Pair Production of Scalar Dyons in Kerr-Newman Black Holes

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CMC, Kim, Sun, Tang, arXiv:1705.10629 [hep-th];
PRD 95 (2017) 044043 [arXiv:1607.02610 [hep-th]]

Outline

- Motivation
- Scalar Dyons Pair Production
- Thermal Interpretation
- Dual CFT Description
- Summary

RN black holes:

CMC, Kim, Lin, Sun, Wu, PRD 85 (2012) 124041 [arXiv:1202.3224 [hep-th]]
CMC, Sun, Tang, Tsai, CQG 32 (2015) 195003 [arXiv:1412.6876 [hep-th]]
Kim, Lee, Yoon, arXiv:1503.00218 [hep-th]

Motivation

- The spontaneous pair production from charged black holes mixtures two independent processes:
  - Schwinger mechanism by electromagnetic force

$$N_S \sim \mathrm{e}^{-\frac{m}{T_S}}, \qquad T_S = 2T_U, \quad T_U = \frac{1}{2\pi}\frac{qE}{m}$$

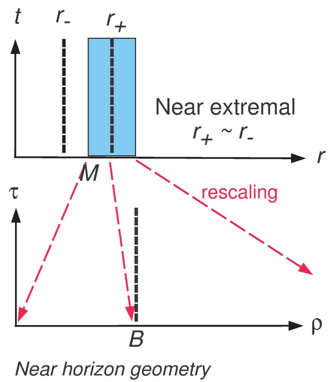
  - Hawking radiation by tunneling through horizon

$$N_H \sim \mathrm{e}^{-\frac{\omega}{T_H}}, \qquad T_H = \frac{\kappa}{2\pi}$$

- Near-extremal Kerr-Newman black holes: dual CFT description & exactly solvable
- Near-horizon region: GR solution & pair production occurs

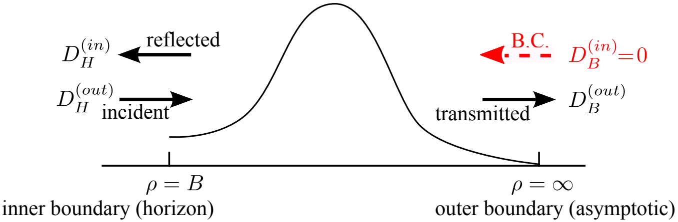
Scalar Dyons Pair Production

- dyonic Kerr-Newman Black Holes: M, Q, P, a
  - Near extremal: $M^2 \sim Q^2 + P^2 + a^2$
- Near horizon geometry: exact solution, AdS structure



Scalar Dyons Pair Production

- Klein-Groden equation of a complex scalar: m, q, p
  - "exact solution": two integration constants c_1, c_2
  - In/Out going modes at horizon and asymptotic: four fluxes
- Outer boundary condition: no incoming flux at asymptotic



CMC, Kim, Lin, Sun, Wu, PRD 85 (2012) 124041 [arXiv:1202.3224 [hep-th]]

Scalar Dyons Pair Production

- Flux conservation for bosonic particles

$$|D_{\text{incident}}| = |D_{\text{reflected}}| + |D_{\text{transmitted}}|$$

- Bogoliubov relation

$$|\alpha|^2 - |\beta|^2 = 1$$

- Vacuum persistence amplitude $|\alpha|^2$ and Mean number of produced pairs $|\beta|^2$

$$|\alpha|^2 \equiv \frac{D_{\text{incident}}}{D_{\text{reflected}}}, \quad |\beta|^2 \equiv \frac{D_{\text{transmitted}}}{D_{\text{reflected}}}$$

- Absorption cross section

$$\sigma_{\text{abs}} \equiv \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{|\beta|^2}{|\alpha|^2}$$

Dyonic Kerr-Newman Black Holes

- Dyonic Kerr-Newman (KN) black holes:
  $M,\ a = J/M,\ Q,\ P$

$$
\begin{aligned}
ds^2 &= -\frac{\Sigma\Delta}{(\hat{r}^2+a^2)^2-\Delta a^2\sin^2\theta}d\hat{t}^2+\frac{\Sigma}{\Delta}d\hat{r}^2+\Sigma d\theta^2 \\
&+ \frac{(\hat{r}^2+a^2)^2-\Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta\left[d\hat{\varphi}-\frac{a(2M\hat{r}-Q^2-P^2)}{(\hat{r}^2+a^2)^2-\Delta a^2\sin^2\theta}d\hat{t}\right]^2 \\
A_{[1]} &= \frac{Q\hat{r}-Pa\cos\theta}{\Sigma}\left(d\hat{t}-a\sin^2\theta d\hat{\varphi}\right)+P\left(\cos\theta\mp 1\right)d\hat{\varphi}
\end{aligned}
$$

$$
\Sigma = \hat{r}^2+a^2\cos^2\theta, \qquad \Delta = \hat{r}^2-2M\hat{r}+a^2+Q^2+P^2
$$

- dual potential: $\bar{A}_{[1]} = A_{[1]}(Q\to P, P\to -Q)$

$$
F_{[2]} = dA_{[1]}, \quad \bar{F}_{[2]} = d\bar{A}_{[1]} \quad \Rightarrow \quad \bar{F}_{[2]} =^{\star} F_{[2]}
$$

Near Horizon Geometry of Dyonic Kerr-Newman

- Near horizon limits: $\varepsilon \to 0, \quad r_0^2 \equiv Q^2 + P^2 + a^2$

$$\hat{\varphi} \to \varphi + \frac{a}{r_0^2 + a^2}\hat{t}, \quad \hat{r} \to r_0 + \varepsilon r, \quad \hat{t} \to \frac{r_0^2 + a^2}{\varepsilon} t, \quad M \to r_0 + \varepsilon^2 \frac{B^2}{2r_0}$$

- Near horizon of near extremal KN black holes

$$\begin{aligned} ds^2 &= (r_0^2 + a^2\cos^2\theta)\left[-(r^2 - B^2)dt^2 + \frac{dr^2}{r^2 - B^2} + d\theta^2\right] \\ &\quad + \frac{(r_0^2 + a^2)^2 \sin^2\theta}{r_0^2 + a^2\cos^2\theta}\left(d\varphi + \frac{2ar_0}{r_0^2 + a^2} r dt\right)^2 \\ A &= -\frac{Q(r_0^2 - a^2\cos^2\theta) - 2Pr_0 a\cos\theta}{r_0^2 + a^2\cos^2\theta} r dt \\ &\quad - \left(\frac{Qr_0 a\sin^2\theta + P(r_0^2 + a^2)\cos\theta}{r_0^2 + a^2\cos^2\theta} \mp P\right) d\varphi \end{aligned}$$

Scalar Dyons Pair Production

- Klein-Gordon equation: $\bar{A} = A(Q \to P, P \to -Q)$

$$(\nabla_\alpha - iqA_\alpha - ip\bar{A}_\alpha)(\nabla^\alpha - iqA^\alpha - ip\bar{A}^\alpha)\Phi - m^2\Phi = 0$$

- ansätz: $\Phi(t, r, \theta, \varphi) = \mathrm{e}^{-i\omega t + i[n \mp (qP - pQ)]\varphi} R(r)\Theta(\theta)$
- decoupled equations

$$\partial_r \left[(r^2 - B^2)\partial_r R\right] + \left(\frac{\left[\omega(r_0^2 + a^2) - (qQ + pP)(Q^2 + P^2)r + 2nar_0 r\right]^2}{(r_0^2 + a^2)^2(r^2 - B^2)} - m^2(r_0^2 + a^2) - \lambda\right) R = 0$$

$$\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta\Theta) - \left(\frac{\left[n(r_0^2 + a^2\cos^2\theta) + (qQ + pP)ar_0\sin^2\theta - (qP - pQ)(r_0^2 + a^2)\cos\theta\right]^2}{(r_0^2 + a^2)^2\sin^2\theta} - m^2a^2\sin^2\theta - \lambda\right)\Theta = 0$$

Scalar Dyons Pair Production

- Exact Solution:

$$\begin{aligned} R(r) &= c_1 (r-B)^{\frac{i}{2}(\tilde{\kappa}-\kappa)} (r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\ & \quad F\left(\frac{1}{2}+i\tilde{\kappa}+i\mu, \frac{1}{2}+i\tilde{\kappa}-i\mu; 1+i\tilde{\kappa}-i\kappa; \frac{1}{2}-\frac{r}{2B}\right) \\ &+ c_2 (r-B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)} (r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\ & \quad F\left(\frac{1}{2}+i\kappa+i\mu, \frac{1}{2}+i\kappa-i\mu; 1-i\tilde{\kappa}+i\kappa; \frac{1}{2}-\frac{r}{2B}\right) \end{aligned}$$

- **three essential parameters**

$$\tilde{\kappa} = \frac{\omega}{B}, \quad \kappa = \frac{(qQ+pP)(Q^2+P^2) - 2nar_0}{r_0^2+a^2},$$

$$\mu = \sqrt{\kappa^2 - m^2(r_0^2+a^2) - \lambda - \frac{1}{4}}$$

Scalar Dyons Pair Production

- Expansion around $r = B$

$$R_H(r) \approx c_H^{(\mathrm{in})}(2B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}(r-B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)} + c_H^{(\mathrm{out})}(2B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)}(r-B)^{\frac{i}{2}(\tilde{\kappa}-\kappa)}$$

  - parameters: $c_H^{(\mathrm{in})} = c_2$ and $c_H^{(\mathrm{out})} = c_1$

- Expansion around $r \to \infty$

$$\begin{aligned} R_B(r) &\approx c_B^{(\mathrm{in})}(r-B)^{-\frac{1}{2}-\frac{i}{2}(\tilde{\kappa}+\kappa)-i\mu}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\ &\quad + c_B^{(\mathrm{out})}(r-B)^{-\frac{1}{2}-\frac{i}{2}(\tilde{\kappa}+\kappa)+i\mu}(r+B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} \\ &\approx c_B^{(\mathrm{in})} r^{-\frac{1}{2}-i\mu} + c_B^{(\mathrm{out})} r^{-\frac{1}{2}+i\mu} \end{aligned}$$

- Condition for existence of propagating modes

$$\mu^2 = \kappa^2 - m^2(r_0^2 + a^2) - \lambda - \frac{1}{4} > 0$$

Scalar Dyons Pair Production

- relations of parameters

$$
\begin{aligned}
c_B^{(\mathrm{in})} &= c_1(2B)^{\frac{1}{2}+i\tilde{\kappa}+i\mu}\frac{\Gamma(1+i\tilde{\kappa}-i\kappa)\Gamma(-2i\mu)}{\Gamma\left(\frac{1}{2}-i\kappa-i\mu\right)\Gamma\left(\frac{1}{2}+i\tilde{\kappa}-i\mu\right)} \\
&\quad +c_2(2B)^{\frac{1}{2}+i\kappa+i\mu}\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(-2i\mu)}{\Gamma\left(\frac{1}{2}+i\kappa-i\mu\right)\Gamma\left(\frac{1}{2}-i\tilde{\kappa}-i\mu\right)} \\
c_B^{(\mathrm{out})} &= c_1(2B)^{\frac{1}{2}+i\tilde{\kappa}-i\mu}\frac{\Gamma(1+i\tilde{\kappa}-i\kappa)\Gamma(2i\mu)}{\Gamma\left(\frac{1}{2}-i\kappa+i\mu\right)\Gamma\left(\frac{1}{2}+i\tilde{\kappa}+i\mu\right)} \\
&\quad +c_2(2B)^{\frac{1}{2}+i\kappa-i\mu}\frac{\Gamma(1-i\tilde{\kappa}+i\kappa)\Gamma(2i\mu)}{\Gamma\left(\frac{1}{2}+i\kappa+i\mu\right)\Gamma\left(\frac{1}{2}-i\tilde{\kappa}+i\mu\right)}
\end{aligned}
$$

Scalar Dyons Pair Production

- Flux formula (radial direction)

$$D = \int d\theta d\varphi \, i\sqrt{-g} g^{rr} (\Phi D_r \Phi^* - \Phi^* D_r \Phi)$$

- Relevant fluxes:

$$D_B^{(\mathrm{in})} = -2(r_0^2 + a^2)\mu \left| c_B^{(\mathrm{in})} \right|^2 \mathfrak{S}$$

$$D_H^{(\mathrm{in})} = -2B(r_0^2 + a^2)(\tilde{\kappa} - \kappa) \left| c_H^{(\mathrm{in})} \right|^2 \mathfrak{S}$$

$$D_B^{(\mathrm{out})} = 2(r_0^2 + a^2)\mu \left| c_B^{(\mathrm{out})} \right|^2 \mathfrak{S}$$

$$D_H^{(\mathrm{out})} = 2B(r_0^2 + a^2)(\tilde{\kappa} - \kappa) \left| c_H^{(\mathrm{out})} \right|^2 \mathfrak{S}$$

- angular part (irrelevant): $\mathfrak{S} = 2\pi \int d\theta \, \sin\theta \, \Theta\Theta^*$

Scalar Dyons Pair Production

- Outer boundary condition: $c_B^{(\mathrm{in})} = 0$

$$c_1 = -c_2 (2B)^{i(\kappa-\tilde{\kappa})} \frac{\Gamma(1 - i\tilde{\kappa} + i\kappa)\Gamma(\frac{1}{2} + i\tilde{\kappa} - i\mu)\Gamma(\frac{1}{2} - i\kappa - i\mu)}{\Gamma(\frac{1}{2} + i\kappa - i\mu)\Gamma(\frac{1}{2} - i\tilde{\kappa} - i\mu)\Gamma(1 - i\kappa + i\tilde{\kappa})}$$

- parameter $c_B^{(\mathrm{out})}$:

$$\begin{aligned} c_B^{(\mathrm{out})} &= -c_2 (2B)^{\frac{1}{2} + i\kappa - i\mu} \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\tilde{\kappa} - \pi\mu)\cosh(\pi\kappa + \pi\mu)} \\ &\quad \times \frac{\Gamma(1 - i\tilde{\kappa} + i\kappa)\Gamma(2i\mu)}{\Gamma(\frac{1}{2} - i\tilde{\kappa} + i\mu)\Gamma(\frac{1}{2} + i\kappa + i\mu)} \end{aligned}$$

Scalar Dyons Pair Production

- Bogoliubov coefficients:

$$|\alpha|^2 = \frac{D_{\text{incident}}}{D_{\text{reflected}}} = \frac{\left|D_H^{(\text{out})}\right|}{\left|D_H^{(\text{in})}\right|} = \frac{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}$$

$$|\beta|^2 = \frac{D_{\text{transmitted}}}{D_{\text{reflected}}} = \frac{\left|D_B^{(\text{out})}\right|}{\left|D_H^{(\text{in})}\right|} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}$$

- Absorption cross section:

$$\sigma_{\text{abs}} = \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{\left|D_B^{(\text{out})}\right|}{\left|D_H^{(\text{out})}\right|} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}$$

Thermal Interpretation

- Mean number of produced pairs: extremal limit $\tilde{\kappa} \to \infty$

$$N = \left(\frac{\mathrm{e}^{-2\pi(\kappa-\mu)} - \mathrm{e}^{-2\pi(\kappa+\mu)}}{1 + \mathrm{e}^{-2\pi(\kappa+\mu)}} \right) \left(\frac{1 - \mathrm{e}^{-2\pi(\tilde{\kappa}-\kappa)}}{1 + \mathrm{e}^{-2\pi(\tilde{\kappa}-\mu)}} \right)$$

- Temperature for Schwinger effect:

$$T_{\mathrm{KN}} = \frac{\bar{m}}{2\pi(\kappa - \mu)} = T_U + \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}$$

- effective mass

$$\bar{m} = m\sqrt{1 + \frac{\lambda + 1/4}{m^2(r_0^2 + a^2)}} = \sqrt{m^2 - \frac{\lambda + 1/4}{2}\mathcal{R}}$$

- Unruh temperature and AdS curvature:

$$T_U = \frac{\kappa}{2\pi\bar{m}(r_0^2 + a^2)}, \qquad \mathcal{R} = -\frac{2}{r_0^2 + a^2}$$

Thermal Interpretation

- Another "temperature": $(\mathcal{R} \to 0 \Rightarrow T_{\mathrm{KN}} = 2T_U, \bar{T}_{\mathrm{KN}} = 0)$

$$\bar{T}_{\mathrm{KN}} = \frac{\bar{m}}{2\pi(\kappa + \mu)} = T_U - \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}$$

- Mean number of produced pairs: $(\mathrm{AdS}_2)\{\mathrm{Rindler}_2\}$

$$\mathcal{N} = \mathrm{e}^{\frac{\bar{m}}{T_{\mathrm{KN}}}} \left(\frac{\mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}} - \mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}}{1 + \mathrm{e}^{-\frac{\bar{m}}{\bar{T}_{\mathrm{KN}}}}} \right) \left\{ \frac{\mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}} \left(1 - \mathrm{e}^{-\frac{\hat{\omega} - q\Phi_H - p\bar{\Phi}_H - n\Omega_H}{T_H}} \right)}{1 + \mathrm{e}^{-\frac{\hat{\omega} - q\Phi_H - p\bar{\Phi}_H - n\Omega_H}{T_H}} \mathrm{e}^{-\frac{\bar{m}}{T_{\mathrm{KN}}}}} \right\}$$

  - $\hat{\omega} = \varepsilon\omega$ (frequency in "original" coordinates), T_H (Hawking temperature), $\Phi_H, \bar{\Phi}_H$ (chemical potentials), Ω_H (angular velocity)

Thermal Interpretation

$$T_H = \frac{B}{2\pi}, \qquad \Omega_H = -\frac{2ar_0B}{r_0^2 + a^2}$$

$$\Phi_H = \frac{Q(Q^2+P^2)B}{r_0^2+a^2}, \qquad \bar{\Phi}_H = \frac{P(Q^2+P^2)B}{r_0^2+a^2}$$

- Hamilton-Jacobi action: $R(r) = e^{iS(r)}$ (WKB)

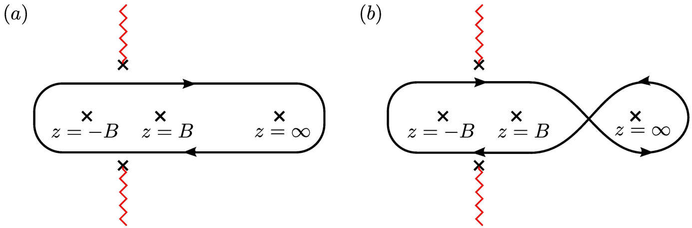
$$S(r) = \int \frac{dr}{r^2 - B^2}\sqrt{(\omega - \kappa r)^2 - \bar{m}^2(r_0^2 + a^2)(r^2 - B^2)}$$

- Residue contributions of the contour integrate at three simple poles: $r = \pm B$ and $r = \infty$

$$S_+ = -\frac{\tilde{\kappa}+\kappa}{2}, \quad S_- = \frac{\tilde{\kappa}-\kappa}{2}, \quad S_\infty = \mu$$

Thermal Interpretation

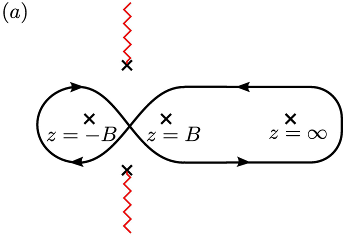
- Contour for Schwinger effect in AdS space



$$N_a = e^{i(-2\pi i)(S_- + S_+ + S_\infty)} = e^{-2\pi(\kappa - \mu)} = e^{-\frac{\bar{m}}{T_{\rm KN}}}$$
$$N_b = e^{i(-2\pi i)(S_- + S_+ - S_\infty)} = e^{-2\pi(\kappa + \mu)} = e^{-\frac{\bar{m}}{\bar{T}_{\rm KN}}}$$

Thermal Interpretation

- Contour for Schwinger effect in Rindler space



$$N_a = e^{i(-2\pi i)(S_- - S_+ - S_\infty)} = e^{-2\pi(\tilde{\kappa}+\mu)}$$
$$N_b = e^{i(-2\pi i)(S_- - S_+ + S_\infty)} = e^{-2\pi(\tilde{\kappa}-\mu)}$$

Dual CFT Description

- Absorption cross section (pair production)

$$\sigma_{\text{abs}} = \frac{\sinh(2\pi\mu)}{\pi^2} \sinh(\pi\tilde{\kappa}-\pi\kappa) \left|\Gamma\left(\frac{1}{2}+i(\mu-\kappa)\right)\right|^2 \left|\Gamma\left(\frac{1}{2}+i(\mu+\tilde{\kappa})\right)\right|^2$$

- from 2-point correlator in 2D CFT

$$\sigma_{\text{abs}} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\tilde{\omega}_L}{2T_L}+\frac{\tilde{\omega}_R}{2T_R}\right) \left|\Gamma\left(h_L + i\frac{\tilde{\omega}_L}{2\pi T_L}\right)\right|^2 \times \left|\Gamma\left(h_R + i\frac{\tilde{\omega}_R}{2\pi T_R}\right)\right|^2$$

- Conformal weight: $h_L = h_R = \frac{1}{2} + i\mu$ (complex)

Dual CFT Description

- Threefold CFT descriptions for Kerr-Newman black holes

CMC, Huang, Sun, Wu, Zou, PRD 82 (2010) 066004 [arXiv:1006.4097 [hep-th]]

- J-picture: $c_L^J = c_R^J = 12J$ (central charges) temperatures

$$T_L^J = \frac{r_+^2 + r_-^2 + 2a^2}{4\pi a(r_+ + r_-)}, \; T_R^J = \frac{r_+ - r_-}{4\pi a} \quad \Rightarrow \quad T_L^J \sim \frac{r_0^2 + a^2}{4\pi a r_0}, \; T_R^J \sim \frac{B}{2\pi a}$$

- Q-picture: $c_L^Q = c_R^Q = \frac{6Q(Q^2+P^2)}{\ell}$ (central charges) temperatures

$$T_L^Q = \frac{(r_+^2 + r_-^2 + 2a^2)\ell}{4\pi Q(r_+ r_- - a^2)}, T_R^Q = \frac{(r_+^2 - r_-^2)\ell}{4\pi Q(r_+ r_- - a^2)} \Rightarrow T_L^Q \sim \frac{(r_0^2 + a^2)\ell}{2\pi Q(Q^2+P^2)}, T_R^Q \sim \frac{r_0 B \ell}{\pi Q(Q^2+P^2)}$$

- P-picture: $c_L^P = c_R^P = \frac{6P(Q^2+P^2)}{\ell}$ (central charges) temperatures

$$T_L^P = \frac{(r_+^2 + r_-^2 + 2a^2)\ell}{4\pi P(r_+ r_- - a^2)}, T_R^P = \frac{(r_+^2 - r_-^2)\ell}{4\pi P(r_+ r_- - a^2)} \Rightarrow T_L^P \sim \frac{(r_0^2 + a^2)\ell}{2\pi P(Q^2+P^2)}, T_R^P \sim \frac{r_0 B \ell}{\pi P(Q^2+P^2)}$$

- geometrical meaning of ℓ: radius of embedded extra circle

Dual CFT Description

- CFT entropy: (for all three pictures)

$$S_{\rm CFT} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) \sim \pi(r_0^2 + a^2 + 2r_0 B)$$

- Black hole entropy and temperature

$$S_{\rm BH} = \pi(r_+^2 + a^2) \quad \Rightarrow \quad S_{\rm BH} \sim \pi(r_0^2 + a^2 + 2r_0 B)$$

$$\hat{T}_H = \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)} \quad \Rightarrow \quad T_H \sim \frac{B}{2\pi}$$

- Identification via first law of thermodynamics ($\delta S_{\rm BH} = \delta S_{\rm CFT}$)

$$\frac{\delta M - \Omega_H \delta J - \Phi_H \delta Q - \bar{\Phi}_H \delta P}{T_H} = \frac{\tilde{\omega}_L}{T_L} + \frac{\tilde{\omega}_R}{T_R}$$

Dual CFT Description

- angular velocity and chemical potential (at $r = B$)

$$\Omega_H = \frac{2ar_0}{r_0^2 + a^2} B, \qquad \Phi_H = -\frac{Q^3 B}{r_0^2 + a^2}$$

- variation of parameters: $\delta M = \omega,\ \delta J = n,\ \delta Q = q,\ \delta P = p$
- CFT “frequencies”:

$$J\text{-picture}(\delta J = n, \delta Q = 0, \delta P = 0): \qquad \tilde{\omega}_R^J = \frac{\omega}{a}, \quad \tilde{\omega}_L^J = n$$

$$Q\text{-picture}(\delta J = 0, \delta Q = q, \delta P = 0): \qquad \tilde{\omega}_R^Q = \frac{2r_0 \ell \omega}{Q(Q^2 + P^2)}, \quad \tilde{\omega}_L^Q = -q\ell$$

$$P\text{-picture}(\delta J = 0, \delta Q = 0, \delta P = p): \qquad \tilde{\omega}_R^P = \frac{2r_0 \ell \omega}{P(Q^2 + P^2)}, \quad \tilde{\omega}_L^P = -p\ell$$

- for all three pictures

$$\frac{\tilde{\omega}_L}{2T_L} = -\pi\kappa, \qquad \frac{\tilde{\omega}_R}{2T_R} = \pi\tilde{\kappa}$$

Summary

- Spontaneous pair production of scalar dyons in near extremal dyonic KN black holes: exact Bogoliubov coefficients
- There is a remarkable thermal interpretation.
- The pair production (unstable mode) is holographically dual to an operator with complex conformal weight.