Soft Heisenberg Hair

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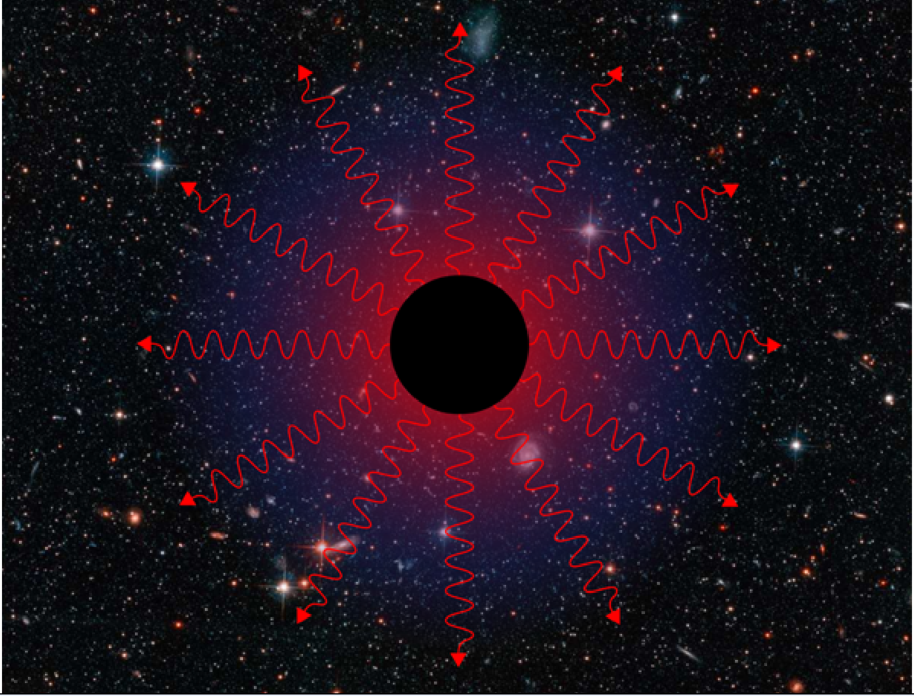
ICGAC = International Conference on Gravitation, Astrophysics, and Cosmology
KIRA = Korea-Italy Relativistic Astrophysics Symposium

FWF

papers: 1603.04824, 1607.00009, 1607.05360, 1611.09783, 1703.02594, 1705.06257, 1705.10605



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based on work with

- Hamid Afshar, Shahin Sheikh-Jabbari [IPM Teheran]
- Martin Ammon [U. Jena]
- Stephane Detournay, Max Riegler [ULB]
- Wout Merbis, Stefan Prohazka, Raphaela Wutte [TU Wien]
- Alfredo Perez, David Tempo, Ricardo Troncoso [CECS Valdivia]
- Hossein Yavartanoo [ITP Beijing]

Outline

Motivation

Problems (and possible resolutions)

Near horizon boundary conditions and soft hair

Proposal for semi-classical BTZ microstates

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plus semi-classical corrections

$$S = S_{\text{BH}} - q\,\ln S_{\text{BH}} + \mathcal{O}(1) \qquad q = \text{number depending on matter}$$

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  [at least any theory of quantum gravity claiming to reproduce (semi-)classical Einstein gravity in limit of small Newton constant]

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Perhaps no need for full knowledge of quantum gravity to account microscopically for black hole entropy (of sufficiently large black holes)

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$$S_{\text{Cardy}} = 2\pi\left(\sqrt{c\Delta^+/6} + \sqrt{c\Delta^-/6}\right) = \frac{A}{4G} = S_{\text{BH}}$$

c: left/right central charges of CFT_2
$\Delta^\pm$: left/right energies of state whose entropy is counted

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- Generalizations in 2+1 gravity/gravity-like theories (Galilean CFT, warped CFT, ...)

warped CFT: Detournay, Hartman, Hofman '12
Galilean CFT: Bagchi, Detournay, Fareghbal, Simon '13; Barnich '13

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Kerr/CFT: Guica, Hartman, Song, Strominger '09; Compere '12

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Hope: near horizon symmetries allow for Cardyology

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additional motivation: information loss

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Perhaps no need for full knowledge of quantum gravity to construct microstates (of sufficiently large non-extremal black holes)
[at least for some observer, not necessarily an asymptotic one]

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  Famous example: BTZ black hole is locally AdS_3, but canonical boundary charges (e.g. mass, angular momentum) differ
  Bañados, Henneaux, Teitelboim, Zanelli '93

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  Donnay, Giribet, Gonzalez, Pino '16
  Afshar, Detournay, Grumiller, Merbis, Perez, Tempo, Troncoso '16

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Hope: soft hair could address black hole entropy puzzles and microstates in a semi-classical framework

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Same conceptual problems as in higher dimension, but technically more manageable

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Properties of Einstein gravity in 2+1 dimensions with negative cc (AdS_3)

- Second order bulk action:

$$I_{\text{EH}} = \frac{1}{16\pi G} \int \mathrm{d}^3x\sqrt{-g}\,\Big(R + \frac{2}{\ell^2}\Big)$$

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- Spectrum of physical states includes BTZ black holes

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$$S_{\text{BH}} = \frac{A}{4G} = \frac{2\pi r_+}{4G} = 2\pi\,\Big(\sqrt{c\Delta^+/6}+\sqrt{c\Delta^-/6}\Big)$$

  $\Delta^\pm = (r_+\pm r_-)^2/(16\ell G) \propto \ell M \pm J$ (M: mass, J: angular momentum)

Near horizon boundary conditions

See Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16 for details

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$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

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  Meaning of coordinates:
  - ρ: radial direction ($\rho = 0$ is horizon)
  - $\varphi \sim \varphi + 2\pi$: angular direction (horizon has S^1 topology)
  - v: (advanced) time
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Near horizon boundary conditions

See Afshar, Detournay, DG, Merbis, Perez, Tempo, Troncoso '16 for details

- Any non-extremal horizon is approximately Rindler near the horizon
- Near horizon line-element with Rindler acceleration a:

$$\mathrm{d}s^2 = -2a\rho\,\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}\rho + \gamma^2\,\mathrm{d}\varphi^2 + \dots$$

Meaning of coordinates:

  - ρ: radial direction ($\rho = 0$ is horizon)
  - $\varphi \sim \varphi + 2\pi$: angular direction (horizon has S^1 topology)
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- This is somewhat unusual, but convenient for our purposes!

Explicit form of our boundary conditions in metric formulation

Note: everything much simpler in Chern–Simons formulation!

Boundary conditions as near horizon expansion of metric

$$g_{tt} = -a^2 r^2 + \mathcal{O}\left(r^3\right)$$

$$g_{\varphi\varphi} = \gamma^2 + \left(\gamma^2 - \ell^2\omega^2\right)\frac{r^2}{\ell^2} + \mathcal{O}\left(r^3\right)$$

$$g_{t\varphi} = a\omega r^2 + \mathcal{O}\left(r^3\right)$$

$$g_{rr} = 1 + \mathcal{O}\left(r^2\right) \quad g_{rt} = \mathcal{O}\left(r^2\right) \quad g_{r\varphi} = \mathcal{O}\left(r^2\right)$$

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$$g_{t\varphi} = \tfrac{1}{4}\,a\omega r^2 - \tfrac{1}{2}\,a\omega\ell^2 + \mathcal{O}\left(\tfrac{1}{r}\right)$$

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Boundary conditions in Chern–Simons formulation

$$A^{\pm} = b_{\pm}^{-1}\big(\,\mathrm{d} + \mathfrak{a}^{\pm}\big)b_{\pm}$$

with fixed $\mathfrak{sl}_2$ group element

$$b_{\pm} = \exp\Big(\pm\frac{r}{2\ell}\left(L_1 - L_{-1}\right)\Big)$$

and 1-form ($\mathcal{J}^{\pm} = \gamma/\ell \pm \omega$)

$$\mathfrak{a}^{\pm} = L_0\left(\pm\mathcal{J}^{\pm}\,\mathrm{d}\varphi - a\,\mathrm{d}t\right) \qquad \delta\mathcal{J}^{\pm} \neq 0 \quad \delta a = 0$$

Consequences of our near horizon boundary conditions

To reduce clutter consider henceforth constant Rindler acceleration, $a = \text{const.}$

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Map

$$P_0 = J_0^+ + J_0^- \qquad P_n = \frac{i}{kn}\,(J_{-n}^+ + J_{-n}^-)\;\text{if}\; n \neq 0 \qquad X_n = J_n^+ - J_n^-$$

yields Heisenberg algebra (with Casimirs $X_0,\, P_0$)

$$[X_n,\, X_m] = [P_n,\, P_m] = [X_0,\, P_n] = [P_0,\, X_n] = 0$$
$$[X_n,\, P_m] = i\delta_{n,m} \quad \text{if}\; n \neq 0$$

Map explains word "Heisenberg" in title and provides first punchline

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- For real J_0 all states in theory regular and have horizon

  Whole spectrum (subject to reality) compatible with regularity!

  Could be used as defining property of our bc's

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- Near horizon Hamiltonian $H \sim J_0^{+} + J_0^{-}$ commutes with all $J_n^{\pm}$

Near horizon Hamiltonian defined as diffeo charge generated by unit translations ∂_v in (advanced) time direction

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- Near horizon Hamiltonian $H \sim J_0^+ + J_0^-$ commutes with all $J_n^\pm$
- Consequence: soft hair!

$$H|\psi\rangle = E|\psi\rangle \quad \Rightarrow \quad H|\tilde{\psi}\rangle = E|\tilde{\psi}\rangle$$

where state $\tilde{\psi}$ is state ψ dressed arbitrarily with soft hair

$$|\tilde{\psi}\rangle = \prod_{n_i^\pm \in \mathbb{Z}^+} J_{n_i^+}^+ J_{n_i^-}^- |\psi\rangle$$

Explains word "soft hair" in title

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- Entropy formula remarkably simple

$$S = 2\pi \left(J_0^+ + J_0^-\right) = T^{-1}\, H$$

also remarkably universal:
generalizes to flat space, higher spins, higher derivatives!

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- Relations to asymptotic Virasoro charges $L^{\pm}$ and sources $\mu^{\pm}$

$$L \sim J^2 + J' \qquad \mu' - \mu J \sim a$$

Twisted Sugawra construction emerges! (yields Brown–Henneaux c)

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Assumptions

For technical details see Afshar, DG, Sheikh-Jabbari, Yavartanoo '17

1. Central charges quantized in integers

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   Needed due to relations like
   $$\mathcal{J}_{cn} \sim \mathcal{W}_n^0$$
   Note non-local relation
   $$\mathcal{W} \sim e^{-2\int \mathcal{J}}$$

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Note twisted periodicity conditions

$$\mathcal{W}^\nu(\varphi + 2\pi) = e^{-2\pi\nu i}\,\mathcal{W}^\nu(\varphi)$$

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   Maldacena, Maoz '00; Lunin, Maldacena, Maoz '02

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   Justification 2: gives nice result

List of all semi-classical BTZ black hole microstates

- Given a BTZ black hole with mass M and angular momentum J (as measured by asymptotic observer) define parameters

$$\Delta_\pm = \frac{1}{2}\left(\ell M \pm J\right) = \frac{c}{6}\left(J_0^\pm\right)^2$$

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BTZ black hole entropy from counting all semi-classical microstates

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- Subleading log corrections also turn out to be correct!

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- We proposed semi-classical set of BTZ black hole microstates
- Their counting reproduces Bekenstein–Hawking entropy
- Also subleading log corrections to entropy are correct

Loose ends:

- Derivation of Bohr-type quantization conditions of c and ν?
- Derivation of black hole/particle correspondence?
- Near horizon field theory beyond semi-classical approximation?

Generalizations:

- Semi-classical microstate construction for cosmological horizons?

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  Neglecting soft gravitons generates information loss Carney, Chaurette, Neuenfeld, Semenoff '17

  Conjectured resolution of information loss problem: include soft gravitons

  Strominger '17

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Thanks for your attention!