frac{1}{M^2 M_{\rm PL}^2}$$

$$U(\Phi) = \frac{V(\phi)}{(1-\xi\kappa^2\phi^2)^2}$$
$$= \frac{\lambda}{4\xi^2} M_{\rm PL}^4 \left[1 - \frac{\lambda M_{\rm PL}^4}{2|\xi|^3 M^2}\frac{1}{\Phi^2} + \cdots\right]$$

higher-derivative terms

$$a := \frac{\lambda}{4}\frac{M_{\rm PL}^2}{\xi^2 M^2}$$



$a \ll 1$

the original Higgs inflation

$a \sim O(1)$

$U \propto 1 - c_0 \Phi^{-2}$

➢ cosmological perturbations in the disformal frame

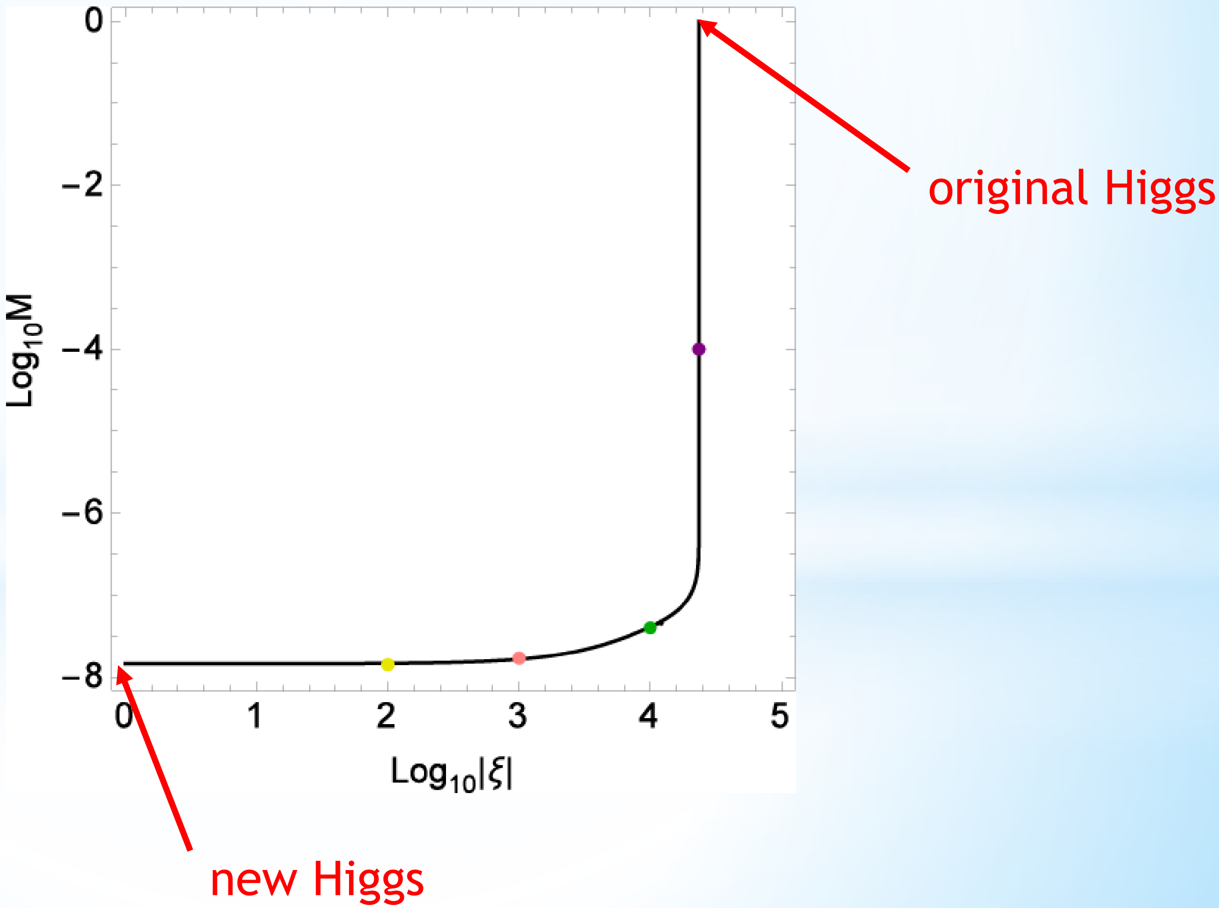
Scalar perturbation

$$P_\zeta \approx 10^{-9}$$ at N=60

N. Makino, M. Sasaki(1991)
J.-O. Gong et al (2011)
Y. Watanabe et al(2015)
H. Motohashi, J. White (2016)

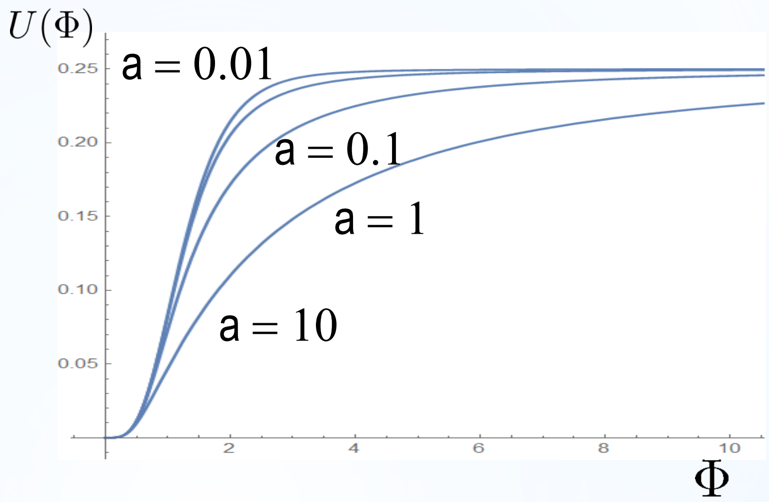
⇩

constrain the relation between M and ξ



Truncated model

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\Phi)^2 - U(\Phi) + \cdots\right]$$



$$a := \frac{\lambda}{4}\frac{M_{\rm PL}^2}{\xi^2 M^2}$$

$$\epsilon = \frac{1}{2}\left(\frac{1}{U}\frac{dU}{d\phi}\right)^2$$

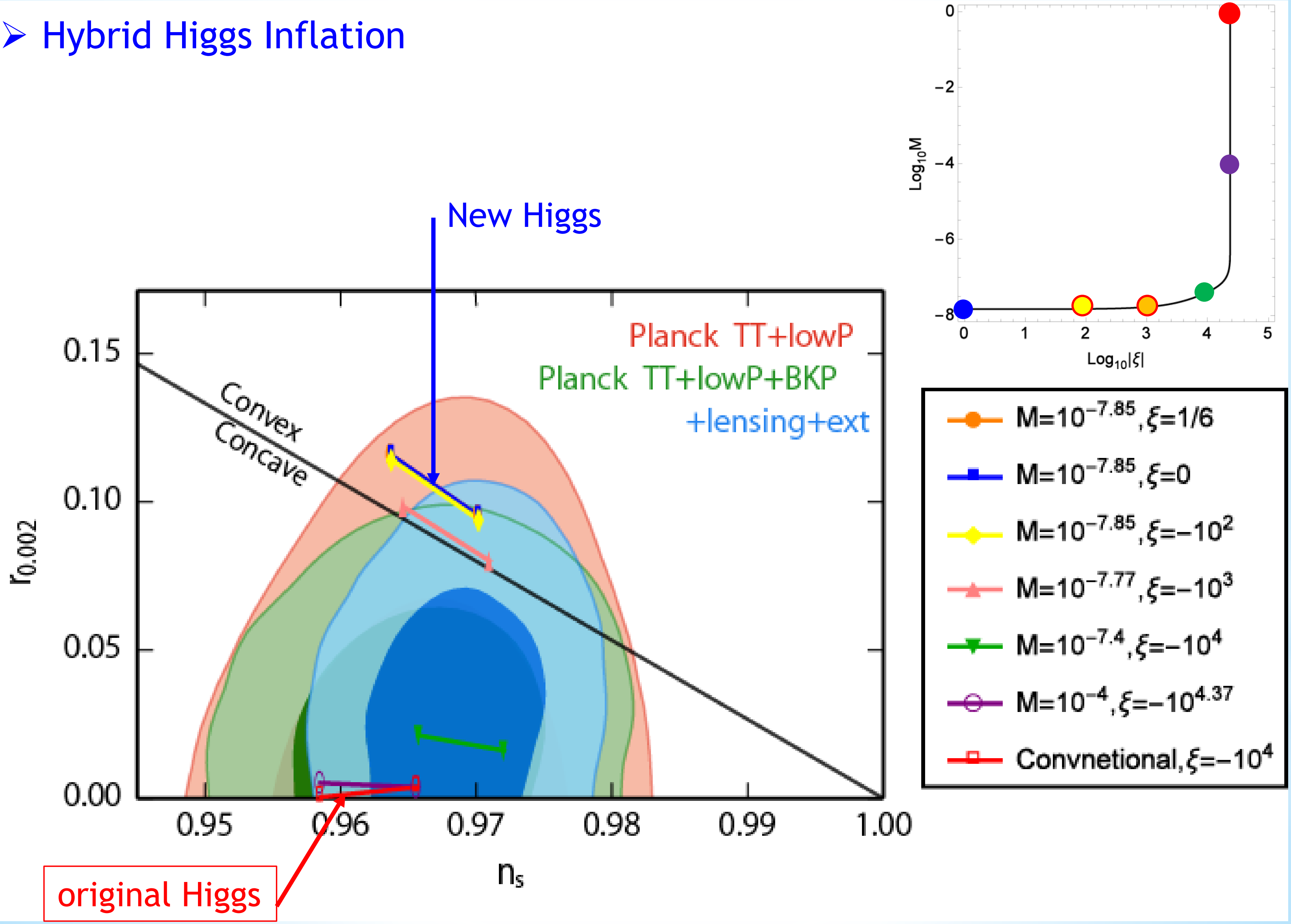
$$\eta = \frac{1}{U}\frac{d^2U}{d\phi^2}$$

$\Rightarrow$

$$n_s \simeq 1 - 6\epsilon + 2\eta$$

$$r \equiv \frac{P_T}{P_\zeta} \simeq 16\epsilon$$

➢ Hybrid Higgs Inflation



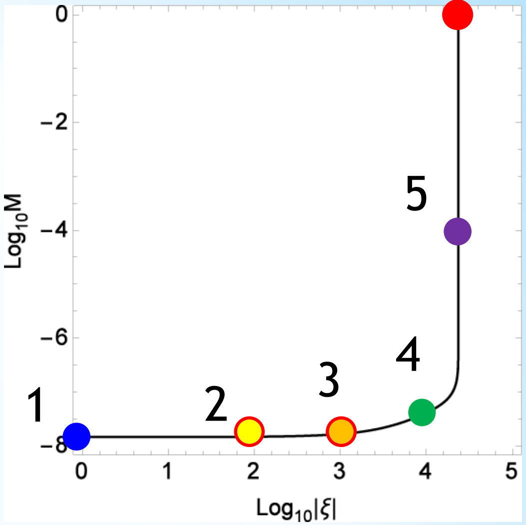
The truncation is valid ?

accuracy

$$\Delta n_s \equiv \frac{|n_s - n_{s\,ADM}|}{n_{s\,ADM}} \times 100$$

$$\Delta r \equiv \frac{|r - r_{ADM}|}{r_{ADM}} \times 100$$

$n_{s\mathrm{ADM}}$
r_{ADM}
Perturbations in original frame based on ADM decomposition



model	$\Delta n_s[\%]$		$\Delta r[\%]$	
	N=60	N=50	N=60	N=50
①	5.8×10^{-2}	9.3×10^{-2}	3.4	4.2
②	7.5×10^{-2}	1.1×10^{-1}	3.6	4.5
③	7.6×10^{-2}	1.1×10^{-1}	3.6	4.6
④	1.2×10^{-1}	1.8×10^{-1}	5.1	6.5
⑤	3.6×10^{-1}	9.3×10^{-2}	3.5	4.4

➢ Comparison with the Generalized Higgs inflation

Generalized Higgs inflation

K.Kamada, T.Kobayashi, T.Takahashi, M.Yamaguchi & J.Yokoyama(2012)

$$S = \Sigma_{i=2}^{5} \int d^4x \sqrt{-g} L_i$$

$$L_2 = K(\phi, X)$$
$$L_3 = -G_3(\phi, X)\Box\phi$$
$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \left(\nabla_\mu\nabla_\nu\phi\right)^2\right]$$
$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)\left(\nabla_\mu\nabla_\nu\phi\right)^2 + \left(\left(\nabla_\mu\nabla_\nu\phi\right)^3\right)\right]$$

$$X = \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$$

$$\begin{cases} K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots \\ G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots \end{cases}$$

$$g_3 = g_5 = 0$$

- Conventional Higgs Inflation $\left(g_4 = \frac{M_P^2}{2} - \xi\frac{\phi^2}{2}\right)$
- New Higgs Inflation $\left(g_5 = \frac{\phi}{2M^2}\right)$
- Running kinetic inflation $(\mathcal{K} = 1 + \kappa\phi^{2n})$
- Higgs G-inflation $\left(h_3 = \frac{\phi}{\mu^4}\right)$

Generalized Higgs inflation

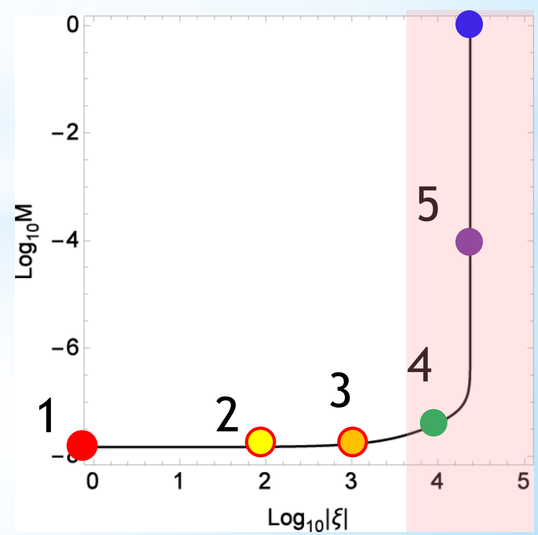
$$\mathcal{U} \equiv \mathcal{K} + \frac{h_4 V}{g_4} \Rightarrow 1 + \frac{V}{M^2(M_P^2 - \xi\phi^2)}$$

$\mathcal{U} \gg 1$

$$\Rightarrow n_s \simeq 1 - \frac{8(2M_P^2 - \xi\phi^2)(2M^2(M_P^2 - \xi\phi^2)\mathcal{U} + V)}{M^2\mathcal{U}^2(M_P^2 - \xi\phi^2)^2} + \frac{8}{\mathcal{U}\phi^2(M_P^2 - \xi\phi^2)}$$

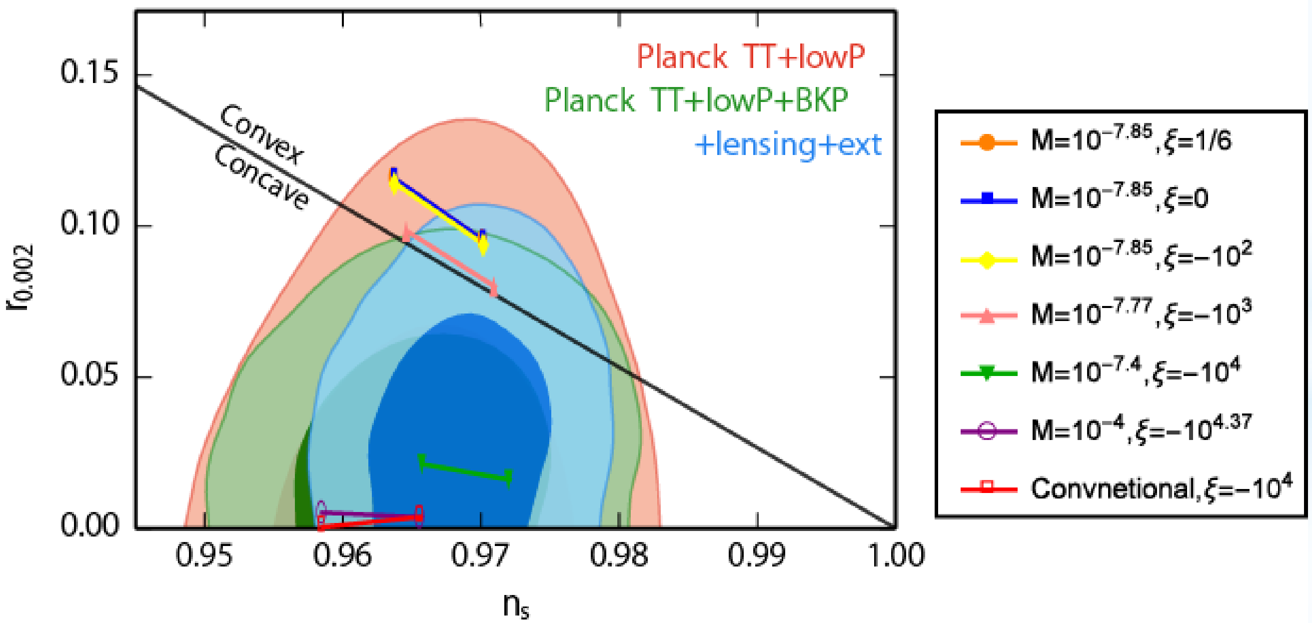
$$\Rightarrow r \simeq \frac{128}{\mathcal{U}\phi^2(M_P^2 - \xi\phi^2)}$$

model		$\Delta n_s[\%]$		$\Delta r[\%]$	
		N=60	N=50	N=60	N=50
①	Disformal	5.8×10^{-2}	9.3×10^{-2}	3.4	4.2
	Generalized	5.0×10^{-2}	7.6×10^{-2}	3.1	3.8
②	Disformal	7.5×10^{-2}	1.1×10^{-1}	3.6	4.5
	Generalized	6.0×10^{-2}	7.9×10^{-2}	3.1	3.8
③	Disformal	7.6×10^{-2}	1.1×10^{-1}	3.6	4.6
	Generalized	6.5×10^{-2}	8.6×10^{-2}	3.2	3.9
④	Disformal	1.2×10^{-1}	1.8×10^{-1}	5.1	6.5
	Generalized	2.8×10^{-1}	3.8×10^{-1}	9	10
⑤	Disformal	3.6×10^{-1}	9.3×10^{-2}	3.5	4.4
	Generalized	4.1×10^{5}	5.1×10^{5}	1.1×10^{7}	1.1×10^{7}



Summary

- In the hybrid Higgs inflation, scalar-tensor ratio is enhanced.
- By use of disfomal transformation, the analysis becomes much simpler.
- The truncation of higher-derivative terms is valid in all parameter region.



Other models ?

Galileon
Horndesky
Generalized Proca
⋮

(disformal) transformation →

EH action
with some potential

Analysis is simple

“Equivalence” between two theories
when we ignore the higher-derivative terms

Some remarks on disformal transformation

- causal structure

conformal transformation : null $\longrightarrow$ null

disformal transformation : null $\not\longrightarrow$ null

A causal structure is changed

- coupling to matter fields

After transformation,
we find the coupling to matter fields

Coupled Quintessence

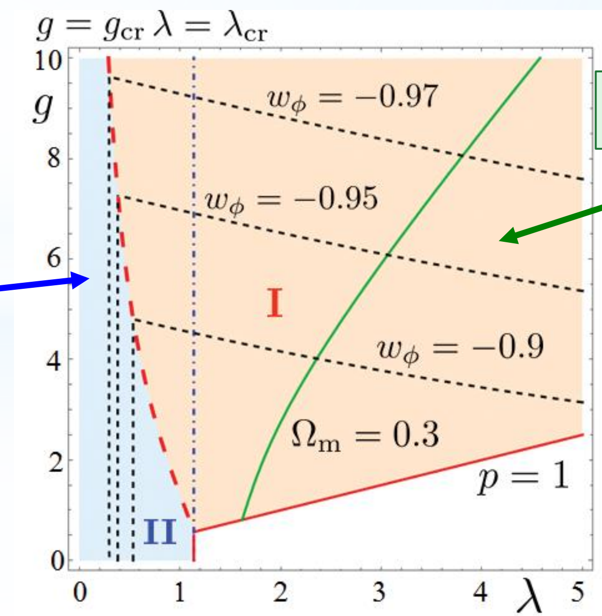
L. Amendola (2000)
S. Panpanich, KM, S. Mizuno (2017)

Coincidence Problem of Dark Energy

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{f(\phi)}\sqrt{1+f(\phi)(\partial\phi)^2} + \frac{1}{f(\phi)} - V(\phi)\right] + \int d^4x \mathcal{L}_m(A^2(\phi)g_{\mu\nu}, \psi_m)$$

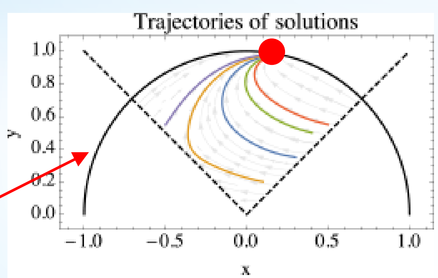
$$f(\phi) = \epsilon f_0 e^{-\mu\phi}, \quad V(\phi) = V_0 e^{-\lambda\phi}, \quad A(\phi) = e^{g\phi}$$

$\epsilon = 0$ (cannonical)
$\epsilon = -1$ (DBI)
$\epsilon = 1$ (D − BIonic)



Scaling solution

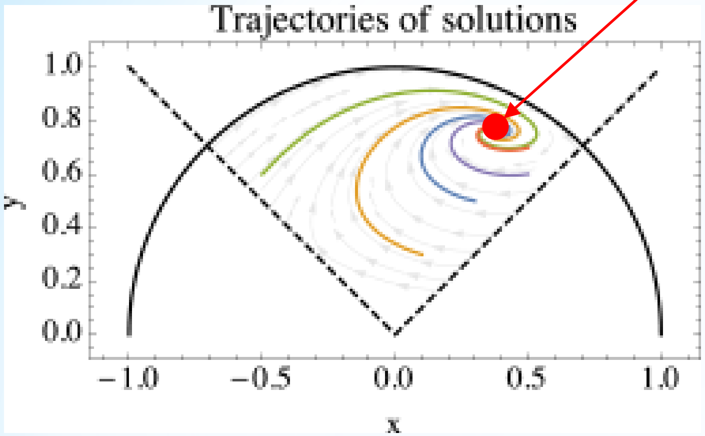
DE dominated

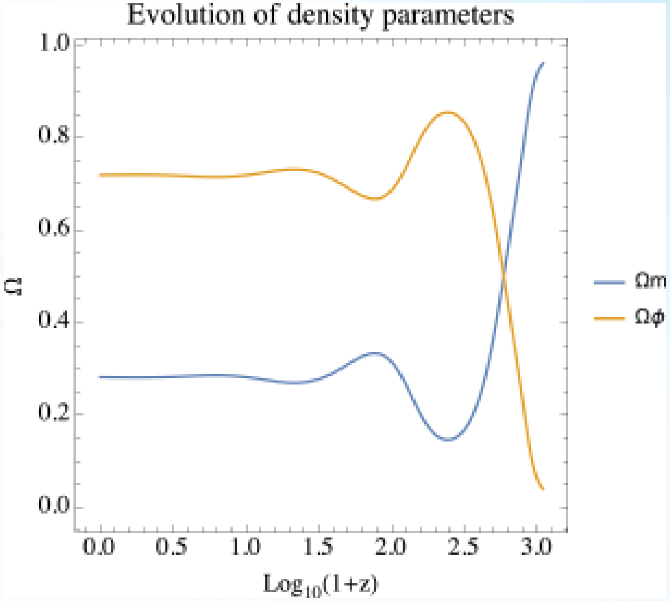


$\Omega_m = 0$

Scaling solution

$\Omega_m \neq 0$





Observational constraints

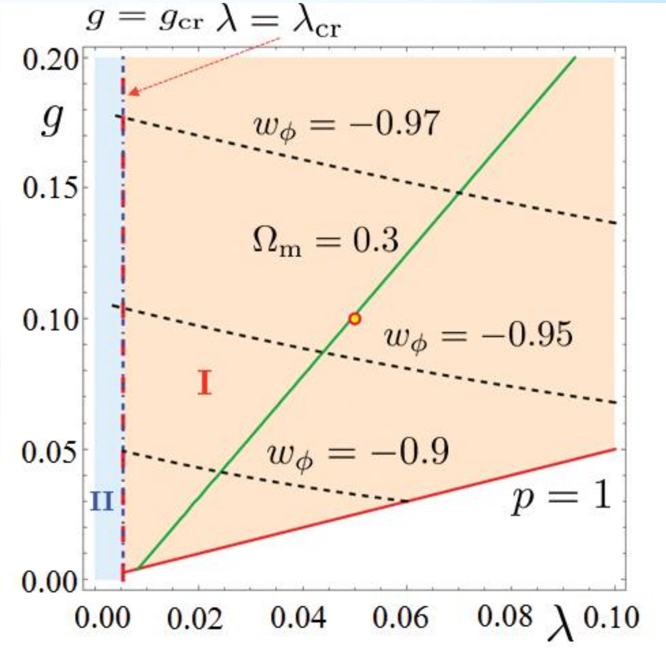
CMB $|g| < 0.13$

the screening at smaller scale

$$f_0 e^{-\mu\phi_0} = \Lambda^{-4} > 3.9 \times 10^{15}\,\mathrm{eV}^{-4}$$

$$f_0 V_0 \approx 1.0 \times 10^5$$

$$\Omega_\phi \approx 0.697\,, \quad \Omega_\mathrm{m} \approx 0.303\,, \quad w_\phi \approx -0.96$$



How about disformal transformation?

Disformal metric coupled to matter fluid

⟹ The coincidence problem could be solved ?

Anything new by disformal transformation ?

- Scale deformation in time direction

  ⟹ New acceleration mechanism ?

- coupling to matter fields

  ⟹ The coincidence problem ?

Some discussions about disformal inflation (or disformal dark energy)

Kaloper(2004),
van de Bruck, Koivisto, Longden (2016)
Zumalacarregui et al (2010)

Thank you for your attention